

MASTER MATHS

STEVE PEScott

10

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NOTE TO TEACHERS

Master Maths 10 covers the mathematics curriculum of Level 10 and Level 10A. The topics and chapters of Level 10A have been noted in the Contents.

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Numbers

Real Numbers

Definitions

Real Numbers: the set of numbers that consist of *rational* numbers and *irrational* numbers.

Rational Numbers: all numbers that can be expressed as fractions. Rational numbers include whole numbers, integers, non-recurring and recurring decimals and fractions.

Examples of rational numbers are:

$6 \left(\frac{6}{1}\right)$, $0.5 \left(\frac{1}{2}\right)$, $3.6 \left(\frac{36}{10}\right)$, $0.\dot{4} \left(\frac{4}{9}\right)$, $8.\overline{57} \left(\frac{849}{99}\right)$, $-18.963 \left(-\frac{18963}{1000}\right)$

The set of rational numbers consists of several subsets:

Natural numbers (or counting numbers): the set of numbers: $\{1, 2, 3, \dots\}$

Whole numbers: the set of natural numbers and zero: $\{0, 1, 2, 3, \dots\}$

Integers: the set of positive and negative whole numbers and zero.

Irrational Numbers: all numbers that cannot be expressed as fractions.

Examples of irrational numbers are π , and surds ($\sqrt{3}$, $\sqrt{75}$, etc)

The following symbols are often used to represent these sets:

R = the set of real numbers

Q = the set of rational numbers

N = the set of natural numbers

W = the set of whole numbers

Z = the set of integers

I = the set of irrational numbers

EXERCISE 1A

The symbol $\in$ means '*is an element of*'. It can be used to state if a number belongs to a particular set of numbers.

Examples

1. The number 6 belongs to the set of whole numbers.

This could be written as:

6 is an element of W (the set of whole numbers)

or

$$6 \in W$$

2. The number -9 belongs to the set of integers.

This could be written as:

-9 is an element of Z (the set of integers)

or

$$-9 \in Z$$

1. State which of the following statements are true(***T***) or false(***F***).

- | | | | |
|---------------------------|------------------------|----------------------|-------------------------|
| (a) $6 \in N$ | (b) $8.5 \in I$ | (c) $\sqrt{8} \in Q$ | (d) $-8.43 \in Z$ |
| (e) $-5 \in W$ | (f) $\pi \in I$ | (g) $16 \in I$ | (h) $\frac{3}{8} \in R$ |
| (i) $-\frac{16}{7} \in Q$ | (j) $0 \in N$ | (k) $-13 \in Q$ | (l) $6\pi \in R$ |
| (m) $25 \in N$ | (n) $\sqrt{5.6} \in R$ | (o) $9.1 \in N$ | (p) $\sqrt{6} \in N$ |
| (q) $16.2 \in Z$ | (r) $-2 \in Z$ | (s) $7 \in W$ | (t) $0 \in W$ |

2. Choose ***all*** the numbers from the following groups that would form a set to make the statement correct.

The symbol $\subset$ means '*is a subset of*'.

Example $0, 5, -6, 3\frac{1}{2}, \sqrt{5}, 4.8, -2 \quad \{\dots\} \subset Z$

Answer $\{0, 5, -6, -2\} \subset Z$

- | | |
|---|-----------------------|
| (a) $5, -8, 7.3, \sqrt{6}, \pi, \frac{1}{2}, 12$ | $\{\dots\} \subset N$ |
| (b) $-3, 4, 2\frac{4}{7}, 8.9, \sqrt{15}, 2\pi$ | $\{\dots\} \subset Q$ |
| (c) $6.4, 10, 0, -4, 7, -9, 5.8$ | $\{\dots\} \subset W$ |
| (d) $2.9, -5.6, -8, 0, \frac{13}{5}, 13$ | $\{\dots\} \subset Z$ |
| (e) $7\pi, 6.9, -5, 0, \frac{8}{9}, -11.8$ | $\{\dots\} \subset R$ |
| (f) $\sqrt{7}, -2.4, 6\frac{4}{11}, 2\pi, 7, \sqrt{13}$ | $\{\dots\} \subset I$ |

Whole Numbers

EXERCISE 1B

1. Find the following numbers.

(a) This four digit number has one more hundred than it has thousands. It has twice as many units as it has tens. The sum of the four digits is 24.

(b) This five digit number has three times as many hundreds as it has thousands and four times as many tens as it has units. The sum of the five digits is 31.

2. Without using a calculator, **add** 100 to each of the following numbers.

(a) 8 (b) 73 (c) 605 (d) 926
(e) 5014 (f) 6918 (g) 14 993 (h) 599 999

3. Without using a calculator, **subtract** 1000 from each of the following numbers.

(a) 2804 (b) 1075 (c) 20 732 (d) 540 036
(e) 200 917 (f) 3 000 000 (g) 1 234 567 (h) 72 000 295

4. Round the following numbers to the nearest 10.

(a) 63 (b) 87 (c) 196 (d) 303
(e) 785 (f) 2799 (g) 9083 (h) 12 996

5. Round these numbers to the nearest 1000.

(a) 4571 (b) 683 (c) 497 (d) 14 625
(e) 29 596 (f) 319 713 (g) 208 496 (h) 99 999

6. Copy and complete this multiplication table.

×		8	9	6			3
4							
						35	
5	10		35				
							27
			42		72		
						50	
11							
			72			40	

7. Without using a calculator solve the following problems.
Remember to use **BODMAS**.

B	BRACKETS
O	OF
D	DIVISION
M	MULTIPLICATION
A	ADDITION
S	SUBTRACTION

Examples

1. $8 + 3 \times 4$
 $= 8 + 12$
 $= \mathbf{20}$

2. $3 \times (2 + 4) - \frac{1}{2} \text{ of } 8$
 $= 3 \times 6 - \frac{1}{2} \text{ of } 8$
 $= 3 \times 6 - 4$
 $= 18 - 4$
 $= \mathbf{14}$

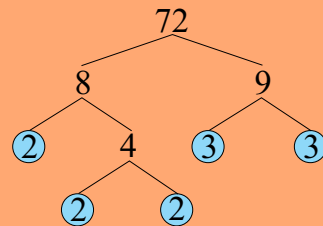
3. $4 \times 4 \div 2 \times 3 \div 12$
 $= 16 \div 2 \times 3 \div 12$
 $= 8 \times 3 \div 12$
 $= 24 \div 12$
 $= \mathbf{2}$

- (a) $4 + 5 \times 2$ (b) $6 \times 3 - 2 \times 4$ (c) $\frac{1}{2} \text{ of } 8 - 4$
 (d) $12 \div 6 - 4 \div 2$ (e) $6 \times (3 + \frac{1}{2} \text{ of } 8)$ (f) $8 - \frac{1}{2} \text{ of } (12 - 4 \times 2)$
 (g) $4 \times (2 + 7) \div \frac{1}{2} \text{ of } 24$ (h) $8 + (12 - 14 \div 2) - 10 \div 5$
 (i) $(6 + 20 \div 10) \times (15 \div \frac{1}{2} \text{ of } 10)$
 (j) $6 \times 3 \div 9 \times 10 \div 2 \times 4 \div 8 \times 6 \div 2$
 (k) $5 + (\frac{1}{4} \text{ of } 12) \times (6 - 12 \div 6) - 8 \div (20 \div 2 - \frac{1}{2} \text{ of } 12)$

8. Write the following numbers as products of their prime factors in index form.

- (a) 64 (b) 36
 (c) 200 (d) 144
 (e) 300 (f) 78
 (g) 1000 (h) 437
 (i) 936 (j) 4096
 (k) 529 (l) 111

Example 72



$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$= \mathbf{2^3 \times 3^2}$$

9. What number is as much more than 11 as it is less than half of 58?

10. Find the missing digits in these problems.

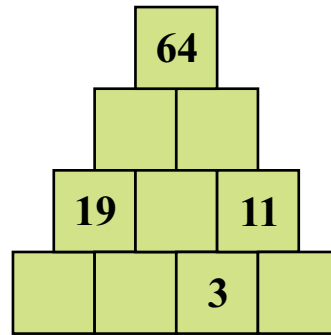
(a)

$$\begin{array}{rcccccc}
 & 8 & 7 & \square & 1 & \square \\
 + & \square & \square & 4 & \square & 5 \\
 \hline
 1 & 3 & 8 & 3 & 9 & 1
 \end{array}$$

(b)

$$\begin{array}{rcccccc}
 & 3 & 2 & \square & 5 & \square \\
 - & \square & \square & 6 & \square & 9 \\
 \hline
 & & 3 & 5 & 0 & 5
 \end{array}$$

11. Each block in this puzzle is equal to the sum of the numbers in the two blocks beneath it. Find all the missing numbers.



Roman Numerals

EXERCISE 1C

I = 1 V = 5 X = 10 L = 50 C = 100 D = 500 M = 1000

Examples

36 = XXXVI

462 = CDLXII

1263 = MCCLXIII

2594 = MMDXCIV

1. Convert the following roman numerals to number form.

(a) XXIV

(b) LVII

(c) XCIX

(d) MDCLXVI

(e) LXXIX

(f) CCXCV

(g) CMLIV

(h) MMMCCXXII

(i) CLIV

(j) CMI

(k) MMX

(l) CDXLIV

2. Convert the following numbers to roman numerals.

(a) 36

(b) 84

(c) 250

(d) 793

(e) 3408

(f) 1111

(g) 2449

(h) 938

(i) 333

(j) 3877

(k) 2020

(l) 672

Fractions

EXERCISE 1D

1. Write the following fractions in their simplest form.

Example

$$\frac{25}{30} \xrightarrow{\div 5} \frac{5}{6}$$

$$\frac{25}{30} = \frac{5}{6}$$

- (a) $\frac{10}{12}$ (b) $\frac{20}{40}$ (c) $\frac{24}{36}$ (d) $\frac{56}{64}$ (e) $\frac{72}{90}$ (f) $\frac{82}{96}$

2. Complete the following equivalent fractions.

Example

$$\frac{2}{3} = \frac{\square}{18} \quad \frac{2}{3} = \frac{\square}{18} \quad \frac{2}{3} = \frac{12}{\square} \quad \frac{2}{3} = \frac{12}{18}$$

- (a) $\frac{4}{5} = \frac{\square}{20}$ (b) $\frac{2}{7} = \frac{8}{\square}$ (c) $\frac{7}{8} = \frac{\square}{64}$ (d) $\frac{5}{9} = \frac{45}{\square}$
- (e) $\frac{3}{4} = \frac{\square}{16} = \frac{15}{\square} = \frac{\square}{28} = \frac{33}{\square} = \frac{\square}{36} = \frac{\square}{80}$
- (f) $\frac{16}{24} = \frac{\square}{48} = \frac{48}{\square} = \frac{\square}{6} = \frac{2}{\square} = \frac{\square}{18} = \frac{14}{\square} = \frac{\square}{66} = \frac{\square}{84}$

3. Change the following fractions to have the same denominator and hence arrange them in order from the smallest to the largest.

Example $\frac{3}{4}, \frac{5}{8}, \frac{3}{5}$ Lowest common denominator = 40

$$\frac{3}{4} = \frac{30}{40}$$

$$\frac{5}{8} = \frac{25}{40}$$

$$\frac{3}{5} = \frac{24}{40}$$

In order from smallest to largest: $\frac{3}{5}, \frac{5}{8}, \frac{3}{4}$

- (a) $\frac{3}{4}, \frac{5}{7}$ (b) $\frac{5}{12}, \frac{3}{8}, \frac{1}{3}$ (c) $\frac{5}{9}, \frac{3}{4}, \frac{7}{12}$ (d) $\frac{2}{3}, \frac{5}{8}, \frac{7}{9}$

4. An amount of money is divided up so that William received one third, Russell received two sevenths and Sue received the remainder of the money.

- (a) Who received the most?
(b) Who received the least?

5. Write the first quantity as a fraction of the second and write in its simplest form.

Ensure that the two quantities are in the same units before writing as a fraction.

Example

$$\begin{aligned} 60\text{c} : \$2 \\ = 60\text{c} : 200\text{c} \\ = \frac{60}{200} \\ = \frac{3}{10} \end{aligned}$$

- (a) 10 cm : 2 m (b) 8 mm : 4 cm (c) 200 kg : 3 tonnes
(d) 60c : \$5 (e) 400 g : 6 kg (f) 50 m : 2 km
(g) 25 mm : 5 cm (h) 520 g : 8 kg (i) 90c : \$3.60

6. Change the following mixed numbers to improper fractions.

Example $4\frac{3}{4} = \frac{19}{4}$

- (a) $3\frac{1}{2}$ (b) $2\frac{3}{5}$ (c) $5\frac{2}{3}$ (d) $6\frac{1}{4}$
(e) $4\frac{5}{6}$ (f) $8\frac{7}{8}$ (g) $9\frac{3}{7}$ (h) $7\frac{7}{12}$

7. Change the following improper fractions to mixed numbers.

- (a) $\frac{9}{2}$ (b) $\frac{13}{3}$ (c) $\frac{27}{5}$ (d) $\frac{63}{10}$ (e) $\frac{33}{4}$
(f) $\frac{41}{6}$ (g) $\frac{76}{9}$ (h) $\frac{53}{8}$ (i) $\frac{97}{7}$ (j) $\frac{145}{6}$

8. Find: (a) $\frac{1}{3}$ of 12 (b) $\frac{2}{3}$ of 12

9. Find: (a) $\frac{1}{5}$ of 20 (b) $\frac{2}{5}$ of 20 (c) $\frac{3}{5}$ of 20 (d) $\frac{4}{5}$ of 20

10. Find the following amounts.

- (a) $\frac{1}{4}$ of 80 kg (b) $\frac{3}{5}$ of \$50 (c) $\frac{5}{6}$ of 36 m
(d) $\frac{3}{4}$ of 200 L (e) $\frac{7}{10}$ of \$300 (f) $\frac{5}{8}$ of 48 cm
(g) $\frac{2}{3}$ of 360 kg (h) $\frac{3}{4}$ of \$120 (i) $\frac{2}{5}$ of 250 g
(j) $\frac{5}{6}$ of \$780 (k) $\frac{3}{4}$ of 960 kg (l) $\frac{11}{20}$ of 2560 km

11. Solve the following problems.

Examples

$$1. \quad \frac{3}{7} + \frac{2}{7} \\ = \frac{5}{7}$$

$$2. \quad \frac{5}{6} + \frac{3}{8} \\ = \frac{20}{24} + \frac{9}{24} \\ = \frac{29}{24} \\ = 1\frac{5}{24}$$

$$3. \quad 2\frac{3}{4} - 1\frac{5}{6} \\ = \frac{11}{4} - \frac{11}{6} \\ = \frac{33}{12} - \frac{22}{12} \\ = \frac{11}{12}$$

(a) $\frac{5}{9} + \frac{2}{9}$

(b) $\frac{7}{11} - \frac{4}{11}$

(c) $\frac{2}{3} + \frac{1}{5}$

(d) $\frac{3}{4} - \frac{1}{8}$

(e) $\frac{3}{8} + \frac{1}{3}$

(f) $\frac{7}{12} - \frac{1}{3}$

(g) $\frac{5}{6} + \frac{3}{10} + \frac{2}{3}$

(h) $\frac{7}{8} + \frac{5}{9} - \frac{5}{12}$

(i) $3\frac{1}{2} + 2\frac{3}{5}$

(j) $4\frac{3}{4} - 2\frac{5}{6}$

(k) $3\frac{2}{3} + 4\frac{1}{6}$

(l) $5\frac{7}{10} - 2\frac{3}{4}$

(m) $8\frac{2}{5} + 6\frac{3}{8}$

(n) $5\frac{2}{7} - 4\frac{5}{8}$

(o) $2\frac{1}{2} + 3\frac{3}{4} - 4\frac{7}{8}$

12. Solve the following problems.

Examples

$$1. \quad \frac{3}{7} \times \frac{2}{5} \\ = \frac{6}{35}$$

$$2. \quad \frac{2}{3} \text{ of } 14 \\ = \frac{2}{3} \times \frac{14}{1} \\ = \frac{28}{3} \\ = 9\frac{1}{3}$$

$$3. \quad \frac{8}{9} \times \frac{3}{16} \\ = \frac{1\cancel{8}}{3\cancel{9}} \times \frac{\cancel{3}^1}{\cancel{16}_2} \\ = \frac{1}{6}$$

$$4. \quad 3\frac{3}{4} \times 2\frac{2}{3} \\ = \frac{15}{4} \times \frac{8}{3} \\ = \frac{5\cancel{15}}{1\cancel{4}} \times \frac{\cancel{8}^2}{\cancel{3}_1} \\ = \frac{10}{1} \\ = 10$$

(a) $\frac{4}{5} \times \frac{6}{7}$

(b) $\frac{5}{8} \times \frac{4}{5}$

(c) $\frac{9}{16} \times \frac{8}{27}$

(d) $\frac{3}{5} \text{ of } 11$

(e) $\frac{2}{7} \text{ of } 20$

(f) $\frac{3}{8} \text{ of } 30$

(g) $3\frac{3}{5} \times 2\frac{1}{12}$

(h) $3\frac{3}{8} \times 3\frac{1}{3}$

(i) $1\frac{7}{8} \times 1\frac{1}{3}$

(j) $7\frac{7}{8} \times 4\frac{8}{9}$

(k) $10\frac{5}{6} \times 5\frac{2}{5}$

(l) $2\frac{7}{16} \times 4\frac{12}{13}$

13. Solve the following problems.

Examples

$$\begin{aligned} 1. \quad & 3 \div \frac{5}{6} \\ &= \frac{3}{1} \div \frac{5}{6} \\ &= \frac{3}{1} \times \frac{6}{5} \\ &= \frac{18}{5} \\ &= 3\frac{3}{5} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{2}{3} \div 4 \\ &= \frac{2}{3} \div \frac{4}{1} \\ &= \frac{2}{3} \times \frac{1}{4} \\ &= \frac{2}{3} \times \frac{1}{4_2} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{4}{5} \div \frac{8}{15} \\ &= \frac{4}{5} \times \frac{15}{8} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 4. \quad & 3\frac{3}{4} \div \frac{3}{16} \\ &= \frac{15}{4} \div \frac{3}{16} \\ &= \frac{15}{4} \times \frac{16}{3} \\ &= \frac{15}{4} \times \frac{16^4}{3_1} \\ &= \frac{20}{1} \\ &= 20 \end{aligned}$$

(a) $8 \div \frac{4}{5}$

(b) $\frac{3}{8} \div 9$

(c) $\frac{4}{7} \div 14$

(d) $\frac{5}{8} \div \frac{3}{4}$

(e) $\frac{15}{16} \div \frac{5}{8}$

(f) $\frac{3}{7} \div \frac{9}{14}$

(g) $\frac{8}{15} \div 6\frac{2}{5}$

(h) $3\frac{3}{8} \div 2\frac{1}{4}$

(i) $6\frac{2}{9} \div 2\frac{2}{3}$

14. (a) How many quarters are in $5\frac{1}{2}$?

(b) How many tenths are in $20\frac{3}{5}$?

(c) How many $\frac{3}{4}$'s are in 15?

(d) How many 5's are in 14?

(e) How many 8's are in 3?

15. Use a calculator to solve the following problems.

(a) $8\frac{2}{3} \times (4\frac{1}{2} + 3\frac{3}{4})$

(b) $\frac{3\frac{3}{5} + 2\frac{1}{16}}{4\frac{2}{5} - 2\frac{3}{15}}$

(c) Find $\frac{2}{3}$ of \$3621

(d) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

(e) $1\frac{2}{3} \times 4\frac{5}{6} \times 7\frac{8}{9}$

(f) $7\frac{1}{2} + \frac{(6\frac{2}{5} + 3\frac{3}{4})}{(5\frac{1}{6} - 2\frac{3}{4})}$

For some calculators it may be necessary to write mixed numbers as improper fractions.

$$4\frac{2}{5} = \frac{22}{5} = 22 \div 5$$

Example 1

A water tank containing 1500 litres of water is three-fifths full. What is the capacity of the tank?

If 1500 litres is three fifths of the tank then one fifth is 500 litres. Therefore the capacity of the tank is **2500 litres**.

Answer: 2500 litres

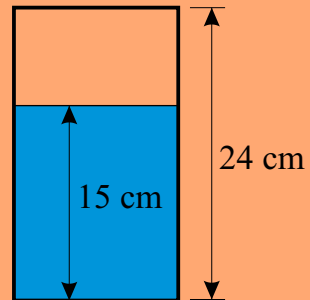
**Example 2**

A cylindrical container is 24 cm high. 500 mL of water is added. The depth of the water is 15 cm. What is the capacity of the container?

The fraction of the container that is holding water is:

$$\frac{15}{24}$$

$$= \frac{5}{8} \text{ (simplest form)}$$



If 500 mL is five eighths of the container then one eighth is 100 mL.

Therefore the capacity of the container is **800 mL**.

Answer: 800 mL

16. (a) 60 mL of ink is inserted into an empty ink cartridge making it three-quarters full. What is the capacity of the ink cartridge?
- (b) An aeroplane had travelled 1400 km. This was seven tenths of the total flight. What is the distance of the total flight?
- (c) A building site received 18 pallets of bricks. After using 4 pallets they had laid 800 bricks. What is the total number of bricks on the 18 pallets?

- (d) Window cleaners were cleaning the windows of a 20 storey building. There were the same number of windows on each storey. It took $1\frac{1}{2}$ hours to clean the windows on 6 stories. How long did it take to clean all the windows on the building?



- (e) William was jogging around a park. It took him 15 minutes to complete 10 laps. He wants to run a total of 18 laps. **How much longer** will it take him to complete the 18 laps if he runs at the same speed?



- (f) Meryl was downloading some files onto her computer. There was a total of 56 MB to download and it took 33 seconds to download 21 MB. How long will it take to download the remainder of the files?
- (g) Damien poured one quarter of a large bucket of flour into a bowl and then used this to make five loaves of bread. Each loaf weighed 250 grams. How many kilograms of flour were in the large bucket?
- (h) Peter is a gardener. He had 24 full barrows of compost delivered. He used four full barrows to pot 72 plants. He found that the compost for 16 of these pots weighed 10 kg. He paid \$756 for the 24 barrows of compost. What was the price of one kilogram of compost?

Write all fractions in their simplest form

17. Meredith earned \$360. She decided to spend one third of her earnings on clothes, save one quarter of her earnings and keep the remainder for weekly expenses.
(a) How many dollars did she keep for her weekly expenses?
(b) What fraction of her earnings did she keep for expenses?
18. Judy had a 2400 litre water tank in her back yard. Before a heavy rain she noticed it was one-third full. After the rain it was three-quarters full.
(a) How many litres of water did the tank receive from the rain?
(b) A week later the tank was half full. There had been no rain in that week. What fraction of the water in the tank had Judy used in that week?
19. A survey showed that four fifths of the people listening to a radio station were under 20 years old. Of these under 20 year olds, three quarters were female. What fraction of the people listening to this radio station were females under 20 years old?
20. Steve, Duncan and Tim are builders. On a particular job they divided up the payment so that Steve received two fifths, Duncan received one third and Tim received the remainder. If Tim received \$1600, what was the total payment for the job?
21. A foreman at a warehouse earned three quarters of the manager's wage and a storeman earned three quarters of a foreman's wage. A foreman earned \$225 more than a storeman each week. What was the weekly salary of each person?
22. The half-life of a radioactive isotope was 20 years. This means that every 20 years the isotope decays to half the amount present at the start of the 20 years.
A scientist found 3 grams of the isotope. How much would have been present 100 years earlier?
23. Amelia joined in a fun-run. She noted that at 2 o'clock she had completed one third of the run and at 2:20 she had completed three fifths of the run.
(a) How long did it take her to complete the whole run?
(b) At what time did she finish?

Decimal Numbers

EXERCISE 1E

1. Find the following numbers.

(a) ➤ This three digit number is between 3 and 4.

➤ It has five tenths.

➤ The sum of the three digits is 10.

(b) ➤ This five digit number is between 15 and 16.

➤ It has two hundredths.

➤ It has twice as many tenths as hundredths.

➤ It has twice as many hundredths as thousandths.

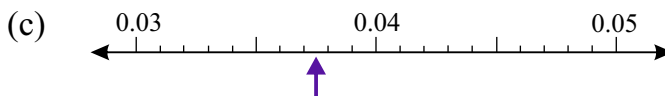
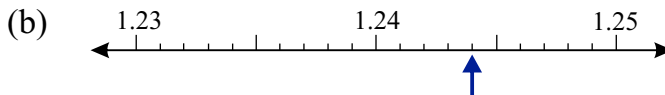
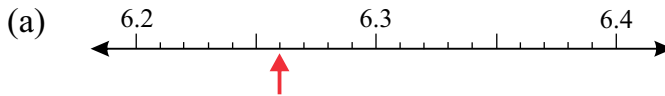
2. How many tenths is twelve hundredths?

3. Place the decimal point in the following number so that it has seven thousandths: 123456789

4. What number is midway between 3.8 and 3.98?

5. What number is midway between 17.81 and 17.92?

6. Read the number indicated on each of the following scales.



7. Add five thousandths to 8.8888.

8. Is this statement true or false: 6 millionths = 0.000006

9. Subtract five hundredths from 34.8379

10. Complete this pattern: 200, 40, 8, 1.6, 0.32, __, __.

11. Write as a decimal number: eighty-nine thousandths

12. Write in words: 0.0357

13. Copy and complete the following table showing conversions between fractions and decimals.

<i>Fraction</i>	$\frac{1}{2}$	$\frac{1}{4}$		$\frac{1}{10}$	$\frac{3}{10}$		$\frac{1}{100}$		$\frac{32}{100}$		$\frac{1}{1000}$
<i>Decimal</i>			0.75			0.7		0.09		0.53	

14. Convert the following fractions to decimal numbers.

- (a) $\frac{7}{100}$ (b) $\frac{9}{1000}$ (c) $\frac{17}{100}$ (d) $\frac{11}{1000}$ (e) $6\frac{9}{10}$
 (f) $12\frac{29}{1000}$ (g) $3\frac{157}{1000}$ (h) $\frac{21}{10\,000}$ (i) $\frac{9}{10\,000}$ (j) $23\frac{579}{10\,000}$

15. Convert the following decimal numbers to fractions.

- (a) 0.003 (b) 0.039 (c) 0.2357 (d) 0.0591 (e) 4.31
 (f) 0.00007 (g) 8.0092 (h) 5.2001 (i) 0.00059 (j) 9.043431

16. Convert the following fractions to decimal numbers.

Examples

$$1. \quad \frac{5}{8} \quad \frac{5}{8} = 5 \div 8 \\ = \mathbf{0.625}$$

$$2. \quad 4\frac{5}{8} \\ = \mathbf{4.625}$$

- (a) $\frac{3}{8}$ (b) $7\frac{1}{8}$ (c) $\frac{3}{5}$ (d) $11\frac{4}{5}$ (e) $\frac{11}{20}$
 (f) $\frac{9}{16}$ (g) $6\frac{3}{16}$ (h) $11\frac{36}{50}$ (i) $9\frac{39}{200}$ (j) $2\frac{413}{500}$

17. Convert the following fractions to decimal numbers (recurring decimals).

Examples

$$1. \quad \frac{2}{3} = 2 \div 3 \\ = 0.666\ldots \\ = \mathbf{0.\dot{6}}$$

$$2. \quad \frac{5}{12} = 5 \div 12 \\ = 0.41666\ldots \\ = \mathbf{0.41\dot{6}}$$

$$3. \quad \frac{3}{7} = 3 \div 7 \\ = 0.428571428571\ldots \\ = \mathbf{0.428571\overline{}}$$

$$4. \quad 8\frac{2}{3} \\ = \mathbf{8.\dot{6}}$$

$$5. \quad 15\frac{5}{12} \\ = \mathbf{15.41\dot{6}}$$

$$6. \quad 27\frac{3}{7} \\ = \mathbf{27.428571\overline{}}$$

- (a) $\frac{1}{3}$ (b) $2\frac{5}{6}$ (c) $\frac{2}{9}$ (d) $13\frac{3}{11}$ (e) $\frac{13}{15}$
 (f) $\frac{11}{13}$ (g) $5\frac{6}{7}$ (h) $9\frac{19}{21}$ (i) $3\frac{528}{999}$ (j) $2\frac{529}{600}$

To convert a recurring decimal to a fraction follow the steps shown below.

Examples

1. $0.\overline{43}$

Step 1 Let $x = 0.\overline{43}$

Step 2 If there is one digit recurring, multiply both sides by 10.
If there are two digits recurring, multiply both sides by 100.
If there are three digits recurring, multiply both sides by 1000.
etc.

Label these two equations **A** and **B**.

In this example multiply both sides by 100 because there are two digits recurring.

$$x = 0.\overline{43}$$

$$\text{or } x = 0.434343.. \quad \text{———— } A$$

Multiply both sides by 100:

$$100x = 43.4343... \quad \text{———— } B$$

Step 3 Subtract equation **A** from equation **B** and simplify.

$$100x - x = 43.4343.. - 0.4343..$$

$$99x = 43$$

$$x = \frac{43}{99}$$

2. $0.5\dot{6}\dot{2}$

Step 1 $x = 0.5\dot{6}\dot{2}$ ——— **A**

Step 2 $10x = 5.6\dot{2}$ ——— **B**

Step 3 $10x - x = 5.62222.. - 0.56222..$

$$9x = 5.06$$

$$x = \frac{5.06}{9}$$

$$x = \frac{506}{900}$$

Multiply top and bottom by 100
to eliminate the decimal number
from the fraction

$$x = \frac{253}{450}$$

Simplest form

3. $\overline{0.615384}$

Step 1 $x = \overline{0.615384}$ ————— **A**

Step 2 $1000000x = 615384.615384$ ——— **B**

Step 3 $1000000x - x = 615384.615384.. - 0.615384..$
 $999999x = 615384$

$$x = \frac{615384}{999999}$$

$$x = \frac{8}{13} \quad \text{Simplest form}$$

18. Convert the following decimal numbers to fractions.

(a) $0.\dot{4}$

(b) $0.\dot{7}$

(c) $0.1\dot{6}$

(d) $0.8\dot{3}$

(e) $0.\overline{45}$

(f) $0.2\dot{6}$

(g) $0.\overline{36}$

(h) $0.\overline{135}$

(i) $0.5\overline{27}$

(j) $0.38\overline{23}$

(k) $0.\overline{208}$

(l) $0.\overline{714285}$

(m) $0.58\dot{3}$

(n) $0.9523\overline{80}$

(o) $0.4\overline{225}$

(p) $0.45\overline{75}$

19. Round the following numbers to the nearest tenth.

Examples

1. 0.416
 $= 0.4$

2. 3.6629
 $= 3.7$

3. 62.75
 $= 62.8$

(a) 0.3561

(b) 6.719

(c) 0.0831

(d) 12.0095

(e) 13.8791

(f) 0.5965

(g) 0.9567

(h) 15.96711

20. Round the following numbers to the nearest hundredth.

(a) 0.7862

(b) 10.5109

(c) 9.5682

(d) 90.909090

(e) 124.5677

(f) 6.59615

(g) 129.9967

(h) 67.9084

21. Round the following numbers to three decimal places.

Examples

1. 0.4162
 $= 0.416$

2. 6.71383
 $= 6.714$

3. 62.75
 $= 62.750$

(a) 0.44567

(b) 9.60532

(c) 0.06782

(d) 100.80349

(e) 8.10568

(f) 45.0196

(g) 7.9897

(h) 0.099999

(i) 56.23

(j) 7.8

(k) 23.4015

(l) 2

(m) 4.6996

(n) 29.9995

(o) 25.0008

(p) 5.80135

22. Write the following numbers in scientific notation (standard form).

Examples

$$\begin{array}{l} \text{1. } 78\,900\,000 \\ = 7.89 \times 10^7 \end{array}$$

$$\begin{array}{l} \text{2. } 0.000\,056\,31 \\ = 5.631 \times 10^{-5} \end{array}$$

- | | | |
|--------------------|-------------------|-------------------|
| (a) 5 430 000 | (b) 60 901 000 | (c) 230 000 000 |
| (d) 4100 | (e) 7 000 000 000 | (f) 0.0006 |
| (g) 0.000 067 54 | (h) 0.000 784 5 | (i) 0.000 000 041 |
| (j) 56 000 000 000 | (k) 0.000 000 077 | (l) 0.000 897 1 |

23. Write the following quantities in scientific notation.

- (a) The mass of the Sun is:
1 989 100 000 000 000 000 000 000 000 kg.
- (b) The largest ocean is the Pacific Ocean with an area of:
181 300 000 km².
- (c) The largest continent is Asia with an area of 44 493 000 km².
- (d) The mass of an electron is:
0.000 000 000 000 000 000 000 000 000 910 9 kg.
- (e) The mass of a proton is:
0.000 000 000 000 000 000 000 000 001 672 kg.

24. Solve the following problems without using a calculator.

Examples

When **multiplying** by a power of 10 move the decimal point to the **right** the same number of places as the number of zeros.

$$\begin{array}{l} \text{1. } 4.56 \times 1000 \\ = 4560 \end{array}$$

$$\begin{array}{l} \text{2. } 0.003189 \times 10\,000 \\ = 31.89 \end{array}$$

When **dividing** by a power of 10 move the decimal point to the **left** the same number of places as the number of zeros.

$$\begin{array}{l} \text{3. } 3113.6 \div 100 \\ = 31.136 \end{array}$$

$$\begin{array}{l} \text{4. } 27000 \div 100 \\ = 270 \end{array}$$

- | | |
|-----------------------------|------------------------------|
| (a) 8.7942×1000 | (b) $56.6 \times 10\,000$ |
| (c) 0.0006×1000 | (d) 0.00785×1000 |
| (e) $0.089 \times 100\,000$ | (f) 45×1000 |
| (g) $45.2 \div 1000$ | (h) $4504.6 \div 100$ |
| (i) $30\,000 \div 100\,000$ | (j) $10\,402.6 \div 10\,000$ |
| (k) $56.78 \div 100\,000$ | (l) $0.05 \div 100$ |

Example

Write the first quantity as a fraction of the second in its simplest form. (There cannot be decimal numbers in fractions)

$$4.5 \text{ m} : 6.5 \text{ m}$$

$$= \frac{4.5}{6.5} \quad \text{Multiply the numerator and denominator by 10 to eliminate the decimal point}$$

$$= \frac{45}{65}$$

$$= \frac{9}{13} \quad \text{Fraction in its simplest form}$$

25. Write the first quantity as a fraction of the second. Write answers in their simplest form without decimal points.
- (a) 2.4 kg : 3.6 kg (b) 3.2 m : 5.6 m (c) 2.55 L : 4.59 L
 (d) 0.8 g : 4.4 g (e) 1.5 L : 2.42 L (f) 0.05 m : 2.45 m
 (g) 2.455 g : 3.865 g (h) 0.12 m : 3.8 m (i) 2.852 m : 8.644 m
26. A truck is carrying a 8.55 tonne load of wheat. It unloads 4.75 tonnes at a wheat silo.
- (a) What fraction of the load was unloaded?
 (b) What fraction of the load remains?
27. A rechargeable battery when full charged holds 46.2 amp-hours of current. 25.2 amp-hours have been used. What fraction of the battery's charge is remaining?
28. In a science experiment Rachel added 0.465 litres of hydrochloric acid to 1.355 litres of water. She filled five *equal* containers with this solution and had 0.025 litres remaining. How many litres did each of the containers hold?

29. In the qualifying laps of a car race Fordy completed a lap in 58.379 seconds. This was twenty-three thousandths of a second faster than Toyer. Toyer's lap time was thirty-seven hundredths of a second slower than Holda's lap time. What was Holda's lap time?



30. (a) A stroboscope at a party is flashing light at the rate of 84 flashes every minute. What is the time (in seconds) between flashes? Give answer to three decimal places.
(b) The stroboscope is reset so that the time between flashes is 1.25 seconds. How many flashes occur each minute?
31. Geologists calculated that a rock surface exposed to weather was eroding at the rate of six-thousandths of a millimetre every 100 years. They compared a rock that had been exposed to the weather to a rock that had not been exposed to the weather. The exposed rock was 12 millimetres thinner than the unexposed rock. How old are the rocks?
32. It is known that metals expand when heated. A metal used on a train track expanded four-thousandths of a millimetre per metre of track for every tenth of a degree Celsius that the temperature rises. On a particular day the minimum temperature was 6.8°C and the maximum temperature was 27.3°C . How many millimetres would a 12.5 metre length of track expand?
33. A delicate mechanical instrument moved 1.5 mm for every 20 turns of an adjusting control knob.
(a) How many mm would the instrument move for the following number of turns of the control knob?
(i) 1 turn (ii) 12 turns (iii) 0.2 turns
(b) How many turns of the control knob are needed to move the instrument the following amounts?
(i) 0.6 mm (ii) 0.003 mm (ii) 0.045 mm

Integers (Directed Numbers)

EXERCISE 1F

Remember

$$++ = +$$

$$+- = -$$

$$-- = +$$

$$-+ = -$$

The positive sign does not need to be shown:

$$^+3 = 3$$

Examples

$$\begin{aligned} 1. \quad 2 + ^+3 \\ = 2 + 3 \\ = 5 \end{aligned}$$

$$\begin{aligned} 2. \quad -2 + -4 \\ = -2 - 4 \\ = -6 \end{aligned}$$

$$\begin{aligned} 3. \quad 5 - ^+3 \\ = 5 - 3 \\ = 2 \end{aligned}$$

$$\begin{aligned} 4. \quad -3 - -4 \\ = -3 + 4 \\ = 1 \end{aligned}$$

1. Solve the following problems without using a calculator.

(a) $8 - ^+10$

(b) $-4 + 7$

(c) $-6 + -5$

(d) $-11 + ^+3$

(e) $-5 - 7$

(f) $-1 - 3 + 2$

(g) $-6 - -3$

(h) $8 - -9$

(i) $9 - 13$

(j) $-6 - 15$

(k) $-11 + 20$

(l) $7 - 10 + 13$

(m) $-3 - -5 - 4$

(n) $-13 + 26$

(o) $21 - -15$

(p) $-1 - -1 - -1 - -1$

When multiplying or dividing *two positive numbers* the answer is **positive**.

When multiplying or dividing *two negative numbers* the answer is **positive**.

When multiplying or dividing a *positive* and a *negative number* the answer is **negative**.

Examples

$$\begin{aligned} 1. \quad 5 \times 6 \\ = 30 \end{aligned}$$

$$\begin{aligned} 2. \quad -2 \times -4 \\ = 8 \end{aligned}$$

$$\begin{aligned} 3. \quad 5 \times -3 \\ = -15 \end{aligned}$$

$$\begin{aligned} 4. \quad -3 \times 4 \\ = -12 \end{aligned}$$

$$\begin{aligned} 5. \quad 24 \div 6 \\ = 4 \end{aligned}$$

$$\begin{aligned} 6. \quad -20 \div -4 \\ = 5 \end{aligned}$$

$$\begin{aligned} 7. \quad 18 \div -3 \\ = -6 \end{aligned}$$

$$\begin{aligned} 8. \quad -36 \div 12 \\ = -4 \end{aligned}$$

2. Solve the following problems without using a calculator.

(a) 6×-2

(b) -8×3

(c) -6×-3

(d) -4×5

(e) 7×-6

(f) -9×4

(g) -2×-9

(h) -5×11

(i) $18 \div -3$

(j) $24 \div -8$

(k) $-15 \div -3$

(l) $36 \div -9$

(m) $-45 \div -5$

(n) $-28 \div 7$

(o) $30 \div -10$

(p) $-8 \times -3 \div -6$

3. Copy and complete the following addition and multiplication tables.

(a)

+	6					
		-10				19
11						
-17					-2	
			13	9		49
			-39			
		14			31	

(b)

×	5					
		16				24
-7						
4					36	
			-33	66		132
			24			
		-40			45	

4. Solve the following problems without using a calculator.

Remember to use **BODMAS**.

(a) $3 + -2 \times -5$

(b) $-2 \times -2 + -2 \div -2$

(c) $(4 - -3) \times -2$

(d) $-8 \times -3 - -2 \times -3$

(e) $-5 \times (-2 - 3) \div -1$

(f) $6 + -2 \times -3 - -2$

(g) $-1 \times -1 \div -1 + -1 - -1$

(h) $-1 - -1 + -1 \div -1 \times -1$

(i) $-4 + 3 \times -2 + 4$

(j) $-3 \times (-4 + 5) + -2$

(k) $-6 \times -3 \div -2 \times -4 \div -12$

(l) $5 + -2 - -3 + -5 - 6$

(m) $(-1)^5$

(n) $(-2)^3$

(o) $(-3)^2$

(p) $(-10)^4$

(q) $\frac{1}{4}$ of $-8 + 6 \times -2 - 2 \times -5$

(r) $8 + \frac{1}{2}$ of $-16 - \frac{1}{2}$ of $(3 \times 4 - -2)$

5. Find the missing numbers in the following equations.

(a) $7 + \square = 4$

(b) $5 - \square = 8$

(c) $\square \times -3 = 15$

(d) $-8 \div \square = -2$

(e) $-6 - \square = 4$

(f) $-11 + \square = -19$

(g) $-13 - \square = -5$

(h) $\square - 6 = -11$

(i) $\square + 7 = -5$

(j) $\square - 4 = 5$

(k) $3 + \square - 5 = -4$

(l) $4 - \square + 2 = -4$

(m) $5 \times \square - 4 = -14$

(n) $-2 \times \square \times -3 = -24$

(o) $(5 + \square) \div -3 = -2$

(p) $8 \times (\square - 3) = -40$

(q) $4 \times (3 + \square) \times -2 = -16$

(r) $-3 \times (-10 - \square) \times -2 = -12$

6. Find the following numbers.

- (a) 4 more than -8 (b) 8 less than -1
 (c) 5 less than 2 (d) 8 more than -5
 (e) 6 more than half of -20 (f) 5 less than a quarter of -16
 (g) Midway between -6 and 2 (h) Midway between -11 and 6?

7. Solve the following problems without using a calculator.

- (a) $2.6 - 3.4$ (b) $-3.6 + 2.1$ (c) $-4.5 - 2.3$
 (d) $3.7 - 4.2$ (e) $5.6 - 4.8$ (f) $1.3 - 3.7$
 (g) $-5.6 - 2.4$ (h) $-7.8 - 1.7$ (i) $-4.3 + 7.9$
 (j) $5.7 - 9.8$ (k) $-4.5 + 2.8$ (l) $-10.6 + 5.9$
 (m) $5.6 - 13.0$ (n) $-8.0 + 6.7$ (o) $-13.6 + 17.2$

8. Solve the following problems given that:

$$a = -4, b = -3, c = 6$$

- (a) $a + b$ (b) $a - c$ (c) $c - b$
 (d) $b - a$ (e) $3a$ (f) $2a + 3c$
 (g) $2a \times 5b$ (h) $ab \div c$ (i) $2abc$
 (j) $4c + 5a$ (k) $2a - 3b - 2c$ (l) $(2a)^2$
 (m) $\frac{4b + 6a}{2c}$ (n) $\frac{2c + 3a}{b}$ (o) $\frac{3a - 3c}{2b}$

9. Copy and complete this magic square where the sum of the numbers in each row, column and diagonal is 14.

1			
13		-5	2
	9		11
	5		3

10. The average of three numbers A , B and C is -1.

C is 13 more than A .

B is twice A .

Find A , B and C .

11. A patrol boat was patrolling a length of north-south coastline.

The boat left its base and travelled north for 86 km. It then turned around and travelled south 157 km. It then travelled north 188 km then south 212 km. How far, and in what direction, is it from its base?

Problem Solving

EXERCISE 1G

- Stephanie runs laps of an oval every morning. After a certain number of laps she has completed one quarter of her run. After one more lap she has completed one third of her run.
How many laps does she run?

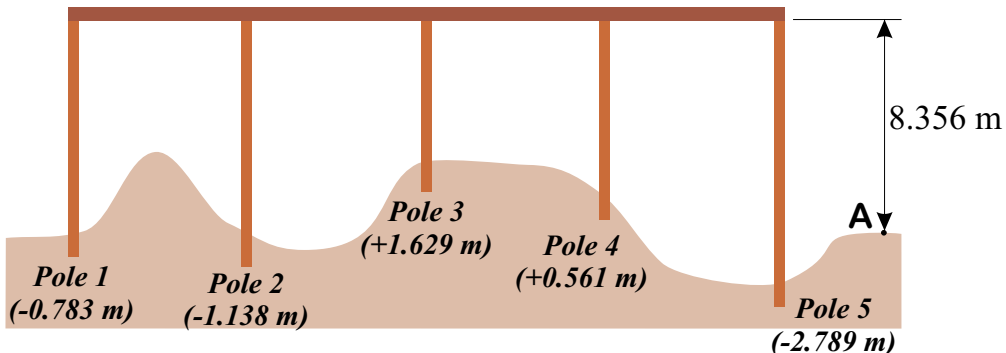
- John grew a pumpkin. He cut one fifth off and made soup. He gave one quarter of the pumpkin to his neighbour. He roasted one sixth of the pumpkin. The remaining piece of pumpkin weighed 4.6 kg. What did the pumpkin weigh?

- Sue had three dogs - Pixie, Hubbsie and Footsie. She had to weigh them but the dogs wouldn't stand on the scales by themselves. She had to weigh them two at a time. Pixie and Hubbsie weighed 7.9 kg. Pixie and Footsie weighed 7.0 kg. Hubbsie and Footsie weighed 10.3 kg. What did each dog weigh?



- A brick company made two sizes of bricks - standard and large. The standard brick weighs 2.6 kg. 170 standard bricks weighed the same as 130 large bricks.
 - What does a large brick weigh?
 - A load of bricks weighed 430 kg and there were twice as many small bricks as large ones. How many small bricks were there?
- An aeroplane has two flashing lights. One flashes every 4.9 seconds and the other flashes every 6.3 seconds. What is the time between the two lights flashing simultaneously?
- A truck is three-quarters full of sand. The total weight of the truck and sand is 5.9 tonnes. It unloads some sand and it is now one-third full. The total weight of the truck and sand is now 4.4 tonnes. What is the weight of the empty truck?

7. A piece of wood 1.364 m long is cut from a 6.820 m long log. What ***fraction*** of the log remains? Give answer as a fraction in its simplest form with no decimals.
8. A log of wood has one quarter of its length cut off. The remaining piece of wood is 4.05 m long. How long was the original log?
9. A wind turbine was producing 4.75 MW (megawatts) of power which was five eighths of full capacity. What is the full capacity of the wind turbine?
10. A load of recycled waste was sorted into the different materials. One quarter of the load was glass and it was removed. One third of the remaining waste was paper and it was removed. One half of the remaining waste was plastic. The remainder of the waste was organic matter. If there was 0.45 tonnes of organic matter, what was the total weight of the load of waste?
11. Jessie had red hair that was 28 cm long. She dyed it black. Her hair grows 0.8 cm per week.
- What fraction of her hair will be black after 12 weeks?
 - After how many weeks will her hair be half black?
12. A builder needs to calculate the length of five posts that are supporting a beam. The underside of the beam is 8.356 m above a reference point **A**. The distance vertically to the base of each of the poles from point **A** is shown on the diagram below. A positive measurement means the base of the pole is above point **A**, a negative measurement means the base of the pole is below point **A**. Find the length of each of the poles.



PROBLEM SOLVING

Solar Hot Water System

Donna is thinking of replacing her electric hot water system with a solar hot water system. She wanted to calculate how long it would take for the savings made by using a solar system to pay for the cost of its installation. Use the information below to calculate how many **months** it will take for the solar hot water system to pay for itself.

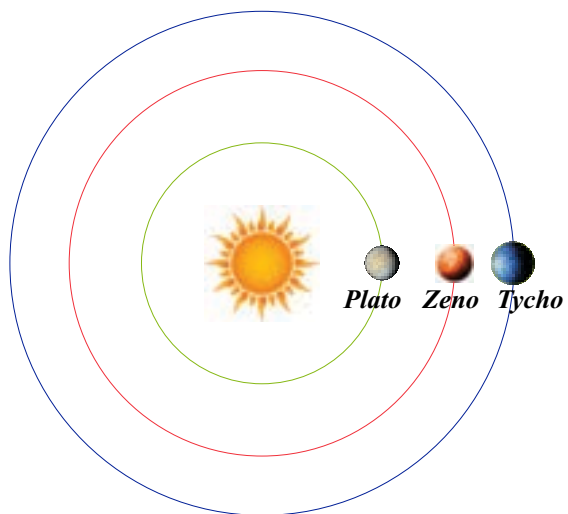
1. The cost of installing the solar system = \$2160
2. It costs Donna \$25 per month to heat water using her electric hot water system.
3. The solar system is expected to heat four fifths of Donna's hot water needs. The remainder will be heated by an electric backup that runs at the same rate as her old system.

Planets

Three planets Plato, Zeno and Tycho orbit a sun.
The time taken by each planet to complete one orbit of the sun is:

Plato - 1.2 years
Zeno - 2.7 years
Tycho - 4.5 years

In this diagram all planets are aligned. In how many years will all three planets be next aligned in these same positions?

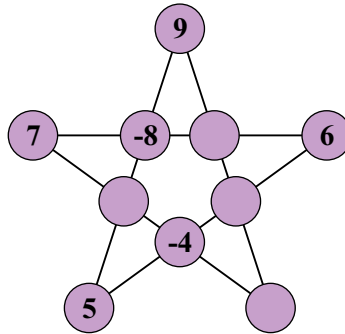


PUZZLES

1. Find the words from this chapter that would be spelt by the following digits on a standard telephone.

(a) 37228466 (b) 686237 (c) 4683437 (d) 263627

2. Copy and complete this puzzle by putting numbers in the empty circles so that the sum of the numbers on each line is the same.



3. Julie has an aviary with four perches - 1, 2, 3 and 4 as shown here.
She has four breeds of birds - budgerigars, canaries, finches and parrots.
Each breed of bird sits on one of the perches.
Julie has a total of 29 birds.

PERCH 1

PERCH 2

PERCH 3

PERCH 4

Use the information below to find how many of each breed of bird she owns and on which perch each breed sits.

- (a) The finches sit directly above the budgerigars.
(b) The canaries are on perch 4.
(c) There are eight birds on perch 1.
(d) There are two more finches than there are canaries.
(e) There are 10 budgerigars.
4. Unscramble the letters from the following phrases to find terms from this chapter.
- | | |
|---|--|
| <p>(a) MEAL BURNER</p> <p>(c) A BURNT NUMERAL</p> <p>(e) INSECT NOTIFICATION</p> | <p>(b) MURDERING A CIRCLE</p> <p>(d) BIANNUAL TREMOR</p> <p>(f) NORMAL MANURE</p> |
|---|--|

CHAPTER REVIEW

1. State which of the following statements are true(T) or false(F).

- (a) $8 \in W$ (b) $-4 \in N$ (c) $\sqrt{25} \in Q$ (d) $-7 \in Z$
 (e) $\frac{7}{5} \in Q$ (f) $2\pi \in R$ (g) $0 \in N$ (h) $\sqrt{10} \in I$

2. Choose **all** the numbers from the following group that would make the statements below correct.

$5, -7, \sqrt{8}, \frac{9}{7}, -\frac{5}{3}, 7\pi, 8.2, -5.6$

- (a) $\{...\} \subset R$ (b) $\{...\} \subset Q$ (c) $\{...\} \subset N$
 (d) $\{...\} \subset W$ (e) $\{...\} \subset Z$ (f) $\{...\} \subset I$

3. Without using a calculator, **add** 1000 to the following numbers.

- (a) 56 (b) 8254 (c) 19 943 (d) 24 599 104

4. Without using a calculator, **subtract** 100 from the following numbers.

- (a) 7831 (b) 1045 (c) 120 719 (d) 500 003

5. Without using a calculator solve the following problems.

- (a) $4 + 5 \times 6 - 2$ (b) $(8 + 6 \div 3) + \frac{1}{3} \text{ of } 24 - 6$
 (c) $(3 + 5) \times (12 - \frac{1}{3} \text{ of } 15)$ (d) $8 + 4 \times (9 - \frac{1}{4} \text{ of } 12) - 18 \div 3$
 (e) $8 + 4 - 6 - 7 + 5 - 9 + 2$ (f) $8 \div 4 \times 3 \times 5 \div 6 \times 4 \div 2 \div 10$

6. Write 1800 as a product of its prime factors in index form.

7. Convert the following roman numbers to numeral form.

- (a) CXXI (b) CDLXIV (c) CCXLVII (d) MMDCCCLXIV

8. Convert the following numbers to roman numerals.

- (a) 256 (b) 784 (c) 1442 (d) 2834

9. Complete the following equivalent fractions.

$$\frac{28}{36} = \frac{\square}{18} = \frac{\square}{9} = \frac{\square}{45} = \frac{21}{\square} = \frac{\square}{81} = \frac{42}{\square}$$

10. Change the following fractions to have the denominator and hence arrange them in order from the smallest to the largest

$$\frac{4}{9}, \frac{3}{8}, \frac{5}{6}$$

- 11.** Write the first quantity as a fraction of the second and write in its simplest form.
 (a) 80 mm : 2 m (b) 65 kg : 2.5 tonnes (c) 3.5 m : 8.5 m
- 12.** Change $6\frac{3}{4}$ to an improper fraction.
- 13.** Change $\frac{59}{7}$ to a mixed number.
- 14.** Find the following amounts.
 (a) $\frac{3}{4}$ of 48 kg (b) $\frac{5}{9}$ of \$2790 (c) $\frac{7}{8}$ of 328.48 m
- 15.** Solve the following problems without using a calculator.
 (a) $\frac{3}{11} + \frac{5}{11}$ (b) $\frac{5}{12} - \frac{1}{6}$ (c) $\frac{3}{4} + \frac{2}{9}$
 (d) $2\frac{3}{5} + 3\frac{1}{4}$ (e) $5\frac{2}{7} - 3\frac{3}{5}$ (f) $6\frac{3}{4} - 3\frac{2}{3} + 2\frac{1}{2}$
 (g) $\frac{4}{7} \times \frac{2}{5}$ (h) $\frac{9}{16} \times \frac{8}{15}$ (i) $2\frac{7}{9} \times 5\frac{2}{5}$
 (j) $8 \div \frac{2}{3}$ (k) $\frac{7}{12} \div \frac{5}{6}$ (l) $2\frac{5}{6} \div 2\frac{11}{12}$
- 16.** Use a calculator to solve: $13\frac{5}{8} \div (2\frac{7}{9} + 4\frac{6}{11})$
- 17.** An empty container has 600 mL of liquid poured into it. The container is now three-fifths full. What is the capacity of the container?
- 18.** Two people set out on a hike. They walk one third of the total distance on the first day. They walk one third of the remaining distance on the second day. They walk one half of the remaining distance on the third day. On the last day they walked the remaining 4 km. What is the total distance walked?
- 19.** (a) Add twenty-three thousandths to 3.8965.
 (b) What number is midway between 6.874 and 7.008?
 (c) Write 2.0673 in fraction form.
 (d) Subtract five hundredths from 6.928.
 (e) Convert $4\frac{3}{5}$ to a decimal number.
 (f) Convert $6\frac{7}{8}$ to a decimal number.

20. Convert the $4\frac{2}{7}$ to a decimal number.
21. (a) Convert $0.\overline{38}$ to a fraction.
(b) Convert $0.4\dot{5}$ to a fraction.
22. Round the following numbers to the nearest hundredth.
(a) 4.5672 (b) 6.0129 (c) 0.8963 (d) 34.2851
23. Round the following numbers to one decimal place.
(a) 89.07 (b) 7.939 (c) 12.3798 (d) 0.9711
24. Write the following numbers in scientific notation.
(a) 34 500 000 (b) 8 900 000 (c) 0.000 067 (d) 0.000 000 02
25. Solve the following problems without using a calculator.
(a) $0.006\ 78 \times 100\ 000$ (b) $560\ 000 \div 10\ 000\ 000$
26. (a) Jock's lap time in a car race was 56.785 seconds. Eric's lap time was sixty-seven thousandths of a second faster than Jock's. What was Eric's lap time?
(b) A container held 5.585 L of water. A coloured dye was added at the rate of 0.155 L per minute for twenty-five minutes. The resultant mixture was poured into five equal containers. What volume of the mixture is in each of the five containers?
27. Solve the following problems without using a calculator.
(a) $6 - 10$ (b) $-9 + 6$ (c) $8 - -5$ (d) $-10 - 8$
(e) $-17 - -12$ (f) $3.8 - 7.4$ (g) $-5.6 - -6.7$ (h) $-7.9 + 13.4$
(i) -8×-6 (j) $20 \div -5$ (k) $6 - 4 \times -2$ (l) $-6 \times -4 \div -3$
28. Solve the following problems given that: $a = -5$, $b = 4$, $c = -2$
(a) $2a - 3b + c$ (b) $\frac{4a + 3b}{2c}$ (c) $\frac{2ab + 3bc}{2b}$
29. The temperature in a coolroom in a shop is 18.2°C before the refrigerator is turned on. The refrigerator is turned on and the room cools at the rate of 1.4° every minute. How many minutes will it take the coolroom to reach at temperature of -5.6°C ?

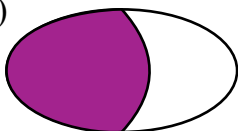
Percentages

Percentages

EXERCISE 2A

1. Choose which is the best estimate of the percentage of each of the following shapes that is shaded.

(a)



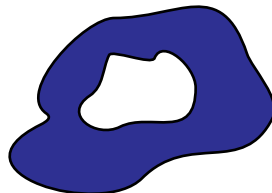
A 20% **B** 40%
C 60% **D** 80%

(b)



A 10% **B** 20%
C 30% **D** 40%

(c)



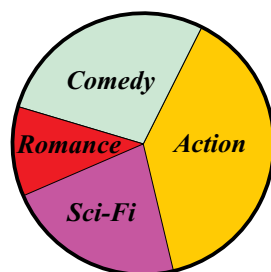
A 20% **B** 40%
C 60% **D** 80%

2. (a) The percentage of salt in a sample of sea water was 3.38%.
What was the percentage of water in this sample?
- (b) In the salt from this sea water sample there was a number of different salt ions found. The main ions were chloride (55.03%), sodium (30.59%) and sulfate (7.58%). What percentage of the salt was other salt ions?
3. The 2016 Australian census lists the following religious affiliations and the percentage of the Australian population that follow them:

Roman Catholic (22.6%), **Anglican** (13.3%), **Other Christian** (16.3%), **Islam** (2.6%), **Buddhism** (2.4%), **Hinduism** (1.9%), **Other religion** (1.7%)

What percentage of the Australian population do not follow a religion?

4. The pie graph shown here represents the favourite movie genre of a number of people. From the list below choose which percentage represents each genre.



11% 12% 22% 28% 31% 38% 41%

Percentages, Fractions and Decimals

EXERCISE 2B

1. Convert the following fractions to percentages.

Examples

*To convert a fraction to a percentage:
- multiply by 100.*

1. $\frac{3}{5}$

$$= \frac{3}{5} \times 100$$

$$= 60\%$$

2. $\frac{7}{8}$

$$= \frac{7}{8} \times 100$$

$$= 87.5\%$$

(a) $\frac{1}{10}$

(b) $\frac{3}{10}$

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

(e) $\frac{1}{5}$

(f) $\frac{3}{4}$

(g) $\frac{7}{20}$

(h) $\frac{13}{50}$

(i) $\frac{12}{25}$

(j) $\frac{1}{8}$

(k) $\frac{7}{16}$

(l) $1\frac{3}{16}$

(m) $2\frac{17}{20}$

(n) $1\frac{3}{8}$

(o) $3\frac{32}{125}$

2. Convert the following percentages to fractions in their simplest form.

Examples

*To convert a percentage to a fraction:
- divide by 100.*

1. 58%

$$= \frac{58}{100}$$

$$= \frac{29}{50}$$

2. 45.5%

$$= \frac{45.5}{100}$$

$$= \frac{91}{200}$$

(a) 22%

(b) 95%

(c) 12%

(d) 120%

(e) 2%

(f) 15.5%

(g) 62.5%

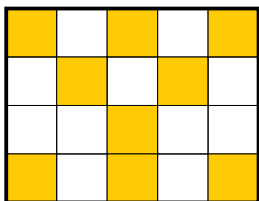
(h) 20.75%

(i) 0.2%

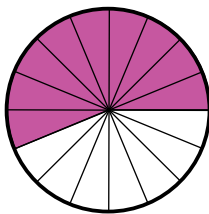
(j) 5.3%

3. What percentage of each of the following shapes is shaded?

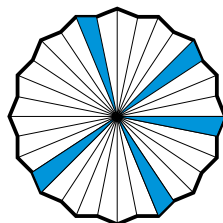
(a)



(b)



(c)



4. Convert the following decimals to percentages.

Example $0.21 \quad 0.21 \times 100$
 $= 0.21 \downarrow$
 $= 21\%$

*To convert a decimal to a percentage - multiply by 100.
 Move the decimal point 2 places to the right.*

- (a) 0.33 (b) 0.19 (c) 0.87 (d) 0.06 (e) 0.4
 (f) 0.295 (g) 0.056 (h) 0.008 (i) 2 (j) 0.1057

5. Convert the following percentages to decimals.

Example $37\% \quad 37 \div 100$
 $= 37 \downarrow$
 $= 0.37$

*To convert a percentage to a decimal - divide by 100.
 Move the decimal point 2 places to the left.*

- (a) 59% (b) 24% (c) 76.2% (d) 30% (e) 8%
 (f) 1.25% (g) 100% (h) 65.4% (i) 5.8% (j) 0.25%

6. Copy and complete this table.

Write all fractions in their simplest form.

One line has been completed as an example.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
10%		
	$\frac{1}{4}$	
		0.2
	$\frac{9}{10}$	
66%		
	$\frac{9}{50}$	
		0.32
	$\frac{17}{20}$	
9%		
	$\frac{3}{100}$	
		0.045
	$\frac{1}{3}$	
	$\frac{2}{3}$	
	$\frac{3}{16}$	
68.75%		
		0.555
0.22%		

Finding Percentages

EXERCISE 2C

1. Write the first quantity as a percentage of the second.

Examples

Make sure the units are the same

1. 2 kg, 10 kg

$$= \frac{2}{10} \times 100$$

$$= \mathbf{20\%}$$

2. 60 cents, \$2

$$= 60 \text{ cents, } 200 \text{ cents}$$

$$= \frac{60}{200} \times 100$$

$$= \mathbf{30\%}$$

3. 85 cm, 5 m

$$= 85 \text{ cm, } 500 \text{ cm}$$

$$= \frac{85}{500} \times 100$$

$$= \mathbf{17\%}$$

(a) 8 kg, 20 kg

(b) 75 mm, 300 mm

(c) \$45, \$500

(d) 40 cents, \$8

(e) 400 mm, 2 m

(f) 150 g, 3 kg

(g) 36 sec, 1 min

(h) 12 min, 2 hours

(i) 50 mm, 20 cm

(j) 240 mL, 3 L

(k) 380 kg, 2 tonnes

(l) 81 cm, 3 m

(m) \$3.20, \$2.50

(n) 156 kg, 120 kg

(o) 72 min, 1 hour

(p) 1.2 min, 50 sec

(q) 1.6 tonnes, 800 kg

(r) 28 cm, 160 mm

2. Write the first quantity as a percentage of the second.

Give answers correct to one decimal place.

(a) 85 kg, 128 kg

(b) 46 sec, 3 min

(c) 6.8 kg, 85.3 kg

(d) 28 mm, 3.7 cm

(e) 56 cm, 1.6 m

(f) 126 kg, 0.72 tonnes

(g) 650 mL, 2.53 L

(h) 3.8 m, 89 cm

(i) 895 kg, 0.4 tonnes

(j) 49 sec, 0.9 min

(k) 0.7 m, 260 mm

(l) 1000 sec, 25 min

3. Samples of soil were taken from two farms affected by salinity.

Sample **A** weighed 3.5 kg and contained 7.26 grams of salt.

Sample **B** weighed 4.6 kg and contained 9.3 grams of salt.

(a) Calculate the percentage of salt in each sample. Give answers correct to three decimal places.

(b) Which sample had the higher concentration of salt?

4. The population of Great Britain is approximately 61 000 000.

The population of London is approximately 7 200 000.

What percentage of Great Britain's population live in London?

Give answer correct to one decimal place.

Write the percentages for questions 5 - 8 correct to one decimal place

5. The population of Victoria is approximately 5 100 000.
The population of Melbourne is approximately 3 800 000.
- What percentage of Victoria's population live in Melbourne?
 - Write Victoria's population as a percentage of London's population (question 4).
 - The table below shows the approximate number of people in each age group in Victoria. What percentage of Victoria's population are in each age group?

<i>Age Group</i>	0 - 4	5 - 14	15 - 24	25 - 54	55 - 64	65 -
<i>Number of People</i>	321 300	688 500	693 600	2 152 200	561 000	683 400

6. The area of Victoria is $227\,600\text{ km}^2$.
The area of Great Britain is $244\,111\text{ km}^2$.
The area of England is $130\,439\text{ km}^2$.
- Write Victoria's area as a percentage of the area of Great Britain.
 - Write England's area as a percentage of the area of Victoria.
7. (a) Charles was 146 cm tall at the start of the year and 153 cm at the end of the year. What was his growth as a percentage of his height at the start of the year?
- (b) Rose's baby weighed 3.9 kg at birth and after three months weighed 5.2 kg. What was its percentage growth?
8. A water reservoir has a capacity of 500 ML. It is holding 38 ML.
- Write the amount of water in the reservoir as a percentage of its capacity.
 - Recycled water is added to the reservoir at the rate of 2.5 ML per day. What percentage of the water in the reservoir will be recycled water after the following number of days?
(i) one day (ii) four days (iii) a week
 - After how many days will the percentage of recycled water in the reservoir be 50%?

Percentage of Quantities

EXERCISE 2D

1. Without using a calculator find the following quantities.

Example 50% of \$20
 $= \frac{1}{2}$ of \$20
 $= \$10$

- | | | |
|-------------------------------|-------------------------------|---------------------------------|
| (a) 50% of 8 m | (b) 25% of \$40 | (c) 10% of \$50 |
| (d) $33\frac{1}{3}\%$ of 9 kg | (e) $66\frac{2}{3}\%$ of 9 kg | (f) 75% of 12 m |
| (g) 50% of \$120 | (h) 25% of \$400 | (i) $33\frac{1}{3}\%$ of 30 km |
| (j) 10% of 250 kg | (k) 50% of \$8000 | (l) 20% of 500 kg |
| (m) 50% of \$5 | (n) 10% of \$3 | (o) $66\frac{2}{3}\%$ of \$3.90 |

2. Use a calculator to find the following quantities.

Example 43% of \$350
 $= \frac{43}{100} \times \$350$
 $= \$150.50$

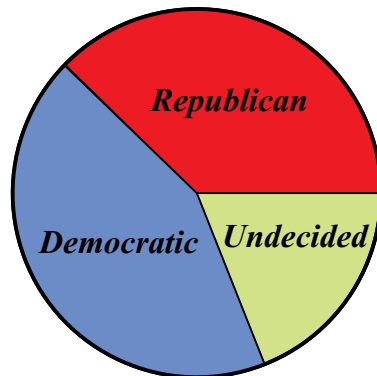
- | | | |
|-------------------|----------------------|------------------------|
| (a) 62% of 450 kg | (b) 19% of 850 m | (c) 23% of \$2580 |
| (d) 47% of \$2400 | (e) 82% of 3675 m | (f) 11% of 3.5 m |
| (g) 6.5% of \$750 | (h) 8.5% of \$42 000 | (i) 12.75% of \$84 700 |

3. Tayla had \$3500 in a bank account that earned 6% interest per year. How much interest would Tayla receive after one year?
4. Barney's annual income is \$48 000. He receives a pay rise of 3.2%. What will be his new annual income?
5. Neil has a farm that is 2500 hectares. He presently farms nut trees on 65% of his farm and citrus trees on the remainder. He wants to change this to 70 % citrus and the remainder nut trees. How many hectares of nut trees will he need to change to citrus?
6. The area of a rainforest was decreasing by 1.8% each year. If the area of the rainforest was calculated to be 150 000 hectares, find its area two years later.

7. A landcare group were hoping to increase the area of a wetland by 12.5% each year for two years (the area at the end of each year would be 12.5% larger than at the start of that year). The wetland was $40\,000\text{ m}^2$. What will be the expected area of wetland at the end of two years?



8. A poll was taken before a US election and showed that 37.9% of those polled would vote for the Republican Party, 43.4% would vote for the Democratic Party and the remainder had not decided. If this information was to be displayed on a pie chart, what angle should be used for each section? Round each angle to the nearest degree.



9. (a) The share price of a particular stock was \$14.67 at the start of trading on Monday. It rose by 1.9% over the day. What was the share price (to the nearest cent) at the close of trading on Monday?
- (b) On Tuesday the price fell by 2.4%. What was the share price (to the nearest cent) at the close of trading on Tuesday?
- (c) On Wednesday the share price fell by 0.8%, on Thursday it rose by 3.5% and on Friday it rose by 2.1%. What was the share price (to the nearest cent) at the close of trading on Friday?

Example 1

An \$850 computer is discounted by 10%. Find the discounted price.

There are two ways this can be calculated.

Method 1

Find 10% of \$850 and subtract this from \$850.

Step 1 $10\% \text{ of } \$850 = 0.1 \times \$850 = \$85$

Step 2 $\$850 - \$85 = \$765$

Method 2

Subtracting 10% from \$850 is the same as finding 90% of \$850.

$90\% \text{ of } \$850 = 0.9 \times \$850 = \$765$

Example 2

The height of a 20 m tree increases by 5% in a year. What is the height of the tree at the end of the year?

There are two ways this can be calculated.

Method 1

Find 5% of 20 m and add this to 20 m.

Step 1 $5\% \text{ of } 20 \text{ m} = 0.05 \times 20 \text{ m} = 1 \text{ m}$

Step 2 $20 \text{ m} + 1 \text{ m} = 21 \text{ m}$

Method 2

Adding 5% to 20 m is the same as finding 105% of 20 m.

$105\% \text{ of } 20 \text{ m} = 1.05 \times 20 \text{ m} = 21 \text{ m}$

10. (a) What percentage of the original amount remains after a reduction of:
 (i) 30% (ii) 15% (iii) 8% (iv) 27.5% (v) 6.4%
- (b) What percentage of the original amount is present after an increase of:
 (i) 20% (ii) 17% (iii) 6% (iv) 45.6% (v) 3.8%

11. Find the selling price of the following items after they have been discounted.
- (a) A \$250 dress discounted by 15%.
 - (b) A \$875 guitar discounted by 20%.
 - (c) A \$80 book discounted by 17.5%.
 - (d) A \$65 basketball discounted by 25%.
12. The volume of a metal ball increased by 2% after being heated.
What is the volume of a 65 cm^3 ball after it is heated?
13. A farmer used a new fertilizer that would increase his crop yield by 5%.
Before using the fertilizer
the farmer's crop yield was 2860 kg.
What could the farmer expect
his crop yield to be after
using the fertilizer?



14. Graeme's car averaged 14.5 km per litre of petrol. A combined ethanol/petrol fuel was expected to increase the km/L by 8%.
How many kilometres should Graeme's car average per litre using the ethanol/petrol fuel?

Example

What percentage is the same as 60% of 75%.

This can be calculated by writing the percentages as decimals.

$$60\% = 0.6, 75\% = 0.75$$

$$60\% \text{ of } 75\% = 0.6 \times 0.75$$

$$= 0.45$$

$$= \mathbf{45\%}$$

15. What percentage is the same as the following.
- (a) 70% of 80%
 - (b) 55% of 60%
 - (c) 90% of 65%
 - (d) 80% of 120%
 - (e) 36% of 65%
 - (f) 86% of 112%
 - (g) 8% of 46%
 - (h) 65% of 5%
 - (i) 40% of 60 % of 90%
 - (j) 65% of 80% of 70%
 - (k) 55% of 95% of 74%
 - (l) 65% of 110% of 120%

16. Find the following.
- (a) 20% of 70% of \$350
 - (b) 65% of 80% of 800 kg
 - (c) 85% of 55% of 5600 L
 - (d) 42% of 88% of \$30 000
 - (e) 60% of 125% of 700 kg
 - (f) 25.4% of 68.5% of \$400 000
17. A shop offered a discount of 20% off a \$450 camera.
The camera still didn't sell so the shop reduced the price by 20% of the discount price. What is the new sale price of the camera?
18. A clothing shop discounted the price of a \$950 dress by 20%. It then offered 30% off the discount price claiming: '**50% off**'
- (a) Is the '**50% claim**' correct?
 - (b) What is the actual discount percentage?
19. A shop achieved a profit of \$15 000 in its first year.
In the second year the profit increased by 6%.
The third year profit was 80% of the second year's profit.
- (a) What percentage of the first year's profit is the profit in the third year?
 - (b) What is the profit in the third year?
20. A desalination plant was going to process 120 ML of salt water each year. One litre of salt water weighs one kilogram. 3.6% of the salt water is salt. 60% of the salt taken from the salt water was going to be used commercially and the remainder returned to the sea. How many kilograms of salt will be returned to the sea each year?
21. A biscuit company decreased the size of its '*Crunchie Choc*' biscuits by 10% but increased the number of biscuits in each packet by 10%. The price of each packet remained the same. Are customers that buy '*Crunchie Choc*' biscuits getting the same value for money? Explain.
22. A shop is advised that if it reduces its prices by 20% it will sell 25% more items. Should the shop follow this advice? Explain.

Finding 100%

EXERCISE 2E

To find 100% of a quantity knowing a percentage of it - first find 1% then find 100%

Example

A petrol tank is 35% full. There are 14 litres in the tank.
How many litres does the tank hold?

Step 1 Find 1%:

$$14 \div 35 = 0.4 \text{ litres}$$

Step 2 Find 100%:

$$0.4 \times 100 = \mathbf{40 \text{ litres}}$$

- Find 100% of the following quantities.

(a) 15% is \$75	(b) 20% is 45 kg
(c) 24% is \$300	(d) 72% is 450 g
(e) 5% is \$350	(f) 10.5% is \$84
- A pump used 5% of its fuel pumping 600 litres of water. How many litres could be pumped by the full tank of fuel?
- A computer program was scanning a computer to find viruses. It took 30 seconds to scan 6% of the computer. How long would it take to scan the whole computer?
- A motorcycle cost \$1936 after a 12% discount. What was the full price of the motorcycle?
- A wheat silo is 28% full. 4800 kg of wheat are added to a silo taking it to 44% full. What is the capacity of the wheat silo?
- A theatre has 75 seats added increasing its capacity by 3%. How many seats are in the theatre after the new seats are added?
- A painter took 15 hours to complete 30% of a job. How much longer will he take to finish the job?

Simple Interest

EXERCISE 2F

The following formula can be used to find the amount of *simple interest* earned on an investment or needs to be paid on money borrowed.

$$SI = \frac{PRT}{100}$$

Where SI = the amount of *simple interest*
 P = the original amount or *principal*
 R = the interest *rate* per year (or per annum p.a.)
 T = the number of years or *term* of the investment or the loan

Example

Naomi invests \$5000 for 3 years at 5.5% p.a. simple interest. Calculate the amount of interest she will earn.

$$P = 5000, R = 5.5, T = 3$$

$$\begin{aligned} SI &= \frac{PRT}{100} \\ &= \frac{5000 \times 5.5 \times 3}{100} \\ &= \text{\$825} \end{aligned}$$

1. Calculate the amount of simple interest that would be earned on the following investments.
 - (a) \$3500 invested for 2 years at 6% p.a.
 - (b) \$600 invested for 3 years at 4.5% p.a.
 - (c) \$20 000 invested for 1² year at 11% p.a.
 - (d) \$6750 invested for 5 years at 8.5% p.a.
 - (e) \$1200 invested for 2 years at 9% p.a.
 - (f) \$8000 invested for 6 months at 10% p.a.

2. For the following loans calculate:

- (i) the amount of simple interest that will be paid
- (ii) the total amount of money that needs to be repaid
- (a) \$4000 is borrowed for 2 years at 7.5% p.a.
- (b) \$12 000 is borrowed for 1 year at 6% p.a.
- (c) \$2500 is borrowed for 5 years at 12.5% p.a.
- (d) \$1850 is borrowed for 10 years at 12% p.a.
- (e) \$20 000 is borrowed for $3\frac{1}{2}$ years at 7.5% p.a.
- (f) \$6500 is borrowed for 6 months at 8.25% p.a.

Compound Interest

EXERCISE 2G

If the interest earned on an investment is added onto the investment at the end of each year this is called **compound interest**.

Example

\$5000 is invested for 3 years earning compound interest of 6% p.a. To find the value of the investment at the end of the 3 years follow the steps below.

Step 1 Calculate the interest earned in the first year.

$$\text{Interest} = 6\% \text{ of } \$5000 = \$300$$

Step 2 Add this interest onto the principal.

$$5000 + 300 = \$5300$$

Step 3 Calculate the interest earned in the second year using \$5300 as the principal.

$$\text{Interest} = 6\% \text{ of } \$5300 = \$318$$

Step 4 Add this interest onto the principal

$$5300 + 318 = \$5618$$

Step 5 Calculate the interest earned in the third year using \$5618 as the principal.

$$\text{Interest} = 6\% \text{ of } \$5618 = \$337.08$$

Step 6 Add this interest onto the principal

$$5618 + 337.08 = \$5955.08$$

The investment is worth **\$5955.08**

1. For the following investments where the interest is compounded calculate:
- (i) the amount the investment is worth at the end of the time period
 - (ii) the amount of interest earned
- (a) \$4000 is invested at 5% p.a. for 2 years.
 - (b) \$2000 is invested at 8% p.a. for 3 years
 - (c) \$12 000 is invested at 7.5% p.a. for 3 years
 - (d) \$45 000 is invested at 8.75% p.a. for 4 years

An easier way to calculate compound interest is to use the following formula.

$$A = P\left(1 + \frac{R}{100}\right)^T$$

Where A = the **amount** the investment is worth at the end of the time period

P = the **principal** invested

R = the percentage interest **rate**

T = the **time period** (eg. number of years) of the investment

Example

\$20 000 is invested at 6.5% p.a. for 5 years with the interest compounded. Calculate:

- (i) the amount the investment is worth at the end of the time period
- (ii) the amount of interest earned

(i) $A = P\left(1 + \frac{R}{100}\right)^T$

$$A = 20\,000 \times \left(1 + \frac{6.5}{100}\right)^5$$

$$= 20\,000 \times (1.065)^5$$

$$= \$27\,401.73$$

(ii) Interest earned = $27\,401.73 - 20\,000$

$$= \$7401.73$$

The y^x or $\wedge$ button on a calculator can be used

2. Use this formula to check the answers for question 1.
3. Clancy invests \$800 for 12 years at an interest rate of 7.25% p.a. that is compounded yearly.
- What will his investment be worth at the end of the 12 years?

Not all investments and loans have the interest calculated and compounded annually.

For example many credit cards calculate and compound the interest *daily*.

To calculate the amount to be paid, the same formula can be used but the interest rate and time period need to be altered.

Example

Nerada owes \$500 on her credit card. The annual interest rate is 9.5% and the interest is calculated daily. How much will she owe at the end of 30 days?

$$A = P\left(1 + \frac{R}{100}\right)^T$$

A = the *amount* owing

P = *principal*

R = *daily interest rate*

T = number of *days*

$$R = \frac{9.5}{365} \quad \leftarrow \text{days in a year (ignore leap year)}$$

$$= 0.0260\% \text{ (four decimal places)}$$

$$T = 30$$

$$A = 500\left(1 + \frac{0.0260}{100}\right)^{30}$$

$$A = \mathbf{\$503.91}$$

4. Calculate the amount owing on the following credit cards.

- \$600 on a credit card for 30 days with an annual interest rate of 10.5% with interest compounded daily.
- \$1200 on a credit card for 60 days with an annual interest rate of 12.5% with interest compounded daily.
- \$2000 on a credit card for 90 days with an annual interest rate of 8.5% with interest compounded daily.

Other periods that are used to calculate compound interest are monthly (12 in a year) and quarterly (four in a year).

Example

\$4000 is invested for three years in a bank that is offering an annual interest rate of 6% compounded **quarterly**.

$$A = P\left(1 + \frac{R}{100}\right)^T$$

A = the **amount** the investment is worth

P = \$4000

R = **quarterly interest rate** = $\frac{6}{4} = 1.5\%$

T = number of **quarters** = $3 \times 4 = 12$

$$A = 4000\left(1 + \frac{1.5}{100}\right)^{12}$$

$$A = \mathbf{\$4782.47}$$

5. Beryl wants to invest \$5000 for five years. Calculate the value of her investment for the following interest rates.
 - (a) 8% compounded annually
 - (b) 8% compounded quarterly
 - (c) 8% compounded monthly
 - (d) 8% compounded daily
6. Compare the answers from question 5. Comment on the affect the period of compounding has on the final value of the investment.
7. Geoff borrows \$8000 for four years. The annual interest rate is 9.5% and the interest is calculated and compounded quarterly. Find the total amount Geoff needs to pay back.
8. Xavier wants to invest \$2000 for eight years. Find the total amount his investment would be worth if invested in the following ways.
 - (a) An annual interest rate of 5.7% compounded daily
 - (b) An annual interest rate of 5.8% compounded monthly
 - (c) An annual interest rate of 5.9% compounded quarterly
 - (d) An annual interest rate of 6.0% compounded annually

Percentage Errors

When using measurements in calculations it is very important not to round the measurements as this can lead to inaccuracies in the calculated values.

Example

The dimensions of a rectangle were measured to be:
length = 8.8 cm and width = 5.7 cm

Using these values the exact value of the area of the rectangle can be calculated:

$$\begin{aligned}\text{area} &= \text{length} \times \text{width} \\ &= 8.8 \times 5.7 \\ &= 50.16 \text{ cm}^2\end{aligned}$$

If the measurements are rounded to the nearest whole number before the area is calculated, the result is:

$$\begin{aligned}\text{area} &= \text{length} \times \text{width} \\ &= 9 \times 6 \\ &= 54 \text{ cm}^2\end{aligned}$$

The **error** between the exact value and the value obtained using the rounded values is:

$$\begin{aligned}\text{error} &= 54 - 50.16 \\ &= 3.84\end{aligned}$$

The **percentage error** created by rounding before calculating the area can be found:

$$\begin{aligned}\text{percentage error} &= \frac{\text{error}}{\text{exact value}} \times 100 \\ &= \frac{3.84}{50.16} \times 100 \\ &= 7.7\% \text{ (one decimal place)}\end{aligned}$$

EXERCISE 2H

Find the percentage errors (correct to one decimal place) between the following calculations using the exact values and the calculations using the measurements rounded to the nearest *whole number*.

1. (a) The area of a rectangle 4.8 cm long and 7.7 cm wide.
 (b) The area of a square with side length 8.2 m.
 (c) The volume of a box 6.3 cm long, 5.1 cm wide and 2.7 cm high.
 (d) The volume of cube with side length 6.4 m.
 (e) The total surface area of a prism 12.8 cm long, 8.1 cm wide and 6.7 cm high.

2. The speed of a dog that ran 87.4 m in 8.6 seconds.

Use the formula:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$



3. The kinetic energy (KE) of an object with mass, m , travelling at velocity, v , can be found using the formula:

$$KE = \frac{1}{2}mv^2$$

Calculate the percentage error between the exact kinetic energy and kinetic energy using the rounded values of a ball with mass 2.8 kg travelling at 7.6 m/sec.

4. The distance travelled by a falling object that is dropped can be calculated using this formula:

$$\text{distance} = \frac{1}{2}at^2$$

Where a is the acceleration and t is the time it has fallen.

Calculate the percentage error between the exact calculated distance and the distance using rounded values of an object that has fallen for 6.75 seconds with an acceleration of 9.81 m/sec².

PROBLEM SOLVING

In an Australian Rules Football competition a goal is worth six points and a behind worth one point. The ladder is calculated by first considering the number of wins each team has achieved and secondly their percentage. A team's percentage is calculated using the following formula:

$$\text{Percentage} = \frac{\text{total points kicked by the team}}{\text{total points kicked against the team}} \times 100$$

The columns in the ladder are **Team**, number of games played (**P**), number of wins (**W**), number of draws (**D**), number of losses (**L**), total number of points for (**Pts For**), total number of points against (**Pts Ag**), percentage (**Per**) and points (**Pts** - teams get four points for a win and two points for a draw). The headings for a ladder are shown below.

<i>Team</i>	<i>P</i>	<i>W</i>	<i>D</i>	<i>L</i>	<i>Pts For</i>	<i>Pts Ag</i>	<i>Per</i>	<i>Pts</i>
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Consider a competition between the following 12 teams:

Blues, Bombers, Bulldogs, Cats, Demons, Hawks, Kangaroos, Lions, Magpies, Saints, Swans, Tigers

The results from the first round of their competition were as shown below. Scores show the number of goals, number of behinds and the total number of points.

Blues 12.8 (80) defeated ***Bombers*** 10.7 (67)
Cats 15.11 (101) defeated ***Bulldogs*** 12.17 (89)
Hawks 13.13 (91) defeated ***Demons*** 13.10 (88)
Kangaroos 8.17 (65) defeated ***Lions*** 9.2 (56)
Saints 18.12 (120) defeated ***Magpies*** 13.23 (101)
Swans 13.6 (84) defeated ***Tigers*** 11.17 (83)

The ladder for the first round is shown on the next page.

<i>Team</i>	<i>P</i>	<i>W</i>	<i>D</i>	<i>L</i>	<i>Pts For</i>	<i>Pts Ag</i>	<i>Per</i>	<i>Pts</i>
<i>Blues</i>	1	1	-	-	80	67	119.4	4
<i>Saints</i>	1	1	-	-	120	101	118.8	4
<i>Kangaroos</i>	1	1	-	-	65	56	116.1	4
<i>Cats</i>	1	1	-	-	101	89	113.5	4
<i>Hawks</i>	1	1	-	-	91	88	103.4	4
<i>Swans</i>	1	1	-	-	84	83	101.2	4
<i>Tigers</i>	1	-	-	1	83	84	98.8	-
<i>Demons</i>	1	-	-	1	88	91	96.7	-
<i>Bulldogs</i>	1	-	-	1	89	101	88.1	-
<i>Lions</i>	1	-	-	1	56	65	86.1	-
<i>Magpies</i>	1	-	-	1	101	120	84.2	-
<i>Bombers</i>	1	-	-	1	67	80	83.8	-

The results from the second round of the competition are shown below.

Lions 22.13 (145) defeated *Bulldogs* 19.9 (123)
Cats 18.17 (125) defeated *Saints* 15.21 (111)
Magpies 12.7 (79) defeated *Demons* 11.10 (76)
Kangaroos 17.12 (114) defeated *Blues* 13.13 (91)
Bombers 9.22 (76) defeated *Tigers* 4.9 (33)
Swans 11.15 (81) defeated *Hawks* 8.11 (59)

1. Compile the ladder for the competition after the second round.
Remember to add the points for and point against onto the points from round one.
2. Make up your own scores for the third round of the competition.
3. Using your scores compile the ladder after the third round.
Remember to add the points onto the previous totals.

PUZZLES

1. A father has one son and one daughter. He is 75% older than his son and twice his daughter's age. His son is six years older than his daughter. Find the age of the father, son and daughter.

2. Four friends (Adela, Bella, Celia and Dahlia) are in different mathematics classes. The names of the teachers of the mathematics classes are Ms. Addet, Mr. Bargraff, Ms. Cuebrute and Mr. Devisa. From the information below find each girl's birthday, the name of each girl's mathematics teacher and how many students are in their mathematics classes.
 - Dahlia's birthday is two days after Adela's birthday.
 - There are 20 students in Bella's class.
 - The girl in the largest class has her birthday on June 2.
 - The girl in Mr. Bargraff's class has her birthday on June 3.
 - There are 16 students in Celia's class.
 - Celia's birthday is on June 3.
 - Adela's teacher is a male.
 - The girl in Ms. Cuebrute's class has her birthday in June 1.
 - There is 10% more students in Dahlia's class than Bella's class.
 - There is a total of 81 students in the four classes.
 - Their birthdays are on consecutive days.

3. Find the words below in this word puzzle.

DECIMAL
PERCENTAGE
DISCOUNT
FRACTION
INTEREST
COMPOUND
SIMPLE
ERROR
DAILY
PRINCIPAL
RATE
TERM
QUARTERLY
MONTHLY

Y	L	R	E	T	R	A	U	Q	E	D
U	Y	A	L	M	I	T	M	P	N	E
I	L	N	P	D	Y	T	E	U	M	T
L	H	N	M	I	F	L	O	R	S	O
A	T	E	I	S	C	P	I	E	M	L
M	N	H	S	C	M	N	R	A	T	E
I	O	P	M	O	F	E	I	O	D	I
C	M	I	C	U	T	E	R	R	O	R
E	G	A	T	N	E	C	R	E	P	Y
D	N	O	I	T	C	A	R	F	H	R

CHAPTER REVIEW

1. Sterling silver is a mixture of silver (Ag) and copper (Cu) with 92.5% of the mixture being silver.
What percentage of sterling silver is copper?
2. Statistics on Australia's population show that 70.9% of the population were born in Australia, 4.3% were born in England, 2.1% were born in New Zealand and 1.1% were born in China.
What percentage of the population were born in countries other than the four listed above?
3. (a) Convert the following fractions to percentages: (i) $\frac{4}{5}$ (ii) $\frac{5}{8}$
(b) Convert the following decimals to percentages: (i) 0.765 (ii) 0.025
(c) Convert the following percentages to fractions in their simplest form: (i) 45% (ii) 8.2% (iii) 15.75%
(d) Convert the following percentages to decimals: (i) 6.8% (ii) 84.1%
4. Write the first quantity as a percentage of second.
(a) 75 g, 1.2 kg (b) 70 cm, 3.5 m (c) 256 kg, 400 kg
5. Write the first quantity as a percentage of the second.
Give answers correct to one decimal place.
(a) 35 m, 62 m (b) 564 mm, 4 m (c) 86 cm, 760 mm
6. Tarkyn scored 31 points out of his teams score of 58 in a game of basketball. What percentage of the team's total did Tarkyn score?
Give answer correct to one decimal place.
7. The area of mainland Australia is 7 614 500 km². The area of land that drains into the Murray-Darling Basin is about one million square kilometres. What percentage of mainland Australia's area drains into the Murray-Darling Basin?
Give answer correct to one decimal place.
8. Without using a calculator find the following quantities.
(a) 25% of \$840 (b) 10% of 640 kg (c) 50% of 64.8 m
(d) 20% of 500 m (e) $33\frac{1}{3}\%$ of \$96 (f) $66\frac{2}{3}\%$ of 63 kg

9. Find the following quantities.
(a) 24% of 520 kg (b) 6.5% of \$4500 (c) 17.2% of \$12 000
10. A shop made a profit of \$45 600. The owners were hoping for a 2.5% increase in their profits in the following year. What is their expected profit in that year?
11. Find the discounted prices of the following items.
(a) A \$620 bed is discounted by 12%.
(b) A \$4200 sofa is discounted 17.5%.
12. A town's population is 6200. In one year the population grew by 2% and in the following year it grew by a further 3%. What is the new population of the town? Give answer correct to the nearest person.
13. In a school 30% of the students play a musical instrument. Of the students who play a musical instrument 40% play guitar.
(a) What percentage of the students in the school play guitar?
(b) If there are 950 students at the school, how many play guitar?
14. A fuel tank contains 280 litres of fuel which is 35% full. What is the capacity of the fuel tank?
15. A container ship is 74% full. It can hold another 3900 tonnes. What is the capacity of the ship?
16. How much **simple interest** is earned if \$3500 is invested at an annual simple interest rate of 5% for four years?
17. \$8000 is invested for five years earning **compound interest** of 8.5% per annum compounded annually. What will the investment be worth after the five years?
18. Frieda has \$1250 on her credit card. The interest rate is 12% per annum and compounds **daily**. How much will be owing on Frieda's credit card after 30 days?
19. Karl has a fish tank that is 84 cm long, 36 cm wide and 28 cm high. He guesses the dimensions to be 80 cm $\times$ 30 cm $\times$ 30 cm. What is the percentage error between the actual volume and the volume calculated using Karl's estimate of the dimensions? Give answer to one dec. pl..

Surds

Rational and Irrational Numbers

Definitions

Rational numbers can be expressed in the form $\frac{a}{b}$.
Rational numbers are whole numbers, numbers with a definite number of decimal places or recurring decimals.

Examples 15, $8.5 (= \frac{17}{2})$, $-\frac{7}{8}$, $\sqrt{16} (= 4)$, $0.\dot{3} (= \frac{1}{3})$,

$0.0871 (= \frac{871}{10\,000})$, $0.41\dot{6} (= \frac{5}{12})$, $0.\overline{428571} (= \frac{3}{7})$

Irrational numbers cannot be expressed in the form $\frac{a}{b}$.
Irrational numbers are infinitely non-recurring decimals.
Those that can be written in the form $\sqrt{a}$ are called **surds**.

