Logarithms

Logarithms

Logarithms are the inverse or opposite of exponentials (indices).
The abbreviation for logarithms is ***log***.

Example

exponent (index or power)

$$2^3 = 8$$

base number ***basic numeral (number)***

In words: 2 to the power of 3 equals 8

basic numeral (number) ***exponent (index or power)***

$$\log_2 8 = 3$$

base number

In words: The logarithm (log) with base 2 of 8 equals 3

The logarithm of a number is the exponent (index) to which the base number must be raised to give the basic numeral (number).

Examples

1. $5^3 = 125$

$\log_5 125 = 3$

2. $4^2 = 16$

$\log_4 16 = 2$

3. $7^4 = 2401$

$\log_7 2401 = 4$

4. $10^3 = 1000$

$\log_{10} 1000 = 3$

5. $2^8 = 256$

$\log_2 256 = 8$

6. $9^5 = 59\ 049$

$\log_9 59\ 049 = 5$

7. $6^1 = 6$

$\log_6 6 = 1$

8. $3^0 = 1$

$\log_3 1 = 0$

9. $19^3 = 6859$

$\log_{19} 6859 = 3$

EXERCISE 5A

1. Write the following exponential equations in logarithmic form.

(a) $3^4 = 81$ (b) $5^2 = 25$ (c) $10^2 = 100$ (d) $8^4 = 4096$

(e) $7^0 = 1$ (f) $4^5 = 1024$ (g) $9^1 = 9$ (h) $20^3 = 8000$

2. Write the following logarithmic equations in exponential form.

(a) $\log_2 32 = 5$ (b) $\log_4 64 = 3$ (c) $\log_6 7776 = 5$

(d) $\log_8 512 = 3$ (e) $\log_7 343 = 3$ (f) $\log_{10} 10\ 000 = 4$

(g) $\log_3 27 = 3$ (h) $\log_9 1 = 0$ (i) $\log_{25} 390\ 625 = 4$

3. Find x in the following equations.

(a) $2^x = 64$ (b) $\log_3 x = 5$ (c) $x^3 = 216$ (d) $\log_x 16 = 4$

(e) $12^2 = x$ (f) $\log_5 625 = x$ (g) $4^x = 256$ (h) $\log_8 32\ 768 = x$

Evaluating Logarithms

Example 1 Evaluate $\log_3 729$

Step 1 Let $\log_3 729 = x$

Step 2 Write in exponential form: $3^x = 729$

Step 3 Find x : Trial and error is one method:

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

9 27 81 243 729

$$3^x = 3^6$$

$$\boldsymbol{x = 6}$$

Example 2 Evaluate the log with base 5 of 78 125

Step 1 Write in log form: $\log_5 78\,125$

Step 2 Let $\log_5 78\,125 = x$

Step 3 Write in exponential form: $5^x = 78\,125$

Step 4 Find x : Trial and error is one method:

$$5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$$

25 125 625 3125 15 625 78 125 390 625

$$5^x = 5^8$$

$$\boldsymbol{x = 8}$$

EXERCISE 5B

1. Evaluate the following logarithms.
(a) $\log_4 4096$ (b) $\log_7 16\,807$ (c) $\log_9 6561$ (d) $\log_2 8192$

2. (a) Evaluate the log with base 3 of 2187.
(b) Evaluate the log with base 5 of 3125.
(c) Evaluate the log with base 10 of 1 000 000.
(d) Evaluate the log with base 12 of 248 832.

Logarithm Laws

Logarithm Law 1

$\log_a m + \log_a n = \log_a mn$

Examples

1. $\log_3 8 + \log_3 4$

$= \mathbf{\log_3 32}$

2. $\log_x a + \log_x b$

$= \mathbf{\log_x ab}$

3. $\log_7 8 + \log_7 d$

$= \mathbf{\log_7 8d}$

4. $\log_4 6a + \log_4 5b$

$= \mathbf{\log_4 30ab}$

5. $\log_x st$

$= \mathbf{\log_x s + \log_x t}$

6. $\log_8 15$

$= \mathbf{\log_8 5 + \log_8 3}$

EXERCISE 5C

1. Use Logarithm Law 1 to simplify the following.

(a) $\log_5 7 + \log_5 6$ (b) $\log_2 3 + \log_2 9$ (c) $\log_7 5 + \log_7 8$

(d) $\log_n 2 + \log_n 23$ (e) $\log_6 a + \log_6 c$ (f) $\log_x 4p + \log_x 3q$

(g) $\log_3 x + \log_3 x$ (h) $\log_{10} 3m + \log_{10} 5m$ (i) $\log_x 2ab^2 + \log_x 9a^3b^4$

2. Solve for x in the following equations.

(a) $\log_4 9 + \log_4 x = \log_4 36$ (b) $\log_7 6 + \log_7 x = \log_7 48$

(c) $\log_a 35 = \log_a x + \log_a 7$ (d) $\log_6 56 = \log_6 x + \log_6 28$

(e) $\log_5 24 = \log_5 2 + \log_x 12$ (f) $\log_n ab = \log_n a + \log_n x$

(g) $\log_{10} 7 + \log_{10} x = \log_{10} 91$ (h) $\log_n x + \log_n 13 = \log_n 312$

Logarithm Law 2

$$\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$$

Examples

1. $\log_3 9 - \log_3 4$

$= \mathbf{\log_3\left(\frac{9}{4}\right)}$

2. $\log_4 7 - \log_4 12$

$= \mathbf{\log_4\left(\frac{7}{12}\right)}$

3. $\log_2 16 - \log_2 12$

$= \log_2\left(\frac{16}{12}\right)$

$= \mathbf{\log_2\left(\frac{4}{3}\right)}$

4. $\log_7 18 - \log_7 9$

$= \log_7\left(\frac{18}{9}\right)$

$= \mathbf{\log_7 2}$

5. $\log_n 6a - \log_n 4$

$= \log_n\left(\frac{6a}{4}\right)$

$= \mathbf{\log_n\left(\frac{3a}{2}\right)}$

6. $\log_m 8ab - \log_m 6a$

$= \log_m\left(\frac{8ab}{6a}\right)$

$= \mathbf{\log_m\left(\frac{4b}{3}\right)}$

EXERCISE 5D

1. Use Logarithm Law 2 to simplify the following.

(a) $\log_4 5 - \log_4 9$ (b) $\log_3 8 - \log_3 5$ (c) $\log_{10} 25 - \log_{10} 15$

(d) $\log_5 36 - \log_5 48$ (e) $\log_6 32 - \log_6 8$ (f) $\log_x 8d - \log_x d$

(g) $\log_3 27x - \log_3 36x$ (h) $\log_2 8mn - \log_2 16m$ (i) $\log_x 6a^3b^2 - \log_x 9ab^4$

2. Solve for x in the following equations

(a) $\log_4 18 - \log_4 x = \log_4\left(\frac{3}{4}\right)$ (b) $\log_7\left(\frac{7}{5}\right) = \log_7 42 - \log_7 x$

(c) $\log_3 x - \log_3 35 = \log_3\left(\frac{3}{7}\right)$ (d) $\log_6\left(\frac{2a}{9}\right) = \log_6 8ab - \log_6 x$

(e) $\log_2\left(\frac{3m}{4n}\right) = \log_2 81m^2n - \log_2 x$ (f) $\log_3 x - \log_3 36a = \log_3\left(\frac{9a^2}{2}\right)$

Logarithm Law 3

$$\log_a m^n = n\log_a m$$

Examples

1. $\log_5 7^3$

$= \mathbf{3\log_5 7}$

2. $\log_6 81$

$= \log_6 9^2$

$= \mathbf{2\log_6 9}$

3. $\log_6 x^5$

$= \mathbf{5\log_6 x}$

4. $5\log_7 6^8$

$= \mathbf{40\log_7 6}$

5. $8\log_6 729$

$= 8\log_6 3^6$

$= \mathbf{48\log_6 9}$

6. $a\log_m b^c$

$= \boldsymbol{ac\log_m b}$

EXERCISE 5E

1. Use Logarithm Law 3 to simplify the following.

(a) $\log_4 3^7$ (b) $\log_3 125$ (c) $\log_a n^c$

(d) $6\log_5 7^8$ (e) $9\log_2 2401$ (f) $5\log_x 8^d$

(g) $3n\log_{10} 2^p$ (h) $4\log_6 256$ (i) $8\log_4 9^5$

2. Solve for x in the following equations.

(a) $\log_5 x = 3\log_5 10$ (b) $4\log_7 3 = \log_7 x$

(c) $\log_3 x = a\log_3 b$ (d) $5\log_6 2 = \log_6 4x$

(e) $\log_2 59\,049 = 5\log_2 x$ (f) $\log_4 9x = 7\log_4 3$

Logarithm Law 4

$\log_a a = 1$

Examples

1. $\log_5 5$

$= \mathbf{1}$

2. $\log_x x$

$= \mathbf{1}$

3. $7\log_6 6$

$= 7 \times 1$

$= \mathbf{7}$

EXERCISE 5F

Use Logarithm Law 4 to simplify the following.

1. $\log_4 4$ **2.** $3\log_7 7$ **3.** $x\log_5 5$ **4.** $a\log_b b$

Logarithm Law 5

$\log_a 1 = 0$

Examples

1. $\log_7 1$

$= \mathbf{0}$

2. $\log_x 1$

$= \mathbf{0}$

3. $9\log_6 1$

$= 9 \times 0$

$= \mathbf{0}$

EXERCISE 5G

Use Logarithm Law 5 to simplify the following.

1. $\log_4 1$ **2.** $4\log_7 1$ **3.** $x\log_5 1$ **4.** $n\log_b 1$

Logarithm Law 6

$$\log_a m = \frac{\log_b m}{\log_b a}$$

Example

Write $\log_2 7$ with a base of 10.

$$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2}$$

EXERCISE 5H

1. Write the following logarithms with a base of 6.

(a) $\log_4 3$ (b) $\log_7 12$ (c) $\log_a n$

2. Write the following logarithms with a base of n.

(a) $\log_5 15$ (b) $\log_{10} 9$ (c) $\log_x m$

Using All The Logarithm Laws

Examples

Simplify the following.