Examples $\pi (= 3.14159265.....)$, $\sqrt{7} (= 2.64575131.....)$

EXERCISE 3A

1. Which of the following numbers are:

(a) rational numbers (b) irrational numbers

9.76, 0.5, $\sqrt{25}$, $0.\dot{6}$, $\frac{7}{17}$, $\frac{\pi}{2}$, $0.75\overline{32}$, -3.7311, $\sqrt{11}$, $\sqrt{529}$

2. Use a calculator to find the value of the following surds correct to five decimal places.

(a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) $\sqrt{29}$ (d) $\sqrt{156}$ (e) $\sqrt{5} + \sqrt{3} - \sqrt{2}$
(f) $\sqrt{13} + \sqrt{7} - \sqrt{23}$ (g) $\sqrt{37} - \sqrt{11} + \sqrt{19}$ (h) $\sqrt{3}(\sqrt{5} + \sqrt{7})$

3. Use a calculator to find the exact value of the following numbers.

(a) $\sqrt{169}$ (b) $\sqrt{42.25}$ (c) $\sqrt{243.36}$ (d) $\sqrt{3364}$ (e) $\sqrt{\frac{841}{1369}}$

4. Without using a calculator find the following square roots.

Example $\sqrt{9} = 3$

- | | | | | |
|------------------|----------------------------|----------------------------|-----------------------------|-----------------------------|
| (a) $\sqrt{49}$ | (b) $\sqrt{100}$ | (c) $\sqrt{64}$ | (d) $\sqrt{36}$ | (e) $\sqrt{4}$ |
| (f) $\sqrt{25}$ | (g) $\sqrt{81}$ | (h) $\sqrt{144}$ | (i) $\sqrt{121}$ | (j) $\sqrt{16}$ |
| (k) $\sqrt{400}$ | (l) $\sqrt{\frac{16}{25}}$ | (m) $\sqrt{\frac{49}{64}}$ | (n) $\sqrt{\frac{81}{100}}$ | (o) $\sqrt{\frac{36}{169}}$ |

5. Guess the value of the following surds (correct to one decimal place) and check this using a calculator.

Example $\sqrt{11}$ is between $\sqrt{9}$ and $\sqrt{16}$.
The answer will be between 3 and 4 = **3.3**

- | | | | | |
|-----------------|-----------------|-----------------|------------------|------------------|
| (a) $\sqrt{30}$ | (b) $\sqrt{90}$ | (c) $\sqrt{5}$ | (d) $\sqrt{105}$ | (e) $\sqrt{135}$ |
| (f) $\sqrt{56}$ | (g) $\sqrt{78}$ | (h) $\sqrt{14}$ | (i) $\sqrt{24}$ | (j) $\sqrt{44}$ |

Simplifying Surds

EXERCISE 3B

1. Simplify the following square roots.

Rule $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$

$3\sqrt{5}$ means $3 \times \sqrt{5}$

Examples

$$\begin{aligned} 1. \quad & \sqrt{16 \times 25} \\ &= \sqrt{16} \times \sqrt{25} \\ &= 4 \times 5 \\ &= \mathbf{20} \end{aligned}$$

$$\begin{aligned} 2. \quad & \sqrt{1600} \\ &= \sqrt{16 \times 100} \\ &= \sqrt{16} \times \sqrt{100} \\ &= 4 \times 10 \\ &= \mathbf{40} \end{aligned}$$

$$\begin{aligned} 3. \quad & 8\sqrt{25 \times 49} \\ &= 8\sqrt{25} \times \sqrt{49} \\ &= 8 \times 5 \times 7 \\ &= \mathbf{280} \end{aligned}$$

- | | | |
|----------------------------|-----------------------------|---------------------------------------|
| (a) $\sqrt{49 \times 64}$ | (b) $\sqrt{100 \times 25}$ | (c) $\sqrt{36 \times 144}$ |
| (d) $\sqrt{4 \times 81}$ | (e) $\sqrt{9 \times 121}$ | (f) $\sqrt{400}$ |
| (g) $\sqrt{2500}$ | (h) $\sqrt{8100}$ | (i) $5\sqrt{25 \times 64}$ |
| (j) $6\sqrt{4 \times 121}$ | (k) $9\sqrt{10 \times 000}$ | (l) $4\sqrt{144 \times 36 \times 64}$ |

2. Simplify the following square roots.

Rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Example $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$

(a) $\sqrt{\frac{49}{64}}$

(b) $\sqrt{\frac{9}{100}}$

(c) $\sqrt{\frac{4}{121}}$

(d) $\sqrt{\frac{36}{169}}$

3. Simplify the following surds.

Hint: Find two factors such that one is a **square number**:

$2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2$, etc.

4, 9, 16, 25, 36, 49, 64, 81, 100, etc

Write this factor first

Examples

$$\begin{aligned} 1. \quad & \sqrt{63} \\ &= \sqrt{9 \times 7} \\ &= \sqrt{9} \times \sqrt{7} \\ &= 3 \times \sqrt{7} \\ &= 3\sqrt{7} \end{aligned}$$

$$\begin{aligned} 2. \quad & \sqrt{96} \quad \text{OR} \quad \sqrt{96} \\ &= \sqrt{4 \times 24} &= \sqrt{16 \times 6} \\ &= \sqrt{4} \times \sqrt{24} &= \sqrt{16} \times \sqrt{6} \\ &= 2 \times \sqrt{24} &= 4 \times \sqrt{6} \\ &= 2 \times \sqrt{4 \times 6} &= 4\sqrt{6} \\ &= 2 \times \sqrt{4} \times \sqrt{6} \\ &= 2 \times 2 \times \sqrt{6} \\ &= 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} 3. \quad & 3\sqrt{18} \\ &= 3\sqrt{9 \times 2} \\ &= 3 \times \sqrt{9} \times \sqrt{2} \\ &= 3 \times 3 \times \sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$

(a) $\sqrt{20}$

(b) $\sqrt{27}$

(c) $\sqrt{200}$

(d) $\sqrt{45}$

(e) $\sqrt{50}$

(f) $\sqrt{48}$

(g) $\sqrt{56}$

(h) $\sqrt{98}$

(i) $\sqrt{75}$

(j) $\sqrt{80}$

(k) $\sqrt{125}$

(l) $\sqrt{216}$

(m) $\sqrt{180}$

(n) $\sqrt{405}$

(o) $3\sqrt{54}$

(p) $5\sqrt{28}$

(q) $2\sqrt{90}$

(r) $6\sqrt{250}$

(s) $5\sqrt{32}$

(t) $4\sqrt{300}$

(u) $3\sqrt{245}$

(v) $6\sqrt{448}$

(w) $10\sqrt{468}$

(x) $7\sqrt{972}$

4. Write the following as entire surds.

Examples

$$\begin{aligned} 1. \quad 2\sqrt{10} &= 2 \times \sqrt{10} \\ &= \sqrt{4} \times \sqrt{10} \\ &= \sqrt{40} \end{aligned}$$

$$\begin{aligned} 2. \quad 7\sqrt{3} &= 7 \times \sqrt{3} \\ &= \sqrt{49} \times \sqrt{3} \\ &= \sqrt{147} \end{aligned}$$

- | | | | |
|------------------|------------------|------------------|-----------------|
| (a) $2\sqrt{2}$ | (b) $2\sqrt{3}$ | (c) $10\sqrt{5}$ | (d) $5\sqrt{6}$ |
| (e) $2\sqrt{10}$ | (f) $7\sqrt{6}$ | (g) $3\sqrt{11}$ | (h) $6\sqrt{2}$ |
| (i) $4\sqrt{7}$ | (j) $8\sqrt{2}$ | (k) $12\sqrt{3}$ | (l) $5\sqrt{7}$ |
| (m) $2\sqrt{17}$ | (n) $3\sqrt{30}$ | (o) $4\sqrt{11}$ | (p) $9\sqrt{3}$ |

5. Simplify the following expressions.

Examples

$$\begin{aligned} 1. \quad \sqrt{5} \times \sqrt{7} &= \sqrt{35} \end{aligned}$$

$$\begin{aligned} 2. \quad 3\sqrt{2} \times 7\sqrt{5} &= 3 \times \sqrt{2} \times 7 \times \sqrt{5} \\ &= 3 \times 7 \times \sqrt{2} \times \sqrt{5} \\ &= 21\sqrt{10} \end{aligned}$$

$$\begin{aligned} 3. \quad 5\sqrt{3} \times 2\sqrt{6} &= 10\sqrt{18} \\ &= 10\sqrt{9 \times 2} \\ &= 10 \times \sqrt{9} \times \sqrt{2} \\ &= 10 \times 3 \times \sqrt{2} \\ &= 30\sqrt{2} \end{aligned}$$

Always check if the surd can be simplified.

- | | | |
|-----------------------------------|-----------------------------------|-----------------------------------|
| (a) $\sqrt{5} \times \sqrt{3}$ | (b) $\sqrt{10} \times \sqrt{3}$ | (c) $3\sqrt{5} \times \sqrt{2}$ |
| (d) $\sqrt{10} \times 5\sqrt{7}$ | (e) $\sqrt{5} \times \sqrt{5}$ | (f) $3\sqrt{3} \times 4\sqrt{3}$ |
| (g) $3\sqrt{7} \times 5\sqrt{3}$ | (h) $6\sqrt{11} \times 2\sqrt{3}$ | (i) $\sqrt{6} \times \sqrt{2}$ |
| (j) $4\sqrt{2} \times \sqrt{10}$ | (k) $2\sqrt{6} \times 5\sqrt{3}$ | (l) $\sqrt{15} \times \sqrt{3}$ |
| (m) $3\sqrt{3} \times 2\sqrt{21}$ | (n) $2\sqrt{5} \times 3\sqrt{10}$ | (o) $5\sqrt{3} \times 7\sqrt{3}$ |
| (p) $\sqrt{5} \times 2\sqrt{15}$ | (q) $3\sqrt{14} \times 2\sqrt{7}$ | (r) $5\sqrt{12} \times 2\sqrt{3}$ |

6. Simplify each surd before multiplying the following expressions.

Example

$$\begin{aligned}
 & 2\sqrt{20} \times 3\sqrt{48} \\
 &= 2\sqrt{4 \times 5} \times 3\sqrt{16 \times 3} \\
 &= 2 \times \sqrt{4} \times \sqrt{5} \times 3 \times \sqrt{16} \times \sqrt{3} \\
 &= 2 \times 2 \times \sqrt{5} \times 3 \times 4 \times \sqrt{3} \\
 &= 4 \times \sqrt{5} \times 12 \times \sqrt{3} \\
 &= 48\sqrt{15}
 \end{aligned}$$

(a) $3\sqrt{12} \times 2\sqrt{20}$

(b) $2\sqrt{27} \times 10\sqrt{18}$

(c) $5\sqrt{40} \times 3\sqrt{32}$

(d) $2\sqrt{50} \times \sqrt{120}$

(e) $6\sqrt{72} \times 3\sqrt{200}$

(f) $3\sqrt{1000} \times 6\sqrt{75}$

(g) $3\sqrt{45} \times 2\sqrt{99}$

(h) $3\sqrt{96} \times 5\sqrt{180}$

(i) $4\sqrt{80} \times 5\sqrt{128}$

(j) $\sqrt{500} \times \sqrt{600}$

(k) $3\sqrt{54} \times 5\sqrt{900}$

(l) $2\sqrt{196} \times 5\sqrt{484}$

7. Simplify the following expressions.

Examples

$$\begin{aligned}
 1. \quad & \frac{\sqrt{40}}{\sqrt{8}} \\
 &= \frac{\sqrt{40}}{\sqrt{8}} \\
 &= \sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{8\sqrt{50}}{2\sqrt{20}} \\
 &= \frac{4\sqrt{50}}{\sqrt{20}} \\
 &= 4\sqrt{\frac{50}{20}} \\
 &= 4\sqrt{\frac{5}{2}} \\
 &= \frac{4\sqrt{5}}{\sqrt{2}}
 \end{aligned}$$

(a) $\frac{\sqrt{24}}{\sqrt{6}}$

(b) $\frac{8\sqrt{50}}{4\sqrt{2}}$

(c) $\frac{\sqrt{48}}{\sqrt{30}}$

(d) $\frac{8\sqrt{85}}{\sqrt{20}}$

(e) $\frac{10\sqrt{30}}{\sqrt{24}}$

(f) $\frac{8\sqrt{50}}{5\sqrt{32}}$

(g) $\frac{6\sqrt{27}}{2\sqrt{3}}$

(h) $\sqrt{\frac{80}{32}}$

8. Simplify the following expressions.

Examples

$$\begin{aligned} 1. \quad & 3\sqrt{2} + 5\sqrt{2} \\ &= 8\sqrt{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & 7\sqrt{3} - 9\sqrt{3} \\ &= -2\sqrt{3} \end{aligned}$$

$$\begin{aligned} 3. \quad & 3\sqrt{2} + 5\sqrt{3} + 7\sqrt{2} - 3\sqrt{3} \\ &= \boxed{3\sqrt{2}} + \boxed{5\sqrt{3}} + \boxed{7\sqrt{2}} - \boxed{3\sqrt{3}} \\ &= 10\sqrt{2} + 2\sqrt{3} \end{aligned}$$

Surds can only be added or subtracted if they are the same.

$$\begin{aligned} 4. \quad & \sqrt{48} + \sqrt{27} \\ &= \sqrt{16 \times 3} + \sqrt{9 \times 3} \\ &= \sqrt{16} \times \sqrt{3} + \sqrt{9} \times \sqrt{3} \\ &= 4\sqrt{3} + 3\sqrt{3} \\ &= 7\sqrt{3} \end{aligned}$$

$$\begin{aligned} 5. \quad & 2\sqrt{20} + 3\sqrt{7} - \sqrt{45} \\ &= 2\sqrt{4 \times 5} + 3\sqrt{7} - \sqrt{9 \times 5} \\ &= 2 \times \sqrt{4} \times \sqrt{5} + 3\sqrt{7} - \sqrt{9} \times \sqrt{5} \\ &= 2 \times 2 \times \sqrt{5} + 3\sqrt{7} - 3 \times \sqrt{5} \\ &= 4\sqrt{5} + 3\sqrt{7} - 3\sqrt{5} \\ &= \sqrt{5} + 3\sqrt{7} \end{aligned}$$

(a) $7\sqrt{3} + 8\sqrt{3}$

(c) $\sqrt{7} + 2\sqrt{7}$

(e) $3\sqrt{11} - \sqrt{11}$

(g) $4\sqrt{6} + 2\sqrt{6} - 3\sqrt{6}$

(i) $5\sqrt{2} + 2\sqrt{2} - 7\sqrt{2}$

(k) $3\sqrt{2} - 7\sqrt{2} + 4\sqrt{2}$

(m) $2\sqrt{3} + 3\sqrt{5} + 3\sqrt{3}$

(o) $3\sqrt{5} + 2\sqrt{3} + \sqrt{5} + \sqrt{3}$

(q) $6\sqrt{2} + 3\sqrt{5} + 6\sqrt{5} - 3\sqrt{2}$

(s) $\sqrt{20} + \sqrt{45}$

(u) $\sqrt{28} - \sqrt{63}$

(w) $3\sqrt{2} + 5\sqrt{18} - 4\sqrt{32}$

(y) $5\sqrt{11} + \sqrt{63} - 2\sqrt{99}$

(b) $9\sqrt{5} - 3\sqrt{5}$

(d) $5\sqrt{7} - 7\sqrt{7}$

(f) $2\sqrt{5} + 5\sqrt{5} + 3\sqrt{5}$

(h) $5\sqrt{10} - 2\sqrt{10} + \sqrt{10}$

(j) $4\sqrt{3} + \sqrt{3} - 7\sqrt{3}$

(l) $6\sqrt{3} - 10\sqrt{3} + 2\sqrt{3}$

(n) $\sqrt{7} + 2\sqrt{3} + 7\sqrt{3}$

(p) $3\sqrt{11} + 2\sqrt{7} - 2\sqrt{11} + 5\sqrt{7}$

(r) $7\sqrt{3} + 6\sqrt{7} - 4\sqrt{3} - 7\sqrt{7}$

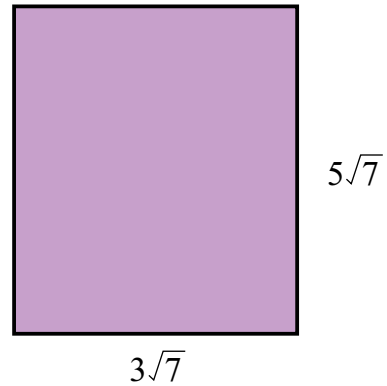
(t) $\sqrt{75} - \sqrt{12}$

(v) $2\sqrt{54} - 3\sqrt{24}$

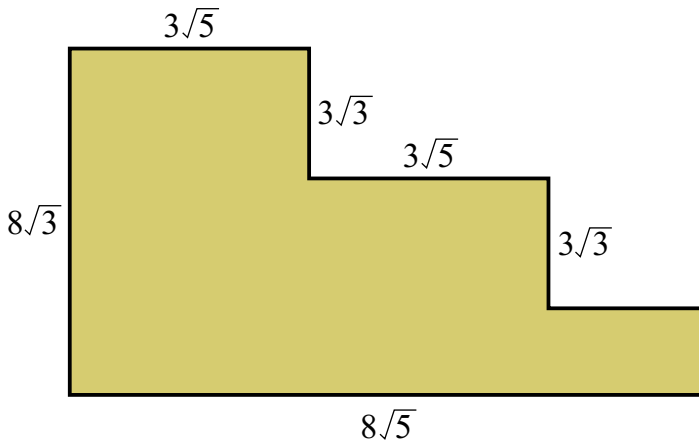
(x) $3\sqrt{20} + \sqrt{125} - 6\sqrt{5}$

(z) $6\sqrt{112} + 2\sqrt{63} - 5\sqrt{343}$

9. (a) Write an expression, in its simplest form, for the perimeter of this rectangle.
- (b) Write an expression, in its simplest form, for the area of this rectangle.



10. (a) Write an expression, in its simplest form, for the perimeter of the shape shown below.
- (b) Write an expression, in its simplest form, for the area of the shape shown below.



Surds and the Distributive Law

EXERCISE 3C

The Distributive Law

$a(b + c) = ab + ac$ (each term in the bracket is multiplied by the term outside the bracket)

Examples

Expand the brackets using the distributive law and simplify.

$$\begin{aligned} 1. \quad & \sqrt{3}(\sqrt{2} + \sqrt{5}) \\ &= \sqrt{3} \times \sqrt{2} + \sqrt{3} \times \sqrt{5} \\ &= \sqrt{6} + \sqrt{15} \end{aligned}$$

$$\begin{aligned} 2. \quad & 3\sqrt{6}(2\sqrt{2} + 4\sqrt{5}) \\ &= 3\sqrt{6} \times 2\sqrt{2} + 3\sqrt{6} \times 4\sqrt{5} \\ &= 6\sqrt{12} + 12\sqrt{30} \\ &= 6\sqrt{4 \times 3} + 12\sqrt{30} \\ &= 6\sqrt{4} \times \sqrt{3} + 12\sqrt{30} \\ &= 6 \times 2 \times \sqrt{3} + 12\sqrt{30} \\ &= 12\sqrt{3} + 12\sqrt{30} \end{aligned}$$

1. Expand and simplify where possible.

(a) $2(\sqrt{5} + \sqrt{3})$

(c) $\sqrt{3}(\sqrt{2} - \sqrt{11})$

(e) $\sqrt{5}(\sqrt{6} + \sqrt{5})$

(g) $\sqrt{3}(4 + \sqrt{3})$

(i) $\sqrt{2}(\sqrt{8} - \sqrt{22})$

(k) $\sqrt{3}(3\sqrt{3} + \sqrt{5})$

(m) $\sqrt{7}(2\sqrt{3} + \sqrt{14})$

(o) $\sqrt{5}(4\sqrt{3} - 3\sqrt{7})$

(q) $3\sqrt{2}(\sqrt{5} + \sqrt{7})$

(s) $-5\sqrt{7}(2 + 3\sqrt{7})$

(u) $3\sqrt{5}(2\sqrt{5} - 3\sqrt{7})$

(w) $5\sqrt{3}(3\sqrt{2} + 3\sqrt{5})$

(b) $3(\sqrt{7} - \sqrt{2})$

(d) $\sqrt{2}(\sqrt{5} + \sqrt{7})$

(f) $\sqrt{6}(\sqrt{2} - \sqrt{3})$

(h) $\sqrt{5}(\sqrt{10} - \sqrt{20})$

(j) $\sqrt{6}(\sqrt{2} + \sqrt{30})$

(l) $\sqrt{5}(2\sqrt{7} + \sqrt{3})$

(n) $\sqrt{2}(5\sqrt{8} + 3\sqrt{6})$

(p) $\sqrt{2}(3\sqrt{10} + 5\sqrt{6})$

(r) $4\sqrt{3}(\sqrt{3} + 2\sqrt{5})$

(t) $-7\sqrt{3}(2\sqrt{5} - \sqrt{12})$

(v) $2\sqrt{3}(3\sqrt{2} + 2\sqrt{8})$

(x) $3\sqrt{7}(2\sqrt{14} - 4\sqrt{7})$

The Distributive Law for Two Brackets

$$(a + b)(c + d)$$

Remember FOIL (First Outer Inner Last)

$$(a + b)(c + d) = ac + ad + bc + bd$$

Examples

Expand the brackets using the distributive law and simplify.

1. $(\sqrt{5} + 2\sqrt{3})(\sqrt{2} + \sqrt{5})$
 $= \sqrt{5} \times \sqrt{2} + \sqrt{5} \times \sqrt{5} + 2\sqrt{3} \times \sqrt{2} + 2\sqrt{3} \times \sqrt{5}$
 $= \sqrt{10} + \sqrt{25} + 2\sqrt{6} + 2\sqrt{15}$
 $= \sqrt{10} + 5 + 2\sqrt{6} + 2\sqrt{15}$
2. $(\sqrt{7} + 3\sqrt{3})(2\sqrt{3} - 5\sqrt{7})$
 $= \sqrt{7} \times 2\sqrt{3} + \sqrt{7} \times -5\sqrt{7} + 3\sqrt{3} \times 2\sqrt{3} + 3\sqrt{3} \times -5\sqrt{7}$
 $= 2\sqrt{21} - 5\sqrt{49} + 6\sqrt{9} - 15\sqrt{21}$
 $= 2\sqrt{21} - 5 \times 7 + 6 \times 3 - 15\sqrt{21}$
 $= -13\sqrt{21} - 35 + 18$
 $= -13\sqrt{21} - 17$

2. Expand and simplify.

- | | |
|--|--|
| (a) $(\sqrt{3} + \sqrt{2})(\sqrt{5} + \sqrt{2})$ | (b) $(4 + \sqrt{5})(3 + \sqrt{5})$ |
| (c) $(\sqrt{7} + \sqrt{3})(\sqrt{2} + \sqrt{7})$ | (d) $(\sqrt{3} + 6)(2 + \sqrt{3})$ |
| (e) $(5 + \sqrt{3})(4 - \sqrt{3})$ | (f) $(3 + \sqrt{2})(\sqrt{3} - \sqrt{2})$ |
| (g) $(\sqrt{3} - \sqrt{2})(\sqrt{2} + \sqrt{5})$ | (h) $(\sqrt{2} - \sqrt{5})(\sqrt{3} - \sqrt{2})$ |
| (i) $(\sqrt{3} - \sqrt{7})(\sqrt{7} - \sqrt{2})$ | (j) $(2 - \sqrt{5})(\sqrt{5} - 3)$ |
| (k) $(6 + \sqrt{3})(6 - \sqrt{3})$ | (l) $(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})$ |
| (m) $(2\sqrt{5} + \sqrt{3})(3\sqrt{5} + \sqrt{2})$ | (n) $(3\sqrt{3} + \sqrt{2})(2\sqrt{3} + \sqrt{2})$ |
| (o) $(3\sqrt{5} + \sqrt{7})(2\sqrt{5} - 2)$ | (p) $(4 - 3\sqrt{5})(2\sqrt{5} + 2)$ |
| (q) $(2\sqrt{5} - 3\sqrt{2})(3\sqrt{5} + 2\sqrt{2})$ | (r) $(3\sqrt{7} - 2\sqrt{5})(2\sqrt{7} - 3\sqrt{5})$ |
| (s) $(4\sqrt{3} + 2\sqrt{7})(3\sqrt{3} - 5\sqrt{7})$ | (t) $(2\sqrt{5} - 3\sqrt{2})(5\sqrt{2} + 7\sqrt{5})$ |
| (u) $(2\sqrt{3} + 3\sqrt{5})(2\sqrt{3} - 3\sqrt{5})$ | (v) $(5\sqrt{2} + 3\sqrt{7})(5\sqrt{2} - 3\sqrt{7})$ |
| (w) $(7\sqrt{2} + 5\sqrt{5})(7\sqrt{2} - 5\sqrt{5})$ | (x) $(9\sqrt{5} + 2\sqrt{2})(9\sqrt{5} - 2\sqrt{2})$ |

Rationalising the Denominator

When a fraction has a surd (irrational number) on the denominator it is usual to change the denominator to a rational number.

This is called ***rationalising the denominator***.

The method used to rationalise the denominator is to multiply the fraction by the surd on the denominator divided by itself.

This is the same as multiplying by 1.

Example 1 $\frac{2}{\sqrt{5}}$

The surd on the denominator is $\sqrt{5}$ so multiply by $\frac{\sqrt{5}}{\sqrt{5}}$.

$$\begin{aligned} & \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} \\ &= \frac{2\sqrt{5}}{\sqrt{25}} \\ &= \frac{2\sqrt{5}}{5} \end{aligned}$$

Example 2 $\frac{3\sqrt{7}}{\sqrt{2}}$

The surd on the denominator is $\sqrt{2}$ so multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$$\begin{aligned} & \frac{3\sqrt{7}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{3\sqrt{7} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} \\ &= \frac{3\sqrt{14}}{2} \end{aligned}$$

EXERCISE 3D

1. Write the following fractions with a rational denominator.

(a) $\frac{3}{\sqrt{7}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{4}{\sqrt{5}}$ (d) $\frac{5}{\sqrt{3}}$ (e) $\frac{2}{\sqrt{11}}$

(f) $\frac{2\sqrt{7}}{\sqrt{5}}$ (g) $\frac{3\sqrt{5}}{\sqrt{2}}$ (h) $\frac{5\sqrt{2}}{\sqrt{3}}$ (i) $\frac{4\sqrt{6}}{\sqrt{5}}$ (j) $\frac{3\sqrt{5}}{\sqrt{3}}$

2. Write the following fractions with a rational denominator and simplify.

Example $\frac{3\sqrt{8}}{\sqrt{3}}$

$$\begin{aligned} & \frac{3\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{3\sqrt{24}}{\sqrt{9}} \\ &= \frac{3\sqrt{4 \times 6}}{3} \\ &= \frac{3\sqrt{4} \times \sqrt{6}}{3} \\ &= \frac{\cancel{3} \times 2 \times \sqrt{6}}{\cancel{3}} \\ &= 2\sqrt{6} \end{aligned}$$

(a) $\frac{3\sqrt{5}}{\sqrt{10}}$ (b) $\frac{2\sqrt{6}}{\sqrt{8}}$ (c) $\frac{3\sqrt{3}}{\sqrt{6}}$ (d) $\frac{2\sqrt{10}}{\sqrt{6}}$ (e) $\frac{6\sqrt{12}}{\sqrt{8}}$

3. Write the following fractions with a rational denominator and simplify.

Example $\frac{5\sqrt{7}}{3\sqrt{5}}$

$$\begin{aligned} & \frac{5\sqrt{7}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\cancel{5}\sqrt{35}}{3 \times \cancel{5}} \\ &= \frac{\sqrt{35}}{3} \end{aligned}$$

(a) $\frac{6\sqrt{5}}{3\sqrt{2}}$ (b) $\frac{3\sqrt{7}}{2\sqrt{5}}$ (c) $\frac{2\sqrt{10}}{3\sqrt{3}}$ (d) $\frac{5\sqrt{7}}{3\sqrt{3}}$ (e) $\frac{5\sqrt{8}}{3\sqrt{2}}$

(f) $\frac{6\sqrt{7}}{5\sqrt{3}}$ (g) $\frac{8\sqrt{5}}{5\sqrt{2}}$ (h) $\frac{3\sqrt{7}}{7\sqrt{3}}$ (i) $\frac{6\sqrt{5}}{5\sqrt{3}}$ (j) $\frac{7\sqrt{8}}{4\sqrt{2}}$

4. Write the following fractions with a rational denominator and simplify.

Example $\frac{3\sqrt{2}+\sqrt{7}}{2\sqrt{3}}$

$$\begin{aligned} & \frac{3\sqrt{2}+\sqrt{7}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}(3\sqrt{2}+\sqrt{7})}{2\sqrt{3} \times \sqrt{3}} \\ &= \frac{3\sqrt{6}+\sqrt{21}}{2 \times 3} \\ &= \frac{3\sqrt{6}+\sqrt{21}}{6} \end{aligned}$$

(a) $\frac{2\sqrt{3}+\sqrt{5}}{3\sqrt{3}}$

(b) $\frac{5-3\sqrt{2}}{2\sqrt{7}}$

(c) $\frac{5\sqrt{7}+\sqrt{3}}{5\sqrt{11}}$

(d) $\frac{5\sqrt{5}-3\sqrt{2}}{7\sqrt{2}}$

(e) $\frac{3\sqrt{2}+7\sqrt{11}}{3\sqrt{2}}$

(f) $\frac{5\sqrt{7}-3\sqrt{5}}{2\sqrt{5}}$

(g) $\frac{8\sqrt{2}+3\sqrt{5}}{4\sqrt{3}}$

(h) $\frac{4\sqrt{5}-3\sqrt{7}}{2\sqrt{7}}$

(i) $\frac{7\sqrt{10}+3\sqrt{8}}{2\sqrt{2}}$

5. Write the following fractions with a rational denominator and simplify.

When the denominator consists of two terms the fraction can be rationalised by multiplying the numerator and denominator by the same expression with the opposite sign (this is called the **conjugate**).

Example 1 $\frac{3}{3+\sqrt{2}}$ The conjugate of $(3+\sqrt{2})$ is $(3-\sqrt{2})$

$$\begin{aligned} &= \frac{3}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} \\ &= \frac{3(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})} \\ &= \frac{9-3\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2} \\ &= \frac{9-3\sqrt{2}}{7} \end{aligned}$$

Example 2 $\frac{2\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{2}}$ The conjugate of $(\sqrt{5} - \sqrt{2})$ is $(\sqrt{5} + \sqrt{2})$

$$\begin{aligned}
 &= \frac{2\sqrt{3} + 3\sqrt{5}}{\sqrt{5} - \sqrt{2}} \times \frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} + \sqrt{2}} \\
 &= \frac{(2\sqrt{3} + 3\sqrt{5})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})} \\
 &= \frac{2\sqrt{3} \times \sqrt{5} + 2\sqrt{3} \times \sqrt{2} + 3\sqrt{5} \times \sqrt{5} + 3\sqrt{5} \times \sqrt{2}}{\sqrt{5} \times \sqrt{5} + \sqrt{5} \times \sqrt{2} - \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{2}} \\
 &= \frac{2\sqrt{15} + 2\sqrt{6} + 3\sqrt{25} + 3\sqrt{10}}{\sqrt{25} + \sqrt{10} - \sqrt{10} - \sqrt{4}} \\
 &= \frac{2\sqrt{15} + 2\sqrt{6} + 3 \times 5 + 3\sqrt{10}}{5 - 2} \\
 &= \frac{2\sqrt{15} + 2\sqrt{6} + 15 + 3\sqrt{10}}{3}
 \end{aligned}$$

(a) $\frac{5}{5 + \sqrt{3}}$

(b) $\frac{2}{\sqrt{7} - 2}$

(c) $\frac{4}{9 + \sqrt{2}}$

(d) $\frac{\sqrt{7}}{5 + \sqrt{2}}$

(e) $\frac{\sqrt{10}}{3 - \sqrt{2}}$

(f) $\frac{\sqrt{11}}{7 + \sqrt{5}}$

(g) $\frac{5}{\sqrt{7} - \sqrt{3}}$

(h) $\frac{8}{\sqrt{11} + \sqrt{5}}$

(i) $\frac{1}{\sqrt{13} - \sqrt{7}}$

(j) $\frac{2 - \sqrt{3}}{\sqrt{5} - \sqrt{3}}$

(k) $\frac{\sqrt{3} + 4}{\sqrt{7} + \sqrt{2}}$

(l) $\frac{8 - \sqrt{7}}{\sqrt{11} + \sqrt{5}}$

(m) $\frac{\sqrt{7} + \sqrt{5}}{\sqrt{5} - \sqrt{3}}$

(n) $\frac{\sqrt{2} - \sqrt{3}}{\sqrt{7} + \sqrt{5}}$

(o) $\frac{3\sqrt{3} - \sqrt{5}}{\sqrt{11} - \sqrt{7}}$

(p) $\frac{2\sqrt{2} + 3\sqrt{3}}{\sqrt{7} - \sqrt{3}}$

(q) $\frac{4\sqrt{2} + 3\sqrt{5}}{\sqrt{5} - \sqrt{2}}$

(r) $\frac{6\sqrt{3} - 9\sqrt{5}}{\sqrt{5} + \sqrt{3}}$

6. Write the following fractions with a rational denominator and simplify.

Example

$$\begin{aligned}
 & \frac{2\sqrt{5} + 3\sqrt{3}}{3\sqrt{5} - 2\sqrt{2}} \quad \text{The conjugate of } (3\sqrt{5} - 2\sqrt{2}) \text{ is } (3\sqrt{5} + 2\sqrt{2}) \\
 &= \frac{2\sqrt{5} + 3\sqrt{3}}{3\sqrt{5} - 2\sqrt{2}} \times \frac{3\sqrt{5} + 2\sqrt{2}}{3\sqrt{5} + 2\sqrt{2}} \\
 &= \frac{(2\sqrt{5} + 3\sqrt{3})(3\sqrt{5} + 2\sqrt{2})}{(3\sqrt{5} - 2\sqrt{2})(3\sqrt{5} + 2\sqrt{2})} \\
 &= \frac{2\sqrt{5} \times 3\sqrt{5} + 2\sqrt{5} \times 2\sqrt{2} + 3\sqrt{3} \times 3\sqrt{5} + 3\sqrt{3} \times 2\sqrt{2}}{3\sqrt{5} \times 3\sqrt{5} + 3\sqrt{5} \times 2\sqrt{2} - 2\sqrt{2} \times 3\sqrt{5} - 2\sqrt{2} \times 2\sqrt{2}} \\
 &= \frac{6\sqrt{25} + 4\sqrt{10} + 9\sqrt{15} + 6\sqrt{6}}{9\sqrt{25} + 6\sqrt{10} - 6\sqrt{10} - 4\sqrt{4}} \\
 &= \frac{6 \times 5 + 4\sqrt{10} + 9\sqrt{15} + 6\sqrt{6}}{9 \times 5 - 4 \times 2} \\
 &= \frac{30 + 4\sqrt{10} + 9\sqrt{15} + 6\sqrt{6}}{45 - 8} \\
 &= \frac{30 + 4\sqrt{10} + 9\sqrt{15} + 6\sqrt{6}}{37}
 \end{aligned}$$

(a) $\frac{3 + \sqrt{5}}{3\sqrt{5} + 2\sqrt{2}}$

(b) $\frac{6 - \sqrt{2}}{7\sqrt{2} - 3\sqrt{3}}$

(c) $\frac{\sqrt{5} + 9}{3\sqrt{5} + 2\sqrt{3}}$

(d) $\frac{\sqrt{2} - \sqrt{3}}{5\sqrt{5} - 2\sqrt{3}}$

(e) $\frac{\sqrt{3} + \sqrt{5}}{2\sqrt{7} + 2\sqrt{3}}$

(f) $\frac{\sqrt{5} + \sqrt{11}}{2\sqrt{11} + 2\sqrt{5}}$

(g) $\frac{2\sqrt{7} + 3\sqrt{5}}{3\sqrt{5} - 2\sqrt{3}}$

(h) $\frac{2\sqrt{2} - 3\sqrt{3}}{5\sqrt{3} + 3\sqrt{2}}$

(i) $\frac{3\sqrt{3} - 4\sqrt{5}}{2\sqrt{5} - \sqrt{3}}$

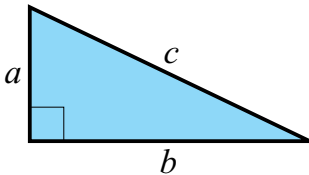
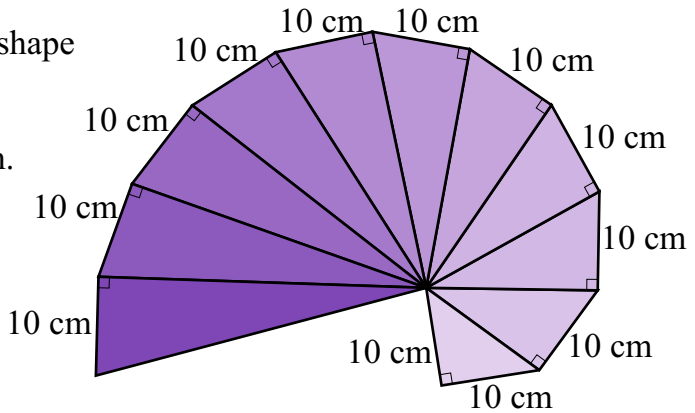
(j) $\frac{2\sqrt{2} + 3\sqrt{3}}{3\sqrt{7} - 4\sqrt{3}}$

(k) $\frac{4\sqrt{2} + 3\sqrt{5}}{4\sqrt{5} - 3\sqrt{2}}$

(l) $\frac{6\sqrt{3} - 9\sqrt{5}}{8\sqrt{5} + 5\sqrt{3}}$

PROBLEM SOLVING

All of the triangles in this shape are right-angled triangles. The shorter sides of the smallest triangle are 10 cm. The outer side of all the triangles is 10 cm. Pythagoras' Theorem can be used to find the length of the other sides.



$$c = \sqrt{a^2 + b^2}$$

A window maker wants to include this shape in a stained glass window. Find the total length (in cm) of the framing metal needed for this stained glass shape. Give answer correct to one decimal place.

PUZZLE

Why can't you have a discussion with surds?

Simplify the following surds and write the letters in spaces to find the answer to this riddle.

$\frac{10}{\sqrt{2}}$
H

$\frac{70}{2\sqrt{7}}$
A

$\sqrt{27}$
E

$\sqrt{8}$
T

$\frac{15\sqrt{21}}{5\sqrt{3}}$
I

$6\sqrt{5} - 4\sqrt{5}$
Y

$\sqrt{245}$
O

$10\sqrt{3} - 8\sqrt{3}$
L

$3\sqrt{2} + 4\sqrt{2}$
R

$2\sqrt{2}$

$5\sqrt{2}$

$3\sqrt{3}$

$2\sqrt{5}$

$5\sqrt{7}$

$7\sqrt{2}$

$3\sqrt{3}$

$\frac{18}{\sqrt{3}}$
N

$3\sqrt{7}$

$7\sqrt{2}$

$7\sqrt{2}$

$5\sqrt{7}$

$2\sqrt{2}$

$3\sqrt{7}$

$7\sqrt{5}$

$6\sqrt{3}$

$5\sqrt{7}$

$2\sqrt{3}$

CHAPTER REVIEW

1. Which of the following are *irrational* numbers.

$$0.\dot{8} \quad \sqrt{729} \quad \sqrt{89} \quad \frac{\pi}{5} \quad 0.3\overline{26} \quad \frac{5}{9} \quad \sqrt{124} \quad 0.876\dot{5}$$

2. Simplify the following surds.

$$(a) \sqrt{50} \quad (b) \sqrt{96} \quad (c) 3\sqrt{27} \quad (d) 7\sqrt{45}$$

3. Write the following terms as entire surds.

$$(a) 5\sqrt{5} \quad (b) 3\sqrt{6} \quad (c) 10\sqrt{3} \quad (d) 4\sqrt{7}$$

4. Simplify the following expressions.

$$\begin{array}{lll} (a) \sqrt{6} \times 3\sqrt{5} & (b) 2\sqrt{6} \times 3\sqrt{10} & (c) 5\sqrt{15} \times 2\sqrt{6} \\ (d) 3\sqrt{20} \times 2\sqrt{18} & (e) \frac{6\sqrt{8}}{2\sqrt{2}} & (f) \frac{10\sqrt{48}}{2\sqrt{28}} \end{array}$$

5. Simplify the following expressions.

$$\begin{array}{ll} (a) 3\sqrt{5} + 2\sqrt{5} & (b) 6\sqrt{7} - 3\sqrt{7} \\ (c) 8\sqrt{2} - 5\sqrt{2} - 6\sqrt{2} & (d) 5\sqrt{3} + 7\sqrt{7} - 2\sqrt{3} - \sqrt{7} \\ (e) \sqrt{3} + \sqrt{12} - \sqrt{48} & (f) 2\sqrt{20} + 2\sqrt{45} - 3\sqrt{80} \end{array}$$

6. Expand and simplify where possible.

$$\begin{array}{ll} (a) 5(\sqrt{3} + \sqrt{5}) & (b) \sqrt{5}(\sqrt{5} - \sqrt{2}) \\ (c) \sqrt{3}(2\sqrt{5} + 3\sqrt{3}) & (d) 3\sqrt{5}(2\sqrt{7} - 3\sqrt{5}) \\ (e) (\sqrt{3} + \sqrt{2})(\sqrt{2} - \sqrt{5}) & (f) (3\sqrt{7} - 5\sqrt{2})(2\sqrt{7} + 3\sqrt{2}) \end{array}$$

7. Write the following fractions with rational denominators and simplify.

$$\begin{array}{lll} (a) \frac{2}{\sqrt{3}} & (b) \frac{3\sqrt{2}}{\sqrt{7}} & (c) \frac{5\sqrt{7}}{3\sqrt{11}} \\ (d) \frac{2\sqrt{5} - 3\sqrt{2}}{5\sqrt{2}} & (e) \frac{3\sqrt{2} + 2\sqrt{11}}{5\sqrt{2}} & (f) \frac{3\sqrt{5}}{4 + \sqrt{5}} \\ (g) \frac{2\sqrt{2} + 3\sqrt{5}}{\sqrt{7} + \sqrt{5}} & (h) \frac{4\sqrt{5} - 3\sqrt{7}}{2\sqrt{7} - 5} & (i) \frac{7\sqrt{5} + 3\sqrt{2}}{5\sqrt{2} + 3\sqrt{5}} \end{array}$$

Indices

Indices

Example

base number

index, power or exponent

exponent form or index form

factor form

basic numeral

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

EXERCISE 4A

1. Write the following in exponent form.

Examples

1. $5 \times 5 \times 5$
2. $a \times a \times a \times a \times a \times a$
3. $-6 \times -6 \times -6 \times -6 \times -6$
4. 5 to the power of 8

Answers

1. 5^3
2. a^6
3. $(-6)^5$
4. 5^8

- | | |
|---|---|
| (a) $7 \times 7 \times 7 \times 7$ | (b) $9 \times 9 \times 9 \times 9 \times 9 \times 9$ |
| (c) $2 \times 2 \times 2 \times 2 \times 2$ | (d) $12 \times 12 \times 12 \times 12 \times 12$ |
| (e) $-3 \times -3 \times -3 \times -3$ | (f) $-8 \times -8 \times -8 \times -8 \times -8$ |
| (g) $y \times y \times y$ | (h) $a \times a \times a \times a \times a \times a \times a \times a \times a$ |
| (i) $m \times m$ | (j) $-p \times -p \times -p \times -p \times -p \times -p$ |
| (k) 5 to the power of 7 | (l) 7 to the power of 4 |
| (m) 10 to the power of 15 | (n) k to the power of 12 |
| (o) a to the power of 10 | (p) x to the power of 3 |

2. Write the following in factor form.

Examples

1. 2^4

2. x^5

3. $5a^3b^2$

Answers

1. $2 \times 2 \times 2 \times 2$

2. $x \times x \times x \times x \times x$

3. $5 \times a \times a \times a \times b \times b$

(a) 3^6

(b) 5^7

(c) 10^4

(d) 8^3

(e) a^4

(f) x^7

(g) y^5

(h) b^9

(i) $7a^2b^5$

(j) $3x^4y^3$

(k) $2m^2n$

(l) $3a^2b^3c^4$

3. Write the following in exponent form.

Example

$$\begin{aligned}
 &3 \times a \times a \times 2 \times b \times a \times b \times a \\
 &= 3 \times 2 \times a \times a \times a \times a \times b \times b \\
 &= \mathbf{6a^4b^2}
 \end{aligned}$$

(a) $2 \times a \times 2 \times a \times b \times b \times b$

(b) $3 \times x \times 5 \times x \times y \times x \times y \times x \times y$

(c) $m \times m \times n \times 4 \times n \times 2 \times n \times n \times m \times 2 \times n \times m$

(d) $5 \times a \times 2 \times c \times b \times c \times a \times b \times 3 \times a \times b \times c \times c \times c$

(e) $3x \times 2y \times -2x \times 5y \times -2x$

4. Evaluate the following.

Examples

1. 5^3

$= 5 \times 5 \times 5$

$= \mathbf{125}$

2. $(-4)^5$

$= -4 \times -4 \times -4 \times -4 \times -4$

$= \mathbf{-1024}$

3. $2^3 + 3^2$

$= 2 \times 2 \times 2 + 3 \times 3$

$= \mathbf{17}$

(a) 10^4

(b) 8^3

(c) 7^5

(d) $(-5)^4$

(e) -6^5

(f) $(-2)^9$

(g) $6^4 - 5^4$

(h) $(-3)^3 + (-2)^6$

(i) $8^1 + 8^2 + 8^3$

(j) $1^7 - 1^8$

(k) $(-2)^7 - (-4)^6 + (-8)^2$

(l) $5^5 - (-6)^6 + (-7)^7$

5. Evaluate the following, giving answers correct to one decimal place.

(a) $3.7^4 + 5.2^5$

(b) $5.6^3 + (-5.7)^3$

(c) $2.75^6 - 3.15^5$

(d) $\frac{(-8.85)^7 + 9.3^7}{5.6^5}$

(e) $\frac{13.73^7 + (-9.6)^9}{156.5^4}$

(f) $\frac{(4.3^2 + 1.56^7)^5}{(4.1^4 + 2.46^6)^3}$

6. Find the value of x in the following equations.

(a) $2^x = 16$

(b) $3^x = 2187$

(c) $6^x = 1296$

(d) $2.5^x = 15.625$

(e) $4^x = 65\,536$

(f) $6^x = 46\,656$

Index Laws

EXERCISE 4B

Examples

Index Law 1

$$a^m \times a^n = a^{m+n}$$

$$\begin{aligned} 1. \quad x^4 \times x^3 \\ &= x^{(4+3)} \\ &= x^7 \end{aligned}$$

$$\begin{aligned} 2. \quad m^6 \times m \\ &= m^6 \times m^1 \\ &= m^{(6+1)} \\ &= m^7 \end{aligned}$$

$$\begin{aligned} 3. \quad 3a^3b^5 \times 2ab^4 \\ &= 6a^4b^9 \end{aligned}$$

$$\begin{aligned} 4. \quad 5x^3y^2 \times x^5y^2 \times 4xy^3 \\ &= 20x^9y^7 \end{aligned}$$

1. Simplify the following.

(a) $a^5 \times a^3$

(b) $n^4 \times n^4$

(c) $y^7 \times y^4$

(d) $b^6 \times b$

(e) $m^3 \times m^2 \times m^4$

(f) $x^4 \times x^5 \times x$

(g) $a \times a^6 \times a^7$

(h) $x^{10} \times x^5 \times x$

(i) $y^2 \times y \times y^5 \times y$

2. Simplify the following.

(a) $3a^5 \times 6a^3$

(b) $2x^4 \times 4x^7$

(c) $2m^3 \times 3m \times m^6$

(d) $3n^6 \times 2n^5 \times 2n^7$

(e) $2a^2b^2 \times a^3b^5$

(f) $xy \times 6x^3y^6$

(g) $4a^5y^6 \times 3a^2y$

(h) $2a^3b^4c \times 7a^4b^5c$

(i) $3m^5n^2 \times 2mn \times 2m^2n^5$

(j) $6x^5y^3z^4 \times 2x^3yz^2 \times xy^2z^2$

(k) $3a^4b^3c \times 8a^6bc^7$

(l) $5m^5n^3 \times 4m^6n^4 \times 2n \times 3m^6$

EXERCISE 4C

Examples

$$\begin{aligned}
 1. \quad x^6 \div x^4 \\
 &= x^{6-4} \\
 &= x^2
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{m^7}{m^4} \\
 &= m^{7-4} \\
 &= m^3
 \end{aligned}$$

Index Law 2

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\begin{aligned}
 3. \quad 8x^6y^7 \div 4x^5y \\
 &= \frac{8}{4} x^{6-5} y^{7-1} \\
 &= 2xy^6
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{18m^8n^6}{12m^3n^4} \\
 &= \frac{\overset{3}{\cancel{18}}}{\underset{2}{\cancel{12}}} m^{8-3} n^{6-4} \\
 &= \frac{3}{2} m^5 n^2
 \end{aligned}$$

1. Simplify the following.

(a) $x^5 \div x^2$

(b) $a^8 \div a^5$

(c) $n^9 \div n^3$

(d) $m^{10} \div m^3$

(e) $b^{11} \div b^8$

(f) $y^6 \div y$

(g) $p^7 \div p$

(h) $w^4 \div w^3$

(i) $\frac{m^{12}}{m^5}$

(j) $\frac{x^{13}}{x^4}$

(k) $\frac{n^{15}}{n}$

(l) $\frac{m^{11}}{m^{10}}$

2. Simplify the following.

(a) $10x^6y^7 \div 5x^3y^2$

(b) $16m^8n^9 \div 4m^5n^3$

(c) $24a^7b^5 \div 8a^4b^2$

(d) $16m^7n^6 \div 12m^5n^3$

(e) $8x^{10}y^8 \div 12x^7y^5$

(f) $20a^{11}b^7 \div 15a^8b^4$

(g) $12x^6y^6 \div 8x^4y$

(h) $20m^8n^{11} \div 24mn^{10}$

(i) $30a^8b^{12} \div 24a^5b$

(j) $\frac{18x^8y^4}{9x^5y^3}$

(k) $\frac{24m^8n^{12}}{20mn}$

(l) $\frac{24a^{10}b^{10}}{28a^9b}$

(m) $\frac{36m^8n^{11}}{12m^6n^5}$

(n) $\frac{32x^{15}y^{10}}{48x^9y^9}$

(o) $\frac{42m^7n^9}{14m^6n^8}$

(p) $\frac{54h^7i^{14}}{81h^6i^9}$

(q) $\frac{27a^7b^5}{54ab^5}$

(r) $\frac{48a^8b^7c^6}{36a^7b^7c^5}$

EXERCISE 4D

Index Law 3

$$(a^m)^n = a^{m \times n}$$

Examples

$$\begin{aligned} 1. (x^3)^4 &= x^{3 \times 4} \\ &= x^{12} \end{aligned}$$

$$\begin{aligned} 2. (2m^2)^3 &= 2^3 m^{2 \times 3} \\ &= 8m^6 \end{aligned}$$

1. Simplify the following.

(a) $(m^2)^6$

(b) $(x^3)^5$

(c) $(a^4)^6$

(d) $(c^6)^3$

(e) $(p^5)^6$

(f) $(b^7)^3$

(g) $(n^9)^4$

(h) $(y^4)^5$

2. Expand the brackets in the following.

(a) $(3x^2)^2$

(b) $(2m^3)^4$

(c) $(3m^5)^3$

(d) $(4n^6)^2$

(e) $(-6c^7)^2$

(f) $(-2n^4)^5$

(g) $(-3a^4)^3$

(h) $-(3u^6)^4$

(i) $(6a^3)^3$

(j) $-(5y^5)^3$

(k) $-(8c^4)^2$

(l) $-(-4y^3)^5$

EXERCISE 4E

Index Law 4

$$\begin{aligned} (a \times b)^n &= (ab)^n \\ &= a^n b^n \end{aligned} \quad \begin{aligned} (a \div b)^n &= \left(\frac{a}{b}\right)^n \\ &= \frac{a^n}{b^n} \end{aligned}$$

Examples

$$\begin{aligned} 1. (x^3 \times y^5)^4 &= (x^3 y^5)^4 \\ &= (x^3)^4 (y^5)^4 \\ &= x^{12} y^{20} \end{aligned}$$

$$\begin{aligned} 2. (2m^2 n^5)^3 &= 2^3 (m^2)^3 (n^5)^3 \\ &= 8m^6 n^{15} \end{aligned}$$

$$\begin{aligned} 3. (a^3 \div b^6)^5 &= \left(\frac{a^3}{b^6}\right)^5 \\ &= \frac{(a^3)^5}{(b^6)^5} \\ &= \frac{a^{15}}{b^{30}} \end{aligned}$$

$$\begin{aligned} 4. \left(\frac{3m^2 n^3}{5x^3 y^5}\right)^2 &= \frac{3^2 (m^2)^2 (n^3)^2}{5^2 (x^3)^2 (y^5)^2} \\ &= \frac{9m^4 n^6}{25x^6 y^{10}} \end{aligned}$$

1. Expand the brackets and simplify the following.

(a) $(a^2 \times b^3)^4$

(b) $(x^4 \times y^7)^2$

(c) $(p^2 \times q^9)^4$

(d) $(u^5 v^3)^6$

(e) $(m^5 n^8 o^6)^5$

(f) $(a^6 b^3 c^9)^3$

(g) $(3x^3 y^5)^2$

(h) $(4a^3 b)^2$

(i) $(-5mn)^6$

(j) $(8a^5 b^3)^2$

(k) $(-2a^4 bc^3)^6$

(l) $(9p^4 q^7 r^3)^2$

(m) $-(2m^3 n^6)^3$

(n) $-(3x^4 y^5)^4$

(o) $-(5u^6 v^7)^3$

(p) $-(-a^3 b^4)^3$

(q) $-(-4m^6 n^3)^4$

(r) $-(-2a^3 b^2)^7$

(s) $(6x^4 yz^5)^3$

(t) $(7a^4 b^3 c)^4$

(u) $(5p^3 q^7 r^8)^5$

2. Expand the brackets and simplify the following.

(a) $(a^5 \div b^2)^3$

(b) $(m^7 \div n^3)^2$

(c) $(x^7 \div y^5)^3$

(d) $\left(\frac{m^5}{n^4}\right)^2$

(e) $\left(\frac{a^6}{b^7}\right)^3$

(f) $\left(\frac{e^4}{r^2}\right)^5$

(g) $\left(\frac{m^3 n^4}{p^4 q^3}\right)^4$

(h) $\left(\frac{a^6 b^3}{x^2 y^8}\right)^2$

(i) $\left(\frac{m^5 p^3}{n^4 q^2}\right)^6$

(j) $\left(\frac{2a^3 b^4}{m^2 n^5}\right)^3$

(k) $\left(\frac{x^4}{3y^3 z^5}\right)^3$

(l) $\left(\frac{4m^2 n^5}{p^5 q^4}\right)^2$

(m) $\left(\frac{4m^3 n^6}{3a^2 b}\right)^3$

(n) $\left(\frac{5x^3 y^4}{6z^7}\right)^2$

(o) $\left(\frac{3a^4 b^9}{7x^5 y^7}\right)^2$

(p) $\left(\frac{2abc^3}{3x^2 y^3 z^4}\right)^3$

(q) $\left(\frac{5}{m^2 n^3 p^2}\right)^4$

(r) $\left(\frac{7mn^7}{10p^4}\right)^3$

EXERCISE 4F

Examples

Index Law 5

$$a^0 = 1$$

$$1. \quad 4^0 = 1$$

$$2. \quad (a^2 b^3)^0 = 1$$

$$\begin{aligned} 3. \quad 3x^0 &= 3 \times x^0 \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

1. Simplify the following.

(a) y^0

(b) 5^0

(c) p^0

(d) 68^0

(e) $(5m)^0$

(f) $(3x)^0$

(g) $(7ab)^0$

(h) $(6xyz)^0$

2. Simplify the following.

- (a) $5m^0$ (b) $3a^0$ (c) $4(3c^3)^0$ (d) $2a(3b^2)^0$
 (e) $3^0 + 8^0$ (f) $2x^0 + (4x)^0$ (g) $(3m^0)^2$ (h) $5(n^5)^0$
 (i) $4x^0 - (2x)^0$ (j) $5x^2y^0 + 2(xy^0)^2$ (k) $4x^0y^0z^0$ (l) $(3c^0)^2 - 2c^0$

3. Simplify the following.

- (a) $\frac{m^6}{m^6}$ (b) $\frac{x^2z^5}{x^2z^3}$
 (c) $\frac{6m^5n^4}{3m^2n^4}$ (d) $\frac{8a^4b^7}{10a^4b}$
 (e) $\frac{12u^3v^3}{6u^3v^3}$ (f) $\frac{8m^8n^4}{12m^7n^4}$
 (g) $\frac{18x^4y^3z^5}{24x^4yz^4}$ (h) $\frac{20a^5b^4c}{10a^4b^3c}$

Example

$$\begin{aligned} & \frac{4m^5n^3}{3m^3n^3} \\ &= \frac{4}{3} m^{5-3} n^{3-3} \\ &= \frac{4}{3} m^2 n^0 \\ &= \frac{4}{3} m^2 \times 1 \\ &= \frac{4}{3} m^2 \end{aligned}$$

EXERCISE 4G

Examples

1. x^{-5}
 $= \frac{1}{x^5}$

2. $5n^{-6}$
 $= \frac{5}{n^6}$

3. $2m^4n^{-3}$
 $= \frac{2m^4}{n^3}$

Index Law 6

$$a^{-m} = \frac{1}{a^m}$$

4. $\frac{2}{3} x^2 y^{-4}$
 $= \frac{2x^2}{3y^4}$

5. $\frac{1}{6} a^3 b^{-5} c^{-2}$
 $= \frac{a^3}{6b^5c^2}$

1. Write the following with positive indices.

- (a) x^{-8} (b) 3^{-4} (c) a^{-2} (d) e^{-10} (e) c^{-1}
 (f) $3m^{-4}$ (g) xy^{-2} (h) $4mn^{-7}$ (i) $6x^2y^{-3}$ (j) $8a^{-2}b^9$

2. Write the following with positive indices.

(a) $3m^2n^{-6}$

(b) $7a^3b^{-5}$

(c) $a^4b^3c^{-6}$

(d) $6x^{-5}y^3$

(e) $\frac{3}{4}m^2n^{-3}$

(f) $\frac{5}{7}a^3b^{-4}$

(g) $\frac{1}{2}x^{-4}y^3$

(h) $\frac{1}{3}x^3yz^{-2}$

(i) $\frac{4}{9}a^5b^{-3}c$

(j) $\frac{2}{7}p^4q^{-3}r^{-2}$

(k) $\frac{3}{8}a^{-2}b^{-3}c^{-2}$

(l) $\frac{1}{5}v^{-3}w^2x^{-7}$

Using All The Index Laws

EXERCISE 4H

1. Simplify the following.

Examples

$$\begin{aligned} 1. \quad & \frac{x^3 \times x^6}{x^4} \\ &= \frac{x^9}{x^4} \\ &= x^5 \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{4m \times 6m^5}{12m^2} \\ &= \frac{24m^6}{12m^2} \\ &= 2m^4 \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{(2x^3)^3}{6x^4} \\ &= \frac{2^3x^9}{6x^4} \\ &= \frac{8x^9}{6x^4} \\ &= \frac{4x^5}{3} \end{aligned}$$

(a) $\frac{x^5 \times x^6}{x^3}$

(b) $\frac{m^6 \times m^2}{m}$

(c) $\frac{c^7 \times c}{c^5}$

(d) $\frac{a^2b^3 \times a^6b^4}{a^3b^2}$

(e) $\frac{m^5n^3 \times m^4n^6}{m^4n}$

(f) $\frac{6x^2 \times 2x^5}{4x^3}$

(g) $\frac{5m^4 \times 4m^3}{10m^5}$

(h) $\frac{3x^7 \times 6x^6}{2x^4}$

(i) $\frac{8c^6 \times 3c}{16c^4}$

(j) $\frac{b^3 \times 2b^4}{8b^5}$

(k) $\frac{4x \times 5x^3}{12x^2}$

(l) $\frac{9n^2 \times 4n^4}{6n^6}$

(m) $\frac{(4x^3)^2}{2x^3}$

(n) $\frac{(3m^4)^2}{6m^5}$

(o) $\frac{(2x^4)^4}{10x}$

2. Simplify the following.

Examples

$$\begin{aligned}
 1. \quad & \frac{(x^2y^3)^2 \times (x^3y)^3}{(x^3y^2)^2 \times x^4y^5} \\
 &= \frac{x^4y^6 \times x^9y^3}{x^6y^4 \times x^4y^5} \\
 &= \frac{x^{13}y^9}{x^{10}y^9} \\
 &= x^3y^0 \\
 &= x^3
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{3(x^4y^3)^3}{(2x^2y)^3} \times \frac{4(x^5y^2)^4}{(x^3y^2)^2} \\
 &= \frac{3x^{12}y^9}{8x^6y^3} \times \frac{4x^{20}y^8}{x^6y^4} \\
 &= \frac{12x^{32}y^{17}}{8x^{12}y^7} \\
 &= \frac{3}{2} x^{20}y^{10}
 \end{aligned}$$

$$(a) \quad \frac{(x^4y^2)^3 \times (xy^5)^3}{(x^2y)^4 \times x^2y^7}$$

$$(b) \quad \frac{m^5n^8 \times (m^4n^2)^3}{(m^2n^4)^2 \times (m^6n^3)^2}$$

$$(c) \quad \frac{(a^5b^3)^4 \times (a^3b^2)^3}{(a^2b^3)^2 \times (a^4b)^3}$$

$$(d) \quad \frac{3(x^4y^3)^2 \times (4x^3y^6)^2}{2(xy^2)^2 \times 6x^2y^5}$$

$$(e) \quad \frac{(3m^5n^3)^2 \times 8(m^4n^2)^3}{2(m^3n^2)^2 \times (3m^5n^2)^3}$$

$$(f) \quad \frac{(3x^5y^3)^3 \times (4x^3y^6)^2}{2(x^6y^2)^2 \times (6x^2y^5)^2}$$

$$(g) \quad \frac{(x^2y^3)^4}{(3xy^2)^2} \times \frac{6(x^3y^6)^3}{(2x^3y^5)^2}$$

$$(h) \quad \frac{(3m^6)^2}{(2m^3)^3} \times \frac{4(n^5)^2}{3(mn^2)^3}$$

$$(i) \quad \frac{6a^3b^5}{(2a^2b^3)^4} \times \frac{(4a^5b^6)^2}{(3a^2b)^2}$$

$$(j) \quad \frac{(10x^3)^2}{(4y^2)^3} \times \frac{(8y^5)^2}{(5x)^2}$$

$$(k) \quad \frac{4(m^3n^2)^4}{(3m^2n)^3} \times \frac{(9m^3n^6)^2}{(2mn^3)^5}$$

$$(l) \quad \frac{(2a^3b^4)^3}{4(a^5b^{10})^3} \times \frac{(3a^2b^5)^4}{(9ab)^2}$$

3. Simplify the following.

Examples

$$\begin{aligned}
 1. \quad & \frac{3a^2}{5} \div \frac{9a}{10} \\
 &= \frac{3a^2}{5} \times \frac{10}{9a} \\
 &= \frac{30a^2}{45a} \\
 &= \frac{2a}{3}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{(2mn^4)^2}{(3m^2n^3)^2} \div \frac{(6m^2n^3)^2}{(3m^2n^4)^3} \\
 &= \frac{4m^2n^8}{9m^4n^6} \div \frac{36m^4n^6}{27m^6n^{12}} \\
 &= \frac{4m^2n^8}{9m^4n^6} \times \frac{27m^6n^{12}}{36m^4n^6} \\
 &= \frac{1}{1} \frac{4m^2n^8}{9m^4n^6} \times \frac{1}{3} \frac{27m^6n^{12}}{36m^4n^6} \\
 &= \frac{1m^8n^{20}}{3m^8n^{12}} \\
 &= \frac{n^8}{3}
 \end{aligned}$$

$$(a) \quad \frac{4x^4}{3} \div \frac{28x}{15}$$

$$(b) \quad \frac{15c^7}{8} \div \frac{5c^3}{16}$$

$$(c) \quad \frac{20}{9a^3} \div \frac{15}{18a^6}$$

$$(d) \quad \frac{12a^5}{9b^3} \div \frac{4}{18b^3}$$

$$(e) \quad \frac{7x^7}{8y^3} \div \frac{21x}{4y^5}$$

$$(f) \quad \frac{30x^4}{25y^3} \div \frac{6x^4}{5y^3}$$

$$(g) \quad \frac{6x^3y^5}{(5x^2y^4)^2} \div \frac{3x^8y^5}{25(x^5y^5)^2}$$

$$(h) \quad \frac{(2x^2y^3)^5}{(3xy^2)^2} \div \frac{(4xy^2)^3}{(6x^3y)^2}$$

$$(i) \quad \frac{(8a^3b^7)^3}{(5a^4b^5)^3} \div \frac{(2a^2b^3)^9}{(10a^8b^7)^3}$$

$$(j) \quad \frac{(3m^2n^3)^4}{(2mn^3)^6} \div \frac{(3m^2n^3)^3}{(4m^5n^{10})^2}$$

Fractional Indices

What does $x^{\frac{1}{2}}$ mean?

$$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x^{(\frac{1}{2} + \frac{1}{2})}$$

$$= x^1$$

$$x^{\frac{1}{2}} \times x^{\frac{1}{2}} = x$$

Remembering surds:

$$\sqrt{x} \times \sqrt{x} = x$$

Therefore $x^{\frac{1}{2}} = \sqrt{x}$ This is called the square root of x .

What does $x^{\frac{1}{3}}$ mean?

$$x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x^{(\frac{1}{3} + \frac{1}{3} + \frac{1}{3})}$$

$$= x^1$$

$$x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = x$$

$x^{\frac{1}{3}}$ is the cube root of x . This is written as $\sqrt[3]{x}$.

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

Similarly:

$$x^{\frac{1}{4}} = \sqrt[4]{x}, \quad x^{\frac{1}{5}} = \sqrt[5]{x}, \quad x^{\frac{1}{6}} = \sqrt[6]{x}, \text{ etc}$$

Note: the square root of x could be written as $\sqrt[2]{x}$ but it is commonly abbreviated to $\sqrt{x}$.

In general:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EXERCISE 4I

1. Write the following surds in index form.

(a) $\sqrt[3]{x}$ (b) $\sqrt[7]{a}$ (c) $\sqrt[8]{m}$ (d) $\sqrt[12]{z}$ (e) $\sqrt[a]{n}$ (f) $\sqrt[y]{x}$

2. Write the following terms in surd form.

(a) $x^{\frac{1}{6}}$ (b) $y^{\frac{1}{9}}$ (c) $m^{\frac{1}{10}}$ (d) $a^{\frac{1}{13}}$ (e) $n^{\frac{1}{x}}$ (f) $z^{\frac{1}{a}}$

Example 1

$$2^3 = 8$$

Therefore $2 = \sqrt[3]{8} = 8^{\frac{1}{3}}$

In general if: $x^n = y$
then $x = \sqrt[n]{y} = y^{\frac{1}{n}}$

Example 2

Use a calculator to find $243^{\frac{1}{5}}$

Some calculators have a $\wedge$ button to raise a number to a power.

Other calculators have a y^x button.

Remember to use brackets around the fraction when raising to a fractional index.

Press the following buttons on the calculator.

2 4 3 $\wedge$ (1 $\div$ 5) ENTER

$$243^{\frac{1}{5}} = 3$$

EXERCISE 4J

1. Evaluate the following without using a calculator.

- (a) $16^{\frac{1}{4}}$ (b) $27^{\frac{1}{3}}$ (c) $64^{\frac{1}{3}}$ (d) $1000^{\frac{1}{3}}$ (e) $64^{\frac{1}{6}}$

2. Use a calculator to evaluate the following.

- (a) $729^{\frac{1}{6}}$ (b) $65\,536^{\frac{1}{8}}$ (c) $4096^{\frac{1}{4}}$ (d) $59\,049^{\frac{1}{5}}$

Example Evaluate $\sqrt[6]{729}$

This can be evaluated by remembering $\sqrt[6]{729} = 729^{\frac{1}{6}}$ and using the calculator as in question 2.

The other technique is to use the $\sqrt{x}$ function on the calculator. On some calculators this is found by pressing the **MATH** button. Other calculators have a $\sqrt{x}$ button.

Find this function on your calculator and evaluate $\sqrt[6]{729}$.

$$\sqrt[6]{729} = 3$$

3. Use a calculator to evaluate the following.

- (a) $\sqrt[3]{12\,167}$ (b) $\sqrt[4]{6561}$ (c) $\sqrt[9]{19\,683}$ (d) $\sqrt[4]{83\,521}$
 (e) $\sqrt[3]{32\,768}$ (f) $\sqrt[13]{8192}$ (g) $\sqrt[8]{5\,764\,801}$ (h) $\sqrt[9]{10\,077\,696}$

4. Use a calculator to evaluate the following, giving answers correct to two decimal places.

- (a) $3^{\frac{3}{4}}$ (b) $17^{\frac{5}{2}}$ (c) $13^{\frac{2}{9}}$ (d) $23^{\frac{4}{3}}$ (e) $31^{\frac{5}{7}}$

Example Write the following in index form and use a calculator to evaluate to two decimal places.

$$\begin{aligned} & (\sqrt[3]{7})^4 \\ &= (7^{\frac{1}{3}})^4 \\ &= 7^{\frac{4}{3}} \quad (\text{index law 3}) \\ &= \mathbf{13.39} \end{aligned}$$

5. Write the following in index form and use a calculator to evaluate to two decimal places.

- (a) $(\sqrt[4]{5})^5$ (b) $(\sqrt[3]{6})^2$ (c) $(\sqrt[5]{9})^8$ (d) $(\sqrt[7]{6})^{11}$ (e) $(\sqrt[4]{12})^7$

Index Laws and Fractional Indices

EXERCISE 4K

1. Use **Index Law 1** to simplify the following. Write the indices in their simplest form.

$$\begin{array}{llll}
 \text{(a)} \ x^{\frac{1}{4}} \times x^{\frac{1}{4}} & \text{(b)} \ x^{\frac{1}{5}} \times x^{\frac{2}{5}} & \text{(c)} \ a^{\frac{3}{7}} \times a^{\frac{2}{7}} & \text{(d)} \ n^{\frac{2}{9}} \times n^{\frac{7}{9}} \\
 \text{(e)} \ x^{\frac{1}{3}} \times x^{\frac{1}{6}} & \text{(f)} \ a^{\frac{2}{5}} \times a^{\frac{1}{4}} & \text{(g)} \ n^{\frac{1}{2}} \times n^{\frac{2}{3}} & \text{(h)} \ m^{\frac{5}{4}} \times m^{\frac{5}{3}} \\
 \text{(i)} \ 2x^{\frac{2}{3}} \times 3x^{\frac{7}{5}} & \text{(j)} \ 5n^{\frac{5}{7}} \times 4n^{\frac{2}{3}} & \text{(k)} \ 6a^{\frac{9}{2}} \times \frac{1}{3}a^{\frac{2}{5}} & \text{(l)} \ \frac{3}{4}y^{\frac{2}{5}} \times \frac{4}{5}y^{\frac{7}{4}}
 \end{array}$$

2. Use **Index Law 2** to simplify the following. Write the indices in their simplest form.

$$\begin{array}{llll}
 \text{(a)} \ x^{\frac{3}{4}} \div x^{\frac{1}{4}} & \text{(b)} \ x^{\frac{4}{7}} \div x^{\frac{2}{7}} & \text{(c)} \ a^{\frac{5}{8}} \div a^{\frac{1}{8}} & \text{(d)} \ n^{\frac{5}{6}} \div n^{\frac{2}{3}} \\
 \text{(e)} \ x^{\frac{8}{3}} \div x^{\frac{2}{3}} & \text{(f)} \ a^{\frac{11}{3}} \div a^{\frac{5}{6}} & \text{(g)} \ 8n^{\frac{2}{3}} \div 2n^{\frac{1}{2}} & \text{(h)} \ 12m^{\frac{5}{3}} \div 3m^{\frac{5}{4}} \\
 \text{(i)} \ \frac{16x^{\frac{2}{5}}}{12x^{\frac{1}{6}}} & \text{(j)} \ \frac{8x^{\frac{15}{4}}}{12x^{\frac{7}{4}}} & \text{(k)} \ \frac{16n^{\frac{13}{2}}}{24n^{\frac{5}{3}}} & \text{(l)} \ \frac{48x^{\frac{9}{4}}}{32x^{\frac{8}{7}}}
 \end{array}$$

3. Use **Index Law 3** to simplify the following. Write the indices in their simplest form.

$$\begin{array}{llll}
 \text{(a)} \ (x^{\frac{3}{4}})^4 & \text{(b)} \ (x^{\frac{4}{7}})^2 & \text{(c)} \ (a^{\frac{5}{8}})^4 & \text{(d)} \ (n^{\frac{5}{6}})^9 \\
 \text{(e)} \ (x^{\frac{8}{3}})^{\frac{3}{2}} & \text{(f)} \ (a^{\frac{11}{3}})^{\frac{3}{22}} & \text{(g)} \ (n^{\frac{4}{3}})^{\frac{9}{2}} & \text{(h)} \ (m^{\frac{5}{3}})^{\frac{12}{25}}
 \end{array}$$

4. Use **Index Law 4** to expand the brackets and simplify the following. Write the indices in their simplest form.

$$\begin{array}{llll}
 \text{(a)} \ (x^{\frac{3}{4}} \times y^{\frac{5}{2}})^4 & \text{(b)} \ (x^{\frac{5}{8}} \times y^{\frac{1}{6}})^2 & \text{(c)} \ (a^{\frac{9}{4}} \times b^{\frac{7}{2}})^8 & \text{(d)} \ (n^{\frac{5}{6}} \times m^{\frac{8}{3}})^9 \\
 \text{(e)} \ (x^{\frac{10}{3}} \times y^{\frac{8}{9}})^{\frac{3}{2}} & \text{(f)} \ (a^{\frac{11}{3}} \times b^{\frac{3}{8}})^{\frac{24}{33}} & \text{(g)} \ (n^{\frac{4}{3}} \div m^{\frac{9}{2}})^{\frac{6}{5}} & \text{(h)} \ (x^{\frac{5}{3}} \div y^{\frac{5}{7}})^{\frac{42}{5}}
 \end{array}$$

5. Use **Index Law 5** to simplify the following. Write the indices in their simplest form.

$$\begin{array}{llll}
 \text{(a)} \ (x^{\frac{3}{4}})^0 & \text{(b)} \ (x^0)^{\frac{4}{7}} & \text{(c)} \ (a^{\frac{5}{7}}b^{\frac{2}{3}})^0 & \text{(d)} \ (n^{\frac{3}{4}}m^0)^8
 \end{array}$$

6. Use **Index Law 6** to write the following with positive indices.

(a) $x^{-\frac{3}{4}}$

(b) $x^{\frac{-4}{7}}y^{\frac{5}{6}}$

(c) $a^{\frac{5}{7}}b^{-\frac{2}{3}}$

(d) $m^{-\frac{6}{5}}n^{-\frac{3}{4}}$

7. Use all the index laws to simplify the following.

(a) $\frac{x^{\frac{3}{4}} \times x^{\frac{3}{4}}}{x^{\frac{1}{2}}}$

(b) $\frac{m^{\frac{3}{5}} \times m^{\frac{6}{5}}}{m^{\frac{9}{5}}}$

(c) $\frac{c^{\frac{1}{2}} \times c^{\frac{3}{4}}}{c}$

(d) $\frac{a^{\frac{1}{2}}b^3 \times a^2b^{\frac{1}{4}}}{a^{\frac{1}{4}}b^{\frac{1}{2}}}$

(e) $\frac{m^{\frac{7}{3}}n^3 \times m^{\frac{8}{3}}n^6}{m^4n^{\frac{1}{2}}}$

(f) $\frac{6x^2 \times 2x^{\frac{3}{2}}}{4x^3}$

(g) $\frac{6m^{\frac{1}{3}} \times 4m^{\frac{5}{6}}}{8m^{\frac{2}{3}}}$

(h) $\frac{3x^{\frac{1}{2}} \times 6x^{\frac{5}{3}}}{9x^{\frac{5}{6}}}$

(i) $\frac{8c^{\frac{5}{4}} \times 3c^{\frac{2}{3}}}{16c^{\frac{3}{2}}}$

(j) $\frac{(4x^{\frac{3}{2}})^2}{8x^{\frac{3}{4}}}$

(k) $\frac{(4m^{\frac{3}{2}})^{\frac{5}{2}}}{8m^{\frac{3}{2}}}$

(l) $\frac{(9x^{\frac{5}{4}})^{\frac{3}{2}}}{3x^{\frac{3}{16}}}$

(m) $\frac{(x^{\frac{5}{9}}y^{\frac{7}{6}})^3 \times (xy^{\frac{5}{4}})^4}{(x^{\frac{5}{8}}y)^4 \times x^{\frac{3}{2}}y^{\frac{1}{2}}}$

(n) $\frac{m^{\frac{1}{3}}n^{\frac{2}{5}} \times (m^{\frac{5}{9}}n^{\frac{7}{15}})^3}{(m^{\frac{1}{3}}n^{\frac{3}{5}})^{\frac{1}{2}} \times (m^{\frac{5}{6}}n^{\frac{10}{9}})^2}$

(o) $\frac{(a^{\frac{3}{4}}b^{\frac{7}{2}})^4 \times (a^{\frac{5}{6}}b^{\frac{5}{9}})^3}{(a^{\frac{5}{8}}b^{\frac{1}{4}})^2 \times (a^{\frac{5}{3}}b)^{\frac{1}{2}}}$

(p) $\frac{3(x^{\frac{1}{2}}y^{\frac{5}{2}})^2 \times (4x^{\frac{3}{4}}y^{\frac{2}{3}})^2}{2(x^{\frac{2}{3}}y^{\frac{1}{6}})^3 \times 6x^{\frac{1}{2}}y^{\frac{2}{3}}}$

(q) $\frac{(3m^{\frac{3}{8}}n^{\frac{5}{6}})^2 \times 8(m^{\frac{3}{2}}n^{\frac{5}{8}})^{\frac{2}{3}}}{2(m^{\frac{1}{3}}n^{\frac{4}{9}})^{\frac{3}{4}} \times (3m^{\frac{2}{3}}n^{\frac{1}{6}})^3}$

(r) $\frac{(3x^{\frac{5}{3}}y^{\frac{7}{9}})^3 \times (4x^{\frac{8}{3}}y^{\frac{11}{6}})^{\frac{3}{2}}}{2(x^{\frac{5}{6}}y^{\frac{10}{9}})^{\frac{3}{5}} \times (6x^{\frac{1}{2}}y^{\frac{5}{8}})^2}$

(s) $\frac{(x^{\frac{1}{4}}y^{\frac{3}{8}})^4}{(3x^{\frac{1}{2}}y^{\frac{3}{4}})^2} \times \frac{6(x^{\frac{2}{3}}y^{\frac{5}{6}})^3}{(2x^{\frac{1}{4}}y^{\frac{3}{2}})^2}$

(t) $\frac{(3m^{\frac{3}{2}})^2}{(2m^{\frac{5}{6}})^3} \times \frac{4(n^{\frac{6}{5}})^2}{3(m^{\frac{5}{9}}n^{\frac{5}{12}})^3}$

Scientific Notation (Standard Form)

EXERCISE 4L

1. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

$$\begin{aligned} 1. \quad 6000 \\ &= 6 \times 1000 \\ &= \mathbf{6 \times 10^3} \end{aligned}$$

$$\begin{aligned} 2. \quad 935\,000 \\ &= 9.35 \times 100\,000 \\ &= \mathbf{9.35 \times 10^5} \end{aligned}$$

(a) 9000

(b) 50 000

(c) 700 000 000

(d) 275 000

(e) 320 000

(f) 82 100 000

(g) 365 470 000

(h) 230 000 000 000

(i) 50 001

(j) 6587

(k) 254.34

(l) 1537.8

2. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

$$\begin{aligned} 1. \quad 0.0076 \\ &= 7.6 \div 1000 \\ &= 7.6 \times \frac{1}{1000} \\ &= \mathbf{7.6 \times 10^{-3}} \end{aligned}$$

$$\begin{aligned} 2. \quad 0.000\,000\,582 \\ &= 5.82 \div 10\,000\,000 \\ &= 5.82 \times \frac{1}{10\,000\,000} \\ &= \mathbf{5.82 \times 10^{-7}} \end{aligned}$$

(a) 0.0005

(b) 0.000 009

(c) 0.000 000 004

(d) 0.000 13

(e) 0.006 92

(f) 0.000 057 9

(g) 0.002 045

(h) 0.000 006

(i) 0.000 000 068 1

(j) 0.000 020 204

(k) 0.000 000 000 2

(l) 0.000 000 005 7

3. Convert the following numbers into numeral form.

(a) 3.6×10^4

(b) 8×10^6

(c) 6.71×10^5

(d) 2.455×10^8

(e) 7.201×10^7

(f) 3.82×10^{11}

(g) 2.7×10^{-3}

(h) 7×10^{-5}

(i) 9.3×10^{-4}

(j) 4.83×10^{-7}

(k) 5.917×10^{-9}

(l) 1.008×10^{-10}

4. Use a calculator to evaluate the following, given that:

$$A = 3.8 \times 10^8, B = 2.1 \times 10^6, C = 5.2 \times 10^{-7}, D = 6.4 \times 10^{-5}$$

Give answers in scientific notation to one decimal place.

(a) ABC (b) B^3 (c) $\frac{A}{D}$ (d) $\sqrt{CD}$ (e) $\left(\frac{AB}{CD}\right)^2$ (f) $\sqrt{\frac{AC}{BD}}$

5. Give the following answers in scientific notation.

(a) How many seconds are in one non-leap year?

(b) The speed of light is 3×10^8 metres per second (m/s).

A **light year** is the distance that light travels in a year.

How many metres is one light year?

(c) How many grains of sand each with a volume of 10^{-12} m^3 would be on a beach that is 200 m long, 50 m wide and 3 m deep?

6. Atoms consist of three particles - electrons, protons and neutrons.

Write the following in scientific notation.

(a) The mass of an electron is

0.000 000 000 000 000 000 000 000 000 000 910 9 kg.

(b) The mass of a proton is

0.000 000 000 000 000 000 000 000 001 672 6 kg.

(c) The mass of a neutron is

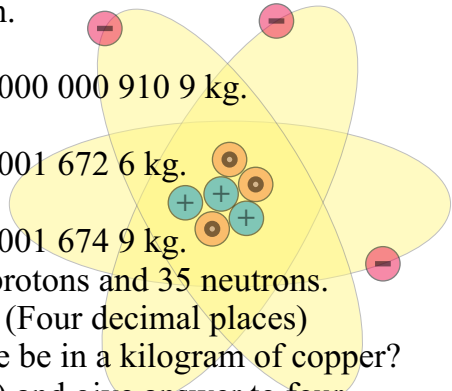
0.000 000 000 000 000 000 000 000 001 674 9 kg.

(d) A copper atom has 29 electrons, 29 protons and 35 neutrons.

Calculate the mass of a copper atom. (Four decimal places)

(e) How many copper atoms would there be in a kilogram of copper?

Use the rounded answer from part (d) and give answer to four decimal places.



7. The area of a certain bacteria is 10^{-12} m^2 . How many of these bacteria would be needed to cover the head of a pin - a circle with diameter 1.0 mm? Give answer to four decimal places.



Use the following information in the questions below.

Mass of the Earth = 5.98×10^{24} kg

Mass of the Moon = 7.36×10^{22} kg

Mass of the Sun = 1.99×10^{30} kg

Distance from Earth to Moon = 3.84×10^8 m

Distance from Earth to Sun = 1.49×10^{11} m

Diameter of the Earth = 1.28×10^7 m

Diameter of the Moon = 3.48×10^6 m

8. The formula used to find the size of the gravitational force between two objects is:

$$F = \frac{GM_1M_2}{d^2}$$

F = the gravitational force (unit of Newtons - N)

$G = 6.67 \times 10^{-11}$

M_1 = the mass of one of the objects (in kg)

M_2 = the mass of the other object (in kg)

d = the distance between the two objects (in m)

Find the size of the gravitational force between the following:
(Give answers in scientific notation correct to two decimal places)

(a) the Sun and the Earth

(b) the Earth and the Moon

9. The formula used to find the velocity required for a rocket to escape the gravitational field of a planet (escape velocity) is:

$$v = \sqrt{\frac{2GM}{R}}$$

v = escape velocity (m/s)

$G = 6.67 \times 10^{-11}$

M = the mass of the planet (kg)

R = the radius of the planet (m)

Calculate the escape velocity (in m/s rounded to the nearest whole number) for a rocket being launched from:

(a) the Earth (b) the Moon



Exponential Growth and Decay

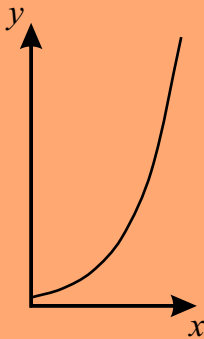
Exponential growth is the term given to an amount that **increases** at a rate that is proportional to its size.

Exponential decay is the term given to an amount that **decreases** at a rate that is proportional to its size.

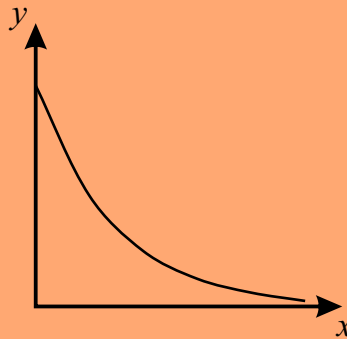
Examples of exponential growth are compound interest, some animal populations and bacterial growth.

Examples of exponential decay are radioactive decay and some depreciations.

Graphs showing exponential growth and decay are shown below.



*exponential
growth*



*exponential
decay*

A formula for **exponential growth** is:

$$y = a \times b^x$$

where y = the amount after x time intervals

a = the initial amount

$b = (1 + r)$ where r is the growth rate as a decimal

x = the number of time intervals

A formula for **exponential decay** is:

$$y = a \times b^x$$

where y = the amount after x time intervals

a = the initial amount

$b = (1 - r)$ where r is the decay rate as a decimal

x = the number of time intervals

The term **exponential** relates to the variable x being the **exponent**.

Examples

1. The population of a town is 5000 people and is increasing by 4% each year. Calculate the population in 5 years.
This is exponential growth because the increase is proportional to the size.

$$y = a \times b^x$$

$$a = 5000$$

$$b = (1 + 0.04) = 1.04 \quad (4\% \text{ as a decimal is } 0.04)$$

$$x = 5 \text{ (the time intervals are each one year)}$$

$$y = 5000 \times (1.04)^5$$

$$y = \mathbf{6083} \text{ (rounded to the nearest person)}$$

2. A colony of bacteria doubled in size every 20 minutes.
If there were initially 100 bacteria, how many would there be in 2 hours?
This is exponential growth because the increase is proportional to the size.

$$y = a \times b^x$$

$$a = 100$$

$$b = (1 + 1) = 2 \quad (\text{Doubling in size is the same as an increase of } 100\% - \text{ as a decimal is } 1)$$

$$x = 6 \text{ (the time intervals are each 20 minutes, so there are 6 time intervals in 2 hours)}$$

$$y = 100 \times 2^6$$

$$y = \mathbf{6400}$$

3. A computer was purchased for \$5000. Its value depreciated by 40% each year. Find the value of the computer after 3 years.
This is exponential decay because the decrease is proportional to its size.

$$y = a \times b^x$$

$$a = 5000$$

$$b = (1 - 0.4) = 0.6 \quad (40\% \text{ as a decimal is } 0.4)$$

$$x = 3 \text{ (the time intervals are each one year)}$$

$$y = 5000 \times 0.6^3$$

$$y = \mathbf{\$1080}$$

EXERCISE 4M

1. The population of a town is 6500 people and is increasing by 5% each year. Calculate the population in six years.
2. The population of people on an island was 3000 and was decreasing by 4% each year. Calculate the population in five years.
3. Novak bought a house for \$240 000 and was hoping its value would increase by 8% each year. Based on this growth rate calculate the value of the house in 10 years. Round to the nearest dollar.
4. The population of frogs in a wetland was increasing by 2% each year. It was estimated that there were 800 frogs in the wetland. Calculate the number of frogs in 20 years.
5. Kevin bought a car for \$15 000 and was told its value would depreciate by 10% each year. Calculate the value of the car in 5 years to the nearest dollar.
6. The population of a bacteria colony doubled every 30 minutes. If there were initially 400 bacteria, how many would there be after 5 hours?
7. Xena invested \$4000 in a bank account that accumulated a compound interest of 6% per year. What would be the value of the investment after 8 years? Round to the nearest dollar.
8. Tim bred mice. His mice numbers doubled every 3 months. If he had 10 mice, how many would he have after 2 years?
9. The **half-life** of a radioactive substance is defined as the time taken for the radioactive substance to decay by one half.
 - (a) The half-life of a particular radioactive material was 50 years. An industrial process produced 100 kg of this substance. How many kilograms of this substance would remain in 200 years?
 - (b) The half-life of plutonium-239 (the radioactive waste produced in nuclear power stations) is about 24 000 years. A nuclear power station produces 32 tonnes of plutonium-239 in a year. How many tonnes will still exist in 120 000 years?

10. The population of bacteria on an item of food after it was taken out of a refrigerator could be calculated by the following formula:

$$N = 2000 \times 3^t$$

Where N = the number of bacteria

and t = the number of hours after food was taken out of the refrigerator

- (a) How many bacteria would be on the food after the following number of hours? (Give answers in scientific notation correct to one decimal place)
- (i) 1 hour (ii) 3 hours (iii) 24 hours
- (b) After how many hours would there be 4.374×10^6 bacteria on the food?

11. A small population of rare marsupials was relocated to an island to protect them from predators. It was found that the number of marsupials on the island after being relocated could be approximated by the following formula:

$$N = 80 \times 1.15^t$$

Where N = number of marsupials

and t = number of years after relocation

- (a) How many marsupials were originally relocated to the island?
This can be found by letting $t = 0$.
- (b) How many marsupials would be on the island 10 years after the relocation? Give answer to the nearest whole number.
- (c) After how many years were there more than 200 of the marsupials on the island?
- (d) What was the percentage increase of marsupials each year?



12. The population of bacteria in a lake was unsafe but was measured to be decreasing after a heavy rain. The number of bacteria could be calculated using the following formula:

$$N = 5000 \times 0.8^t$$

Where N = the number of bacteria per cubic metre
and t = the number of hours after the heavy rain

- How many bacteria would there be per cubic metre the following number of hours after the rain?
(i) 1 hour (ii) 3 hours (iii) 24 hours
- What was the percentage decrease per hour?
- After how many hours would there be the safe number of less than 200 bacteria per cubic metre?

13. The amount of a radioactive material could be calculated by the following formula:

$$M = M_0 \times 0.5^{0.025t}$$

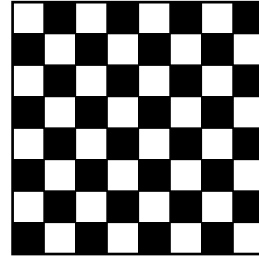
Where M_0 = the initial mass of the radioactive material (kg)
 M = mass of the radioactive material t years after it was produced

50 kg of this radioactive material is produced.

- How many kilograms of this material will there be after the following number of years?
Give answers correct to one decimal place.
(i) 5 years (ii) 10 years (iii) 100 years
- After how many years will there be half of the initial amount of the radioactive material?

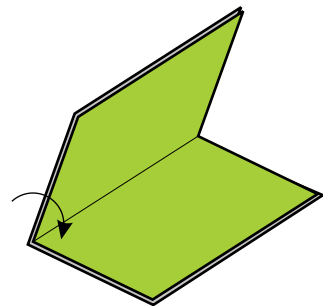
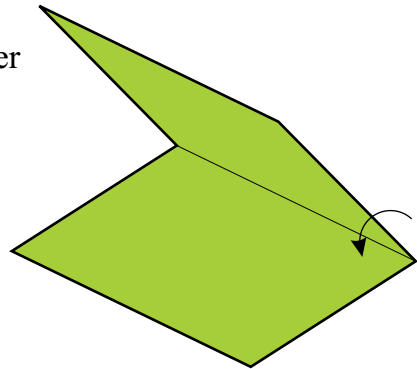


14. The game of chess was, according to a fable, invented by an Indian mathematician. The emperor of India at the time was so pleased with the game he told the mathematician he could have anything in his kingdom he wished. The mathematician asked for rice - one grain on the first square of the chess board, two on the second square, four on the third square, etc. Each square to have twice the number of grains of rice as the previous square.



A chess board is an 8×8 square as shown.

- How many grains of rice would be on the eighth square?
 - What is the total number of grains of rice on the first ten squares?
 - How many grains of rice would be on the last square?
 - Write a formula that could be used to calculate the number of grains of rice (G) on a particular square. Let N represent the number of the square ($N = 1 - 64$)
15. Many people believe that it is impossible to fold a piece of paper in half more than seven times. Although it is very difficult, it is however possible to achieve more than 10 folds (using a large enough sheet of paper) and mathematically 12 folds are possible. Assume for the following questions it is possible to keep folding a piece of paper in half and that the piece of paper being folded is 0.1 mm thick.
- What is the thickness of the folded paper after:
 - four folds?
 - eight folds?
 - What is the least number of folds (**A - D**) for the thickness to be higher than Mount Everest (8848 m)?
A 27 **B** 56 **C** 127 **D** 527
 - Verify this answer.
 - Write a formula that could be used to calculate the thickness (T) after F folds.
 - Guess how many folds would be needed for the folded paper to reach the Moon (3.84×10^8 m) then calculate it.



PROBLEM SOLVING

Barra was investigating setting up a fish farm. He wanted to calculate whether it was going to be profitable. He was basing his calculations on the following information.

- (a) He would start with 5000 fish.
- (b) The number of fish at the end of each year could be calculated by using the following formula.

$$N_E = N_S \times 2.35^{1.9}$$

where N_E = the number of fish at the end of a year

and N_S = the number of fish at the start of a year

- (c) He planned to sell 70% of the fish at the end of each year for \$1.30 per fish.
- (d) His costs each year would be \$40 per 100 fish (based on N_E) and \$8000 per year to pay for set up costs.

1. Construct a table that shows the following figures for the first five years of operation.
 - (a) The number of fish at the start of each year.
 - (b) The number of fish at the end of each year.
 - (c) The number of fish sold.
 - (d) The total income each year.
 - (e) The total expenses each year.
 - (f) The profit for each year.
2. Construct a spreadsheet displaying this information.

PUZZLES

1. Find values of x and y ($x \neq y$) that satisfy the equation: $x^y = y^x$
2. Find values of x and y ($x \neq y$) that satisfy the equation: $x^y = y$

CHAPTER REVIEW

1. Write the following in exponential form.

(a) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ (b) $2 \times a \times a \times a \times b \times 3 \times a \times a \times b$

2. Evaluate the following.

(a) 3^3 (b) $2^4 + 5^2$ (c) $(5 + 2)^3$

3. Write the following with positive indices.

(a) x^{-6} (b) $8m^2n^{-3}$ (c) $\frac{2}{5}x^5y^{-2}z^{-4}$

4. Simplify the following.

(a) $x^5 \times x^4$ (b) $3m^6n^2 \times 4m^5n^4$ (c) $(a^3)^4$ (d) $(2m^3n^2)^5$
 (e) $(-a^2b^3)^3$ (f) $-(3p^2q^3r^2)^4$ (g) $(3m)^0$ (h) $6a^0 + (2b)^0$
 (i) $\frac{a^6 \times 6a^7}{2a^3}$ (j) $\frac{(3x^2y^5)^4}{(9x^3y^2)^2}$ (k) $\frac{(2m^3n^5)^4}{(3mn^3)^3} \times \frac{(9m^5n^6)^2}{(4m^9n^{11})^2}$

5. Write $(\sqrt[6]{11})^7$ in index form.

6. Write $x^{\frac{1}{7}}$ in surd form.

7. Evaluate $32^{\frac{1}{5}}$ without using a calculator.

8. Use a calculator to evaluate $28\,561^{\frac{1}{4}}$.

9. Use a calculator to evaluate $55^{\frac{5}{3}}$ correct to two decimal places.

10. Simplify the following.

(a) $\frac{(2x^{\frac{3}{4}})^4}{8x^{\frac{3}{2}}}$ (b) $\frac{(16m^{\frac{5}{2}})^{\frac{3}{2}}}{8m^{\frac{7}{4}}}$ (c) $\frac{(x^{\frac{5}{3}}y^{\frac{7}{6}})^3 \times (2x^{\frac{5}{2}}y^{\frac{5}{6}})^4}{(x^{\frac{5}{2}}y)^4 \times x^{\frac{3}{2}}y^{\frac{1}{2}}}$

11. Convert the following numbers into scientific notation.

(a) 8 620 000 000 (b) 0.000 000 057 2

12. Convert the following numbers into numeral form.

(a) 6.71×10^4 (b) 3.01×10^{-6}

13. If $x = 5.6 \times 10^{13}$ and $y = 2.5 \times 10^{-6}$, find $(xy)^2$.

14. In a national park there were 120 wedge-tailed eagles. This number was increasing by 2% each year. How many eagles will there be in 10 years?

Logarithms

Logarithms

Logarithms are the inverse or opposite of exponentials (indices).
The abbreviation for logarithms is *log*.

Example

Diagram illustrating the exponential equation $2^3 = 8$. The components are labeled as follows:

- base number**: points to the base 2
- exponent (index or power)**: points to the exponent 3
- basic numeral (number)**: points to the result 8

In words: 2 to the power of 3 equals 8

Diagram illustrating the logarithmic equation $\log_2 8 = 3$. The components are labeled as follows:

- base number**: points to the base 2
- basic numeral (number)**: points to the argument 8
- exponent (index or power)**: points to the result 3

In words: The logarithm (log) with base 2 of 8 equals 3

The logarithm of a number is the exponent (index) to which the base number must be raised to give the basic numeral (number).

Examples

1. $5^3 = 125$

$\log_5 125 = 3$

2. $4^2 = 16$

$\log_4 16 = 2$

3. $7^4 = 2401$

$\log_7 2401 = 4$

4. $10^3 = 1000$

$\log_{10} 1000 = 3$

5. $2^8 = 256$

$\log_2 256 = 8$

6. $9^5 = 59\,049$

$\log_9 59\,049 = 5$

7. $6^1 = 6$

$\log_6 6 = 1$

8. $3^0 = 1$

$\log_3 1 = 0$

9. $19^3 = 6859$

$\log_{19} 6859 = 3$

EXERCISE 5A

1. Write the following exponential equations in logarithmic form.

(a) $3^4 = 81$

(b) $5^2 = 25$

(c) $10^2 = 100$

(d) $8^4 = 4096$

(e) $7^0 = 1$

(f) $4^5 = 1024$

(g) $9^1 = 9$

(h) $20^3 = 8000$

2. Write the following logarithmic equations in exponential form.

(a) $\log_2 32 = 5$

(b) $\log_4 64 = 3$

(c) $\log_6 7776 = 5$

(d) $\log_8 512 = 3$

(e) $\log_7 343 = 3$

(f) $\log_{10} 10\,000 = 4$

(g) $\log_3 27 = 3$

(h) $\log_9 1 = 0$

(i) $\log_{25} 390\,625 = 4$

3. Find x in the following equations.

(a) $2^x = 64$

(b) $\log_3 x = 5$

(c) $x^3 = 216$

(d) $\log_x 16 = 4$

(e) $12^2 = x$

(f) $\log_5 625 = x$

(g) $4^x = 256$

(h) $\log_8 32\,768 = x$

Evaluating Logarithms

Example 1 Evaluate $\log_3 729$

Step 1 Let $\log_3 729 = x$

Step 2 Write in exponential form: $3^x = 729$

Step 3 Find x : Trial and error is one method:

$$\begin{array}{ccccccc} 3 & \times & 3 & \times & 3 & \times & 3 & \times & 3 & \times & 3 & = & 3^6 \\ 9 & 27 & 81 & 243 & 729 & & & & & & & & \end{array}$$

$$3^x = 3^6$$

$$x = 6$$

Example 2 Evaluate the log with base 5 of 78 125

Step 1 Write in log form: $\log_5 78\,125 = x$

Step 2 Let $\log_5 78\,125 = x$

Step 3 Write in exponential form: $5^x = 78\,125$

Step 4 Find x : Trial and error is one method:

$$\begin{array}{ccccccccccc} 5 & \times & 5 & \times & 5 & \times & 5 & \times & 5 & \times & 5 & \times & 5 & \times & 5 & = & 5^8 \\ 25 & 125 & 625 & 3125 & 15625 & 78125 & 390625 & & & & & & & & & & \end{array}$$

$$5^x = 5^8$$

$$x = 8$$

EXERCISE 5B

1. Evaluate the following logarithms.

(a) $\log_4 4096$ (b) $\log_7 16\,807$ (c) $\log_9 6561$ (d) $\log_2 8192$

2. (a) Evaluate the log with base 3 of 2187.

(b) Evaluate the log with base 5 of 3125.

(c) Evaluate the log with base 10 of 1 000 000.

(d) Evaluate the log with base 12 of 248 832.

Logarithm Laws

Logarithm Law 1

$$\log_a m + \log_a n = \log_a mn$$

Examples

$$1. \log_3 8 + \log_3 4$$

$$= \log_3 32$$

$$2. \log_x a + \log_x b$$

$$= \log_x ab$$

$$3. \log_7 8 + \log_7 d$$

$$= \log_7 8d$$

$$4. \log_4 6a + \log_4 5b$$

$$= \log_4 30ab$$

$$5. \log_x st$$

$$= \log_x s + \log_x t$$

$$6. \log_8 15$$

$$= \log_8 5 + \log_8 3$$

EXERCISE 5C

1. Use Logarithm Law 1 to simplify the following.

(a) $\log_5 7 + \log_5 6$

(b) $\log_2 3 + \log_2 9$

(c) $\log_7 5 + \log_7 8$

(d) $\log_n 2 + \log_n 23$

(e) $\log_6 a + \log_6 c$

(f) $\log_x 4p + \log_x 3q$

(g) $\log_3 x + \log_3 x$

(h) $\log_{10} 3m + \log_{10} 5m$

(i) $\log_x 2ab^2 + \log_x 9a^3b^4$

2. Solve for x in the following equations.

(a) $\log_4 9 + \log_4 x = \log_4 36$

(b) $\log_7 6 + \log_7 x = \log_7 48$

(c) $\log_a 35 = \log_a x + \log_a 7$

(d) $\log_6 56 = \log_6 x + \log_6 28$

(e) $\log_5 24 = \log_5 2 + \log_5 12$

(f) $\log_n ab = \log_n a + \log_n x$

(g) $\log_{10} 7 + \log_{10} x = \log_{10} 91$

(h) $\log_n x + \log_n 13 = \log_n 312$

Logarithm Law 2

$$\log_a m - \log_a n = \log_a \left(\frac{m}{n} \right)$$

Examples

1. $\log_3 9 - \log_3 4$

$$= \log_3 \left(\frac{9}{4} \right)$$

2. $\log_4 7 - \log_4 12$

$$= \log_4 \left(\frac{7}{12} \right)$$

3. $\log_2 16 - \log_2 12$

$$= \log_2 \left(\frac{16}{12} \right)$$

$$= \log_2 \left(\frac{4}{3} \right)$$

4. $\log_7 18 - \log_7 9$

$$= \log_7 \left(\frac{18}{9} \right)$$

$$= \log_7 2$$

5. $\log_n 6a - \log_n 4$

$$= \log_n \left(\frac{6a}{4} \right)$$

$$= \log_n \left(\frac{3a}{2} \right)$$

6. $\log_m 8ab - \log_m 6a$

$$= \log_m \left(\frac{8ab}{6a} \right)$$

$$= \log_m \left(\frac{4b}{3} \right)$$

EXERCISE 5D

1. Use Logarithm Law 2 to simplify the following.

(a) $\log_4 5 - \log_4 9$

(b) $\log_3 8 - \log_3 5$

(c) $\log_{10} 25 - \log_{10} 15$

(d) $\log_5 36 - \log_5 48$

(e) $\log_6 32 - \log_6 8$

(f) $\log_x 8d - \log_x d$

(g) $\log_3 27x - \log_3 36x$

(h) $\log_2 8mn - \log_2 16m$

(i) $\log_x 6a^3b^2 - \log_x 9ab^4$

2. Solve for x in the following equations

(a) $\log_4 18 - \log_4 x = \log_4 \left(\frac{3}{4} \right)$

(b) $\log_7 \left(\frac{7}{5} \right) = \log_7 42 - \log_7 x$

(c) $\log_3 x - \log_3 35 = \log_3 \left(\frac{3}{7} \right)$

(d) $\log_6 \left(\frac{2a}{9} \right) = \log_6 8ab - \log_6 x$

(e) $\log_2 \left(\frac{3m}{4n} \right) = \log_2 81m^2n - \log_2 x$

(f) $\log_3 x - \log_3 36a = \log_3 \left(\frac{9a^2}{2} \right)$

Logarithm Law 3

$$\log_a m^n = n \log_a m$$

Examples

1. $\log_5 7^3$

$= 3\log_5 7$

2. $\log_6 81$

$= \log_6 9^2$

$= 2\log_6 9$

3. $\log_6 x^5$

$= 5\log_6 x$

4. $5\log_7 6^8$

$= 40\log_7 6$

5. $8\log_6 729$

$= 8\log_6 3^6$

$= 48\log_6 9$

6. $a\log_m b^c$

$= ac\log_m b$

EXERCISE 5E

1. Use Logarithm Law 3 to simplify the following.

(a) $\log_4 3^7$

(b) $\log_3 125$

(c) $\log_a n^c$

(d) $6\log_5 7^8$

(e) $9\log_2 2401$

(f) $5\log_x 8^d$

(g) $3n\log_{10} 2^p$

(h) $4\log_6 256$

(i) $8\log_4 9^5$

2. Solve for x in the following equations.

(a) $\log_5 x = 3\log_5 10$

(b) $4\log_7 3 = \log_7 x$

(c) $\log_3 x = a\log_3 b$

(d) $5\log_6 2 = \log_6 4x$

(e) $\log_2 59\,049 = 5\log_2 x$

(f) $\log_4 9x = 7\log_4 3$

Logarithm Law 4

$$\log_a a = 1$$

Examples

1. $\log_5 5$

$= 1$

2. $\log_x x$

$= 1$

3. $7\log_6 6$

$= 7 \times 1$

$= 7$

EXERCISE 5F

Use Logarithm Law 4 to simplify the following.

1. $\log_4 4$

2. $3\log_7 7$

3. $x\log_5 5$

4. $a\log_b b$

Logarithm Law 5

$$\log_a 1 = 0$$

Examples

1. $\log_7 1$

$= 0$

2. $\log_x 1$

$= 0$

3. $9\log_6 1$

$= 9 \times 0$

$= 0$

EXERCISE 5G

Use Logarithm Law 5 to simplify the following.

1. $\log_4 1$

2. $4\log_7 1$

3. $x\log_5 1$

4. $n\log_b 1$

Logarithm Law 6

$$\log_a m = \frac{\log_b m}{\log_b a}$$

Example

Write $\log_2 7$ with a base of 10.

$$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2}$$

EXERCISE 5H

1. Write the following logarithms with a base of 6.

(a) $\log_4 3$

(b) $\log_7 12$

(c) $\log_a n$

2. Write the following logarithms with a base of n .

(a) $\log_5 15$

(b) $\log_{10} 9$

(c) $\log_x m$

Using All The Logarithm Laws

Examples

Simplify the following.

$$1. \log_3 9 + \log_5 125$$

$$= \log_3 3^2 + \log_5 5^3$$

$$= 2\log_3 3 + 3\log_5 5 \quad (\text{Law 3})$$

$$= 2 \times 1 + 3 \times 1 \quad (\text{Law 4})$$

$$= \mathbf{5}$$

$$2. \log_5 \left(\frac{25}{5^x} \right)$$

$$= \log_5 25 - \log_5 5^x \quad (\text{Law 2})$$

$$= \log_5 5^2 - \log_5 5^x$$

$$= 2\log_5 5 - x\log_5 5 \quad (\text{Law 3})$$

$$= 2 \times 1 - x \times 1 \quad (\text{Law 4})$$

$$= \mathbf{2 - x}$$

$$3. 3\log_2 3 + 2\log_2 6 - 2\log_2 9$$

$$= \log_2 3^3 + \log_2 6^2 - \log_2 9^2 \quad (\text{Law 3})$$

$$= \log_2 27 + \log_2 36 - \log_2 81$$

$$= \log_2 27 \times 36 - \log_2 81 \quad (\text{Law 1})$$

$$= \log_2 972 - \log_2 81$$

$$= \log_2 \left(\frac{972}{81} \right) \quad (\text{Law 2})$$

$$= \mathbf{\log_2 12}$$

Examples continued

$$\begin{aligned}
4. & 3\log_7 4 - 3\log_7 12 + 3\log_7 3 \\
&= \log_7 4^3 - \log_7 12^3 + \log_7 3^3 \quad (\text{Law 3}) \\
&= \log_7 64 - \log_7 1728 + \log_7 27 \\
&= \log_7 \left(\frac{64}{1728} \right) + \log_7 27 \quad (\text{Law 2}) \\
&= \log_7 \left(\frac{1}{27} \right) + \log_7 27 \quad (\text{Cancel fraction}) \\
&= \log_7 \left(\frac{1}{27} \times 27 \right) \quad (\text{Law 1}) \\
&= \log_7 1 \\
&= 0 \quad (\text{Law 5})
\end{aligned}$$

EXERCISE 5I

Simplify the following.

- | | |
|---|--|
| 1. $\log_4 16 + \log_7 343$ | 2. $\log_2 32 - \log_9 729$ |
| 3. $\frac{\log_5 15\,625}{\log_{10} 10\,000}$ | 4. $\frac{\log_3 729}{\log_2 1024}$ |
| 5. $\log_8 \left(\frac{512}{8^n} \right)$ | 6. $\log_2 \left(\frac{2^m}{512} \right)$ |
| 7. $\log_3 (243 \times 3^x)$ | 8. $\log_2 (4^a \times 16^b)$ |
| 9. $4\log_5 4 + 2\log_5 8 - 12\log_5 2$ | 10. $8\log_8 3 - 8\log_8 9 + 2\log_8 81$ |
| 11. $5\log_6 2 + 5\log_6 5 - 5\log_6 10$ | 12. $5\log_2 4 + 3\log_2 8 - 3\log_2 64$ |
| 13. $3\log_3 4 - 2\log_3 16 + 2\log_3 2$ | 14. $7\log_2 8 - 2\log_2 4 - 2\log_2 256$ |

Examples

Simplify the following.

$$\begin{aligned}
 & 1. \log_2(4x + 8) - \log_2(x + 2) \\
 &= \log_2\left(\frac{4x + 8}{x + 2}\right) \quad (\text{Law 2}) \\
 &= \log_2\left[\frac{4(x + 2)}{x + 2}\right] \quad (\text{Factorise numerator}) \\
 &= \log_2\left[\frac{\cancel{4(x + 2)}}{\cancel{(x + 2)}}\right] \quad (\text{Cancel factors}) \\
 &= \log_2 4 \\
 &= \log_2 2^2 \\
 &= 2\log_2 2 \quad (\text{Law 3}) \\
 &= 2 \times 1 \quad (\text{Law 4}) \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 & 2. \log_4 x + \log_4 3x - \log_4 5x \\
 &= \log_4(x \times 3x) - \log_4 5x \quad (\text{Law 1}) \\
 &= \log_4 3x^2 - \log_4 5x \\
 &= \log_4\left(\frac{3x^2}{5x}\right) \quad (\text{Law 2}) \\
 &= \log_4\left(\frac{3x}{5}\right) \quad (\text{Simplify})
 \end{aligned}$$

Examples continued

$$\begin{aligned}
3. \quad & 3\log_4 a + 2\log_4 8a - 3\log_4 4a \\
&= \log_4 a^3 + \log_4 (8a)^2 - \log_4 (4a)^3 \quad (\text{Law 3}) \\
&= \log_4 a^3 + \log_4 64a^2 - \log_4 64a^3 \\
&= \log_4 64a^5 - \log_4 64a^3 \quad (\text{Law 1}) \\
&= \log_4 \left(\frac{64a^5}{64a^3} \right) \quad (\text{Law 2}) \\
&= \log_4 a^2 \quad (\text{Simplify})
\end{aligned}$$

$$\begin{aligned}
4. \quad & 3\log_2 x + 2\log_2 4y - 3\log_2 2xy \\
&= \log_2 x^3 + \log_2 (4y)^2 - \log_2 (2xy)^3 \quad (\text{Law 3}) \\
&= \log_2 x^3 + \log_2 (4^2 y^2) - \log_2 (2^3 x^3 y^3) \\
&= \log_2 x^3 + \log_2 16y^2 - \log_2 8x^3 y^3 \\
&= \log_2 16x^3 y^2 - \log_2 8x^3 y^3 \quad (\text{Law 1}) \\
&= \log_2 \left(\frac{16x^3 y^2}{8x^3 y^3} \right) \quad (\text{Law 2}) \\
&= \log_2 \left(\frac{2}{y} \right) \quad (\text{Simplify}) \\
&= \log_2 2 - \log_2 y \quad (\text{Law 2}) \\
&= 1 - \log_2 y \quad (\text{Law 3})
\end{aligned}$$

EXERCISE 5J

Simplify the following.

1. $\log_3(3x + 3) - \log_3(x + 1)$
2. $\log_2(12 - 4n) - \log_2(3 - n)$
3. $\log_2(20a + 8) - \log_2(10a + 4)$
4. $\log_5(15 + 3m) - \log_5(10 + 2m)$
5. $\log_3(2x^2 + 6x) - \log_3(6x + 18)$
6. $\log_4(12x^2 + 8xy) - \log_4(3x^2y + 2xy^2)$
7. $\log_3x + \log_32x - \log_33x$
8. $\log_54a + \log_56a - \log_58a$
9. $2\log_4n + \log_42n - \log_44n$
10. $4\log_{10}2x + 2\log_{10}3x - 2\log_{10}12x$
11. $2\log_68m - 3\log_64m + \log_66m$
12. $8\log_73x - 4\log_79x - 4\log_7x$
13. $3\log_26x + 2\log_24y - 3\log_212xy$
14. $3\log_33a + 2\log_3b - 2\log_39ab$
15. $4\log_22x^2 + 2\log_24y^3 - 2\log_28x^3y^3$
16. $2\log_28m^6 + 3\log_24n^4 - 12\log_22mn$
17. $4\log_42x^3 + 2\log_48y^4 - 4\log_44x^3y^2$
18. $2\log_24x^4y^5 - 3\log_22x^3y^3 - 2\log_22xy^2$

Examples

Solve for x in the following equations.

1. $2\log_2 4x - \log_2 x = \log_2 8$

$$\log_2 (4x)^2 - \log_2 x = \log_2 8$$

$$\log_2 16x^2 - \log_2 x = \log_2 8$$

$$\log_2 \left(\frac{16x^2}{x} \right) = \log_2 8$$

$$\log_2 16x = \log_2 8$$

$$16x = 8$$

$$x = \frac{1}{2}$$

2. $2\log_3 8x - \log_3 4x = \log_3 (6x + 10)$

$$\log_3 (8x)^2 - \log_3 4x = \log_3 (6x + 10)$$

$$\log_3 64x^2 - \log_3 4x = \log_3 (6x + 10)$$

$$\log_3 \left(\frac{64x^2}{4x} \right) = \log_3 (6x + 10)$$

$$\log_3 16x = \log_3 (6x + 10)$$

$$16x = 6x + 10$$

$$10x = 10$$

$$x = 1$$

EXERCISE 5K

Solve for x in the following equations.

1. $\log_6 4 + \log_6 x = \log_6 8$

2. $\log_7 12x - \log_7 3 = 2\log_7 6$

3. $\log_2 6x + \log_2 4x = \log_2 12x$

4. $2\log_3 3x - \log_3 x = \log_3 18$

5. $4\log_5 2 + 2\log_5 3 = \log_5 6x$

6. $2\log_2 6x - \log_2 4x = \log_2 (7x + 4)$

7. $3\log_6 4x - 2\log_6 2x = \log_6 2(3x + 5)$

8. $3\log_3 6x - 2\log_3 3x = \log_3 2(3x + 7)$

9. $5\log_5 2x + 2\log_5 4x = 6\log_5 2x$

10. $5\log_7 4x - 4\log_7 2x = \log_7 8(7x + 9)$

Logarithms with Base 10

$$\log_{10} x$$

Logarithms with base 10 are often used in science, engineering, navigation and have many practical applications.

Logarithms with base 10 are written as *base 10 log*, *log base 10*, *base-10 log*, *log to the base 10* or *log with base 10*.

Because *base 10 log* is used so often it is called the *common logarithm*.

The reason it is used so often is largely because we use the decimal system. For this reason it is sometimes called the decimal logarithm.

Because it is used so often, when writing the abbreviation of a base 10 logarithm it is not necessary to write the subscript 10.

It is sufficient to write **logx**. Sometimes the abbreviations **Logx** or **lgx** are used.

$$\text{log}x \text{ means } \log_{10}x$$

As will be seen later in the chapter, using base 10 logarithms can condense a large range of numbers to a manageable scale.

Scientific calculators have a button that finds log to the base 10 of numbers:

LOG

It is sometimes easier to find log to the base 10 for very large and very small numbers when comparing them or representing them on a graph.

Scientific notation is the usual way of firstly writing very large and very small numbers.

Examples

$$740\,000\,000\,000\,000\,000\,000\,000\,000\,000 = 7.4 \times 10^{32}$$

$$0.000\,000\,000\,000\,000\,000\,000\,000\,872 = 8.72 \times 10^{-25}$$

Log to the base 10 can now be found easily:

$$\log(7.4 \times 10^{32}) = \log 7.4 + \log 10^{32} \quad (\text{Law 1})$$

$$= \log 7.4 + 32 \log 10 \quad (\text{Law 3})$$

$$= \log 7.4 + 32 \quad (\text{Law 4})$$

$$= 0.87 + 32$$

$$= \mathbf{32.87}$$

*From calculator
rounded to two
decimal places*

$$\log(8.72 \times 10^{-25}) = \log 8.72 + \log 10^{-25} \quad (\text{Law 1})$$

$$= \log 8.72 - 25 \log 10 \quad (\text{Law 3})$$

$$= \log 8.72 - 25 \quad (\text{Law 4})$$

$$= 0.94 - 25$$

$$= \mathbf{-24.06}$$

*From calculator
rounded to two
decimal places*

Note: Log can be found by entering the full number in scientific notation into a calculator.

EXERCISE 5L

1. Convert the following numbers to scientific notation then find the log of each number. Give logs correct to two decimal places.

- (a) 98 500 000 000 000 (b) 1 620 000 000 000 000 000 000
 (c) 364 000 000 000 000 000 (d) 85 711 000 000 000 000 000 000
 (e) 0.000 000 000 000 8 (f) 0.000 000 000 000 000 000 43
 (g) 0.000 000 000 756 8 (h) 0.000 000 000 000 222

2. Find the log of the following numbers.
 Give logs correct to two decimal places.

- (a) 7.2×10^{15} (b) 3.09×10^{-17} (c) 1.984×10^{25}
 (d) 5.89×10^{-12} (e) 4.321×10^{35} (f) 9.76×10^{-14}

3. Copy and complete the following table.
 Give logs correct to two decimal places.
 The first line is completed as an example.

x	x (<i>Scientific Notation</i>)	$\log x$
1400	1.4×10^3	3.15
14 000		
140 000		
1 400 000		
14 000 000		
140 000 000		
1 400 000 000		

4. Use this table to help answer the following.

When a number (x) is multiplied by 10 what happens to:

- (a) the power of 10 of that number represented in scientific notation?
 (b) the log of that number?

5. Given that $\log 3200 = 3.51$, find the following without using a calculator.

- (a) $\log 32\ 000$ (b) $\log 320\ 000$ (c) $\log 3\ 200\ 000$

6. Given that $\log(2.8 \times 10^6) = 6.45$, find the following without using a calculator.

(a) $\log(2.8 \times 10^7)$

(b) $\log(2.8 \times 10^8)$

(c) $\log(2.8 \times 10^{12})$

7. Given that $\log(9.1 \times 10^{11}) = 11.96$, find x in the following.

(a) $\log x = 12.96$

(b) $\log x = 13.96$

(c) $\log x = 14.96$

(d) $\log x = 17.96$

(e) $\log x = 20.96$

(f) $\log x = 24.96$

Example

Use a calculator to help find x in the following equation.
Write answer in scientific notation correct to two decimal places.

$$\log x = 13.26$$

Remember the following:

$$\log x = a \quad (\log_{10} x = a)$$

$$x = 10^a$$

$$\log x = 13.26$$

$$\log_{10} x = 13.26$$

$$x = 10^{13.26}$$

$$= 18\,197\,008\,586\,099$$

$$x = 1.82 \times 10^{13}$$

Use calculator

8. Use a calculator to help find x in the following equations.

Write answer in scientific notation correct to two decimal places.

(a) $\log x = 15.77$

(b) $\log x = 12.82$

(c) $\log x = 24.06$

(d) $\log x = 17.39$

(e) $\log x = 35.49$

(f) $\log x = -9.45$

(g) $\log x = -7.33$

(h) $\log x = -15.21$

(i) $\log x = -19.34$

It can be seen that a large range of numbers can be reduced to a manageable range by using the logarithms of the numbers.

Examples

1. Mercury is 5.8×10^7 km from the Sun.
 Earth is 1.5×10^8 km from the Sun.
 Neptune is 4.5×10^9 km from the Sun.
 The distance from the Sun to the nearest star is 4.0×10^{13} km.

$$\text{Log}(5.8 \times 10^7) = 7.76$$

$$\text{Log}(1.5 \times 10^8) = 8.18$$

$$\text{Log}(4.5 \times 10^9) = 9.65$$

$$\text{Log}(4.0 \times 10^{13}) = 13.60$$

2. The three main subatomic particles are electrons, protons and neutrons. The masses of these are shown below.

$$\text{Mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$\text{Mass of proton} = 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Mass of neutron} = 1.67 \times 10^{-27} \text{ kg}$$

$$\text{Log}(9.11 \times 10^{-31}) = -30.04$$

$$\text{Log}(1.67 \times 10^{-27}) = -26.78$$

Let's now investigate a few areas where logarithms are used.

The Richter Scale

The **Richter scale** is a base-10 logarithmic scale that is used to measure the magnitude of earthquakes.

The Richter scale was invented in 1935 by Charles F. Richter in the California Institute of Technology.

The magnitude of an earthquake is related to how much energy is released by the quake.

Instruments called seismographs detect movement in the earth.

The intensity of an earthquake, ***R*** on the Richter scale, is calculated by using the following formula:

$$R = \log \left(\frac{A}{A_0} \right)$$

A = the measure of the amplitude of the earthquake wave

***A*₀** = the amplitude of the smallest detectable wave
(or standard wave)

The intensity of an earthquake will typically measure between 2 and 10 on the Richter scale.

Any earthquakes with a Richter rating below a 5 are fairly minor - they may shake the ground a bit, but are seldom strong enough to cause much damage.

Earthquakes with a Richter rating of between 5 and 7.9 are much more severe, and any earthquake above an 8 is likely to cause massive damage.

The highest rating ever recorded for an earthquake is 9.5 during the 1960 Valdivia earthquake in Chile.

To simplify the calculations let the amplitude of the standard wave, A_0 , equal 1.

Example 1

Find the magnitude on the Richter scale of an earthquake with an amplitude that is 450 times the standard wave.

Give answer correct to one decimal place.

$$A = 450A_0$$

$$\text{Let } A_0 = 1$$

$$A = 450$$

$$R = \log \left(\frac{A}{A_0} \right)$$

$$R = \log \left(\frac{450}{1} \right)$$

$$= \log 450$$

$$R = 2.7$$

Example 2

Find A for an earthquake that measured 6.1 on the Richter scale.

$$R = \log \left(\frac{A}{A_0} \right)$$

$$6.1 = \log \left(\frac{A}{A_0} \right)$$

$$\text{Let } A_0 = 1$$

$$6.1 = \log A$$

$$A = 10^{6.1}$$

$$= 1\,258\,925$$

$$A = 1.3 \times 10^6 \text{ (One decimal place)}$$

To simplify the calculations let the amplitude of the standard wave, A_0 , equal 1.

EXERCISE 5M

- Find the magnitude on the Richter scale of earthquakes with the following amplitudes, given that $A_0 = 1$.
Give answers correct to one decimal place.

(a) 700	(b) 5000	(c) 8.7×10^5
(d) 2.9×10^4	(e) 7.7×10^7	(f) 3.2×10^8
- Find the amplitude, A , of the earthquakes that had the following measurements on the Richter scale.
Where required give answers in scientific notation correct to one decimal place.

(a) 3.7	(b) 1.6	(c) 5.2	(d) 8.3	(e) 4.6
---------	---------	---------	---------	---------
- Find the amplitude, A , of the earthquakes that had the following measurements on the Richter scale.
Give answers in scientific notation correct to one decimal place.

(a) 2.4	(b) 3.4	(c) 4.4	(d) 5.4	(e) 6.4
---------	---------	---------	---------	---------
- What happens to the value for the amplitude, A , when the measurement on the Richter scale increases by 1?
 - Use this fact and the answers from question 3 to find the amplitude of an earthquake that measured 7.4 on the Richter scale.
- What would happen to the measurement on the Richter scale if the magnitude of an earthquake is increased by a factor of:

(a) 100	(b) 1000	(c) 10^4	(d) 10^5	(e) 10^6
---------	----------	------------	------------	------------
- An earthquake measures 4.5 on the Richter scale. Two days later another earthquake is measured that is 1000 times the magnitude of the first earthquake.
What was the reading on the Richter scale of the second earthquake?
- Why is it convenient for the Richter scale to be a logarithmic scale?

RICHTER SCALE

Magnitude	Description	What It Feels Like	Frequency
Less than 2.0	Micro	Most people would not feel them.	Millions per year
2.0 - 2.9	Minor	A few people would feel them. No building damage.	Over 1 million per year
3.0 - 3.9	Minor	Some people feel them. Objects inside can be seen shaking.	Over 100 000 per year
4.0 - 4.9	Light	Most people would feel them. Indoor objects shake or fall to the floor.	10 000 - 15 000 per year
5.0 - 5.9	Moderate	Everyone feels them. Damage to some buildings.	1000 - 1500 per year
6.0 - 6.9	Strong	Widespread shaking. Building damage.	100 - 150 per year
7.0 - 7.9	Major	Widespread damage in most areas.	10 - 20 per year
8.0 - 8.9	Great	Widespread damage in large areas.	About 1 per year
9.0 - 9.9	Great	Severe damage to most buildings.	1 per 5 - 50 years
10.0 and over	Massive	Never recorded.	Never recorded.

Decibels

Decibels are units of sound level that are calculated by finding the base-10 logarithm of sound intensity.

Sound intensity is measured in units of watts per square metre. The units of decibels (dB) are named after Alexander Graham Bell who invented the telephone.

One decibel is a tenth of a bel (B). The unit of bel is rarely used. The following formula is used to calculate sound level (***SL***) for a given sound intensity (***I***).

$$SL = 10\log I$$

The difference in sound level (***DSL***) between sound intensity 1 (***I*₁**) and sound intensity 2 (***I*₂**) can be written as the following formula:

$$\begin{aligned} DSL &= 10\log I_1 - 10\log I_2 \\ &= 10(\log I_1 - \log I_2) \end{aligned}$$

$$DSL = 10\log \left(\frac{I_1}{I_2} \right)$$

Examples of Sound Sources and Their Sound Levels

<i>Sound Source</i>	<i>Sound Level (dB)</i>
Jet aircraft, 50 m away	140
Threshold of pain	130
Threshold of discomfort	120
Rock concert	115
Football crowd, at stadium	110
Vacuum cleaner, 1m away	70
Normal talking, 1m away	60
Quiet library	40

Example 1

The sound intensity of an old loud machine was found to be twice that of a new model.

- (a) What is the difference in sound levels (dB) of the two machines?
Give answer to the nearest dB.
- (b) If the sound level of the old machine was 88 dB, find the sound level of the new model.

Answers

- (a) Use the following symbols:

The sound intensity of the old machine = I_1

The sound intensity of the new machine = I_2

$$I_1 = 2I_2$$

$$\begin{aligned} DSL &= 10\log\left(\frac{I_1}{I_2}\right) \\ &= 10\log\left(\frac{2I_2}{I_2}\right) \\ &= 10\log\left(\frac{2\cancel{I_2}}{\cancel{I_2}}\right) \quad I_2\text{s can be cancelled} \\ &= 10\log 2 \end{aligned}$$

$$DSL = 3\text{dB} \quad (\text{Rounded to nearest dB})$$

- (b) Sound level of the new model = $88 - 3 = 85 \text{ dB}$

Example 2

The sound level of a lawn mower was measured at 90 dB at the distance of 1 m. Ear protection reduced the sound level by 15 dB. What percentage of the sound intensity is still heard by the person using the lawn mower?

Round answer to the nearest percentage.

$$DSL = 15\text{dB}$$

$$DSL = 10\log\left(\frac{I_1}{I_2}\right)$$

$$15 = 10\log\left(\frac{I_1}{I_2}\right)$$

$$1.5 = \log\left(\frac{I_1}{I_2}\right)$$

$$\frac{I_1}{I_2} = 10^{1.5}$$

$$\frac{I_1}{I_2} = 31.62..$$

$$I_1 = 31.62.. \times I_2$$

$$I_2 = \frac{1}{31.62..} \times I_1$$

$$\frac{1}{31.62..} = 3\%$$

$$I_2 = 3\% \text{ of } I_1$$

97% of the sound is absorbed by the ear protection.



EXERCISE 5N

1. The sound intensity of a petrol lawn mower was measured to be five times the sound intensity of an electric mower.
Round answers to the nearest dB.
 - (a) What is the difference in sound levels (dB) of the two mowers?
 - (b) If the sound level of the petrol lawn mower was 92 dB, what was the sound level of the electric mower?
2. A group of four musicians often played music in a room at the back of a house. There was a few complaints from neighbours so they decided to add sound insulation to the room. They were told the sound intensity outside the room would then be one tenth of the intensity without insulation.
 - (a) What is the difference in sound levels (dB) before and after the insulation was installed?
 - (b) If the sound level was 85 dB before, what would be the sound level after the insulation was installed?
3. A listening device used to assist in recording the sound of animals in the wild claimed to magnify the animal sounds by a factor of eight (the recorded sounds would be eight times the intensity of the animal sounds).
 - (a) What is the difference in sound levels of the recorded sounds and the actual sounds?
 - (b) The actual animal sounds were 45 dB. What was the level of the recorded sounds?
4. Traffic noise from a freeway at a nearby house was measured at 91 dB. Sound barriers were installed and the sound level was measured at 85 dB. What was the percentage reduction in sound intensity?
Give answer to the nearest percentage.
5. Double glazing was reported to decrease the sound level from 86 dB to 82 dB. What was the percentage reduction in sound intensity?
Give answer to the nearest percentage.
6. A hearing aid was reported to increase the sound level from 60 dB to 64 dB. What was the percentage increase in sound intensity?
Give answer to the nearest percentage.

pH

The pH of a liquid is the measure of the acidity of the liquid. It is based on the concentration of hydrogen ions (H^+) in the liquid. The pH scale is a base-10 logarithmic scale.

$$\text{pH} = -\log(C_{H^+})$$

where C_{H^+} is the concentration of hydrogen ions measured in moles per litre
(1 mole = 6×10^{23} molecules)

Example 1

Find the pH of a liquid that has hydrogen ions with a concentration of 0.004 moles per litre.

Give answer correct to one decimal place.

$$\begin{aligned}\text{pH} &= -\log(C_{H^+}) \\ &= -\log(0.004)\end{aligned}$$

$$\text{pH} = 2.4$$

Example 2

Find the concentration of hydrogen ions in a liquid that has a pH of 8.3.

Give answer in scientific notation correct to one decimal place.

$$\begin{aligned}\text{pH} &= -\log(C_{H^+}) \\ 8.3 &= -\log(C_{H^+}) \\ -8.3 &= \log(C_{H^+})\end{aligned}$$

$$C_{H^+} = 10^{-8.3}$$

$$C_{H^+} = 5.0 \times 10^{-9} \text{ moles per litre}$$

EXERCISE 50

- Find the pH of the liquids with the following concentrations of hydrogen ions (moles per litre).
Give answers correct to one decimal place.

(a) 0.005	(b) 0.083	(c) 0.000 009
(d) 7.2×10^{-6}	(e) 4.1×10^{-11}	(f) 6.6×10^{-4}
- Find the concentration of hydrogen ions (moles per litre) of liquids with the following pH values.
Give answers in scientific notation correct to one decimal place.

(a) 8.2	(b) 12.3	(c) 2.7	(d) 7.1
---------	----------	---------	---------
- Liquids with a pH of 7 are classified as *neutral*.
Liquids with a pH less than 7 are classified as *acidic*.
Liquids with a pH greater than 7 are classified as *alkaline* or *basic*.
 - What is the concentration of hydrogen ions in a neutral liquid?
Give answer in scientific notation.
 - Which of the following liquids are acidic?

Liquid **A** - hydrogen ion concentration = 0.000 045

Liquid **B** - hydrogen ion concentration = 0.078

Liquid **C** - hydrogen ion concentration = 9.2×10^{-10}

Liquid **D** - hydrogen ion concentration = 3.3×10^{-6}



PROBLEM SOLVING

Five earthquakes occurred on Shakey Island.

The five earthquakes and their measurements on the Richter scale are shown below. They are *not* in the order that they occurred.

<i>Earthquake</i>	<i>Size</i>
A	1.3
B	2.3
C	4.3
D	5.3
E	7.3

From the information below find the order that the earthquakes occurred.

- ◆ *The first was not the smallest or the largest.*
- ◆ *The second was 10 times the intensity of the third.*
- ◆ *The fourth was 1000 times the intensity of the fifth.*
- ◆ *The last was 1000 times the intensity of the third.*

PUZZLE

Unscramble the following words or phrases to find words that contain the word **LOG**.

1. **AGRO MILK**
3. **GO LILAC**
5. **GOOK LOB**
7. **BOGY OIL**
9. **GOLF**

2. **THE ONLY COG**
4. **GOOL**
6. **ANGOLA**
8. **LEAN HOG**
10. **PLAY GOO**

CHAPTER REVIEW

1. Write $3^5 = 243$ in logarithmic form.
2. Write $\log_2 16 = 4$ in exponential form.
3. Find x : $\log_4 x = 3$
4. Evaluate $\log_5 125$.
5. Simplify the following.
 - (a) $\log_6 36 + \log_8 4096$
 - (b) $2\log_2 16 + 4\log_2 8 - 8\log_2 4$
 - (c) $2\log_3 27 + \log_3 81 - 5\log_3 9$
 - (d) $\log_2 6x + \log_2 8x - \log_2 12x$
 - (e) $3\log_4 2x + 2\log_4 3xy - 2\log_4 6x$
 - (f) $6\log_2 2x - 3\log_2 x - 3\log_2 4x$
6. Solve for x .
 - (a) $\log_6 64x - 2\log_6 2 = 2\log_6 4$
 - (b) $3\log_4 4x - 2\log_4 2x = \log_4 (8x + 3)$
7. Find $\log_{10}$ of the following numbers correct to two decimal places.
 - (a) 83 000 000
 - (b) 0.000 000 06
 - (c) 8.9×10^7
 - (d) 3.4×10^{-12}
8. Given that $\log(3.7 \times 10^9) = 9.57$, find $\log(3.7 \times 10^{10})$ without using a calculator.
9. An earthquake measured 3.6 on the Richter scale. Two days later another earthquake occurred that was 100 times the intensity of the first. What did the second earthquake measure on the Richter scale?
10. A sound level of a large drill measured 93 dB. Wearing ear protection reduced the sound level to 89 dB. What was the percentage reduction in sound intensity? Round to nearest percentage.
11. What is the pH of a liquid with a hydrogen ion concentration of 5.2×10^{-4} moles per litre? Give answer correct to one decimal place.

Algebra

Algebraic Symbols

EXERCISE 6A

Examples

Write the following statements as algebraic expressions or terms.

1. The sum of x and y
2. a more than 5
3. The product of M and N
4. The total cost of n items that each cost \$8
5. One quarter of Z
6. The sum of $5a$ and $6b$
7. 5 times the sum of c and d
8. The product of k and $l - 6$

Answers

1. $x + y$
2. $5 + a$
3. $M \times N$ or MN
4. $\$8n$
5. $\frac{Z}{4}$
6. $5a + 6b$
7. $5(c + d)$
8. $k(l - 6)$

1. Write the following statements as algebraic expressions or terms.
 - (a) The product of x and z
 - (b) The sum of G and H
 - (c) 2 less than p
 - (d) 7 more than d
 - (e) y more than w
 - (f) The sum of $4t$ and $9w$
 - (g) 7 less than m
 - (h) One third of H
 - (i) The product of t and $5 - q$
 - (j) The total cost of b books that cost \$15 each
 - (k) The product of the sum of a and b and the sum of c and d

All algebraic **terms** consist of a **coefficient** (number) and **pronumerals** (symbols).

The coefficient is written first then the pronumerals are conventionally in alphabetical order.

Examples $6x$: 6 is the coefficient
 x is the pronumeral

$-\frac{3}{8}abc^3$: $-\frac{3}{8}$ is the coefficient
 abc^3 are the pronumerals

pq : 1 is the coefficient
 pq are the pronumerals

Like terms are terms that have the same pronumerals.

Example Group the **like terms** from the following list.

$2a$ $3b$ $6a^2$ b $-\frac{1}{2}a$ $-ab$ $12a^2$ $12ba$ $6a$ $2b$

The groups of **like terms** are:

$2a, -\frac{1}{2}a, 6a$

$3b, b, 2b$

$6a^2, 12a^2$

$-ab, 12ba$

Remember: $ab = ba$
 $(a \times b = b \times a)$

Only **like terms** can be added or subtracted.

Examples 1. $5x + 3y + 8x - y$

$$= \textcircled{5x} + \textcircled{3y} + \textcircled{8x} - \textcircled{y}$$

$$= 13x + 2y$$

2. $5x - 7 - 1 - 8x$

$$= \textcircled{5x} - \textcircled{7} - \textcircled{1} - \textcircled{8x}$$

$$= -3x - 8$$

Algebraic **expressions** are the addition or subtraction of more than one algebraic term.

Examples $4x - 5y$ $3x^2 + 6y^3$ $A + 2C - 8DF^2$

2. Which of the following are: (i) terms? (ii) expressions?

- (a) $4x$ (b) $5p + 2q$ (c) $8(h - g)$ (d) $5mn^6$ (e) $-\frac{3}{4}v^2w^3y^7$
 (f) $5xy - 2z + 4ab$ (g) $5AB^7C^3D^2$ (h) $6 + 5x - 7z$

3. For each of the following terms state the:

(i) coefficient (ii) pronumerals

- (a) $7a$ (b) $34abc^2$ (c) $-2p$ (d) F (e) $\frac{3}{5}m^2n^3$ (f) $-\frac{2}{7}w^2z^5$

4. Match all the like terms from the following list.

$2m$ $4n^2$ $6mn$ n $-4m$ $6m^2$ $\frac{1}{4}n^2$ $4n$ $\frac{1}{4}mn$ $4m^2n$ $3m^2$ $-3nm^2$

5. Simplify the following expressions.

- (a) $3y + 4y$ (b) $8b - 5b$
 (c) $7k + k$ (d) $6z + 5z - 2z$
 (e) $4ab + 7ab$ (f) $5xy - 2xy$
 (g) $9mn - mn - 3mn$ (h) $6xyz + zxy + xzy$
 (i) $6d - 8d$ (j) $2pq^2 - 4pq^2 - pq^2$
 (k) $x^2 + x^2 + x^2$ (l) $4y^2 - 8y^2 + y^2$
 (m) $8a - 10a + a$ (n) $5mn - 2mn - 6nm$
 (o) $-5t - 3t + 8t$ (p) $-6y + 9y - 8y + 4y - 2y - 6y + 11y$
 (q) $-4xy + 5yx - 7xy$ (r) $-cd - 4cd + 9dc - 7dc - 3cd + cd + 5dc$

Examples

1. $6a + 3a = 9a$

2. $5p - p + 2p = 6p$

3. $6k - 9k = -3k$

4. $-2ab + 5ba = 3ab$

6. Simplify the following expressions by collecting all the like terms.

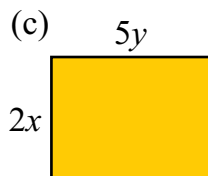
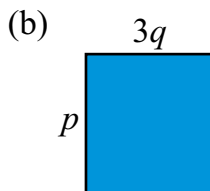
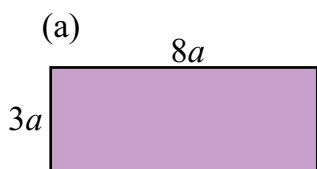
Examples

1. $3a + 4b + 5a + 3b$
 $= 8a + 7b$

2. $-5x - 7y + 3x - 2y$
 $= -2x - 9y$

- (a) $7x + 5y + 2x + 3y$ (b) $4a + 3b + 6b + 3a$
 (c) $4r + 3t + 2r + 6t + r$ (d) $8u + w + 5u + w$
 (e) $6y + 3z + y - z + 4z - 9y$ (f) $10k + 7m - 7k - 4m - 8k - 3m$
 (g) $6w + 13h + 5w - 7h - 4w$ (h) $4a + 5b - 11a - 6b + b - 7a$
 (i) $-2x - 3x - 5y - y - 5x + 6y$ (j) $-3m - 4m + 10m - 3m$
 (k) $3k^2l - 2lk^2 + 2kl^2 - 5kl^2$ (l) $-ab^2 + 4ba^2 - 2a^2b + 6b^2a$
 (m) $-2xyz + 5xzy - 6yxz + 4zxy$ (n) $2x^2y - 3y^2x + 3xy^2 - 5xy^2$
 (o) $3a^2b + 5a^2 - 2b - 2ba^2 + 6b - a^2$
 (p) $-3m^2n + 5n^2m + 2nm^2 - 3mn^2 - 2n^2m + m^2n$

7. Find the **perimeter** of the following shapes.



8. Darnelle earned x dollars per hour. How much would he earn if he worked the following times?

(a) 5 hours (b) 20 hours one week and 15 hours the next week

9. Simplify the following expressions.

Examples

1. $3x \times 2y$
 $= 6xy$

2. $5a \times 4a$
 $= 20a^2$

3. $2p \times 6pq \times 3qr$
 $= 36p^2q^2r$

4. $4c \times 2b \times 5ac$
 $= 40abc^2$

5. $2y \times x \times 3z \times y \times x \times 2z$
 $= 12x^2y^2z^2$

*Always write answers
in alphabetical order*

(a) $5a \times 2b$

(b) $4c \times 2b \times a$

(c) $3y \times 2x \times z$

(d) $4d \times 2d$

(e) $5p \times 6q$

(f) $6u \times 3v \times t$

(g) $3r \times 2q \times 3s$

(h) $2ab \times 3a \times 4b$

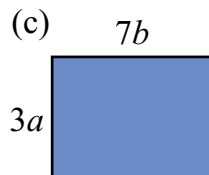
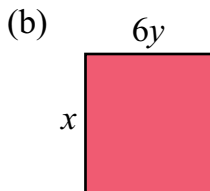
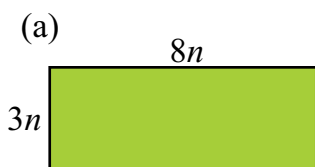
(i) $2xy \times 3yx \times 5z$

(j) $2bc \times 3ac \times 10ba$

(k) $2d \times 3f \times 6e \times 7g$

(l) $3mn \times 2mn \times 2 \times 2l$

10. Find the **area** of the following shapes.



11. Charley could pick a apples per hour. How many apples would she pick in the following times?

(a) 4 hours (b) y hours (c) 5 hours a day for z days

Expanding Algebraic Expressions

EXERCISE 6B

1. Expand the following algebraic expressions.

Examples

Each term in the bracket is multiplied by the term outside the bracket.

$$\begin{aligned} 1. \quad & 3(x + 5) \\ &= 3 \times x + 3 \times 5 \\ &= \mathbf{3x + 15} \end{aligned}$$

$$\begin{aligned} 2. \quad & -4(2a - 5) \\ &= -4 \times 2a + -4 \times \textcircled{-5} \\ &= \mathbf{-8a + 20} \end{aligned}$$

The minus sign makes the number negative.

- | | | |
|-------------------|-------------------|------------------------|
| (a) $3(y + 2)$ | (b) $2(x - 4)$ | (c) $4(m - 6)$ |
| (d) $7(a - 3)$ | (e) $3(4 + 5m)$ | (f) $2(8 + 3d)$ |
| (g) $3(m - n)$ | (h) $6(3p + q)$ | (i) $7(2t + 3u)$ |
| (j) $-3(x + 3)$ | (k) $-5(a + 7)$ | (l) $-8(4 + p)$ |
| (m) $-5(a + b)$ | (n) $-2(6 + 3z)$ | (o) $-7(4y + 5z)$ |
| (p) $-3(x - 5)$ | (q) $-4(a - 6)$ | (r) $-3(2y - 3)$ |
| (s) $-2(3a - 5b)$ | (t) $6(4m - 3n)$ | (u) $5(2a - 3b + 4c)$ |
| (v) $-4(7x - 2y)$ | (w) $-5(3g + 6h)$ | (x) $-4(3x - 2y - 5z)$ |

2. Expand the following algebraic expressions.

Examples

$$\begin{aligned} 1. \quad & x(3x - 1) \\ &= x \times 3x + x \times -1 \\ &= \mathbf{3x^2 - x} \end{aligned}$$

$$\begin{aligned} 2. \quad & -3y(2x - 5y) \\ &= -3y \times 2x + -3y \times -5y \\ &= \mathbf{-6xy + 15y^2} \end{aligned}$$

- | | | |
|---------------------|---------------------|-------------------------|
| (a) $a(2a + 3)$ | (b) $m(2n + 5p)$ | (c) $x(3y - 2z)$ |
| (d) $2p(3q - 7)$ | (e) $2x(3x - 5y)$ | (f) $3a(6b + 7c)$ |
| (g) $5m(3m - 2n)$ | (h) $2b(b - 7c)$ | (i) $2t(3t - 10u)$ |
| (j) $-x(2x + y)$ | (k) $-3a(2a + 3c)$ | (l) $-2x(3a + 2b)$ |
| (m) $-2a(4b - 5)$ | (n) $-3m(2p - 3q)$ | (o) $-5v(2w - 5z)$ |
| (p) $-8m(3n - 2p)$ | (q) $-4d(2a + 3x)$ | (r) $-3a(2a - 5b - 2c)$ |
| (s) $-5a(4a - 2ab)$ | (t) $-4t(3n - 4t)$ | (u) $-2x(5x - 2y + 3z)$ |
| (v) $3x(2xy - 3yz)$ | (w) $-3xy(2x + 3y)$ | (x) $2m(3nm - 2m - 5n)$ |

3. Expand the following algebraic expressions and simplify, where possible, by collecting like terms.

Examples

$$\begin{aligned} 1. \quad & 2(x + 5) + 3(x - 4) \\ &= 2x + 10 + 3x - 12 \\ &= \mathbf{5x - 2} \end{aligned}$$

$$\begin{aligned} 2. \quad & a(2b + 3) + 2a(5 - 3b) \\ &= 2ab + 3a + 10a - 6ab \\ &= \mathbf{-4ab + 13a} \\ & \text{(or } \mathbf{13a - 4ab}) \end{aligned}$$

$$\begin{aligned} 3. \quad & 3m(2m + 3n) - 2n(3m - n) \\ &= 6m^2 + 9mn - 6mn + 2n^2 \\ &= \mathbf{6m^2 + 3mn + 2n^2} \end{aligned}$$

(a) $3(a + 5) + 2(a + 1)$

(c) $5(y + 2) + 4(y - 2)$

(e) $3(x - 5) + 2(x + 1)$

(g) $4(n + 3) - 2(n + 5)$

(i) $y(2y + 3) + 3y(3y - 4)$

(k) $2x(5x - 2) - x(2x - 3)$

(m) $5a(2a + 7) - 4a(3a - 2)$

(o) $2m(6 - 5m) - 3m(2m + 1)$

(q) $8x(3y + 2z) - 3x(5z - 3y)$

(s) $-3a(2a - 2b) - 2b(3a - 2b)$

(b) $2(x + 3) + 3(x + 2)$

(d) $3(m + n) + 2(2m + n)$

(f) $4(y - 3) + 3(2 - y)$

(h) $5(p + 4) - 3(p + 3)$

(j) $2a(a + 4b) + 3a(2a - b)$

(l) $4m(2 - 3n) - 3m(5n - 2)$

(n) $5x(3x - 2y) - 3x(2y - 3x)$

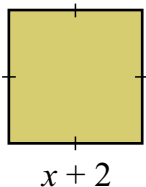
(p) $2c(2c + 3d) + 3c(3d - 2c)$

(r) $3m(4p - 2q) + 2m(3q - 5p)$

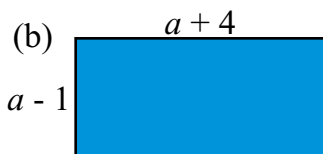
(t) $-5x(2y - 3x) - 6y(4x - 3y)$

4. Find the *perimeter* of these shapes.

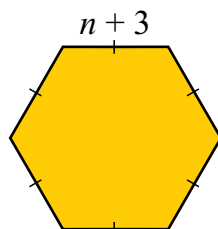
(a)



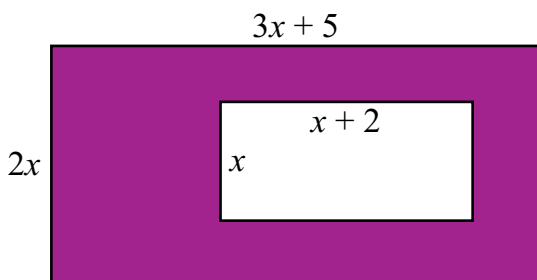
(b)



(c)



5. Find the *area* of the shaded region in this shape.



Expand the following expressions and collect the like terms.

Examples

1. $(x + 3)(x + 5)$

When expanding two brackets, each term in the second bracket must be multiplied by each term in the first bracket.

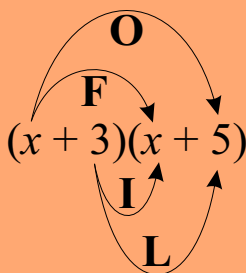
The acronym **FOIL** can be used to help.

First - multiply the **first** term in each bracket

Outer - multiply the **outer** two terms

Inner - multiply the **inner** two terms

Last - multiply the **last** term in each bracket



$$\begin{aligned}
 &(x + 3)(x + 5) \\
 &= x \times x + x \times 5 + 3 \times x + 3 \times 5 \\
 &= x^2 + 5x + 3x + 15 \\
 &= \mathbf{x^2 + 8x + 15}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad &(3y + 5)(2y - 4) \\
 &= 3y \times 2y + 3y \times -4 + 5 \times 2y + 5 \times -4 \\
 &= 6y^2 - 12y + 10y - 20 \\
 &= \mathbf{6y^2 - 2y - 20}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad &(2m - 5n)(3m - 7n) \\
 &= 2m \times 3m + 2m \times -7n + -5n \times 3m + -5n \times -7n \\
 &= 6m^2 - 14mn - 15mn + 35n^2 \\
 &= \mathbf{6m^2 - 29mn + 35n^2}
 \end{aligned}$$

6. Expand and simplify the following expressions.

- | | | |
|----------------------|----------------------|-----------------------|
| (a) $(x + 2)(x + 3)$ | (b) $(y + 5)(y + 1)$ | (c) $(m - 6)(m - 2)$ |
| (d) $(y - 3)(y - 8)$ | (e) $(n + 7)(n - 3)$ | (f) $(x + 10)(x + 5)$ |
| (g) $(w + 2)(w - 4)$ | (h) $(p + 6)(p - 3)$ | (i) $(y - 3)(y - 4)$ |
| (j) $(y + 3)(y - 1)$ | (k) $(m + 4)(m - 3)$ | (l) $(n - 7)(n - 10)$ |
| (m) $(y - 3)(y + 3)$ | (n) $(m - 1)(m + 1)$ | (o) $(p + 6)(p - 6)$ |

7. Expand and simplify the following expressions.

- | | |
|------------------------|------------------------|
| (a) $(2x + 1)(3x + 1)$ | (b) $(3m + 2)(2m + 3)$ |
| (c) $(2x - 3)(5 + 3x)$ | (d) $(2p - 3)(5p - 1)$ |
| (e) $(5c + 2)(c + 3)$ | (f) $(3m - 2)(5 + 2m)$ |
| (g) $(3m - 5)(2m - 1)$ | (h) $(4n + 3)(n - 8)$ |
| (i) $(2c + 5)(3c - 2)$ | (j) $(5d - 6)(2d + 3)$ |
| (k) $(2y - 5)(3y - 2)$ | (l) $(4a - 7)(2a - 7)$ |

8. Expand and simplify the following expressions.

- | | |
|------------------------------|----------------------------|
| (a) $(2x + 3y)(3x + y)$ | (b) $(3a + 2b)(2a + b)$ |
| (c) $(x + y)(2x - y)$ | (d) $(a + b)(a - b)$ |
| (e) $(3m + 2n)(3m - 5n)$ | (f) $(2p - 3q)(4p - 5q)$ |
| (g) $(3w + 2z)(3w - 2z)$ | (h) $(5n + 2p)(3n - 5p)$ |
| (i) $(4x - 5y)(3x - 2y)$ | (j) $(3x + 7y)(2x - 6y)$ |
| (k) $(3xy - 2ab)(3xy + 2ab)$ | (l) $(10a + 7b)(10a - 7b)$ |

9. Expand and simplify the following expressions.

Examples

$$\begin{aligned}
 1. \quad & (x + 3)^2 \\
 &= (x + 3)(x + 3) \\
 &= x^2 + 3x + 3x + 9 \\
 &= x^2 + 6x + 9
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (3a - 4b)^2 \\
 &= (3a - 4b)(3a - 4b) \\
 &= 9a^2 - 12ab - 12ab + 16b^2 \\
 &= 9a^2 - 24ab + 16b^2
 \end{aligned}$$

- | | | |
|-------------------|-------------------|--------------------|
| (a) $(x + 5)^2$ | (b) $(y + 7)^2$ | (c) $(a - 2)^2$ |
| (d) $(m - 4)^2$ | (e) $(2n + 3)^2$ | (f) $(3a + 5)^2$ |
| (g) $(5t - 2)^2$ | (h) $(4d - 7)^2$ | (i) $(8n - 3)^2$ |
| (j) $(a + b)^2$ | (k) $(n - m)^2$ | (l) $(x + y)^2$ |
| (m) $(2x + 3y)^2$ | (n) $(4m - 3n)^2$ | (o) $(2p + 5q)^2$ |
| (p) $(7a - 3b)^2$ | (q) $(8x + 3y)^2$ | (r) $(10p - 3q)^2$ |

10. Expand the following expressions.

Example

$$(3x^2 + 2y)(2x - 5y^3) \\ = 6x^3 - 15x^2y^3 + 4xy - 10y^4$$

- | | |
|------------------------------------|--|
| (a) $(2a + b)(4a^2 + 3b^2)$ | (b) $(x - 5y)(3x^2 - 4y^2)$ |
| (c) $(5m + 2n^2)(3m^2 - 4n^2)$ | (d) $(mn + 2m^2)(3m + 2mn)$ |
| (e) $(3xy - 2xy^2)(3x^2y + 5xy)$ | (f) $(a^2b + 2ab^2)(ab - 3a^2b)$ |
| (g) $(4m^2n - 5mn^2)(3mn + 2m^2n)$ | (h) $(2x^2y^2 + 3xy^2)(4x^2y - 3xy^2)$ |

11. Expand and simplify (where possible) the following expressions.

Examples

$$\begin{aligned} 1. \quad & 3(x + 5)(x - 7) \\ & = (3x + 15)(x - 7) \\ & = 3x^2 - 21x + 15x - 105 \\ & = 3x^2 - 6x - 105 \end{aligned}$$

$$\begin{aligned} 2. \quad & 2a(3a - 2b)(a + 5b) \\ & = (6a^2 - 4ab)(a + 5b) \\ & = 6a^3 + 30a^2b - 4a^2b - 20ab^2 \\ & = 6a^3 + 26a^2b - 20ab^2 \end{aligned}$$

- | | |
|-----------------------------------|-----------------------------------|
| (a) $5(a - 2)(a - 4)$ | (b) $2(m + 3)(m + 4)$ |
| (c) $6(x + 7)(x - 3)$ | (d) $3(y - 6)(y + 6)$ |
| (e) $2(2n + 3m)(3n - m)$ | (f) $3(2c - 3d)(2c - 3d)$ |
| (g) $2x(x + 2y)(3x - 2y)$ | (h) $3m(2m + 3n)(3m + n)$ |
| (i) $2a(3a - 5b)(2a - 3b)$ | (j) $5x(2x + 5y)(3x - 4y)$ |
| (k) $-2x(3xy + 2x^2)(2x^2 - 5xy)$ | (l) $3m(4m^2 + 3mn^2)(2n^2 + mn)$ |

12. Expand and simplify the following expressions.

Example

$$\begin{aligned} & 3(x + 4)^2 \\ & = 3(x + 4)(x + 4) \\ & = (3x + 12)(x + 4) \\ & = 3x^2 + 12x + 12x + 48 \\ & = 3x^2 + 24x + 48 \end{aligned}$$

- | | |
|--------------------|---------------------|
| (a) $2(x + 5)^2$ | (b) $3(y - 2)^2$ |
| (c) $4(a + 1)^2$ | (d) $2(m - 6)^2$ |
| (e) $a(a + b)^2$ | (f) $m(2m - 3n)^2$ |
| (g) $2m(3m + 2)^2$ | (h) $3x(2x - 3y)^2$ |
| (i) $3n(6 - 2n)^2$ | (j) $5a(2a + 3b)^2$ |

13. Expand and simplify the following expressions.

Examples

$$\begin{aligned} 1. \quad & (x+2)(x^2+4x+5) \\ &= x^3 + 4x^2 + 5x + 2x^2 + 8x + 10 \\ &= x^3 + 6x^2 + 13x + 10 \end{aligned}$$

$$\begin{aligned} 2. \quad & (x^2+6x+2)(x-3) \\ &= x^3 - 3x^2 + 6x^2 - 18x + 2x - 6 \\ &= x^3 + 3x^2 - 16x - 6 \end{aligned}$$

(a) $(x+1)(x^2+3x+5)$

(b) $(a+2)(a^2+4a+3)$

(c) $(n-3)(n^2+5n+6)$

(d) $(y+2)(y^2-3y+4)$

(e) $(m-1)(m^2-3m-1)$

(f) $(c+4)(c^2-c-7)$

(g) $(2x+3)(x^2+7x-3)$

(h) $(3a-4)(a^2-3a+5)$

(i) $(2m+4)(2m^2+3m-2)$

(j) $(3n-2)(2n^2+5n-7)$

(k) $(x^2+3x-5)(x+6)$

(l) $(n^2-4n+5)(n+6)$

(m) $(a^2-7a+4)(a+5)$

(n) $(m^2-m-4)(m-3)$

(o) $(x^2+4x+6)(3x+5)$

(p) $(c^2-9c+5)(2c-1)$

(q) $(2m^2+3m+1)(3m+4)$

(r) $(3x^2-x+5)(3x-2)$

(s) $(5a^2-6a-3)(2a+5)$

(t) $(4n^2+2n-3)(5n-2)$

14. Expand and simplify the following expressions.

Example

$$\begin{aligned} & (x+3)(x+5)(x-7) \\ &= (x^2+5x+3x+15)(x-7) \\ &= (x^2+8x+15)(x-7) \\ &= x^3-7x^2+8x^2-56x+15x-105 \\ &= x^3+x^2-41x-105 \end{aligned}$$

(a) $(x+2)(x+3)(x+1)$

(b) $(x+4)(x+2)(x-1)$

(c) $(a+2)(a-5)(a-3)$

(d) $(m-3)(m+5)(m-2)$

(e) $(n+4)(n-6)(n-3)$

(f) $(c-5)(c-2)(c-3)$

(g) $(x+6)(x-7)(x+2)$

(h) $(a+8)(a-10)(a+1)$

(i) $(m+3)(m+2)^2$

(j) $(n-4)(n+3)^2$

(k) $(p-2)(p-3)^2$

(l) $(r+4)(r-2)^2$

(m) $(a+2)^2(a-3)$

(n) $(n-3)^2(n+4)$

(o) $(2a+1)(a-3)(2a+3)$

(p) $(3x+2)(2x-5)(x-2)$

(q) $(3n-2)(2n+5)(2n-1)$

(r) $(5n-2)(3n+4)(2n-3)$

(s) $(2a+5)(3a+2)^2$

(t) $(4x-1)^2(5x-2)$

(u) $(x+2)^3$

(v) $(n-3)^3$

(w) $(2m-5)^3$

(x) $(3a+4)^3$

Factorisation

EXERCISE 6C

1. List **all** the factors of the following terms.

Examples

1. 12

Factors: 1, 2, 3, 4, 6, 12

2. $15x$

Factors: 1, 3, 5, 15, x

3. $20ab^2$

Factors: 1, 2, 4, 5, 10, 20, a , b , b^2

4. $-6xy$

Factors: -1, 1, 2, 3, 6, x , y

- (a) 10 (b) $6x$ (c) $18a$ (d) $36x^2$ (e) $24pq^2$
 (f) $-18m$ (g) $30x^2yz$ (h) $-28c^2$ (i) $-27pq$ (j) $-48m^2n^2$

2. Find the highest common factor (HCF) in the following sets of terms.

Examples

1. 8, 12

The factors of 8 are: 1, 2, 4, 8

The factors of 12 are: 1, 2, 3, 4, 6, 12

The highest common factor = 4

2. $-9a$, $-12ab$

The factors of $-9a$ are: -1, 1, 3, 9, a

The factors of $-12ab$ are: -1, 1, 2, 3, 4, 6, 12, a , b

The highest common factor = **$-3a$**

3. $8x$, $10y$

HCF = 2

4. $3x^2y$, xy^2

HCF = **xy**

5. $12x^2$, $-36x$

HCF = **$12x$**

6. $-12ab$, $-18b$

HCF = **$-6b$**

- (a) 16, 20 (b) 18, 27 (c) 20, 30 (d) x^2 , $3x$
 (e) $6a$, $4b$ (f) $12m$, $-16n$ (g) -28, -16 (h) $6x$, $-8x$
 (i) xy , $3x$ (j) ab , $4b$ (k) $12m$, $16mn$ (l) $-15x$, $-25x^2$
 (m) $10x^2$, $-16x$ (n) $12a^2$, $20a$ (o) x^2y , $-2xy$ (p) $4a^2$, $8a$
 (q) $-4x^2$, $-6x$ (r) $24m^2$, $30m$ (s) $-pq$, $-q^2$ (t) $-6m^2$, $-9m$
 (u) $-32g^2$, $16g$ (v) $54ab^2$, $36a^2b^2$ (w) $-8a^2b$, $-24a^2b$

3. Factorise the following expressions by finding the highest common factor (HCF).

Example $6a + 8$ (HCF = 2)
 $= 2 \times 3a + 2 \times 4$
 $= 2(3a + 4)$

- (a) $8p + 10$ (b) $6c - 10$ (c) $12x + 16$ (d) $4a - 14$
 (e) $18 + 12m$ (f) $6p + 12q$ (g) $5m - 15n$ (h) $18q + 27r$

4. Factorise the following expressions by finding the highest common factor (HCF).

Example $12xy - 18x$ (HCF = $6x$)
 $= 6x \times 2y - 6x \times 3$
 $= 6x(2y - 3)$

- (a) $8pq - 12p$ (b) $12xy + 16y$ (c) $3cd + 6c$ (d) $16mn - 20m$
 (e) $2ab + 24a$ (f) $4a + 28ab$ (g) $6x - 18xy$ (h) $25ab - 15b$
 (i) $6abc + 8ab$ (j) $5xyz + 10yz$ (k) $16mnp + 24mp$

5. Factorise the following expressions by finding the highest common factor (HCF).

Example $-10m - 20n$ (HCF = -10)
 $= -10 \times m - 10 \times 2n$
 $= -10(m + 2n)$

- (a) $-9p - 12q$ (b) $-4m - 6n$ (c) $-3a - 12b$ (d) $-4a - 8$
 (e) $-6x - 10y$ (f) $-10b - 6c$ (g) $-20p - 24q$ (h) $-8x - 18$
 (i) $-24 - 12m$ (j) $-6x - 3$ (k) $-36a - 18b$ (l) $-32x - 24y$

6. Factorise the following expressions by finding the highest common factor (HCF).

Examples 1. $3a + 4a^2$ (HCF = a) 2. $-6m^2n - 18m$ (HCF = $-6m$)
 $= a \times 3 + a \times 4a$ $= -6m \times mn + -6m \times 3$
 $= a(3 + 4a)$ $= -6m(mn + 3)$

- (a) $6n^2 + 8n$ (b) $2a^2 - 4a$ (c) $6p^2 - 20p$ (d) $10p^2 + 14pq$
 (e) $6r^2 - 2r$ (f) $8m^2 + 4m$ (g) $12mn + 16m^2$ (h) $-10x^2 - 15x$
 (i) $-6b - 18b^2$ (j) $-2x^2 - 16x$ (k) $-6m^2 - 10mn$ (l) $-14x^2y - 7xy$

Factorising by Grouping

EXERCISE 6D

1. Factorise the following expressions.

Example $2(x + 3) + y(x + 3)$ The common factor is $(x + 3)$
 $= (x + 3)(2 + y)$

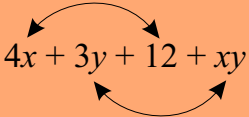
- (a) $2(a + 4) + b(a + 4)$ (b) $4(b - 2) - c(b - 2)$
 (c) $5(d + 3) - c(d + 3)$ (d) $3(x - y) + z(x - y)$
 (e) $7(m + 4) - n(m + 4)$ (f) $6(p - 1) + q(p - 1)$

2. Factorise the following expressions.

Example $\underbrace{3x + 6}_{\text{HCF} = 3} + \underbrace{xy + 2y}_{\text{HCF} = y}$
 $= 3(x + 2) + y(x + 2)$ The common factor is $(x + 2)$
 $= (x + 2)(3 + y)$

- (a) $2a + 6 + ab + 3b$ (b) $3m - 6 + mn - 2n$
 (c) $5p + 10 + pq + 2q$ (d) $4x + 12 + xy + 3y$
 (e) $6a + 9 + 2ab + 3b$ (f) $6p + 6 + pq + q$

3. Factorise the following expressions by grouping the terms so the common factors can be found.

Example $4x + 3y + 12 + xy$

 $= 4x + 12 + 3y + xy$
 $= 4(x + 3) + y(3 + x)$ ← $(x + 3) = (3 + x)$
 $= (x + 3)(4 + y)$

These two pairs of terms have common factors. Rearrange the expression so these two pairs are together.

- (a) $3y + 5z + yz + 15$ (b) $2b + 7c + 14 + cb$
 (c) $5m - 3n - 15 + nm$ (d) $4c + 5d + 20 + cd$
 (e) $3n - m - 3 + mn$ (f) $4y - 3x - 12 + xy$
 (g) $ab - 6 + 3b - 2a$ (h) $mn + 20 + 5n + 4m$
 (i) $x^2 + 3y + 3x + xy$ (j) $ab + 21 + 3a + 7b$

4. Factorise the following expressions.

Example $\underbrace{3x + 12}_{\text{HCF} = 3} - \underbrace{xy - 4y}_{\text{HCF} = -y}$
 $= 3(x + 4) - y(x + 4)$ The common factor is $(x + 4)$
 $= (x + 4)(3 - y)$

- (a) $4m + 8 - nm - 2n$ (b) $3b + 15 - cb - 5c$
 (c) $2x + 8 - xy - 4y$ (d) $mn + 3m - 3n - 9$
 (e) $ab + 5a - 4b - 20$ (f) $gh + 3g - 7h - 21$
 (g) $cd + 4c - 6d - 24$ (h) $4p + 20 - qp - 5q$
 (i) $x^2 + 2x - xy - 2y$ (j) $a^2 + ab - 3a - 3b$
 (k) $n^2 + 5n - pn - 5p$ (l) $6w + w^2 - 6y - wy$

5. Factorise the following expressions.

Examples 1. $4x - 8 - xy + 2y$ 2. $5m + 3n - 15 - mn$
 $= 4(x - 2) - y(x - 2)$ $= 5m - 15 + 3n - mn$
 $= (x - 2)(4 - y)$ $= 5(m - 3) - n(-3 + m)$
 $= 5(m - 3) - n(m - 3)$
 $= (m - 3)(5 - n)$

- (a) $3a - 9 - ab + 3b$ (b) $2m - 10 - nm + 5n$
 (c) $4p - 4 - pq + q$ (d) $3x - 12 - xy + 4y$
 (e) $2y - 4 - yz + 2z$ (f) $5b - 15 - cb + 3c$
 (g) $6m - 30 - mn + 5n$ (h) $4a - 12 - ab + 3b$
 (i) $3a + 5b - 15 - ab$ (j) $6n + 3p - 18 - np$
 (k) $x^2 + 3y - 3x - xy$ (l) $a^2 - ab - 4a + 4b$

6. Factorise the following expressions.

- (a) $mn + 3m + 3n + 9$ (b) $ab - 2a - 7b + 14$
 (c) $3a + 18 - 6c - ac$ (d) $pq - 56 + 7p - 8q$
 (e) $ab + dc + ac + db$ (f) $nm - 10m + 50 - 5n$
 (g) $15 - 5a - 3b + ab$ (h) $3ab + 6ac - 7b - 14c$
 (i) $x^2 + 5x - xy - 5y$ (j) $a^2 + 3b - ab - 3a$
 (k) $pq + 10 - 5q - 2p$ (l) $wz - yz + 6y - 6w$
 (m) $b^2 - cb - cd + db$ (n) $cd - 5c - 6d + 30$
 (o) $2xy + 3x^2 - 2yz - 3xz$ (p) $6a^2 - 15a - 10b + 4ab$
 (q) $4pq - 8p^2 + 5q - 10p$ (r) $amp + a^2b + bmp + ab^2$

Factorising the Difference of Perfect Squares (DOPS)

EXERCISE 6E

The factors of algebraic expressions of the form $a^2 - b^2$ are $(a - b)$ and $(a + b)$.

$$a^2 - b^2 = (a - b)(a + b)$$

Examples

$$\begin{aligned} 1. \quad x^2 - y^2 \\ = (x - y)(x + y) \end{aligned}$$

$$\begin{aligned} 2. \quad 16 - a^2 \\ = 4^2 - a^2 \\ = (4 - a)(4 + a) \end{aligned}$$

1. Factorise the following expressions.

(a) $m^2 - n^2$

(b) $A^2 - B^2$

(c) $v^2 - w^2$

(d) $y^2 - 9$

(e) $25 - c^2$

(f) $x^2 - 49$

(g) $p^2 - 100$

(h) $g^2 - 16$

(i) $b^2 - 1$

2. Factorise the following expressions.

Example

$$\begin{aligned} 4x^2 - 25y^2 \\ = (2x)^2 - (5y)^2 \\ = (2x - 5y)(2x + 5y) \end{aligned}$$

(a) $4a^2 - 9b^2$

(b) $16p^2 - 49q^2$

(c) $100x^2 - 81y^2$

(d) $9n^2 - m^2$

(e) $64g^2 - 121h^2$

(f) $49a^2 - 64b^2$

(g) $25x^2 - y^2$

(h) $36a^2 - b^2$

(i) $169n^2 - 225m^2$

3. Factorise the following expressions.

Example

$$\begin{aligned} 8x^2 - 32y^2 \\ = 8(x^2 - 4y^2) \\ = 8[x^2 - (2y)^2] \\ = 8(x - 2y)(x + 2y) \end{aligned}$$

(a) $27n^2 - 3m^2$

(b) $32a^2 - 2b^2$

(c) $24x^2 - 54y^2$

(d) $8a^2 - 18b^2$

(e) $50g^2 - 2h^2$

(f) $32p^2 - 98q^2$

(g) $20x^2 - 5y^2$

(h) $200a^2 - 8b^2$

(i) $45n^2 - 20m^2$

4. Factorise and simplify the following expressions.

Example

$$\begin{aligned}(x-1)^2 - 9 &= [(x-1) - 3][(x-1) + 3] \\ &= (x-1-3)(x-1+3) \\ &= (\mathbf{x-4})(\mathbf{x+2})\end{aligned}$$

- | | | |
|---------------------|--------------------|--------------------|
| (a) $(x-3)^2 - 4$ | (b) $(a+1)^2 - 16$ | (c) $(n-3)^2 - 25$ |
| (d) $(m+6)^2 - 1$ | (e) $(y-5)^2 - 25$ | (f) $(x+9)^2 - 81$ |
| (g) $(c-7)^2 - 100$ | (h) $(n+4)^2 - 49$ | (i) $(y-8)^2 - 64$ |

5. Factorise and simplify the following expressions.

Example

$$\begin{aligned}(x-1)^2 - (x+4)^2 &= [(x-1) - (x+4)][(x-1) + (x+4)] \\ &= (x-1-x-4)(x-1+x+4) \\ &= \mathbf{-5(2x+3)}\end{aligned}$$

- | | |
|-------------------------|-------------------------|
| (a) $(x+3)^2 - (x+2)$ | (b) $(a+5)^2 - (a-2)^2$ |
| (c) $(n-4)^2 - (n-2)^2$ | (d) $(m+5)^2 - (m+7)^2$ |
| (e) $(c-3)^2 - (c-6)^2$ | (f) $(x-3)^2 - (x-8)^2$ |
| (g) $(y+1)^2 - (y-1)^2$ | (h) $(a-b)^2 - (a+b)^2$ |

6. Factorise and simplify the following expressions.

Example

$$\begin{aligned}16(x+3)^2 - 25(x+2)^2 &= [4(x+3)]^2 - [5(x+2)]^2 \\ &= [4(x+3) - 5(x+2)][4(x+3) + 5(x+2)] \\ &= (4x+12-5x-10)(4x+12+5x+10) \\ &= \mathbf{(-x+2)(9x+22)}\end{aligned}$$

- | | |
|------------------------------|-------------------------------|
| (a) $9(x+1)^2 - 4(x+2)^2$ | (b) $25(a-2)^2 - 16(a+3)^2$ |
| (c) $49(n-3)^2 - 36(n-4)^2$ | (d) $9(m+2)^2 - 81(m-5)^2$ |
| (e) $64(y+5)^2 - 100(y-2)^2$ | (f) $144(x+2)^2 - 64(x+4)^2$ |
| (g) $16(a+3)^2 - 121(a-5)^2$ | (h) $169(n-4)^2 - 225(n-2)^2$ |

7. Factorize the following expressions.

Example

$$\begin{aligned} & x^2 - 11 \\ &= x^2 - (\sqrt{11})^2 \\ &= (x - \sqrt{11})(x + \sqrt{11}) \end{aligned}$$

(a) $x^2 - 5$

(b) $a^2 - 13$

(c) $c^2 - 2$

(d) $n^2 - 23$

8. Factorize the following expressions.

Example

$$\begin{aligned} & 3x^2 - 15 \\ &= 3(x^2 - 5) \\ &= 3[x^2 - (\sqrt{5})^2] \\ &= 3(x - \sqrt{5})(x + \sqrt{5}) \end{aligned}$$

(a) $2x^2 - 14$

(b) $5a^2 - 30$

(c) $3n^2 - 9$

(d) $7m^2 - 70$

9. Factorise the following expressions.

(a) $4x^2 - 7$

(b) $25a^2 - 21$

(c) $18x^2 - 10$

(d) $n^2 - 3m^2$

(e) $4n^2 - 5m^2$

(f) $7p^2 - q^2$

(g) $3x^2 - 5y^2$

(h) $8a^2 - 24b^2$

(i) $3x^2 - 45y^2$

10. Factorise the following expressions and simplify where possible.

(a) $2a^2 - 18$

(b) $25a^2 - 121b^2$

(c) $a^2b^2 - x^2y^2$

(d) $\frac{x^2}{4} - \frac{y^2}{16}$

(e) $25c^2 - \frac{4}{d^2}$

(f) $\frac{36x^2}{49} - \frac{64}{9y^2}$

(g) $\frac{x^2}{4} - 3$

(h) $\frac{m^2}{7} - \frac{n^2}{13}$

(i) $\frac{m^2}{7n^2} - \frac{5}{a^2}$

(j) $\frac{(x+3)^2}{25} - \frac{(y-4)^2}{49}$

(k) $\frac{9(x-5)^2}{25} - \frac{4(y+7)^2}{81}$

(l) $\frac{3x^2}{11} - \frac{7y^2}{51}$

(m) $3(n-5)^2 - 300$

(n) $5(b+2)^2 - 80$

(o) $22 - 46m^2$

(p) $x^4 - y^4$

(q) $\theta^2 - \phi^2$

(r) $21x^2 - 63y^2$

Factorising Quadratic Trinomials with $a = 1$

Quadratic trinomials are algebraic expressions of the general form:

$$ax^2 + bx + c$$

Quadratic expressions have the highest power = 2 (eg. x^2 , y^2 , etc)

Trinomial expressions have three terms.

The following steps can be used to find the factors of a quadratic trinomial with $a = 1$.

$$x^2 + bx + c$$

Step 1 Find two numbers that multiply to give c and add to give b .

Step 2 If these two numbers are m and n , then the factors are:

$$(x + m)(x + n)$$

EXERCISE 6F

Example $x^2 + 8x + 12$

Step 1 Two numbers that multiply to give 12 and add to give 8:

2 and 6

Step 2 The two factors are:

$$(x + 2)(x + 6)$$

1. Factorise the following quadratic trinomials.

(a) $x^2 + 5x + 6$

(b) $x^2 + 7x + 12$

(c) $n^2 + 9n + 14$

(d) $a^2 + 10a + 16$

(e) $p^2 + 11p + 18$

(f) $a^2 + 14a + 48$

(g) $y^2 + 8y + 16$

(h) $x^2 + 11x + 24$

(i) $n^2 + 10n + 24$

(j) $x^2 + 14x + 24$

(k) $a^2 + 11a + 10$

(l) $y^2 + 9y + 8$

Example

$$x^2 - 8x + 15$$

Step 1 Two numbers that multiply to give 15 and add to give -8:
-3 and -5**Step 2** The two factors are:

$$(x - 3)(x - 5)$$

2. Factorise the following quadratic trinomials.

(a) $x^2 - 7x + 12$

(b) $m^2 - 8m + 16$

(c) $a^2 - 11a + 24$

(d) $c^2 - 11c + 30$

(e) $z^2 - 12z + 32$

(f) $x^2 - 17x + 30$

(g) $n^2 - 15n + 36$

(h) $y^2 - 16y + 28$

Example

$$x^2 + 3x - 10$$

Step 1 Two numbers that multiply to give -10 and add to give 3:
5 and -2**Step 2** The two factors are:

$$(x + 5)(x - 2)$$

3. Factorise the following quadratic trinomials.

(a) $x^2 + 5x - 24$

(b) $a^2 + 7a - 30$

(c) $n^2 - 7n - 30$

(d) $c^2 - 8c - 20$

(e) $m^2 + 6m - 16$

(f) $y^2 + 4y - 12$

(g) $x^2 + 7x - 18$

(h) $a^2 - 5a - 24$

(i) $y^2 + 5y - 36$

(j) $x^2 - 3x - 40$

(k) $n^2 + n - 42$

(l) $a^2 + a - 12$

(m) $x^2 - x - 20$

(n) $g^2 - g - 30$

4. Factorise the following quadratic trinomials.

(a) $x^2 + 15x + 54$

(b) $c^2 - 13c + 40$

(c) $z^2 - 7z - 18$

(d) $a^2 - 12a + 35$

(e) $x^2 + 16x + 60$

(f) $n^2 - 15n + 56$

(g) $y^2 - 5y - 66$

(h) $b^2 - b - 72$

(i) $x^2 - 25x + 100$

(j) $m^2 - 20m + 99$

(k) $a^2 + 2a - 63$

(l) $x^2 - 23x + 132$

(m) $y^2 + 18y - 40$

(n) $r^2 + 12r + 32$

(o) $a^2 + 7a + 12$

(p) $m^2 - 3m - 18$

Factorising Quadratic Trinomials with $a \neq 1$

The following steps can be used to find the factors of a quadratic expression with $a \neq 1$.

$$ax^2 + bx + c$$

Step 1 Find two factors of a (a_1 and a_2) and two factors of c (c_1 and c_2).

Step 2 Arrange these factors as shown here.

$$\begin{array}{cc} a_1 & c_1 \\ a_2 & c_2 \end{array}$$

Step 3 'Cross multiply' these factors and add the products.

$$\begin{array}{cc} a_1 & c_1 \\ & \times \\ a_2 & c_2 \end{array}$$

$$(a_1 \times c_2) + (a_2 \times c_1)$$

Step 4 If the sum of the products is equal to b then the factors are:

$$(a_1x + c_1)(a_2x + c_2)$$

Step 5 If the sum of the products does not equal b then other factors and/or combinations need to be tried until it equals b .

Example 1 $2x^2 + 13x + 15$

Step 1 The factors of 2 are (1 and 2)
The factors of 15 are (1 and 15) and (3 and 5)

Step 2, The factors could be arranged in many ways. Try

Step 3 combinations until the sum of the products of the 'cross multiplication' equals 13.

Step 5 $\begin{array}{cc} 1 & 1 \\ 2 & 15 \end{array}$ sum of 'cross multiplication' = $15 + 2 = 17$

$\begin{array}{cc} 1 & 15 \\ 2 & 1 \end{array}$ sum of 'cross multiplication' = $1 + 30 = 31$

$\begin{array}{cc} 1 & 3 \\ 2 & 5 \end{array}$ sum of 'cross multiplication' = $5 + 6 = 11$

$\begin{array}{cc} 1 & 5 \\ 2 & 3 \end{array}$ sum of 'cross multiplication' = $3 + 10 = 13$

Step 4 Factors are: $(x + 5)(2x + 3)$

Example 2

$6x^2 + 11x - 10$

Step 1 The factors of 6 are (1 and 6) and (2 and 3).

The factors of -10 are (1 and -10), (-1 and 10), (2 and -5) and (-2 and 5).

Step 2, The factors could be arranged in many ways. Try

Step 3 combinations until the sum of the products of the
and 'cross multiplication' equals 11.

Step 5

$$\begin{array}{r} 1 \times 1 \\ 6 \times -10 \end{array} \text{ sum of 'cross multiplication' } = -10 + 6 = -4$$

$$\begin{array}{r} 1 \times -1 \\ 6 \times 10 \end{array} \text{ sum of 'cross multiplication' } = 10 - 6 = 4$$

$$\begin{array}{r} 1 \times 2 \\ 6 \times -5 \end{array} \text{ sum of 'cross multiplication' } = -5 + 12 = 7$$

$$\begin{array}{r} 1 \times -2 \\ 6 \times 5 \end{array} \text{ sum of 'cross multiplication' } = 5 - 12 = -7$$

$$\begin{array}{r} 2 \times 1 \\ 3 \times -10 \end{array} \text{ sum of 'cross multiplication' } = -20 + 3 = -17$$

$$\begin{array}{r} 2 \times -1 \\ 3 \times 10 \end{array} \text{ sum of 'cross multiplication' } = 20 - 3 = 17$$

$$\begin{array}{r} 2 \times 2 \\ 3 \times -5 \end{array} \text{ sum of 'cross multiplication' } = -10 + 6 = -4$$

$$\begin{array}{r} 2 \times -2 \\ 3 \times 5 \end{array} \text{ sum of 'cross multiplication' } = 10 - 6 = 4$$

$$\begin{array}{r} 2 \times 5 \\ 3 \times -2 \end{array} \text{ sum of 'cross multiplication' } = -4 + 15 = \mathbf{11}$$

Step 4 Factors are: $(2x + 5)(3x - 2)$

EXERCISE 6G

1. Factorise the following quadratic trinomials.

- | | |
|------------------------|------------------------|
| (a) $2x^2 + 7x + 6$ | (b) $2x^2 + 7x + 5$ |
| (c) $5x^2 + 49x + 36$ | (d) $3x^2 - 8x + 4$ |
| (e) $5x^2 - 8x + 3$ | (f) $3x^2 - 13x + 4$ |
| (g) $6x^2 + 25x + 14$ | (h) $8x^2 - 19x + 6$ |
| (i) $10x^2 + 33x + 20$ | (j) $15x^2 + 28x + 12$ |

2. Factorise the following quadratic trinomials.

- | | |
|------------------------|-----------------------|
| (a) $3x^2 + 13x - 10$ | (b) $4x^2 + 4x - 15$ |
| (c) $6x^2 + x - 12$ | (d) $6x^2 + 7x - 20$ |
| (e) $14x^2 + 31x - 10$ | (f) $6x^2 - 5x - 4$ |
| (g) $4x^2 - 13x - 12$ | (h) $12x^2 + 7x - 10$ |
| (i) $2x^2 - x - 6$ | (j) $6x^2 - 7x - 5$ |

3. Factorise the following quadratic trinomials.

- | | |
|------------------------|-------------------------|
| (a) $6x^2 - 29x + 28$ | (b) $6x^2 - 7x - 10$ |
| (c) $9x^2 + 6x - 8$ | (d) $12x^2 - 47x + 35$ |
| (e) $10x^2 - 39x + 36$ | (f) $24x^2 - 18x - 15$ |
| (g) $15x^2 + 47x - 10$ | (h) $12x^2 - 145x + 12$ |

Perfect Squares

EXERCISE 6H

1. Factorise the following quadratic trinomials.

- | | |
|----------------------|-----------------------|
| (a) $x^2 + 2x + 1$ | (b) $x^2 + 4x + 4$ |
| (c) $x^2 - 2x + 1$ | (d) $x^2 + 6x + 9$ |
| (e) $x^2 + 10x + 25$ | (f) $x^2 - 10x + 25$ |
| (g) $x^2 - 18x + 81$ | (h) $x^2 + 20x + 100$ |

2. All the quadratic trinomials in question 1 are perfect squares.

The general form of this type of perfect square is:

$x^2 + 2ax + a^2$	or	$x^2 - 2ax + a^2$
$= (x + a)^2$		$= (x - a)^2$

Complete the following perfect squares and factorise.

- | | |
|--------------------------|---------------------------|
| (a) $x^2 + \square + 36$ | (b) $x^2 - \square + 49$ |
| (c) $x^2 + 8x + \square$ | (d) $x^2 - 16x + \square$ |

Completing the Square

Many quadratic trinomials cannot be factorised by using the techniques used in exercises 6F and 6G.