1. $\log_3 9 + \log_5 125$

$= \log_3 3^2 + \log_5 5^3$

$= 2\log_3 3 + 3\log_5 5$ ***(Law 3)***

$= 2 \times 1 + 3 \times 1$ ***(Law 4)***

$=$ **5**

2. $\log_5 \left(\frac{25}{5^x}\right)$

$= \log_5 25 - \log_5 5^x$ ***(Law 2)***

$= \log_5 5^2 - \log_5 5^x$

$= 2\log_5 5 - x\log_5 5$ ***(Law 3)***

$= 2 \times 1 - x \times 1$ ***(Law 4)***

$= \mathbf{2 - x}$

3. $3\log_2 3 + 2\log_2 6 - 2\log_2 9$

$= \log_2 3^3 + \log_2 6^2 - \log_2 9^2$ ***(Law 3)***

$= \log_2 27 + \log_2 36 - \log_2 81$

$= \log_2 27 \times 36 - \log_2 81$ ***(Law 1)***

$= \log_2 972 - \log_2 81$

$= \log_2 \left(\frac{972}{81}\right)$ ***(Law 2)***

$= \mathbf{\log_2 12}$

Examples continued

4. $3\log_7 4 - 3\log_7 12 + 3\log_7 3$

$= \log_7 4^3 - \log_7 12^3 + \log_7 3^3$ ***(Law 3)***

$= \log_7 64 - \log_7 1728 + \log_7 27$

$= \log_7\left(\frac{64}{1728}\right) + \log_7 27$ ***(Law 2)***

$= \log_7\left(\frac{1}{27}\right) + \log_7 27$ ***(Cancel fraction)***

$= \log_7\left(\frac{1}{27} \times 27\right)$ ***(Law 1)***

$= \log_7 1$

$=$ **0** ***(Law 5)***

EXERCISE 5I

Simplify the following.

1. $\log_4 16 + \log_7 343$

2. $\log_2 32 - \log_9 729$

3. $\dfrac{\log_5 15\,625}{\log_{10} 10\,000}$

4. $\dfrac{\log_3 729}{\log_2 1024}$

5. $\log_8\left(\dfrac{512}{8^n}\right)$

6. $\log_2\left(\dfrac{2^m}{512}\right)$

7. $\log_3(243 \times 3^x)$

8. $\log_2(4^a \times 16^b)$

9. $4\log_5 4 + 2\log_5 8 - 12\log_5 2$

10. $8\log_8 3 - 8\log_8 9 + 2\log_8 81$

11. $5\log_6 2 + 5\log_6 5 - 5\log_6 10$

12. $5\log_2 4 + 3\log_2 8 - 3\log_2 64$

13. $3\log_3 4 - 2\log_3 16 + 2\log_3 2$

14. $7\log_2 8 - 2\log_2 4 - 2\log_2 256$

Examples

Simplify the following.

1. $\log_2(4x + 8) - \log_2(x + 2)$

$= \log_2\left(\frac{4x+8}{x+2}\right)$ ***(Law 2)***

$= \log_2\left[\frac{4(x+2)}{x+2}\right]$ ***(Factorise numerator)***

$= \log_2\left[\frac{4\cancel{(x+2)}}{\cancel{(x+2)}}\right]$ ***(Cancel factors)***

$= \log_2 4$

$= \log_2 2^2$

$= 2\log_2 2$ ***(Law 3)***

$= 2 \times 1$ ***(Law 4)***

$= \mathbf{2}$

2. $\log_4 x + \log_4 3x - \log_4 5x$

$= \log_4(x \times 3x) - \log_4 5x$ ***(Law 1)***

$= \log_4 3x^2 - \log_4 5x$

$= \log_4\left(\frac{3x^2}{5x}\right)$ ***(Law 2)***

$= \mathbf{\log_4\left(\frac{3x}{5}\right)}$ ***(Simplify)***

Examples continued

3. $3\log_4 a + 2\log_4 8a - 3\log_4 4a$

$= \log_4 a^3 + \log_4 (8a)^2 - \log_4 (4a)^3$ ***(Law 3)***

$= \log_4 a^3 + \log_4 64a^2 - \log_4 64a^3$

$= \log_4 64a^5 - \log_4 64a^3$ ***(Law 1)***

$= \log_4 \left(\frac{64a^5}{64a^3}\right)$ ***(Law 2)***

$= \mathbf{\log_4 a^2}$ ***(Simplify)***

4. $3\log_2 x + 2\log_2 4y - 3\log_2 2xy$

$= \log_2 x^3 + \log_2 (4y)^2 - \log_2 (2xy)^3$ ***(Law 3)***

$= \log_2 x^3 + \log_2 (4^2 y^2) - \log_2 (2^3 x^3 y^3)$

$= \log_2 x^3 + \log_2 16y^2 - \log_2 8x^3 y^3$

$= \log_2 16x^3 y^2 - \log_2 8x^3 y^3$ ***(Law 1)***

$= \log_2 \left(\frac{16x^3 y^2}{8x^3 y^3}\right)$ ***(Law 2)***

$= \log_2 \left(\frac{2}{y}\right)$ ***(Simplify)***

$= \log_2 2 - \log_2 y$ ***(Law 2)***

$= \mathbf{1 - \log_2 y}$ ***(Law 3)***

EXERCISE 5J

Simplify the following.