Example 1 $x^2 + 6x + 2$

Many of these can be factorised by completing the perfect square of the first two terms and then using the difference of two squares. This technique is demonstrated using the steps shown below.

$$x^2 + 6x + 2$$

Step 1 Complete the square of the first two terms.

$$x^2 + 6x + 9$$

$\underbrace{x^2 + 6x + 2}_{\nearrow}$

The easiest way to complete the square is to halve the coefficient of x and square it.

$$6 \div 2 = 3$$

$$3^2 = 9$$

Step 2 Subtract the same amount that has been added to complete the square. This keeps the expression the same.

$$x^2 + 6x + 9 - 9 + 2$$

Step 3 Factorise the perfect square and simplify the remainder of the expression.

$$\begin{aligned} & (x^2 + 6x + 9) - 9 + 2 \\ &= (x + 3)^2 - 7 \end{aligned}$$

Step 4 Factorise as a difference of perfect squares (exercise 6E).

$$\begin{aligned} & (x + 3)^2 - 7 \\ &= (x + 3)^2 - (\sqrt{7})^2 \\ &= [(x + 3) - \sqrt{7}][(x + 3) + \sqrt{7}] \\ &= (x + 3 - \sqrt{7})(x + 3 + \sqrt{7}) \end{aligned}$$

Example 2 $x^2 - 12x - 5$

Step 1

and Step 2 $x^2 - 12x + 36 - 36 - 5$

Step 3 $(x^2 - 12x + 36) - 41$
 $= (x - 6)^2 - 41$

Step 4 $(x - 6)^2 - (\sqrt{41})^2$
 $= [(x - 6) - \sqrt{41}][(x - 6) + \sqrt{41}]$
 $= (x - 6 - \sqrt{41})(x - 6 + \sqrt{41})$

EXERCISE 6I

1. Factorise the following quadratic trinomials by completing the square.

- | | |
|-----------------------|----------------------|
| (a) $x^2 + 2x - 1$ | (b) $x^2 - 8x + 9$ |
| (c) $x^2 - 10x - 4$ | (d) $x^2 + 4x - 6$ |
| (e) $x^2 + 6x + 3$ | (f) $x^2 - 14x - 8$ |
| (g) $x^2 - 20x + 23$ | (h) $x^2 + 16x + 31$ |
| (i) $x^2 + 40x + 123$ | (j) $x^2 - 32x - 11$ |

2. Factorise the following quadratic trinomials by completing the square or by inspection if possible.

Example $x^2 + 22x + 112$

Step 1

and Step 2 $x^2 + 22x + 121 - 121 + 112$

Step 3 $(x^2 + 22x + 121) - 9$
 $= (x + 11)^2 - 9$

Step 4 $(x + 11)^2 - 3^2$
 $= [(x + 11) - 3][(x + 11) + 3]$
 $= (x + 8)(x + 14)$

- | | |
|-----------------------|-----------------------|
| (a) $x^2 + 22x + 117$ | (b) $x^2 + 20x + 91$ |
| (c) $x^2 - 24x + 128$ | (d) $x^2 - 20x + 36$ |
| (e) $x^2 + 28x + 195$ | (f) $x^2 - 26x + 165$ |

Algebraic Fractions

EXERCISE 6J

1. Simplify the following algebraic fractions.

Examples

$$\begin{aligned} 1. \quad & \frac{8a}{12} \\ &= \frac{2\cancel{8}a}{3\cancel{12}} \\ &= \frac{2a}{3} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{5x}{25x} \\ &= \frac{\cancel{5}x}{5\cancel{25}\cancel{x}} \\ &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{16b}{10b^2} \\ &= \frac{8\cancel{16}b}{5\cancel{10}b^2b} \\ &= \frac{8}{5b} \end{aligned}$$

(a) $\frac{6x}{8}$ (b) $\frac{10a}{12}$ (c) $\frac{5m}{15}$ (d) $\frac{8}{14y}$ (e) $\frac{24}{30n}$

(f) $\frac{6m}{20m}$ (g) $\frac{15a}{20a}$ (h) $\frac{18b}{24b}$ (i) $\frac{14x}{18x}$ (j) $\frac{12n}{48n}$

(k) $\frac{8x}{18x^2}$ (l) $\frac{21y^2}{7y}$ (m) $\frac{16a}{32a^2}$ (n) $\frac{20m}{25m^2}$ (o) $\frac{6nm}{14nm^2}$

2. Simplify the following algebraic fractions by first factorising the numerator and/or denominator and then cancelling.

Examples

$$\begin{aligned} 1. \quad & \frac{5x + 15}{10} \\ &= \frac{\cancel{5}(x + 3)}{\cancel{2}5} \\ &= \frac{x + 3}{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{4n}{n^2 - 3n} \\ &= \frac{4\cancel{n}}{\cancel{n}(n - 3)} \\ &= \frac{4}{n - 3} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{4c + 8}{6c + 12} \\ &= \frac{\cancel{2}4(c + 2)}{\cancel{3}6(c + 2)} \\ &= \frac{2}{3} \end{aligned}$$

(a) $\frac{2x - 4}{10}$ (b) $\frac{6y + 18}{12}$ (c) $\frac{8y - 24}{12}$ (d) $\frac{15}{5a + 20}$

(e) $\frac{12b - 60}{28}$ (f) $\frac{16}{10p + 30}$ (g) $\frac{6x + 6}{6}$ (h) $\frac{5}{5y - 25}$

(i) $\frac{12a - 36}{6}$ (j) $\frac{8}{8 - 4m}$ (k) $\frac{5x}{x^2 + x}$ (l) $\frac{3y}{y^2 - 4y}$

(m) $\frac{m^2 - 5m}{3m}$ (n) $\frac{4a^2 - 6a}{8a}$ (o) $\frac{12n}{6n + 12n^2}$ (p) $\frac{2m^2 - 4m}{3m - 6}$

(q) $\frac{6a + 24}{4a^2 + 16a}$ (r) $\frac{3x - 6}{4x - 8}$ (s) $\frac{2a + 10}{5a + 25}$ (t) $\frac{6n^2 - 30n}{3n - 15}$

3. Simplify the following algebraic fractions by first factorising the numerator and/or denominator and then cancelling.

Examples

$$\begin{aligned} 1. \quad & \frac{3x+6}{x^2+5x+6} \\ &= \frac{3(x+2)}{(x+3)(x+2)} \\ &= \frac{3}{x+3} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{2n^2+8n}{n^2-2n-24} \\ &= \frac{2n(n+4)}{(n-6)(n+4)} \\ &= \frac{2n}{n-6} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{a^2-7a+12}{a^2-9} \\ &= \frac{(a-4)(a-3)}{(a-3)(a+3)} \\ &= \frac{a-4}{a+3} \end{aligned}$$

$$(a) \quad \frac{4x+16}{x^2+9x+20}$$

$$(b) \quad \frac{5n+15}{n^2+7n+12}$$

$$(c) \quad \frac{2m^2+10m}{m^2+7m+10}$$

$$(d) \quad \frac{6n^2-18n}{n^2-10n+21}$$

$$(e) \quad \frac{4y-20}{y^2-6y+5}$$

$$(f) \quad \frac{c^2+2c-24}{5c-20}$$

$$(g) \quad \frac{n^2-8n+15}{n^2-9n+18}$$

$$(h) \quad \frac{a^2+8a+12}{a^2+10a+24}$$

$$(i) \quad \frac{p^2+5p+4}{p^2+7p+6}$$

$$(j) \quad \frac{x^2+9x+14}{x^2-3x-10}$$

$$(k) \quad \frac{x^2-2x-15}{x^2-x-20}$$

$$(l) \quad \frac{x^2+x-42}{x^2-4x-12}$$

$$(m) \quad \frac{x^2+12x+35}{x^2-25}$$

$$(n) \quad \frac{a^2-9a+18}{a^2-9}$$

$$(o) \quad \frac{x^2-36}{x^2+4x-60}$$

4. Simplify the following algebraic fractions.

Examples

$$\begin{aligned} 1. \quad & \frac{2x}{3} + \frac{5x}{3} \\ &= \frac{7x}{3} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{3a}{4} + \frac{7a}{4} \\ &= \frac{10a}{4} \\ &= \frac{5a}{2} \end{aligned}$$

$$(a) \quad \frac{3x}{5} + \frac{4x}{5}$$

$$(b) \quad \frac{2a}{3} + \frac{5a}{3}$$

$$(c) \quad \frac{m}{7} + \frac{2m}{7}$$

$$(d) \quad \frac{5c}{11} + \frac{c}{11}$$

$$(e) \quad \frac{5x}{8} + \frac{7x}{8}$$

$$(f) \quad \frac{3a}{4} + \frac{7a}{4}$$

$$(g) \quad \frac{m}{6} + \frac{5m}{6}$$

$$(h) \quad \frac{7i}{8} + \frac{9i}{8}$$

5. Simplify the following algebraic fractions by finding a common denominator before adding or subtracting.

Examples

$$\begin{aligned} 1. \quad & \frac{2x}{3} + \frac{5x}{6} \\ &= \frac{4x}{6} + \frac{5x}{6} \\ &= \frac{9x}{6} \\ &= \frac{3x}{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{4a}{3} - \frac{3a}{5} \\ &= \frac{20a}{15} - \frac{9a}{15} \\ &= \frac{11a}{15} \end{aligned}$$

(a) $\frac{3x}{5} + \frac{2x}{15}$ (b) $\frac{2y}{3} + \frac{5y}{9}$ (c) $\frac{2a}{5} + \frac{3a}{10}$ (d) $\frac{3n}{4} - \frac{5n}{12}$

(e) $\frac{a}{2} - \frac{3a}{8}$ (f) $\frac{4x}{5} - \frac{7x}{15}$ (g) $\frac{5n}{18} + \frac{2n}{9}$ (h) $\frac{9u}{14} - \frac{3u}{7}$

(i) $\frac{2x}{3} + \frac{3x}{4}$ (j) $\frac{3a}{5} + \frac{3a}{4}$ (k) $\frac{d}{2} + \frac{d}{3}$ (l) $\frac{m}{3} + \frac{m}{4}$

(m) $\frac{c}{3} - \frac{c}{5}$ (n) $\frac{2x}{3} + \frac{5x}{8}$ (o) $\frac{3n}{4} - \frac{2n}{7}$ (p) $\frac{9c}{8} - \frac{c}{5}$

6. Simplify the following algebraic fractions.

Examples

$$\begin{aligned} 1. \quad & \frac{5x+2}{2} + \frac{3x-5}{2} \\ &= \frac{5x+2+3x-5}{2} \\ &= \frac{8x-3}{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{2a+5}{3} + \frac{4a+7}{3} \\ &= \frac{2a+5+4a+7}{3} \\ &= \frac{6a+12}{3} \\ &= \frac{6(a+2)}{3} \\ &= 2(a+2) \end{aligned}$$

(a) $\frac{x+2}{3} + \frac{3x-1}{3}$ (b) $\frac{n+5}{2} + \frac{4n-3}{2}$ (c) $\frac{c-3}{5} + \frac{c+6}{5}$

(d) $\frac{2x+5}{4} + \frac{4x-1}{4}$ (e) $\frac{3a+5}{6} + \frac{6a+4}{6}$ (f) $\frac{6x+7}{3} + \frac{3x-1}{3}$

7. Simplify the following algebraic fractions by finding a common denominator before adding or subtracting.

Examples

$$\begin{aligned}
 1. \quad & \frac{2x+3}{2} + \frac{x+1}{4} \\
 &= \frac{2(2x+3)}{4} + \frac{x+1}{4} \\
 &= \frac{2(2x+3) + x+1}{4} \\
 &= \frac{4x+6+x+1}{4} \\
 &= \frac{5x+7}{4}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{10x+1}{6} - \frac{2x-3}{2} \\
 &= \frac{10x+1}{6} - \frac{3(2x-3)}{6} \\
 &= \frac{10x+1-3(2x-3)}{6} \\
 &= \frac{10x+1-6x+9}{6} \\
 &= \frac{4x+10}{6} \\
 &= \frac{2(2x+5)}{6} \\
 &= \frac{2x+5}{3}
 \end{aligned}$$

(a) $\frac{2x+3}{2} + \frac{x+1}{4}$

(b) $\frac{3n+2}{3} + \frac{2n+1}{6}$

(c) $\frac{7x+1}{6} - \frac{3x+1}{3}$

(d) $\frac{5a-1}{6} - \frac{a+1}{2}$

(e) $\frac{8x+3}{12} - \frac{2x+3}{6}$

(f) $\frac{10a+3}{15} - \frac{a+3}{5}$

(g) $\frac{7x+1}{2} + \frac{3x+1}{3}$

(h) $\frac{n+2}{3} + \frac{4n+1}{4}$

(i) $\frac{2x-1}{2} + \frac{3x+2}{5}$

(j) $\frac{x+1}{3} + \frac{2x-1}{5}$

(k) $\frac{3n-1}{2} - \frac{n+2}{3}$

(l) $\frac{2a+3}{4} - \frac{a+3}{3}$

(m) $\frac{3x-1}{5} - \frac{2x-1}{4}$

(n) $\frac{5x+2}{3} - \frac{3x-2}{2}$

(o) $\frac{6a+1}{5} - \frac{2a-3}{6}$

(p) $\frac{3x+1}{4} - \frac{2x-1}{7}$

(q) $\frac{3a+4}{8} + \frac{6a-5}{3}$

(r) $\frac{6x+8}{7} + \frac{3x-7}{5}$

(s) $\frac{3x-4}{9} - \frac{2x-3}{7}$

(t) $\frac{5x-2}{8} - \frac{3x-2}{6}$

(u) $\frac{6a+7}{9} - \frac{2a-3}{8}$

8. Simplify the following expressions by cancelling first.

Examples

$$\begin{aligned} 1. \quad & \frac{3x}{5} \times \frac{10}{9x} \\ &= \frac{\cancel{3}x}{\cancel{5}} \times \frac{10^{\cancel{2}}}{\cancel{9}x} \\ &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{x+1}{6} \times \frac{12}{x+1} \\ &= \frac{\cancel{x+1}}{\cancel{1}6} \times \frac{12^{\cancel{2}}}{\cancel{x+1}} \\ &= \frac{2}{1} \\ &= 2 \end{aligned}$$

(a) $\frac{4a}{7} \times \frac{21}{16a}$

(b) $\frac{3}{8n} \times \frac{4n}{9}$

(c) $\frac{9c}{10} \times \frac{5}{18c}$

(d) $\frac{24m}{25} \times \frac{15}{6m}$

(e) $\frac{12x}{7} \times \frac{28}{15x}$

(f) $\frac{6a}{35} \times \frac{15}{14a}$

(g) $\frac{x+2}{12} \times \frac{6}{x+2}$

(h) $\frac{3}{a-4} \times \frac{a-4}{15}$

(i) $\frac{8}{x-1} \times \frac{x-1}{20}$

(j) $\frac{x+5}{12} \times \frac{18}{x+5}$

(k) $\frac{n-3}{8} \times \frac{12}{n-3}$

(l) $\frac{10}{x-6} \times \frac{x-6}{15}$

9. Simplify the following expressions by factorising and cancelling before multiplying.

Examples

$$\begin{aligned} 1. \quad & \frac{4x-8}{15} \times \frac{10}{2x-4} \\ &= \frac{2\cancel{4}(\cancel{x-2})}{\cancel{3}15} \times \frac{10^{\cancel{2}}}{\cancel{2}(\cancel{x-2})} \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{x+3}{5} \times \frac{x^2+2x-8}{x^2+7x+12} \\ &= \frac{\cancel{x+3}}{5} \times \frac{(x+4)(x-2)}{(\cancel{x+3})(x+4)} \\ &= \frac{x-2}{5} \end{aligned}$$

(a) $\frac{4x-8}{15} \times \frac{5}{2x-4}$

(b) $\frac{3}{4x-16} \times \frac{2x-8}{5}$

(c) $\frac{3x-6}{10} \times \frac{5}{6x-12}$

(d) $\frac{2n+10}{9} \times \frac{3}{3n+15}$

(e) $\frac{2a-14}{15} \times \frac{3}{3a-21}$

(f) $\frac{4}{6x+30} \times \frac{4x+20}{9}$

(g) $\frac{3x+3}{6x+18} \times \frac{9x+27}{x+1}$

(h) $\frac{x^2+2x}{3} \times \frac{9}{x^2+5x+6}$

(i) $\frac{6n}{2n^2+6n} \times \frac{n^2+9n+18}{n^2+11n+30}$

$$(j) \frac{3x^2 - 6x}{x^2 - 9x + 20} \times \frac{2x - 8}{5x^2 - 10x}$$

$$(k) \frac{a^2 - 9}{a^2 + 8a + 15} \times \frac{5a + 25}{6a^2 - 18a}$$

$$(l) \frac{n^2 - 36}{n^2 + n - 30} \times \frac{3n^2 - 15n}{n^2 - n - 30}$$

$$(m) \frac{x^2 - 49}{x^2 + 2x - 63} \times \frac{x^2 + 4x - 45}{x^2 + 2x - 35}$$

10. Convert the following division problems to multiplications before simplifying.

Example

$$\begin{aligned} & \frac{2x^2 + 10x}{x^2 + 7x + 10} \div \frac{x^2 + 5x - 14}{x^2 - 4} \\ &= \frac{2x^2 + 10x}{x^2 + 7x + 10} \times \frac{x^2 - 4}{x^2 + 5x - 14} \\ &= \frac{2x(x+5)}{(x+2)(x+5)} \times \frac{(x-2)(x+2)}{(x+7)(x-2)} \\ &= \frac{2x}{(x+7)} \end{aligned}$$

When changing from a division to a multiplication problem invert the second fraction

$$(a) \frac{2x}{5} \div \frac{4x}{15}$$

$$(b) \frac{15}{8a} \div \frac{10}{6a}$$

$$(c) \frac{3x+6}{7} \div \frac{6x+12}{35}$$

$$(d) \frac{25}{6n-42} \div \frac{20}{3n-21}$$

$$(e) \frac{x^2+2x}{8} \div \frac{x^2+6x+8}{4}$$

$$(f) \frac{a^2+a}{3} \div \frac{a^2+5a+4}{9}$$

$$(g) \frac{12}{4x+8} \div \frac{6}{x^2+5x+6}$$

$$(h) \frac{n^2-3n}{6} \div \frac{n^2-8n+15}{8}$$

$$(i) \frac{x^2-4}{5} \div \frac{x^2+7x+10}{10}$$

$$(j) \frac{x^2+2x-8}{x^2-2x} \div \frac{x^2+x-12}{3x}$$

$$(k) \frac{x^2-25}{x^2-8x+15} \div \frac{x^2+8x+15}{x^2-9}$$

$$(l) \frac{x^2-64}{x^2-4x-96} \div \frac{x^2+x-72}{x^2-3x-108}$$

Creating Formulae

EXERCISE 6K

Write the following statements as formulae.

Examples

1. x is the product of p and q .

$$x = pq$$

2. Velocity (v) is equal to the sum of initial speed (u) and the product of acceleration (a) and time (t).

$$v = u + at$$

1. C is equal to the sum of A and B .
2. g is equal to the product of h and d .
3. L is equal to M divided by N .
4. S is equal to the product of A and $(E + F)$.
5. Z is equal to $3P$ divided by $(Q - T)$.
6. b is equal to the sum of c^2 and n^2 .
7. The area of a rectangle (A) is equal to the product of the length (L) and width (W).
8. The area of a circle (A) is equal to the product of π and the square of the radius (r).
9. Power (P) is equal to the product of voltage (V) and current (I).
10. Force (F) is equal to the product of mass (m) and acceleration (a).
11. Magnetic force (F) is equal to the product of magnetic strength (B), current (I) and length (l).
12. Speed (s) is equal to distance travelled (d) divided by time (t).
13. Energy (E) is equal to the product of mass (m) and the square of the speed of light (c).
14. Force (F) is equal to the product of mass (m) and the square of velocity (v) divided by radius (R).
15. The time for a pendulum to swing once (T) is equal to the product of 2π and the square root of [the pendulum length (l) divided by the acceleration due to gravity (g)].
16. The escape velocity of a space rocket (v) is equal to the square root of [the product of 2, the gravitational constant (G) and mass of the Earth (M), divided by the radius of the Earth (R)]

Transposition

EXERCISE 6L

Transpose the following formulae to make the pronumeral in the brackets the subject.

Examples

1. $a + b = c$ (a)

Subtract b from both sides

$$a + b = c$$

$$\boxed{-b} \quad \boxed{-b}$$

$$a = c - b$$

3. $\frac{2x}{y} + 3z = K$ (x)

Subtract $3z$ from both sides

$$\frac{2x}{y} + 3z = K$$

$$\boxed{-3z} \quad \boxed{-3z}$$

$$\frac{2x}{y} = K - 3z$$

Multiply both sides by y

$$\frac{2x}{y} = K - 3z$$

$$\boxed{\times y} \quad \boxed{\times y}$$

$$2x = y(K - 3z)$$

Divide both sides by 2

$$2x = y(K - 3z)$$

$$\boxed{\div 2} \quad \boxed{\div 2}$$

$$x = \frac{y(K - 3z)}{2}$$

2. $3x - y = z$ (x)

Add y to both sides

$$3x - y = z$$

$$\boxed{+y} \quad \boxed{+y}$$

$$3x = z + y$$

Divide both sides by 3

$$3x = z + y$$

$$\boxed{\div 3} \quad \boxed{\div 3}$$

$$x = \frac{z + y}{3}$$

4. $p - q = r$ (q)

Subtract p from both sides

$$p - q = r$$

$$\boxed{-p} \quad \boxed{-p}$$

$$-q = r - p$$

Multiply both sides by -1

$$-q = r - p$$

$$\boxed{\times -1} \quad \boxed{\times -1}$$

$$q = -1(r - p)$$

$$q = -r + p$$

$$q = p - r$$

Examples continued

5. $x^2 + y = z$ (x)

Subtract y from both sides

$$x^2 + y = z$$

$$\boxed{-y} \quad \boxed{-y}$$

$$x^2 = z - y$$

Square root both sides

$$\sqrt{x^2} = \sqrt{z - y}$$

$$x = \sqrt{z - y}$$

6. $v^2 = m^2 + 2aB$ (a)

Swap sides to make a on the left side

$$m^2 + 2aB = v^2$$

Subtract m^2 from both sides

$$m^2 + 2aB = v^2$$

$$\boxed{-m^2} \quad \boxed{-m^2}$$

$$2aB = v^2 - m^2$$

Divide both sides by $2B$

$$2aB = v^2 - m^2$$

$$\boxed{\div 2B} \quad \boxed{\div 2B}$$

$$a = \frac{v^2 - m^2}{2B}$$

1. $x + y = A$ (x)

3. $z - B = c$ (z)

5. $3a - b = C$ (a)

7. $uc + t = h$ (t)

9. $F = ma$ (m)

11. $EFG = H$ (F)

13. $v = u + at$ (u)

15. $ut + am = s$ (t)

17. $3(a + b) = C$ (a)

19. $L = GT - RA$ (T)

21. $y^2 + t = Z$ (y)

23. $\frac{A + C}{D} = F$ (A)

25. $F = \frac{9C}{5} + 32$ (C)

27. $A = \frac{(a + b)}{2} h$ (a)

2. $5p = q$ (p)

4. $x/m = R$ (x)

6. $am + n = P$ (m)

8. $X = Y + A$ (Y)

10. $F = ma$ (a)

12. $F = BIl$ (B)

14. $v = u + at$ (a)

16. $t - v = u$ (v)

18. $G = k^2 + 3mp$ (m)

20. $x^2 = P$ (x)

22. $MN = h^2 - 2pQ$ (h)

24. $\frac{m}{F} - C = F$ (m)

26. $K = \frac{4}{3} hR^2$ (R)

28. $F = \frac{GM}{R^2}$ (R)

Substitution

EXERCISE 6M

Substitute the given values into the following equations to find the unknown quantity.

Example $S = \frac{3r + 5t^2}{h}$ Find S if $r = 8$, $t = 6$ and $h = 10$

$$S = \frac{3r + 5t^2}{h}$$

$$S = \frac{3 \times 8 + 5 \times 6^2}{10}$$

$$S = \frac{24 + 5 \times 36}{10}$$

$$S = \frac{24 + 180}{10}$$

$$S = \frac{204}{10}$$

$$S = 20.4$$

1. $y = mx + c$

Find y if $m = 3$, $x = 5$ and $c = 7$

2. $v = u + at$

Find v if $u = 15$, $a = 10$ and $t = 5$

3. $F = ma$

Find F if $m = 8.8$ and $a = 6.5$

4. $s = b - rt^2$

Find s if $b = 36$, $r = 5$ and $t = 2$

5. $h = 3r^2 + 5(m^2 + r)$

Find h if $r = 4$ and $m = 3$

6. $E = \frac{1}{2}mv^2 + mgh$

Find E if $m = 5$, $v = 6$, $g = 10$ and $h = 8$

7. $P = I^2R - i^2r$

Find P if $I = 12$, $R = 3$, $i = 4$ and $r = 2$

8. $v = \sqrt{u^2 - n^2}$

Find v if $u = 13$ and $n = 12$

9. $F = \frac{mv^2}{R}$

Find F if $m = 8$, $v = 6$ and $R = 9$

10. $x = \frac{b + \sqrt{b^2 - 4ac}}{2a}$

Find x if $b = 13$, $a = 2$ and $c = 15$

11. $I = \frac{PRT}{100}$

Find I if $P = 4000$, $R = 5.56$ and $T = 20$

EXERCISE 6N

1. The formula used to calculate the distance travelled, d , by an object with an acceleration, a , for time, t , after an initial velocity, u , is:

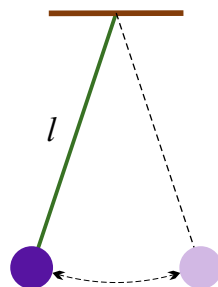
$$d = ut + \frac{1}{2}at^2$$

- (a) Use this formula to calculate the distance travelled by a car that accelerates at 1 m/s^2 for 10 seconds after travelling with an initial velocity of 6 m/s .
- (b) Transpose this equation to make acceleration, a , the subject.
- (c) A car accelerates over a distance of 66 metres for 6 seconds after having an initial velocity of 5 m/s . Calculate the acceleration.
2. The time, T , (in seconds) for a pendulum to complete one swing is given by the following formula.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

l = the length of the pendulum (in metres)

$g = 10$



- (a) Find the time for a 0.4 m long pendulum to complete one swing. Give answer correct to one decimal place.
- (b) A child is sitting on a swing that is 3 metres long. Find the time for the child to complete one swing.
- (c) Transpose the equation to make l the subject.
- (d) Find the length of a pendulum that takes 2.68 seconds to complete one swing. Give answer correct to two decimal places.

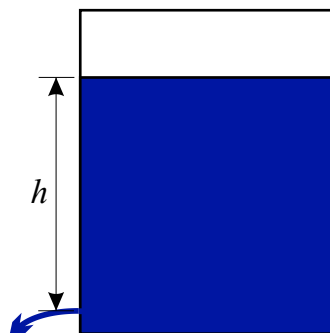


3. The velocity, v , (in m/s) that water spurts from a hole in a container can be calculated using the following formula.

$$v = \sqrt{2gh}$$

h = the height (in metres)
of the water level
above the hole

$g = 10$

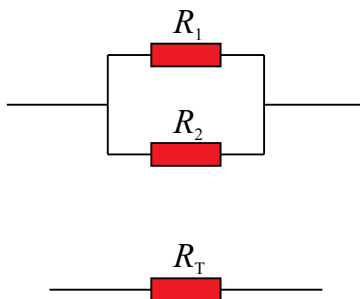


- (a) Calculate the velocity that water spurts from a hole that is the following distances below the water level in a container. Give answers correct to one decimal place.
- (i) 0.5 m (ii) 4.7 m (iii) 20 cm (iv) 85 cm
- (b) Transpose the equation to make h the subject.
- (c) The velocity of water spurting from a hole in a container is measured to be 5.5 m/s. What distance is the hole below the water level? Give answer in metres to one decimal place.

4. In an electric circuit if two resistors (R_1 and R_2) are connected as shown here they can be replaced by one resistor R_T .

The size of R_T can be calculated by using the following formula.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$



- (a) Use this formula to find the value of R_T for the following values of R_1 and R_2 . The unit of resistance is ohms (Ω).
- (i) $R_1 = 10 \Omega$, $R_2 = 10 \Omega$
- (ii) $R_1 = 20 \Omega$, $R_2 = 5 \Omega$
- (iii) $R_1 = 10 \Omega$, $R_2 = 6 \Omega$
- (b) Find the size of R_2 for the following values of R_1 and R_T .
- (i) $R_1 = 4 \Omega$, $R_T = 2 \Omega$
- (ii) $R_1 = 20 \Omega$, $R_T = 15 \Omega$

PROBLEM SOLVING

1. Expand the following algebraic expressions.

(a) $(a + b)^2$ (b) $(a + b)^3$ (c) $(a + b)^4$

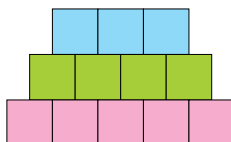
2. Use these expansions to complete the coefficients shown below.

(a) $(a + b)^2 = \square a^2 + \square ab + \square b^2$

(b) $(a + b)^3 = \square a^3 + \square a^2b + \square ab^2 + \square b^3$

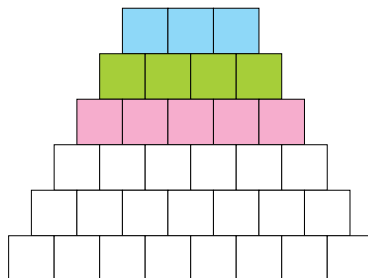
(c) $(a + b)^4 = \square a^4 + \square a^3b + \square a^2b^2 + \square ab^3 + \square b^4$

3. Arrange these coefficients in a diagram as shown here.



4. (a) Describe the pattern that connects adjacent rows of this diagram.

(b) Use this pattern to find the numbers that would make up the next three rows of this diagram.

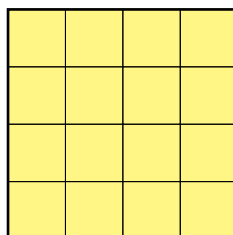


5. Use the numbers in this diagram to find the following expansions.

(a) $(a + b)^5$ (b) $(a + b)^6$ (c) $(a + b)^7$

PUZZLE

Using the letters **A**, **B**, **C** and **D** fill in the squares of this grid so that each letter only appears once in each row, column and diagonal.



CHAPTER REVIEW

1. Expand the following algebraic expressions.

(a) $5(x - 3)$ (b) $-2(a - 3b)$ (c) $5x(2y - 3z)$

2. Expand and simplify the following algebraic expressions.

(a) $2(m + 3) + 5(m - 2)$ (b) $6(x - 3) - 2(x - 5)$
 (c) $(x - 3)(x + 5)$ (d) $(2y + 3)(4y - 7)$
 (e) $(3n + 2m)(2n + 5m)$ (f) $(3a + 2b)(3a - 2b)$
 (g) $(2x + 3)^2$ (h) $(3p - 4q)^2$
 (i) $(3a^2b + 5b^2)(4ab^2 - 2a^2)$ (j) $4x(2x - 5y)(3x + 4y)$
 (k) $2m(3m + 4n)^2$ (l) $(a + 4)(2a^2 - 3a + 5)$
 (m) $(x - 2)(x + 3)(x - 5)$ (n) $(2x + 3)^2(3x - 5)$

3. Factorise the following expressions.

(a) $10xy - 12y$ (b) $-6a - 9b$ (c) $3(a - 4) + b(a - 4)$

4. Factorise the following expressions.

(a) $15x + 10 + 3xy + 2y$ (b) $6ab - 8a + 9b - 12$
 (c) $ab + 10 + 5b + 2a$ (d) $a^2 + 2a - ab - 2b$

5. Factorise the following expressions and simplify where possible.

(a) $a^2 - b^2$ (b) $x^2 - 16$ (c) $25m^2 - 49n^2$
 (d) $(m + 1)^2 - 36$ (e) $(x + 2)^2 - (x - 5)^2$ (f) $64(c - 3)^2 - 81(c + 2)^2$
 (g) $x^2 - 7$ (h) $16n^2 - 13$ (i) $12(y - 3)^2 - 33$

6. Factorise the following expressions.

(a) $a^2 + 11a + 18$ (b) $n^2 + 14n + 24$ (c) $p^2 - 11p + 24$
 (d) $x^2 - 11x + 28$ (e) $a^2 + 5a - 36$ (f) $n^2 - n - 12$
 (g) $2x^2 + 5x - 12$ (h) $12x^2 + 35x + 8$ (i) $15x^2 + 47x - 10$
 (j) $x^2 - 16x + 64$ (k) $x^2 + 24x + 144$

7. Factorise the following quadratic trinomials by completing the square.

(a) $x^2 + 4x - 6$ (b) $x^2 - 6x + 7$ (c) $x^2 + 10x - 11$

8. Simplify the following.

(a) $\frac{16m}{20m}$

(b) $\frac{8}{16 - 8m}$

(c) $\frac{6n^2 - 30n}{3n - 15}$

(d) $\frac{6x^2 - 18x}{x^2 - 10x + 21}$

(e) $\frac{3x}{5} + \frac{6x}{5}$

(f) $\frac{x-2}{4} + \frac{2x-3}{3}$

(g) $\frac{3x+3}{6x+18} \times \frac{9x+27}{x+1}$

(h) $\frac{n^2-36}{n^2+n-30} \times \frac{3n^2-15n}{n^2-n-30}$

(i) $\frac{a^2+a}{3} \div \frac{a^2+5a+4}{9}$

(j) $\frac{n^2-7n+12}{n^2+n-20} \div \frac{n^2-9n+18}{n^2-n-30}$

9. Write the following statements as formulae.

(a) Y is equal to the sum of A and B .

(b) P is equal to the product of m and c^2 .

(c) Volume (V) is equal to the product of area (A) and length (l).

10. Transpose the following formulae to make the pronumeral in the brackets the subject.

(a) $m + n = p$ (m)

(b) $5a = B$ (a)

(c) $4r - k = h$ (r)

(d) $y/5 + x = A$ (y)

(e) $2(x + y) = z$ (y)

(f) $c^2 - G = N$ (c)

(g) $\frac{3T+V}{W} = D$ (T)

(h) $P = \frac{F+2A^2}{B} + 3$ (F)

11. Substitute the given values into the following equations to find the unknown quantity.

(a) $a = 2b - c$

Find a if $b = 5$ and $c = 3$

(b) $m = nt + v$

Find m if $n = 5$, $t = 3$ and $v = 8$

(c) $k = er^2 - f$

Find k if $e = 10$, $r = 3$ and $f = 12$

(d) $G = 2ab^2 + 3c^2$

Find G if $a = 3$, $b = 2$ and $c = 4$

(e) $P = \sqrt{d^2 + m^2}$

Find P if $d = 6$ and $m = 8$

(f) $B = \frac{at^2 + abt}{mt}$

Find B if $a = 4$, $t = 3$, $b = 5$ and $m = 2$

Solving Equations

Solving Linear Equations

There are three main methods of solving linear equations.

In methods 1 and 2 use the inverse operations

$+$ and $-$ are *inverse operations*
 $\times$ and $\div$ are *inverse operations*

Method 1 Backtracking

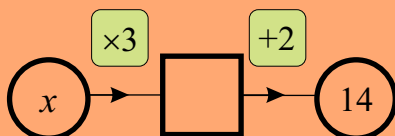
Step 1 Write a flowchart starting with x .

Step 2 Backtrack to find x by using the inverse operations.

Example $3x + 2 = 14$

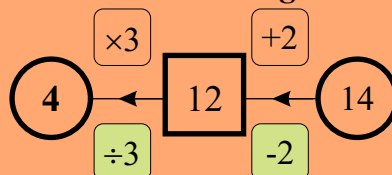
Step 1

Flowchart



Step 2

Backtracking

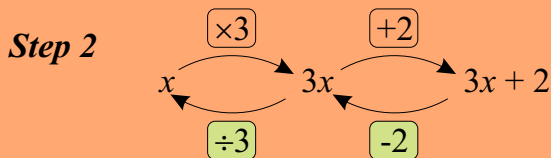
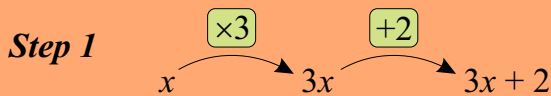


Answer $x = 4$

Method 2 *Doing the same to both sides of the equation using the inverse operations.*

- Step 1** Write *in order* the operations that are being performed on x .
- Step 2** Find the inverse operations that would 'undo' the operations in the *reverse order*.
- Step 3** Perform the inverse operations on *both sides* of the equation.

Example $3x + 2 = 14$



Step 3

$$\begin{array}{rcl}
 3x + 2 & = & 14 \\
 \boxed{-2} & \boxed{-2} & \\
 \hline
 3x & = & 12 \\
 \boxed{\div 3} & \boxed{\div 3} &
 \end{array}$$

Answer $x = 4$

Method 3 Trial and Error (Guess, Check and Improve)**Example**

$3x + 2 = 14$

Guess 1 $x = 5$

$$3x + 2 = 3 \times 5 + 2$$

$$= 17 \text{ (Too high)}$$

Guess 2 $x = 3$

$$3x + 2 = 3 \times 3 + 2$$

$$= 11 \text{ (Too low)}$$

Guess 3 $x = 4$

$$3x + 2 = 3 \times 4 + 2$$

$$= 14$$

Answer $x = 4$

Remember Check your answer by substituting into the original equation.

Example $3x + 2 = 14$ **Answer :** $x = 4$

In *left* side of the equation: $3 \times 4 + 2 = 14$
 $=$ *right* side of the equation

EXERCISE 7A

1. Choose one of the three methods to solve these equations.

(a) $5x = 30$

(b) $a + 7 = 15$

(c) $c - 4 = 21$

(d) $\frac{n}{3} = 6$

(e) $8 + m = 26$

(f) $4b = 28$

(g) $y - 7 = 15$

(h) $a + 6 = -2$

(i) $d - 3 = -5$

(j) $4y = -20$

(k) $\frac{a}{5} = -7$

(l) $p - 8 = -3$

(m) $3a + 5 = 11$

(n) $4a + 2 = 26$

(o) $5d + 7 = 32$

(p) $2x - 4 = 10$

(q) $3m - 6 = 18$

(r) $4c - 3 = 33$

(s) $2m + 3 = -5$

(t) $3n + 2 = -16$

(u) $5t + 6 = -4$

(v) $3a - 8 = 7$

(w) $4x - 7 = -19$

(x) $5y - 2 = -22$

(y) $\frac{a}{4} + 2 = 6$

(z) $\frac{n}{6} - 5 = -7$

2. Choose one of the three methods to solve these equations.

(a) $\frac{n+5}{3} = 4$

(b) $\frac{a+7}{6} = 3$

(c) $\frac{m+6}{4} = 4$

(d) $\frac{x-6}{3} = 3$

(e) $\frac{y-1}{10} = 2$

(f) $\frac{w-8}{2} = 11$

Remember Check your answer by substituting into the original equation.

3. Choose one of the three methods to solve these equations.

(a) $\frac{4n}{3} = 16$

(b) $\frac{2a}{5} = 4$

(c) $\frac{6x}{7} = 12$

(d) $\frac{3m}{8} = 6$

(e) $\frac{3c}{5} = -12$

(f) $\frac{7n}{9} = -21$

(g) $\frac{n}{3} + 2 = 4$

(h) $\frac{a}{4} + 5 = 8$

(i) $\frac{b}{8} + 7 = 11$

(j) $\frac{m}{6} - 4 = 2$

(k) $\frac{d}{6} + 2 = -4$

(l) $\frac{v}{5} - 3 = -4$

(m) $\frac{n+1}{3} + 2 = 8$

(n) $\frac{a+2}{5} + 1 = 3$

(o) $\frac{b+3}{4} + 3 = 7$

(p) $\frac{m-2}{6} + 3 = 10$

(q) $\frac{x-3}{3} - 4 = 5$

(r) $\frac{c-2}{7} - 5 = -6$

(s) $\frac{e-8}{6} - 4 = -5$

(t) $\frac{f+6}{7} + 5 = -3$

(u) $\frac{s-5}{6} + 3 = -9$

4. Choose one of the three methods to solve these equations.

(a) $\frac{2n+4}{5} + 3 = 9$

(b) $\frac{3a+2}{5} + 2 = 6$

(c) $\frac{5m-6}{4} + 6 = 17$

(d) $\frac{7x-3}{2} - 3 = 20$

(e) $\frac{3x+5}{2} + 5 = 6$

(f) $\frac{4a-3}{5} - 2 = -5$

(g) $\frac{4n+5}{7} + 3 = -2$

(h) $\frac{9m-7}{7} - 5 = -6$

5. Choose one of the three methods to solve these equations.

(a) $\frac{8n+6}{9} + 3 = 5$

(b) $\frac{10m-3}{3} + 2 = 5$

(c) $\frac{5n+7}{2} - 8 = -1$

(d) $\frac{6k+7}{4} - 4 = -6$

6. Solve these equations.

Example

$$14 - 3x = 8$$

$$14 - 3x = 8$$

$$\boxed{-14}$$

$$\boxed{-14}$$

Subtract 14 from both sides

$$-3x = -6$$

$$\boxed{\div -3}$$

$$\boxed{\div -3}$$

Divide both sides by -3

$$x = 2$$

(a) $8 - 3x = -1$

(b) $15 - 4a = 7$

(c) $11 - m = 2$

(d) $7 - 4y = -5$

(e) $10 - n = 16$

(f) $12 - 7m = 26$

(g) $8 - 3g = -10$

(h) $9 - 5d = 24$

(i) $12 - 6u = -12$

7. Solve these equations.

Example

$$3(5x + 4) = 27$$

$$15x + 12 = 27$$
 Expand the brackets

$$\boxed{-12}$$

$$\boxed{-12}$$

Subtract 12 from both sides

$$15x = 15$$

$$\boxed{\div 15}$$

$$\boxed{\div 15}$$

Divide both sides by 15

$$x = 1$$

(a) $2(3a + 4) = 20$

(b) $5(2m + 7) = 55$

(c) $3(5d + 6) = 48$

(d) $4(4a - 3) = 36$

(e) $2(3x - 8) = -4$

(f) $5(3y + 7) = 20$

(g) $6(4 + 2n) = -12$

(h) $4(5a + 6) = -36$

(i) $5(5 - 3r) = 40$

(j) $2(6 - 3b) = 0$

(k) $3(5 - 4a) = 39$

(l) $3(5 - 6x) = -39$

(m) $\frac{2(3n + 4)}{5} = 4$

(n) $\frac{5(3a + 2)}{11} = 5$

(o) $\frac{2(5m - 6)}{9} = 2$

(p) $\frac{3(6x - 7)}{7} = 15$

(q) $\frac{3(5m - 6)}{4} + 6 = 9$

(r) $\frac{4(2x - 3)}{7} - 6 = -2$

8. Solve these equations.

Examples

When there are unknown terms on both sides of the equation, subtract the smaller term from both sides of the equation then solve.

1. $6x - 5 = 2x + 7$

$2x$ is the smaller of the unknown terms

$$6x - 5 = 2x + 7$$

$-2x$

$-2x$

$$4x - 5 = 7$$

$$4x = 12$$

$$x = 3$$

2. $3(2n + 4) = 6(3n + 8)$

Expand brackets

$$6n + 12 = 18n + 48$$

$6n$ is the smaller of the unknown terms

$$6n + 12 = 18n + 48$$

$-6n$

$-6n$

$$12 = 12n + 48$$

Rearrange the equation

$$12n + 48 = 12$$

$$12n = -36$$

$$n = -3$$

Remember Check your answer by substituting into the original equation.

(a) $3x + 6 = x + 16$

(c) $4a + 7 = 3a + 9$

(e) $6m + 4 = 3m + 10$

(g) $5m - 6 = 3m$

(i) $2a + 3 = 3a - 4$

(k) $a + 3 = 3a - 7$

(m) $4y - 7 = 9y - 2$

(o) $2x - 9 = 6x + 7$

(q) $4(3x + 4) = 2(3x + 5)$

(s) $4(5w + 3) = 3(3w - 7)$

(u) $3(2x - 6) = 4(5x - 1)$

(w) $2(6m + 7) = 7(3m - 7)$

(y) $7(2k + 5) = 6(3k + 5)$

(b) $5x - 3 = x + 1$

(d) $5n - 6 = 4n - 2$

(f) $6y - 11 = 4y - 5$

(h) $6t + 9 = 3t$

(j) $4c + 5 = 5c + 2$

(l) $2m + 7 = 5m - 2$

(n) $3d + 5 = 8d + 10$

(p) $4n + 5 = 11n + 19$

(r) $5(3y - 4) = 2(2y + 1)$

(t) $4(7b + 5) = 5(5b + 1)$

(v) $2(2n + 5) = 3(3n - 5)$

(x) $5(6y - 3) = 7(5y - 10)$

(z) $6(6n - 5) = 4(5n + 9)$

9. Solve the following equations.

(a) $\frac{24}{x} + 3 = 6$

(b) $\frac{18}{a} - 5 = 4$

(c) $\frac{36}{y} + 7 = 4$

(d) $\frac{7}{2a} + 1 = 8$

(e) $\frac{10}{3m} - 3 = -8$

Example $\frac{12}{x} + 5 = 8$

$$\frac{12}{x} = 3$$

$$\frac{12}{x} = \frac{3}{1}$$

$$\frac{x}{12} = \frac{1}{3} \quad \text{Invert both sides}$$

$$x = 4$$

10. Solve the following equations. Answers may be fractions - write these in their simplest form.

(a) $12x = 3$

(b) $10m = 6$

(c) $8n = 22$

(d) $9y = -15$

(e) $6n - 5 = 3$

(f) $5x + 3 = 1$

(g) $3 - 2a = 8$

(h) $3(x + 4) = 5$

(i) $5n + 4 = 2n - 1$

(j) $\frac{2n}{5} = 14$

(k) $\frac{5x}{3} = 2$

(l) $\frac{2a + 5}{3} = 6$

(m) $\frac{4m}{7} - 3 = 7$

(n) $\frac{3(x - 2)}{4} = -1$

(o) $\frac{5u + 2}{6} + 3 = 2$

(p) $3n = \frac{2}{5}$

(q) $2x - 3 = \frac{1}{4}$

(r) $5m + 6 = \frac{7}{3}$

(s) $\frac{5x}{2} - 1 = -\frac{2}{3}$

(t) $\frac{a - 2}{3} - \frac{5}{2} = \frac{3}{4}$

(u) $3(v - \frac{5}{2}) = \frac{6}{5}$

(v) $\frac{3}{x} = \frac{5}{7}$

(w) $\frac{7}{n} + 2 = \frac{12}{5}$

(x) $\frac{7}{n - 2} - 3 = \frac{3}{4}$

11. Solve the following equations.

(a) $\frac{1}{a} + \frac{1}{a} = 3$

(b) $\frac{1}{x} + \frac{5}{x} = 2$

(c) $\frac{1}{y} + \frac{2}{3y} = 7$

(d) $\frac{1}{3a} + \frac{3}{4a} = \frac{5}{2}$

(e) $\frac{1}{5x} - \frac{3}{4} = \frac{6}{5}$

(f) $\frac{3}{2y} - \frac{2}{5} = \frac{1}{y}$

(g) $\frac{4}{3(x + 1)} = \frac{2}{9}$

(h) $\frac{x}{2x - 3} = \frac{3}{5}$

(i) $\frac{4x}{3x + 5} = \frac{2}{3}$

EXERCISE 7B

1. Write the following statements as equations then solve.

Example

If six is added to three times this number the answer is 21.

Let n represent the number:

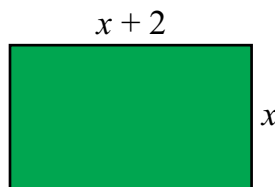
$$3n + 6 = 21$$

$$3n = 15$$

$$n = 5$$

- If five is added to twice this number the answer is 13.
- If seven is subtracted from four times this number the answer is 25.
- If four is added to a quarter of this number the answer is 9.
- If six is added to this number and the result is multiplied by three the answer is 33.
- If nine is subtracted from this number and the result is multiplied by seven the answer is 21.
- If eight is added to three times this number and the result is multiplied by five the answer is 55.
- If six is subtracted from four times this number and the result is multiplied by three the answer is 18.
- If seven is added to three times this number the result is the same as if two is added to four times the number.
- If six is subtracted from five times this number the result is the same as if eight is added to three times the number.

2. A park is in the shape of a rectangle with its length 2 kilometres longer than its width. The total perimeter of the park is 24 kilometres. Find the length and width of the park.



3. Charmaine is 5 years older than her brother Shaheen.
- If Shaheen's age is x , write an expression for Charmaine's age.
 - Write an expression for the sum of their ages.
 - If the sum of their ages is 29, find their ages.

4. Neela is three years older than her sister, seven years older than her brother and one third her mother's age. The sum of their ages is 80. How old is Neela?
5. A 5.4 metre long timber plank is cut into three pieces, **A**, **B** and **C**. Piece **A** is 1.2 shorter than **B** and 1.8 m shorter than **C**. How long is each piece of timber?
6. Kyle, a builder, and his apprentice, Leith, built a carport. Leith was paid \$12 less than Kyle per hour. The total cost to build the carport was \$1456. If Kyle and Leith worked for 26 hours, how much does Leith get paid per hour?
7. Jo works 16 hours a week in a restaurant. One week her boss incorrectly typed 61 weeks when calculating her pay. She was overpaid \$675. What is her normal weekly pay?
8. A car petrol tank is full. On one trip two fifths of the petrol is used. On the next trip one third of the tank capacity is used. There were 16 litres remaining in the tank. What is the capacity of the tank?
9. George is a green grocer. He buys 126 kg of apples that he wants to divide into 5 kg and 2 kg bags. How many of each sized bag would he have if:
 - (a) he wanted the same number of 5 kg and 2 kg bags?
 - (b) he wanted twice as many 2 kg bags as 5 kg bags?
 - (c) he wanted 7 more 2 kg bags than 5 kg bags?
10. A farmer had two barrels of equal size. One was one tenth full with wheat, the other was one eighth full with wheat. He emptied the wheat from both containers and weighed it to be 10.8 kg. What weight of wheat would each container hold?
11. Kerrie had six equally sized containers of orange juice. She emptied them into a large bowl and added 2.4 litres of water. She then divided the mixture into 20 equally sized smaller containers. From one of these smaller containers she filled three 200 mL glasses. How many litres of orange juice were in each of the six original orange juice containers?

Solving Quadratic Equations

Example 1

Solve the quadratic equation $x^2 + 3x = 0$

Step 1 Factorise the left side of the equation $x(x + 3) = 0$

Step 2 Recognise that if the product of two terms equals zero then at least one of those terms must be zero. In this example the two terms are x and $(x + 3)$.

Therefore $x = 0$ or $x + 3 = 0$

$$x = -3$$

Step 3 Write the two solutions: $x = 0$ or -3

Example 2

Solve the quadratic equation $x^2 = 8x$

Step 1 Transpose the equation $x^2 - 8x = 0$

Step 2 Solve as in example 1 $x(x - 8) = 0$
 $x = 0$ or $x - 8 = 0$
 $x = 0$ or 8

Example 3

Solve the quadratic equation $3x^2 - 5x = 0$

Step 1 Factorise $x(3x - 5) = 0$

Step 2 Solve as in example 1 $x = 0$ or $3x - 5 = 0$
 $3x = 5$
 $x = 0$ or $\frac{5}{3}$

EXERCISE 7C

1. Solve the following equations.

(a) $x^2 - 6x = 0$

(b) $x^2 + 7x = 0$

(c) $x^2 + x = 0$

(d) $x^2 = 3x$

(e) $x^2 = -10x$

(f) $x^2 = x$

(g) $2x^2 - 7x = 0$

(h) $3x^2 + 8x = 0$

(i) $7x^2 = 9x$

(j) $8x^2 + 24x = 0$

(k) $-6x^2 - 14x = 0$

(l) $12x^2 = -16x$

Example 1Solve the equation $x^2 - 9 = 0$ **Refer to Factorising the Difference of Perfect Squares (p 148)****Step 1** Factorise $(x - 3)(x + 3) = 0$ **Step 2** One of these factors must be zero
 $x - 3 = 0$ or $x + 3 = 0$
 $x = 3$ or -3 **Example 2**Solve the equation $4(x + 2)^2 - 25 = 0$ **Refer to Factorising the Difference of Perfect Squares (p 149)****Step 1** Factorise and simplify
 $[2(x + 2) - 5][2(x + 2) + 5] = 0$
 $(2x + 4 - 5)(2x + 4 + 5) = 0$
 $(2x - 1)(2x + 9) = 0$ **Step 2** One of these factors must be zero
 $2x - 1 = 0$ or $2x + 9 = 0$
 $2x = 1$ or $2x = -9$
 $x = \frac{1}{2}$ or $-\frac{9}{2}$ **Example 3**Solve the equation $x^2 - 3 = 0$ **Refer to Factorising the Difference of Perfect Squares (p 150)****Step 1** Factorise $(x - \sqrt{3})(x + \sqrt{3}) = 0$ **Step 2** One of these factors must be zero
 $x - \sqrt{3} = 0$ or $x + \sqrt{3} = 0$
 $x = \sqrt{3}$ or $-\sqrt{3}$ **2. Solve the following equations.**

(a) $x^2 - 16 = 0$

(b) $x^2 - 100 = 0$

(c) $4x^2 - 25 = 0$

(d) $49x^2 - 64 = 0$

(e) $3x^2 - 12 = 0$

(f) $8x^2 - 50 = 0$

(g) $(x + 3)^2 - 9 = 0$

(h) $(x - 3)^2 - 1 = 0$

(i) $16(x - 3)^2 - 25 = 0$

(j) $81(x + 5)^2 - 16 = 0$

(k) $(x - 4)^2 - (x + 1)^2 = 0$

(l) $9(x + 2)^2 - 36(x - 5)^2 = 0$

(m) $x^2 - 11 = 0$

(n) $4x^2 - 7 = 0$

(o) $12x^2 - 18 = 0$

(p) $18x^2 - 30 = 0$

Example 1Solve the equation $x^2 - 3x - 10 = 0$ **Refer to Factorising Quadratic Trinomials $a = 1$ (p 151)****Step 1** Factorise $(x - 5)(x + 2) = 0$ **Step 2** One of these factors must be zero
 $x - 5 = 0$ or $x + 2 = 0$
 $x = 5$ or -2 **Example 2**Solve the equation $6x^2 - x - 12 = 0$ **Refer to Factorising Quadratic Trinomials $a \neq 1$ (p 153)****Step 1** Factorise $(2x - 3)(3x + 4) = 0$ **Step 2** One of these factors must be zero
 $2x - 3 = 0$ or $3x + 4 = 0$
 $x = \frac{3}{2}$ or $-\frac{4}{3}$ **Example 3**Solve the equation $x^2 - 12x + 36 = 0$ **Step 1** Factorise $(x - 6)^2 = 0$ **Step 2** **$x = 6$** **3. Solve the following equations.**

(a) $x^2 + 9x + 14 = 0$

(c) $x^2 - 15x + 36 = 0$

(e) $x^2 - 7x - 30 = 0$

(g) $x^2 - x - 20 = 0$

(i) $5x^2 + 49x + 36 = 0$

(k) $3x^2 + 13x - 10 = 0$

(m) $6x^2 + x - 12 = 0$

(o) $14x^2 + 31x - 10 = 0$

(q) $4x^2 - 13x - 12 = 0$

(s) $x^2 - 2x + 1 = 0$

(u) $x^2 + 10x + 25 = 0$

(w) $x^2 - 18x + 81 = 0$

(b) $x^2 + 11x + 24 = 0$

(d) $x^2 + 4x - 12 = 0$

(f) $x^2 - 3x - 40 = 0$

(h) $x^2 - 16x + 28 = 0$

(j) $3x^2 - 8x + 4 = 0$

(l) $4x^2 + 4x - 15 = 0$

(n) $6x^2 + 7x - 20 = 0$

(p) $6x^2 - 5x - 4 = 0$

(r) $12x^2 + 7x - 10 = 0$

(t) $x^2 + 6x + 9 = 0$

(v) $x^2 - 10x + 25 = 0$

(x) $x^2 + 20x + 100 = 0$

The Quadratic Formula

In general, to find the solutions to an equation of the form:

$$ax^2 + bx + c = 0$$

the following formula can be used:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is called the
quadratic formula

$\pm$ means + or -

Example 1

Solve the equation

$$x^2 + 8x - 20 = 0$$

Step 1 Find a , b and c

$$a = 1, b = 8, c = -20$$

Step 2 Substitute into the quadratic formula and simplify

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times -20}}{2 \times 1} \\ &= \frac{-8 \pm \sqrt{64 - -80}}{2} \\ &= \frac{-8 \pm \sqrt{144}}{2} \\ &= \frac{-8 \pm 12}{2} \\ &= \frac{-8 + 12}{2} \quad \text{or} \quad \frac{-8 - 12}{2} \end{aligned}$$

$$x = 2 \quad \text{or} \quad -10$$

Example 2

Solve the equation

$$15x^2 - 14x - 8 = 0$$

Step 1 Find a , b and c

$$a = 15, b = -14, c = -8$$

Step 2 Substitute into the quadratic formula and simplify

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-14) \pm \sqrt{(-14)^2 - 4 \times 15 \times -8}}{2 \times 15} \\ &= \frac{14 \pm \sqrt{196 - -480}}{30} \\ &= \frac{14 \pm \sqrt{676}}{30} \\ &= \frac{14 \pm 26}{30} \\ &= \frac{14 + 26}{30} \text{ or } \frac{14 - 26}{30} \\ &= \frac{40}{30} \text{ or } \frac{-12}{30} \\ x &= \frac{4}{3} \text{ or } -\frac{2}{5} \end{aligned}$$

Example 3

Solve the equation

$$x^2 - 16 = 0$$

Step 1 Find a , b and c

$$a = 1, b = 0, c = -16$$

Step 2 Substitute into the quadratic formula and simplify

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{0 \pm \sqrt{0^2 - 4 \times 1 \times -16}}{2 \times 1} \\ &= \frac{0 \pm \sqrt{64}}{2} \\ x &= 4 \text{ or } -4 \end{aligned}$$

Example 4

Solve the equation

$$3x^2 + 4x - 2 = 0$$

Step 1 Find a , b and c

$$a = 3, b = 4, c = -2$$

Step 2 Substitute into the quadratic formula and simplify

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4 \times 3 \times -2}}{2 \times 3} \\ &= \frac{-4 \pm \sqrt{16 - -24}}{6} \\ &= \frac{-4 \pm \sqrt{40}}{6} \\ &= \frac{-4 \pm \sqrt{4 \times 10}}{6} \\ &= \frac{-4 \pm 2\sqrt{10}}{6} \\ &= \frac{2(-2 \pm \sqrt{10})}{6} \\ &= \frac{-2 \pm \sqrt{10}}{3} \\ x &= \frac{-2 + \sqrt{10}}{3} \quad \text{or} \quad \frac{-2 - \sqrt{10}}{3} \end{aligned}$$

4. Use the *quadratic formula* to solve the following equations.

(a) $x^2 + 4x + 3 = 0$

(b) $x^2 + 7x + 10 = 0$

(c) $-x^2 - 10x - 16 = 0$

(d) $-x^2 + 8x + 48 = 0$

(e) $2x^2 - 7x - 15 = 0$

(f) $3x^2 + 11x - 20 = 0$

(g) $6x^2 - 17x + 12 = 0$

(h) $-8x^2 - 6x + 35 = 0$

(i) $9x^2 - 30x + 25 = 0$

(j) $-4x^2 + 28x - 49 = 0$

(k) $x^2 + 22x + 121 = 0$

(l) $4x^2 - 25 = 0$

(m) $x^2 + 3x - 5 = 0$

(n) $2x^2 + x - 11 = 0$

(o) $-3x^2 + 11x + 9 = 0$

(p) $5x^2 - 11x + 4 = 0$

Worted Problems

EXERCISE 7D

Example Stan and Mia run around a park that has a perimeter of 1200 m. They start at the same point but run in opposite directions. Stan runs at 5 m/s and Mia runs at 3 m/s. After how long will they meet?

Solution Let the distance that Stan runs before they meet = x
Therefore the distance Mia runs = $1200 - x$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Time taken by Stan} = \frac{x}{5}$$

$$\text{Time taken by Mia} = \frac{1200 - x}{3}$$

$$\text{These are the same therefore: } \frac{x}{5} = \frac{1200 - x}{3}$$

$$3x = 5(1200 - x)$$

$$3x = 6000 - 5x$$

$$8x = 6000$$

$$x = 750 \text{ seconds}$$

1. Chrissy bought a sandwich, two pieces of fruit and two biscuits for \$4.40. Perry bought a sandwich, three pieces of fruit and three biscuits for \$5.70. What is the price of a sandwich?
2. Two yachts are 32 km apart sailing towards each other. One is sailing 8 km/h faster than the other. After two hours they pass each other. Find the speed of both yachts.
3. A rectangle has a perimeter of 96 cm. Its length is five times its width. Find the area of the rectangle.
4. Tatiana wants to invest \$80 000. An investment advisor suggests two types of investment. A high risk investment that earns 12% per year and a safer investment that earns 7% per year. How much money should she put into each investment to earn a total of \$6600 per year from the investments?

5. A father is now seven times his daughter's age. In eight years he will be four times his daughter's age. What are their present ages?
6. A batter in cricket scores 37, 81, 47, 62 and 5 in five innings. What score does she need to make in her next innings to have an average of 50?
7. Cadel is a canoeist who knows he can paddle at a constant speed of 5 m/s on still water. He competes in a race on the Murray River. The canoeists start at Echuca, paddle upstream to a picnic area then back to Echuca. Cadel takes 65 minutes to get to the picnic area and 35 minutes to get back to Echuca.
- (a) How fast is the Murray River flowing?
- (b) How far is the picnic area from Echuca?



8. A farmer had three pumps that he could use to fill a drinking trough from a nearby river. One pump filled the trough in 60 minutes, another took 40 minutes and the third pump took 24 minutes. How long would it take to fill the trough if all three pumps were used at once?
9. Kyle is a painter. He and his apprentice, Lee, have the job of painting the rooms in a motel. Kyle can paint a room in 6 hours and if Lee helps it takes them 4 hours. How long would it take Lee to paint a room?
10. How many litres of a 20% acid solution must be added to 2 litres of 12% acid solution to make a 15% acid solution?
11. Cara rides her bike to school. She rides at an average speed of 32 km/h going to school and 20 km/h going home. The total riding time is 39 minutes. How far is the school from Cara's home?

Inequalities

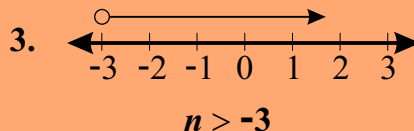
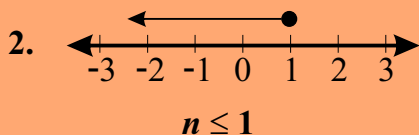
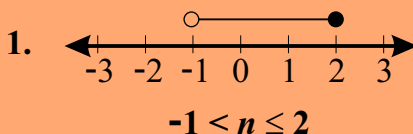
Regions on a number line can be described as shown in the examples below.

Note: The solid dot $\bullet$ indicates the number is included in the region.

The open dot $\circ$ indicates the number is **not** included in the region.

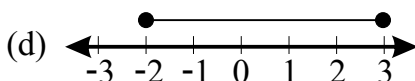
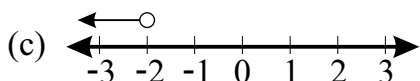
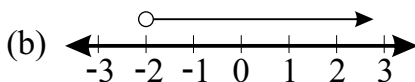
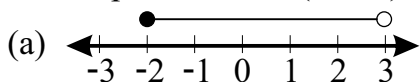
Symbols: $>$ - greater than
 $<$ - less than
 $\geq$ - greater than or equal to
 $\leq$ - less than or equal to

Examples



EXERCISE 7E

1. Match the regions shown on the following number lines with the descriptions below (A - D).



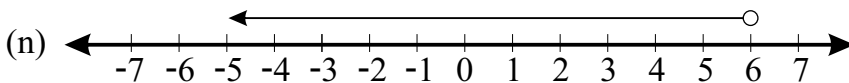
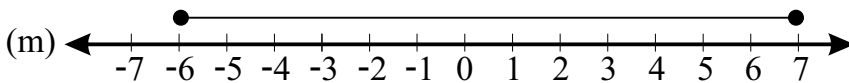
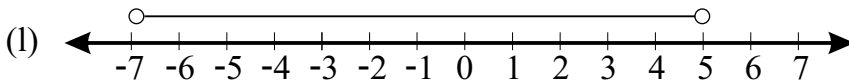
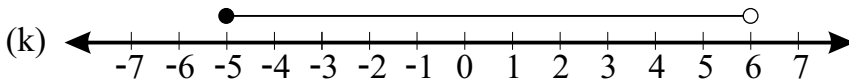
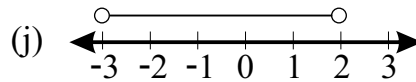
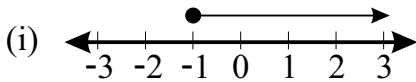
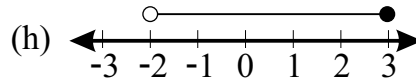
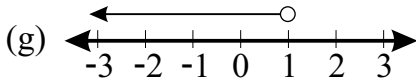
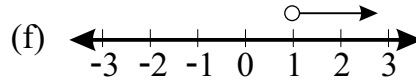
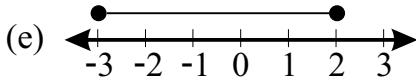
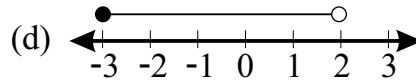
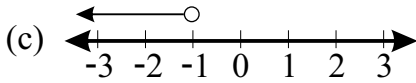
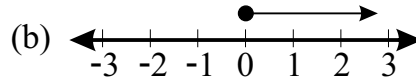
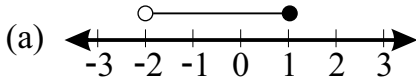
A $n < -2$

B $-2 \leq n < 3$

C $n > -2$

D $-2 \leq n \leq 3$

2. Write the descriptions of the regions shown on the number lines below.



3. Draw number lines showing the regions described below.

(a) $-5 < n < 4$

(b) $n > 2$

(c) $-2 < n \leq 4$

(d) $n \geq -5$

(e) $n \leq 3$

(f) $n < -1$

(g) $3 \leq n < 8$

(h) $-4 < n \leq 2$

(i) $-3 \leq n$

(j) $-6 \leq n \leq 0$

(k) $n > 0$

(l) $n \leq 0$

(m) $0 \leq n < 7$

(n) $-7 \leq n \leq -3$

(o) $-2 \leq n < 8$

4. Find the regions described by the inequalities below.

Examples

$$1. \quad n + 5 > 2$$

$$\boxed{-5} \quad \boxed{-5}$$

$$n > -3$$

$$2. \quad \frac{2(3n - 4)}{5} + 3 \leq 11$$

$$\boxed{-3} \quad \boxed{-3}$$

$$\frac{2(3n - 4)}{5} \leq 8$$

$$\boxed{\times 5} \quad \boxed{\times 5}$$

$$2(3n - 4) \leq 40$$

$$6n - 8 \leq 40$$

$$\boxed{+8} \quad \boxed{+8}$$

$$6n \leq 48$$

$$\boxed{\div 6} \quad \boxed{\div 6}$$

$$n \leq 8$$

(a) $n - 6 > 8$

(b) $n + 7 < 3$

(c) $5n \geq -20$

(d) $\frac{n}{2} \leq 4$

(e) $2n + 4 \geq 10$

(f) $3n - 7 \leq -13$

(g) $4(n - 6) < 12$

(h) $5(2n + 9) > 5$

(i) $\frac{n - 5}{3} \leq -2$

(j) $\frac{n + 2}{4} - 1 \geq -5$

(k) $\frac{2(n - 5)}{7} < -6$

(l) $\frac{n - 6}{3} + 9 < -9$

(m) $\frac{5(2n - 7)}{2} + 8 \leq 18$

(n) $\frac{3(2n + 7)}{4} - 8 > -17$

(o) $\frac{2(2n - 7)}{5} + 3 \geq 17$

(p) $\frac{2(3n + 4)}{5} + 1 < -19$

5. Find the regions described by the inequalities below.

Examples

1. $5 - 4n > -15$

-5

-5

$$-4n > -20$$

$\div -4$

$\div -4$

$$n < 5$$

If an inequation is divided or multiplied by a negative number the inequality symbol is reversed.

$>$ changed to $<$

$<$ changed to $>$

$\geq$ changed to $\leq$

$\leq$ changed to $\geq$

2. $8 + 3n \leq 5(n + 4)$

$$8 + 3n \leq 5n + 20$$

$$8 \leq 2n + 20$$

$$-12 \leq 2n$$

$$-6 \leq n$$

$$n \geq -6$$

If the terms on either side of an inequality are swapped the inequality symbol is reversed.

$>$ changed to $<$

$<$ changed to $>$

$\geq$ changed to $\leq$

$\leq$ changed to $\geq$

(a) $6 - n > 2$

(c) $6 - 5n \geq -19$

(e) $3(2 - 5n) \geq -24$

(g) $2(6 - \frac{n}{3}) < 20$

(i) $\frac{5 - n}{3} \leq 4$

(k) $\frac{2(5 - n)}{7} < -2$

(m) $4n + 6 \geq 7n - 3$

(o) $5n - 7 < 9n + 13$

(q) $4(5 + 6n) > 3(10n + 5)$

(b) $7 - 4n < 3$

(d) $-9 - 2n \leq 3$

(f) $4(3 - 2n) \leq 28$

(h) $2(6 - \frac{n}{3}) < -4$

(j) $\frac{2 - n}{4} - 1 \geq 2$

(l) $\frac{5(7 - 3n)}{2} - 6 \leq -11$

(n) $3n - 1 > 5n + 9$

(p) $2(5 - 3n) \geq 6n + 58$

(r) $4(3n - 8) \leq 5(4n + 7)$

Worded Problems

Example 1

- (a) Cheryl runs at a speed of **at least** 6 m/sec. Write an inequality stating the distance she runs.
 (b) If she runs for 10 minutes, how far will she run?

$$\text{distance} = \text{speed} \times \text{time}$$

$$d = s \times t$$

Answers

- (a) Given that she runs at a speed of at least 6 m/sec, the distance she runs will be at least $6 \times t$.

$$d \geq 6 \times t$$

$$d \geq 6t \text{ (where } t \text{ is in seconds)}$$

- (b) If she ran for 10 minutes (600 sec) then:

$$d \geq 6 \times 600$$

$$d \geq 3600 \text{ m}$$

Example 2

- (a) Liam also enjoys running and knows his running speed varies between 5 m/sec and 9 m/sec. Write an inequality stating the time it takes Liam to run a certain distance.
 (b) How long will it take Liam to run 4.5 km?

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$t = \frac{d}{s}$$

Answers

- (a) The time taken will be longest when running at 5 m/sec and shortest when running at 9 m/sec. The time taken will be in this range.

$$\frac{d}{9} \leq t \leq \frac{d}{5}$$

- (b) For $d = 4.5 \text{ km}$ (4500 m):

$$\frac{4500}{9} \leq t \leq \frac{4500}{5}$$

$$500 \leq t \leq 900 \text{ (seconds)}$$

Example 3

Asha has \$400 in her bank account. She plans to save \$75 per week until she has a total of **at least** \$1300 she needs to buy a bicycle. Write this as an inequality and solve.

Let n = the number of weeks that Asha saves \$75 per week

The total amount of money Asha will have in her bank account after n weeks is:

$$400 + 75n$$

Asha wants this total to be at least \$1300.

$$400 + 75n \geq 1300$$

Solve for n : $400 + 75n \geq 1300$

$$75n \geq 900$$

$$n \geq 12$$

So Asha needs to save for at least 12 weeks.

Example 4

Joel has \$2000 in his bank account. \$820 is deposited into his account each week by his employer and he withdraws at least \$450 each week.

- Write an inequality relating the amount Joel has in his bank account, A , after n weeks.
- Find out how much Joel will have in his bank account after 12 weeks.

Answers

- If Joel withdrew \$450 each week then the amount in his account would be:

$$A = 2000 + 820n - 450n$$

$$A = 2000 + 370n$$

But because he withdraws at least this amount then he will have this amount or less. So:

$$A \leq 2000 + 370n$$

Example 4 continued

(b) After 12 weeks :

$$A \leq 2000 + 370 \times 12$$

$$A \leq 2000 + 4440$$

$$A \leq \$6440$$

EXERCISE 7F

1. (a) Finnigan regularly jogs at a speed of **at least** 5 m/sec. Write an inequality stating the distance, d , in metres, he jogs in t seconds.
 (b) If he runs for 20 minutes, how far will he run?
 (c) How long will it take Finnigan to run a distance of 12 km?
 Give answer in minutes.
2. Anja enjoys riding her bicycle and noted her riding speed varies between 24 km/h and 42 km/h.
 (a) Convert these speeds to m/min.
 (b) Write an inequality stating the time, t , in minutes, it takes Anja to ride a certain distance, d , in metres. Use the speed in m/min.
 (c) How long would it take Anja to ride 28 km? Give answer in minutes.
3. Charlie has \$3000 in his bank account. He plans to save **at least** \$250 per week.
 (a) Write an inequality stating the amount, A , Charlie will have in his bank account after n weeks.
 (b) After how many weeks, w , will Charlie have \$10 000 in his bank account?
4. Vera has \$4000 in her bank account. \$500 is paid into her bank account each week and she withdraws **at least** \$600 each week.
 (a) Write an inequality stating the amount, A , Vera has in her bank account after w weeks.
 (b) After how many weeks will Vera have nothing in her bank account?

5. The total weight of a truck and its load cannot exceed 4000 kg.
The truck without any load weighs 2400 kg.
The truck was to have n crates each weighing 80 kg loaded onto it.
(a) Write an inequality for this information.
(b) Solve this inequality.
6. Kyle makes wooden toys. The material cost for each toy is \$15 and he sells each toy for *at least* \$90.
(a) Write an inequality stating the amount, A , Kyle makes on producing n toys.
(b) Find the amount Kyle will earn each week if he sells nine toys each week.
(c) Kyle wanted to change these amounts to earn at least \$720 per week from producing eight toys. Using p to represent the price of each toy write an inequality for this information and solve it for p .



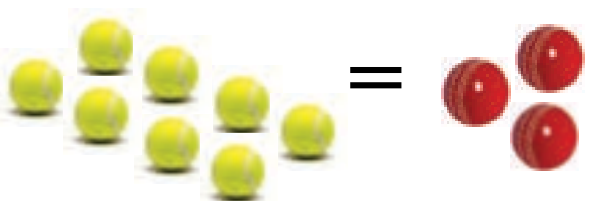
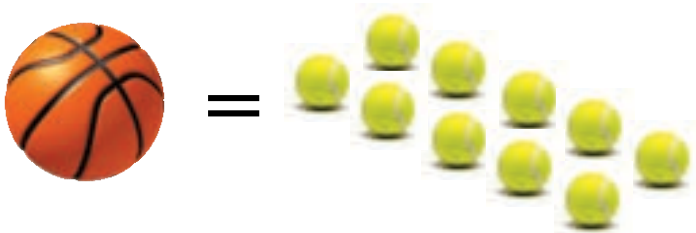
7. A eucalyptus tree grew by between 50 cm and 2 m each year depending on the weather conditions. A eucalyptus tree was 2 m tall when it was planted.
(a) Write an inequality stating the height, H , in metres, of the tree after n years.
(b) Solve this inequality to find the height range of the tree after 12 years.
(c) Solve the inequality to find the number of years it would take for the tree to reach its full height of 30 m.

PROBLEM SOLVING

The diagrams below show the equivalent *masses* of basketballs, tennis balls, golf balls and cricket balls.
For example one basketball weighs the same as ten tennis balls.
The following fact is also true:

Three tennis balls plus four golf balls plus two cricket balls is the same mass as one basketball plus 80 grams

Find the mass of each of these sports balls.



PUZZLE

Each shape in this puzzle represents a number.
The sum of two rows and a column are shown.
Find the number represented by each shape
and complete the table.

■	■	■	●	8
●	▲	■	▲	
●	▲	▲	●	14
●	■	●	■	
			13	

CHAPTER REVIEW

1. Solve the following equations.

- | | | | |
|---------------------------------------|---|---------------------------------|-----------------------|
| (a) $x + 5 = 1$ | (b) $n - 6 = -4$ | (c) $7a = 42$ | (d) $\frac{y}{4} = 5$ |
| (e) $3m + 7 = 16$ | (f) $2x - 4 = -20$ | (g) $2(3x + 4) = -4$ | |
| (h) $8a - 3 = -7$ | (i) $3(m - 2) + 7 = -10$ | (j) $8 - 3(2n + 1) = 6$ | |
| (k) $\frac{2n + 3}{5} = 5$ | (l) $\frac{3c - 5}{7} = -2$ | (m) $\frac{m - 4}{6} + 3 = 8$ | |
| (n) $\frac{2x + 3}{3} - 6 = 1$ | (o) $\frac{4(n + 3)}{5} = 8$ | (p) $\frac{3(2c - 4)}{2} = -14$ | |
| (q) $\frac{3(x - 4)}{5} + 2 = -1$ | (r) $\frac{4(2m + 5)}{5} - 17 = 5$ | | |
| (s) $5m + 3 = 3m + 7$ | (t) $2n - 6 = 5n - 12$ | | |
| (u) $2(2a + 5) = 3(3a - 5)$ | (v) $3(2y - 6) = 4(5y - 1)$ | | |
| (w) $\frac{7}{2a} + 1 = 8$ | (x) $\frac{7}{n - 2} - 3 = \frac{3}{4}$ | | |
| (y) $\frac{4x}{3x + 5} = \frac{2}{5}$ | (z) $\frac{1}{y} + \frac{2}{3y} = 8$ | | |

2. (a) If three is added to five times this number and the result is multiplied by two, the answer is 26. Write this as an equation then solve.
- (b) Bruce cut a 45 m length of rope into three pieces. The longest was twice the length of the shortest and the other piece was 5 m longer than the shortest. Find the length of the three pieces of rope.
- (c) A tea producer wants to make a blend of two teas (**A** and **B**) that will cost \$15 per kilogram. Tea **A** costs \$12.20 per kg and tea **B** costs \$16.20 per kg. How many grams of each tea should be used in each kilogram of the blend?

3. Solve the following equations.

- | | |
|--------------------------|--------------------------|
| (a) $x^2 + 13x = 0$ | (b) $15x^2 = 12x$ |
| (c) $x^2 - 64 = 0$ | (d) $4(x + 1)^2 - 9 = 0$ |
| (e) $x^2 - 15 = 0$ | (f) $20x^2 - 15 = 0$ |
| (g) $x^2 - 9x + 20 = 0$ | (h) $x^2 + 2x - 24 = 0$ |
| (i) $2x^2 - 7x - 30 = 0$ | (j) $12x^2 + x - 20 = 0$ |
| (k) $x^2 + 16x + 64 = 0$ | (l) $-x^2 - 9x - 18$ |
| (m) $x^2 + 4x - 10 = 0$ | (n) $-2x^2 + 7x + 5 = 0$ |

4. Find the regions described by the inequalities below and draw them on number lines.

(a) $2n + 5 > 13$

(b) $3(4n - 7) \leq -45$

(c) $\frac{4(n+7)}{3} \geq 8$

(d) $\frac{5(2n-3)}{3} + 2 < 7$

(e) $5 - 3n \leq 23$

(f) $6 + 2n > 5n - 3$

5. The cost of hiring a car was \$40 plus 20 cents per km driven.

Gerry wanted to hire a car and spend no more than \$250.

(a) Write an inequality stating amount Gerry will spend on the car hire after driving k kilometres.

(b) Solve it to find how many kilometres Gerry can drive.

Polynomials

Polynomials

The word **polynomial** comes from the Greek terms **poly** (many) and **nomen** (name or term).

So polynomial means **many terms**.

A polynomial is an algebraic expression that contains several terms that can be a combination of numbers (coefficients) and/or variables (pronumerals) with **positive whole number** indices (exponents).

Examples

The following expressions are all polynomials.

$$7a$$

$$3x^4 + 5x^2 - x + 8$$

$$8x^2 + 3y^4$$

$$m^3p^3 - 6m^2n^3$$

$$6x^3y^7z + 8x^4y^3 - 7yz^6$$

The following expressions are **not** polynomials

$$\sqrt{x}$$

$$3y^{-5} + 3$$

$$\frac{m^3}{n^2} + 2p^4 - 6m^5$$

These are not polynomials because they have non-positive indices or indices that are not whole numbers.

Degree of a Term

For a term with more than one variable the **degree** of the term is defined as the sum of the indices of the variables.

Examples

$$x^3y^4 \text{ has a degree of } 7 \text{ (} 3 + 4 \text{)}$$

$$5ab^8 \text{ has a degree of } 9 \text{ (} 1 + 8 \text{)}$$

For a term with only one variable the **degree** of the term is defined as the size of the index of that variable.

Examples

$$5x^8 \text{ has a degree of } 8$$

$$6a \text{ has a degree of } 1 \text{ (} a = a^1 \text{)}$$

$$9 \text{ has a degree of } 0 \text{ (} 1 = x^0 \text{)}$$

Degree of a Polynomial

The degree of a polynomial is defined as the degree of the term with the highest degree.

Examples $3x^4 + 5x^2 - 7$ is a degree 4 polynomial

Polynomials are written with the terms in order from the term with the highest degree first and the other terms in order of descending degrees.

Examples 1. $3x^2 + 5x^4 - 3 + 4x$

This polynomial would be written with the terms in the following order:

$$5x^4 + 3x^2 + 4x - 3$$

$$2. \quad 8 + 6y^5 - 2y^4 - 3y^6 + \frac{3}{4}y$$

This polynomial would be written with the terms in the following order:

$$-3y^6 + 6y^5 - 2y^4 + \frac{3}{4}y + 8$$

EXERCISE 8A

1. Which of the following expressions are polynomials?

- A** $3y - 6x$ **B** $7a^2b^3 + 6$ **C** $\frac{7}{3}m^6 - 6n^7$ **D** $7y^2z^{-3} + 6x^2y^6$
E $6y^4 - 5x^{\frac{3}{4}}$ **F** $2x^5 - 3x^4 + 9$ **G** $2\frac{x}{y} + 5y$ **H** 9

2. (i) Arrange the terms in the following expressions in order from the term with the highest degree to the term with the lowest degree.

(ii) State the degree of these polynomials.

(a) $6x^3 + 4x - 7 + x^4$

(b) $2m^2 - 6m^5 + 9 + 5m^4$

(c) $a - 4a^3 + 2a^2 - 5a^6 + 3a^4$

(d) $9 + 5x^5 - 2x^2 + \frac{7}{3}x^4 - 3x^7 + x^3$

As was shown in Chapter 6 a number of expressions can be multiplied to form a single polynomial.

Example 1 $(3x + 7)(2x - 5)$
 $= 6x^2 - 15x + 14x - 35$ (*Using FOIL p140*)
 $= 6x^2 - x - 35$

Example 2 $(2x - 5)(x + 6)(3x - 2)$
 $= (2x^2 + 12x - 5x - 30)(3x - 2)$
 $= (2x^2 + 7x - 30)(3x - 2)$
 $= 6x^3 - 4x^2 + 21x^2 - 14x - 90x + 60$
 $= 6x^3 + 17x^2 - 104x + 60$

EXERCISE 8B

Expand the following expressions and state the degree of the resultant polynomial.

- | | |
|---------------------------------------|---------------------------------------|
| 1. $(5x + 4)(2x - 3)$ | 2. $(3a - 5)(4 - a)$ |
| 3. $a(4a - 1)(2a + 3)$ | 4. $n(2n + 5)(n + 6)$ |
| 5. $3x(3x - 4)(5 - x)$ | 6. $2x(2x - 7)(3x + 4)$ |
| 7. $(n + 1)(n - 3)(n + 4)$ | 8. $(a + 4)(5 - a)(2a - 3)$ |
| 9. $(2x - 1)(3x + 2)(5 - 2x)$ | 10. $(3x - 4)(3x + 4)(2x - 5)$ |
| 11. $(2x - 5)(2x + 5)(3x + 4)$ | 12. $(3x - 1)(5x - 4)(4x - 7)$ |
| 13. $m(m - 4)(m + 1)(m - 3)$ | 14. $m(m - 3)(2 - m)(2m - 4)$ |
| 15. $2n(n - 5)(2n + 1)(3n - 2)$ | 16. $2x(x - 4)(5 - 2x)(3x - 2)$ |
| 17. $(x + 4)(2x - 3)(3x - 1)(x - 4)$ | 18. $(2x - 3)(3x - 4)(4x - 3)(x + 5)$ |
| 19. $(3 - x)(4x - 5)(1 - 3x)(2x - 1)$ | 20. $(3x + 5)(3 - 5x)(6 - x)(x - 4)$ |

Factorising Polynomials

In Chapter 6 several techniques were used to find the factors of quadratic polynomials (degree of 2).

Factorising polynomials with degrees more than 2 require different techniques.

One way to find the factors of these polynomials is to use a technique called the **factor theorem**.

The **factor theorem** states that if $(x - a)$ is a factor of a polynomial then if a is substituted into the polynomial the result is 0.

This fact is used 'in reverse' to find the first factor:

find a by trial and error then the factor will be $(x - a)$.

To use the factor theorem to factorise a polynomial follow the steps in the examples below.

At this stage only polynomials with degree 3 (called cubics) will be studied.

Example 1

Factorise the following polynomial:

$$x^3 - 2x^2 - 5x + 6$$

Step 1 Find all the factors of the **last term** (the term without a variable called the **constant term**) and list these in order from the smallest to the largest. In this example the last term is 6.

$$x^3 - 2x^2 - 5x + 6$$

The factors of 6 are: 1, -1, 2, -2, 3, -3, 6, -6

Step 2 Substitute the smallest factor (1) into the polynomial.

$$x^3 - 2x^2 - 5x + 6$$

$$\begin{aligned} x = 1: \quad & 1^3 - 2 \times 1^2 - 5 \times 1 + 6 \\ & = 1 - 2 - 5 + 6 \\ & = 0 \end{aligned}$$

Because the answer is 0 this means that $a = 1$ and $(x - 1)$ is a factor. If the answer had not been 0 then you need to move onto the next factor and repeat this step. This will be demonstrated in a later example.

Step 3 Divide this factor into the polynomial. To do this follow the steps below.

Step 3a Draw the division as shown here.

$$\text{factor} \rightarrow x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \leftarrow \text{polynomial}$$

Step 3b Find what the first term in the factor needs to be multiplied by to get the first term in the polynomial.
In this example $x \times x^2 = x^3$

$$\begin{array}{c} x^2 \\ \curvearrowright \\ x - 1 \end{array} \overline{) x^3 - 2x^2 - 5x + 6}$$

Step 3c Write this as shown below.

$$x - 1 \overline{) \begin{array}{c} x^2 \\ x^3 - 2x^2 - 5x + 6 \end{array}}$$

Step 3d Multiply each term in the factor by this term and write this as shown below.

$$\begin{array}{c} \times \\ x - 1 \end{array} \overline{) \begin{array}{c} x^2 \\ x^3 - 2x^2 - 5x + 6 \end{array}} \\ \hline = \quad x^3 - x^2$$

Step 3e Subtract this from the polynomial as shown below.

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6
 \end{array}$$

$$\begin{aligned}
 & -2x^2 - (-x^2) \\
 &= -2x^2 + x^2 \\
 &= -x^2
 \end{aligned}$$

Step 3f Find what the first term in the factor needs to be multiplied by to get the first term in this expression.
In this example $x \times -x = -x^2$

$$\begin{array}{r}
 x^2 \\
 \textcircled{x} - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6
 \end{array}$$

$\times -x \rightarrow \textcircled{-x^2} - 5x + 6$

Step 3g Write this as shown below.

$$\begin{array}{r}
 x^2 - x \\
 x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6
 \end{array}$$

Step 3h Multiply each term in the factor by this term and write this as shown below.

$$\begin{array}{r}
 \times \\
 \textcircled{-x} \\
 x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6 \\
 = \\
 -x^2 + x
 \end{array}$$

Step 3i Subtract this from the expression above it as shown below.

$$\begin{array}{r}
 x^2 - x \\
 x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6 \\
 \underline{-(-x^2 + x)} \\
 -6x + 6
 \end{array}$$

Step 3j Find what the first term in the factor needs to be multiplied by to get the first term in this expression.
In this example $x \times -6 = -6x$

$$\begin{array}{r}
 x^2 - x \\
 \textcircled{x} - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6 \\
 \underline{-(-x^2 + x)} \\
 -6x + 6
 \end{array}$$

$\times -6$
→
 $\textcircled{-6x} + 6$

Step 3k Write this as shown below.

$$\begin{array}{r}
 x^2 - x - 6 \\
 x - 1 \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6 \\
 \underline{-(-x^2 + x)} \\
 -6x + 6
 \end{array}$$

Step 3l Multiply each term in the factor by this term and write this as shown below.

$$\begin{array}{r}
 \begin{array}{c} \times \\ \downarrow \end{array} \begin{array}{r} x^2 - x - 6 \\ \hline x^3 - 2x^2 - 5x + 6 \\ - x^3 - x^2 \\ \hline -x^2 - 5x + 6 \\ - -x^2 + x \\ \hline -6x + 6 \\ = -6x + 6 \end{array} \\
 x - 1 \quad \begin{array}{c} \downarrow \\ \downarrow \end{array}
 \end{array}$$

Step 3m Subtract this from the expression above it as shown below.

$$\begin{array}{r}
 \begin{array}{c} x^2 - x - 6 \\ \hline x^3 - 2x^2 - 5x + 6 \\ - x^3 - x^2 \\ \hline -x^2 - 5x + 6 \\ - -x^2 + x \\ \hline -6x + 6 \\ - -6x + 6 \\ \hline 0 \end{array} \\
 x - 1 \quad \begin{array}{c} \downarrow \\ \downarrow \end{array}
 \end{array}$$

Step 3n The result should be 0. We now have two factors of the polynomial as shown below.

$$\begin{array}{r}
 \textcircled{x^2 - x - 6} \\
 \textcircled{x - 1} \overline{) x^3 - 2x^2 - 5x + 6} \\
 \underline{-x^3 - x^2} \\
 -x^2 - 5x + 6 \\
 \underline{-x^2 + x} \\
 -6x + 6 \\
 \underline{-6x + 6} \\
 0
 \end{array}$$

Step 4 The two factors are: $(x - 1)(x^2 - x - 6)$
 The second factor $(x^2 - x - 6)$ can be further factorised using the techniques developed in Chapter 6 to factorise quadratic equations:

$$x^2 - x - 6 = (x - 3)(x + 2)$$

Step 5 The polynomial is now fully factorised:

$$x^3 - 2x^2 - 5x + 6 = (x - 1)(x - 3)(x + 2)$$

Remember: The order that the factors are given does not matter:

$$\begin{aligned}
 & (x - 1)(x - 3)(x + 2) \\
 &= (x - 1)(x + 2)(x - 3) \\
 &= (x + 2)(x - 1)(x - 3), \text{ etc}
 \end{aligned}$$

Example 2

Factorise the following polynomial:

$$2x^3 - 3x^2 - 18x - 8$$

Step 1 Find all the factors of the **last term** (the term without a variable called the constant term) and list these in order from the smallest to the largest. In this example the last term is -8.

$$2x^3 - 3x^2 - 18x - 8$$

The factors of -8 are: 1, -1, 2, -2, 4, -4, 8, -8

Step 2 Substitute the smallest factor (1) into the polynomial.

$$2x^3 - 3x^2 - 18x - 8$$

$$\begin{aligned} x = 1: \quad & 2 \times 1^3 - 3 \times 1^2 - 18 \times 1 - 8 \\ & = 2 - 3 - 18 - 8 \\ & = -27 \end{aligned}$$

Because the answer is not 0 we need to keep trying the factors until we find one that results in an answer of 0.

$$\begin{aligned} x = -1: \quad & 2 \times -1^3 - 3 \times -1^2 - 18 \times -1 - 8 \\ & = -2 - 3 + 18 - 8 \\ & = 5 \end{aligned}$$

Still not 0 so try the next factor.

$$\begin{aligned} x = 2: \quad & 2 \times 2^3 - 3 \times 2^2 - 18 \times 2 - 8 \\ & = 16 - 12 - 36 - 8 \\ & = -40 \end{aligned}$$

Still not 0 so try the next factor.

$$\begin{aligned} x = -2: \quad & 2 \times -2^3 - 3 \times -2^2 - 18 \times -2 - 8 \\ & = -16 - 12 + 36 - 8 \\ & = 0 \end{aligned}$$

Because the answer is 0 this means that $a = -2$ and $(x - -2) = (x + 2)$ is a factor.

Step 3 Divide this factor into the polynomial.

Step 3a

$$x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8}$$

Step 3b

$$\begin{array}{r} 2x^2 \\ \curvearrowright \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \end{array}$$

Step 3c

$$\begin{array}{r} 2x^2 \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \end{array}$$

Step 3d

$$\begin{array}{r} 2x^2 \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\ 2x^3 + 4x^2 \end{array}$$

Step 3e

$$\begin{array}{r} 2x^2 \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\ - 2x^3 + 4x^2 \\ \hline -7x^2 - 18x - 8 \end{array}$$

Step 3f

$$\begin{array}{r} 2x^2 \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\ - 2x^3 + 4x^2 \\ \hline -7x^2 - 18x - 8 \\ \times -7x \quad \rightarrow \quad -7x^2 \end{array}$$

Step 3g

$$x + 2 \sqrt{\frac{2x^2 - 7x}{2x^3 - 3x^2 - 18x - 8} - \frac{2x^3 + 4x^2}{-7x^2 - 18x - 8}}$$

Step 3h

$$\begin{array}{r}
 \times \\
 x+2 \overline{) \begin{array}{r} 2x^2 - 7x \\ 2x^3 - 3x^2 - 18x - 8 \\ \underline{-2x^3 + 4x^2} \\ -7x^2 - 18x - 8 \\ \underline{-7x^2 - 14x} \end{array} } \\
 =
 \end{array}$$

Step 3i

$$\begin{array}{r} 2x^2 - 7x \\ x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\ \underline{-2x^3 + 4x^2} \\ -7x^2 - 18x - 8 \\ \underline{-7x^2 - 14x} \\ -4x - 8 \end{array}$$

Step 3j

$$\begin{array}{r}
 2x^2 - 7x \\
 \hline
 x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\
 \underline{- 2x^3 + 4x^2} \\
 -7x^2 - 18x - 8 \\
 \underline{- -7x^2 - 14x} \\
 -4x - 8
 \end{array}$$

Step 3k

$$\begin{array}{r}
 2x^2 - 7x - 4 \\
 x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\
 \underline{- 2x^3 + 4x^2} \\
 -7x^2 - 18x - 8 \\
 \underline{- -7x^2 - 14x} \\
 -4x - 8
 \end{array}$$

Step 3l

$$\begin{array}{r}
 \times \\
 \swarrow \\
 2x^2 - 7x - 4 \\
 x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\
 \underline{- 2x^3 + 4x^2} \\
 -7x^2 - 18x - 8 \\
 \underline{- -7x^2 - 14x} \\
 -4x - 8 \\
 = \\
 -4x - 8
 \end{array}$$

Step 3m

$$\begin{array}{r}
 2x^2 - 7x - 4 \\
 x + 2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\
 \underline{- 2x^3 + 4x^2} \\
 -7x^2 - 18x - 8 \\
 \underline{- -7x^2 - 14x} \\
 -4x - 8 \\
 \underline{- -4x - 8} \\
 0
 \end{array}$$

Step 3n The result is 0. We now have two factors of the polynomial as shown below.

$$\begin{array}{r}
 \overline{2x^2 - 7x - 4} \\
 x+2 \overline{) 2x^3 - 3x^2 - 18x - 8} \\
 \underline{- 2x^3 + 4x^2} \\
 -7x^2 - 18x - 8 \\
 \underline{- -7x^2 - 14x} \\
 -4x - 8 \\
 \underline{- -4x - 8} \\
 0
 \end{array}$$

Step 4 The two factors are: $(x + 2)(2x^2 - 7x - 4)$
 The second factor $(2x^2 - 7x - 4)$ can be further factorised using the techniques developed in Chapter 6 to factorise quadratic equations:

$$2x^2 - 7x - 4 = (2x + 1)(x - 4)$$

Step 5 The polynomial is now fully factorised:

$$2x^3 - 3x^2 - 18x - 8 = (x + 2)(2x + 1)(x - 4)$$

Example 3

Factorise the following polynomial:

$$x^3 + 7x^2 - 36$$

Step 1 Find all the factors of the **last term** (the term without a variable called the constant term) and list these in order from the smallest to the largest. In this example the last term is -36.

$$x^3 + 7x^2 - 36$$

The factors of -36 are: 1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 9, -9, 12, -12, 18, -18, 36, -36

Step 2 Substitute the smallest factor (1) into the polynomial.

$$x^3 + 7x^2 - 36$$

$$\begin{aligned} x = 1: \quad & 1^3 + 7 \times 1^2 - 36 \\ & = 1 + 7 - 36 \\ & = -28 \end{aligned}$$

Because the answer is not 0 we need to keep trying the factors until we find one that results in an answer of 0.

$$\begin{aligned} x = -1: \quad & -1^3 + 7 \times -1^2 - 36 \\ & = -1 + 7 - 36 \\ & = -30 \end{aligned}$$

Still not 0 so try the next factor.

$$\begin{aligned} x = 2: \quad & 2^3 + 7 \times 2^2 - 36 \\ & = 8 + 28 - 36 \\ & = 0 \end{aligned}$$

Because the answer is 0 this means that $a = 2$ and $(x - 2)$ is a factor.

Step 3 Divide this factor into the polynomial.

Step 3a

$$x - 2 \overline{) x^3 + 7x^2 - 36}$$

$$x - 2 \overline{) x^3 + 7x^2 + 0x - 36}$$

When there are one or more terms 'missing' add in a zero of these terms.