1. $\log_3(3x + 3) - \log_3(x + 1)$

2. $\log_2(12 - 4n) - \log_2(3 - n)$

3. $\log_2(20a + 8) - \log_2(10a + 4)$

4. $\log_5(15 + 3m) - \log_5(10 + 2m)$

5. $\log_3(2x^2 + 6x) - \log_3(6x + 18)$

6. $\log_4(12x^2 + 8xy) - \log_4(3x^2y + 2xy^2)$

7. $\log_3 x + \log_3 2x - \log_3 3x$

8. $\log_5 4a + \log_5 6a - \log_5 8a$

9. $2\log_4 n + \log_4 2n - \log_4 4n$

10. $4\log_{10} 2x + 2\log_{10} 3x - 2\log_{10} 12x$

11. $2\log_6 8m - 3\log_6 4m + \log_6 6m$

12. $8\log_7 3x - 4\log_7 9x - 4\log_7 x$

13. $3\log_2 6x + 2\log_2 4y - 3\log_2 12xy$

14. $3\log_3 3a + 2\log_3 b - 2\log_3 9ab$

15. $4\log_2 2x^2 + 2\log_2 4y^3 - 2\log_2 8x^3y^3$

16. $2\log_2 8m^6 + 3\log_2 4n^4 - 12\log_2 2mn$

17. $4\log_4 2x^3 + 2\log_4 8y^4 - 4\log_4 4x^3y^2$

18. $2\log_2 4x^4y^5 - 3\log_2 2x^3y^3 - 2\log_2 2xy^2$

Examples

Solve for x in the following equations.

1. $2\log_2 4x - \log_2 x = \log_2 8$

$\log_2 (4x)^2 - \log_2 x = \log_2 8$

$\log_2 16x^2 - \log_2 x = \log_2 8$

$\log_2 \left(\frac{16x^2}{x}\right) = \log_2 8$

$\log_2 16x = \log_2 8$

$16x = 8$

$\boldsymbol{x = \frac{1}{2}}$

2. $2\log_3 8x - \log_3 4x = \log_3 (6x + 10)$

$\log_3 (8x)^2 - \log_3 4x = \log_3 (6x + 10)$

$\log_3 64x^2 - \log_3 4x = \log_3 (6x + 10)$

$\log_3 \left(\frac{64x^2}{4x}\right) = \log_3 (6x + 10)$

$\log_3 16x = \log_3 (6x + 10)$

$16x = 6x + 10$

$10x = 10$

$\boldsymbol{x = 1}$

EXERCISE 5K

Solve for x in the following equations.

1. $\log_6 4 + \log_6 x = \log_6 8$

2. $\log_7 12x - \log_7 3 = 2\log_7 6$

3. $\log_2 6x + \log_2 4x = \log_2 12x$

4. $2\log_3 3x - \log_3 x = \log_3 18$

5. $4\log_5 2 + 2\log_5 3 = \log_5 6x$

6. $2\log_2 6x - \log_2 4x = \log_2 (7x + 4)$

7. $3\log_6 4x - 2\log_6 2x = \log_6 2(3x + 5)$

8. $3\log_3 6x - 2\log_3 3x = \log_3 2(3x + 7)$

9. $5\log_5 2x + 2\log_5 4x = 6\log_5 2x$

10. $5\log_7 4x - 4\log_7 2x = \log_7 8(7x + 9)$

Logarithms with Base 10

$$\log_{10} x$$

Logarithms with base 10 are often used in science, engineering, navigation and have many practical applications.
Logarithms with base 10 are written as ***base 10 log***, ***log base 10***, ***base-10 log***, ***log to the base 10*** or ***log with base 10***.
Because ***base 10 log*** is used so often it is called the ***common logarithm***.
The reason it is used so often is largely because we use the decimal system. For this reason it is sometimes called the decimal logarithm.
Because it is used so often, when writing the abbreviation of a base 10 logarithm it is not necessary to write the subscript 10.
It is sufficient to write **log*x***. Sometimes the abbreviations **Log*x*** or **lg*x*** are used.

log*x* means $\log_{10} x$

As will be seen later in the chapter, using base 10 logarithms can condense a large range of numbers to a manageable scale.

Scientific calculators have a button that finds log to the base 10 of numbers:

LOG

It is sometimes easier to find log to the base 10 for very large and very small numbers when comparing them or representing them on a graph.
Scientific notation is the usual way of firstly writing very large and very small numbers.