Step 3b

$$\begin{array}{r} x^2 \\ x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \end{array}$$

Step 3c

$$x - 2 \overline{) \begin{array}{r} x^2 \\ x^3 + 7x^2 + 0x - 36 \end{array}}$$

Step 3d

$$x - 2 \overline{) \begin{array}{r} x^2 \\ x^3 + 7x^2 + 0x - 36 \\ x^3 - 2x^2 \end{array}}$$

Step 3e

$$x - 2 \overline{) \begin{array}{r} x^2 \\ x^3 + 7x^2 + 0x - 36 \\ -x^3 - 2x^2 \\ \hline 9x^2 + 0x - 36 \end{array}}$$

Step 3f

$$\begin{array}{r} x^2 \\ x - 2 \overline{) \begin{array}{r} x^3 + 7x^2 + 0x - 36 \\ -x^3 - 2x^2 \\ \hline 9x^2 + 0x - 36 \end{array}} \\ \times 9x \rightarrow 9x^2 + 0x - 36 \end{array}$$

Step 3g

$$\begin{array}{r}
 x^2 + 9x \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36
 \end{array}$$

Step 3h

$$\begin{array}{r}
 \times \\
 x^2 + 9x \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 = 9x^2 - 18x
 \end{array}$$

Step 3i

$$\begin{array}{r}
 x^2 + 9x \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36
 \end{array}$$

Step 3j

$$\begin{array}{r}
 x^2 + 9x \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36
 \end{array}$$

$\times 18 \rightarrow 18x - 36$

Step 3k

$$\begin{array}{r}
 x^2 + 9x + 18 \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36
 \end{array}$$

Step 3l

$$\begin{array}{r}
 \times \\
 \swarrow \quad \searrow \\
 x^2 + 9x + \textcircled{18} \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36 \\
 = 18x - 36
 \end{array}$$

Step 3m

$$\begin{array}{r}
 x^2 + 9x + 18 \\
 x - 2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36 \\
 \underline{-18x - 36} \\
 0
 \end{array}$$

Step 3n The result is 0. We now have two factors of the polynomial as shown below.

$$\begin{array}{r}
 \overline{x^2 + 9x + 18} \\
 x-2 \overline{) x^3 + 7x^2 + 0x - 36} \\
 \underline{-x^3 - 2x^2} \\
 9x^2 + 0x - 36 \\
 \underline{-9x^2 - 18x} \\
 18x - 36 \\
 \underline{-18x - 36} \\
 0
 \end{array}$$

Step 4 The two factors are: $(x - 2)(x^2 + 9x + 18)$
 The second factor $(x^2 + 9x + 18)$ can be further factorised using the techniques developed in Chapter 6 to factorise quadratic equations:

$$x^2 + 9x + 18 = (x + 6)(x + 3)$$

Step 5 The polynomial is now fully factorised:

$$x^3 + 7x^2 - 36 = (x - 2)(x + 6)(x + 3)$$

Example 4

Factorise the following polynomial.

$$2x^3 + 16x^2 - 18x$$

Always look for a common factor first when factorising a polynomial.

Step 1 Find the highest common factor.

$$2x^3 + 16x^2 - 18x$$

The highest common factor in all the terms is **2x**.

Step 2 'Take out' the highest common factor.

$$2x(x^2 + 8x - 9)$$

Step 3 Factorise the remaining polynomial. In this example the polynomial is a quadratic so techniques from Chapter 6 can be used.

$$\begin{aligned} &2x(x^2 + 8x - 9) \\ &= 2x(x + 9)(x - 1) \end{aligned}$$

EXERCISE 8C

Factorise the following polynomials.

- | | |
|-------------------------------|-------------------------------|
| 1. $x^3 + 5x^2 + 2x - 8$ | 2. $x^3 - 13x^2 + 47x - 35$ |
| 3. $x^3 + 10x^2 - 27x - 36$ | 4. $x^3 + x^2 - 24x + 36$ |
| 5. $3x^3 - 4x^2 - 12x + 16$ | 6. $2x^3 + 7x^2 - 19x - 60$ |
| 7. $3x^3 - 14x^2 + 32$ | 8. $x^3 - 6x^2 + 32$ |
| 9. $x^3 - 13x - 12$ | 10. $x^3 - 21x + 20$ |
| 11. $x^3 + 5x^2 - 24x$ | 12. $4x^3 + 8x^2 - 32x$ |
| 13. $6x^3 + 23x^2 - 6x - 8$ | 14. $6x^3 - 25x^2 - 31x + 30$ |
| 15. $8x^3 + 50x^2 + 67x - 20$ | 16. $9x^3 - 48x^2 - 44x + 48$ |

Solving Polynomial Equations

We will only investigate solving equations with cubic polynomials but the techniques could be used to solve polynomials of any degree.

Example 1

Solve the following equation: $x^3 - 3x^2 - 6x + 8 = 0$

To solve this equation the polynomial needs to be factorised. The steps to factorise a polynomial have been detailed in previous pages.

For the purpose of this exercise the factors will be given.

$$x^3 - 3x^2 - 6x + 8 = (x - 1)(x + 2)(x - 4)$$

Therefore: $(x - 1)(x + 2)(x - 4) = 0$

For the left side of the equation to equal 0 one of the factors must be equal to 0.

So there are three possible solutions:

$$\text{Either } (x - 1) = 0 \text{ or } (x + 2) = 0 \text{ or } (x - 4) = 0$$

Solve these:

$$\begin{array}{ccccc} x - 1 = 0 & & x + 2 = 0 & & x - 4 = 0 \\ x = 1 & \text{or} & x = -2 & \text{or} & x = 4 \end{array}$$

Solutions: $x = 1, -2 \text{ or } 4$

Example 2

Solve the following equation: $x^3 + x^2 - 8x - 12 = 0$

$$\begin{aligned} x^3 + x^2 - 8x - 12 &= (x + 2)(x + 2)(x - 3) \\ &= (x + 2)^2(x - 3) \end{aligned}$$

Therefore: $(x + 2)^2(x - 3) = 0$

For this example there are two possible solutions:

Either $(x + 2) = 0$ or $(x - 3) = 0$

Solve these:

$$\begin{array}{ccc} x + 2 = 0 & \text{or} & x - 3 = 0 \\ \mathbf{x = -2} & & \mathbf{x = 3} \end{array}$$

Solutions: $\mathbf{x = -2 \text{ or } 3}$

Example 3

Solve the following equation: $x^3 - 12x^2 + 48x - 64 = 0$

$$\begin{aligned} x^3 - 12x^2 + 48x - 64 &= (x - 4)(x - 4)(x - 4) \\ &= (x - 4)^3 \end{aligned}$$

Therefore: $(x - 4)^3 = 0$

For this example there is one possible solution:

$$(x - 4) = 0$$

Solve this: $x - 4 = 0$

$$\mathbf{x = 4}$$

Solution: $\mathbf{x = 4}$

Example 4

Solve the following equation: $12x^3 + 17x^2 + 2x - 3 = 0$

$$12x^3 + 17x^2 + 2x - 3 = (3x - 1)(4x + 3)(x + 1)$$

Therefore: $(3x - 1)(4x + 3)(x + 1) = 0$

For this example there are three possible solutions:

Either $(3x - 1) = 0$ or $(4x + 3) = 0$ or $(x + 1) = 0$

Solve these:

$$\begin{array}{llll} 3x - 1 = 0 & \text{or} & 4x + 3 = 0 & \text{or} & x + 1 = 0 \\ 3x = 1 & & 4x = -3 & & x = -1 \\ x = \frac{1}{3} & & x = -\frac{3}{4} & & \end{array}$$

Solutions: $x = \frac{1}{3}, -\frac{3}{4} \text{ or } -1$

EXERCISE 8D

Solve the following equations. The factors for each cubic are given.

1. $x^3 - 3x^2 - 10x + 24 = 0$

Factors: $x^3 - 3x^2 - 10x + 24 = (x - 2)(x + 3)(x - 4)$

2. $x^3 + 8x^2 + 5x - 50 = 0$

Factors: $x^3 + 8x^2 + 5x - 50 = (x + 3)^2(x - 2)$

3. $x^3 + 21x^2 + 147x + 343 = 0$

Factors: $x^3 + 21x^2 + 147x + 343 = (x + 7)^3$

4. $6x^3 - 37x^2 + 32x + 15 = 0$

Factors: $6x^3 - 37x^2 + 32x + 15 = (2x - 3)(3x + 1)(x - 5)$

5. $32x^3 - 64x^2 + 10x + 25 = 0$

Factors: $32x^3 - 64x^2 + 10x + 25 = (4x - 5)^2(2x + 1)$

Sketching Polynomials

The shape of the curve of a polynomial depends on the degree of the polynomial.

All polynomials have a number of turning points.

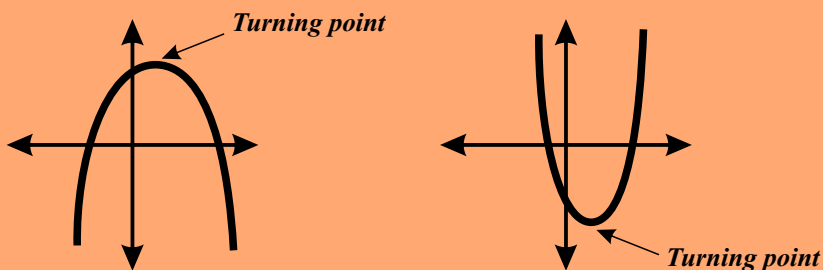
The turning points of a curve are the points where the curve changes from going up to going down or vice versa.

The number of turning points depends on the degree of the polynomial.

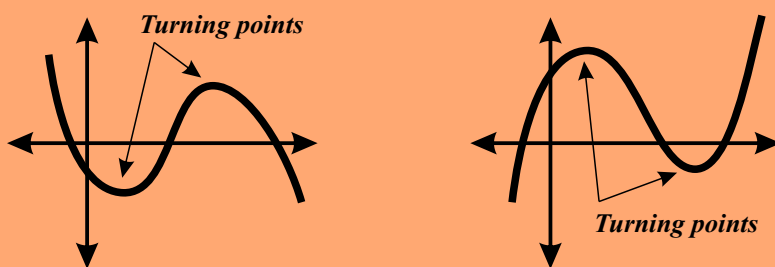
The number of turning points is one less than the degree of the polynomial.

Examples

A polynomial with a degree of 2 (quadratic) has one turning point.

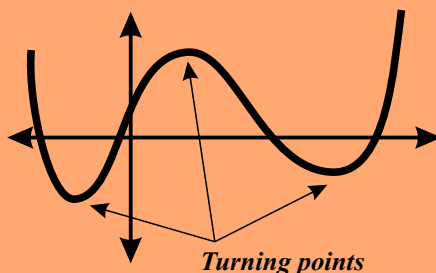


A polynomial with a degree of 3 (cubic) has two turning points.



A polynomial with a degree of 4 (quartic) has three turning points.

Et cetera



The curves of **quadratics** are called **parabolas** and these will be studied in detail in a later chapter.

The curves of **cubic** polynomials will be studied now.

To sketch any curve of a polynomial, key points of the curve need to be identified.

The key points of any curve are the points at which the curve crosses the axes and the turning points.

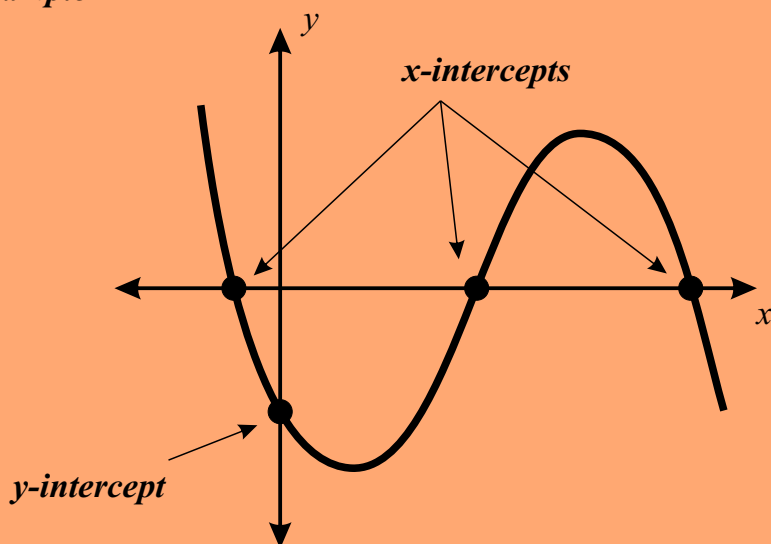
The points where a curve crosses the **x-axis** are called the **x-intercepts**.

The point where a curve crosses the **y-axis** is called the **y-intercept**.

The easiest way to sketch the curve of a polynomial is to find the x - and y - intercepts. Finding the coordinates of the turning points will be studied later.

*The x -intercepts occur where $y = 0$.
The y -intercept occurs where $x = 0$.*

Example



Example 1

Sketch the following curve: $y = x^3 - 4x^2 + x + 6$

Step 1 Find the x -intercepts.

To do this the polynomial needs to be factorised.

A technique for factorising a polynomial was detailed earlier in this chapter.

The factors of $x^3 - 4x^2 + x + 6$ are $(x + 1)(x - 2)(x - 3)$

$$\text{So } y = (x + 1)(x - 2)(x - 3)$$

The x -intercepts occur where $y = 0$.

$$\text{So } 0 = (x + 1)(x - 2)(x - 3)$$

$$\text{or } (x + 1)(x - 2)(x - 3) = 0$$

This can be solved as detailed in the last section on p221.

The solutions are:

$$x = -1, 2 \text{ and } 3$$

Step 2 Find the y -intercept.

The y -intercept occurs where $x = 0$.

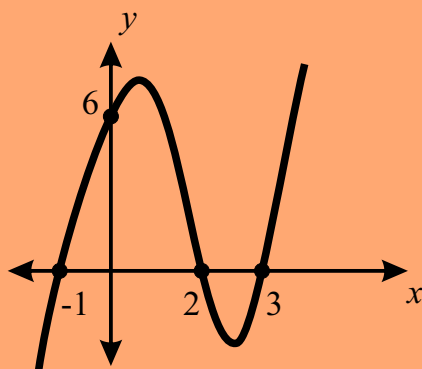
$$y = x^3 - 4x^2 + x + 6$$

$$x = 0 \quad y = 0^3 - 4 \times 0^2 + 0 + 6$$

$$y = 6$$

Step 3 Plot the x - and y - intercepts on x - and y - axes.

Step 4 Sketch a smooth curve connecting all the intercept points. Note the coordinates of the turning points are not known so just sketch a neat and smooth curve.



Example 2

Sketch the following curve:

$$y = x^3 - 7x^2 + 15x - 9$$

Step 1 Find the x -intercepts by factorising the cubic polynomial.The factors of $x^3 - 7x^2 + 15x - 9$ are $(x - 3)(x - 3)(x - 1)$

$$y = (x - 3)(x - 3)(x - 1)$$

$$y = (x - 3)^2(x - 1)$$

 x -intercepts occur where $y = 0$

$$0 = (x - 3)^2(x - 1)$$

$$(x - 3)^2(x - 1) = 0$$

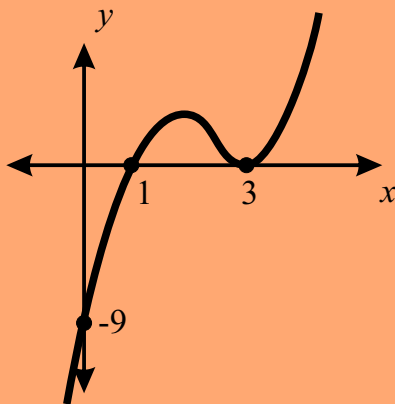
$$x = 3, 1$$

Note there are only two x -intercepts.When there are only two x -intercepts **a turning point occurs at the factor that was repeated**. In this example $x = 3$ **Step 2** Find the y -intercept.The y -intercept occurs where $x = 0$.

$$y = x^3 - 7x^2 + 15x - 9$$

$$x = 0 \quad y = 0^3 - 7 \times 0^2 + 15 \times 0 - 9$$

$$y = -9$$

Step 3 & 4 Sketch the curve.

Example 3

Sketch the following curve: $y = -9x^3 + 24x^2 + 44x + 16$

Step 1 Find the x -intercepts by factorising the cubic polynomial.

The factors of $-9x^3 + 24x^2 + 44x + 16$ are $(3x + 2)^2(-x + 4)$

$$y = (3x + 2)^2(-x + 4)$$

x -intercepts occur where $y = 0$

$$0 = (3x + 2)^2(-x + 4)$$

$$(3x + 2)^2(-x + 4) = 0$$

$$x = -\frac{2}{3}, 4$$

There are only two x -intercepts so one must be a turning point and this is the repeated factor: $x = -\frac{2}{3}$

Step 2 Find the y -intercept.

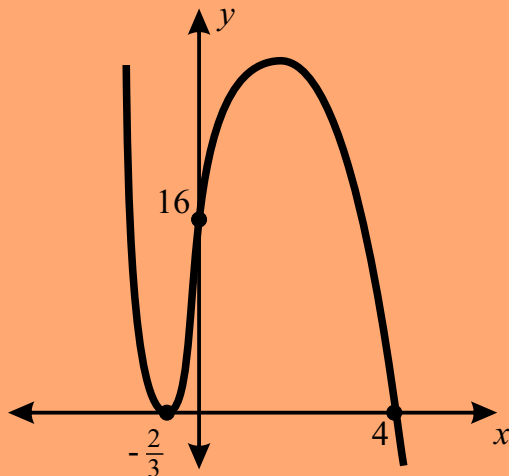
The y -intercept occurs where $x = 0$

$$y = -9x^3 + 24x^2 + 44x + 16$$

$$x = 0 \quad y = -9 \times 0^3 + 24 \times 0^2 + 44 \times 0 + 16$$

$$y = 16$$

Step 3 & 4 Sketch the curve.



Example 4

Sketch the following curve: $y = x^3 + 2x^2 - 3x$

Step 1 Find the x -intercepts by factorising the cubic polynomial.

The factors of $x^3 + 2x^2 - 3x$ are $x(x + 3)(x - 1)$

$$y = x(x + 3)(x - 1)$$

x -intercepts occur when $y = 0$

$$0 = x(x + 3)(x - 1)$$

$$x(x + 3)(x - 1) = 0$$

$$x = 0, -3 \text{ and } 1$$

Step 2 Find the y -intercept.

The y -intercept occurs when $x = 0$

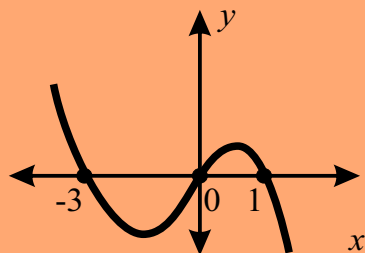
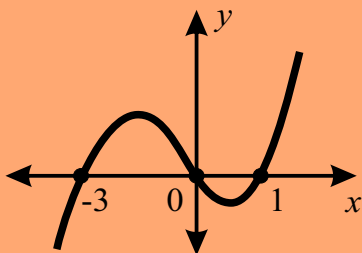
$$y = x^3 + 2x^2 - 3x$$

$$x = 0 \quad y = 0^3 + 2 \times 0^2 - 3 \times 0$$

$$y = 0$$

Step 3 & 4 Sketch the curve.

Note with these intercept points there are two possible curves that could be drawn.



Step 5 In this situation another point on the curve needs to be found.

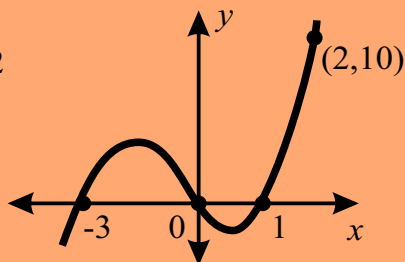
Any point can be found. Use $x = 2$ and find y when $x = 2$.

$$y = x^3 + 2x^2 - 3x$$

$$x = 2 \quad y = 2^3 + 2 \times 2^2 - 3 \times 2$$

$$y = 10$$

The point $(2, 10)$ can only be possible on the first curve.



Example 5

Sketch the following curve: $y = x^3 - 6x^2 + 12x - 8$

Step 1 Find the x -intercepts by factorising the cubic polynomial.

The factors of $x^3 - 6x^2 + 12x - 8$ are $(x - 2)(x - 2)(x - 2)$

$$y = (x - 2)^3$$

x -intercepts occur when $y = 0$

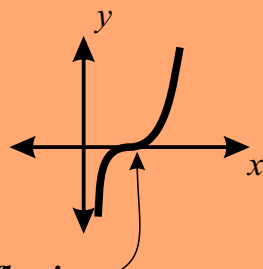
$$0 = (x - 2)^3$$

$$(x - 2)^3 = 0$$

$$x = 2$$

There is only one x -intercept.

This means the curve has a shape as shown here. The point of the curve at the x -intercept is called the **point of inflection**.



Step 2 Find the y -intercept.

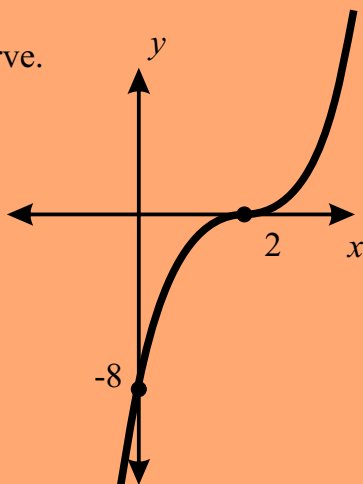
The y -intercept occurs when $x = 0$

$$y = x^3 - 6x^2 + 12x - 8$$

$$x = 0 \quad y = 0^3 - 6 \times 0^2 + 12 \times 0 - 8$$

$$y = -8$$

Step 3 & 4 Sketch the curve.



EXERCISE 8E

1. Sketch the curves of the following equations showing all intercept points.
The factors for each cubic polynomial are given.

(a) $y = x^3 - 2x^2 - 5x + 6$

Factors: $x^3 - 2x^2 - 5x + 6 = (x + 2)(x - 1)(x - 3)$

(b) $y = x^3 - 6x^2 + 11x - 6$

Factors: $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$

(c) $y = x^3 - 9x^2 + 26x - 24$

Factors: $x^3 - 9x^2 + 26x - 24 = (x - 2)(x - 3)(x - 4)$

(d) $y = x^3 - 3x^2 + 2x$

Factors: $x^3 - 3x^2 + 2x = x(x - 1)(x - 2)$

(e) $y = x^3 - x^2 - 6x$

Factors: $x^3 - x^2 - 6x = x(x + 2)(x - 3)$

(f) $y = -x^3 - 2x^2 + 5x + 6$

Factors: $-x^3 - 2x^2 + 5x + 6 = (x + 2)(x - 3)(-x - 1)$

(g) $y = -x^3 + 4x^2 - 3x$

Factors: $-x^3 + 4x^2 - 3x = x(-x + 1)(x - 3)$

(h) $y = x^3 + 6x^2 + 12x + 8$

Factors: $x^3 + 6x^2 + 12x + 8 = (x + 2)^3$

(i) $y = -x^3 + 5x^2 - 3x - 9$

Factors: $-x^3 + 5x^2 - 3x - 9 = (x - 3)^2(-x - 1)$

(j) $y = 6x^3 + 19x^2 + x - 6$

Factors: $6x^3 + 19x^2 + x - 6 = (3x + 2)(2x - 1)(x + 3)$

2. Sketch the curves of the following equations showing all intercept points.
The cubic polynomials will need to be factorised first.

(a) $y = x^3 - 4x^2 - x + 4$

(b) $y = x^3 + 6x^2 + 11x + 6$

(c) $y = x^3 + x^2 - 5x + 3$

(d) $y = x^3 + 2x^2 - 4x - 8$

The Remainder Theorem

If a polynomial is divided by a linear polynomial $(x - a)$ the result will be a **quotient** and a **remainder**.

Example

$2x^3 + 5x^2 - 6x + 7$ is divided by $(x + 1)$

$$\begin{array}{r}
 \text{quotient} \swarrow \\
 \begin{array}{r}
 2x^2 + 3x - 9 \\
 x + 1 \overline{) 2x^3 + 5x^2 - 6x + 7} \\
 \underline{- 2x^3 + 2x^2} \\
 3x^2 - 6x + 7 \\
 \underline{- 3x^2 + 3x} \\
 -9x + 7 \\
 \underline{- -9x - 9} \\
 16
 \end{array} \\
 \searrow \text{remainder}
 \end{array}$$

As we have seen earlier in this chapter if the linear polynomial $(x - a)$ is a factor of the polynomial then the remainder is 0.
(**Factor Theorem**)

The **Remainder Theorem** states that if a is substituted into the polynomial then the result is equal to the remainder.

In this example the linear polynomial is $(x + 1) = (x - (-1))$ so $a = -1$.
Substituting -1 into the polynomial:

$$\begin{aligned}
 \text{Polynomial} &= 2x^3 + 5x^2 - 6x + 7 \\
 x = -1: \quad &= 2 \times (-1)^3 + 5 \times (-1)^2 - 6 \times -1 + 7 \\
 &= 2 \times -1 + 5 \times 1 - 6 \times -1 + 7 \\
 &= -2 + 5 + 6 + 7 \\
 &= 16 \text{ (the remainder)}
 \end{aligned}$$

EXERCISE 8F

1. Find the quotient and remainder when each of the following polynomials is divided by the linear polynomials shown.
 - (a) $x^3 - 3x^2 - 9x + 25$ is divided by $(x - 4)$
 - (b) $3x^3 + 4x^2 - x - 7$ is divided by $(x + 2)$
 - (c) $2x^3 - 5x^2 + 6$ is divided by $(x - 3)$
 - (d) $2x^3 + 3x^2 - 7x + 19$ is divided by $(x + 5)$
2. Use the remainder theorem to confirm the answers in question 1.
3. Use the remainder theorem to find the remainder when each of the following polynomials is divided by the linear polynomials shown.
 - (a) $4x^3 - 2x^2 - 3x + 17$ is divided by $(x - 2)$
 - (b) $3x^3 + 5x^2 - 8x - 12$ is divided by $(x + 6)$
 - (c) $2x^3 - 9x^2 - x + 19$ is divided by $(x - 7)$
 - (d) $5x^3 - 2x^2 - 8x - 14$ is divided by $(x - 3)$
 - (e) $x^3 + 10x^2 - 13x - 25$ is divided by $(x + 4)$
 - (f) $9x^3 - 17x^2 - 3x + 15$ is divided by $(x - 8)$
 - (g) $11x^3 + 7x^2 - 17x - 21$ is divided by $(x + 12)$

PROBLEM SOLVING

Find a , b and c in the following equation.

$$(x - a)(x - b)(x - c) = ax^3 - cx^2 + bx + 2$$

PUZZLE

Expand the following.

Hint: look at the other terms.

$$(x - a)(x - b)(x - c) \dots\dots\dots (x - y)(x - z)$$

CHAPTER REVIEW

1. Which of the following expressions are polynomials?

- A** $7a - 6b$ **B** 11 **C** $\sqrt{n}$ **D** $\frac{8m}{9}$ **E** $3x^5 + 9y^6$
F $5x^3 - 2x^7 + 6x - 2$ **G** $m^3 + 9m^{-4} + 6m$ **H** $\frac{2x}{7y}$

2. State the degree of the following polynomials.

- (a)** $3x^4 + 6x^2 + 5x - 1$ **(b)** $n^2 + 9n^5 + 4n - 7n^3$

3. Expand the following expressions and state the degree of the resultant polynomial.

- (a)** $(3x + 4)(2x - 7)$ **(b)** $(2a - 5)(3 - 8a)$
(c) $x(x + 5)(3x - 2)$ **(d)** $6m(3m - 1)(3m + 1)$
(e) $(x + 3)(x - 4)(2x - 5)$ **(f)** $x(2x + 1)(x + 3)(3x - 5)$

4. Sketch the curves of the following polynomials clearly showing the intercept points.

The polynomials will need to be factorised first to find the x -intercepts.

- (a)** $x^3 - x^2 - 17x - 15$ **(b)** $6x^3 - 25x^2 + 21x + 10$
(c) $x^3 - 4x^2 - 11x - 6$ **(d)** $x^3 + 2x^2 - 8x$
(e) $x^3 + 3x^2 + 3x + 1$ **(f)** $6x^3 - 23x^2 + 11x + 12$

5. Find the quotient and remainder when each of the following polynomials is divided by the linear polynomials shown.

- (a)** $x^3 - 7x^2 - 11x - 31$ is divided by $(x - 3)$
(b) $4x^3 - x^2 + 9$ is divided by $(x + 2)$

6. Use the remainder theorem to find the remainder when each of the following polynomials is divided by the linear polynomials shown.

- (a)** $2x^3 - 5x^2 + 10x - 13$ is divided by $(x - 4)$
(b) $5x^3 - 8x^2 - 14x + 23$ is divided by $(x + 1)$
(c) $3x^3 - 2x^2 - 8x - 7$ is divided by $(x - 3)$

Linear Relationships

Linear Relationships

EXERCISE 9A

Linear relationships are relationships that when graphed produce **straight lines**.

1. (a) If $y = 2x + 5$, find y if $x = 3$
 (b) If $a = 4b - 17$, find a if $b = 4$
 (c) If $M = 3N + 4$, find M if $N = -6$
 (d) If $C = 8 - 2D$, find C if $D = 7$

2. Copy and complete the following tables for the given rules.

(a) $n = m + 2$

m	-3	-2	-1	0	1	2	3
n							

(b) $y = 2x - 5$

x	-3	-2	-1	0	1	2	3
y							

(c) $M = 3 - 4N$

N	-3	-2	-1	0	1	2	3
M							

Example

$$y = 3x + 5$$

x	-3	-2	-1	0	1	2	3
y							

Answer

x	-3	-2	-1	0	1	2	3
y	-4	-1	2	5	8	11	14

3. Find the rule connecting the variables in each of the tables below.

Example

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

Answer: $y = 2x + 1$

Hint: The y values increase by the coefficient of x when the x values increase by 1.

(a)

x	-3	-2	-1	0	1	2	3
y	-12	-8	-4	0	4	8	12

(b)

B	-3	-2	-1	0	1	2	3
A	0	1	2	3	4	5	6

3. (c)

c	-3	-2	-1	0	1	2	3
d	-5	-4	-3	-2	-1	0	1

(d)

M	-3	-2	-1	0	1	2	3
N	-7	-5	-3	-1	1	3	5

(e)

a	-3	-2	-1	0	1	2	3
b	-8	-5	-2	1	4	7	10

(f)

P	-3	-2	-1	0	1	2	3
Q	3	2	1	0	-1	-2	-3

(g)

x	-3	-2	-1	0	1	2	3
y	-13	-9	-5	-1	3	7	11

(h)

m	-3	-2	-1	0	1	2	3
n	6	5	4	3	2	1	0

(i)

C	-3	-2	-1	0	1	2	3
D	8	6	4	2	0	-2	-4

(j)

x	-3	-2	-1	0	1	2	3
y	10	7	4	1	-2	-5	-8

4. Find the equation connecting the variables in each of the tables below.

Example

x	0	3	8	15	25
y	3	9	19	33	53

x	0	3	8	15	25
y	3	9	19	33	53

$\xrightarrow{+3}$ $\xrightarrow{+5}$ $\xrightarrow{+7}$ $\xrightarrow{+10}$
 $\xrightarrow{+6}$ $\xrightarrow{+10}$ $\xrightarrow{+14}$ $\xrightarrow{+20}$

Note: the increase in y values is twice the increase in the x values. The coefficient of x in the equation will therefore be 2.

Answer: $y = 2x + 3$

(a)

x	0	2	7	14	19
y	-1	5	20	41	56

(b)

x	-5	-1	4	12	21
y	3	7	12	20	29

(c)

x	0	3	8	16	22
y	-6	0	10	26	38

(d)

x	-2	3	9	13	24
y	14	9	3	-1	-12

(e)

x	-5	0	3	8	15
y	26	11	2	-13	-34

(f)

x	0	4	11	19	21
y	-8	12	47	87	97

(g)

x	0	4	10	16	28
y	3	13	28	43	73

(h)

x	-3	6	12	21	30
y	-3	0	2	5	8

5. (a) Copy and complete this table

for the relationship:

B is equal to 11 more than
five times A

A	0	5	12	22	38
B					

(b) Complete the equation: $B = \underline{\hspace{2cm}}$.

(c) Find B when $A = 65$.

(d) Find A when $B = 251$.

6. A farm's water tank holds 30 000 litres and is full. The farm uses 600 litres each day.

(a) Copy and complete this table

showing the volume of water (V),
in litres, N days after it was full.

N	0	2	6	12	24
V					

Assume no water is added to the tank.

(b) Complete the equation: $V = \underline{\hspace{2cm}}$.

(c) Find the volume of water in the tank after 35 days.

(d) After how many days will the tank be empty?

7. A meteorologist released a weather balloon that recorded the air temperature every 100 m above sea level. The results for the first 500 metres are shown in this table. H represents the height, in metres, above sea level and T is the temperature in $^{\circ}\text{C}$.

H	0	100	200	300	400	500
T	24.0	23.5	23.0	22.5	22.0	21.5

(a) Write a relationship between T and H : $T = \underline{\hspace{2cm}}$.

(b) Find the temperature at a height of 1200 metres.

(c) At what height would the temperature be 0°C ?

8. The fuel consumption of a pump was dependant on the volume of water being pumped. The table below shows the fuel consumption, F (in mL/min), for different volumes of water being pumped, V (in L/sec).

V	0	1	2	3	4	5
F	10.0	12.5	15.0	17.5	20.0	22.5

(a) Write a relationship between F and V : $F = \underline{\hspace{2cm}}$.

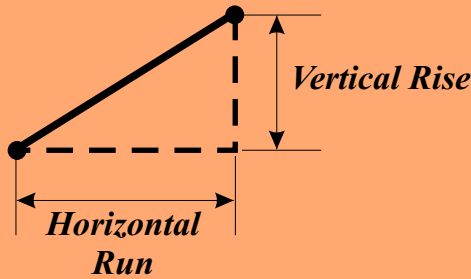
(b) Find the fuel consumption when 12 L/sec are being pumped.

Gradient

Definition

The **gradient** of a line between two points is defined as:

$\frac{\text{the vertical **rise** between the two points}}{\text{the horizontal **run** between the two points}}$

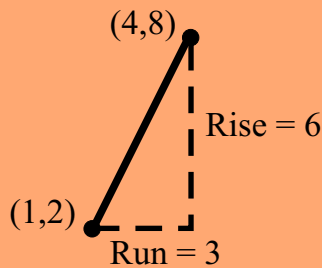
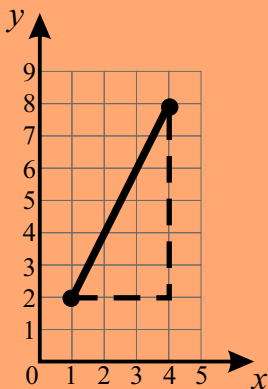


The symbol for gradient is m

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

Example

Find the gradient of the line joining the points (1,2) and (4,8)



$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

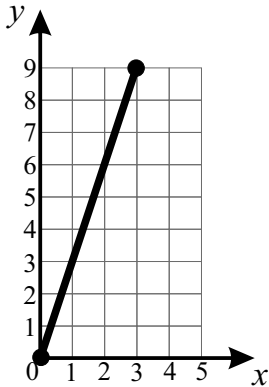
$$m = \frac{6}{3}$$

$$m = 2$$

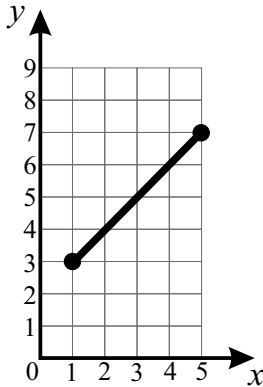
EXERCISE 9B

1. Find the gradient of the lines shown on the graphs below.

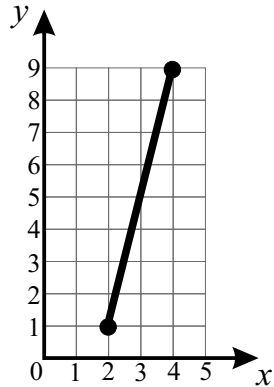
(a)



(b)



(c)



2. Find the gradient of the lines joining the following pairs of points.

(a) (0,0) and (3,6)

(b) (1,0) and (6,5)

(c) (2,3) and (5,6)

(d) (3,2) and (6,8)

(e) (0,4) and (2,12)

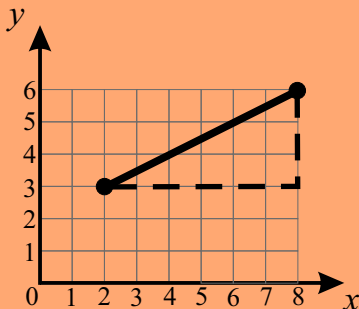
(f) (3,1) and (9,7)

(g) (4,2) and (6,8)

(h) (3,0) and (11,16)

3. Find the gradient of the lines joining the following pairs of points.

Example (2,3) and (8,6)



Rise = 3

Run = 6

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{3}{6}$$

$$m = \frac{1}{2}$$

(a) (0,0) and (4,2)

(b) (1,2) and (7,5)

(c) (2,3) and (8,6)

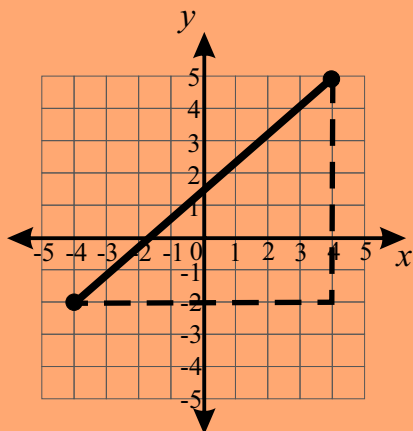
(d) (1,4) and (13,8)

(e) (0,4) and (10,7)

(f) (2,5) and (11,11)

4. Find the gradient of the lines joining the following pairs of points.

Example $(-4, -2)$ and $(4, 5)$



$$\text{Rise} = 7$$

$$\text{Run} = 8$$

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{7}{8}$$

(a) $(-1, 2)$ and $(4, 7)$

(c) $(3, -2)$ and $(10, 5)$

(e) $(-2, -3)$ and $(5, 1)$

(g) $(-4, -4)$ and $(6, 8)$

(i) $(-3, 0)$ and $(0, 5)$

(b) $(-4, -2)$ and $(1, 8)$

(d) $(-1, 2)$ and $(7, 6)$

(f) $(-5, 3)$ and $(7, 7)$

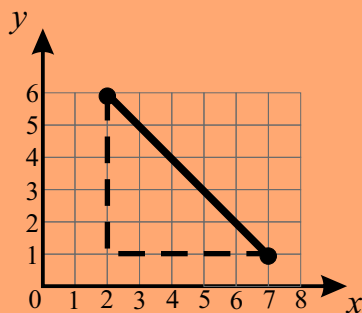
(h) $(-1, -6)$ and $(4, -1)$

(j) $(-8, 5)$ and $(-1, 8)$

Negative Gradient

If a line slopes down when going from left to right it has a **negative** gradient.

Example Find the gradient of the line joining $(2, 6)$ and $(7, 1)$



$$\text{Rise} = -5$$

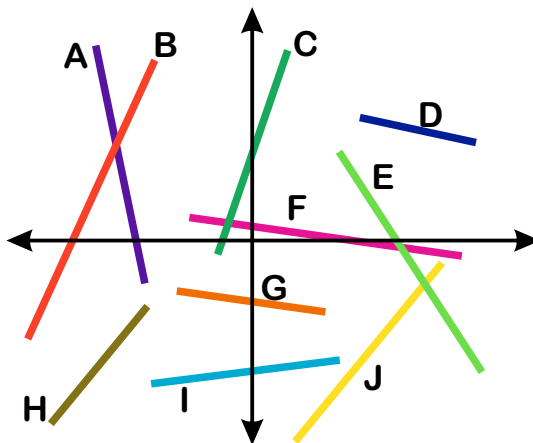
$$\text{Run} = 5$$

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{-5}{5}$$

$$m = -1$$

5. Which of the lines on this graph have a **negative** gradient.



6. Plot the following pairs of points on a graph and connect each pair with a straight line. State which of these lines will have a negative gradient and calculate these negative gradients.
- | | |
|------------------------|-----------------------|
| (a) (1,4) and (5,0) | (b) (2,8) and (4,4) |
| (c) (-2,1) and (3,-4) | (d) (-1,-1) and (3,1) |
| (e) (-2,-3) and (2,-5) | (f) (0,3) and (5,0) |
| (g) (2,0) and (6,-2) | (h) (-3,-4) and (4,0) |

Calculating Gradient

The gradient of the line connecting points (x_1, y_1) and (x_2, y_2) can be found by using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples

Find the gradient of the line joining the following pairs of points.

1. (2,-3) and (5,7)

$$(x_1, y_1) = (2, -3)$$

$$(x_2, y_2) = (5, 7)$$

$$\begin{aligned} m &= \frac{7 - (-3)}{5 - 2} \\ &= \frac{7 + 3}{3} \end{aligned}$$

$$m = \frac{10}{3}$$

2. (-5,4) and (-2,-3)

$$(x_1, y_1) = (-5, 4)$$

$$(x_2, y_2) = (-2, -3)$$

$$\begin{aligned} m &= \frac{(-3) - 4}{(-2) - (-5)} \\ &= \frac{-3 - 4}{-2 + 5} \end{aligned}$$

$$m = -\frac{7}{3}$$

EXERCISE 9C

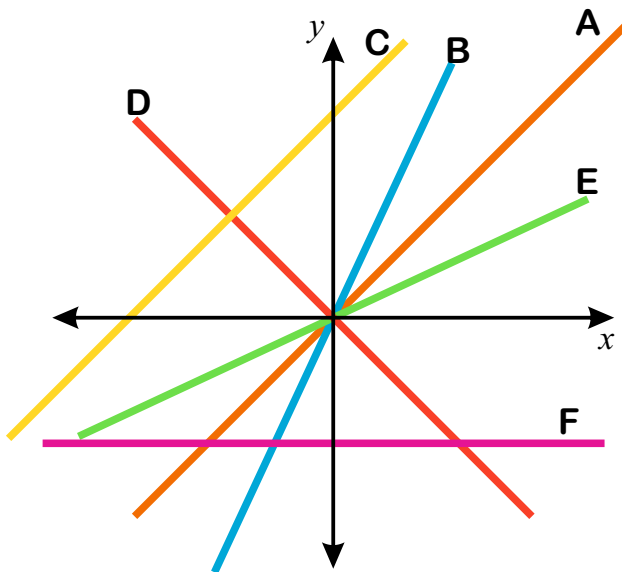
1. Calculate the gradient of the lines joining the following pairs of points.

- | | |
|-----------------------|------------------------|
| (a) (2,4) and (8,10) | (b) (0,3) and (4,11) |
| (c) (1,3) and (8,17) | (d) (0,0) and (5,8) |
| (e) (2,3) and (7,9) | (f) (3,4) and (10,8) |
| (g) (-1,2) and (4,5) | (h) (-2,-3) and (5,0) |
| (i) (0,4) and (6,0) | (j) (1,8) and (7,-4) |
| (k) (-4,5) and (3,6) | (l) (-2,-5) and (6,4) |
| (m) (0,8) and (7,-4) | (n) (2,2) and (12,-7) |
| (o) (-2,8) and (5,3) | (p) (-3,-1) and (7,-5) |
| (q) (-4,0) and (0,-8) | (r) (-1,6) and (0,0) |
| (s) (2,5) and (7,5) | (t) (-3,-4) and (5,-4) |

2. (a) Which axis (x or y) is 1(s) and 1(t) parallel to?
 (b) What is the gradient of a line that is parallel to the x -axis?

3. On the graph shown below, line **A** has a gradient of 1.

- (a) Which of the other lines (**B-F**) has a gradient of $\frac{1}{2}$?
 (b) Which of the other lines has a gradient of -1?
 (c) Which of the other lines has a gradient of 2?
 (d) Which of the other lines is parallel to line **A**?
 (e) Which of the other lines has a gradient of 0?



***x*- and *y*-Intercepts**

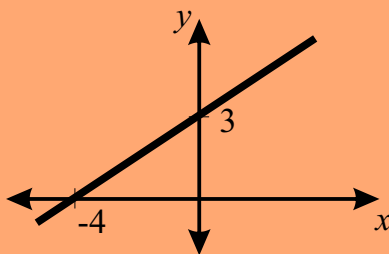
Definitions

The *x*-intercept is the point where a line crosses the *x*-axis.

The *y*-intercept is the point where a line crosses the *y*-axis.

Examples

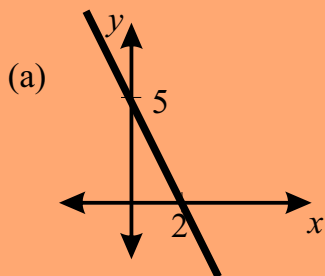
1. For the straight line shown on this graph:
the *x*-intercept = -4
the *y*-intercept = 3



The coordinates of the *x*-intercept are (-4,0)

The coordinates of the *y*-intercept are (0,3)

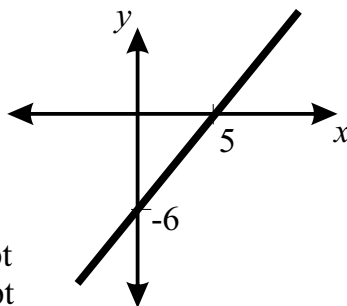
2. (a) Sketch the straight line that has the following intercepts:
x-intercept = 2
y-intercept = 5
(b) State the coordinates of the intercept points.



- (b) *x*-intercept = (2,0)
y-intercept = (0,5)

EXERCISE 9D

1. For the straight line shown on this graph:
(a) state the *x*-intercept
(b) state the *y*-intercept
(c) state the coordinates of the *x*-intercept
(d) state the coordinates of the *y*-intercept



2. Sketch the straight lines that have the following x - and y -intercepts.

- (a) x -intercept = 3, y -intercept = 2
- (b) x -intercept = -4, y -intercept = 6
- (c) x -intercept = 5, y -intercept = -1
- (d) x -intercept = -3, y -intercept = -5
- (e) x -intercept = 1, y -intercept = -6
- (f) x -intercept = -7, y -intercept = -2

3. State the coordinates of the intercept points in question 2.

4. Sketch the straight lines that have the following coordinates for their intercept points.

- (a) (4,0) and (0,-3)
- (b) (-5,0) and (0,5)
- (c) (-1,0) and (0,7)
- (d) (6,0) and (0,2)

When the equation for a straight line is known, the intercept points can be found by using the following technique.
The straight line can then be sketched.

Note - At the x -intercept $y = 0$
At the y -intercept $x = 0$

Step 1 Find the x -intercept by substituting $y = 0$ into the equation

Step 2 Find the y -intercept by substituting $x = 0$ into the equation

Example 1

Find the x -intercept and y -intercept of the straight line with the following equation and sketch the straight line.

$$3x + 2y = 6$$

Step 1 Find x -intercept by substituting $y = 0$ into the equation

$$3x + 2 \times 0 = 6$$

$$3x = 6$$

$$x = 2$$

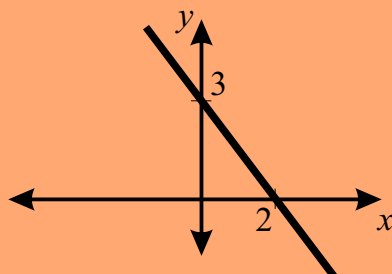
Step 2 Find y -intercept by substituting $x = 0$ into the equation

$$3 \times 0 + 2y = 6$$

$$2y = 6$$

$$y = 3$$

The straight line can now be sketched.



Example 2

Find the x -intercept and y -intercept of the straight line with the following equation and sketch the straight line.

$$y = 2x + 8$$

Step 1 Find x -intercept by substituting $y = 0$ into the equation

$$0 = 2x + 8$$

$$2x + 8 = 0$$

$$2x = -8$$

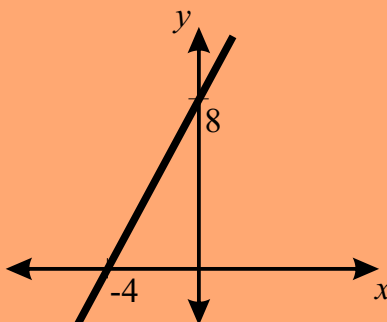
$$x = -4$$

Step 2 Find y -intercept by substituting $x = 0$ into the equation

$$y = 2 \times 0 + 8$$

$$y = 8$$

The straight line can now be sketched.



5. Find the x -intercept and y -intercept of the straight lines with the following equations and sketch the lines.

- | | |
|--------------------|--------------------|
| (a) $x + y = 3$ | (b) $2x + y = 4$ |
| (c) $3x + 3y = 6$ | (d) $2x + 5y = 10$ |
| (e) $3x + 8y = 24$ | (f) $x + 5y = 10$ |
| (g) $4x + 2y = 12$ | (h) $3x + y = 9$ |
| (i) $x - y = 2$ | (j) $x - 2y = 4$ |
| (k) $3x - 4y = 12$ | (l) $4x - 3y = 24$ |
| (m) $x + 2y = 1$ | (n) $2x - y = 1$ |

6. Find the x -intercept and y -intercept of the straight lines with the following equations and sketch the lines.

- | | |
|-------------------|-------------------|
| (a) $y = 3x - 6$ | (b) $y = 2x - 4$ |
| (c) $y = x - 3$ | (d) $y = x + 4$ |
| (e) $y = x - 1$ | (f) $y = 2x + 6$ |
| (g) $y = 3x - 12$ | (h) $y = 4x + 8$ |
| (i) $y = 2x - 10$ | (j) $y = 10 - 5x$ |
| (k) $y = 12 - 6x$ | (l) $y = 8 - 8x$ |
| (m) $y = 4x + 2$ | (n) $y = 6x - 3$ |

$$y = mx + c$$

Equations of straight lines are often written in the form:

$$y = mx + c$$

where $m = \text{the gradient}$ and $c = \text{the } y\text{-intercept}$

Examples

Equation	Gradient	y -intercept
$y = 2x + 5$	2	5
$y = -3x + 1$	-3	1
$y = \frac{2}{3}x - 4$	$\frac{2}{3}$	-4
$y = x$	1	0

EXERCISE 9E

1. State the gradient and y -intercept of the straight lines with the following equations.

(a) $y = 3x + 4$

(c) $y = x + 5$

(e) $y = 2x - 5$

(g) $y = -3x + 2$

(i) $y = -x - 2$

(k) $y = 2x + \frac{1}{2}$

(m) $y = \frac{1}{3}x + 2$

(o) $y = 5 + 2x$

(q) $y = 6 + 3x$

(s) $y = \frac{x}{3} + 1$

(b) $y = 2x + 6$

(d) $y = x - 1$

(f) $y = 6x - 7$

(h) $y = -4x - 1$

(j) $y = -5x$

(l) $y = -7x - \frac{3}{5}$

(n) $y = -\frac{3}{4}x - \frac{1}{4}$

(p) $y = 3 - x$

(r) $y = 2 + 8x$

(t) $y = \frac{3x}{5} - 4$

2. Write the equations of the straight lines with the gradients and y -intercepts shown in this table.

	Gradient	y-intercept
(a)	5	3
(b)	2	-1
(c)	-4	5
(d)	-2	-7
(e)	1	-2
(f)	3	0
(g)	$\frac{1}{3}$	-2
(h)	2	$-\frac{2}{9}$

3. Rearrange the letters of the phrases in capital letters to make this sentence make sense!

Straight lines that are ALL PEARL have the same RED GIANT.

4. State which of the following straight lines are parallel to the line with equation: $y = 3x - 4$.

A $y = 4x - 3$

C $y = x - 4$

E $y = 3x - 1$

G $y = 4x$

I $y = 2 + 3x$

B $y = 3x + 4$

D $y = x + 3$

F $y = 3x$

H $y = 2 - 3x$

J $y = 3x + 7$

5. Transpose the following equations to make y the subject and hence find the gradient and y -intercept.

- (a) $y - 3x - 5 = 0$
- (b) $y + 2x + 6 = 0$
- (c) $y - x + 3 = 0$
- (d) $y + x - 2 = 0$
- (e) $2y - 6x - 8 = 0$
- (f) $2y - 2x + 4 = 0$
- (g) $3y + 3x - 9 = 0$
- (h) $4y - 8x - 12 = 0$
- (i) $2y + 8x - 5 = 0$
- (j) $3y - 5x + 12 = 0$
- (k) $5y - x - 1 = 0$
- (l) $3y + 4x = 0$
- (m) $6y - 12x + 15 = 0$
- (n) $9y + 12 - 6x = 0$

Example

$$3y - 2x + 6 = 0$$

Subtract 6 from both sides

$$3y - 2x + 6 = 0$$

$$\boxed{-6} \quad \boxed{-6}$$

$$3y - 2x = -6$$

Add $2x$ to both sides

$$3y - 2x = -6$$

$$\boxed{+2x} \quad \boxed{+2x}$$

$$3y = 2x - 6$$

Divide all terms by 3

$$3y = 2x - 6$$

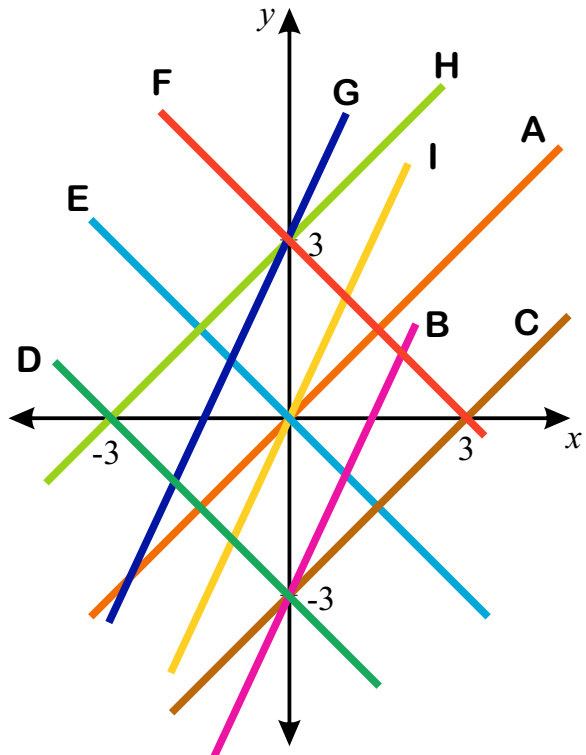
$$\boxed{\div 3} \quad \boxed{\div 3} \quad \boxed{\div 3}$$

$$y = \frac{2}{3}x - 2$$

Gradient = $\frac{2}{3}$, **y -intercept** = -2

6. On the graph shown here, the equation of line **A** is $y = x$. Match the following equations with the lines on the graph.

- (a) $y = 2x$
- (b) $y = 2x + 3$
- (c) $y = x + 3$
- (d) $y = -x$
- (e) $y = 2x - 3$
- (f) $y = -x - 3$
- (g) $y = x - 3$
- (h) $y = -x + 3$



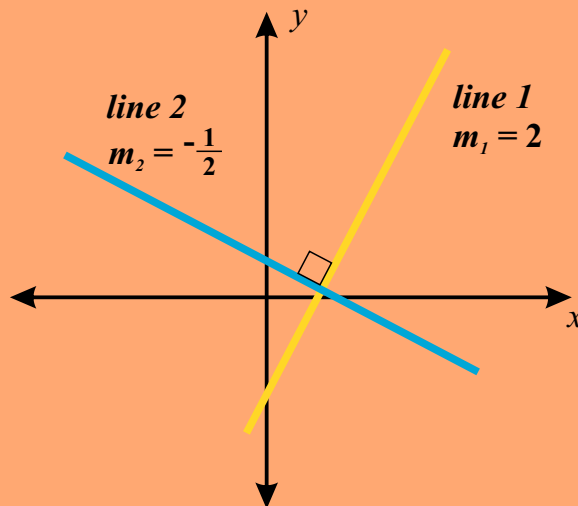
Perpendicular Lines

If two straight lines are perpendicular (are at right angles to each other) the product of their gradients is equal to **-1**.

If two lines, **line 1** (gradient = m_1) and **line 2** (gradient = m_2), are perpendicular then:

$$m_1 \times m_2 = -1$$

Example 1



These two lines are perpendicular because the product of their gradients is equal to -1.

$$m_1 = 2$$

$$m_2 = -\frac{1}{2}$$

$$m_1 \times m_2 = 2 \times -\frac{1}{2}$$

$$m_1 \times m_2 = -1$$

Example 2

What would be the gradient of a line that is perpendicular to a line with gradient -4?

Answer Let the gradient of line 1 = $m_1 = -4$
and gradient of line 2 = m_2

$$m_1 \times m_2 = -1$$

$$-4 \times m_2 = -1$$

Divide both sides by -4

$$m_2 = \frac{-1}{-4}$$

$$m_2 = \frac{1}{4}$$

An easy way to find the gradient of a line that is perpendicular to a line with a known gradient is to follow these steps:

Step 1 Write the known gradient as a fraction.

Step 2 Invert this fraction and change the sign.

Example 3

Find the gradients of lines that are perpendicular to the lines with the following gradients.

1. $m_1 = 5$

2. $m_1 = -\frac{7}{3}$

Step 1 As a fraction $5 = \frac{5}{1}$

Step 1 Already a fraction

Step 2 Invert $\frac{5}{1} \rightarrow \frac{1}{5}$

Step 2 Invert $-\frac{7}{3} \rightarrow -\frac{3}{7}$

Change the sign $\rightarrow -\frac{1}{5}$

Change the sign $\rightarrow \frac{3}{7}$

$$m_2 = -\frac{1}{5}$$

$$m_2 = \frac{3}{7}$$

Example 4 Find the gradient of a line that is perpendicular to the line with the following equation:

$$y = 6x - 7$$

Step 1 Identify the gradient of this line and label it m_1 .

$$m_1 = 6$$

Step 2 Use the method in example 3 to find m_2 .

$$m_2 = -\frac{1}{6}$$

Example 5 Find the gradient of a line that is perpendicular to the line with the following equation:

$$3y + 2x - 8 = 0$$

Step 1 Transpose the equation into the form $y = mx + c$

$$3y + 2x - 8 = 0$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

Step 2 Identify the gradient of this line and label it m_1 .

$$m_1 = -\frac{2}{3}$$

Step 3 Use the method in example 3 to find m_2 .

$$m_2 = \frac{3}{2}$$

EXERCISE 9F

1. Find the gradient of a line that is perpendicular to lines with the following gradients.

- (a) 5 (b) -3 (c) $\frac{1}{4}$ (d) $-\frac{1}{7}$ (e) $-\frac{5}{2}$ (f) $\frac{3}{8}$

2. Find the gradient of a line that is perpendicular to the straight lines with each of the following equations.

- | | |
|----------------------------|-----------------------------|
| (a) $y = 4x - 5$ | (b) $y = -x + 1$ |
| (c) $y = x$ | (d) $y = -7x - 3$ |
| (e) $y = \frac{3}{4}x - 5$ | (f) $y = -\frac{1}{7}x + 8$ |
| (g) $y = 5 - 2x$ | (h) $y = 3 + 5x$ |
| (i) $y = 6 + \frac{7}{5}x$ | (j) $y = 4 - \frac{2}{9}x$ |

3. Find the gradient of a line that is perpendicular to the straight lines with each of the following equations.

- | | |
|--------------------------|-----------------------|
| (a) $y - 3x + 1 = 0$ | (b) $y + 4x - 7 = 0$ |
| (c) $y - x = 0$ | (d) $y + x + 2 = 0$ |
| (e) $2y + x + 1 = 0$ | (f) $4y - x + 3 = 0$ |
| (g) $6y + 7x - 5 = 0$ | (h) $8y - 3x + 5 = 0$ |
| (i) $12y + 11x + 17 = 0$ | (j) $3y - 8x + 4 = 0$ |

Finding Equations of Straight Lines

Example 1

Find the equation of the straight line joining the points $(-3, -13)$ and $(4, 8)$.

Step 1 Find the gradient

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{8 - (-13)}{4 - (-3)} \\ &= \frac{21}{7} \\ m &= 3 \end{aligned}$$

Step 2 Substitute the gradient and values for x and y from either of the points into $y = mx + c$ to find c .

$$\begin{aligned} -13 &= 3 \times -3 + c \\ -13 &= -9 + c \\ -4 &= c \\ c &= -4 \end{aligned}$$

Step 3 Substitute the values for m and c into $y = mx + c$

$$y = 3x - 4$$

Example 2

Find the equation of the straight line that is **parallel** to the straight line with the equation $y = 2x - 4$ and passes through the point $(-1, 5)$.

Step 1 Recognise that if the straight line is parallel to the straight line with equation $y = 2x - 4$ then it has the same gradient and the gradient of this line is 2.
So the equation of the line will be:

$$y = 2x + c$$

Step 2 Substitute the coordinates of the point this line passes through into this equation to find c . The point is $(-1, 5)$.

$$y = 2x + c$$

$$5 = 2 \times -1 + c$$

$$5 = -2 + c$$

$$7 = c$$

$$c = 7$$

Step 3 Write the equation:

$$y = 2x + 7$$

Example 3

Find the equation of the straight that is **perpendicular** to the straight line with the equation $y = -3x - 5$ and passes through the point (1,7).

Step 1 Recognise that if the straight line is perpendicular to the straight line with equation $y = -3x - 5$ then the product of their gradients is -1. The gradient of this straight line is -3 so the gradient of the perpendicular line is $\frac{1}{3}$.

So the equation of the line will be: $y = \frac{1}{3}x + c$

Step 2 Substitute the coordinates of the point the line passes through into this equation to find c . The point is (1,7).

$$y = \frac{1}{3}x + c$$

$$7 = \frac{1}{3} \times 1 + c$$

$$7 = \frac{1}{3} + c$$

$$7 - \frac{1}{3} = c$$

$$\frac{20}{3} = c$$

$$c = \frac{20}{3}$$

Step 3 Write the equation:

$$y = \frac{1}{3}x + \frac{20}{3}$$

EXERCISE 9G

1. Find the equations of the straight lines joining the following pairs of points.

(a) (1,5) and (4,11)	(b) (-1,-5) and (5, 19)
(c) (-3,-5) and (-1,-3)	(d) (2,-1) and (6,-13)
(e) (-5,3) and (4,-6)	(f) (-2,4) and (6,8)
(g) (4,-4) and (12,-6)	(h) (-6,3) and (3,9)
(i) (2,4) and (6,12)	(j) (-15,3) and (-5,1)

2. Find the equations of the straight lines with the following x - and y -intercepts.

(a) x -intercept = 4, y -intercept = -8
(b) x -intercept = 6, y -intercept = 12
(c) x -intercept = -8, y -intercept = 4
(d) x -intercept = -10, y -intercept = -4

3. Find the x - and y -intercepts of the following straight lines.

(a) gradient of 4 and passes through the point (1,-2)
(b) gradient of -3 and passes through the point (2,-12)
(c) gradient of 6 and passes through the point (-3,-18)
(d) gradient of 4 and passes through the point (-2,5)

4. (a) Find the equation of the straight line that is parallel to the straight line with equation $y = 3x + 5$ and passes through the point (3,7).
(b) Find the equation of the straight line that is parallel to the straight line with equation $y = 4x + 6$ and passes through the point (2, 5).
(c) Find the equation of the straight line that is parallel to the straight line with equation $y = -2x - 1$ and passes through the point (3,-6).
(d) Find the equation of the straight line that is parallel to the straight line with equation $y = -5x + 2$ and passes through the point (-6,-2).
(e) Find the equation of the straight line that is parallel to the straight line with equation $y = 2x + 7$ and passes through the point (4,8).
(f) Find the equation of the straight line that is parallel to the straight line with equation $y = \frac{2}{5}x + 1$ and passes through the point (10,-3).
(g) Find the equation of the straight line that is parallel to the straight line with equation $y = -\frac{7}{3}x$ and passes through the point (12,0).

5. (a) Find the equation of the straight line that is perpendicular to the straight line with equation $y = \frac{1}{3}x - 7$ and passes through the point (2,5).
- (b) Find the equation of the straight line that is perpendicular to the straight line with equation $y = 4x - 2$ and passes through the point (8,3).
- (c) Find the equation of the straight line that is perpendicular to the straight line with equation $y = \frac{5}{2}x - 1$ and passes through the point (10,-2).
- (d) Find the equation of the straight line that is perpendicular to the straight line with equation $y = -\frac{1}{5}x$ and passes through the point (2,-7).
- (e) Find the equation of the straight line that is perpendicular to the straight line with equation $y = -\frac{4}{7}x + 3$ and passes through the point (4,0).
- (f) Find the equation of the straight line that is perpendicular to the straight line with equation $y = \frac{8}{3}x + 1$ and passes through the point (4,-6).
- (g) Find the equation of the straight line that is perpendicular to the straight line with equation $y = \frac{4}{9}x - 8$ and passes through the point (5,-4).

6. A triangle is formed by connecting the points **A**, **B** and **C** with straight lines. The coordinates of **A**, **B** and **C** are shown here.

$$\mathbf{A} = (-2, -1)$$

$$\mathbf{B} = (3, 9)$$

$$\mathbf{C} = (6, -5)$$

Show that this is a right-angled triangle and state at which of the points the right-angle occurs.

7. A rhombus is formed by connecting the following points **D**, **E**, **F** and **G** with straight lines.

- (a) On graph paper plot these points and connect them to show the rhombus.
- (b) Find the gradient of each of the diagonals.
- (c) What can you deduce about the diagonals.

$$\mathbf{D} = (-3, -2)$$

$$\mathbf{E} = (0, 2)$$

$$\mathbf{F} = (5, 2)$$

$$\mathbf{G} = (2, -2)$$

Simultaneous Equations

EXERCISE 9H

1. Accurately draw the following pairs of straight lines and find the point where they intersect.

Example

$$y = x + 2$$

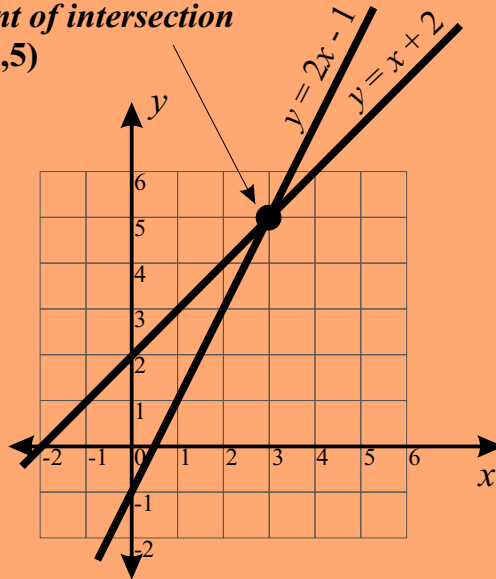
and

$$y = 2x - 1$$

The graphs can be drawn by plotting points or finding the x -intercept and y -intercept.

Remember - it is only necessary to find two points to draw each straight line.

Point of intersection
= (3,5)



- | | |
|------------------------------------|-----------------------------------|
| (a) $y = x$ and $y = 2x - 2$ | (b) $y = 2x$ and $y = x + 3$ |
| (c) $y = -x + 2$ and $y = x - 2$ | (d) $y = 2x - 1$ and $y = x + 1$ |
| (e) $y = -x$ and $y = x + 2$ | (f) $y = 2x - 2$ and $y = -x + 7$ |
| (g) $y = x - 1$ and $y = 2x + 2$ | (h) $y = -2x + 5$ and $y = x - 4$ |
| (i) $y = 2x - 4$ and $y = -3x + 6$ | (j) $y = -x + 1$ and $y = 2x + 7$ |

The point of intersection of two straight lines can be found algebraically. At this point the two equations have the same solution. To find this point algebraically the equations are solved simultaneously.

There are two methods of solving simultaneous equations:

1. **Substitution**
2. **Elimination**

Substitution

The following are examples of solving simultaneous equations using **substitution**.

Follow the steps below.

Step 1 - Label the equations **1** and **2**

Step 2 - Substitute one equation into the other

Step 3 - Solve for x

Step 4 - Substitute x into either equation to solve for y

Example 1 $y = 2x$ and $y = 3x - 4$

Step 1

$y = 2x$		1
$y = 3x - 4$		2

Step 2 and 3 Substitute equation **2** into equation **1**
and solve for x :

$$\begin{array}{r} 3x - 4 = 2x \\ \boxed{- 2x} \quad \boxed{- 2x} \end{array}$$

$$\begin{array}{r} x - 4 = 0 \\ \boxed{+ 4} \quad \boxed{+ 4} \end{array}$$

$$x = 4$$

Step 4 Substitute $x = 4$ into equation **1** (or **2**)
to solve for y .

$$y = 2 \times 4$$

$$y = 8$$

Solution: $x = 4, y = 8$ **(4,8)**

Example 2 $y + 3x = 7$ and $y = 2x - 8$

Step 1

$$\begin{array}{rcl} y + 3x = 7 & \text{---} & \textcircled{1} \\ y = 2x - 8 & \text{---} & \textcircled{2} \end{array}$$

Step 2 and 3 Substitute equation 2 into equation 1
and solve for x :

$$\begin{array}{rcl} 2x - 8 + 3x & = & 7 \\ 5x - 8 & = & 7 \\ \boxed{+ 8} & \boxed{+ 8} & \\ 5x & = & 15 \\ \boxed{\div 5} & \boxed{\div 5} & \\ x & = & 3 \end{array}$$

Step 4 Substitute $x = 3$ into equation 1 (or 2)
to solve for y .

$$\begin{array}{rcl} y + 3 \times 3 & = & 7 \\ y + 9 & = & 7 \\ \boxed{- 9} & \boxed{- 9} & \\ y & = & -2 \end{array}$$

Solution: $x = 3, y = -2$ (3,-2)

2. Solve the following simultaneous equations using substitution.

- | | |
|-----------------------------------|-----------------------------------|
| (a) $y = x$ and $y = 3x - 4$ | (b) $y = 2x$ and $y = 4x - 6$ |
| (c) $y = 2x + 1$ and $y = 4x - 7$ | (d) $y = 3x - 2$ and $y = 5x - 4$ |
| (e) $y = x - 3$ and $y = 2x - 1$ | (f) $y = 2x - 3$ and $y = 4x + 1$ |
| (g) $y = 3x + 2$ and $y = x + 6$ | (h) $y = 4x - 5$ and $y = x + 7$ |

3. Solve the following simultaneous equations using substitution.

- | | |
|-------------------------------------|-----------------------------------|
| (a) $y + 2x = 8$ and $y = x - 1$ | (b) $y + 3x = 7$ and $y = 3x - 5$ |
| (c) $y + 3x = 3$ and $y = 2x + 8$ | (d) $y - 2x = 3$ and $y = x + 4$ |
| (e) $y - 3x = -1$ and $y = 2x + 1$ | (f) $y + 5x = -7$ and $y = x - 1$ |
| (g) $y - 4x = -11$ and $y = 2x - 7$ | (h) $y - x = 0$ and $y = 3x + 4$ |

Example 3 $2y + 3x = 13$ and $y = 2x - 4$

Step 1 $2y + 3x = 13$ ——— ①
 $y = 2x - 4$ ——— ②

Step 2 and 3 Substitute equation 2 into equation 1
 and solve for x :

$$\begin{aligned} 2(2x - 4) + 3x &= 13 \\ 4x - 8 + 3x &= 13 \\ 7x - 8 &= 13 \\ \boxed{+ 8} \quad \boxed{+ 8} & \\ 7x &= 21 \\ \boxed{\div 7} \quad \boxed{\div 7} & \\ x &= 3 \end{aligned}$$

Step 4 Substitute $x = 3$ into equation 2 (easier than 1)
 to solve for y .

$$\begin{aligned} y &= 2 \times 3 - 4 \\ y &= 2 \end{aligned}$$

Solution: $x = 3, y = 2$ (3,2)

4. Solve the following simultaneous equations using substitution.

(a) $3y + 4x = 10$ and $y = 3x - 1$

(b) $2y + 4x = 20$ and $y = 2x - 2$

(c) $2y + 4x = 2$ and $y = 2x + 5$

(d) $3y - 2x = 7$ and $y = 3x + 7$

(e) $3y - 2x = -7$ and $y = 2x - 1$

(f) $2y + 5x = -16$ and $y = 3x + 3$

(g) $4y - 3x = -23$ and $y = 3 - x$

(h) $2y + 5x = -12$ and $y = 2x + 12$

Example 4 $3y - x = 11$ and $y + 2x = 6$

Step 1 $3y - x = 11$ ——— ①
 $y + 2x = 6$ ——— ②

Step 2 and 3 Rearrange equation 2 to make y the subject:

$$\begin{array}{r} y + 2x = 6 \\ -2x \quad -2x \\ \hline y = 6 - 2x \end{array}$$

Substitute into equation 1 and solve for x :

$$\begin{array}{r} 3(6 - 2x) - x = 11 \\ 18 - 6x - x = 11 \\ 18 - 7x = 11 \\ -18 \quad -18 \\ \hline -7x = -7 \\ \div -7 \quad \div -7 \\ \hline x = 1 \end{array}$$

Step 4 Substitute $x = 1$ into equation 1 (or 2) to solve for y .

$$\begin{array}{r} 3y - 1 = 11 \\ +1 \quad +1 \\ \hline 3y = 12 \\ \div 3 \quad \div 3 \\ \hline y = 4 \end{array}$$

Solution: $x = 1, y = 4$ (1,4)

5. Solve the following simultaneous equations using substitution.

(a) $2y - 3x = -1$ and $y - 2x = -1$ (b) $5y + 2x = 14$ and $y - 3x = -4$

(c) $3y - 4x = -6$ and $y + 2x = 8$ (d) $3x + 2y = 5$ and $y + 3x = 7$

(e) $2x - 3y = 12$ and $y - 3x = -11$ (f) $4y + 3x = -18$ and $y + x + 5 = 0$

(g) $2y + 3x = -10$ and $y - x = -5$ (h) $3x - 2y = -1$ and $y + 2x + 10 = 0$

Elimination

The following are examples of solving simultaneous equations using *elimination*.

Follow the steps below.

Step 1 - Label the equations **1** and **2**

Step 2 - Add or subtract the equations to eliminate one of the variables

Step 3 - Solve for the other variable

Step 4 - Substitute into either equation to solve for the other variable

Example 1 $x + y = 7$ and $x - y = -1$

Step 1

$$\begin{array}{rcl} x + y = 7 & \text{---} & \textcircled{1} \\ x - y = -1 & \text{---} & \textcircled{2} \end{array}$$

Step 2 and 3 By **adding** the equations y will be eliminated:

$$x + y + x - y = 7 - 1$$

$$2x = 6$$

$$\boxed{\div 2} \quad \boxed{\div 2}$$

$$x = 3$$

Step 4 Substitute $x = 3$ into equation **1** (or **2**) to solve for y .

$$3 + y = 7$$

$$\boxed{-3} \quad \boxed{-3}$$

$$y = 4$$

Solution: $x = 3, y = 4$ **(3,4)**

Example 2 $3y + x = 11$ and $2y + x = 8$

Step 1 $3y + x = 11$ ———— ①
 $2y + x = 8$ ———— ②

Step 2 and 3 **Subtracting** equation 2 from equation 1 will eliminate x .

$$\begin{aligned}(3y + x) - (2y + x) &= 11 - 8 \\ 3y + x - 2y - x &= 3 \\ y &= 3\end{aligned}$$

Step 4 Substitute $y = 3$ into equation 1 (or 2) to solve for x .

$$\begin{aligned}3 \times 3 + x &= 11 \\ 9 + x &= 11 \\ -9 &\quad -9 \\ x &= 2\end{aligned}$$

Solution: $x = 2, y = 3$ **(2,3)**

6. Solve the following simultaneous equations using elimination.

- | | |
|--|---|
| (a) $x + y = 6$ and $x - y = 2$ | (b) $y + x = -2$ and $y - x = 14$ |
| (c) $3y + x = 7$ and $y - x = 1$ | (d) $2x + y = 9$ and $3x - y = 1$ |
| (e) $5y + 2x = 9$ and $3y - 2x = 15$ | (f) $2y - 3x = -12$ and $4y + 3x = -6$ |
| (g) $4x - 2y = -8$ and $3x + 2y = -13$ | (h) $2x + 3y = -10$ and $3x - 3y = -15$ |

7. Solve the following simultaneous equations using elimination.

- | | |
|---|---------------------------------------|
| (a) $2y + x = 10$ and $y + x = 7$ | (b) $3y + x = 11$ and $y + x = 3$ |
| (c) $6y + x = -11$ and $2y + x = -3$ | (d) $y + 3x = 4$ and $y + x = 2$ |
| (e) $2y + 4x = -14$ and $2y + 3x = -12$ | (f) $4x + 3y = 10$ and $2x + 3y = 14$ |
| (g) $5x + 3y = -24$ and $x + 3y = -12$ | (h) $3x + 2y = 8$ and $x + 2y = 8$ |

Example 3 $5y + 2x = 12$ and $2y + x = 5$

Step 1

$$\begin{array}{rcl} 5y + 2x = 12 & \text{---} & \textcircled{1} \\ 2y + x = 5 & \text{---} & \textcircled{2} \end{array}$$

Step 2 and 3 To eliminate one of the variables the coefficients need to be the same in both equations. In this example this can be achieved by multiplying equation **2** by 2. The coefficients of x will then both be the same.

$$\begin{array}{rcl} 5y + 2x = 12 & \text{---} & \textcircled{1} \\ 4y + 2x = 10 & \text{---} & \textcircled{2} \times 2 \quad \textcircled{2A} \end{array}$$

Subtract equation **2A** from equation **1** to eliminate x .

$$\begin{aligned} (5y + 2x) - (4y + 2x) &= 12 - 10 \\ 5y + 2x - 4y - 2x &= 2 \\ y &= 2 \end{aligned}$$

Step 4 Substitute $y = 2$ into equation **1** (or **2**) to solve for x .

$$\begin{aligned} 5 \times 2 + 2x &= 12 \\ 10 + 2x &= 12 \\ -10 & \quad -10 \\ \hline 2x &= 2 \\ \div 2 & \quad \div 2 \\ \hline x &= 1 \end{aligned}$$

Solution: $x = 1, y = 2$ **(1,2)**

Example 4 $2x + 5y = 12$ and $3x - 2y = -1$

Step 1 $2x + 5y = 12$ ——— ①
 $3x - 2y = -1$ ——— ②

Step 2 and 3 To eliminate one of the variables the coefficients need to be the same in both equations. In this example this can be achieved by multiplying equation 1 by 3 and equation 2 by 2.
 The coefficients of x will then both be the same.

$$6x + 15y = 36 \text{ ——— } \textcircled{1} \times 3 \quad \textcircled{1A}$$

$$6x - 4y = -2 \text{ ——— } \textcircled{2} \times 2 \quad \textcircled{2A}$$

Subtract equation 2A from equation 1A to eliminate x .

$$(6x + 15y) - (6x - 4y) = 36 - (-2)$$

$$6x + 15y - 6x + 4y = 38$$

$$19y = 38$$

$$\boxed{\div 19} \quad \boxed{\div 19}$$

$$y = 2$$

Step 4 Substitute $y = 2$ into equation 1 (or 2) to solve for x .

$$2x + 5 \times 2 = 12$$

$$2x + 10 = 12$$

$$\boxed{- 10} \quad \boxed{- 10}$$

$$2x = 2$$

$$\boxed{\div 2} \quad \boxed{\div 2}$$

$$x = 1$$

Solution: $x = 1, y = 2$ (1,2)

8. Solve the following simultaneous equations using elimination.

- (a) $5y + 2x = 16$ and $2y + x = 7$ (b) $2y + 5x = 11$ and $y + 2x = 5$
 (c) $3y - 2x = 14$ and $y + x = 3$ (d) $5y + 3x = 9$ and $y - x = 5$
 (e) $7y - 2x = -1$ and $2y + x = -5$ (f) $2y + 5x = -2$ and $y - 2x = -10$
 (g) $3y - 4x = 17$ and $y + 2x = -1$ (h) $5y + 6x = 11$ and $2y + 3x = 5$
 (i) $3y + 4x = -4$ and $4y + 2x = -12$ (j) $5y - 2x = -4$ and $3y + x = -9$
 (k) $3y - 4x = 6$ and $y - 2x = 4$ (l) $4y - 3x = 2$ and $3y - x = 4$

9. Solve the following simultaneous equations using elimination.

- (a) $2x + 3y = 7$ and $3x + 2y = 8$ (b) $5y + 2x = -9$ and $2y - 3x = 4$
 (c) $3y - 2x = 12$ and $4y + 3x = -1$ (d) $2y + 5x = 16$ and $3y - 2x = -14$
 (e) $3y - 4x = 2$ and $2y + 5x = -14$ (f) $2y - 3x = -19$ and $3y - 5x = -31$
 (g) $3x + 4y = 0$ and $5x + 6y = -2$ (h) $2y - 7x = -32$ and $5y - 4x = 1$
 (i) $3x + 8y = -20$ and $4x - 5y = 36$ (j) $7x + 9y = -3$ and $5x - 4y = 50$
 (k) $5x - 7y = -71$ and $4x + 6y = 36$ (l) $4y + 3x = 11$ and $5y - 7x = 46$

10. Solve the following simultaneous equations by choosing the most appropriate method - substitution or elimination.

- (a) $x + y = 6$ and $x - y = -2$ (b) $2x + y = 5$ and $y = 3x$
 (c) $3x + y = 2$ and $x + y = 0$ (d) $y + 3x = -4$ and $y = x + 4$
 (e) $3y + 2x = 7$ and $2y + x = 5$ (f) $y = 2x + 11$ and $y = x + 7$
 (g) $2y + 3x = 1$ and $5y + 2x = 8$ (h) $3x + 2y = 9$ and $x + y = 2$
 (i) $5x + y = -13$ and $y - x = -1$ (j) $2y - 3x = 6$ and $2x + 3y = -4$
 (k) $3y - 2x = 5$ and $5y + x - 17 = 0$ (l) $2x + 3y = -8$ and $3x + 5y = -13$
 (m) $3y + 5x = -21$ and $3y - 5x = 9$ (n) $2y = 3x + 2$ and $y = x + 2$
 (o) $2x - 3y = -1$ and $x + y = 2$ (p) $5y + 7x = 9$ and $6y - 2x = -10$

11. Graphics calculators can be used to solve simultaneous equations.

Both equations need to be transposed to make y the subject so they can be plotted. The point of intersection can then be found.

Select a few of the above problems and solve them by using a graphics calculator.

Example $3x + 4y = 11$ and $2x - 3y = -4$

Step 1 Transpose both equations to make y the subject.

$$y = \frac{11}{4} - \frac{3}{4}x \quad \text{and} \quad y = \frac{2}{3}x + \frac{4}{3}$$

Step 2 Graph and find point of intersection: $x = 1, y = 2$

Problem Solving Using Simultaneous Equations

Example

Two groups bought tickets for a concert.

One group had three adults and ten children and their tickets cost a total of \$125.

The other group had four adults and nine children and their tickets cost a total of \$132.

Find the cost of an adult's ticket and a child's ticket.

Step 1 Use symbols to represent the unknown items.

Let A = price of adult's ticket

Let C = price of child's ticket

Step 2 Write the information as equations.

$$3A + 10C = 125 \quad \text{---} \textcircled{1}$$

$$4A + 9C = 132 \quad \text{---} \textcircled{2}$$

Step 3 Solve these simultaneous equations

$$12A + 40C = 500 \quad \text{---} \textcircled{1} \times 4 \quad \textcircled{1A}$$

$$12A + 27C = 396 \quad \text{---} \textcircled{2} \times 3 \quad \textcircled{2A}$$

$$13C = 104 \quad \text{---} \textcircled{1A} - \textcircled{2A}$$

$$C = \$8$$

$$3A + 80 = 125 \quad \text{Substitute into equation 1 (or 2)}$$

$$3A = 45$$

$$A = \$15$$

Answer: Adult ticket = \$15, child's ticket = \$8

EXERCISE 9I

1. Eight people in an office ordered their lunch each day.
One day they ordered three salad rolls and five sandwiches which cost a total of \$27.
The next day they ordered four salad rolls and four sandwiches which cost a total of \$28.
Find the cost of a salad roll and sandwich.
2. Leah bought three shirts and two pairs of jeans for \$275.
Pip bought four shirts and one pairs of jeans for \$225.
Find the price of a shirt and a pair of jeans.
3. Sophie made 12 kg of jam and filled 8 large jars and 24 small jars.
She made another batch of jam that weighed 9 kg and from this filled 10 large jars and 6 small jars.
Find the weight of a large jar and small jar of jam.
4. Terry, a bricklayer, and his apprentice, Damon, are laying the bricks for a house. One day Terry worked for five hours and Damon for three hours and they laid 840 bricks. The next day Terry worked for six hours and Damon for four hours and they laid 1040 bricks.
How many bricks can they each lay in an hour?
5. A zoo had 20 chimpanzees and 8 baboons and each day they ate a total of 128 bananas.
Another 8 chimpanzees and 4 baboons were acquired by the zoo and a total of 184 bananas were then eaten each day.
How many bananas did each chimpanzee and baboon eat each day?



PROBLEM SOLVING

A hospital receives a distress call from a yacht. Someone is seriously ill and a rescue helicopter needs to be sent. There are two helicopters available.

Helicopter **A** is stationed 40 km west of the hospital. To fly in a straight line to the yacht it would pass over a point 20 km north of the hospital.

Helicopter **A** can fly at an average speed of 132 km/h.

Helicopter **B** is stationed 100 km south of the hospital. To fly in a straight line to the yacht it would pass over a point 40 km east of the hospital.

Helicopter **B** can fly at an average speed of 178 km/h.

1. Draw a set of axes to show this information.
2. Use simultaneous equations to find the coordinates of the yacht.
3. Use Pythagoras' Theorem to help find which helicopter could fly to the yacht, rescue the ill passenger and fly them to the hospital in the quickest time?



PUZZLE

Three straight lines (**A**, **B** and **C**) are drawn on a graph. The lines are different colours - red, green and blue. The red line is steeper than line **C**. Line **B** is steeper than line **A**. The green line is the steepest. Match the letters and the colours of the lines.

CHAPTER REVIEW

1. Copy and complete the following tables for the given rules.

(a) $y = 3x - 4$

x	-3	-2	-1	0	1	2	3
y							

(b) $n = 3 - 2m$

m	-3	-2	-1	0	1	2	3
n							

2. For each of the tables of values below, find the rule connecting the variables.

(a)

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

(b)

b	-3	2	5	10	16
a	-4	11	20	35	53

3. Water is pumped into an 8000 litre tank at the rate of 200 litres per minute. The tank initially held 600 litres.

T	0	2	5	10	25
V					

- (a) Complete this table showing the volume of water, V , T minutes after pumping started.
 (b) Complete the equation: $V = \underline{\hspace{2cm}}$.
 (c) After how many minutes would the tank be full?
4. Find the gradient of the line connecting the following pairs of points.
 (a) (0,0) and (2,10) (b) (2,3) and (5,12) (c) (-1,-3) and (1,5)
 (d) (-5,4) and (-1,6) (e) (2,1) and (10,-7) (f) (-2,0) and (5,-4)
5. Sketch the straight lines with the following x - and y -intercepts.
 (a) x -intercept = 2 and y -intercept = -4
 (b) x -intercept = 3 and y -intercept = 2
6. State the **coordinates** of the x -intercept and y -intercept for the straight lines in question 5.
7. State the coordinates of the x -intercept and y -intercept for the straight lines with the following equations.
 (a) $x + y = 5$ (b) $3y - 2x = 6$ (c) $y = x + 1$ (d) $y = 3x - 12$
8. State the gradient and y -intercept of the straight lines with the following equations.
 (a) $y = 3x - 5$ (b) $y = -x + 2$ (c) $y = \frac{2}{3}x - 2$ (d) $y = -4x - \frac{1}{2}$

9. Write the equations for the straight lines with the following gradients and y -intercepts.
- gradient = 2 and y -intercept = 6
 - gradient = -2 and y -intercept = -3
 - gradient = $\frac{3}{4}$, y -intercept = 0
10. Transpose the following equations to make y the subject and hence find the gradient and y -intercept.
- $y + 5x - 3 = 0$
 - $2y - 3x + 6 = 0$
 - $4x - 3y + 12 = 0$
11. Find the equation of the straight lines joining the following pairs of points.
- (-1,2) and (4,12)
 - (-2,12) and (5,-9)
12. Find the equations of the straight lines with the following x - and y -intercepts.
- x -intercept = 3, y -intercept = 6
 - x -intercept = 12, y -intercept = -9
13. Find the x - and y -intercept of the straight line with a gradient of 4 that passes through the point (4,4).
14. (a) Find the equation to the straight line that is parallel to the line with equation $y = -4x - 6$ and passes through the point (3,-7).
 (b) Find the equation to the straight line that is perpendicular to the line with equation $y = \frac{2}{3}x + 1$ and passes through the point (-6,-11).
15. Solve the following simultaneous equations by choosing the most appropriate method - substitution or elimination.
- $y = 3x - 14$ and $y + x = 6$
 - $y + 2x = -3$ and $3y - 2x = 7$
 - $3y - x = 14$ and $2y + 3x = 2$
 - $2y + 5x = -11$ and $3y + 2x = 0$
16. Six mathematics books and nine English books weighed 10.2 kg. Ten mathematics books and three English books weighed 9.8 kg. Find the weight of each book.

Non-linear Relationships

There are many families of curves that have **non-linear relationships**. Non-linear relationships are relationships that when graphed produce graphs that are not straight lines. In this chapter the following curves will be studied: parabolas, rectangular hyperbolas, circles and exponential functions.

Parabolas



EXERCISE 10A

Parabolas are the curves formed by graphing quadratic equations. **Quadratic equations** are equations that have 2 as the highest power.

Examples

The following are quadratic equations.

$$y = x^2$$

$$y = 3x^2 - 4$$

$$h = 7t^2 + 5t - 1$$

$$a = 5 - 6b^2$$

The following are **not** quadratic equations.

$$y = x + 1$$

$$y = 8x^3 - 4x^2 + 3$$

$$a = 5 - 6b^4$$

$$h = 7t^2 + 5t^3 - t$$

1. State which of the following are quadratic equations.

A $y = 5x$

B $y = 2x^2$

C $a = 6 + b^2$

D $m = 3n^3 - n^2$

E $y = 6x + 2$

F $v = 3t^2 - 5$

G $h = 2c^4 + 3c^2 - 6c$

H $y = 7 - 4x + 5x^2$

I $B = 2A^3 + 4A^2 - 5A$

J $d = 4r^2 + 2r - 7$

K $y = 2x^2 + 5x + 8$

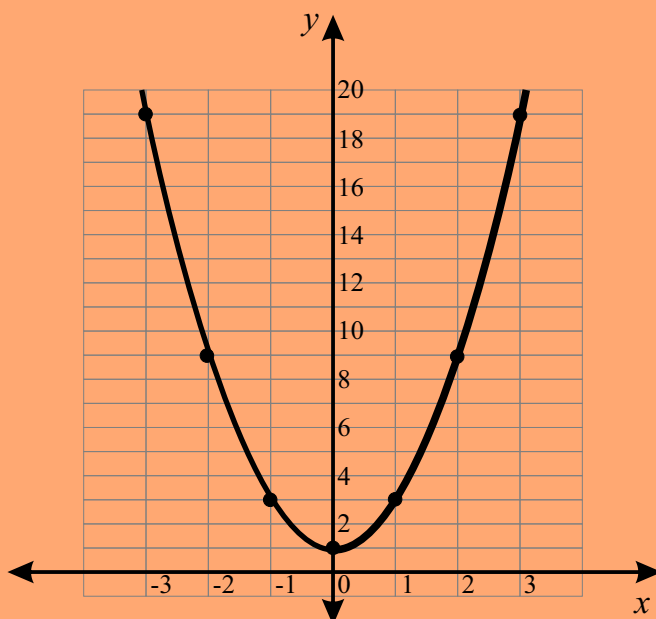
L $z = 7 + 2y^5$

When a quadratic equation is graphed, the resultant curve is called a *parabola*.

Example $y = 2x^2 + 1$

This equation can be graphed by completing a table of values, plotting the points and connecting them with a smooth curve.

x	-3	-2	-1	0	1	2	3
y	19	9	3	1	3	9	19



2. For each of the following quadratic equations, complete the table of values shown, plot the points on a graph and connect the points with a smooth curve.

(a) $y = x^2 - 2x$

x	-2	-1	0	1	2	3	4
y							

(b) $y = x^2 + 4x + 1$

x	-5	-4	-3	-2	-1	0	1
y							

3. (a) Using a graphics calculator, or otherwise, plot on the same axes the following sets of graphs.

A $y = x^2$ **B** $y = 2x^2$ **C** $y = 3x^2$

- (b) Based on these three graphs, what effect does the coefficient of x^2 , a , have on the graph $y = ax^2$, if $a > 1$?

4. (a) Using a graphics calculator, or otherwise, plot on the same axes the following sets of graphs.

A $y = x^2$ **B** $y = \frac{1}{2}x^2$ **C** $y = \frac{1}{3}x^2$

- (b) Based on these three graphs, what effect does the coefficient of x^2 , a , have on the graph $y = ax^2$, if $0 < a < 1$?

5. (a) Using a graphics calculator, or otherwise, plot on the same axes the following sets of graphs.

A $y = -x^2$ **B** $y = -\frac{1}{2}x^2$ **C** $y = -3x^2$

- (b) Based on these three graphs, what effect does the coefficient of x^2 , a , have on the graph $y = ax^2$, if $a < 0$?

Summary of the shape of parabolas.

It can be seen that all parabolas have the same basic shape.

If the coefficient of x^2 is positive, the curve is in the shape of a 'cup'.



If the coefficient of x^2 is negative, the curve is in the shape of a 'hat'.



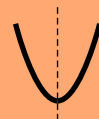
If the coefficient of x^2 is greater than 1, the curve will be thinner.



If the coefficient of x^2 is a fraction less than 1, the curve will be flatter.

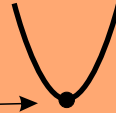


Parabolas are ***symmetrical***.

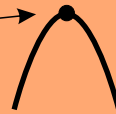


All parabolas have **turning points**.

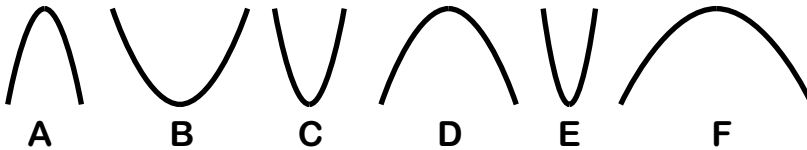
If the parabola is a 'cup' shape, it has a **minimum** turning point.



If the parabola is a 'hat' shape, it has a **maximum** turning point.



6. (a) Which of the following parabolas have a minimum turning point?
 (b) Which of the following parabolas have a maximum turning point?



7. (a) Which of the following parabolas would have a minimum turning point if graphed?
 (b) Which of the following parabolas would have a maximum turning point if graphed?
 (c) Which of the following parabolas would be 'thinner' than the graph of $y = x^2$?
 (d) Which of the following parabolas would be 'flatter' than the graph of $y = x^2$?

A $y = 3x^2$

C $y = -4x^2$

E $y = 3x^2 - 2x - 5$

G $y = -\frac{2}{3}x^2 - 5x + 1$

I $y = 3 + x^2$

K $y = 6 - 2x^2$

M $y = 5 - 4x^2 + 2x$

O $y = -5 - \frac{1}{4}x + \frac{3}{8}x^2$

B $y = x^2 - 5$

D $y = \frac{1}{2}x^2$

F $y = -2x^2 + 6x - 7$

H $y = \frac{1}{3}x^2 - 5$

J $y = 4 - 2x + \frac{3}{4}x^2$

L $y = 5 - 6x - 3x^2$

N $y = -4x + \frac{2}{5}x^2$

P $y = -9x^2 + 5x + 8$

Sketching Parabolas

Parabolas can be sketched knowing several of the following characteristics:

- the axis of symmetry
- the turning point
- x -intercepts (the points where the parabola crosses the x -axis)
- y -intercept (the point where the parabola crosses the y -axis)

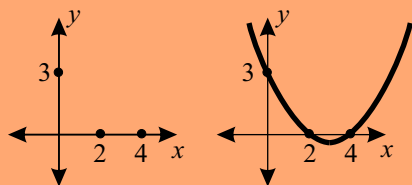
When sketching the parabolas:

- first mark the known points on a set of axes
- then connect them with a *smooth* and *symmetrical* curve.

Examples Sketch the following parabolas

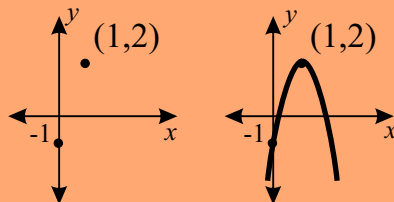
1. This parabola has:

- x -intercepts at 2 and 4
- y -intercept at 3



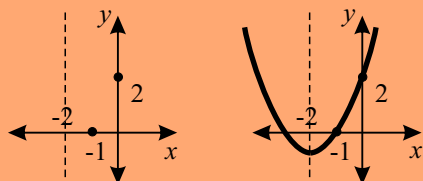
2. This parabola has:

- its turning point at (1,2)
- y -intercept at -1



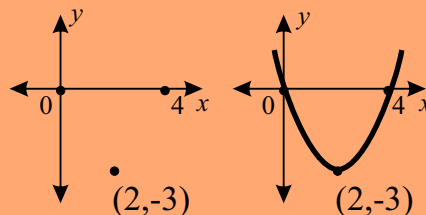
3. This parabola has:

- one x -intercept at -1
- y -intercept at 2
- axis of symmetry at $x = -2$



4. This parabola has:

- its turning point at (2,-3)
- x -intercepts at 0 and 4



EXERCISE 10B

Sketch the following parabolas.

1. This parabola has - x -intercepts at -2 and 2
- y -intercept at -3
2. This parabola has - its turning point at $(2,-1)$
- y -intercept at 4
3. This parabola has - one x -intercept at 2
- y -intercept at -4
- its axis of symmetry at $x = 3$
4. This parabola has - its turning point at $(-1,4)$
- x -intercepts at -3 and 1
5. This parabola has - its axis of symmetry at $x = -3$
- one x -intercept at -1
- y -intercept at 1
6. This parabola has - x -intercepts at 1 and 5
- y -intercept at -4
7. This parabola has - its turning point at $(2,-1)$
- y -intercept at -4
8. This parabola has - x -intercepts at -5 and 1
- y -intercept at -1
9. This parabola has - its turning point at $(-2,3)$
- y -intercept at 5
10. This parabola has - its turning point at $(4,0)$
- y -intercept at 1
11. This parabola has - its axis of symmetry at $x = 2$
- one x -intercept at 5
- y -intercept at -2
12. This parabola has - its axis of symmetry at $x = -3$
- one x -intercept at 3
- y -intercept at -2
13. This parabola has - its axis of symmetry at $x = 0$
- one x -intercept at -1
- y -intercept at -3
14. This parabola has - its axis of symmetry at $x = 0$
- y -intercept at 0
- passes through the point $(2,4)$

Finding x - and y -Intercepts

The y -intercept of a parabola can be found by letting $x = 0$
 The x -intercepts of a parabola can be found by letting $y = 0$

Example 1 $y = x^2 - 5x + 6$

Step 1 Find the y -intercept by letting $x = 0$

$$y = 0^2 - 5 \times 0 + 6$$

$$y = 6$$

Step 2 Find the x -intercepts by letting $y = 0$

$$0 = x^2 - 5x + 6$$

$$x^2 - 5x + 6 = 0$$

This can be solved by following the steps set out in Chapter 7
 - the sections on solving quadratic equations (p 182).

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

There are two solutions:

$$x - 2 = 0 \quad \text{and} \quad x - 3 = 0$$

$$x = 2 \quad \text{and} \quad x = 3$$

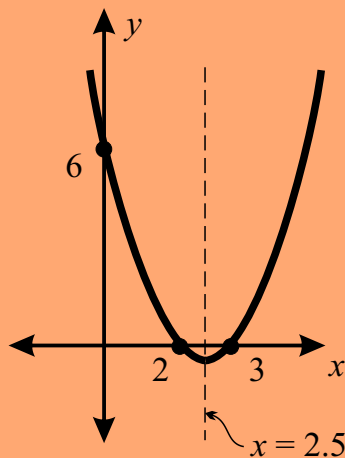
Step 3 The parabola can now be sketched:

y -intercept = 6

x -intercepts = 2 and 3

Note: the axis of symmetry will be midway between the x -intercepts.

In this example the axis of symmetry will be at $x = 2.5$



Example 2 $y = 6x^2 - x - 15$

Step 1 Find the y -intercept by letting $x = 0$
 $y = -15$

Step 2 Find the x -intercepts by letting $y = 0$

$$0 = 6x^2 - x - 15$$

$$6x^2 - x - 15 = 0$$

$$(2x + 3)(3x - 5) = 0$$

There are two solutions:

$$2x + 3 = 0 \quad \text{and} \quad 3x - 5 = 0$$

$$x = -\frac{3}{2} \quad \text{and} \quad x = \frac{5}{3}$$

Step 3 The parabola can now be sketched:

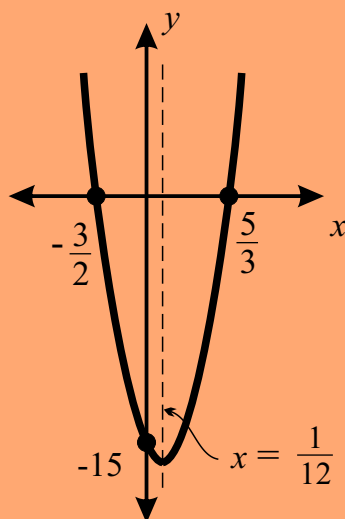
y -intercept = -15

x -intercepts = $-\frac{3}{2}$ and $\frac{5}{3}$

Note: the axis of symmetry will be midway between the x -intercepts.

In this example the axis of symmetry will be at

$$x = \frac{-\frac{3}{2} + \frac{5}{3}}{2} = \frac{1}{12}$$



Any quadratic equation can be solved using the quadratic formula (p185) to find the x-intercepts.

Example 3 $y = -8x^2 - 6x + 35$

Step 1 Find the y-intercept by letting $x = 0$
 $y = 35$

Step 2 Find the x-intercepts by letting $y = 0$ and using the quadratic formula.
 $a = -8$, $b = -6$ and $c = 35$.

$$ax^2 + bx + c = 0$$

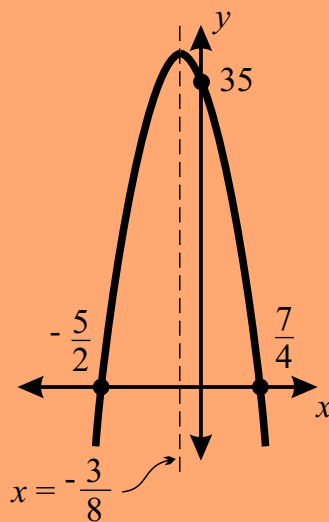
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times -8 \times 35}}{2 \times -8}$$

$$x = \frac{6 \pm 34}{-16} = -\frac{5}{2} \text{ and } \frac{7}{4}$$

Step 3 Axis of symmetry is at

$$x = \frac{-\frac{5}{2} + \frac{7}{4}}{2} = -\frac{3}{8}$$



EXERCISE 10C

- Find the x- and y-intercepts of the graphs for the following equations and sketch the graphs showing all intercepts and the axis of symmetry.
 - $y = x^2 + 4x - 32$
 - $y = x^2 - 14x + 45$
 - $y = x^2 + 7x + 10$
 - $y = -x^2 - 4x + 12$
 - $y = 2x^2 - 19x + 42$
 - $y = -4x^2 - 7x + 15$
 - $y = 6x^2 + 11x - 35$
 - $y = -12x^2 - x + 20$
- Choose several of the equations from question 1 and plot these on a graphics calculator. The scales may need to be changed to view each graph clearly.

Finding the Turning Point Using the Axis of Symmetry

The turning point of a parabola can be found by firstly finding the axis of symmetry then substituting this x -value into the quadratic equation.

Example Find the turning point of the parabola with equation

$$y = x^2 - 4x - 5$$

Step 1 Find the x -intercepts.

$$0 = x^2 - 4x - 5$$

$$0 = (x - 5)(x + 1)$$

x -intercepts are $x = 5$ and -1

Step 2 Find the axis of symmetry (midway between the x -intercepts)

$$x = \frac{5 + -1}{2}$$

$$x = 2$$

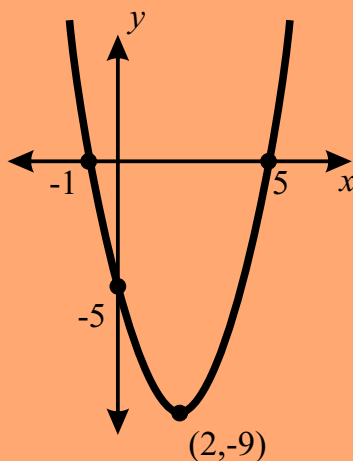
Step 3 Substitute this into the equation to find the y -value of the turning point

$$y = x^2 - 4x - 5$$

$$= 4 - 8 - 5$$

$$y = -9$$

Step 4 The coordinates of the turning point are $(2, -9)$



EXERCISE 10D

Find the coordinates of the turning point for each of the parabolas from question 1 in exercise 10C.

Finding the Turning Point Using $y = a(x - h)^2 + k$

Using the axis of symmetry to find the turning point can only be used when the parabola has x -intercepts. The turning point of any parabola can be found by converting the quadratic equation to the form:

$$y = a(x - h)^2 + k \quad \text{This is called *turning point form*.}$$

The coordinates of the turning point are (h,k).

Example 1 $y = x^2 - 6x - 2$

To convert to **turning point form** it is necessary to **complete the square** of the first two terms (refer to page 156).

$$\begin{aligned} y &= x^2 - 6x - 2 \\ &= x^2 - 6x + 9 - 9 - 2 \\ y &= (x - 3)^2 - 11 \end{aligned}$$

Turning point is **(3,-11)**

Example 2 $y = x^2 + 8x + 3$

$$\begin{aligned} &= x^2 + 8x + 16 - 16 + 3 \\ &= (x + 4)^2 - 13 \\ y &= [x - (-4)]^2 - 13 \end{aligned}$$

Turning point is **(-4,-13)**

Example 3 $y = x^2 + 3x + 8$

$$\begin{aligned} &= x^2 + 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 8 \\ &= x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 8 \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{23}{4} \\ y &= \left[x - \left(-\frac{3}{2}\right)\right]^2 + \frac{23}{4} \end{aligned}$$

Turning point is **$\left(-\frac{3}{2}, \frac{23}{4}\right)$**

Example 4

$$\begin{aligned}
 y &= -x^2 - 12x - 23 \\
 &= -(x^2 + 12x + 23) \\
 &= -(x^2 + 12x + 36 - 36 + 23) \\
 &= -[(x + 6)^2 - 13] \\
 &= -(x + 6)^2 + 13 \\
 &= -[x - (-6)]^2 + 13
 \end{aligned}$$

Turning point is **(-6,13)**

Example 5

$$\begin{aligned}
 y &= -4x^2 + 24x + 36 \\
 &= -4(x^2 - 6x - 9) \\
 &= -4(x^2 - 6x + 9 - 9 - 9) \\
 &= -4[(x - 3)^2 - 18] \\
 &= -4(x - 3)^2 + 72
 \end{aligned}$$

Turning point is **(3,72)**

EXERCISE 10E

1. Find the coordinates of the turning point for the parabolas with the following equations.

(a) $y = x^2 - 10x + 2$

(b) $y = x^2 - 16x + 24$

(c) $y = x^2 + 4x - 3$

(d) $y = x^2 + 6x + 5$

(e) $y = x^2 + 2x - 8$

(f) $y = x^2 - 14x + 1$

(g) $y = x^2 - 20x$

(h) $y = x^2 + 18x$

2. Find the coordinates of the turning point for the parabolas with the following equations.

(a) $y = x^2 - 5x + 3$

(b) $y = x^2 - x + 4$

(c) $y = x^2 + 3x - 8$

(d) $y = x^2 + 9x - 3$

3. Find the coordinates of the turning point for the parabolas with the following equations.

(a) $y = -x^2 + 4x - 5$

(b) $y = -x^2 - 12x - 3$

(c) $y = -2x^2 + 8x - 16$

(d) $y = -3x^2 - 12x + 15$

(e) $y = -2x^2 - 5x + 8$

(f) $y = -4x^2 + 6x - 12$

(g) $y = -5x^2 + 15x - 10$

(h) $y = -6x^2 - 8x + 15$

Turning Point Form

The y -intercept of a parabola can be found by letting $x = 0$.

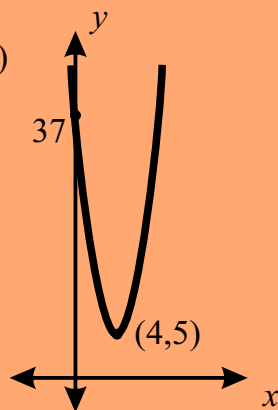
Example 1 Find the turning point and y -intercept of the following equation and sketch the parabola.

$$y = 2(x - 4)^2 + 5$$

The turning point is $(4, 5)$ (Exercise 10E)

The y -intercept is found by letting $x = 0$

$$\begin{aligned} y &= 2(0 - 4)^2 + 5 \\ &= 2 \times (-4)^2 + 5 \\ &= 2 \times 16 + 5 \\ y &= 37 \end{aligned}$$



Remember the general equation of a parabola is $y = a(x - h)^2 + k$

If a is **positive** the parabola is a '**cup**'.

If a is **negative** the parabola is a '**hat**'.

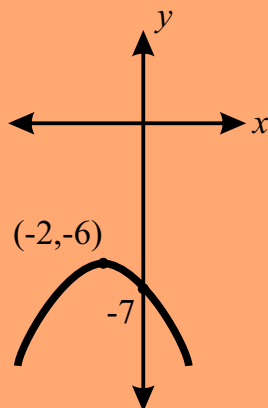
Example 2 Find the turning point and y -intercept of the following equation and sketch the parabola.

$$y = -\frac{1}{4}(x + 2)^2 - 6 \quad (\text{This parabola will be a 'hat' because } a \text{ is negative})$$

The turning point is $(-2, -6)$ (Exercise 10E)

The y -intercept is found by letting $x = 0$

$$\begin{aligned} y &= -\frac{1}{4}(0 + 2)^2 - 6 \\ &= -\frac{1}{4} \times (2)^2 - 6 \\ &= -\frac{1}{4} \times 4 - 6 \\ y &= -7 \end{aligned}$$



EXERCISE 10F

1. Sketch the following parabolas by locating the turning point and y -intercept of each.

(a) $y = 2(x - 3)^2 - 4$

(b) $y = \frac{1}{2}(x - 4)^2 + 1$

(c) $y = -2(x + 1)^2 - 2$

(d) $y = 4(x - 3)^2$

(e) $y = -3(x - 5)^2 - 6$

(f) $y = \frac{1}{3}(x + 3)^2 + 4$

(g) $y = -\frac{1}{4}(x - 8)^2 + 3$

(h) $y = -(x + 4)^2 + 1$

2. By analysing these graphs, which of the parabolas have x -intercepts?

Practical Applications

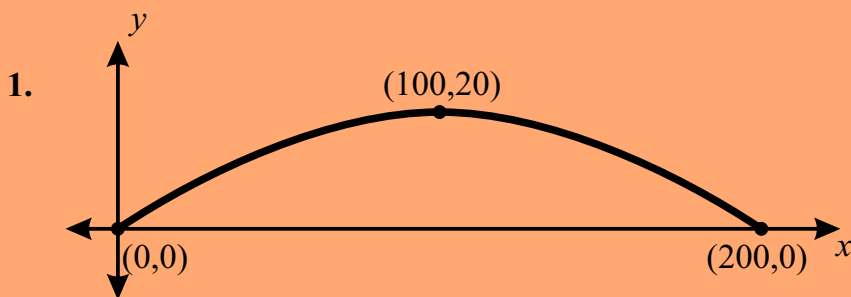
Many shapes that occur naturally and man made are in the shape of parabolas. For example: a piece of string joined to two points, the motion of a ball through the air, a bent piece of plastic, a stream of water spurting from a hose, many styles of bridges, radar dishes, etc.



Example

A golf ball is hit and travels 200 metres before it hits the ground. It just passes over a 20 metre tall tree at the top of its trajectory. Assume that the trajectory of the golf ball is a parabola.

1. Sketch the parabolic trajectory of the ball on coordinate axes using the point that it was hit as the origin. State the coordinates of the turning point and x -intercepts.
2. Find the equation of this parabola.
3. Find the height of the ball 50 metres from the point it was hit.

Answers

2. The turning point form of the equation to a parabola is:

$$y = a(x - h)^2 + k$$

The coordinates of the turning point are (h,k) .

The turning point is $(100,20)$. Substitute these into the equation:

$$y = a(x - 100)^2 + 20$$

To find a , substitute one of the points into this equation.

The easiest point is $(0,0)$.

$$0 = a(0 - 100)^2 + 20$$

$$0 = 10\,000a + 20$$

$$\begin{aligned} a &= -\frac{20}{10\,000} \\ &= -\frac{1}{500} \end{aligned}$$

Complete the equation:

$$y = -\frac{1}{500}(x - 100)^2 + 20$$

Example continued

$$y = -\frac{1}{500}(x - 100)^2 + 20$$

3. Find the height of the ball, y , when $x = 50$.

$$y = -\frac{1}{500}(50 - 100)^2 + 20$$

$$= -\frac{1}{500}(-50)^2 + 20$$

$$= -5 + 20$$

$$y = 15 \text{ metres}$$

Note: the trajectories of objects through air in reality are not perfectly symmetrical parabolas due to spin and air resistance. They are often assumed to be parabolic to make the mathematics simpler.

Assume that all the trajectories in the following problems are parabolic.

EXERCISE 10G

1. A golf ball is hit and travels 100 metres before it hits the ground. It is estimated that the maximum height reached by the ball is 25 metres.
 - (a) Sketch the trajectory of the ball on coordinate axes using the origin as the point the ball was hit.
 - (b) Find the equation of this parabola.
 - (c) Find the height of the ball 20 metres from where it was hit.
2. A footballer kicks the ball from 20 metres out from the goal. The ball just clears the 12 metre high goal post at the top of its trajectory.
 - (a) Sketch the trajectory of the ball on coordinate axes using the origin as the point the ball was kicked.
 - (b) Find the equation of this parabola.
 - (c) Find the height of the ball 5 metres from where it was kicked.

3. The trajectory of two hits in cricket were analysed and the equation for each trajectory is shown below.

Hit **A**: $y = -\frac{2}{125} (x - 50)^2 + 40$

Hit **B**: $y = -\frac{1}{120} (x - 60)^2 + 30$

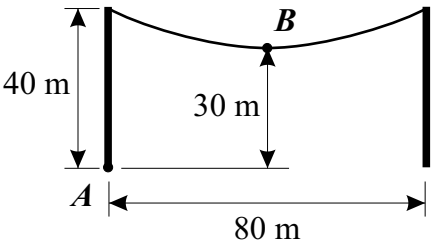
For each hit find:

- (a) the distance travelled by the ball
- (b) the maximum height reached by the ball
- (c) the height of the ball above a fielder standing 30 metres from the batter.



4. A cable hangs between two pylons as shown in this diagram. The pylons are 40 metres high and 80 metres apart. The lowest point of the cable is 30 metres above the ground.

- (a) Using point **A** as the origin, find the equation of the parabolic shape of the cable.
- (b) Copy and complete the table below that states the height of the cable every 10 metres from point **A**. Give answers correct to one decimal place.



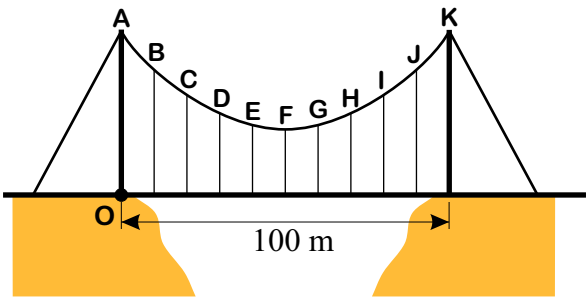
Distance from A (m)	0	10	20	30	40	50	60	70	80
Height above ground (m)									

5. The equation for the parabolic cable on this bridge is:

$$y = \frac{1}{100} (x - 50)^2 + 25$$

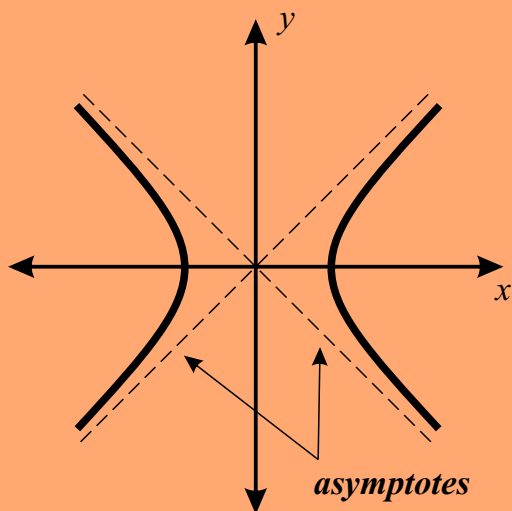
Point **O** is the origin.

- (a) Find the height of the pylons **A** and **K**.
- (b) Find the lengths of the equally spaced supporting cables (**B - J**).



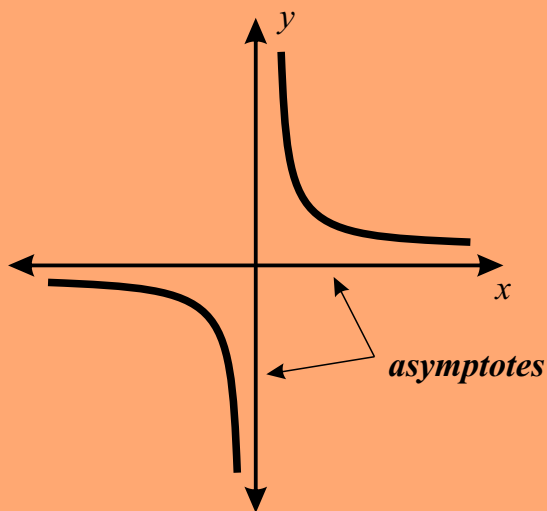
Rectangular Hyperbolas

Hyperbolas (hyperbolae) are a family of curves that approach but never reach lines called asymptotes. An example of a hyperbola is shown below. The asymptotes are shown by dotted lines.



The mathematics of the general shape of these curves will be studied in detail in later year levels.

We will study now hyperbolas that have asymptotes that are the x -axis and y -axis. They are called **rectangular hyperbolas** because the angle between the asymptotes is 90° .



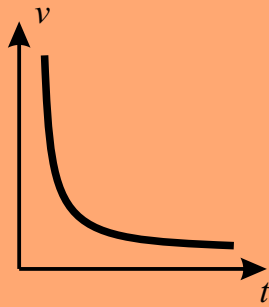
The general relationship for a rectangular hyperbola is:

$$xy = a \quad \text{or} \quad y = \frac{a}{x}$$

This is the relationship in many practical situations. In practical situations only the positive part of the graph applies.

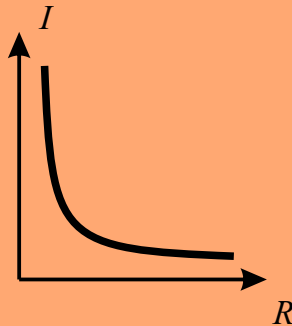
Examples

The relationship between velocity (v) and time (t) for an object travelling over a set distance (d).



$$v = \frac{d}{t}$$

The relationship between current (I) and resistance (R) is a simple electrical circuit where there is a set voltage (V).



$$I = \frac{V}{R}$$

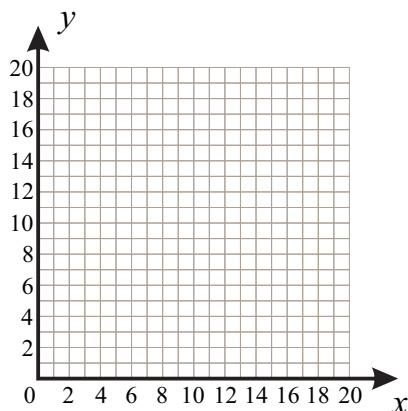
EXERCISE 10H

1. (a) Copy and complete the table of values for following relationship.

$$y = \frac{20}{x}$$

x	1	2	4	5	10	20
y						

- (b) Copy and complete this graph by plotting the points from the above table. Connect the points with a smooth curve.



2. The tables of values below is completed for the given equation.

$$y = \frac{a}{x}$$

x	1	2	4	5	10	20	50
y	100	50	25	20	10	5	2

- (a) Find a .
 (b) Find y for the following values of x .

(i) 25 (ii) 200 (iii) 0.5 (iv) $\frac{1}{4}$

3. The tables of values below is completed for the given equation where P is a constant.

$$M = \frac{P}{Q}$$

Q	0.01	0.5	0.1	1	10	50	100
M	250	5	25	2.5	0.25	0.05	0.025

- (a) Find P .
 (b) Find M for the following values of Q .

(i) 50 (ii) 0.5 (iii) 0.1 (iv) 0.025

4. The current (I), in amps, is measured in an electric circuit for different resistances (R), measured in ohms, and shown in the table below.

I	1	2	4	6	8	12	24
R	24	12	6	4	3	2	1

- (a) The relationship between current and resistance is known to be of the form shown here.

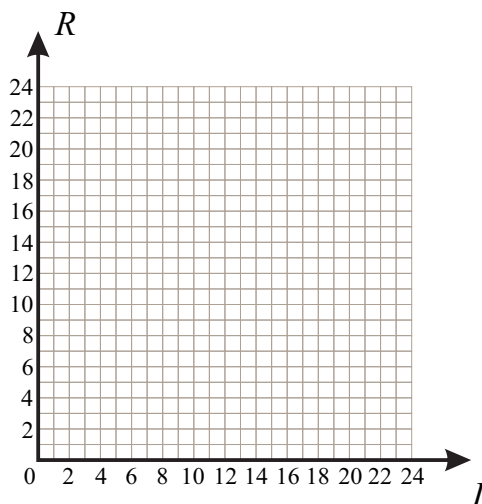
Find V .

$$R = \frac{V}{I}$$

- (b) Find R for the following currents:

(i) 0.1 (ii) 0.2 (iii) 1.2 (iv) 48 (v) 240

- (c) Copy and complete this graph by plotting the points from the above table. Connect the points with a smooth curve.

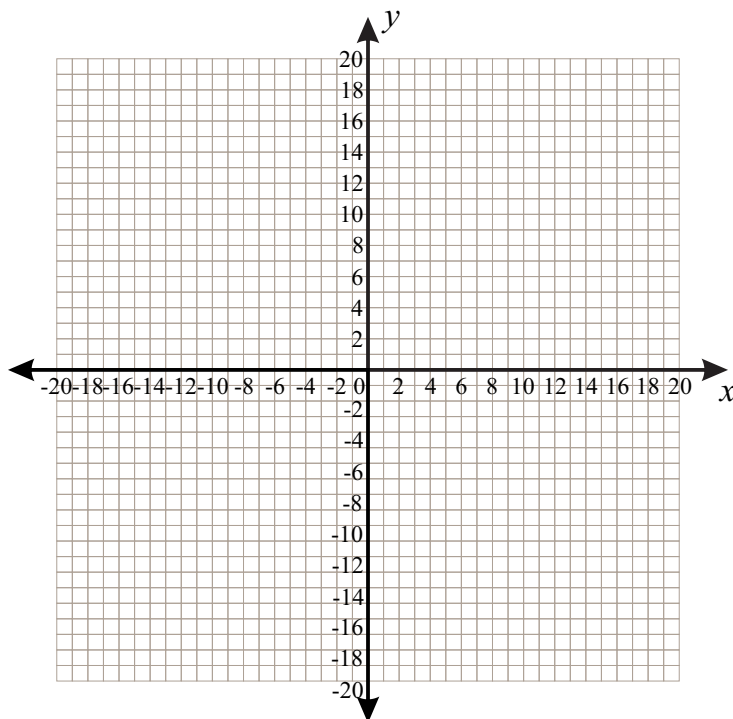


5. (a) Copy and complete the table of values below for this relationship.

$$y = \frac{10}{x}$$

x	-20	-10	-5	-2	-1	-0.5	0.5	1	2	5	10	20
y												

- (b) Copy and complete the graph below by plotting the points from the above table. Connect the points with smooth curves.



6. (a) Copy and complete the table of values below for this relationship.

$$y = \frac{20}{x}$$

x	-20	-10	-5	-4	-2	-1	1	2	4	5	10	20
y												

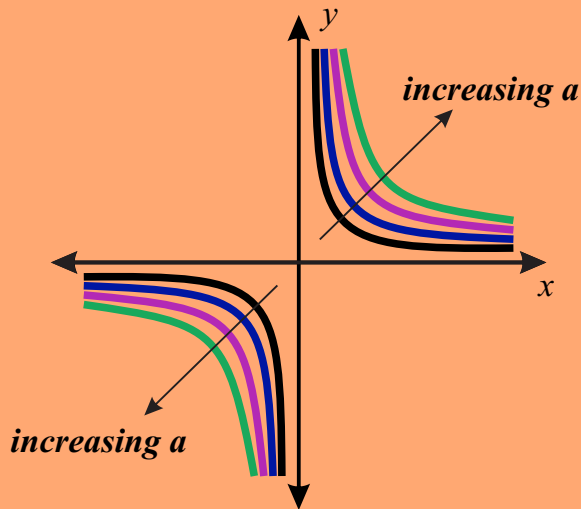
- (b) Plot these points on the same graph as in question 5 and connect with smooth curves.

7. (a) Copy the table of values from question 5 and complete for this relationship.

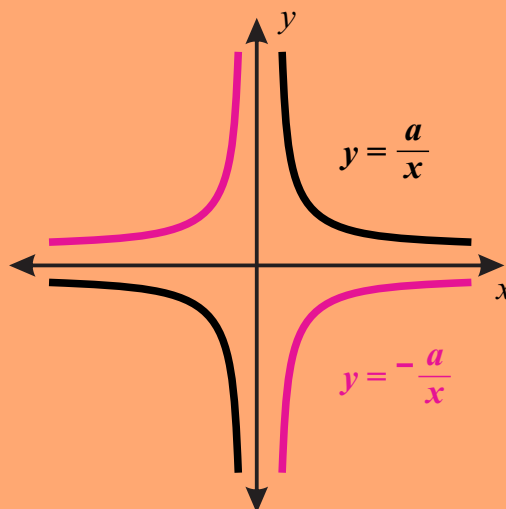
$$y = \frac{-10}{x}$$

- (b) Plot on the same graph and connect with smooth curves.

As seen in questions 5 & 6, changing the value of a in the general equation of a hyperbola $y = \frac{a}{x}$ changes the shape of the curves. The diagram below shows the effect of increasing the value of a on the curve.



It can also be seen from question 7 that a negative sign reflects the hyperbola about the axes.



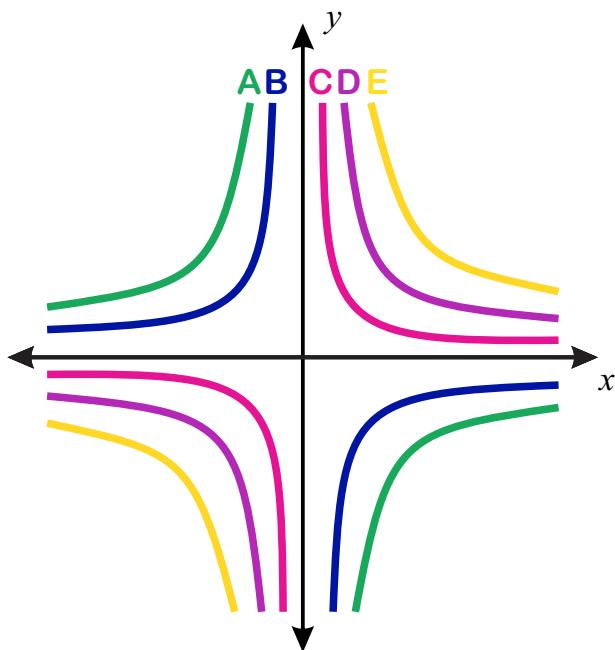
EXERCISE 10I

1. Match the graphs below with the following relationships.

(a) $y = \frac{a}{x}$ (b) $y = -\frac{2a}{x}$

(c) $y = \frac{a}{3x}$ (d) $y = \frac{5a}{x}$

(e) $y = -\frac{a}{2x}$



Note:

$$y = \frac{\frac{1}{3}a}{x}$$

can be written as

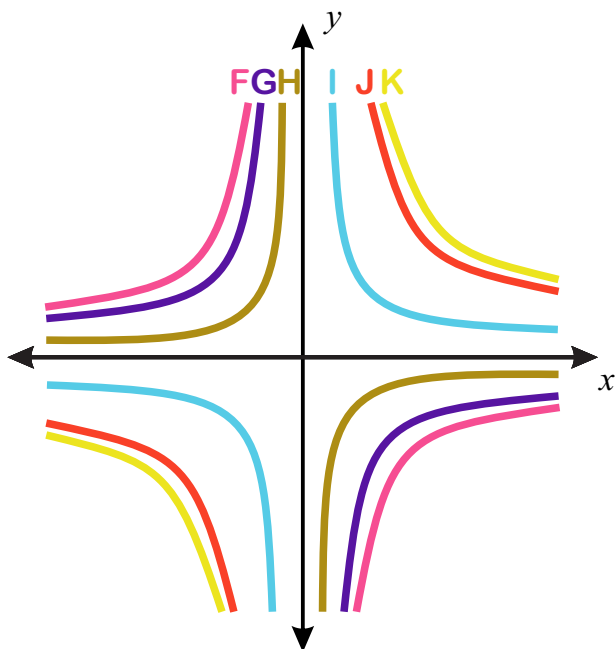
$$y = \frac{a}{3x}$$

2. Match the graphs below with the following relationships.

(a) $y = \frac{5}{x}$ (b) $y = -\frac{3}{x}$

(c) $y = \frac{1}{4x}$ (d) $y = \frac{7}{x}$

(e) $y = -\frac{1}{6x}$ (f) $y = -\frac{1}{2x}$

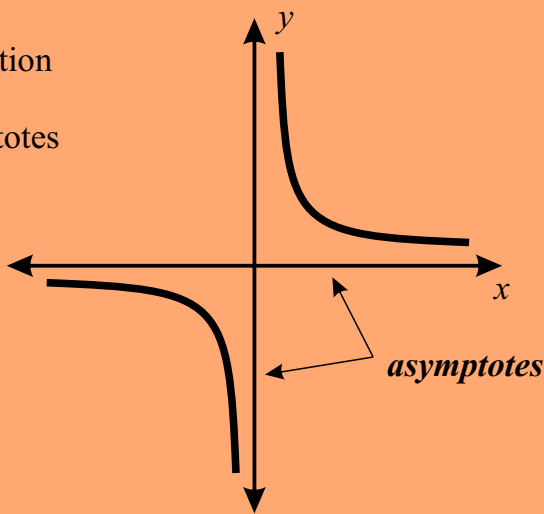


Transforming Hyperbolas

So far we have only considered hyperbolas that have asymptotes that intersect at the origin.

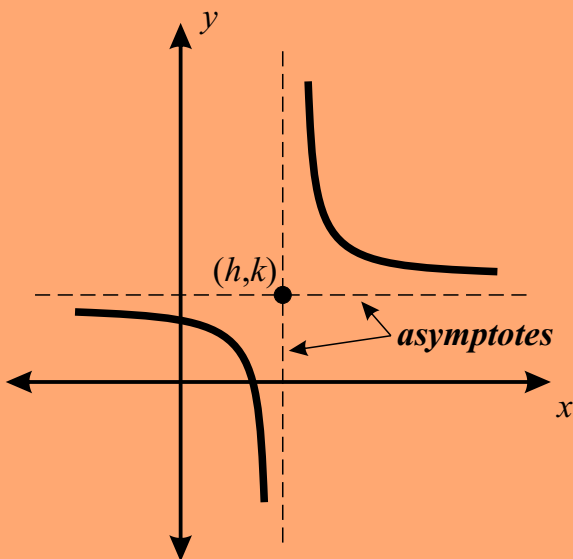
As noted earlier, the equation of a hyperbola with the intersection of the asymptotes at the origin is:

$$y = \frac{a}{x}$$



If a hyperbola is moved so that the intersection of the asymptotes is at the point (h,k) then the equation of the hyperbola will be:

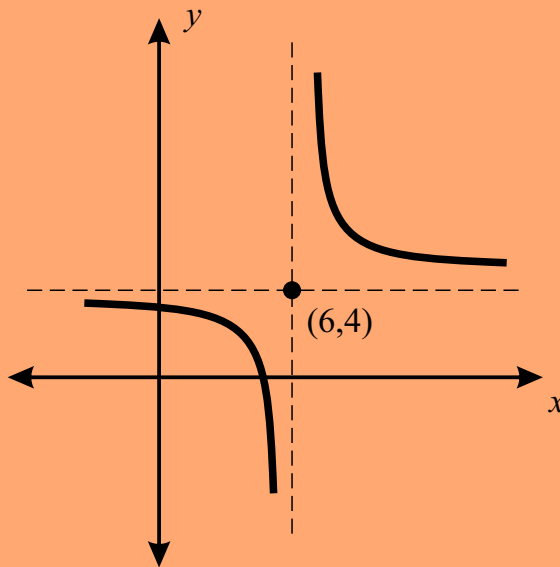
$$y = \frac{a}{(x - h)} + k$$



Example 1

The hyperbola with equation $y = \frac{2}{x}$ has been moved so that the point of intersection of the asymptotes is (6,4) as shown on the graph below.

Write the equation of the hyperbola.

**Answer**

Substitute the values for h and k into the equation of the hyperbola.

$$y = \frac{a}{(x - h)} + k$$

$$h = 6$$

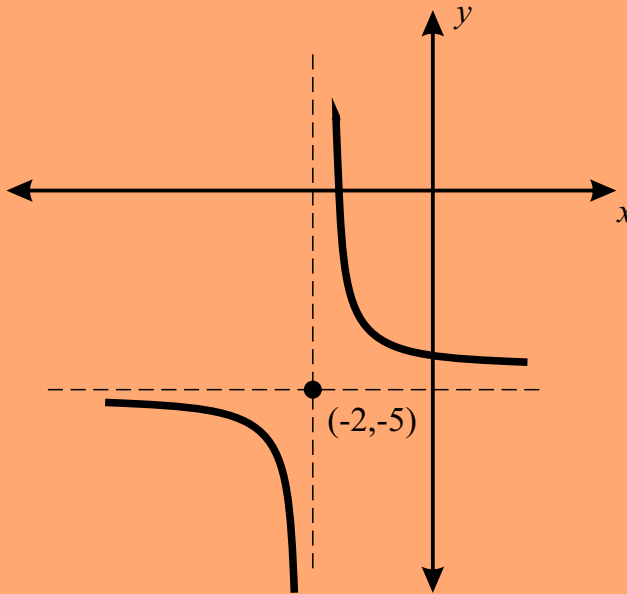
$$k = 4$$

$$y = \frac{2}{(x - 6)} + 4$$

Example 2

The hyperbola with equation $y = \frac{3}{x}$ has been moved so that the point of intersection of the asymptotes is $(-2, -5)$ as shown on the graph below.

Write the equation of the hyperbola.

**Answer**

Substitute the values for h and k into the equation of the hyperbola.

$$y = \frac{a}{(x - h)} + k$$

$$h = -2$$

$$k = -5$$

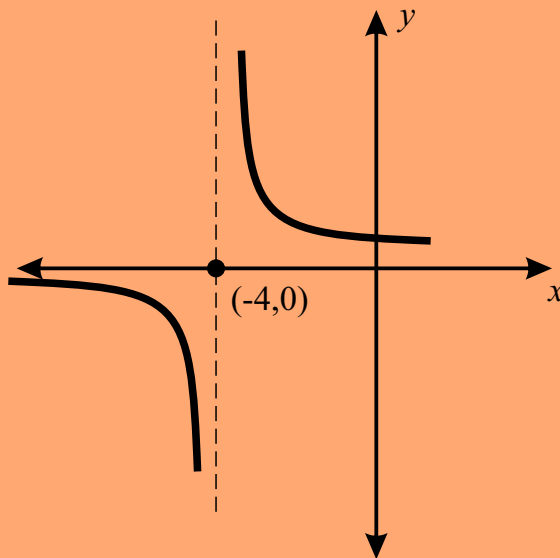
$$y = \frac{3}{(x - -2)} - 5$$

$$y = \frac{3}{(x + 2)} - 5$$

Example 3

The hyperbola with equation $y = \frac{1}{x}$ has been moved so that the point of intersection of the asymptotes is $(-4,0)$ as shown on the graph below.

Write the equation of the hyperbola.

**Answer**

Substitute the values for h and k into the equation of the hyperbola.

$$y = \frac{a}{(x - h)} + k$$

$$h = -4$$

$$k = 0$$

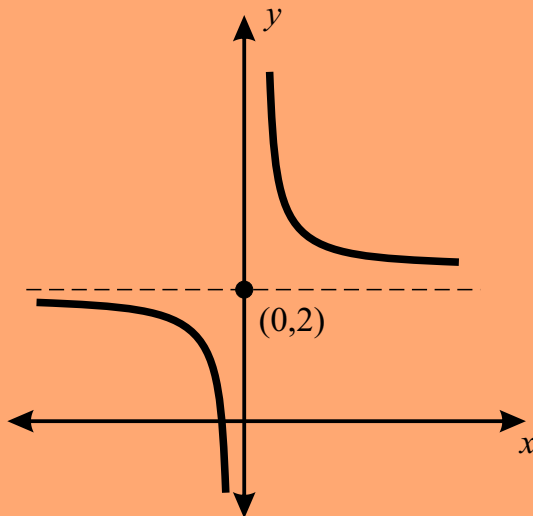
$$y = \frac{1}{(x - -4)} + 0$$

$$y = \frac{1}{(x + 4)}$$

Example 4

The hyperbola with equation $y = \frac{4}{x}$ has been moved so that the point of intersection of the asymptotes is $(0,2)$ as shown on the graph below.

Write the equation of the hyperbola.

**Answer**

Substitute the values for h and k into the equation of the hyperbola.

$$y = \frac{a}{(x - h)} + k$$

$$h = 0$$

$$k = 2$$

$$y = \frac{4}{(x - 0)} + 2$$

$$y = \frac{4}{x} + 2$$

EXERCISE 10J

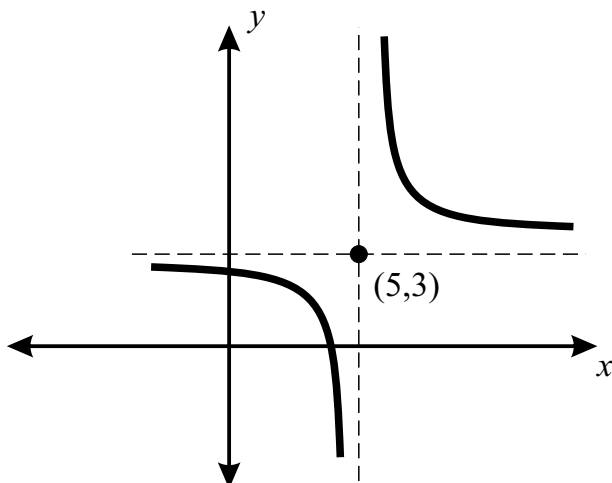
1. Which of the equations below (**A - D**) could be the equation of the hyperbola shown.

A $y = \frac{2}{(x+5)} + 3$

B $y = \frac{2}{(x-3)} + 5$

C $y = \frac{2}{(x-5)} - 3$

D $y = \frac{2}{(x-5)} + 3$



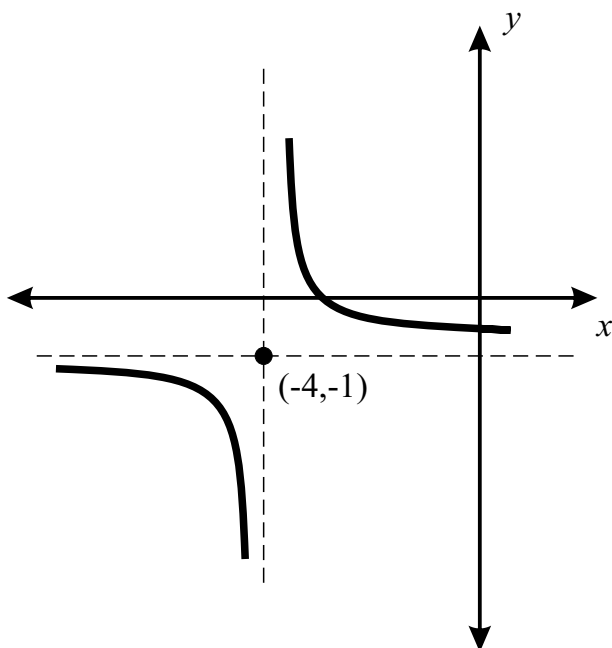
2. Which of the equations below (**A - D**) could be the equation of the hyperbola shown.

A $y = \frac{3}{(x-4)} + 1$

B $y = \frac{3}{(x+1)} - 4$

C $y = \frac{3}{(x+4)} - 1$

D $y = \frac{3}{(x+1)} - 4$



3. Which of the following equations (**A - D**) could be the equation of a hyperbola that has the intersection of its asymptotes at the point (5,-2).

A $y = \frac{1}{(x-5)} + 2$

B $y = \frac{2}{(x-2)} + 5$

C $y = \frac{3}{(x-5)} - 2$

D $y = \frac{4}{(x+5)} - 2$

4. Which of the following equations (**A - D**) could be the equation of a hyperbola that has the intersection of its asymptotes at the point (0,4).

A $y = \frac{5}{(x-4)}$

B $y = \frac{6}{x} + 4$

C $y = \frac{1}{x} - 4$

D $y = \frac{2}{(x+4)}$

5. The hyperbola with equation $y = \frac{1}{x}$ is transposed so that the intersection of its asymptotes is at the points with the following coordinates. Write the equation of the transposed hyperbola for each point.

(a) (3,7) (b) (-5,0) (c) (-1,-5) (d) (0,8) (e) (6,-5)

6. Write the coordinates of the point where the asymptotes intersect for the hyperbolas with following equations and sketch each hyperbola.

(a) $y = \frac{3}{(x-9)} - 1$

(b) $y = \frac{5}{(x+2)} + 7$

(c) $y = \frac{1}{(x-8)}$

(d) $y = \frac{1}{x} + 4$

(e) $y = \frac{3}{(x+1)} - 6$

(f) $y = \frac{9}{(x+8)} - 5$

(g) $y = \frac{12}{(x+2)}$

(h) $y = \frac{6}{x} - 5$

Circles

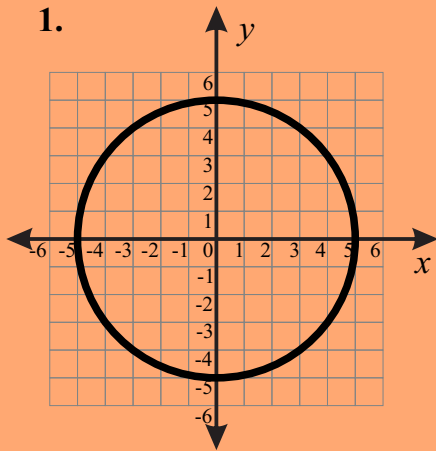
The equation of a circle with radius r and centre at the origin of coordinate axes is:

$$x^2 + y^2 = r^2$$

If the centre of the circle is at coordinates (h,k) then the equation of the circle is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Examples

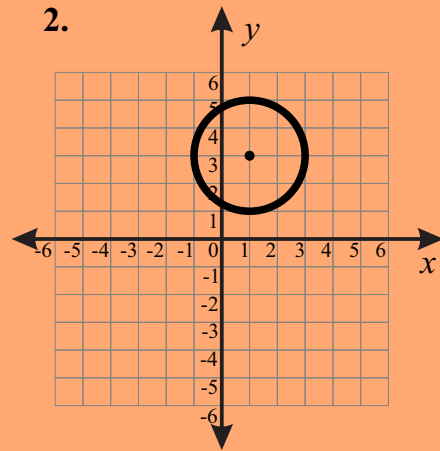


Centre at the origin

Radius = 5

$$x^2 + y^2 = 5^2$$

$$x^2 + y^2 = 25$$



Centre at (1,3)

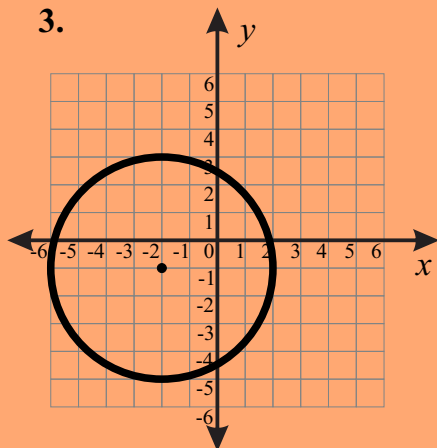
$h = 1, k = 3$

Radius = 2

$$(x - 1)^2 + (y - 3)^2 = 2^2$$

$$(x - 1)^2 + (y - 3)^2 = 4$$

3.

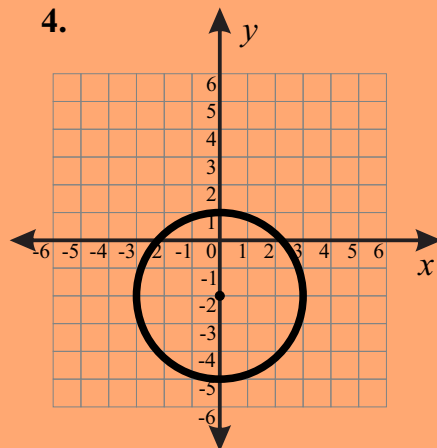
Centre at $(-2, -1)$ $h = -2, k = -1$

Radius = 4

$$(x - -2)^2 + (y - -1)^2 = 4^2$$

$$(x + 2)^2 + (y + 1)^2 = 16$$

4.

Centre at $(0, -2)$ $h = 0, k = -2$

Radius = 3

$$(x - 0)^2 + (y - -2)^2 = 3^2$$

$$x^2 + (y + 2)^2 = 9$$

EXERCISE 10K

1. Write the equation of the circles with the following centres and radii.

(a) Centre $(0, 0)$, radius = 5(b) Centre $(0, 3)$, radius = 4(c) Centre $(4, 0)$, radius = 1(d) Centre $(-5, 0)$, radius = 3(e) Centre $(2, 4)$, radius = 6(f) Centre $(-3, 5)$, radius = 4(g) Centre $(6, -5)$, radius = 7(h) Centre $(7, 9)$, radius = 12(i) Centre $(-5, -8)$, radius = 10(j) Centre $(-6, -1)$, radius = 9

2. Write the coordinates of the centre and the radius of the circles with the following equations.

(a) $x^2 + y^2 = 64$

(b) $(x - 7)^2 + y^2 = 1$

(c) $(x + 3)^2 + (y - 5)^2 = 49$

(d) $x^2 + (y - 2)^2 = 81$

(e) $(x - 6)^2 + (y - 1)^2 = 36$

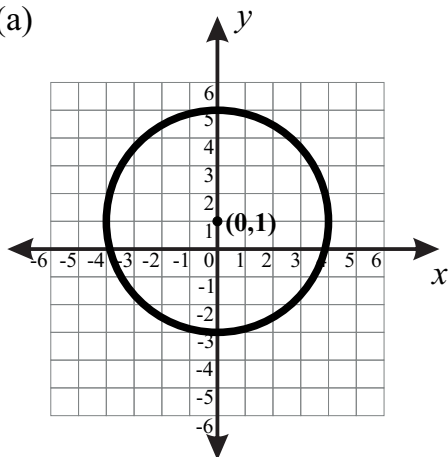
(f) $(x + 2)^2 + (y + 9)^2 = 100$

(g) $(x - 12)^2 + (y + 15)^2 = 144$

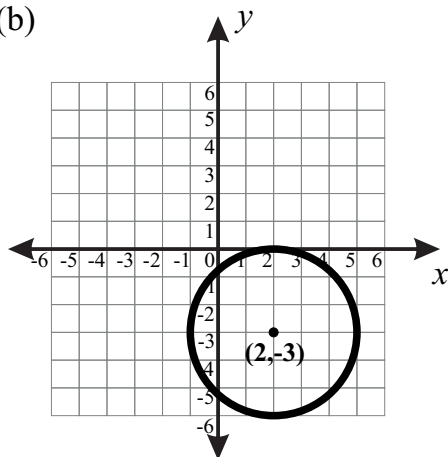
(h) $(x + 13)^2 + (y - 11)^2 = 169$

3. Write the equation of the circles shown below.

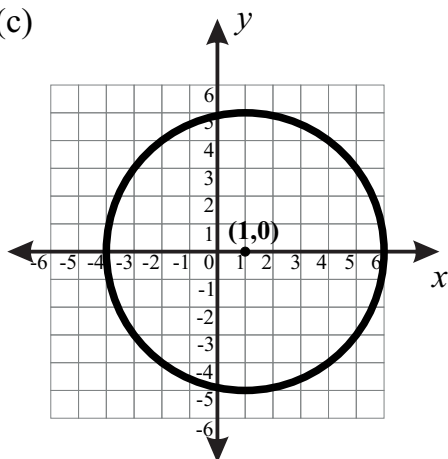
(a)



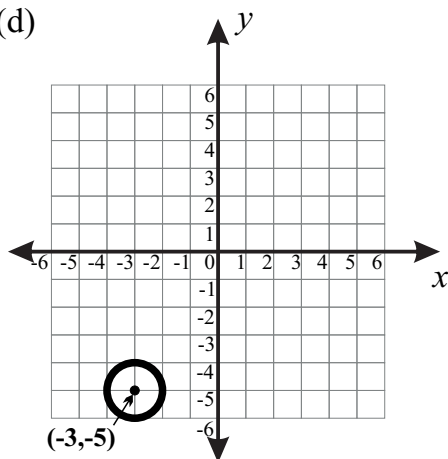
(b)



(c)



(d)



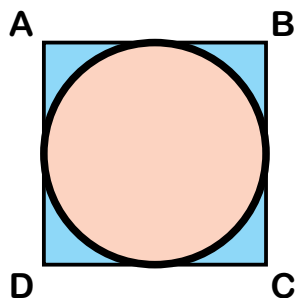
4. Find the equation to a circle that just fits in the square **ABCD**. The coordinates of the corners of the square are:

A = (-2, 4)

B = (8, 4)

C = (8, -6)

D = (-2, -6)



Sometimes the equation of a circle can be given in the form shown in the example below.

$$x^2 + 4x + y^2 - 6y = 14$$

The equation needs to be changed to find the coordinates of the centre and the radius. This is achieved by completing the square as shown on page 156.

Using this example follow the steps shown below.

Step 1 Complete the square for x .

$$x^2 + 4x + y^2 - 6y = 14$$

$$x^2 + 4x + 4 + y^2 - 6y = 14 + 4$$

$$(x + 2)^2 + y^2 - 6y = 18$$

Remember if a number is added to one side of the equation the same number needs to be added to the other side for the two sides of the equation to remain equal.

Step 2 Complete the square for y .

$$(x + 2)^2 + y^2 - 6y + 9 = 18 + 9$$

$$(x + 2)^2 + (y - 3)^2 = 27$$

Step 3 Write the coordinates of the centre of the circle and its radius.

Coordinates of centre = (-2,3)

Radius = $\sqrt{27}$

EXERCISE 10L

Find the coordinates of the centre and the radius of the circles with the following equations.

1. $x^2 - 2x + y^2 - 10y = 10$

2. $x^2 - 6x + y^2 + 20y = -9$

3. $x^2 + y^2 - 12y = -35$

4. $x^2 + 8x + y^2 = 0$

5. $x^2 - 12x + y^2 + 6y = 4$

6. $x^2 - 24x + y^2 + 10y = 0$

7. $x^2 + 14x + y^2 - 16y = -13$

8. $x^2 - 20x + y^2 - 22y = 4$

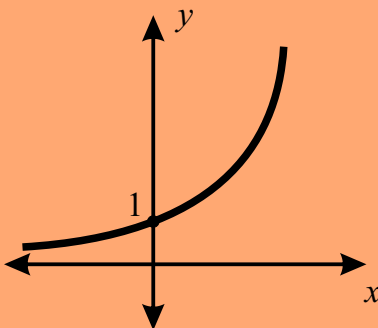
Exponentials

Practical applications of exponential graphs have been studied in detail on pages 95-100.

The basic mathematical equation for an exponential relationship is:

$$y = b^x$$

The shape of the graph of this relationship is:



Key features of this graph are:

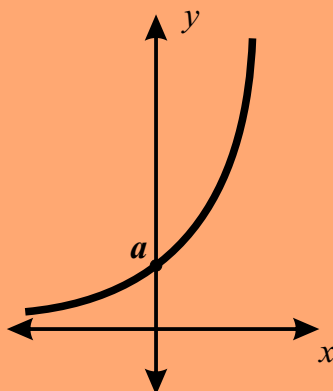
- the y -intercept is 1
- the x -axis is an asymptote

This basic graph can undergo one or more transformations that will change the position and/or the shape of the graph.

1. Multiplied by a constant a .

$$y = a \times b^x$$

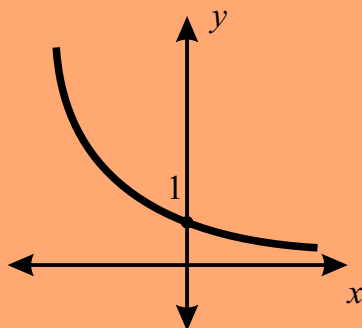
The graph is moved up and stretched.



2. Negative x

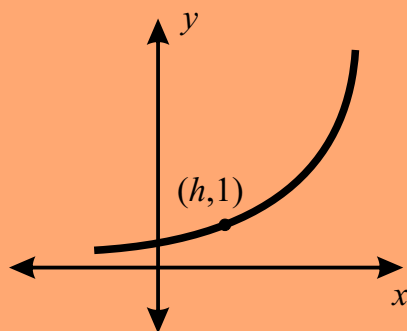
$$y = b^{-x}$$

The graph is reflected about the y -axis.

3. Translated right or left by h .

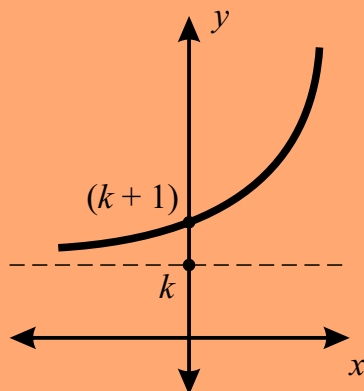
$$y = b^{(x-h)}$$

The point $(h, 1)$ is a point on the graph.
The x -axis is still the asymptote.

4. Translated up or down by k .

$$y = b^x + k$$

$y = k$ is the asymptote



EXERCISE 10M

1. (a) Copy and complete the table of values below for the following relationship.

$$y = 2^x$$

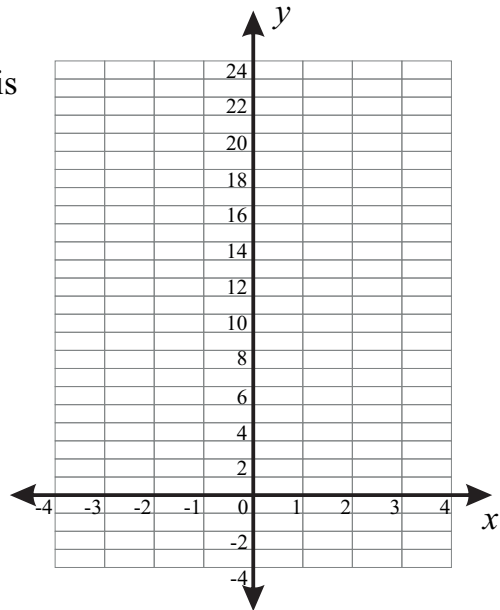
x	-4	-3	-2	-1	0	1	2	3	4
y									

- (b) Draw a set of axes with the x -axis from -4 to 4 and the y -axis from -4 to 24 as shown here.

- (c) Plot the points from the table of values above and connect with a smooth curve.

- (d) Copy the table of values above and complete for the following relationship.

$$y = 2^{-x}$$



- (e) Plot the points from this table on the same set of axes.
- (f) Copy the table of values below and complete for the following relationship.

$$y = 3 \times 2^x$$

x	-4	-3	-2	-1	0	1	2	3
y								

- (g) Plot the points from this table on the same set of axes.

(h) Copy and complete the table of values below for the following relationship.

$y = 2^{(x - 3)}$

x	0	1	2	3	4
y					

- (i) Plot the points from this table on the same set of axes.
(j) On the same axes plot the following relationships.

$y = 2^x + 4$

$y = 2^x - 4$

2. The red line on the graph below is the graph of $y = 3^x$.
Match the other lines with the following relationships.

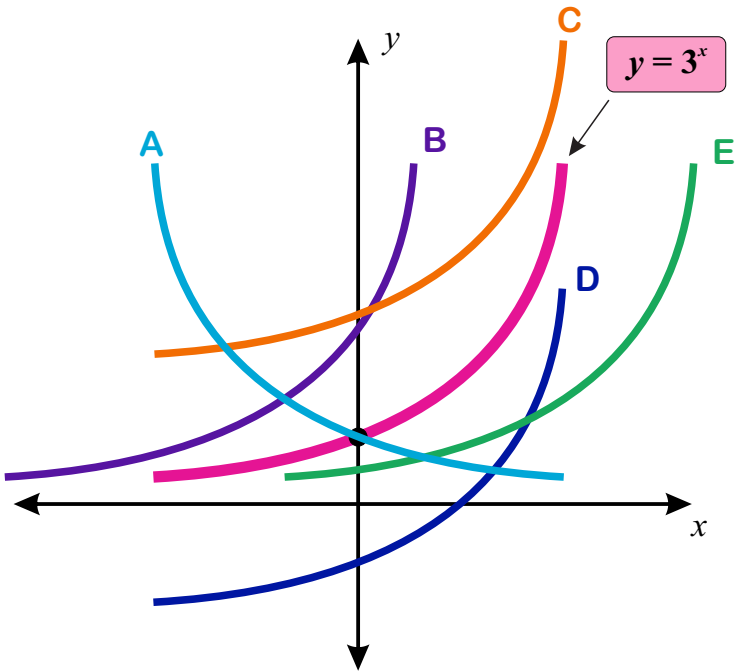
$y = 3^{(x + 2)}$

$y = 3^{(x - 2)}$

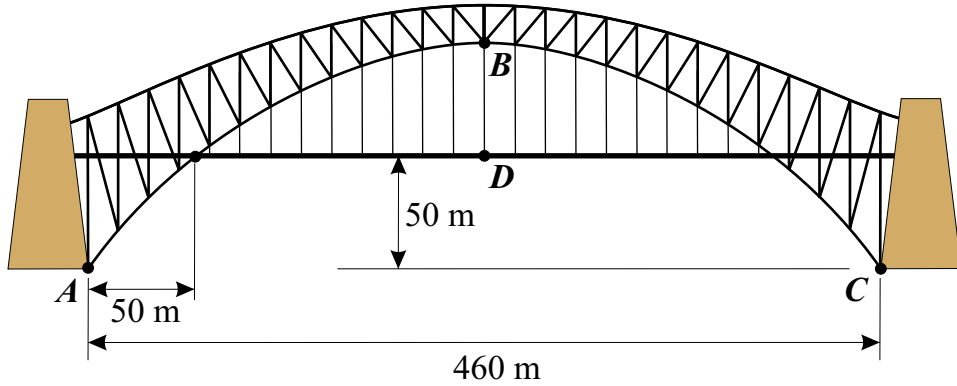
$y = 3^x + 2$

$y = 3^{-x}$

$y = 3^x - 2$



PROBLEM SOLVING



A drawing of the Sydney Harbour Bridge is shown above. The dimensions given are approximate.

1. Use the dimensions shown to write two equations that could be used to find the equation of the parabola ABC . (Use point A as the origin)
2. Solve these simultaneous equations to find the equation of the parabola ABC .
3. Find the length of cable BD . Give answer to the nearest metre.

PUZZLE

The letters P , A , R , B , O and L represent different numbers (all greater than zero). Use the information below to find the numeral value of each letter.

$$P + A + R + A + B + O + L + A = 23$$

$$R + B = P$$

$$A + B = L$$

$$L = 2R$$

CHAPTER REVIEW

1. For each of the following equations, complete the table of values, plot the points on a graph and connect the points with a smooth curve.

(a) $y = x^2 - 4x + 3$

(b) $y = -x^2 - 2x + 1$

x	-1	0	1	2	3	4	5
y							

x	-4	-3	-2	-1	0	1	2
y							

- 2. (a)** Without plotting points, sketch a graph of $y = x^2$.

- (b) On the same graph sketch the following curves:

(i) $y = 2x^2$ (ii) $y = -x^2$ (iii) $y = \frac{1}{2}x^2$ (iv) $y = -2x^2$

3. (a) List which of the following graphs would have a *minimum* turning point.

- (b) List which of the following graphs would have a **maximum** turning point.

A $y = 2x^2 + 3$ **B** $y = -5x^2$ **C** $y = \frac{1}{2}x^2 - 1$ **D** $y = \frac{1}{2}x^2 - 3x + 2$

- 4. Sketch the following parabolas.**

- (a) This parabola has - x -intercepts at -3 and 3
- y -intercept at 2

- (b) This parabola has - its turning point at $(1, 2)$
- y -intercept at 3

- (c) This parabola has - its axis of symmetry at $x = -2$
 - one x -intercept at -5
 - y -intercept at -1

5. Find the x -intercepts, y -intercept and turning point for each of the following parabolas.

(a) $y = x^2 - 8x + 15$

(b) $y = x^2 - 16$

(c) $y = x^2 + 6x - 16$

(d) $y = x^2 - 8x$

6. Find the coordinates of the turning point for the following parabolas.

(a) $y = x^2 + 10x - 9$

(b) $y = x^2 - 5x + 8$

(c) $y = -2x^2 + 12x - 16$

(d) $y = -4x^2 - 6x + 13$

7. Sketch the following parabolas by locating the turning points and y -intercept of each.

(a) $y = 3(x - 5)^2 + 2$

(b) $y = -2(x + 3)^2 - 5$

8. A cricket ball is struck and travels 80 metres before it hits the ground. It is estimated that it reached a maximum height of 25 metres.

(a) Sketch the trajectory of the ball on coordinate axes using the origin as the point the ball was struck.

(b) Find the equation of this parabola.

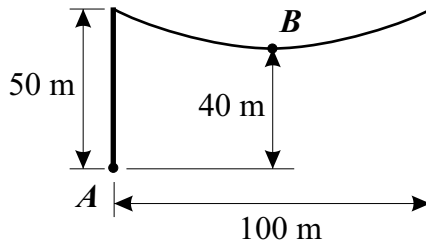
(c) Find the height of the ball 10 metres from where it landed. Give answer in metres correct to one decimal place.

9. A cable hangs between two posts as shown in this diagram.

The posts are 50 metres high and 100 metres apart. The lowest point of the cable is 40 metres above the ground.

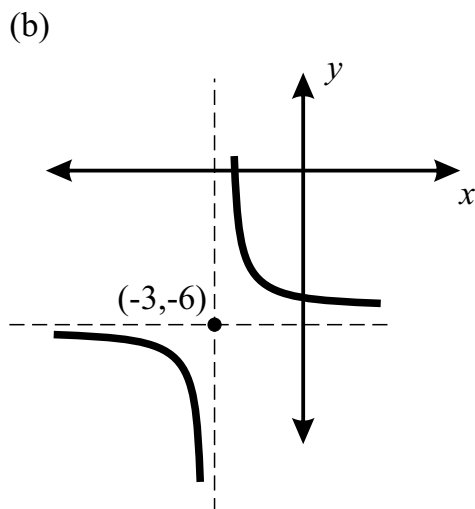
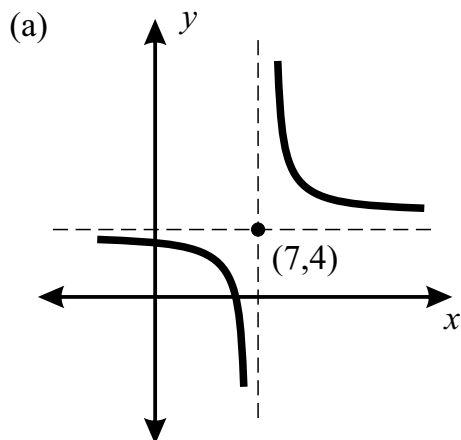
(a) Using point A as the origin, find the equation of the parabolic shape of the cable.

(b) Copy and complete the table below that states the height of the cable every 20 metres from point A.



Distance from A (m)	0	20	40	60	80	100
Height above ground (m)						

10. The rectangular hyperbola with equation $y = \frac{5}{x}$ has been moved and redrawn as shown below. Write the equation for each graph.



11. The hyperbola with equation $y = \frac{3}{x}$ is transposed so that the intersection of the asymptotes is at the point $(-6, 0)$. Write the equation of the transposed hyperbola.

12. Write the coordinates of the point where the asymptotes intersect for the following hyperbolas.

(a) $y = \frac{3}{(x - 5)} + 4$

(b) $y = \frac{6}{x} - 2$

13. Write the equation of the circles with the following centres and radii.

(a) Centre $(2, 0)$, radius = 6

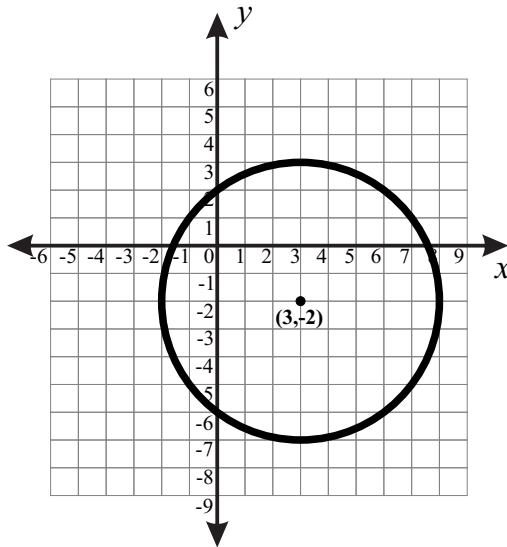
(b) Centre $(7, -5)$, radius = 8

14. Write the coordinates of the centre and the radius of the circles with the following equations.

(a) $(x + 3)^2 + (y - 7)^2 = 81$

(b) $x^2 + (y + 4)^2 = 1$

15. Write the equation of the circle shown on the graph below.



16. The red line on the graph below is the graph of $y = 4^x$.
Match the other lines with the following relationships.

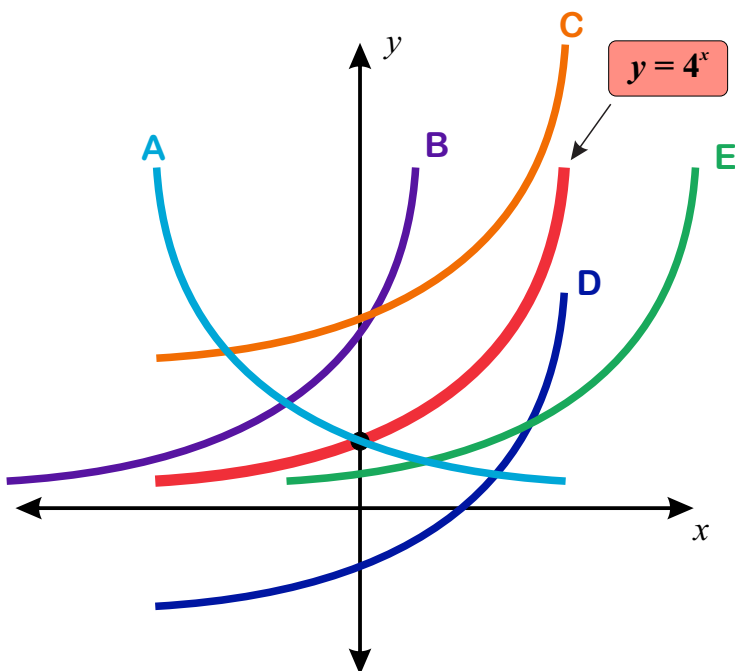
$$y = 4^{(x-3)}$$

$$y = 4^{-x}$$

$$y = 4^{(x+3)}$$

$$y = 4^x - 3$$

$$y = 4^x + 3$$



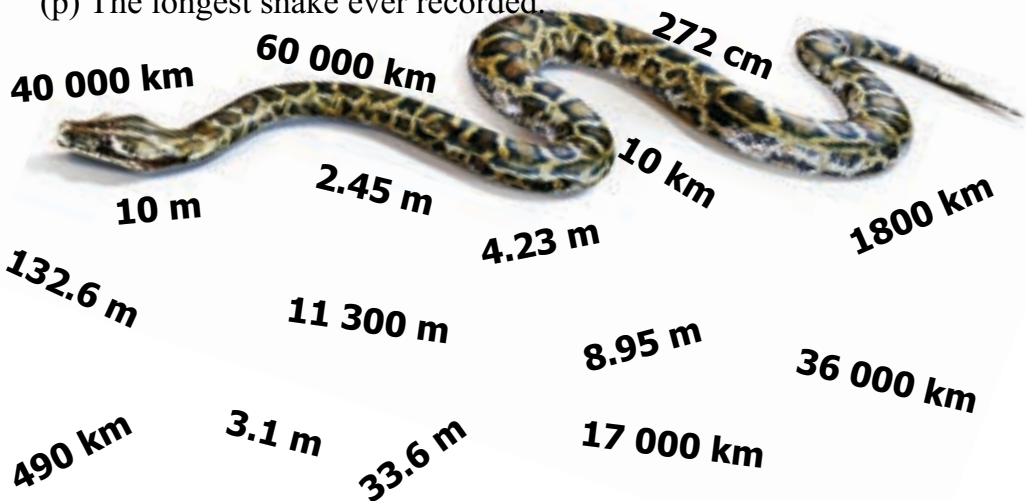
Measurement

Length

EXERCISE 11A

1. From the list below find the length of the following objects.

- (a) The distance from Sydney to London.
- (b) The height of a basketball ring above the court.
- (c) The circumference of the Earth.
- (d) The longest distance travelled by a hang glider.
- (e) The altitude at which satellites orbit.
- (f) The record height jumped by a high-jumper.
- (g) The tallest person ever recorded.
- (h) The length of Australia's coastline (including islands).
- (i) The record distance jumped by a long-jumper.
- (j) The height of the tallest tree ever recorded.
- (k) The highest altitude reached by a bird.
- (l) The length of the longest human hair ever recorded.
- (m) The distance travelled by the Earth in one minute.
- (n) The longest blue whale ever recorded.
- (o) The cruising altitude of a Boeing 747.
- (p) The longest snake ever recorded.



The standard unit of length is the ***metre***.

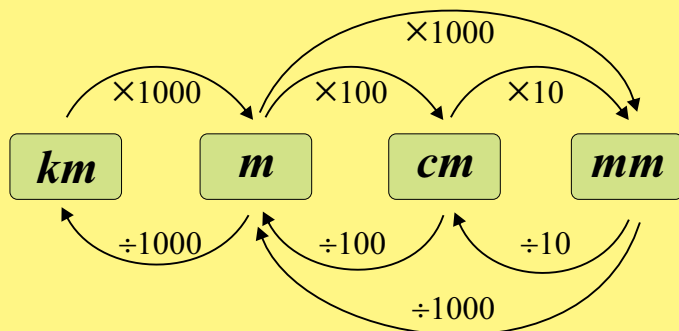
The abbreviation for metre is ***m***.

Other common units used are either multiples or fractions of a metre.

The most common are ***kilometres***, ***centimetres*** and ***millimetres***.

1 kilometre (km) = 1000 metres
1 metre = 100 centimetres (cm)
1 metre = 1000 millimetres (mm)

This diagram can be used to find the conversion factor when converting from one unit to another.



2. Change the following lengths to the units shown in the brackets.

Examples

$$\begin{aligned} 1. \quad & 3.45 \text{ km (m)} \\ & = 3.45 \times 1000 \\ & = \mathbf{3450 \text{ m}} \end{aligned}$$

$$\begin{aligned} 2. \quad & 26.785 \text{ m (mm)} \\ & = 26.785 \times 1000 \\ & = \mathbf{26\,785 \text{ mm}} \end{aligned}$$

$$\begin{aligned} 3. \quad & 9.705 \text{ m (cm)} \\ & = 9.705 \times 100 \\ & = \mathbf{970.5 \text{ cm}} \end{aligned}$$

$$\begin{aligned} 4. \quad & 900 \text{ m (km)} \\ & = 900 \div 1000 \\ & = \mathbf{0.9 \text{ km}} \end{aligned}$$

$$\begin{aligned} 5. \quad & 0.8 \text{ mm (cm)} \\ & = 0.8 \div 10 \\ & = \mathbf{0.08 \text{ cm}} \end{aligned}$$

$$\begin{aligned} 6. \quad & 138.5 \text{ cm (m)} \\ & = 138.5 \div 100 \\ & = \mathbf{1.385 \text{ m}} \end{aligned}$$

- | | | |
|--------------------|-------------------|--------------------|
| (a) 6 cm (mm) | (b) 8 m (cm) | (c) 20 km (m) |
| (d) 2.36 m (mm) | (e) 42.85 cm (mm) | (f) 0.68 km (m) |
| (g) 750 mm (cm) | (h) 75 cm (m) | (i) 76 850 m (km) |
| (j) 0.1 cm (m) | (k) 7.98 m (cm) | (l) 27.6 mm (m) |
| (m) 0.003 m (cm) | (n) 2.357 cm (mm) | (o) 0.5781 km (m) |
| (p) 0.3 km (cm) | (q) 0.43 km (mm) | (r) 0.0017 km (m) |
| (s) 0.0065 cm (mm) | (t) 4.76 cm (m) | (u) 0.006 mm (cm) |
| (v) 6.05 mm (m) | (w) 60 m (km) | (x) 56 450 mm (km) |

3. Change the following lengths to the units shown in the brackets.

Examples

$$\begin{aligned} 1. \quad & 3 \text{ m } 2 \text{ cm (m)} \\ & = \mathbf{3.02 \text{ m}} \end{aligned}$$

$$\begin{aligned} 2. \quad & 2 \text{ m } 18 \text{ mm (m)} \\ & = \mathbf{2.018 \text{ m}} \end{aligned}$$

- | | |
|------------------------|------------------------|
| (a) 2 m 11 cm (m) | (b) 3 cm 2 mm (cm) |
| (c) 6 m 6 cm (m) | (d) 20 m 20 cm (m) |
| (e) 5 km 20 m (km) | (f) 18 m 5 cm (m) |
| (g) 5 m 8 cm (m) | (h) 6 m 23 cm 8 mm (m) |
| (i) 4 m 7 mm (m) | (j) 20 cm 4 mm (cm) |
| (k) 3 m 88 cm 2 mm (m) | (l) 10 m 57 mm (m) |

4. Round the following lengths to the nearest metre.

- | | | |
|-----------------|----------------|-----------------|
| (a) 2.8 m | (b) 66.2 m | (c) 14.09 m |
| (d) 29.7 m | (e) 15.56 m | (f) 20.199 m |
| (g) 3 m 15 cm | (h) 2 m 80 cm | (i) 15 m 56 cm |
| (j) 4 m 4 cm | (k) 8 m 9 cm | (l) 10 m 89 mm |
| (m) 11 m 256 mm | (n) 5 m 780 mm | (o) 48 m 458 mm |

5. Round the following lengths to the nearest centimetre.

- | | | |
|---------------|----------------|----------------|
| (a) 3.4 cm | (b) 2.8 cm | (c) 15.9 cm |
| (d) 1 cm 1 mm | (e) 13 cm 8 mm | (f) 25 cm 6 mm |
| (g) 9 mm | (h) 7 cm 2 mm | (i) 19 cm 9 mm |

6. Janice is a marathon runner. A marathon is 42 km long and Janice wanted to calculate how many strides she would take while running a marathon. She ran 300 metres and counted 400 strides.

- What is the average length of her stride (in mm)?
- How many strides will she take in running a marathon?
- If she completed the marathon in 2 hours 32 minutes 10 seconds, how long would it take her to run 1500 metres running at the same speed? Give answer to the nearest second.

7. Janice decided to compete in the race up the Empire State Building. This race is up 1576 stairs to the 86th floor of the famous building in New York.

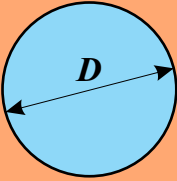
- The average height of each step is 195 mm. How high above the ground (in metres) is the 86th floor?
- Janice completed the race in 11 minutes 5 seconds. What was her rate of running in *steps/second*? Give answer correct to one decimal place.



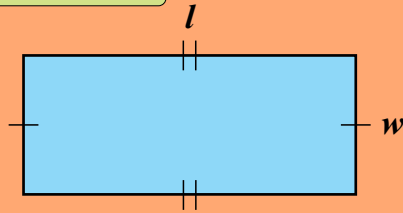
Perimeter

The following rules can be used to find the perimeter of a shape.

Common Shapes



$$\text{Circumference} = \pi D$$



$$\text{Perimeter} = 2l + 2w$$

Arc Length

The length of an arc can be found using the following formula.

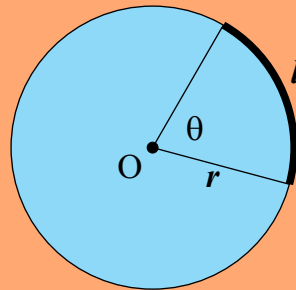
$$l = \frac{\theta}{360} \times 2\pi r$$

where:

l = arc length

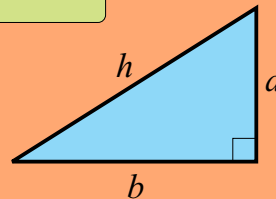
θ = centre angle formed by the arc

r = the radius of the circle



Pythagoras' Theorem

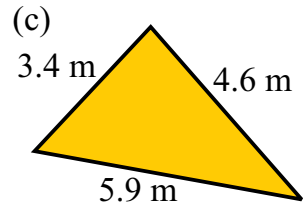
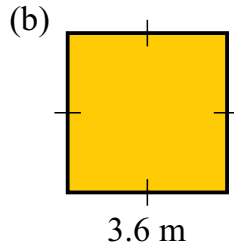
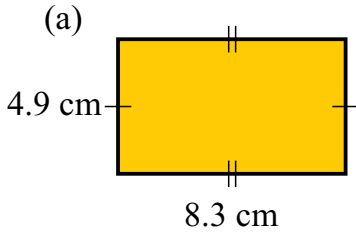
$$h^2 = a^2 + b^2$$



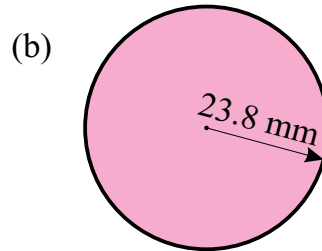
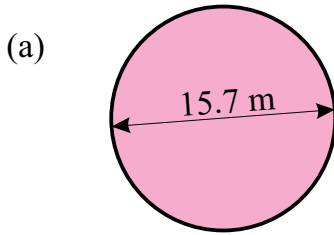
Chapter 12 details Pythagoras' Theorem

EXERCISE 11B

1. Find the perimeter of the following shapes.

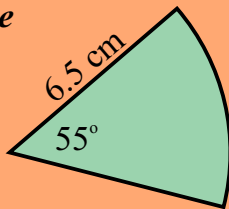


2. Calculate the circumference of each of the circles below.
Give answers correct to one decimal place.



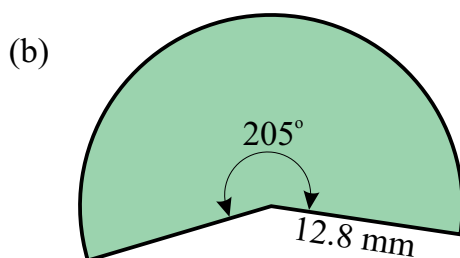
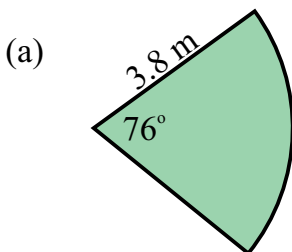
3. Calculate the perimeter of the following shapes.
Give answers correct to one decimal place.

Example



$$\begin{aligned}\text{arc length} &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{55}{360} \times 2\pi \times 6.5 \\ &= 6.2 \text{ cm (one decimal place)}\end{aligned}$$

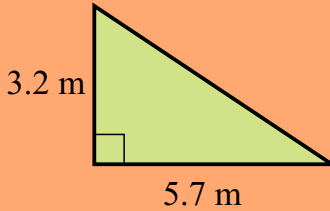
$$\begin{aligned}\text{perimeter} &= 6.5 + 6.5 + 6.2 \\ &= \mathbf{19.2 \text{ cm}}\end{aligned}$$



4. Find the perimeter of the shapes below.
Give answers correct to one decimal place.

Examples

1.



Use Pythagoras' Theorem to find the length of the hypotenuse

$$h^2 = a^2 + b^2$$

$$= 3.2^2 + 5.7^2$$

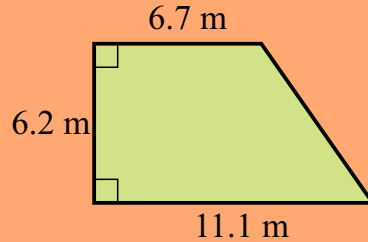
$$h = \sqrt{3.2^2 + 5.7^2}$$

$$h = 6.5 \text{ m (one dec. pl.)}$$

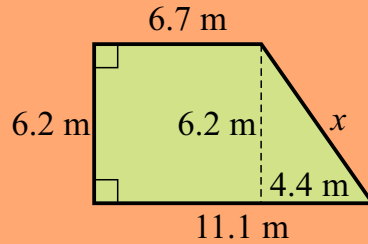
$$\text{perimeter} = 6.5 + 3.2 + 5.7$$

$$= \mathbf{15.4 \text{ m}}$$

2.



Pythagoras' Theorem can be used to find the length of the unknown side



$$x^2 = 6.2^2 + 4.4^2$$

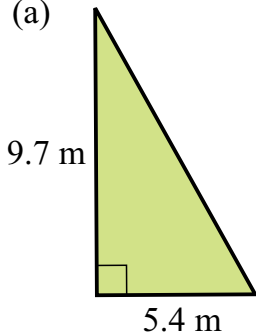
$$x = \sqrt{6.2^2 + 4.4^2}$$

$$x = 7.6 \text{ m (one dec. pl.)}$$

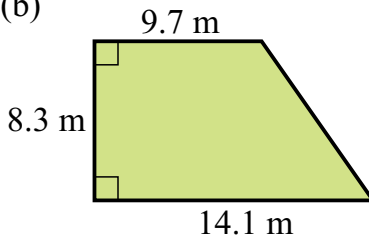
$$\text{perimeter} = 7.6 + 11.1 + 6.2 + 6.7$$

$$= \mathbf{31.6 \text{ m}}$$

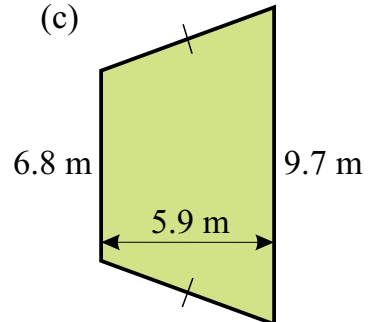
(a)



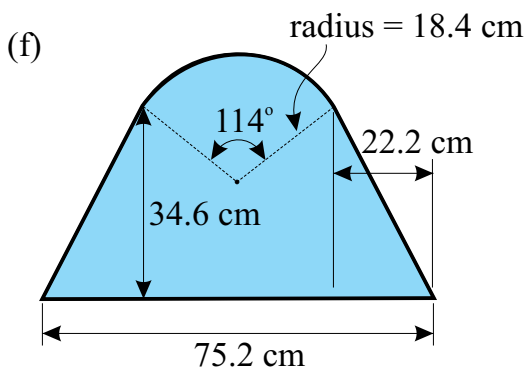
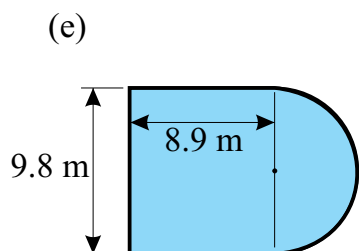
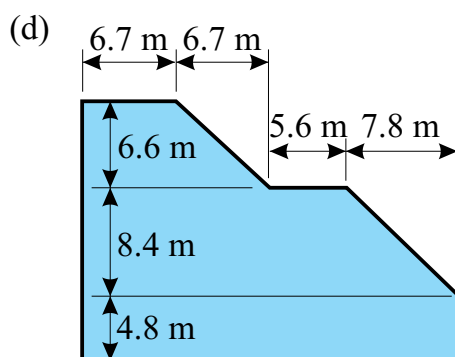
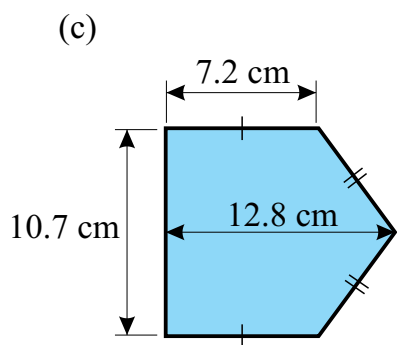
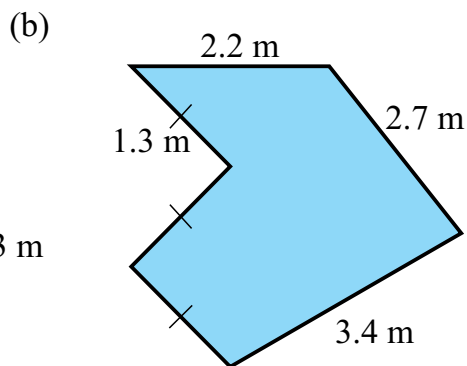
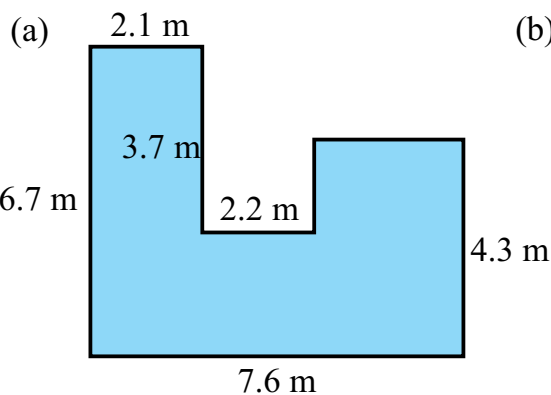
(b)



(c)

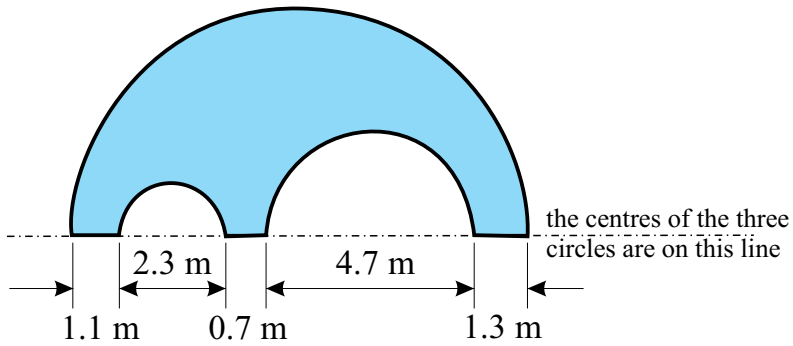


5. Find the perimeter of the following shapes.
Give answers correct to one decimal place.

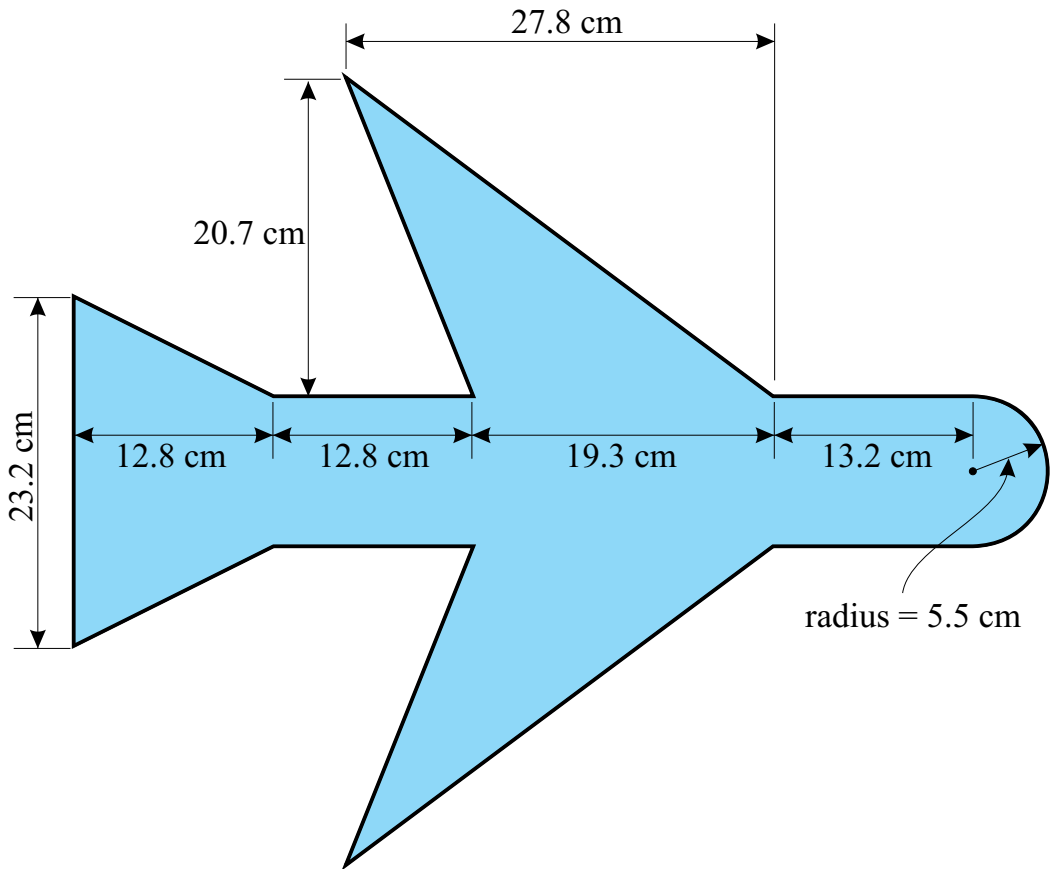


This shape is symmetrical

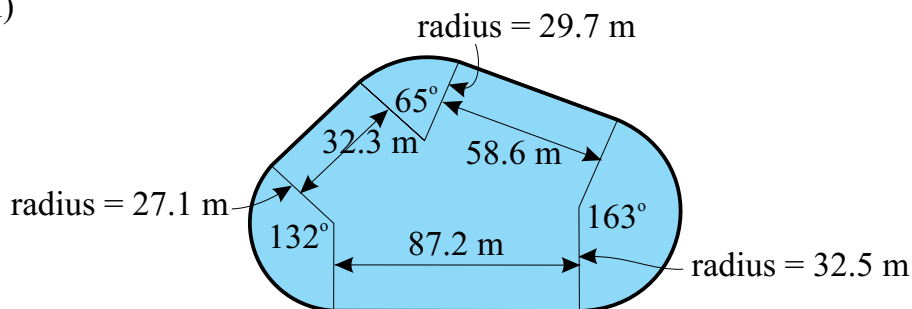
(g)



(h) This shape is symmetrical.