Examples

740 000 000 000 000 000 000 000 000 000 000 = 7.4×10^{32}

0.000 000 000 000 000 000 000 000 872 = 8.72×10^{-25}

Log to the base 10 can now be found easily:

$$\begin{aligned}\log(7.4 \times 10^{32}) &= \log 7.4 + \log 10^{32} \quad (\textbf{\textit{Law 1}})\\ &= \log 7.4 + 32\log 10 \quad (\textbf{\textit{Law 3}})\\ &= \log 7.4 + 32 \quad (\textbf{\textit{Law 4}})\\ &= 0.87 + 32\\ &= \mathbf{32.87}\end{aligned}$$

From calculator rounded to two decimal places

$$\begin{aligned}\log(8.72 \times 10^{-25}) &= \log 8.72 + \log 10^{-25} \quad (\textbf{\textit{Law 1}})\\ &= \log 8.72 - 25\log 10 \quad (\textbf{\textit{Law 3}})\\ &= \log 8.72 - 25 \quad (\textbf{\textit{Law 4}})\\ &= 0.94 - 25\\ &= \mathbf{-24.06}\end{aligned}$$

From calculator rounded to two decimal places

Note: Log can be found by entering the full number in scientific notation into a calculator.

EXERCISE 5L

1. Convert the following numbers to scientific notation then find the log of each number. Give logs correct to two decimal places.

(a) 98 500 000 000 000
(b) 1 620 000 000 000 000 000 000
(c) 364 000 000 000 000 000
(d) 85 711 000 000 000 000 000 000
(e) 0.000 000 000 000 8
(f) 0.000 000 000 000 000 000 43
(g) 0.000 000 000 756 8
(h) 0.000 000 000 000 222

2. Find the log of the following numbers.
Give logs correct to two decimal places.

(a) 7.2×10^{15}
(b) 3.09×10^{-17}
(c) 1.984×10^{25}
(d) 5.89×10^{-12}
(e) 4.321×10^{35}
(f) 9.76×10^{-14}

3. Copy and complete the following table.
Give logs correct to two decimal places.
The first line is completed as an example.

x	*x* (*Scientific Notation*)	*logx*
1400	1.4×10^3	3.15
14 000		
140 000		
1 400 000		
14 000 000		
140 000 000		
1 400 000 000		

4. Use this table to help answer the following.
When a number (x) is multiplied by 10 what happens to:
(a) the power of 10 of that number represented in scientific notation?
(b) the log of that number?

5. Given that log3200 = 3.51, find the following without using a calculator.

(a) log32 000
(b) log320 000
(c) log3 200 000

6. Given that $\log(2.8 \times 10^6) = 6.45$, find the following without using a calculator.

(a) $\log(2.8 \times 10^7)$ (b) $\log(2.8 \times 10^8)$ (c) $\log(2.8 \times 10^{12})$

7. Given that $\log(9.1 \times 10^{11}) = 11.96$, find x in the following.

(a) $\log x = 12.96$ (b) $\log x = 13.96$ (c) $\log x = 14.96$

(d) $\log x = 17.96$ (e) $\log x = 20.96$ (f) $\log x = 24.96$

Example

Use a calculator to help find x in the following equation.
Write answer in scientific notation correct to two decimal places.

$$\log x = 13.26$$

Remember the following:

$$\mathbf{\log x = a \quad (\log_{10} x = a)}$$

$$\mathbf{x = 10^a}$$

$$\log x = 13.26$$
$$\log_{10} x = 13.26$$
$$x = 10^{13.26}$$

Use calculator

$$= 18\ 197\ 008\ 586\ 099$$
$$\mathbf{x = 1.82 \times 10^{13}}$$

8. Use a calculator to help find x in the following equations.
Write answer in scientific notation correct to two decimal places.

(a) $\log x = 15.77$ (b) $\log x = 12.82$ (c) $\log x = 24.06$

(d) $\log x = 17.39$ (e) $\log x = 35.49$ (f) $\log x = -9.45$

(g) $\log x = -7.33$ (h) $\log x = -15.21$ (i) $\log x = -19.34$

It can be seen that a large range of numbers can be reduced to a manageable range by using the logarithms of the numbers.

Examples

1. Mercury is 5.8×10^7 km from the Sun.
Earth is 1.5×10^8 km from the Sun.
Neptune is 4.5×10^9 km from the Sun.
The distance from the Sun to the nearest star is 4.0×10^{13} km.

$\text{Log}(5.8 \times 10^7) = 7.76$
$\text{Log}(1.5 \times 10^8) = 8.18$
$\text{Log}(4.5 \times 10^9) = 9.65$
$\text{Log}(4.0 \times 10^{13}) = 13.60$

2. The three main subatomic particles are electrons, protons and neutrons. The masses of these are shown below.
Mass of electron = 9.11×10^{-31} kg
Mass of proton = 1.67×10^{-27} kg
Mass of neutron = 1.67×10^{-27} kg

$\text{Log}(9.11 \times 10^{-31}) = -30.04$
$\text{Log}(1.67 \times 10^{-27}) = -26.78$

Let's now investigate a few areas where logarithms are used.

The Richter Scale

The ***Richter scale*** is a base-10 logarithmic scale that is used to measure the magnitude of earthquakes.
The Richter scale was invented in 1935 by Charles F. Richter in the California Institute of Technology.
The magnitude of an earthquake is related to how much energy is released by the quake.
Instruments called seismographs detect movement in the earth.
The intensity of an earthquake, ***R*** on the Richter scale, is calculated by using the following formula:

$$R = \log\left(\frac{A}{A_0}\right)$$

A = the measure of the amplitude of the earthquake wave

A_0 = the amplitude of the smallest detectable wave (or standard wave)

The intensity of an earthquake will typically measure between 2 and 10 on the Richter scale.
Any earthquakes with a Richter rating below a 5 are fairly minor - they may shake the ground a bit, but are seldom strong enough to cause much damage.
Earthquakes with a Richter rating of between 5 and 7.9 are much more severe, and any earthquake above an 8 is likely to cause massive damage.

The highest rating ever recorded for an earthquake is 9.5 during the 1960 Valdivia earthquake in Chile.

To simplify the calculations let the amplitude of the standard wave, A_0, equal 1.

Example 1

Find the magnitude on the Richter scale of an earthquake with an amplitude that is 450 times the standard wave.
Give answer correct to one decimal place.

$$A = 450A_0$$

$$\text{Let } A_0 = 1$$

$$A = 450$$

$$R = \log\left(\frac{A}{A_0}\right)$$

$$R = \log\left(\frac{450}{1}\right)$$

$$= \log 450$$

$$R = 2.7$$

Example 2

Find A for an earthquake that measured 6.1 on the Richter scale.

$$R = \log\left(\frac{A}{A_0}\right)$$

$$6.1 = \log\left(\frac{A}{A_0}\right)$$

$$\text{Let } A_0 = 1$$

$$6.1 = \log A$$

$$A = 10^{6.1}$$

$$= 1\ 258\ 925$$

$$A = 1.3 \times 10^6 \quad \text{(One decimal place)}$$

To simplify the calculations let the amplitude of the standard wave, A_0, equal 1.

EXERCISE 5M

1. Find the magnitude on the Richter scale of earthquakes with the following amplitudes, given that $A_0 = 1$.
Give answers correct to one decimal place.

(a) 700 (b) 5000 (c) 8.7×10^5

(d) 2.9×10^4 (e) 7.7×10^7 (f) 3.2×10^8

2. Find the amplitude, A, of the earthquakes that had the following measurements on the Richter scale.
Where required give answers in scientific notation correct to one decimal place.

(a) 3.7 (b) 1.6 (c) 5.2 (d) 8.3 (e) 4.6

3. Find the amplitude, A, of the earthquakes that had the following measurements on the Richter scale.
Give answers in scientific notation correct to one decimal place.

(a) 2.4 (b) 3.4 (c) 4.4 (d) 5.4 (e) 6.4

4. (a) What happens to the value for the amplitude, A, when the measurement on the Richter scale increases by 1?
(b) Use this fact and the answers from question 3 to find the amplitude of an earthquake that measured 7.4 on the Richter scale.

5. What would happen to the measurement on the Richter scale if the magnitude of an earthquake is increased by a factor of:

(a) 100 (b) 1000 (c) 10^4 (d) 10^5 (e) 10^6

6. An earthquake measures 4.5 on the Richter scale. Two days later another earthquake is measured that is 1000 times the magnitude of the first earthquake.
What was the reading on the Richter scale of the second earthquake?

7. Why is it convenient for the Richter scale to be a logarithmic scale?

RICHTER SCALE

Magnitude	Description	What It Feels Like	Frequency
Less than 2.0	Micro	Most people would not feel them.	Millions per year
2.0 - 2.9	Minor	A few people would feel them. No building damage.	Over 1 million per year
3.0 - 3.9	Minor	Some people feel them. Objects inside can be seen shaking.	Over 100 000 per year
4.0 - 4.9	Light	Most people would feel them. Indoor objects shake or fall to the floor.	10 000 - 15 000 per year
5.0 - 5.9	Moderate	Everyone feels them. Damage to some buildings.	1000 - 1500 per year
6.0 - 6.9	Strong	Widespread shaking. Building damage.	100 - 150 per year
7.0 - 7.9	Major	Widespread damage in most areas.	10 - 20 per year
8.0 - 8.9	Great	Widespread damage in large areas.	About 1 per year
9.0 - 9.9	Great	Severe damage to most buildings.	1 per 5 - 50 years
10.0 and over	Massive	Never recorded.	Never recorded.