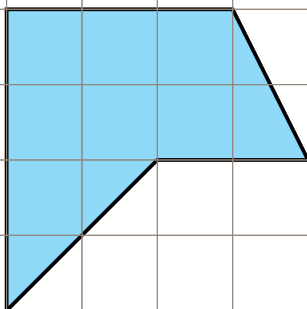


(i)

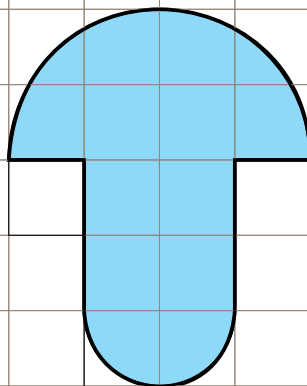


6. The grid below consists of 1 cm squares. Find the perimeter of the shapes on the grid. Give answers correct to one decimal place.

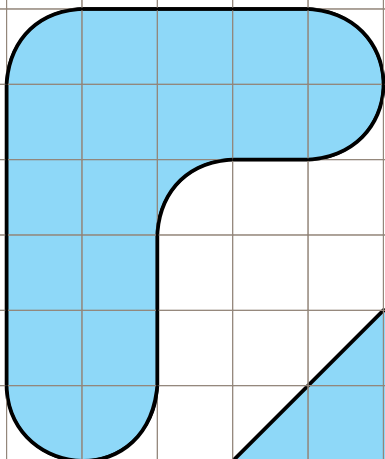
(a)



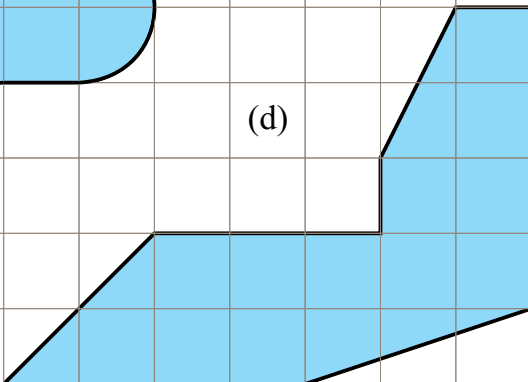
(b)



(c)

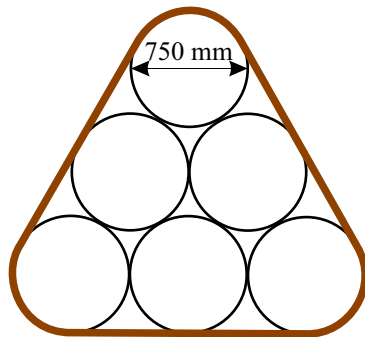


(d)



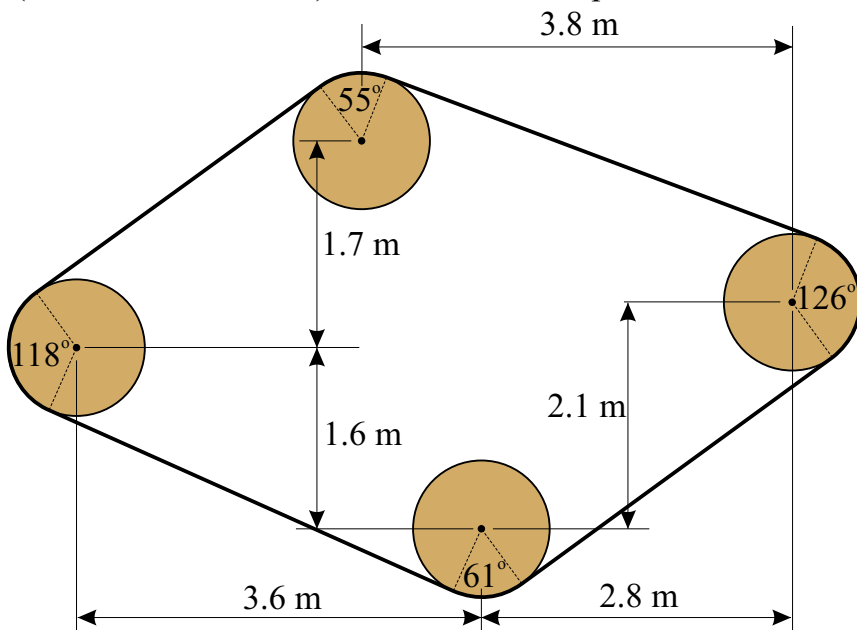
7. (a) The diameter of each wheel on a car was 560 mm. It was recommended that each wheel be replaced after the car had travelled 30 000 km. How many revolutions of each wheel would occur in travelling 30 000 km? Give answer to the nearest revolution.
- (b) A new tyre was produced with a guaranteed life of 25 million revolutions. If the diameter of the tyre was 560 mm, how far could a driver expect to travel on these tyres? Give answer to the nearest km.

8. A strap is used to contain six pipes as shown here.
The pipes have a diameter of 750 mm.
Calculate the length of the strap.
Give answer to the nearest mm.



9. A conveyer belt is shown below. The diameter of each wheel is 1.2 m. The distances between the centres of the wheels are shown. The angles that the belt is in contact with the wheels are shown.

- (a) How long is the belt? Give answer to the nearest mm.
- (b) The belt is moving at a speed of 100 mm/sec. How long will it take (to the nearest second) for the belt to complete one circuit?

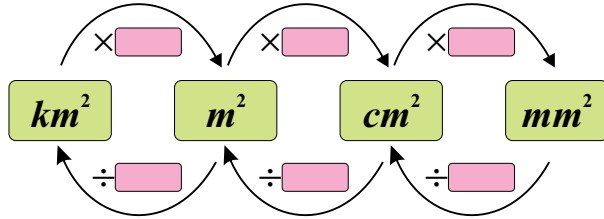


Area - Conversions

EXERCISE 11C

- (a) How many square millimetres are in one square centimetre?
 (b) How many square centimetres are in one square metre?
 (c) How many square metres are in one square kilometre?

- Copy and complete this conversion diagram.



- Convert the following areas to the units shown in the brackets.

- | | |
|--|---|
| (a) 500 mm^2 (cm^2) | (b) 0.2 cm^2 (mm^2) |
| (c) $700\,000 \text{ cm}^2$ (m^2) | (d) 3.4 m^2 (cm^2) |
| (e) $19\,000\,000 \text{ cm}^2$ (m^2) | (f) $60\,000 \text{ cm}^2$ (m^2) |
| (g) 0.031 m^2 (cm^2) | (h) 0.0083 km^2 (m^2) |
| (i) $70\,000\,000 \text{ m}^2$ (km^2) | (j) $630\,000\,000 \text{ mm}^2$ (m^2) |

- A hectare is an area of $10\,000 \text{ m}^2$.

Convert the following areas to hectares (ha).

- | | |
|---------------------------|----------------------------|
| (a) $60\,000 \text{ m}^2$ | (b) $230\,000 \text{ m}^2$ |
| (c) 4000 m^2 | (d) $86\,000 \text{ m}^2$ |
| (e) 3 km^2 | (f) 5.6 km^2 |
| (g) 0.07 km^2 | (h) 320 km^2 |

- A rectangular piece of board is 320 mm wide and 550 mm long. Calculate its area in the following units:

- | | | |
|-------------------|-------------------|------------------|
| (a) mm^2 | (b) cm^2 | (c) m^2 |
|-------------------|-------------------|------------------|

- A rectangular trench is 800 mm wide and 18 m long. Calculate the area of the trench in m^2 .

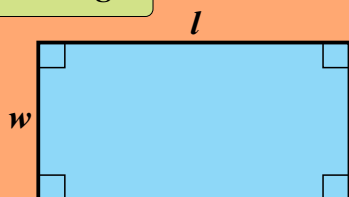
- A rectangular paddock is 500 m long and 350 m wide. How many hectares in this paddock?

- A square property has side lengths of 2.3 km . How many hectares is this property?

Area - Shapes

The following formulae can be used to find the area of common shapes

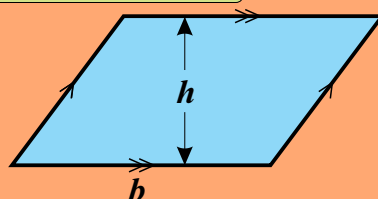
Rectangle



$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= l \times w\end{aligned}$$

$$A = lw$$

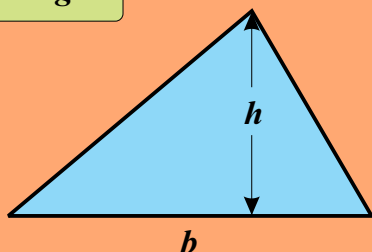
Parallelogram



$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= b \times h\end{aligned}$$

$$A = bh$$

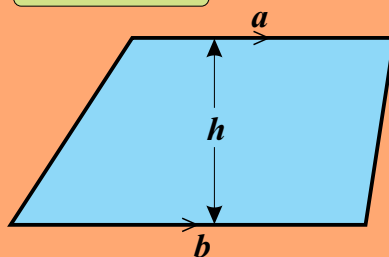
Triangle



$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times b \times h\end{aligned}$$

$$A = \frac{1}{2}bh$$

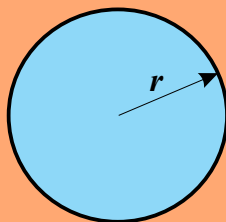
Trapezium



$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (a + b) \times \text{height} \\ &= \frac{1}{2} \times (a + b) \times h\end{aligned}$$

$$A = \frac{1}{2}(a + b)h$$

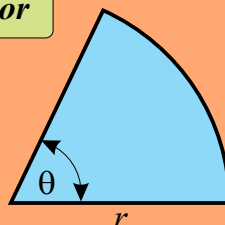
Circle



$$\text{Area} = \pi \times r^2$$

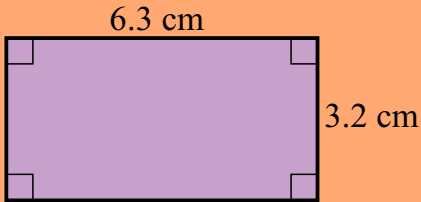
$$A = \pi r^2$$

Sector

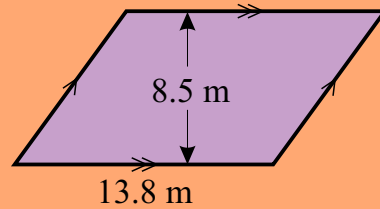


$$\text{Area} = \frac{\theta}{360} \times \pi \times r^2$$

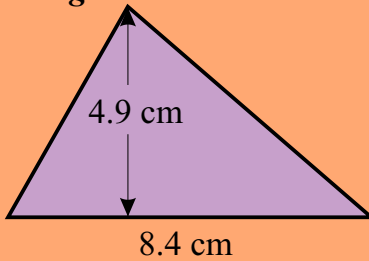
$$A = \frac{\theta}{360} \pi r^2$$

Examples**Rectangle**

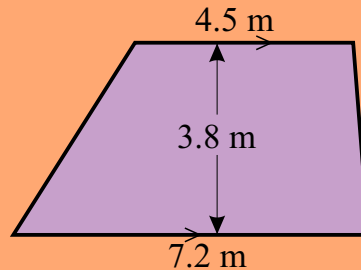
$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 6.3 \times 3.2 \\ \text{Area} &= 20.16 \text{ cm}^2\end{aligned}$$

Parallelogram

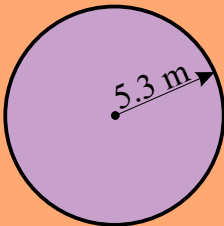
$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= 13.8 \times 8.5 \\ \text{Area} &= 117.3 \text{ m}^2\end{aligned}$$

Triangle

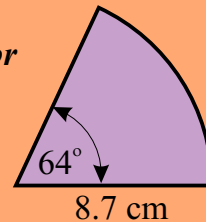
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 8.4 \times 4.9 \\ \text{Area} &= 20.58 \text{ cm}^2\end{aligned}$$

Trapezium

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (a + b) \times \text{height} \\ &= \frac{1}{2} \times (4.5 + 7.2) \times 3.8 \\ \text{Area} &= 22.23 \text{ m}^2\end{aligned}$$

Circle

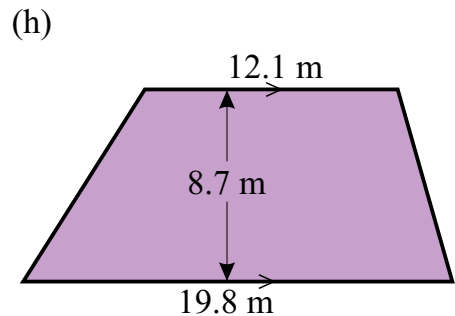
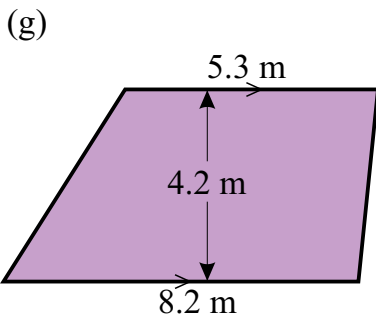
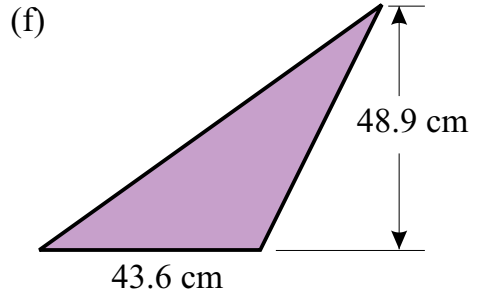
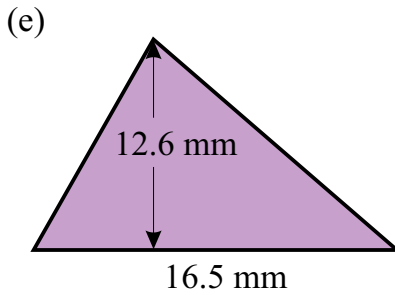
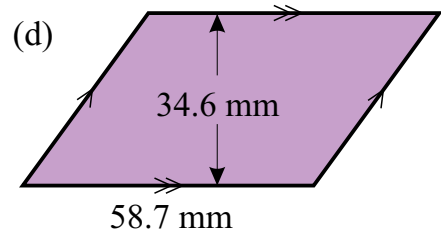
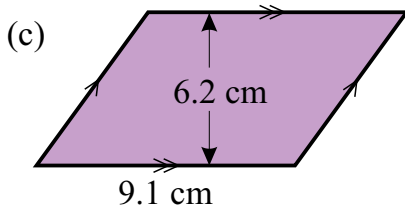
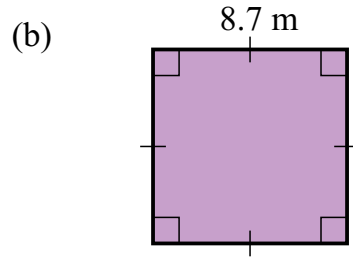
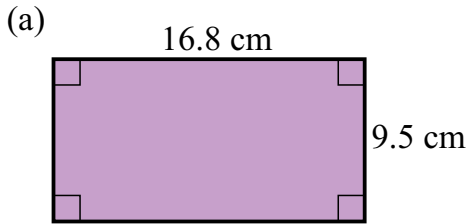
$$\begin{aligned}\text{Area} &= \pi \times r^2 \\ &= \pi \times 5.3^2 \\ \text{Area} &= 88.2 \text{ m}^2 \text{ (1 dec. pl.)}\end{aligned}$$

Sector

$$\begin{aligned}\text{Area} &= \frac{\theta}{360} \times \pi \times r^2 \\ &= \frac{64}{360} \times \pi \times 8.7^2 \\ \text{Area} &= 42.3 \text{ cm}^2 \text{ (1 dec. pl.)}\end{aligned}$$

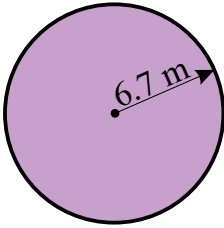
EXERCISE 11D

1. Calculate the area of the following shapes.

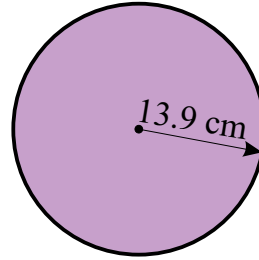


2. Find the area of the following shapes.
Give answers correct to one decimal place.

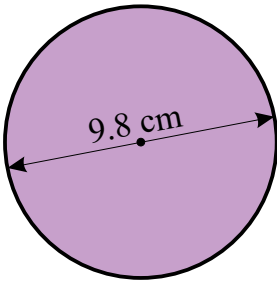
(a)



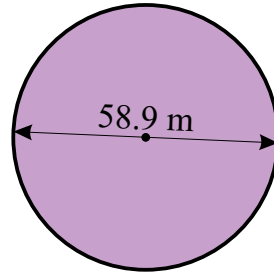
(b)



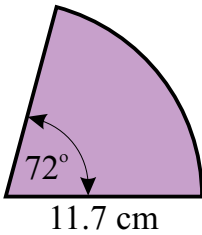
(c)



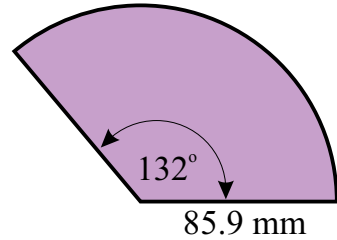
(d)



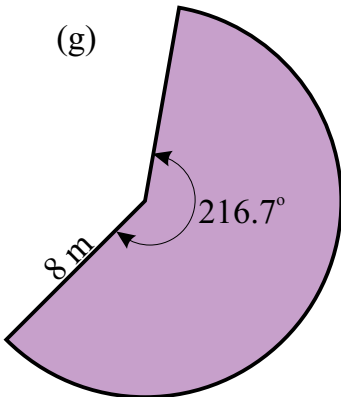
(e)



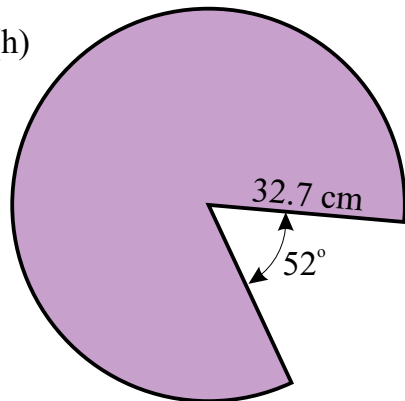
(f)



(g)



(h)



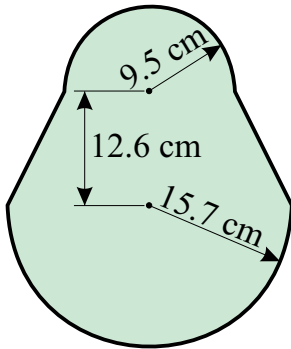
Area - Composite Shapes

EXERCISE 11E

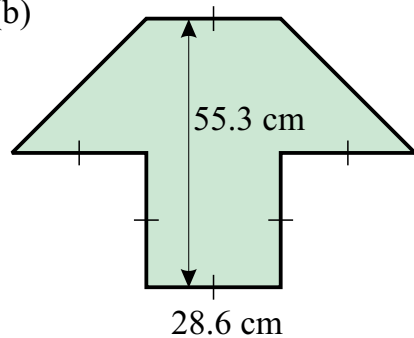
1. Find the area of the following *symmetrical* shapes.

Give answers correct to one decimal place.

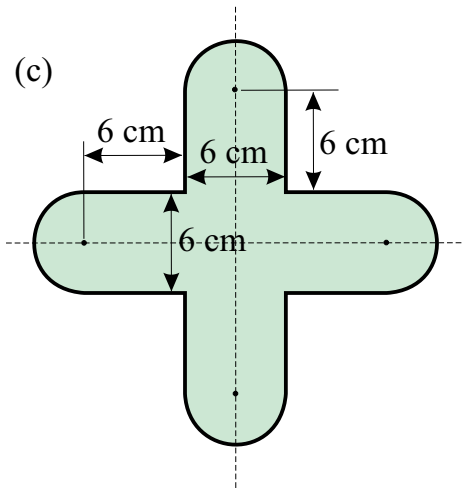
(a)



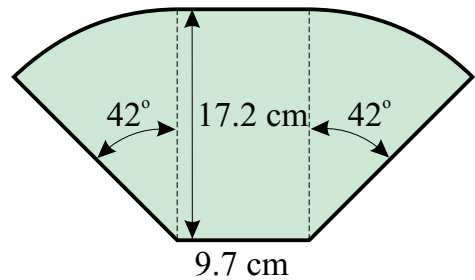
(b)



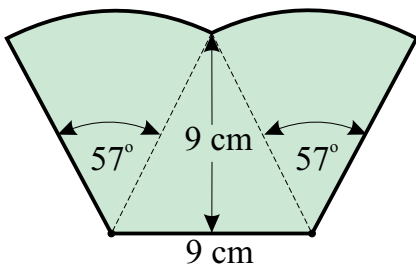
(c)



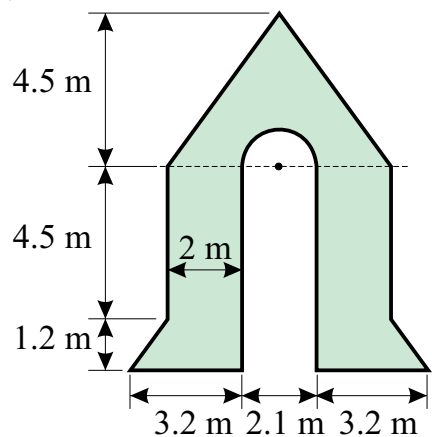
(d)



(e)

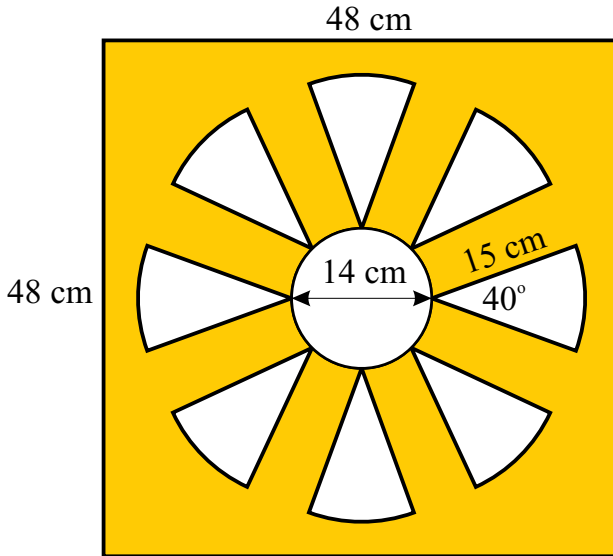


(f)

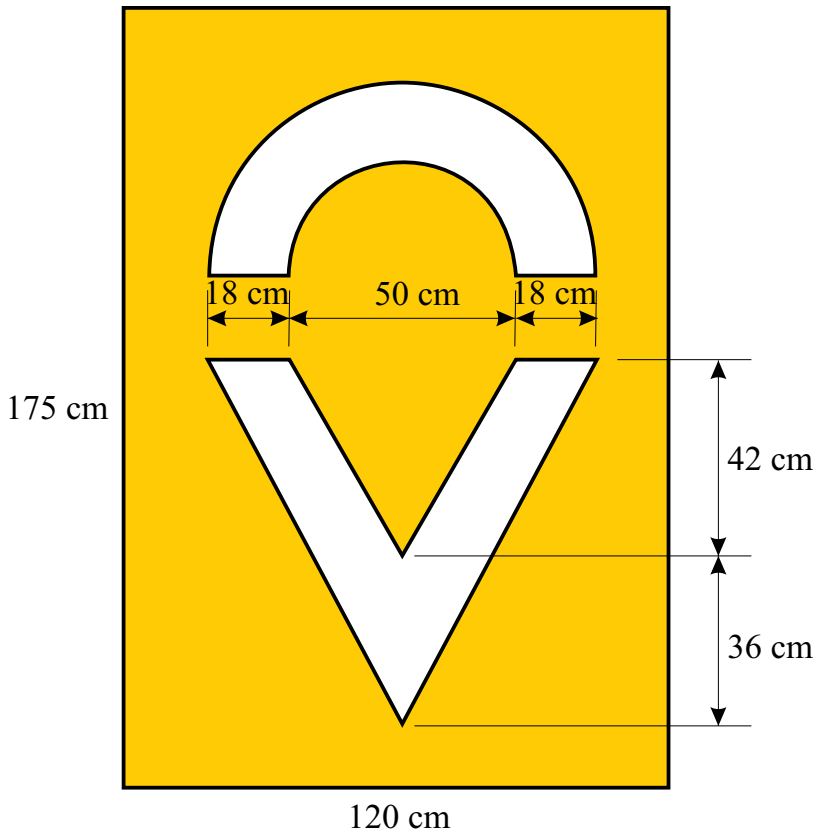


2. Find the area of the shaded regions below.

(a)



(b)



Hero's Formula (Heron's Formula)

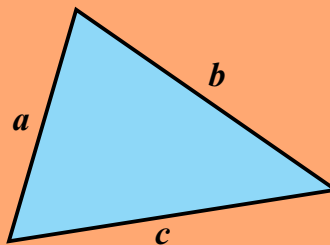
EXERCISE 11F

The area of a triangle can be found by using Hero's formula as shown below.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

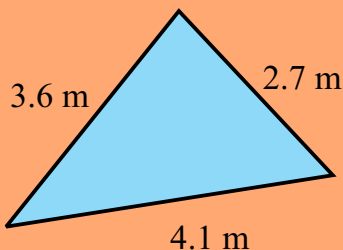
$$\text{where } s = \frac{1}{2}(a+b+c)$$

and a , b and c are the side lengths



Example

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) \\ &= \frac{1}{2} \times (3.6 + 2.7 + 4.1) \\ s &= 5.2 \end{aligned}$$

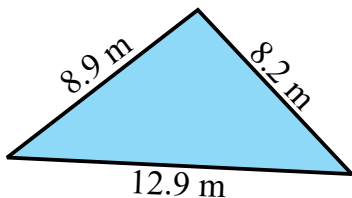


$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{5.2 \times (5.2 - 3.6)(5.2 - 2.7)(5.2 - 4.1)} \end{aligned}$$

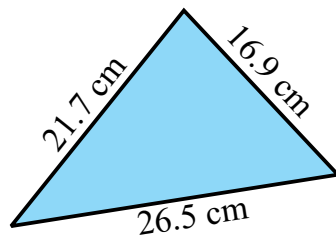
$$\text{Area} = 4.8 \text{ m}^2 \text{ (one decimal place)}$$

Use Hero's formula to calculate the area of the following triangles. Give answers correct to one decimal place.

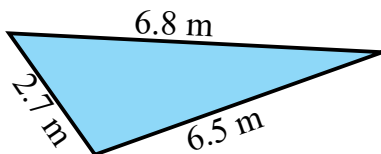
1.



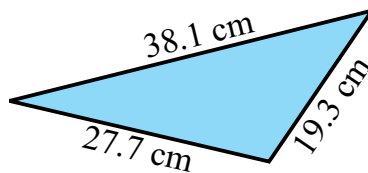
2.



3.



4.



Area - Problem Solving

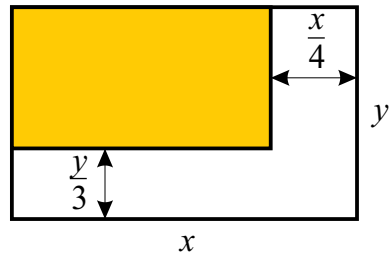
EXERCISE 11G

- (a) A farmer has a property that is 2 km long and 1.5 km wide. How many hectares is the property?

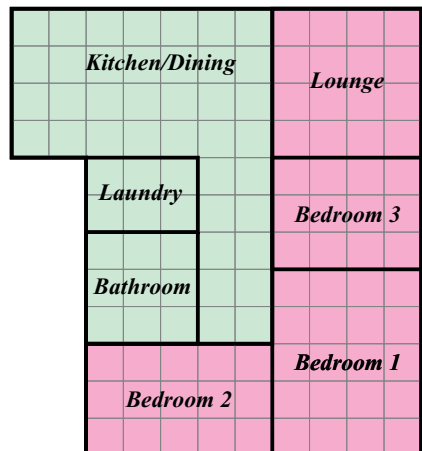
(b) He has three paddocks on this property where his cattle graze. The sizes of the paddocks are $400 \text{ m} \times 400 \text{ m}$, $600 \text{ m} \times 500 \text{ m}$ and $700 \text{ m} \times 200 \text{ m}$. What percentage of the area of his farm is the total area of these three paddocks?
- William has 120 metres of chicken wire that he is going to use to make a yard for his chooks. A keen mathematician, he wanted to calculate the area of the yard for different lengths (l) and widths (w). Calculate the area of the yard for the following dimensions. Assume all the 120 metres of wire are used.

(a) $l = w$ (b) $l = 2w$ (c) $l = 3w$ (d) $l = 4w$ (e) $l = 5w$

- A rectangular piece of board with length x and width y has area A . One quarter of the length is cut off and one third of the width is cut off leaving the shaded region. Which of the following alternatives is an expression for the area of the shaded region?



- A $\frac{7A}{12}$ B $\frac{6A}{7}$ C $\frac{5A}{12}$ D $\frac{5A}{7}$ E $\frac{A}{2}$ F $\frac{A}{7}$ G $\frac{A}{12}$ H $\frac{11A}{12}$
- Emma wants to replace the floor coverings in her house. The floor plan is shown here. The grid shown is 1 metre squares. She wants to cover the floors in the bedrooms and lounge with carpet that costs \$45 per square metre. In the remainder of the house she is going to use timber laminate that costs \$25 per square metre. What is the total cost of the floor coverings?



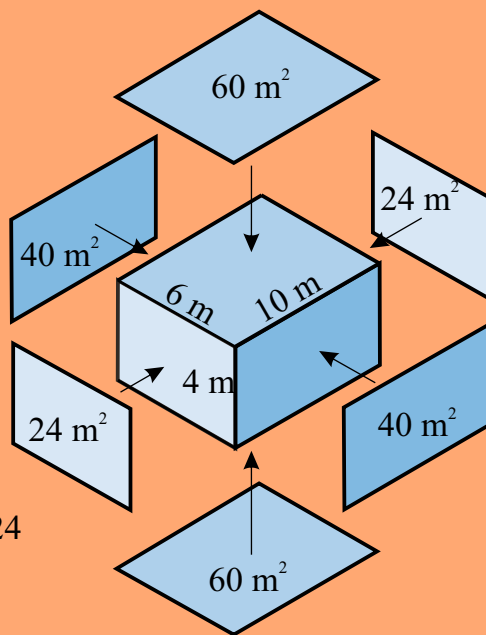
Surface Area

The surface area of any object can be found by adding the areas of all the faces.

Rectangular Prism

Example

Find the area of each face of the prism.
Add the areas of the faces to find the surface area (SA) of the prism.

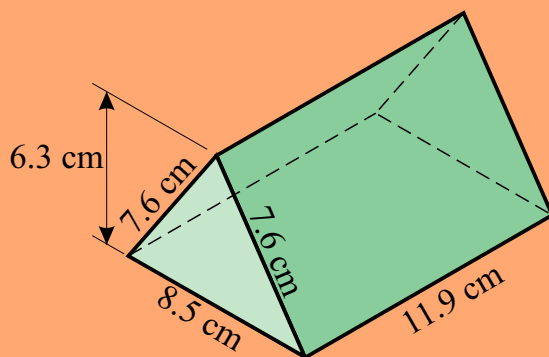


$$\begin{aligned} \text{SA} &= 40 + 60 + 24 + 40 + 60 + 24 \\ &= 248 \text{ m}^2 \end{aligned}$$

Other Prisms

Example

In this example of a triangular prism the surface consists of three rectangles and two triangles.



$$\begin{aligned} \text{SA} &= (8.5 \times 11.9) + (7.6 \times 11.9) + (7.6 \times 11.9) + \\ &\quad \left(\frac{1}{2} \times 6.3 \times 8.5\right) + \left(\frac{1}{2} \times 6.3 \times 8.5\right) \end{aligned}$$

$$\text{SA} = 335.58 \text{ cm}^2$$

Pyramid

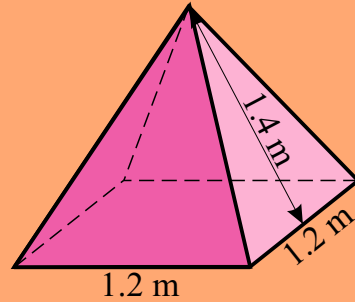
A pyramid consists of four triangles and a base.

In this example the base is a square and the four triangles are all the same size.

$$\begin{aligned}\text{Area of base} &= 1.2 \times 1.2 \\ &= 1.44 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of each triangle} &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} \times 1.2 \times 1.4 \\ &= 0.84 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{SA} &= 1.44 + 4 \times 0.84 \\ &= \mathbf{4.8 \text{ m}^2}\end{aligned}$$



Cylinder

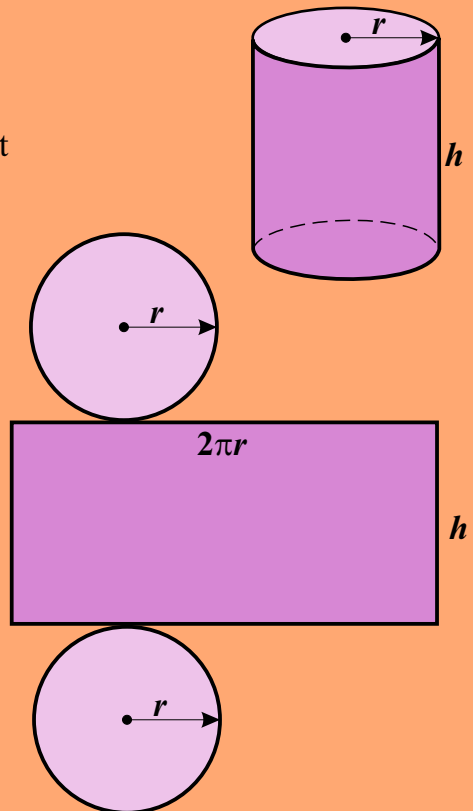
The surface area of a cylinder can be found by considering its net as shown below.

The length of the rectangle that wraps around to form the curved surface = $2\pi r$

SA = area of the two circles + the area of the rectangle

$$\begin{aligned}\text{SA} &= 2 \times \pi r^2 + 2\pi r \times h \\ &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r (r + h)\end{aligned}$$

$$\mathbf{SA = 2\pi r (r + h)}$$



Cone

The surface of a cone can be seen by considering its net as shown below. The circle is its base and the sector is the cone surface opened out.

r = radius of base

s = length of sloping edge

SA = area of circle + area of sector

area of circle = πr^2

The area of the sector can be calculated by realising the curved length of the sector is equal to the circumference of the base = $2\pi r$.

The sector is a part of the circle with radius s and circumference $2\pi s$.

The ratio of the length of the curved part of the sector to the full circumference will be the same as the ratio of the areas.

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{2\pi r}{2\pi s}$$

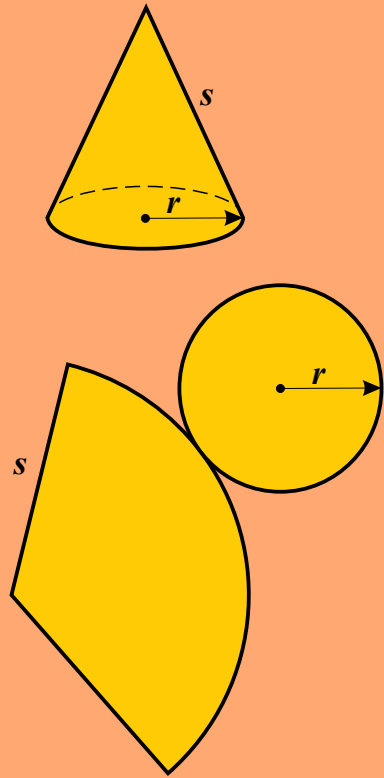
$$\frac{\text{area of sector}}{\pi s^2} = \frac{r}{s}$$

$$\text{area of sector} = \pi s^2 \times \frac{r}{s}$$

$$\text{area of sector} = \pi rs$$

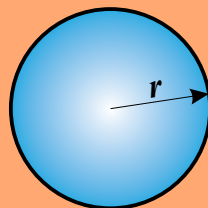
$$\begin{aligned} \text{SA} &= \pi r^2 + \pi rs \\ &= \pi r(r + s) \end{aligned}$$

$$\text{SA} = \pi r(r + s)$$

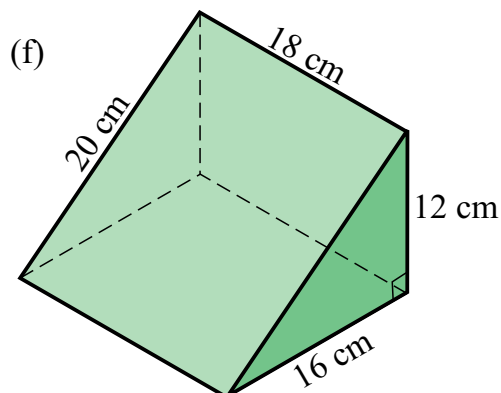
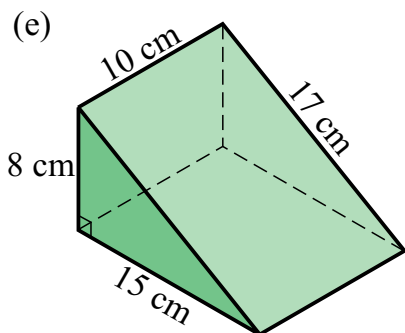
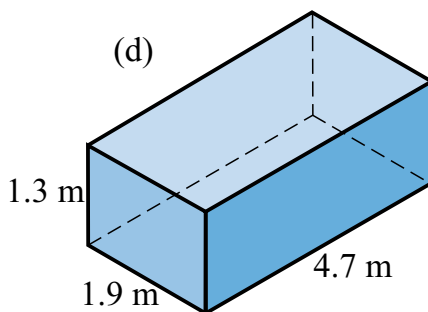
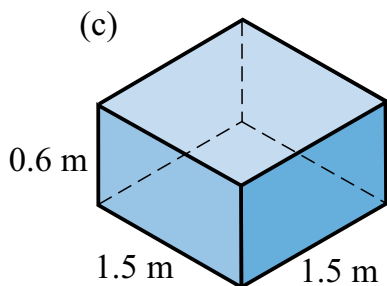
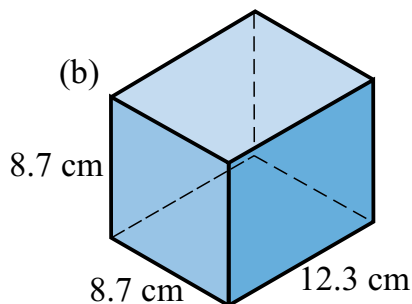
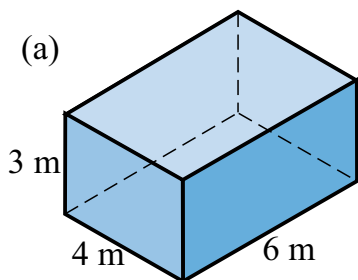


Sphere

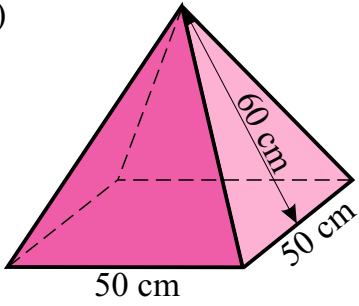
$$SA = 4\pi r^2$$

**EXERCISE 11H**

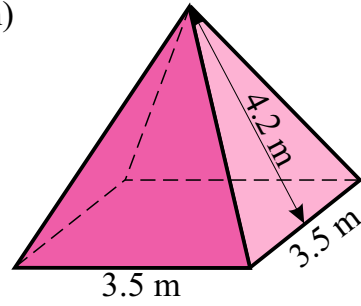
1. Find the surface area of the following objects.



(g)

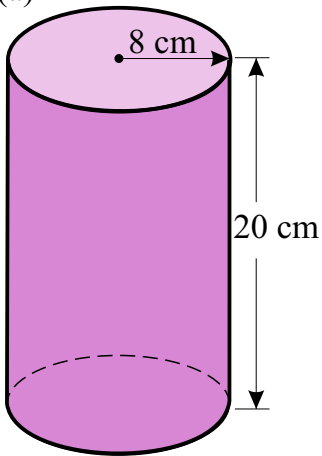


(h)

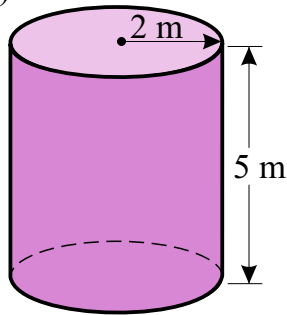


2. Find the surface area of the following objects correct to one decimal place.

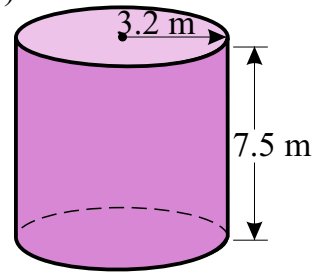
(a)



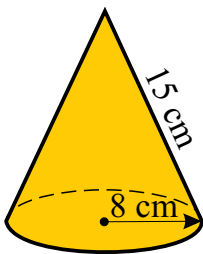
(b)



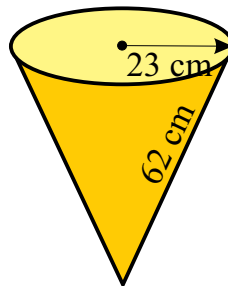
(c)



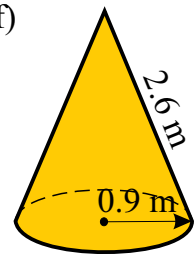
(d)



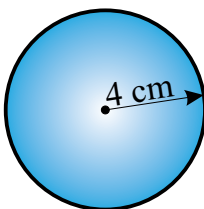
(e)



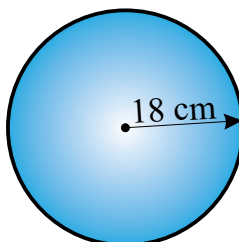
(f)



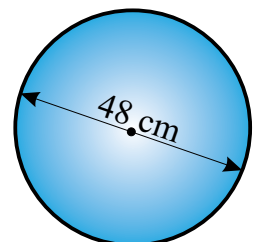
(g)



(h)

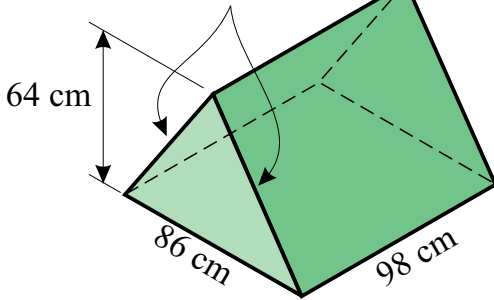


(i)

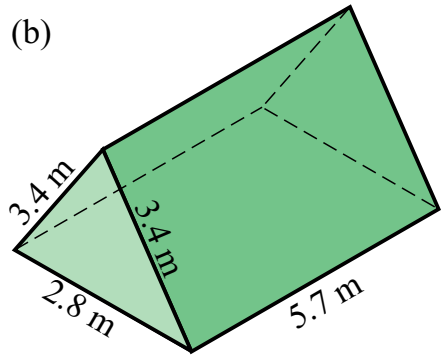


3. Find the surface area of the following objects correct to one decimal place. Pythagoras' Theorem will be needed.

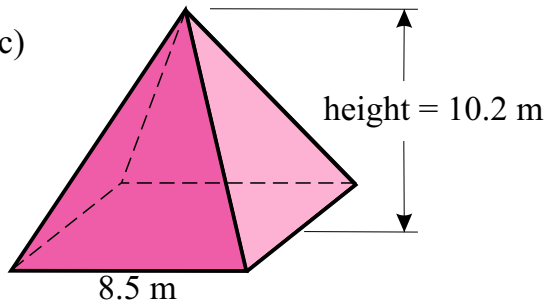
- (a) *These two sides are equal in length*



- (b)



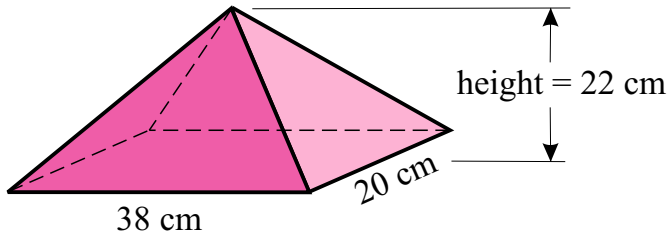
- (c)



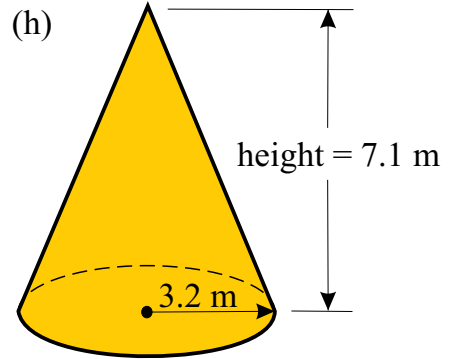
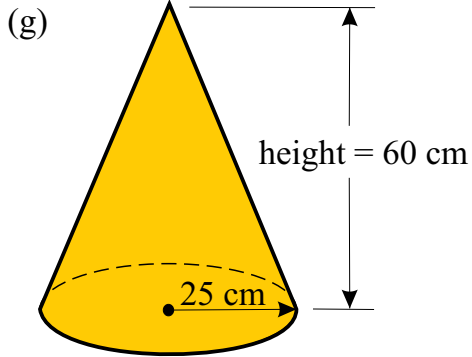
This pyramid has a square base

- (d) A square-based pyramid with base length of 48 cm and vertical height of 32 cm.

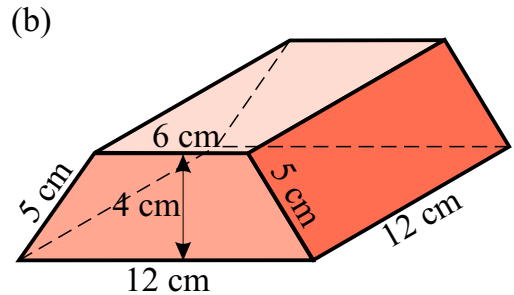
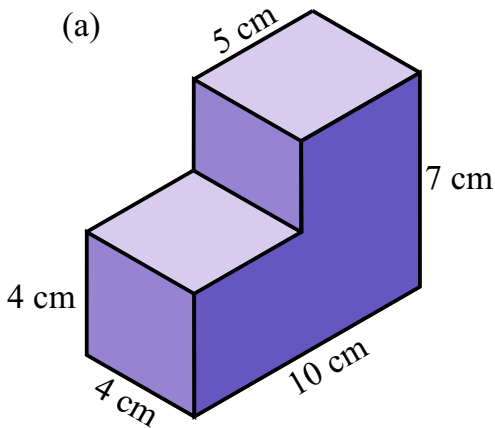
- (e)



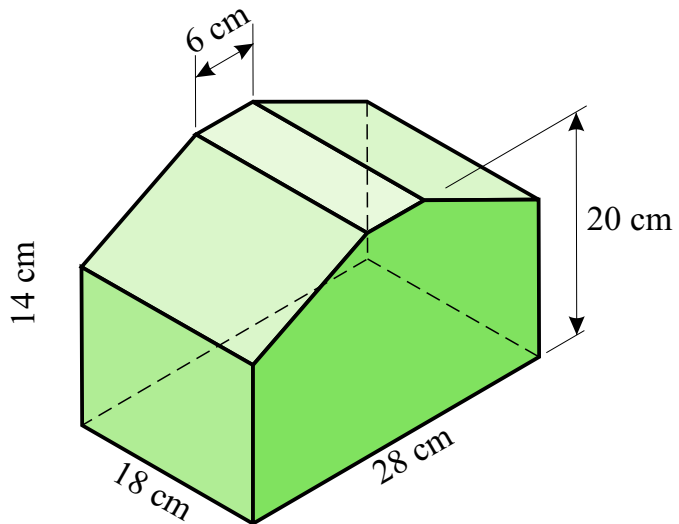
- (f) A pyramid with base dimensions of 8.2 m $\times$ 6.4 m and vertical height of 5.6 m.



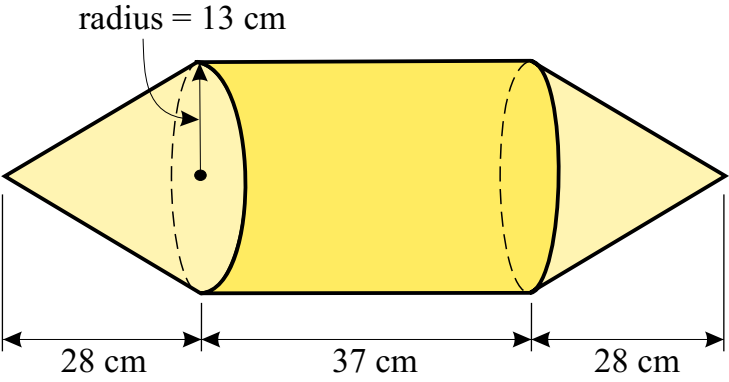
4. Find the surface area of the following objects.
Give answers to one decimal place where necessary.



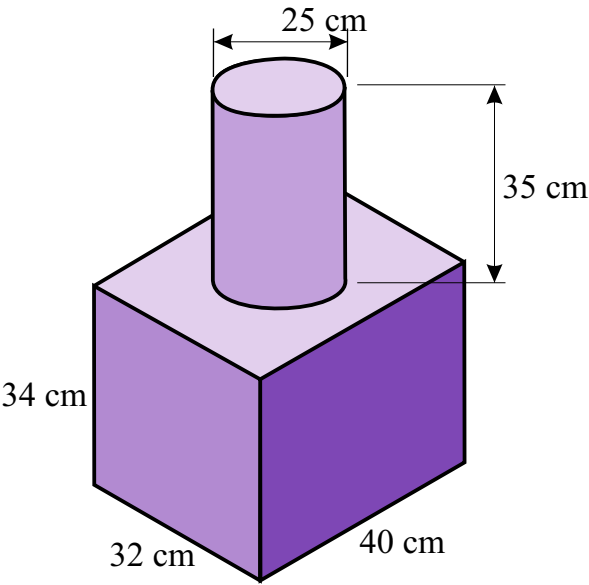
(c) This object is symmetrical.



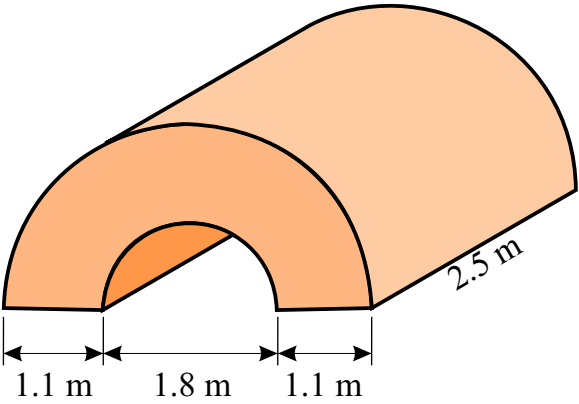
(d)



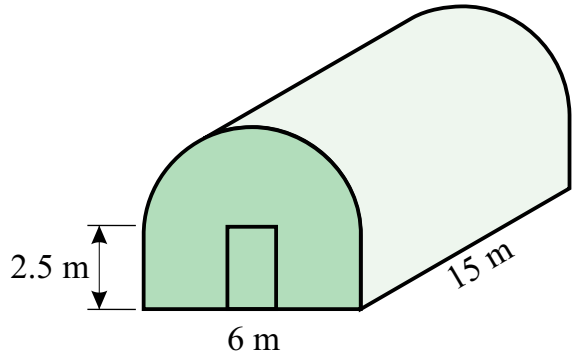
(e)



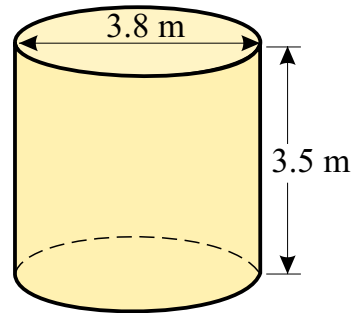
(f)



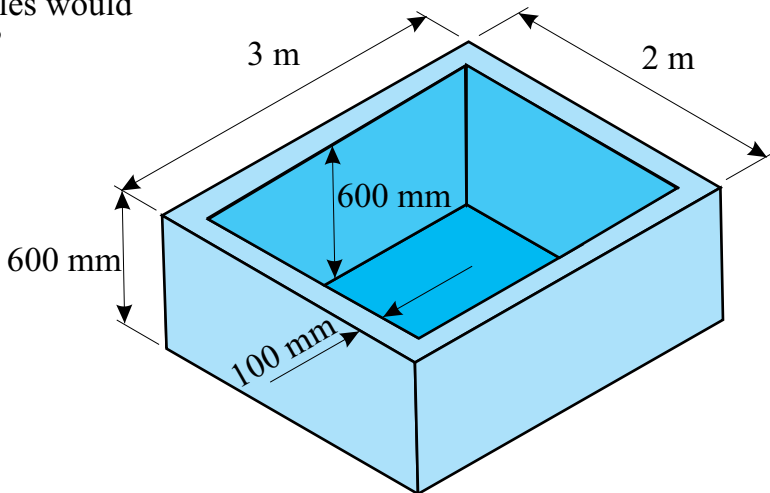
5. Conrad wants to construct the greenhouse shown here. The plastic he is going to use to cover the frame costs \$8.00 per square metre. Calculate the cost to buy the plastic.



6. Rhonda has a cylindrical water tank that is 3.5 m high and has a diameter of 3.8 m. It is leaking so she is going to cover the inside of the tank with fibreglass (not the lid). The fibreglass costs \$60 per square metre. What is the total cost to fibreglass the curved surface and base of the tank?



7. Quinton is making the rectangular spa shown below. The outside dimensions are 3 m $\times$ 2 m and it is 600 mm deep. The wall thickness is 100 mm. Quinton wants to cover all surfaces with tiles (except underneath the spa). He has two tile sizes to choose from:
- 100 mm square tiles or 50 mm square tiles.
- (a) How many 100 mm square tiles would he need?
- (b) How many 50 mm square tiles would he need?



Volume

Common units of volume are mm^3 , cm^3 , *litres* and m^3 .

1 mm^3 is a volume equal to the volume of a cube with side length 1 mm.

1 cm^3 is a volume equal to the volume of a cube with side length 1 cm.

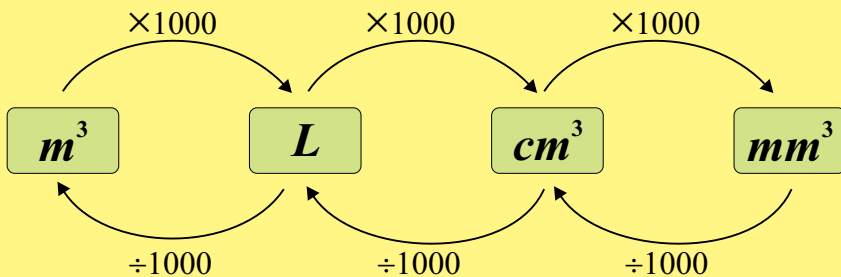
1 litre is a volume equal to the volume of a cube with side length 10 cm.

1 m^3 is a volume equal to the volume of a cube with side length 1 m.

$$\begin{aligned} 1 \text{ cm}^3 &= 1000 \text{ mm}^3 \\ 1 \text{ litre (L)} &= 1000 \text{ cm}^3 \\ 1 \text{ m}^3 &= 1000\,000 \text{ cm}^3 \\ 1 \text{ m}^3 &= 1000 \text{ litres} \\ 1 \text{ cm}^3 &= 1 \text{ mL} \end{aligned}$$

The millilitre (mL) is often used as a unit of volume.
1 millilitre (mL) is equal to 1 cubic centimetre (cm^3).
Cubic centimetre is often abbreviated to c.c.

This diagram can be used to find the conversion factor when converting from one unit to another.



EXERCISE 11I

1. Convert the following volumes to the units shown in the brackets.

- | | |
|--------------------------------|--|
| (a) 50 000 cm ³ (L) | (b) 0.02 cm ³ (mm ³) |
| (c) 600 mL (L) | (d) 8.1 m ³ (L) |
| (e) 0.25 L (mL) | (f) 890 000 mm ³ (cm ³) |
| (g) 300 L (m ³) | (h) 7 mL (mm ³) |
| (i) 8 cm ³ (mL) | (j) 100 000 mm ³ (L) |
| (k) 2.4 L (cm ³) | (l) 0.75 mL (cm ³) |

2. Rani bought 14 bottles of drink for a party she was organising.

Each bottle contained 750 mL.

How many *litres* of drink did she have?

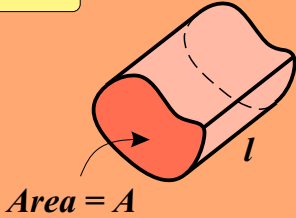
3. Jasmin bought a 2 litre bottle of concentrated fruit juice. She mixed 50 mL of the juice into a glass of water each day. How many days will the 2 litre bottle last?

4. Giaan bought 3 m³ of soil for her garden. Her wheelbarrow holds 100 litres and she could fill the wheelbarrow five times an hour. How many hours will it take her to move all the soil?



Volume of Regular Objects

Prism

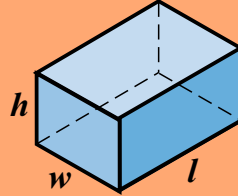


Volume = area of end $\times$ length

$$V = A \times l$$

$$V = Al$$

Rectangular Prism (Cuboid)

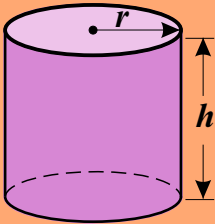


Volume = length $\times$ width $\times$ height

$$V = l \times w \times h$$

$$V = lwh$$

Cylinder

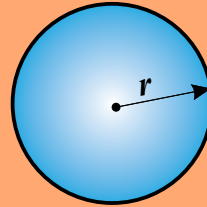


Volume = $\pi r^2 \times$ height

$$V = \pi r^2 \times h$$

$$V = \pi r^2 h$$

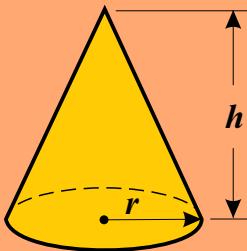
Sphere



Volume = $\frac{4}{3} \pi r^3$

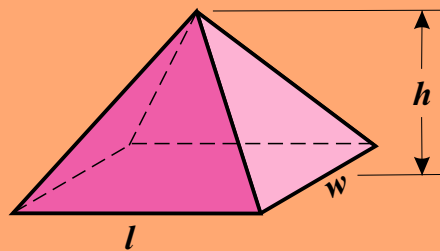
$$V = \frac{4}{3} \pi r^3$$

Cone



$$V = \frac{1}{3} \pi r^2 h$$

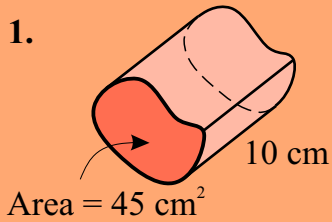
Pyramid



$$V = \frac{1}{3} lwh$$

Examples

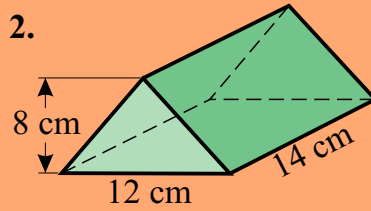
1.

Volume = area of end $\times$ length

$$V = 45 \times 10$$

$$V = \mathbf{450 \text{ cm}^3}$$

2.

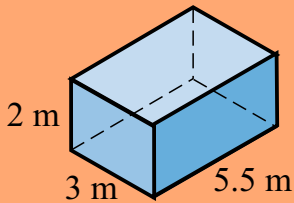
Volume = area of end $\times$ length

$$V = \text{area of triangle} \times \text{length}$$

$$= \frac{1}{2} \times 12 \times 8 \times 14$$

$$V = \mathbf{672 \text{ cm}^3}$$

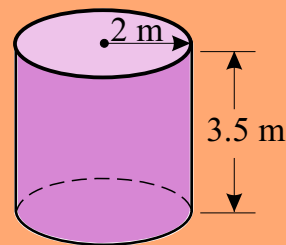
3.

Volume = length $\times$ width $\times$ height

$$V = 5.5 \times 3 \times 2$$

$$V = \mathbf{33 \text{ m}^3}$$

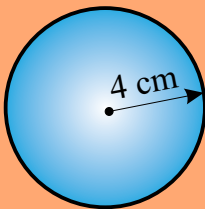
4.

Volume = $\pi r^2 \times$ height

$$V = \pi \times 2^2 \times 3.5$$

$$V = \mathbf{44.0 \text{ m}^3} \text{ (one dec. pl.)}$$

5.

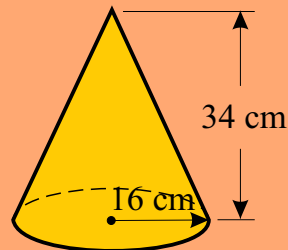


$$\text{Volume} = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \times \pi \times 4^3$$

$$V = \mathbf{268.1 \text{ cm}^3} \text{ (one dec. pl.)}$$

6.



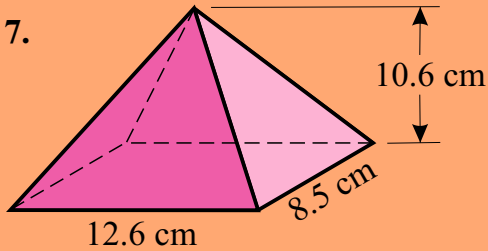
$$\text{Volume} = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \times \pi \times 16^2 \times 34$$

$$V = \mathbf{9114.8 \text{ cm}^3} \text{ (one dec. pl.)}$$

Examples continued

7.



$$\text{Volume} = \frac{1}{3} lwh$$

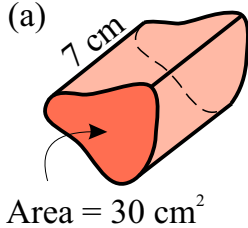
$$V = \frac{1}{3} \times 12.6 \times 8.5 \times 10.6$$

$$V = 378.42 \text{ cm}^3$$

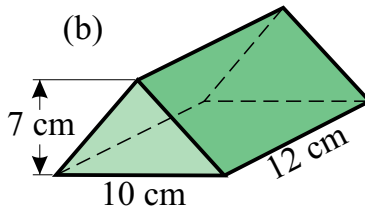
EXERCISE 11J

1. Find the volume of each of the following objects.

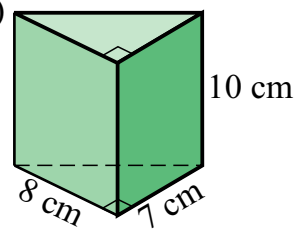
(a)



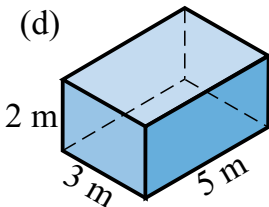
(b)



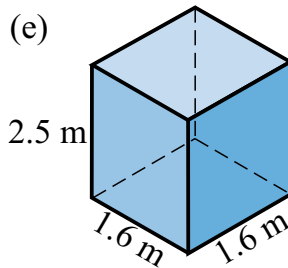
(c)



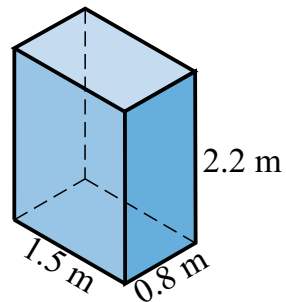
(d)



(e)



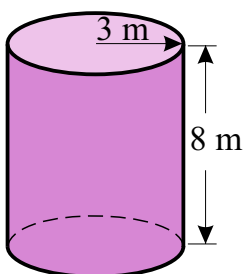
(f)



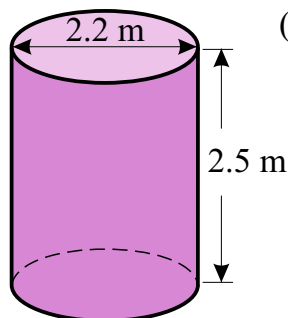
2. Find the volume of each of the following cylinders.

Give answers correct to one decimal place.

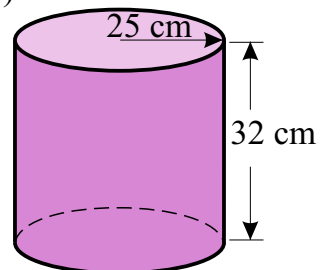
(a)



(b)

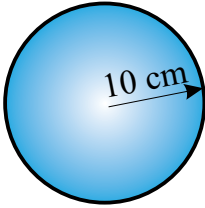


(c)

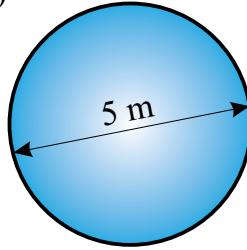


3. Find the volume of each of the following spheres.
Give answers correct to one decimal place.

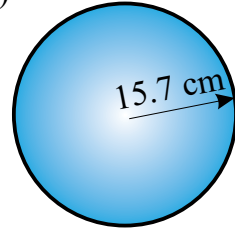
(a)



(b)

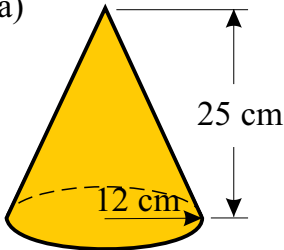


(c)

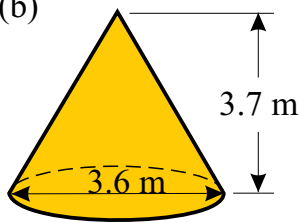


4. Find the volume of each of the following cones.
Give answers correct to one decimal place.

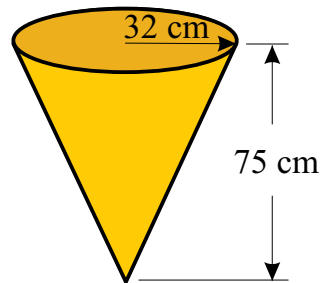
(a)



(b)

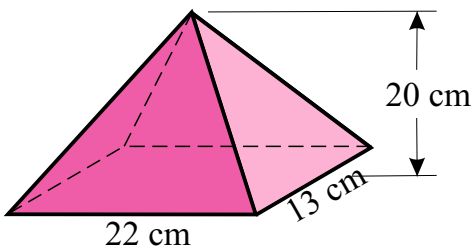


(c)

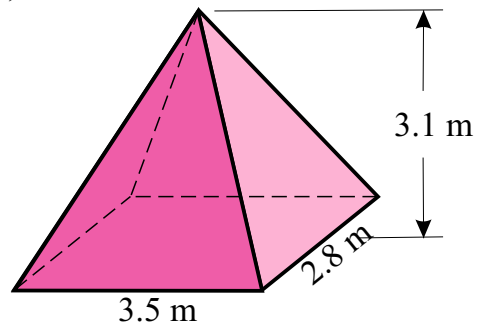


5. Find the volume of each of the following pyramids.
Give answers correct to one decimal place.

(a)

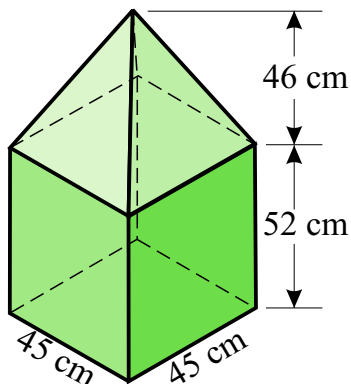


(b)

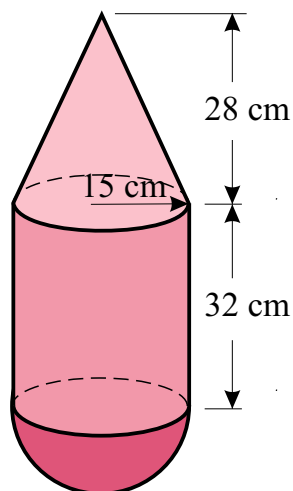


6. Calculate the volume of the following objects.
Give answers correct to one decimal place.

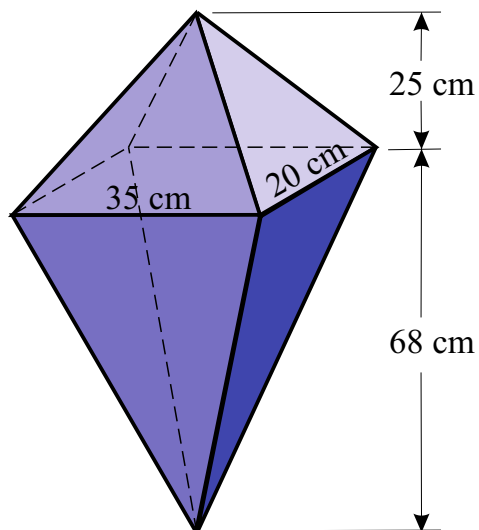
(a)



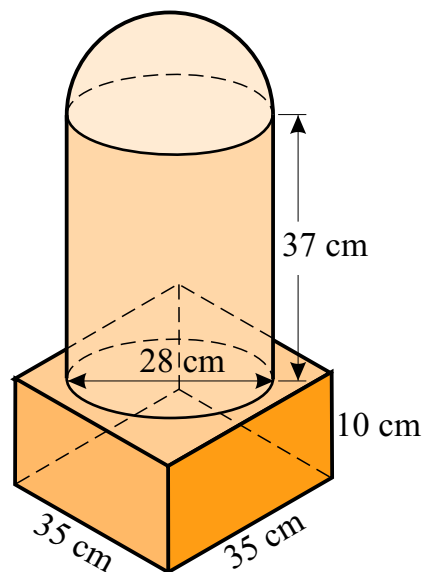
(b)



(c)



(d)



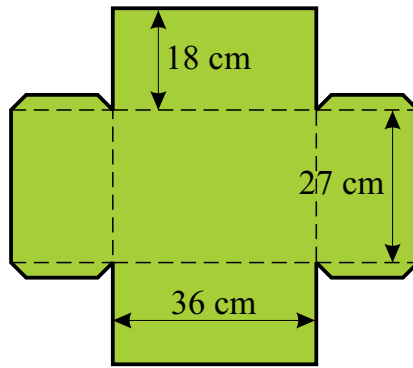
7. (a) Choose which of the following alternatives is the best estimate of the volume of a basketball.

A 2 litres **B** 7 litres **C** 15 litres **D** 27 litres

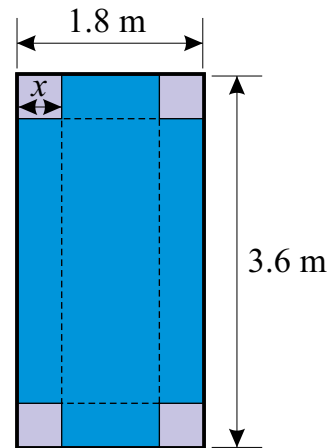
- (b) The **diameter** of a basketball is 24 cm. Calculate the volume of a basketball (in **litres**) correct to one decimal place.



8. The net shown here is of cardboard that is folded to form a box without a lid. What is the volume of the box?



9. Andrew is going to make a rectangular feeding box for his horses by cutting square corners out of a rectangular sheet of metal and folding. The sheet of metal is $3.6 \text{ m} \times 1.8 \text{ m}$. He wants to investigate the effect that the side length, x , of the square cut out, will have on the capacity of the box. Calculate the capacity, in litres, of the box for the following values of x .

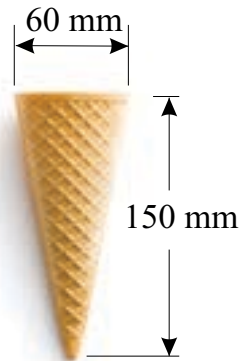


(a) 100 mm (b) 200 mm (c) 300 mm (d) 400 mm (e) 500 mm

10. Emily wanted to move a large stone block in her garden. The stone was 1.5 m long, 800 mm wide and 400 mm high. She needed to calculate how much it weighed to organise a mechanical lifting device. She found a block of the same type of stone that was $200 \text{ mm} \times 100 \text{ mm} \times 60 \text{ mm}$ and it weighed 3 kg. What would the large block weigh?

11. The Moozly breakfast food company want to change the size of their 2 kg box of cereal. The dimensions of the existing box are $300\text{ mm} \times 250\text{ mm} \times 80\text{ mm}$. They want the new box to be 150 mm wide and 320 mm high. Calculate the width of the new box so the volume is the same as the existing box.
12. A producer of nuts and bolts has found that the size of the boxes they use to pack their products is too large. They decide to reduce the length, width and height of the boxes by 10% . What will be the percentage reduction in the volume of the boxes?
13. The I-Scream shop sells ice-creams in cones. They buy cylindrical tubs of ice-cream that have a diameter of 500 mm and are 1.5 m deep.
- (a) How many spherical scoops of ice-cream, 60 mm in diameter, could they get from each tub?

- (b) The I-Scream shop also sells soft ice-cream in cones. The cones have a diameter of 60 mm and are 150 mm deep. The cone is filled with ice-cream till it is level. How many cones could be filled from a 20 litre tub of soft ice-cream?



14. Mulder is a sculptor who is fascinated with pyramids. He buys a block of clay that is in the shape of a cube with side length 1.2 m . He wants to make as many clay pyramids as he can from this block. The pyramids are to be square-based with base length of 400 mm and be 270 mm high. How many of these pyramids could Mulder sculpt from the block?



PROBLEM SOLVING

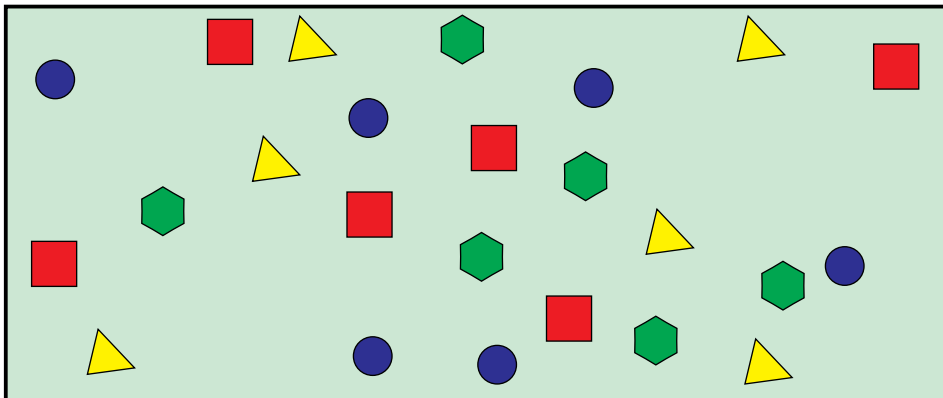
Layla wants to make a fish tank. She has a sheet of glass $2.4 \text{ m} \times 1.2 \text{ m}$ that she is going to cut to make the four sides, base and lid of her fish tank. She wants to choose dimensions that will give the largest volume of water in the tank.

1. Find three different ways that this sheet of glass could be cut, with as little wastage as possible, to make a fish tank with rectangular sides, base and lid. There is one solution that uses all the glass.
2. Calculate how many litres of water each tank would hold.



PUZZLE

Divide this diagram by drawing three straight lines to produce six sections with each section containing a circle, square, triangle and hexagon.



CHAPTER REVIEW

1. Change the following lengths to the units shown in the brackets.

(a) 14 cm (mm)	(b) 8.3 m (cm)	(c) 4 km (m)
(d) 0.56 m (mm)	(e) 9.75 cm (mm)	(f) 1.26 km (m)
(g) 83 mm (cm)	(h) 450 cm (m)	(i) 3450 m (km)
(j) 2.9 cm (m)	(k) 0.055 m (mm)	(l) 35 600 mm (m)

2. Change the following lengths to the units shown in the brackets.

(a) 2 m 11 cm (m)	(b) 3 cm 2 mm (cm)
(c) 6 m 6 cm (m)	(d) 8 m 75 mm (m)

3. Round the following lengths to the nearest metre.

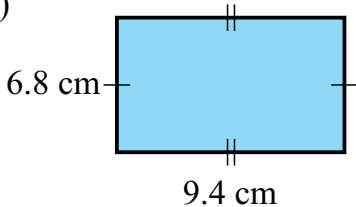
(a) 4.6 m	(b) 7 m 6 cm	(c) 3 m 350 mm
(d) 2 m 78 cm	(e) 0.7852 km	(f) 398 658 mm

4. At a timber yard, sheets of timber laminate, each 5 mm thick, are stacked to a height of 1.6 m. How many sheets of timber laminate are there?

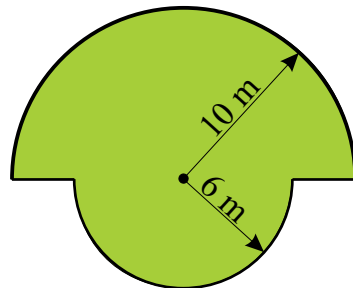
5. Give the following answers to one decimal place
 - (a) Find the circumference of the circle with a diameter of 6.8 m.
 - (b) Find the circumference of the circle with a radius of 23.6 cm.

6. Find the perimeter of the following shapes.
Give answers correct to one decimal place.

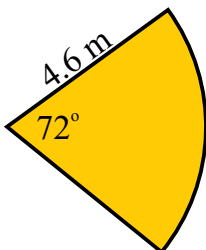
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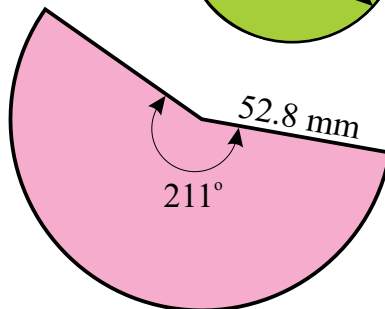
(b)



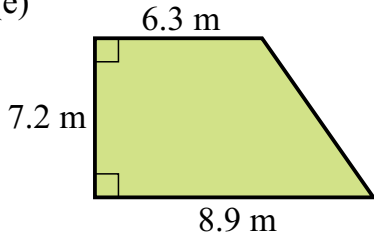
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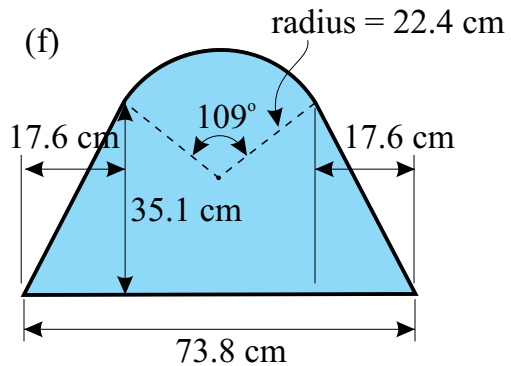
(d)



(e)



(f)



7. The diameter of the wheels on a bicycle is 750 mm. How many times will each wheel turn if the bicycle travels 25 km?

Give answer to the nearest revolution.

8. Convert the following areas to the units shown in the brackets.

(a) 800 mm^2 (cm^2)

(b) 0.26 cm^2 (mm^2)

(c) $150\,000 \text{ cm}^2$ (m^2)

(d) 0.84 m^2 (cm^2)

(e) $70\,000\,000 \text{ m}^2$ (km^2)

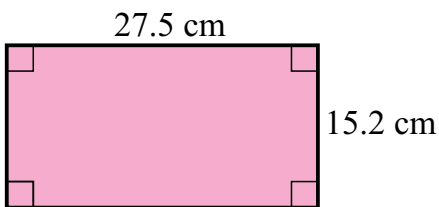
(f) 0.0083 km^2 (m^2)

9. (a) How many hectares is a rectangular paddock $500 \text{ m} \times 300 \text{ m}$?

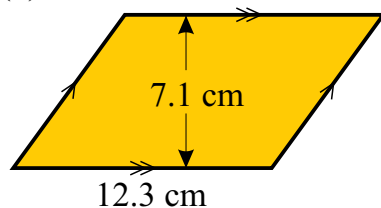
(b) What is the side length of a square park that is 49 hectares?

10. Find the area of the following shapes.

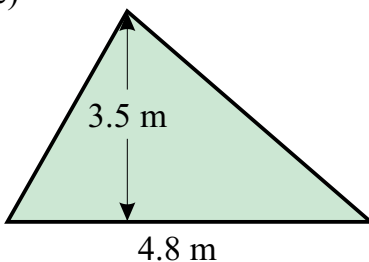
(a)



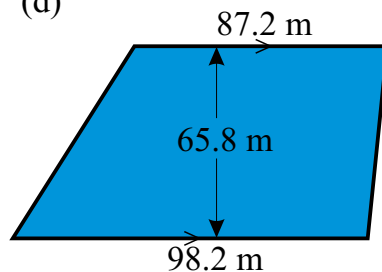
(b)



(c)

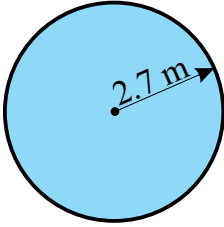


(d)

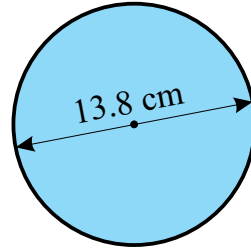


11. Find the area of these shapes correct to one decimal place.

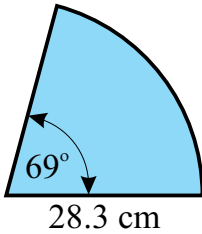
(a)



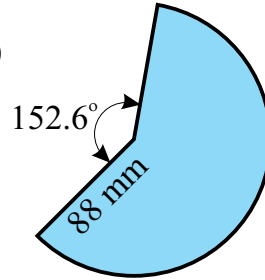
(b)



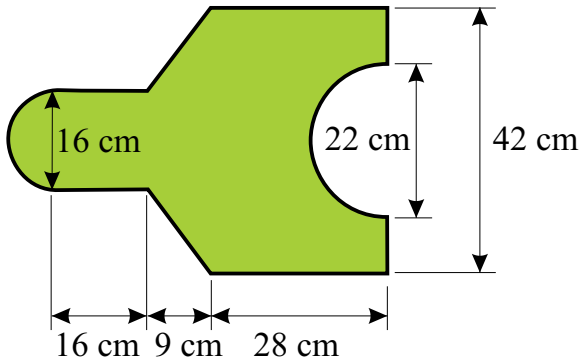
(c)



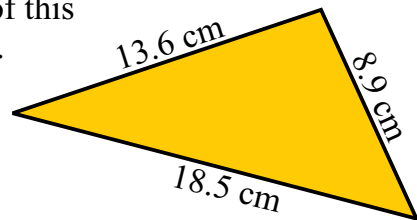
(d)



12. Find the area of the following symmetrical shape.
Give answer correct to one decimal place.

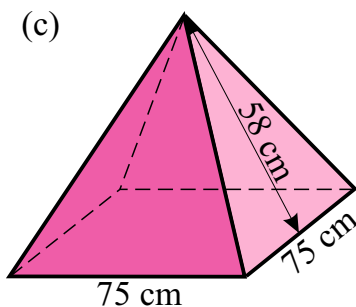
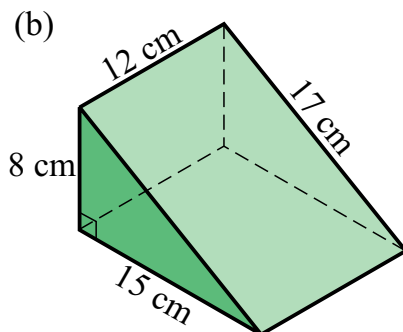
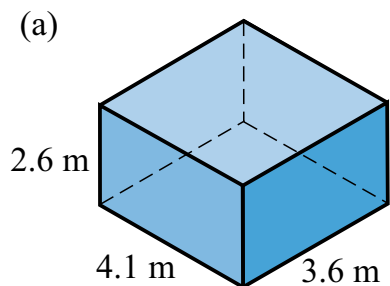


13. Use Hero's formula to find the area of this triangle correct to one decimal place.

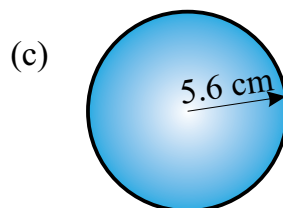
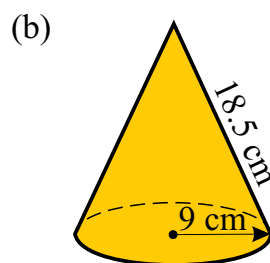
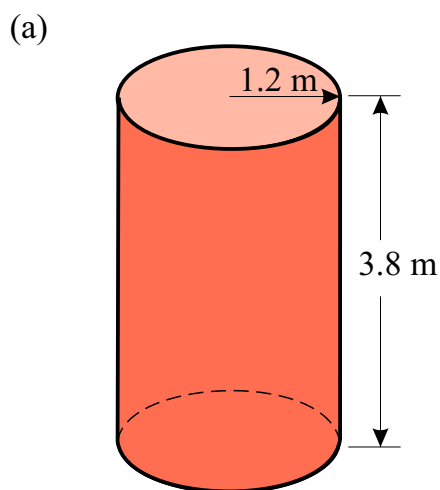


14. Find the side length of a square, correct to one decimal place, that has the same area as a circle with a diameter of 56.8 cm.

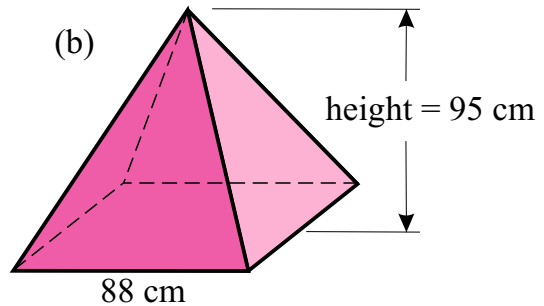
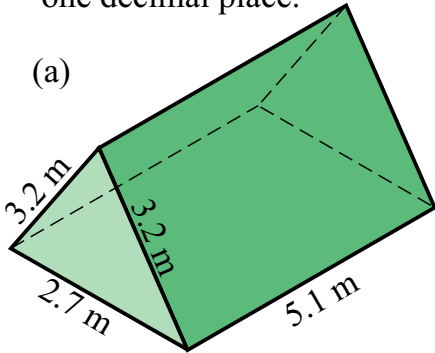
15. Find the surface area of the following objects.



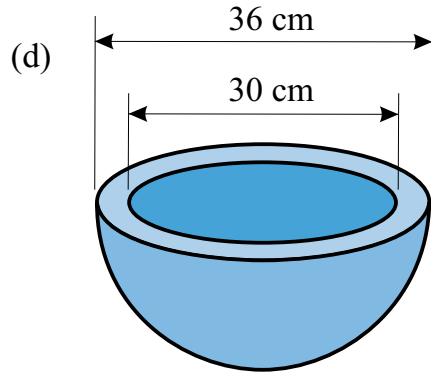
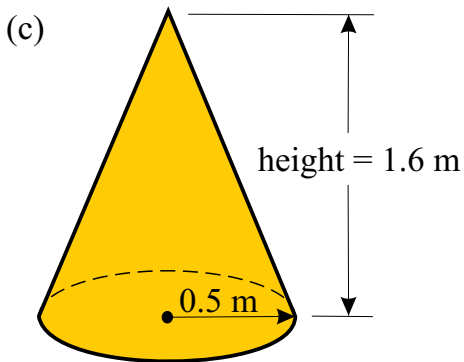
16. Find the surface area of the following objects correct to one decimal place.



17. Find the surface area of the following objects correct to one decimal place.



This pyramid has a square base



18. Convert the following volumes to the units shown in the brackets.

(a) $80\,000\text{ cm}^3$ (L)

(b) 0.035 cm^3 (mm^3)

(c) 700 mL (L)

(d) 3.2 m^3 (L)

(e) 0.035 L (mL)

(f) $90\,000\text{ mm}^3$ (cm^3)

(g) 800 L (m^3)

(h) 0.7 mL (mm^3)

(i) 8 cm^3 (mL)

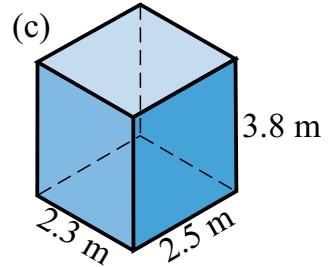
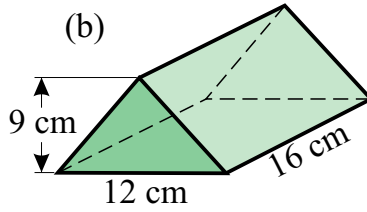
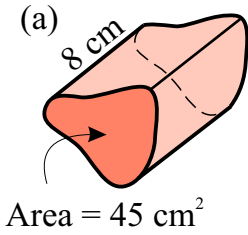
(j) $100\,000\text{ mm}^3$ (L)

(k) 2.4 L (cm^3)

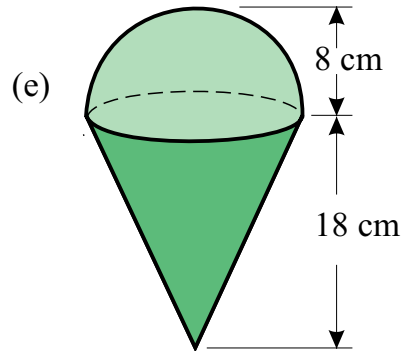
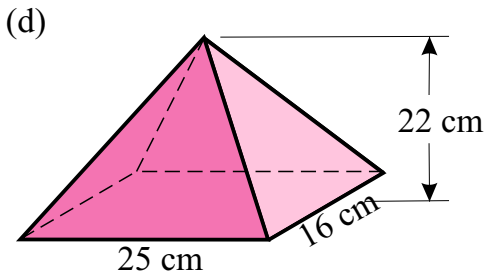
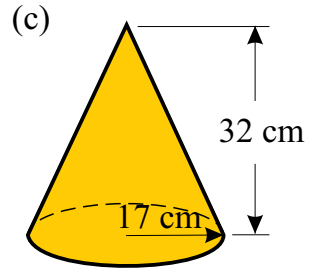
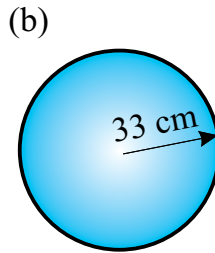
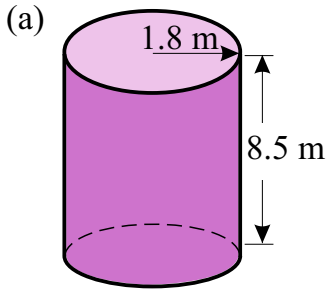
(l) 0.75 mL (cm^3)

19. A bucket holds 10 litres of water. How many buckets full of water would be needed to fill a fish tank that is 1.2 m long, 750 mm wide and 500 mm deep?

20. Find the volume of each of the following objects.



21. Find the volume of each of the following objects.
Give answers correct to one decimal place.



22. Find the volume of a box formed by cutting squares with side length 8 cm out of each corner of a rectangular board 50 cm long and 36 cm wide and folding.

23. Eight spherical balls with diameter 400 mm are dropped into a cylindrical container partially filled with water. The diameter of the cylindrical container is one metre. By how many millimetres will the water level rise? Give answer to nearest mm.