Decibels

Decibels are units of sound level that are calculated by finding the base-10 logarithm of sound intensity.
Sound intensity is measured in units of watts per square metre.
The units of decibels (dB) are named after Alexander Graham Bell who invented the telephone.
One decibel is a tenth of a bel (B). The unit of bel is rarely used.
The following formula is used to calculate sound level (***SL***) for a given sound intensity (***I***).

$$SL = 10\log I$$

The difference in sound level (***DSL***) between sound intensity 1 (I_1) and sound intensity 2 (I_2) can be written as the following formula:

$$DSL = 10\log I_1 - 10\log I_2$$

$$= 10(\log I_1 - \log I_2)$$

$$DSL = 10\log\left(\frac{I_1}{I_2}\right)$$

Examples of Sound Sources and Their Sound Levels

Sound Source	*Sound Level (dB)*
Jet aircraft, 50 m away	140
Threshold of pain	130
Threshold of discomfort	120
Rock concert	115
Football crowd, at stadium	110
Vacuum cleaner, 1m away	70
Normal talking, 1m away	60
Quiet library	40

Example 1

The sound intensity of an old loud machine was found to be twice that of a new model.
(a) What is the difference in sound levels (dB) of the two machines? Give answer to the nearest dB.
(b) If the sound level of the old machine was 88 dB, find the the sound level of the new model.

Answers

(a) Use the following symbols:

The sound intensity of the old machine = $\boldsymbol{I_1}$
The sound intensity of the new machine = $\boldsymbol{I_2}$

$$I_1 = 2I_2$$

$$DSL = 10\log\left(\frac{I_1}{I_2}\right)$$

$$= 10\log\left(\frac{2I_2}{I_2}\right)$$

$$= 10\log\left(\frac{2\cancel{I_2}}{\cancel{I_2}}\right) \quad I_2\text{s can be cancelled}$$

$$= 10\log 2$$

$$\mathbf{DSL = 3dB} \quad \text{(Rounded to nearest dB)}$$

(b) Sound level of the new model = 88 - 3 = **85 dB**

Example 2

The sound level of a lawn mower was measured at 90 dB at the distance of 1 m. Ear protection reduced the sound level by 15 dB. What percentage of the sound intensity is still heard by the person using the lawn mower?
Round answer to the nearest percentage.

$$DSL = 15\text{dB}$$

$$DSL = 10\log\left(\frac{I_1}{I_2}\right)$$

$$15 = 10\log\left(\frac{I_1}{I_2}\right)$$

$$1.5 = \log\left(\frac{I_1}{I_2}\right)$$

$$\frac{I_1}{I_2} = 10^{1.5}$$

$$\frac{I_1}{I_2} = 31.62..$$

$$I_1 = 31.62.. \times I_2$$

$$I_2 = \frac{1}{31.62..} \times I_1$$

$$\frac{1}{31.62..} = 3\%$$

$$I_2 = \textbf{3\% of } I_1$$

97% of the sound is absorbed by the ear protection.

EXERCISE 5N

1. The sound intensity of a petrol lawn mower was measured to be five times the sound intensity of an electric mower.
Round answers to the nearest dB.
(a) What is the difference in sound levels (dB) of the two mowers?
(b) If the sound level of the petrol lawn mower was 92 dB, what was the sound level of the electric mower?

2. A group of four musicians often played music in a room at the back of a house. There was a few complaints from neighbours so they decided to add sound insulation to the room. They were told the sound intensity outside the room would then be one tenth of the intensity without insulation.
(a) What is the difference in sound levels (dB) before and after the insulation was installed?
(b) If the sound level was 85 dB before, what would be the sound level after the insulation was installed?

3. A listening device used to assist in recording the sound of animals in the wild claimed to magnify the animal sounds by a factor of eight (the recorded sounds would be eight times the intensity of the animal sounds).
(a) What is the difference in sound levels of the recorded sounds and the actual sounds?
(b) The actual animal sounds were 45 dB. What was the level of the recorded sounds?

4. Traffic noise from a freeway at a nearby house was measured at 91 dB. Sound barriers were installed and the sound level was measured at 85 dB. What was the percentage reduction in sound intensity?
Give answer to the nearest percentage.

5. Double glazing was reported to decrease the sound level from 86 dB to 82 dB.
What was the percentage reduction in sound intensity?
Give answer to the nearest percentage.

6. A hearing aid was reported to increase the sound level from 60 dB to 64 dB.
What was the percentage increase in sound intensity?
Give answer to the nearest percentage.

pH

The pH of a liquid is the measure of the acidity of the liquid. It is based on the concentration of hydrogen ions (***H***+) in the liquid. The pH scale is a base-10 logarithmic scale.

$$\mathbf{pH = -log(C_{H+})}$$

where C_{H+} is the concentration of hydrogen ions measured in moles per litre
(1 mole = 6×10^{23} molecules)

Example 1

Find the pH of a liquid that has hydrogen ions with a concentration of 0.004 moles per litre.
Give answer correct to one decimal place.

$$\mathbf{pH = -log(C_{H+})}$$

$$= -\log(0.004)$$

$$\mathbf{pH = 2.4}$$

Example 2

Find the concentration of hydrogen ions in a liquid that has a pH of 8.3.
Give answer in scientific notation correct to one decimal place.

$$\mathbf{pH = -log(C_{H+})}$$

$$8.3 = -\log(C_{H+})$$

$$-8.3 = \log(C_{H+})$$

$$C_{H+} = 10^{-8.3}$$

$$\mathbf{C_{H+} = 5.0 \times 10^{-9}\ moles\ per\ litre}$$

EXERCISE 5O

1. Find the pH of the liquids with the following concentrations of hydrogen ions (moles per litre).
Give answers correct to one decimal place.

(a) 0.005 (b) 0.083 (c) 0.000 009

(d) 7.2×10^{-6} (e) 4.1×10^{-11} (f) 6.6×10^{-4}

2. Find the concentration of hydrogen ions (moles per litre) of liquids with the following pH values.
Give answers in scientific notation correct to one decimal place.

(a) 8.2 (b) 12.3 (c) 2.7 (d) 7.1

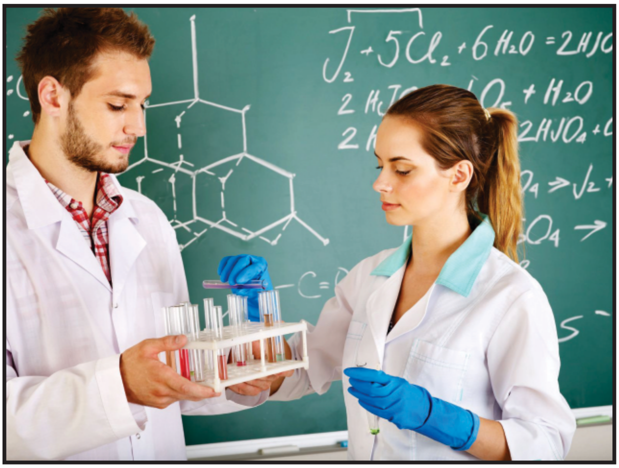
3. Liquids with a pH of 7 are classified as ***neutral***.
Liquids with a pH less than 7 are classified as ***acidic***.
Liquids with a pH greater than 7 are classified as ***alkaline*** or ***basic***.
(a) What is the concentration of hydrogen ions in a neutral liquid?
Give answer in scientific notation.
(b) Which of the following liquids are acidic?

Liquid **A** - hydrogen ion concentration = 0.000 045

Liquid **B** - hydrogen ion concentration = 0.078

Liquid **C** - hydrogen ion concentration = 9.2×10^{-10}

Liquid **D** - hydrogen ion concentration = 3.3×10^{-6}



PROBLEM SOLVING

Five earthquakes occurred on Shakey Island.
The five earthquakes and their measurements on the Richter scale are shown below. They are ***not*** in the order that they occurred.

Earthquake	*Size*
A	1.3
B	2.3
C	4.3
D	5.3
E	7.3

From the information below find the order that the earthquakes occurred.

- ***The first was not the smallest or the largest.***
- ***The second was 10 times the intensity of the third.***
- ***The fourth was 1000 times the intensity of the fifth.***
- ***The last was 1000 times the intensity of the third.***

PUZZLE

Unscramble the following words or phrases to find words that contain the word ***LOG***.

1. ***AGRO MILK***
2. ***THE ONLY COG***
3. ***GO LILAC***
4. ***GOOL***
5. ***GOOK LOB***
6. ***ANGOLA***
7. ***BOGY OIL***
8. ***LEAN HOG***
9. ***GOLF***
10. ***PLAY GOO***

CHAPTER REVIEW

1. Write $3^5 = 243$ in logarithmic form.

2. Write $\log_2 16 = 4$ in exponential form.

3. Find x: $\log_4 x = 3$

4. Evaluate $\log_5 125$.

5. Simplify the following.

(a) $\log_6 36 + \log_8 4096$

(b) $2\log_2 16 + 4\log_2 8 - 8\log_2 4$

(c) $2\log_3 27 + \log_3 81 - 5\log_3 9$

(d) $\log_2 6x + \log_2 8x - \log_2 12x$

(e) $3\log_4 2x + 2\log_4 3xy - 2\log_4 6x$

(f) $6\log_2 2x - 3\log_2 x - 3\log_2 4x$

6. Solve for x.

(a) $\log_6 64x - 2\log_6 2 = 2\log_6 4$

(b) $3\log_4 4x - 2\log_4 2x = \log_4(8x + 3)$

7. Find $\log_{10}$ of the following numbers correct to two decimal places.

(a) 83 000 000

(b) 0.000 000 06

(c) 8.9×10^7

(d) 3.4×10^{-12}

8. Given that $\log(3.7 \times 10^9) = 9.57$, find $\log(3.7 \times 10^{10})$ without using a calculator.

9. An earthquake measured 3.6 on the Richter scale. Two days later another earthquake occurred that was 100 times the intensity of the first. What did the second earthquake measure on the Richter scale?

10. A sound level of a large drill measured 93 dB. Wearing ear protection reduced the sound level to 89 dB. What was the percentage reduction in sound intensity? Round to nearest percentage.

11. What is the pH of a liquid with a hydrogen ion concentration of 5.2×10^{-4} moles per litre? Give answer correct to one decimal place.