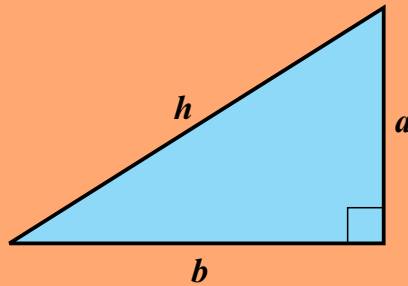
Pythagoras

Pythagoras' Theorem

Pythagoras' Theorem states that:

For any **right-angled triangle** the square of the longest side (the **hypotenuse**) is equal to the sum of the squares of the two shorter sides.

$$h^2 = a^2 + b^2$$

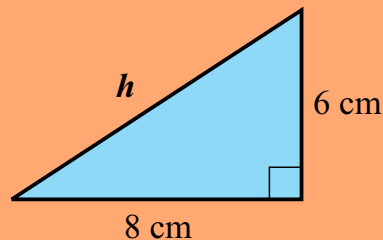


Finding the Hypotenuse

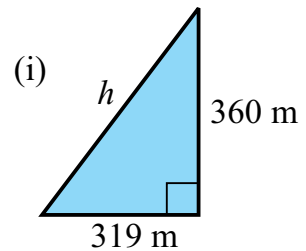
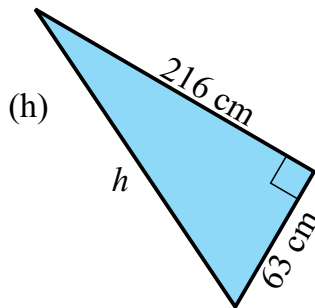
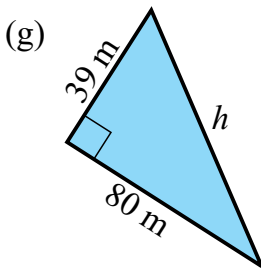
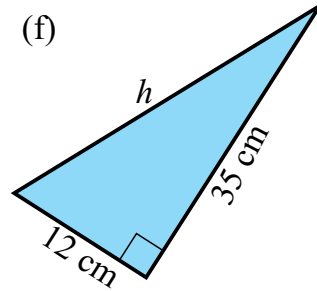
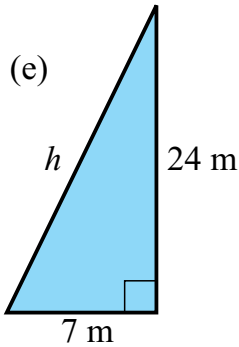
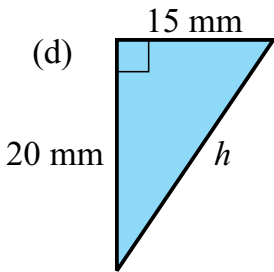
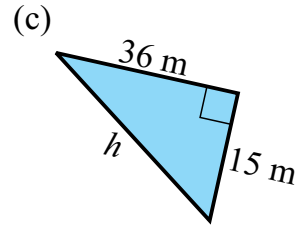
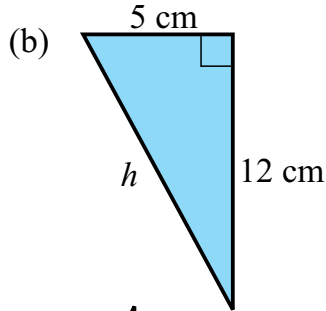
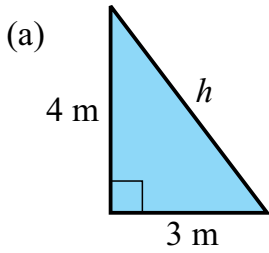
EXERCISE 12A

Example Find the length of the hypotenuse in the following right-angled triangle.

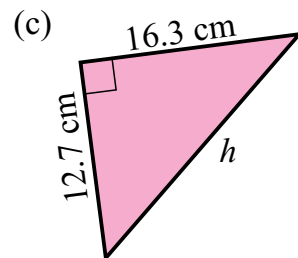
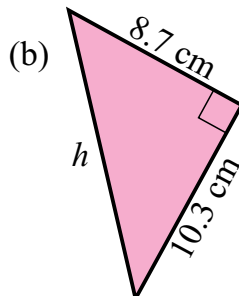
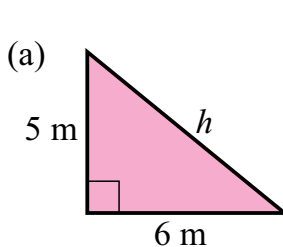
$$\begin{aligned}h^2 &= a^2 + b^2 \\h^2 &= 6^2 + 8^2 \\h^2 &= 36 + 64 \\h^2 &= 100 \\h &= \sqrt{100} \\h &= 10 \text{ cm}\end{aligned}$$



1. Find the length of the hypotenuse in each of the following triangles.

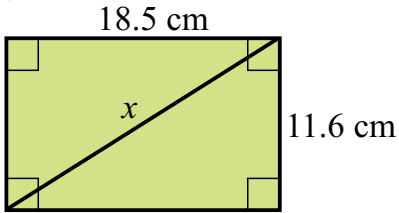


2. Find the length of the hypotenuse in each of the following triangles. Give answers correct to one decimal place.

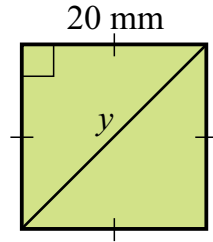


3. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

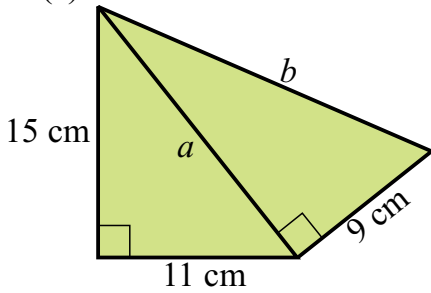
(a)



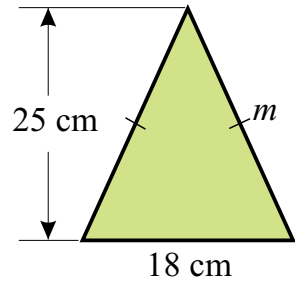
(b)



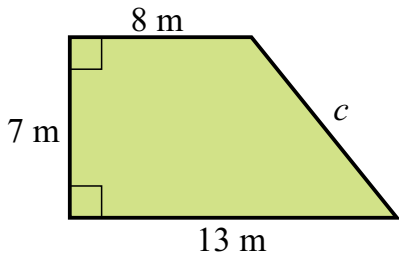
(c)



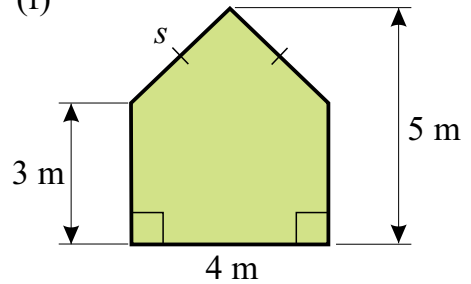
(d)



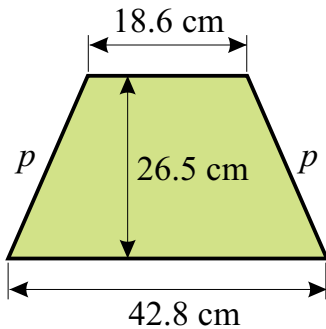
(e)



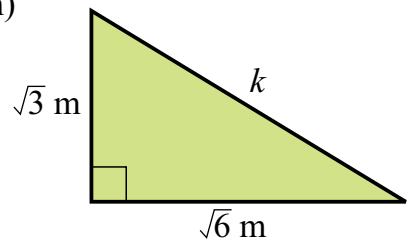
(f)



(g)



(h)



Finding One of the Shorter Sides

EXERCISE 12B

Example Find the length of the unknown side in the following right-angled triangle.

$$h^2 = a^2 + b^2$$

$$25^2 = a^2 + 20^2$$

$$625 = a^2 + 400$$

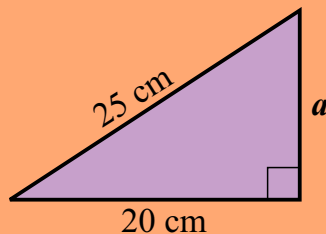
$$625 - 400 = a^2$$

$$225 = a^2$$

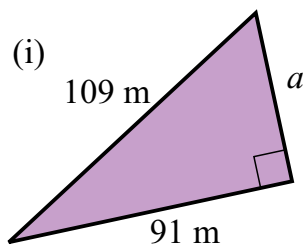
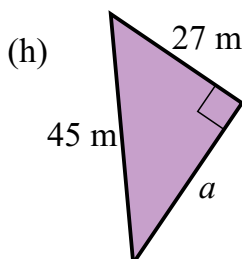
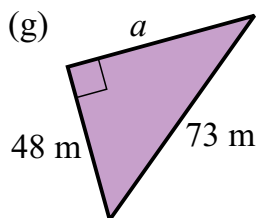
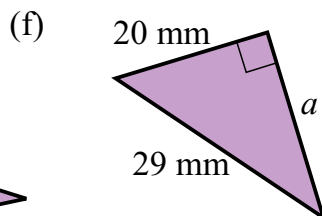
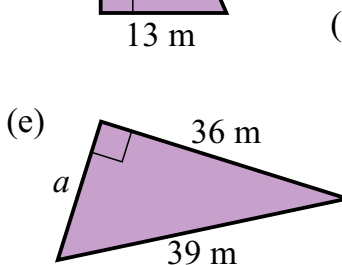
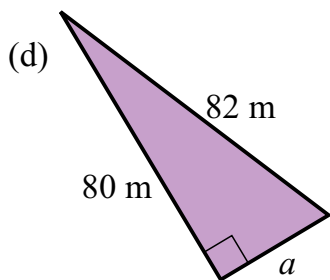
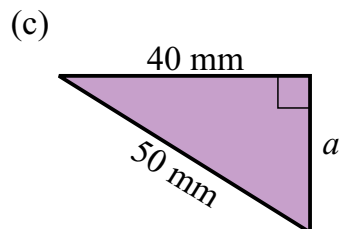
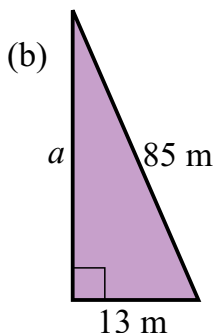
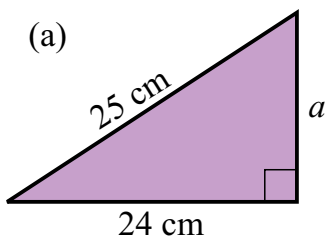
$$a^2 = 225$$

$$a = \sqrt{225}$$

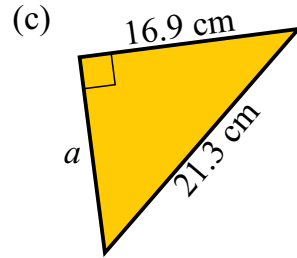
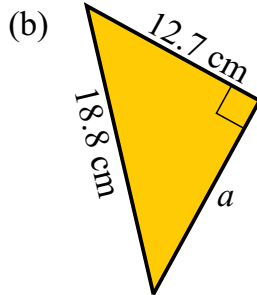
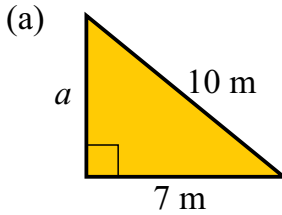
$$a = 15 \text{ cm}$$



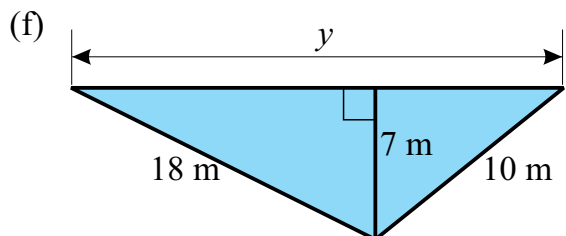
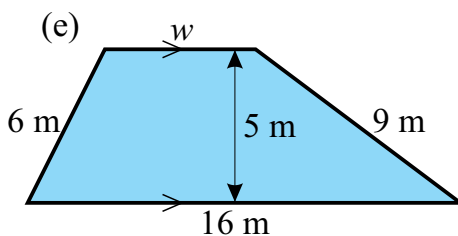
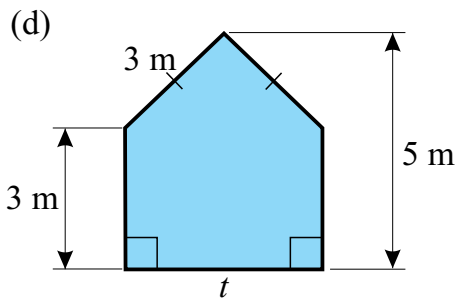
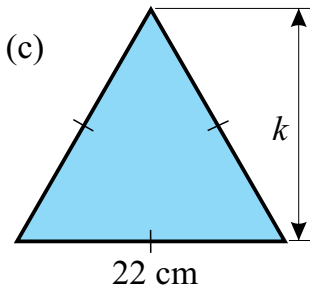
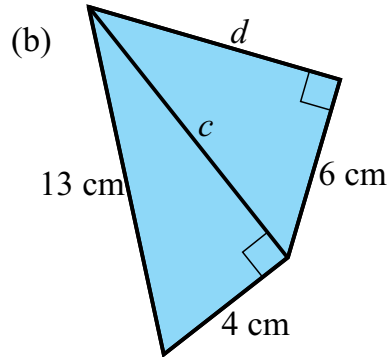
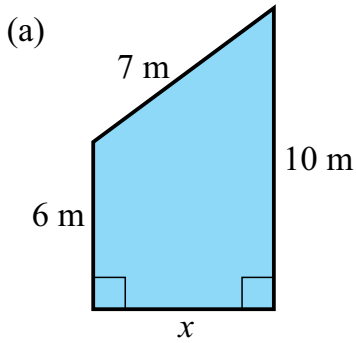
1. Find the length of the unknown side in each of the following triangles.



2. Find the length of the unknown side in each of the following triangles. Give answers correct to one decimal place.



3. Find the unknown lengths in the following shapes. Give answers correct to one decimal place.

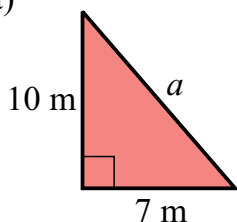


Mixed Problems

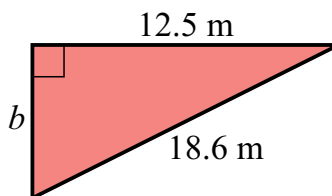
EXERCISE 12C

1. Calculate the length of a diagonal of a square with side length 25 cm. Give answer correct to one decimal place.
2. Calculate the width of a rectangle that is 208 cm long and has a diagonal that is 233 cm long.
3. Find the unknown lengths in the following shapes. Give answers correct to one decimal place.

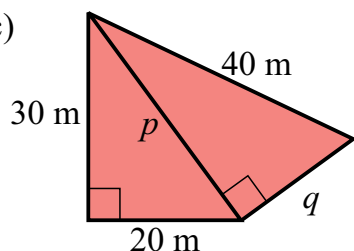
(a)



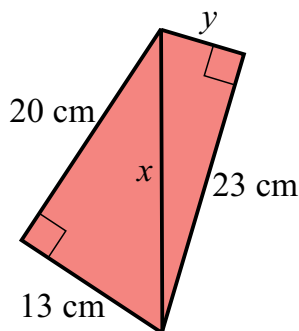
(b)



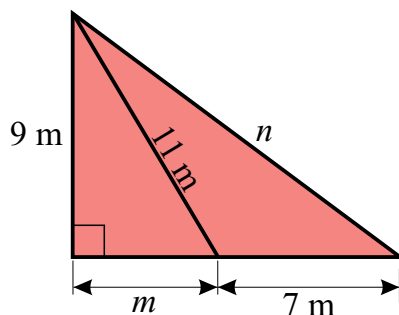
(c)



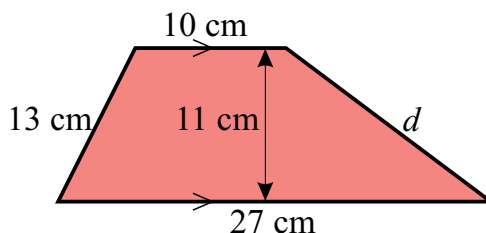
(d)



(e)



(f)



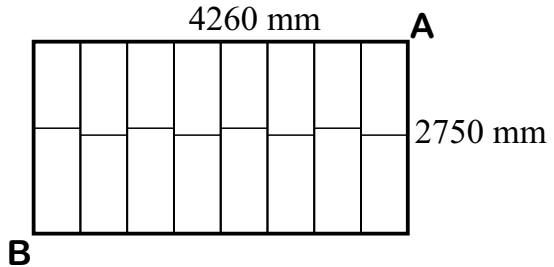
Practical Problems

EXERCISE 12D

1. (a) A naval patrol boat travelled 86 km west from base then 125 km north. How far is it from its base? Give answer correct to one decimal place.
- (b) On another patrol the naval boat travelled 74.3 km on a bearing of $N40^\circ W$ from its base then south 56.9 km where it was due west from its base. It then travelled in a straight line back to base. What is the total distance travelled? Give answer correct to one decimal place.

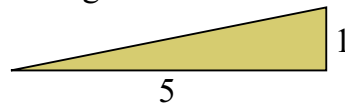


2. A builder has made a wall frame 4260 mm long and 2750 high as shown here.



- (a) What should be the length of the diagonal **AB** (to the nearest mm) for the corners to be right-angles?
- (b) He measured **AB** to be 4580 mm. Which direction should he move point **A** (left or right) for the frame to have right-angled corners?

3. The suggested slope for a ramp was one-in-five. This means the horizontal length is five times the vertical height as shown in this diagram.



What would be the ramp length, l , if the vertical height was 850 mm?

Give answer to the nearest mm.



Pythagoras in 3D

EXERCISE 12E

Example

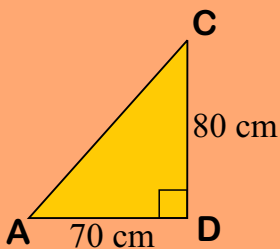
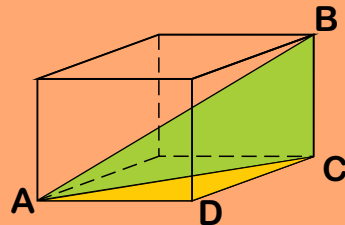
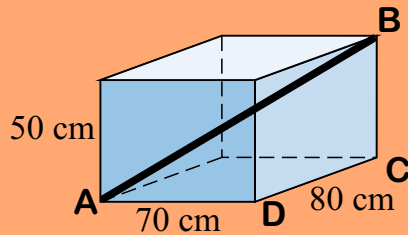
Find the length of the diagonal **AB** in the box shown.

Step 1 - Identify **AB** as the hypotenuse of triangle **ABC**.

Step 2 - Identify **AC** as the hypotenuse of triangle **ACD**.

Step 3 - Calculate **AC**.

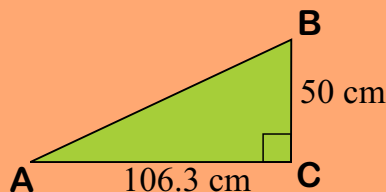
Step 4 - Calculate **AB**.



$$AC^2 = 70^2 + 80^2$$

$$AC^2 = 11300$$

$$AC = 106.3 \text{ cm}$$



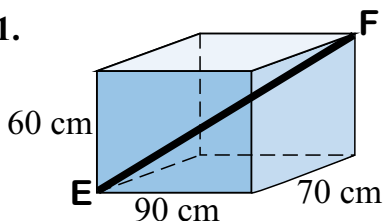
$$AB^2 = 106.3^2 + 50^2$$

$$AB^2 = 13800$$

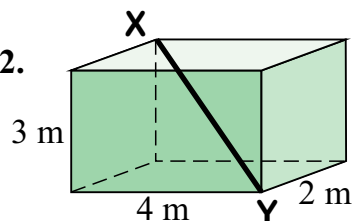
$$AB = 117.5 \text{ cm}$$

Find the length of the diagonals shown in the cuboids below.
Give answers correct to one decimal place.

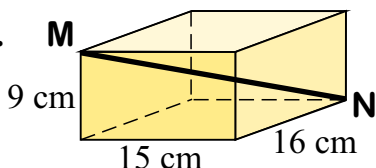
1.



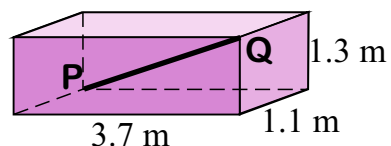
2.



3.

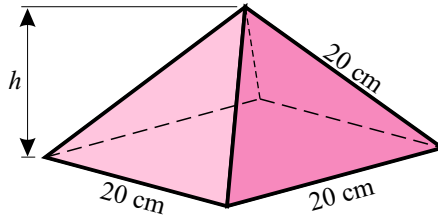


4.



PROBLEM SOLVING

Find the height, h , of this pyramid.
Give answer correct to two decimal places.



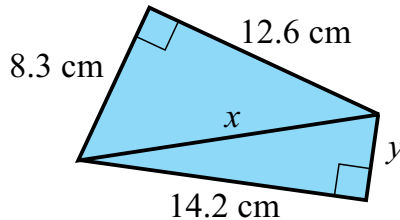
PUZZLE

Which three of the following numbers form a Pythagorean triad (follow Pythagoras Theorem).

15 35 112 144 147
 12 17 39 115 145

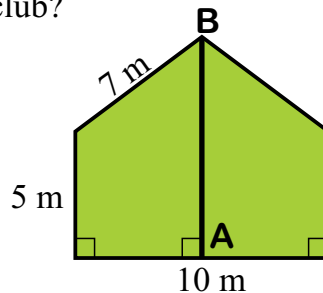
CHAPTER REVIEW

1. Find the unknown lengths.
Give answers correct to one decimal place.

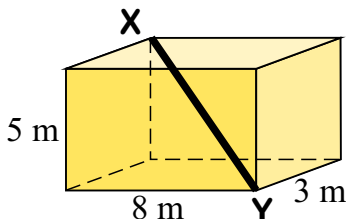


2. A yacht sails 15 km north from the yacht club then 12.5 km west.
How far is the yacht from the yacht club?

3. Find the length of **AB**.

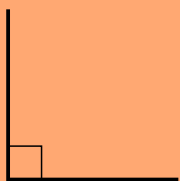


4. Find the length of the **XY** in the cuboid below.

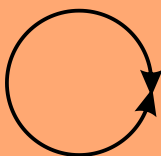


Geometry

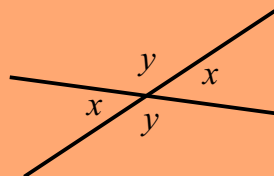
Angles



A **right angle** = 90°



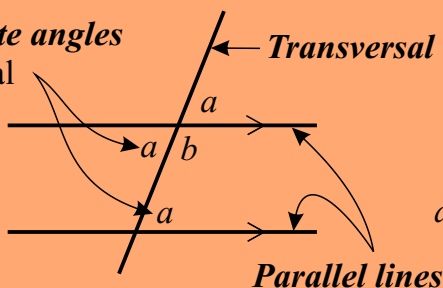
A **revolution** = 360°



Vertically opposite angles are equal and:

$$x + y = 180^\circ$$

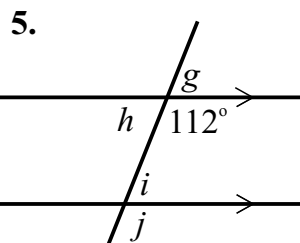
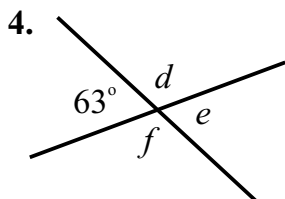
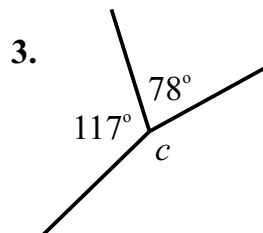
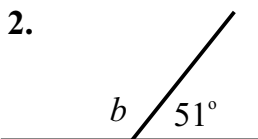
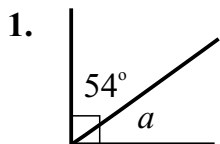
Alternate angles are equal



$$a + b = 180^\circ$$

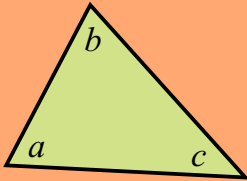
EXERCISE 13A

Find the unknown angles.



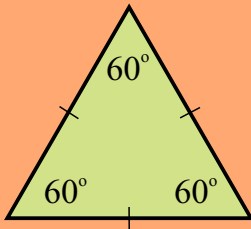
Triangles and Quadrilaterals

TRIANGLES

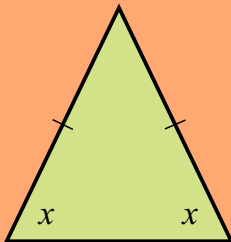


The sum of the angles in a triangle is 180° .

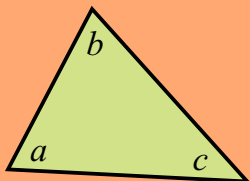
$$a + b + c = 180^\circ$$



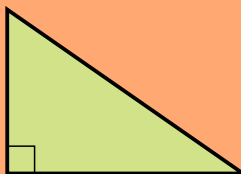
A triangle with all three sides of equal length has all angles of equal size (60°) and is called an ***equilateral triangle***.



A triangle with two sides of equal length has two equal angles (those opposite the equal sides) and is called an ***isosceles triangle***.

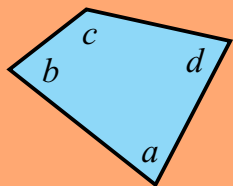


A triangle with all its sides and angles different is called a ***scalene triangle***.



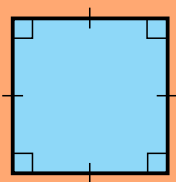
A triangle with one angle equal to 90° is called a ***right-angled triangle***.

QUADRILATERALS

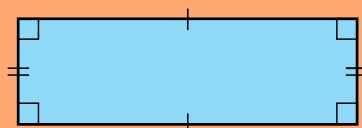


The sum of the angles in a quadrilateral is 360° .

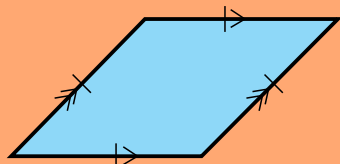
$$a + b + c + d = 360^\circ$$



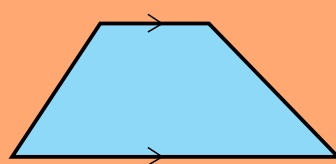
Square



Rectangle

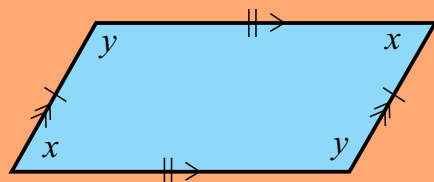


Rhombus



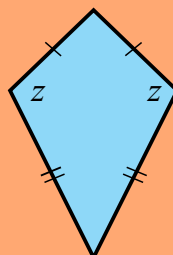
Trapezium

One pair of parallel sides



Parallelogram

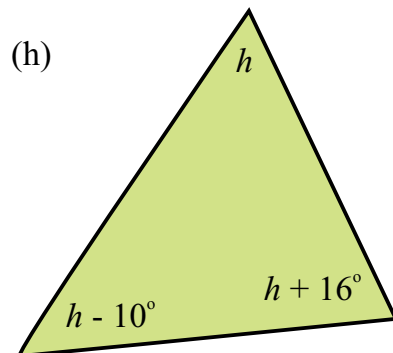
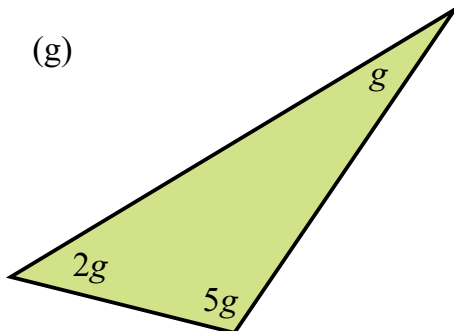
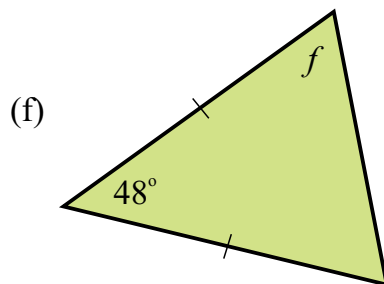
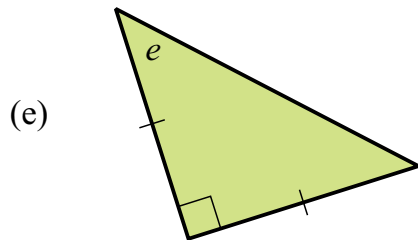
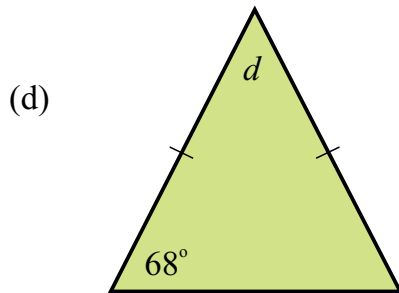
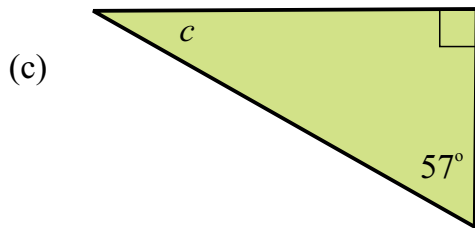
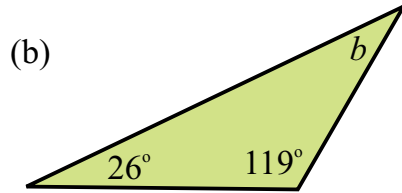
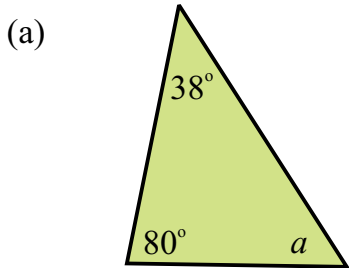
$$x + y = 180^\circ$$



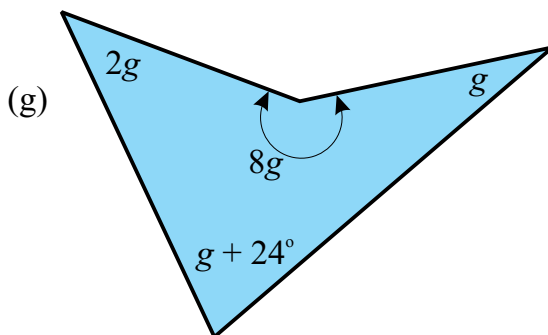
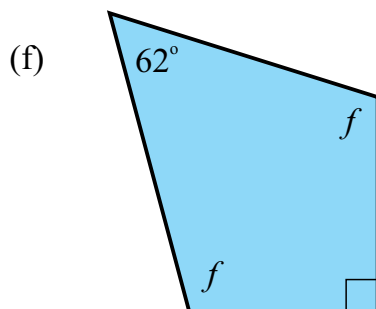
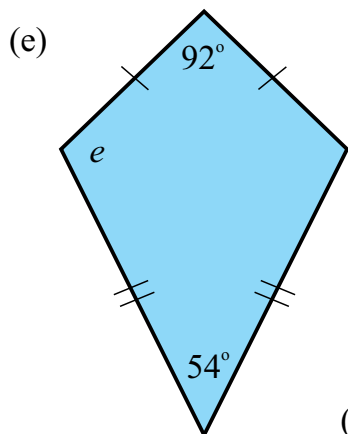
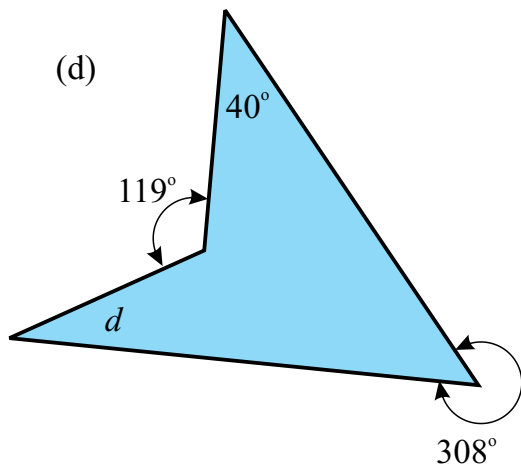
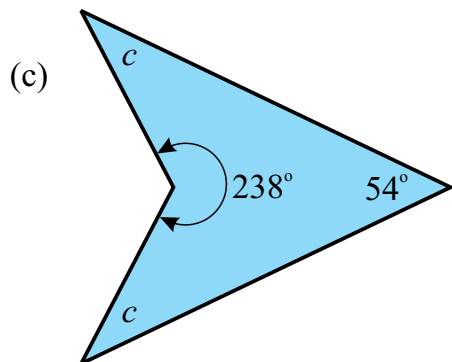
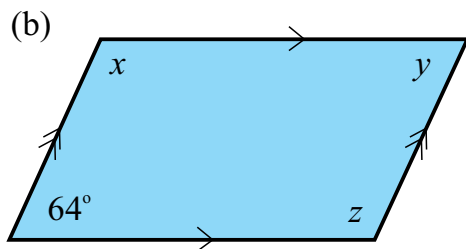
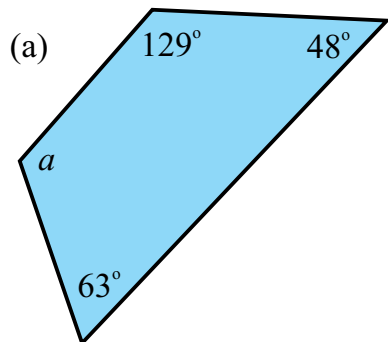
Kite

EXERCISE 13B

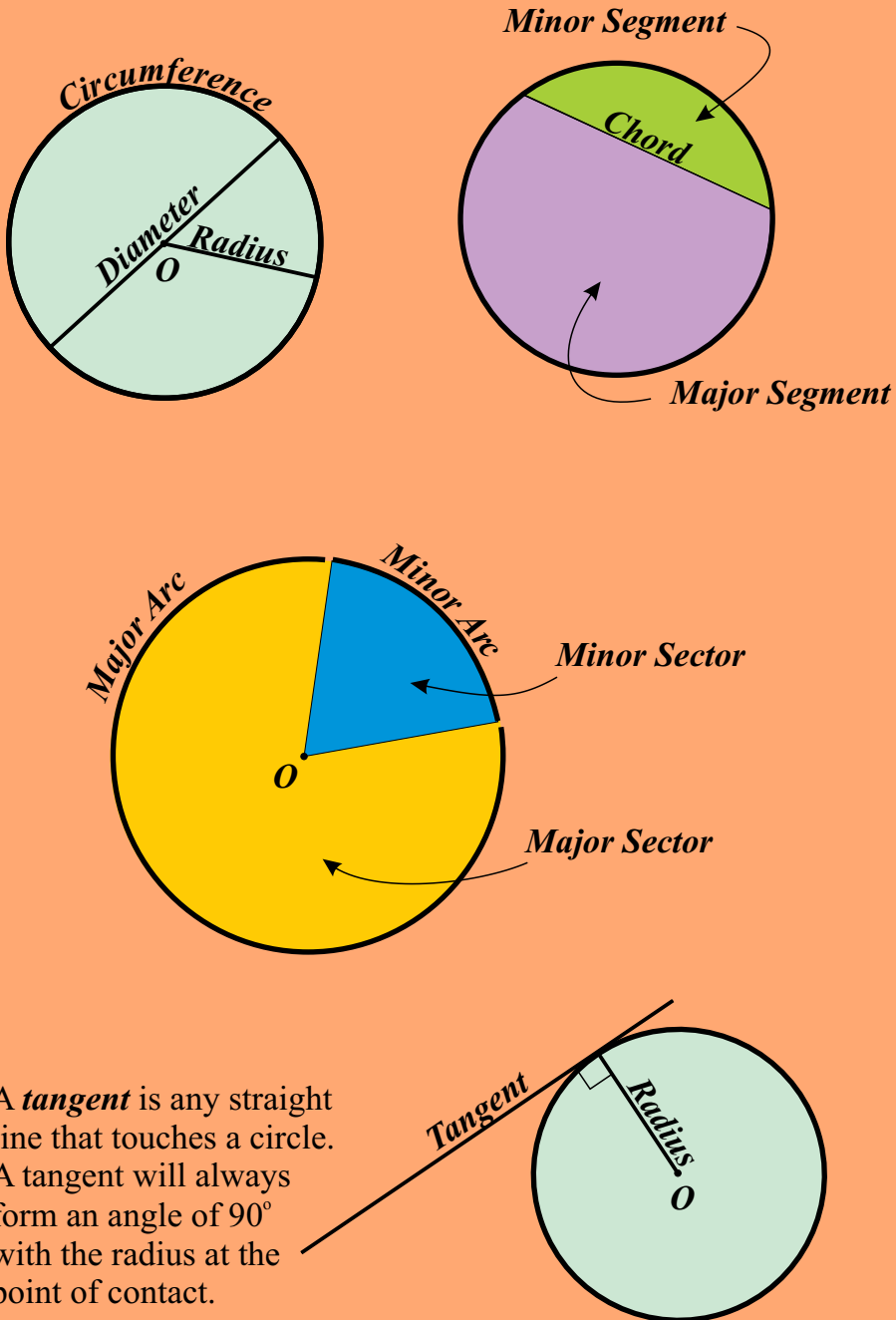
1. Find the unknown angles in the following triangles.



2. Find the unknown angles in the following quadrilaterals.

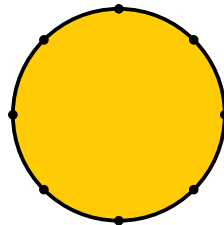


Parts of Circles



EXERCISE 13C

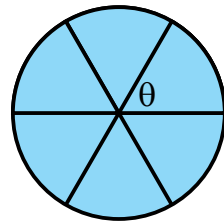
1. How many chords could be drawn in this circle by connecting the points shown? (There are eight points)



2. (a) A circle is divided into six equal sectors as shown.

What is the centre angle, θ , of each sector?

- (b) A circle is divided into 15 equal sectors. What is the centre angle of each of these sectors?

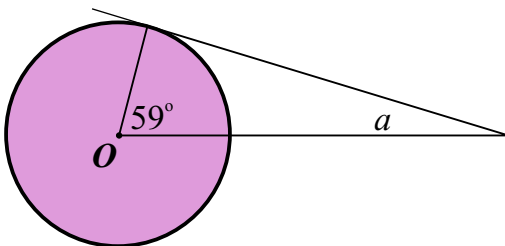


3. Which part of a circle is defined by each of the following descriptions?

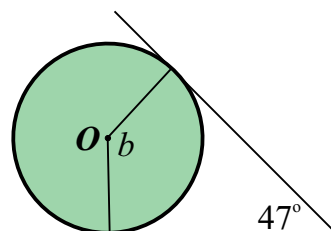
- The area enclosed by a chord and a major arc.
- A straight line joining two points on the circumference and passing through the centre.
- A line joining two points on the circumference and not passing through the centre.
- The area of a circle not included in a major sector.
- A straight line outside of a circle that touches the circle at one point.
- The area enclosed by a chord and an arc that makes an angle of 150° with the centre.
- The area enclosed by two radii and an arc that makes an angle of 200° with the centre.

4. Find the unknown angles.

(a)



(b)



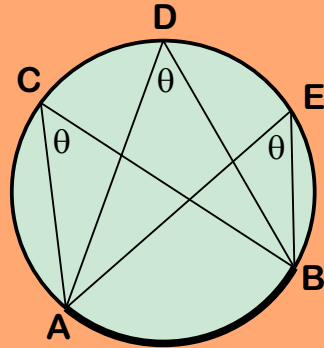
Angles in Circles

Theorem 1

All the angles at the circumference standing on the same arc are equal.

In this diagram, angles formed from arc **AB** to any other point in the same segment are equal.

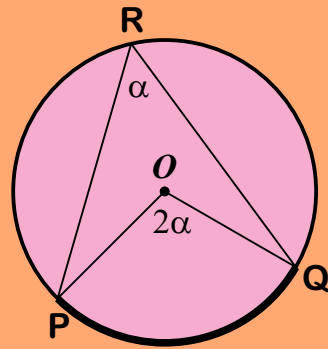
$$\angle ACB = \angle ADB = \angle AEB$$



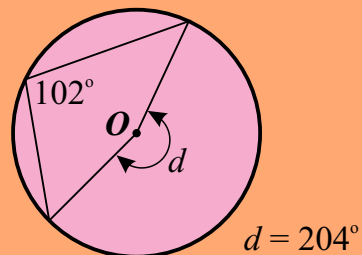
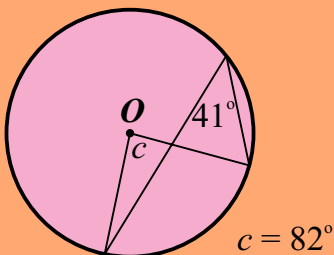
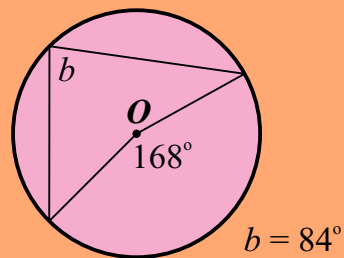
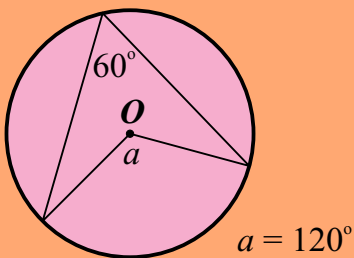
Theorem 2

The angle that an arc forms at the centre of a circle is twice the size of the angle formed at the circumference.

$$\angle POQ = 2 \times \angle PRQ$$

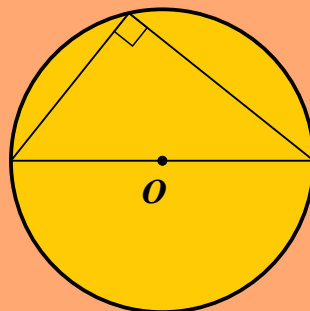


Examples



Theorem 3

The angle formed at the circumference from a diameter of a circle is always a **right angle**.

**Theorem 4**

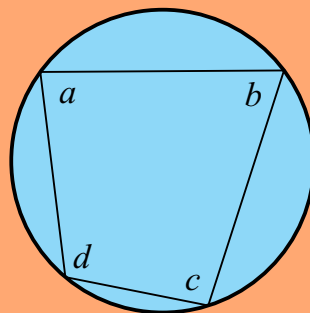
A **cyclic quadrilateral** is a quadrilateral formed in a circle with all its vertices touching the circumference.

The sum of the opposite angles in a cyclic quadrilateral is 180° .

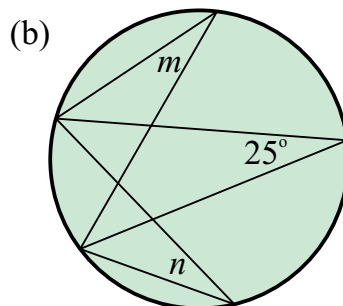
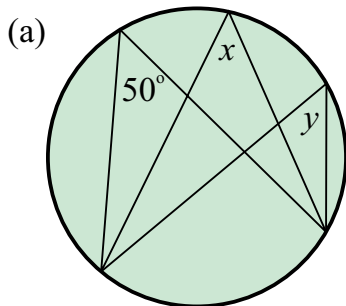
In this diagram:

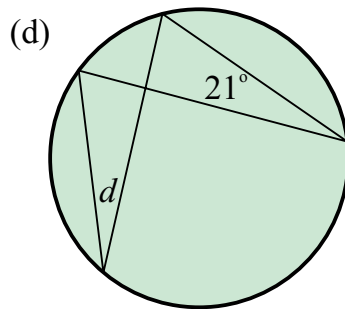
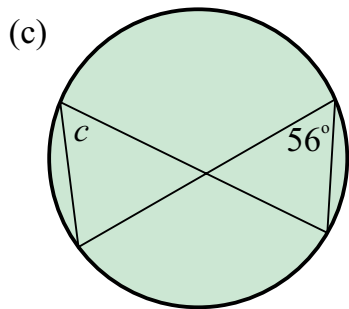
$$a + c = 180^\circ$$

$$b + d = 180^\circ$$

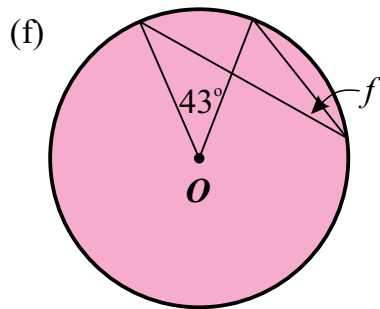
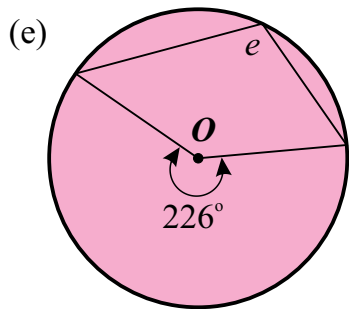
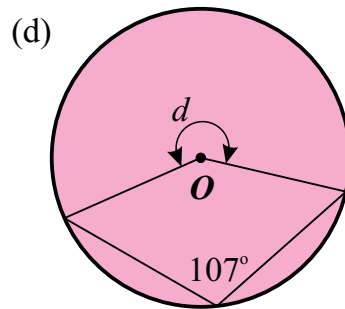
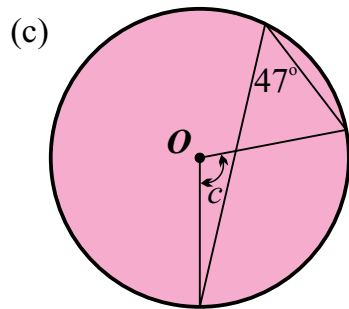
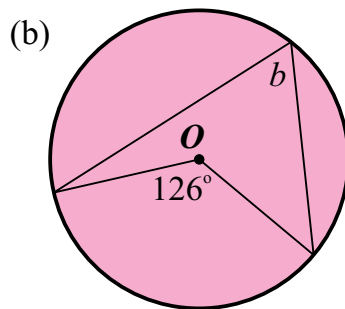
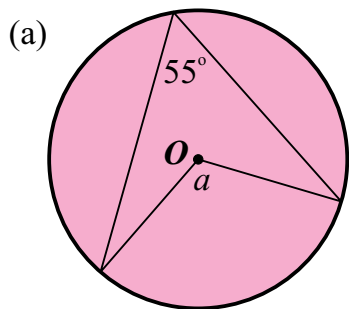
**EXERCISE 13D**

1. Use Theorem 1 to find the unknown angles in the following circles.



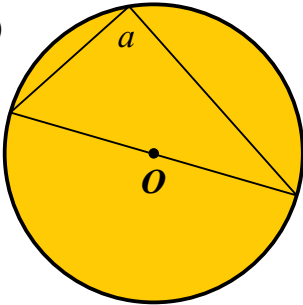


2. Use Theorem 2 to find the unknown angles in the following circles.

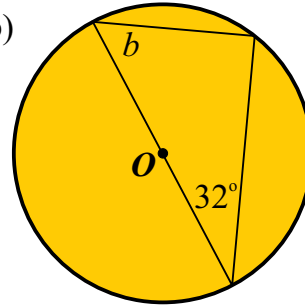


3. Use Theorem 3 to find the unknown angles in the following circles.

(a)

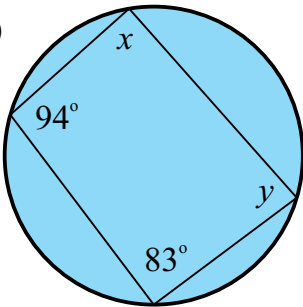


(b)

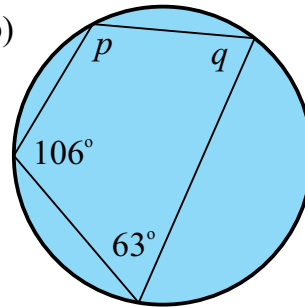


4. Use Theorem 4 to find the unknown angles in the following circles.

(a)

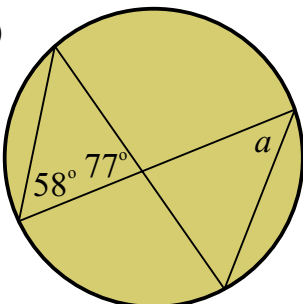


(b)

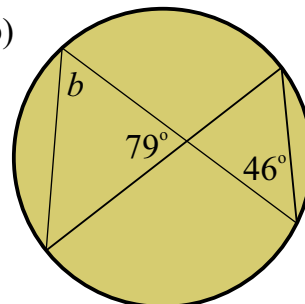


5. Use Theorems 1 - 4 and other geometric relationships to find the unknown angles in the following circles.

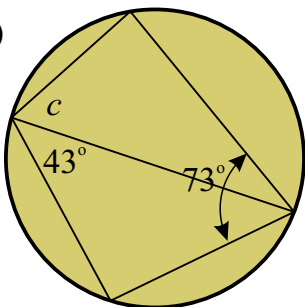
(a)



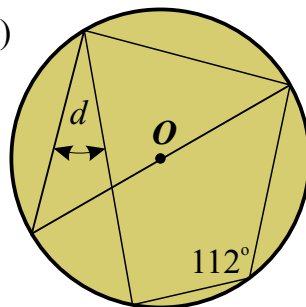
(b)



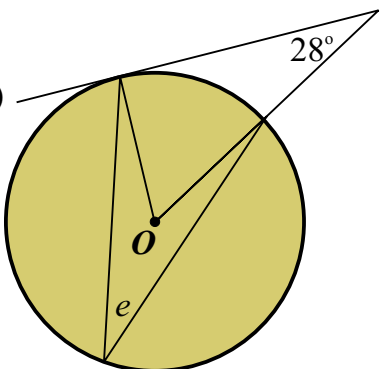
(c)



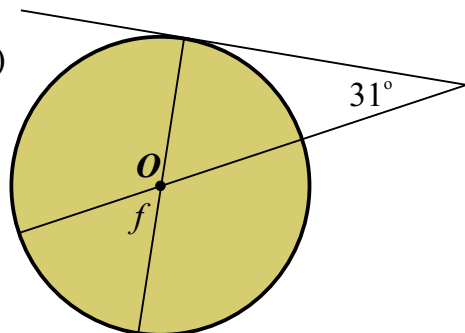
(d)



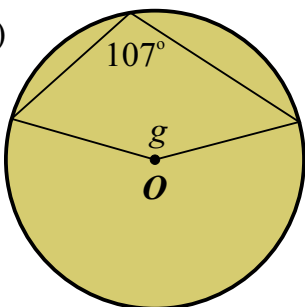
(e)



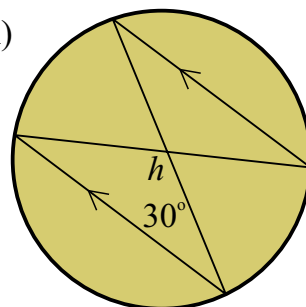
(f)



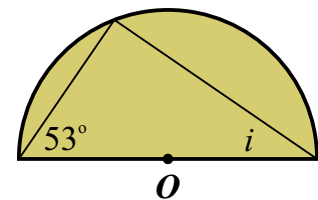
(g)



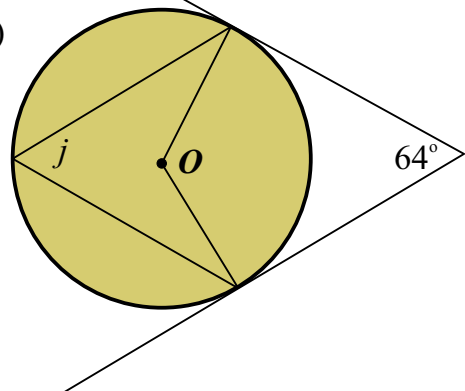
(h)



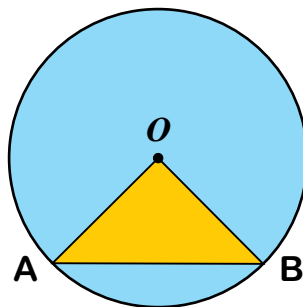
(i)



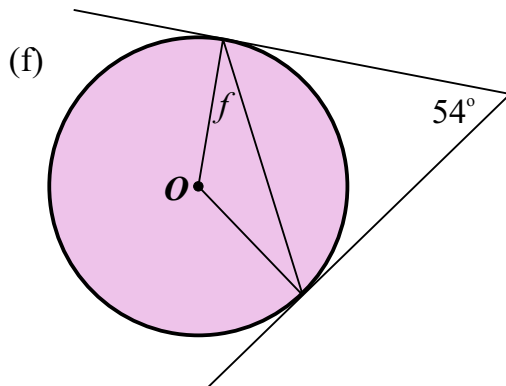
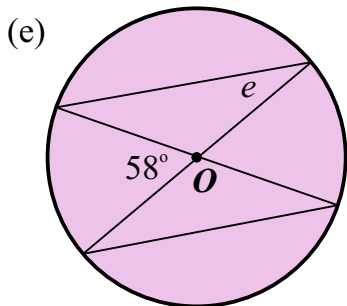
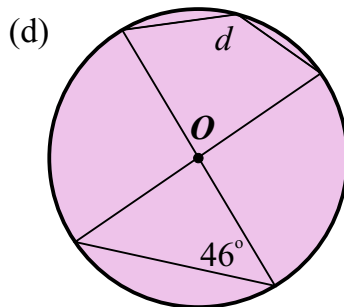
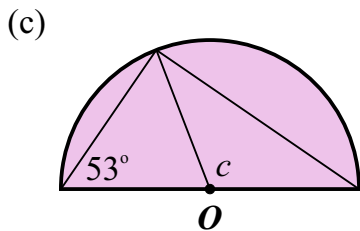
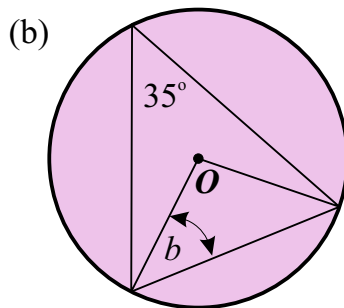
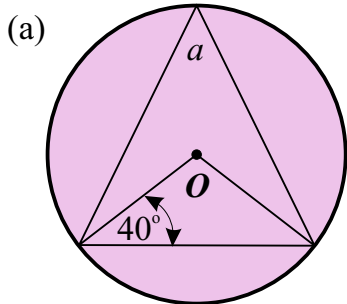
(j)

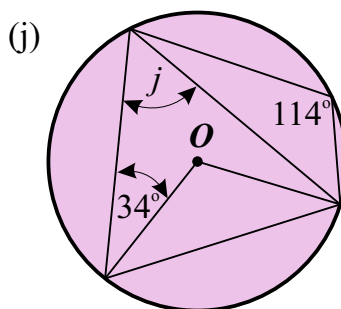
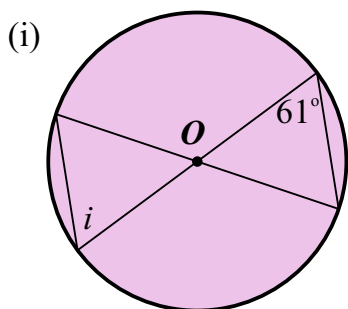
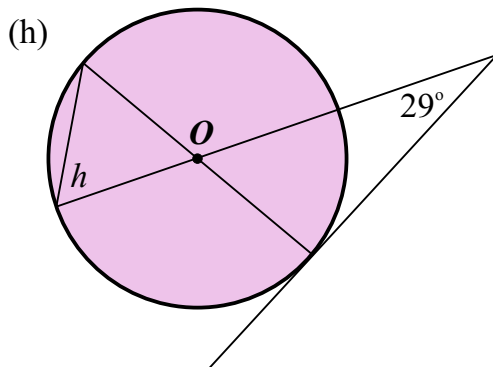
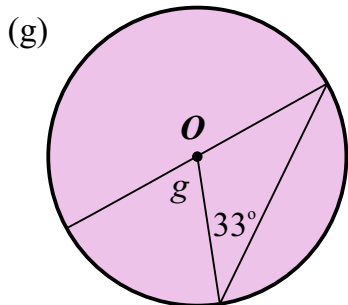


6. (a) What type of triangle is formed using a chord and two radii of a circle?
 (b) Explain why.
 (c) What is the relationship between $\angle OAB$ and $\angle OBA$?



7. Use the result from question 6 to help find the unknown angles in the following circles.





8. Using this diagram complete the following.

(a) What type of triangle has points

A, B and **O** as its vertices?

(b) In terms of x , what is the size of $\angle \mathbf{BAO}$?

(c) In terms of x , what is the size of $\angle \mathbf{AOB}$?

(d) In terms of x , what is the size of $\angle \mathbf{AOD}$?

(e) What type of triangle has points **B, C** and **O** as its vertices?

(f) In terms of y , what is the size of $\angle \mathbf{BCO}$?

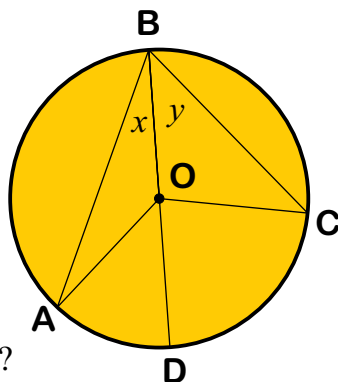
(g) In terms of y , what is the size of $\angle \mathbf{BOC}$?

(h) In terms of y , what is the size of $\angle \mathbf{COD}$?

(i) In terms of x and y , what is the size of $\angle \mathbf{ABC}$?

(j) In terms of x and y , what is the size of $\angle \mathbf{AOC}$? (Use the answers from parts (d) and (h)).

(k) Compare the sizes of $\angle \mathbf{ABC}$ and $\angle \mathbf{AOC}$. Which circle theorem does this prove?



Similar Triangles

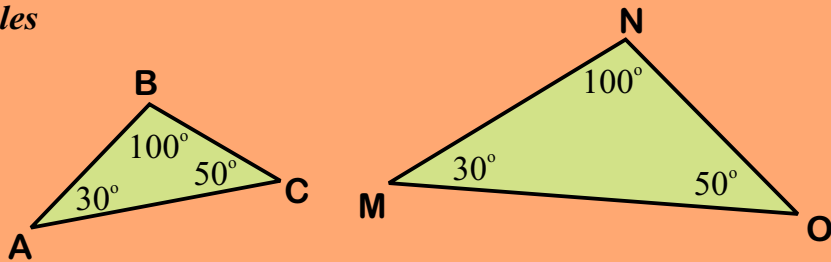
Definition:

Triangles are called *similar triangles* if:

- all their *corresponding angles* are the *same and*
- their *corresponding side lengths* are *different*

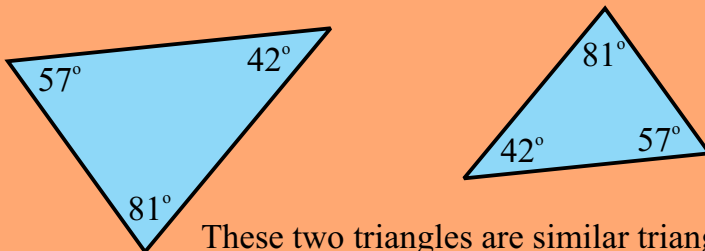
Examples

1.



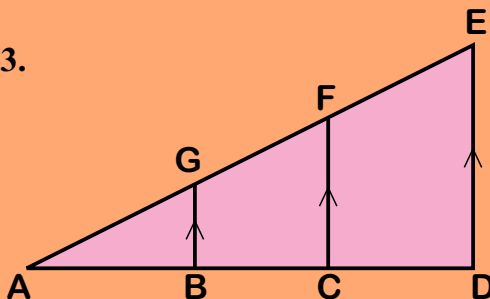
Triangles **ABC** and **MNO** are similar triangles.

2.



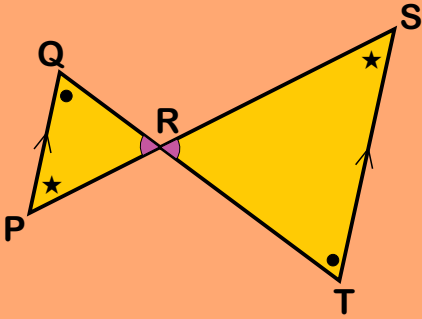
These two triangles are similar triangles.
The first triangle has been rotated and reduced to form the second triangle.

3.



Triangles **ABG**, **ACF** and **ADE** are all similar triangles.

4.

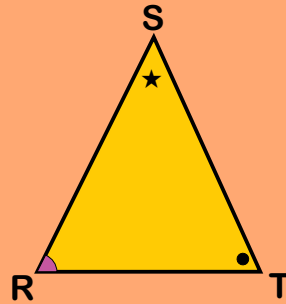
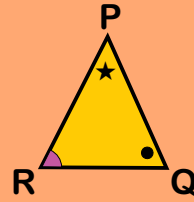


Triangles **QRP** and **TRS**
are similar triangles.

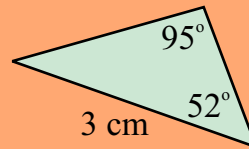
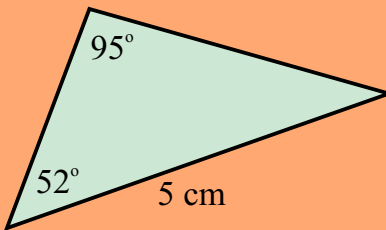
$\angle RPQ = \angle RST$ (alternate angles)

$\angle PQR = \angle STR$ (alternate angles)

$\angle QRP = \angle TRS$ (vertically opposite)



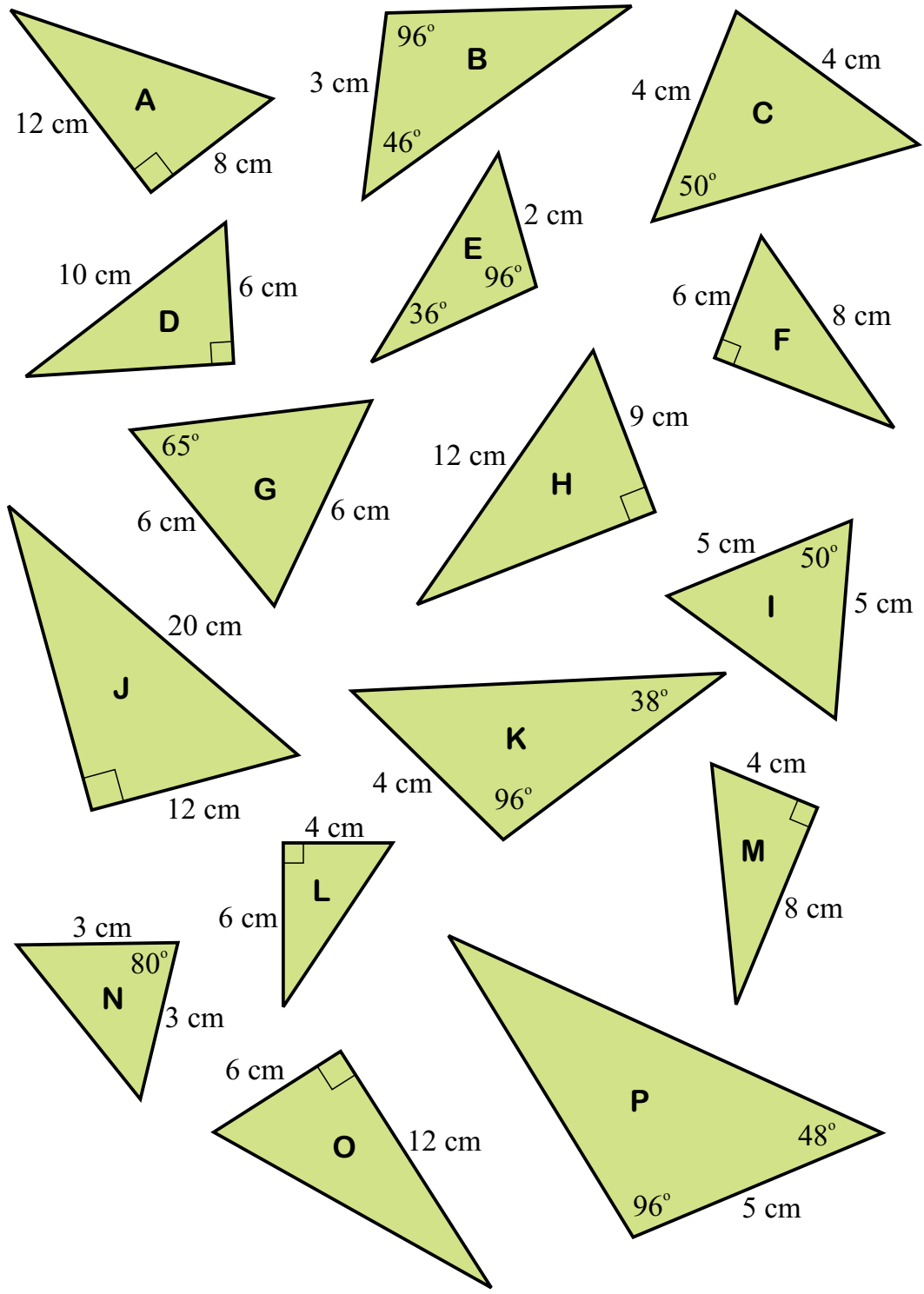
5.



These two triangles are similar triangles.
The first triangle has been reflected,
rotated and reduced to form the second
triangle.

EXERCISE 13E

Match the pairs of similar triangles from the triangles below.



To find unknown lengths in similar triangles:

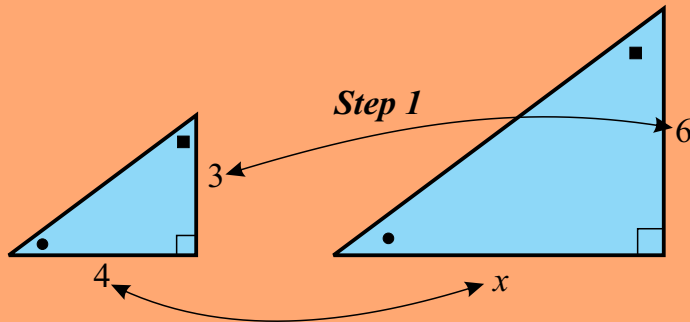
Step 1 Match corresponding sides

Step 2 Write them as equivalent ratios

Step 3 Calculate the unknown value

Examples

1.



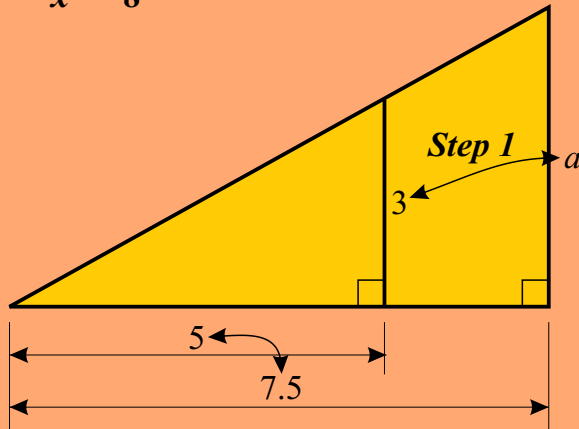
Step 2 $\frac{x}{4} = \frac{6}{3}$

Step 3 $\frac{x}{4} = \frac{6}{3}$ Cross multiply

$$3x = 24$$

$$x = 8$$

2.

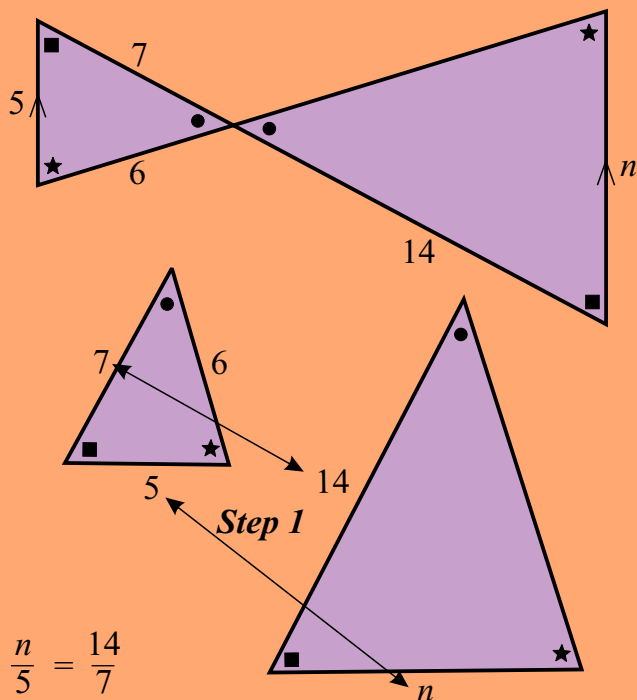


Step 2 $\frac{a}{3} = \frac{7.5}{5}$

Step 3 $5a = 22.5$

$$a = 4.5$$

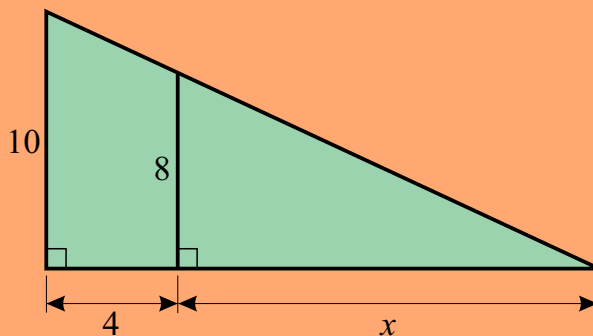
3.



Step 2 $\frac{n}{5} = \frac{14}{7}$

Step 3 $7n = 70$
 $n = 10$

4.



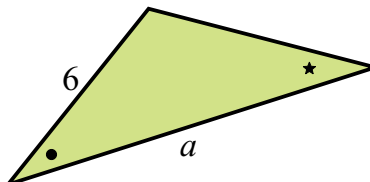
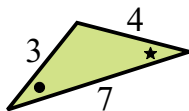
Step 2 $\frac{x}{8} = \frac{x+4}{10}$

Step 3 $10x = 8(x+4)$
 $10x = 8x + 32$
 $2x = 32$
 $x = 16$

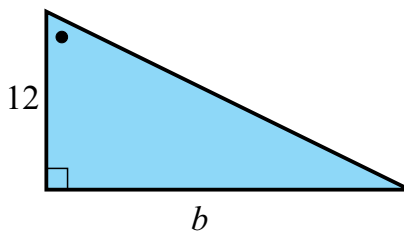
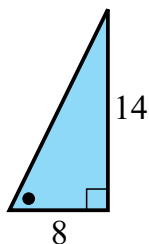
EXERCISE 13F

1. Find the unknown lengths.

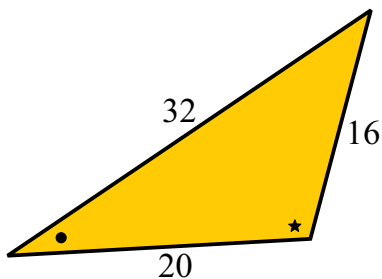
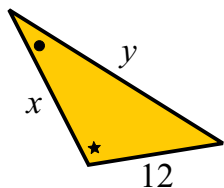
(a)



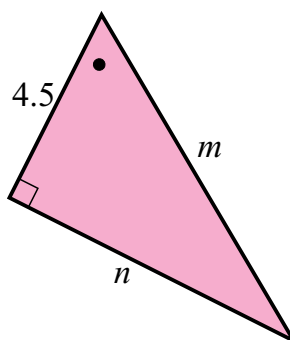
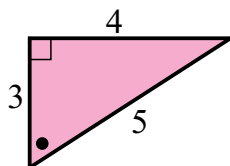
(b)



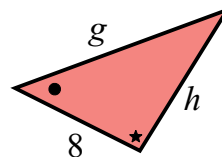
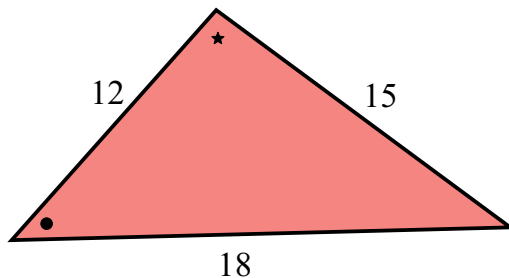
(c)



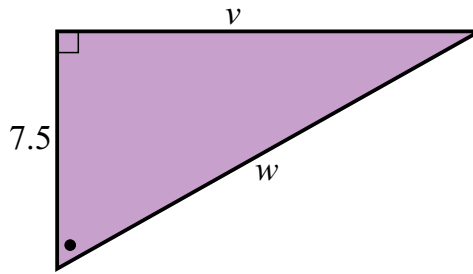
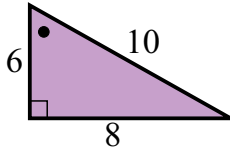
(d)



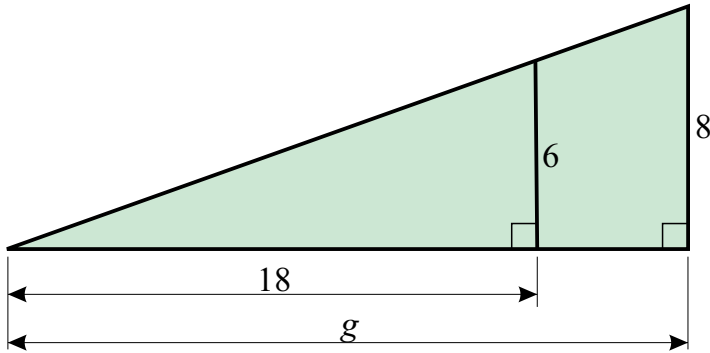
(e)



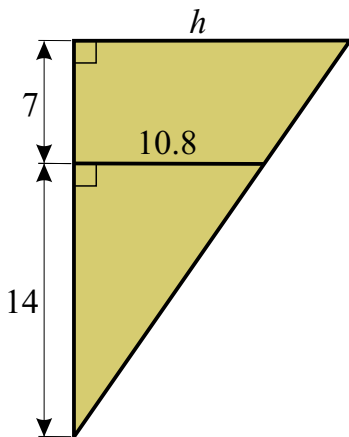
(f)



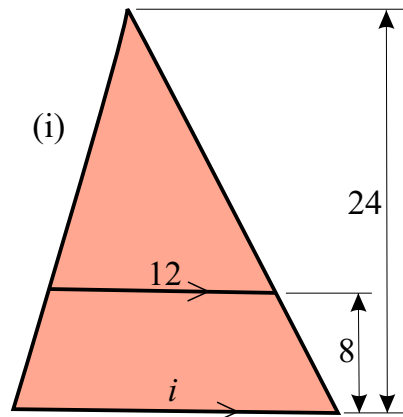
(g)



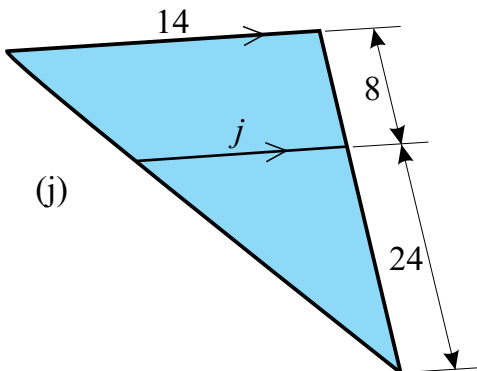
(h)



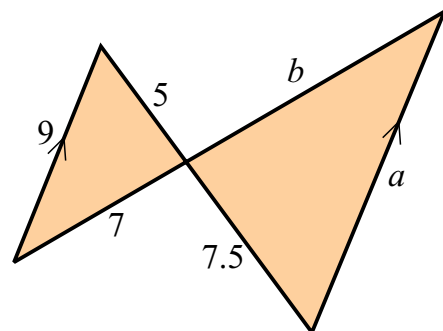
(i)



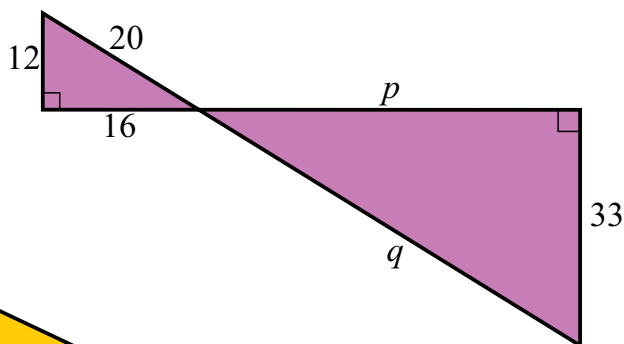
(j)



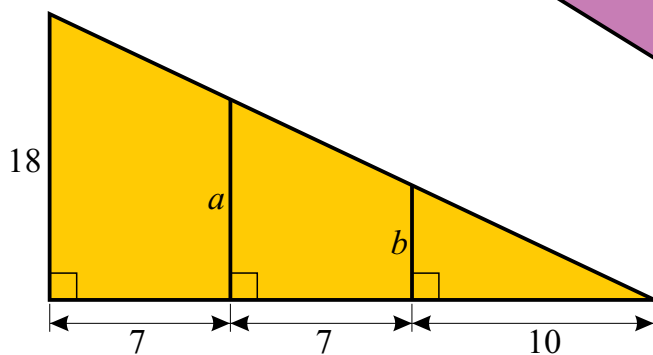
(k)



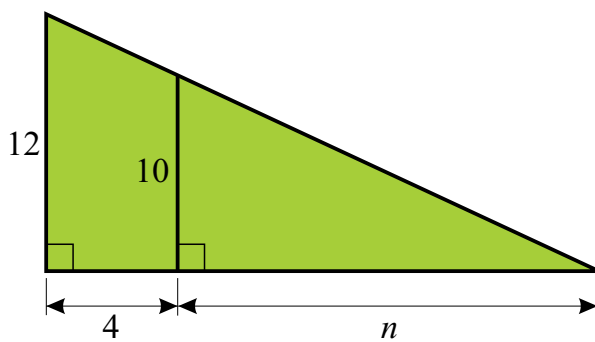
(l)



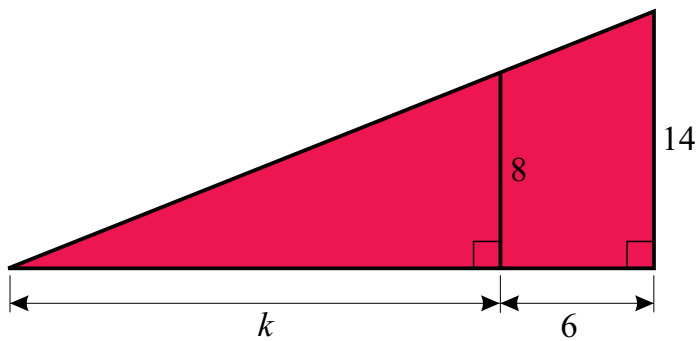
(m)



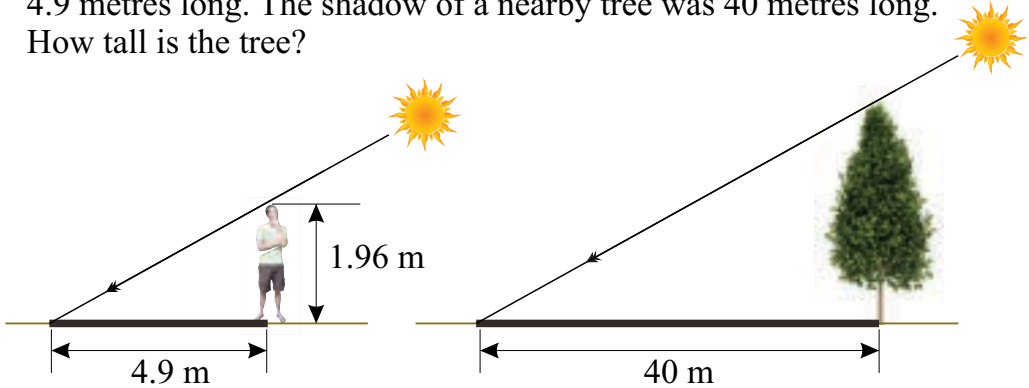
(n)



(o)

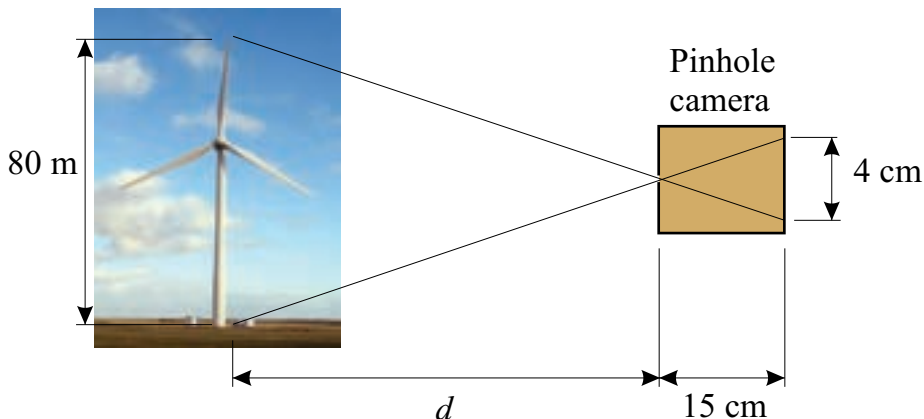


2. James is 1.96 metres tall. He measured his shadow one day to be 4.9 metres long. The shadow of a nearby tree was 40 metres long. How tall is the tree?



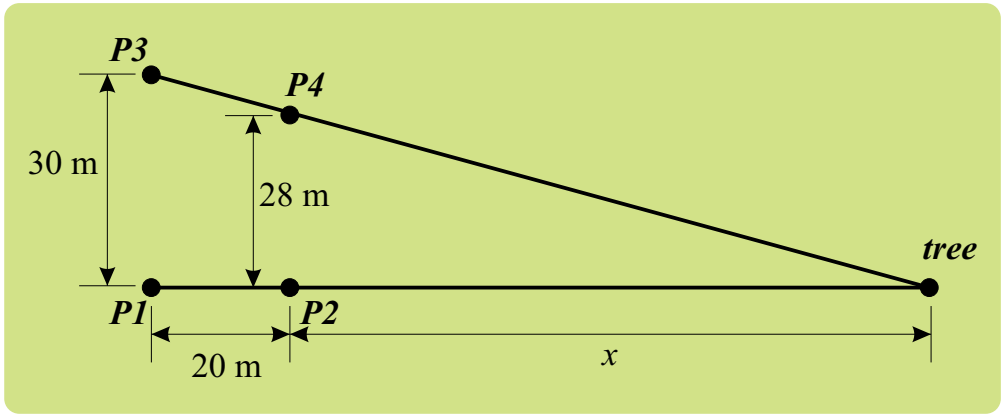
3. Sally shone a torch on a basketball and noticed its shadow to be the same height as a flagpole nearby. The basketball had a diameter of 24 cm. The basketball was 80 cm from the torch. The flagpole was 10 m from the torch. How tall was the flagpole?

4. Michael used a pinhole camera to calculate the distance he was from a wind turbine.



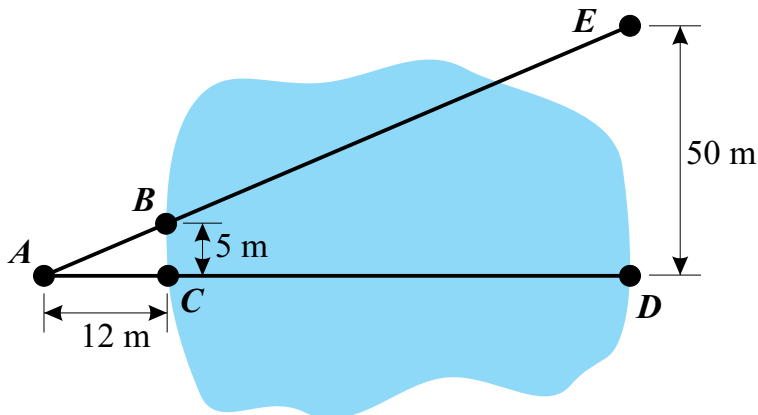
- The wind turbine was 80 metres high.
 The image of the turbine on the back of the camera was 4 cm high.
 The pin hole camera was 15 cm deep.
 How far, d , (in metres) is Michael from the wind turbine?

5. Two hikers were trying to calculate the distance to a tree. They hammered four poles ($P1$ - $P4$) in the ground as shown in the diagram below.



$P1$ and $P2$ are in line with the tree.
 $P3$ and $P4$ are in line with the tree.
 Find the distance, x , from $P2$ to the tree.

6. Fiona and Rob want to calculate the width of a lake. Fiona is standing on one side of the lake and Rob on the other. Fiona puts three stakes (A, B and C) in the ground and Rob puts two stakes (D and E) in the ground as shown on the diagram below.



Use the measurements shown on the diagram to calculate the width of the lake (the distance from C to D).

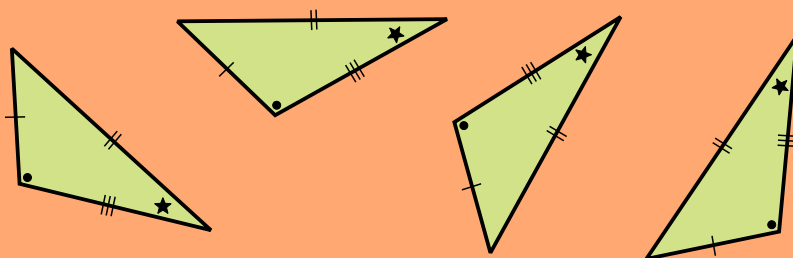
Congruent Triangles

Definition:

Triangles are called **congruent triangles** if they are exactly the same:

- all their **corresponding angle** sizes are the **same and**
- all their **corresponding side lengths** are the **same**

Example: All the triangles below are congruent triangles

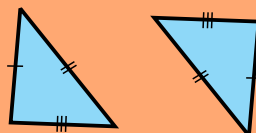


One of three tests can be used to identify congruent triangles.

1. Side-side-side (SSS)

If two triangles have the same side lengths they are congruent.

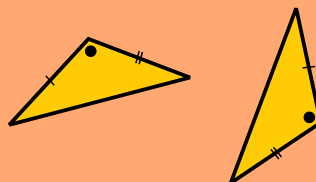
OR



2. Side-angle-side (SAS)

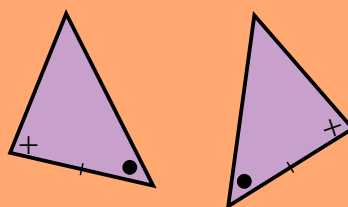
If two side lengths and the contained angle are the same.

OR



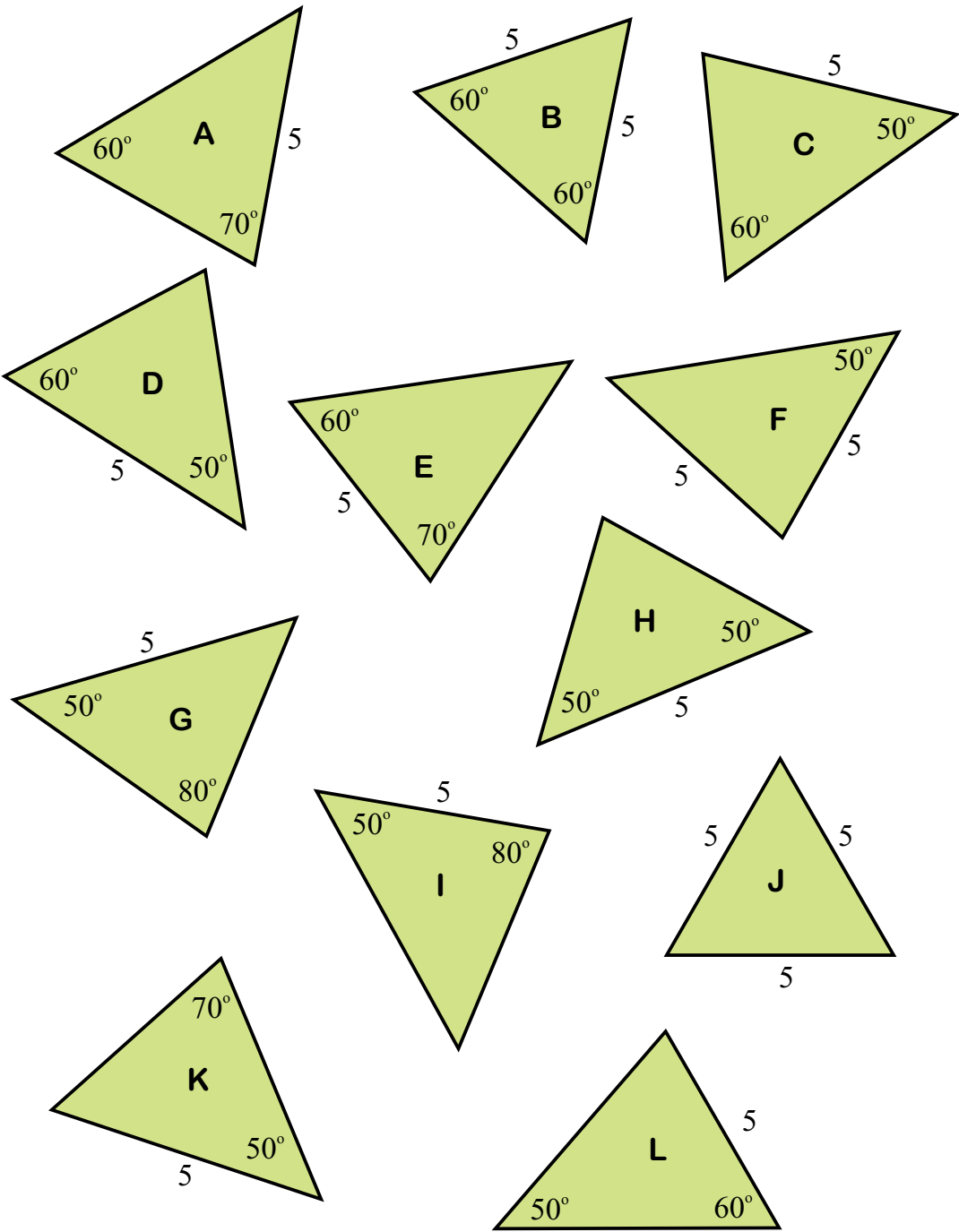
3. Angle-side-angle (ASA)

If two angles and the contained side length are the same.



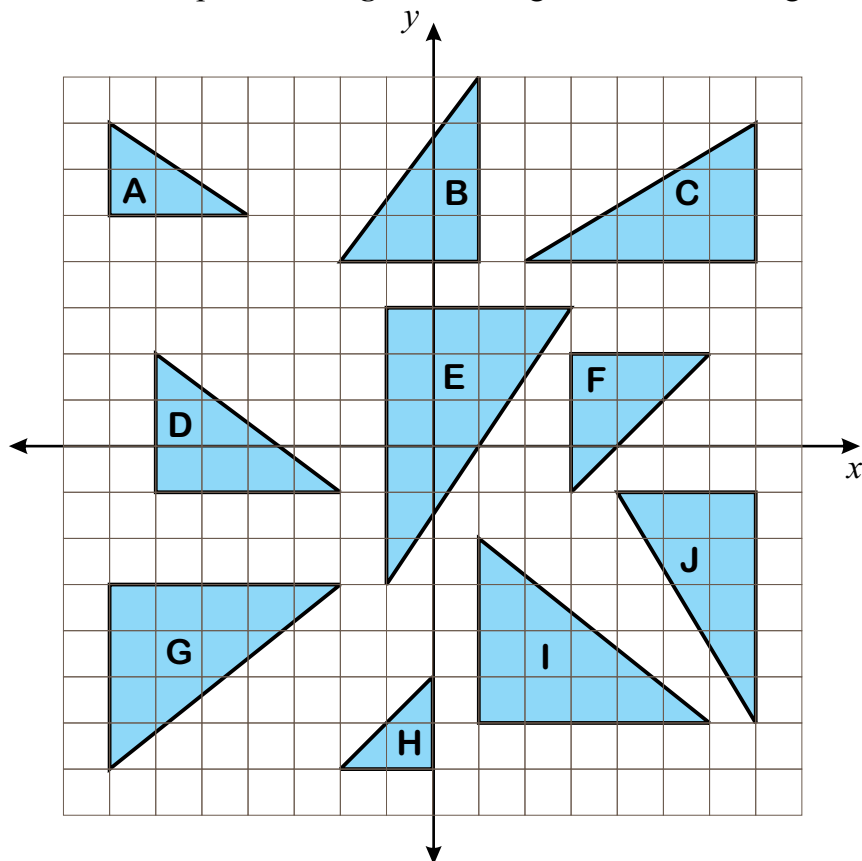
EXERCISE 13G

Match the pairs of congruent triangles from the triangles below.



EXERCISE 13H

- Match all the pairs of *similar* triangles from the triangles below.
- Match all the pairs of *congruent* triangles from the triangle below.



- Mark the following points onto a set of coordinate axes with scales of -8 to 8 on both the x -axis and y -axis.
A(-5,6) **B**(5,8) **C**(-4,2) **D**(0,-3) **E**(0,7) **F**(3,4) **G**(6,-3) **H**(-2,3)
I(-3,6) **J**(4,-3) **K**(-4,-4) **L**(-5,3) **M**(5,5) **N**(-6,-4) **O**(4,8) **P**(3,7)
Q(2,3) **R**(1,3) **S**(4,-6) **T**(6,3) **U**(-1,-6) **V**(-3,0) **W**(-3,-3) **X**(1,-2)
 - Draw triangles by joining the following sets of points:
Triangle 1 - (**A****I****L**), Triangle 2 - (**E****F****P**), Triangle 3 - (**B****O****M**)
Triangle 4 - (**C****N****K**), Triangle 5 - (**H****R****X**), Triangle 6 - (**G****Q****T**),
Triangle 7 - (**D****V****W**), Triangle 8 - (**J****S****U**)
 - State which of the triangles are *similar* triangles.
 - State which of the triangles are *congruent* triangles.

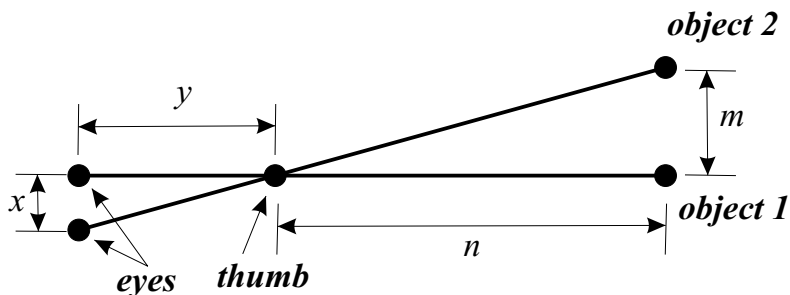
PROBLEM SOLVING

A technique that has been used for hundreds of years to estimate distances is called the 'Rule of Thumb'. It uses the fact that the length of your arm is approximately ten times the distance between your eyes.

1. Ask someone to measure the distance between your eyes (x) and the distance from your body to your upturned thumb when your arm is straight out in front of you (y). See diagram. Record these two lengths.



2. Using the same units for x and y , calculate $\frac{y}{x}$ rounded to the nearest whole number.
(This should be close to 10)
3. This fact can be used to estimate distances by using the following steps.
 - (a) With your arm extended and thumb upturned, close (or cover) your right eye and using your left eye align your thumb with a distant object (**object 1**).
 - (b) Open your right eye, close your left eye and without moving your thumb observe that your thumb is now in line with an object (**object 2**) to the left of the original object (**object 1**).
 - (c) If the distance between these two objects is known (m) then the distance from your thumb to object 1 (n) can be calculated using similar triangles. See diagram below.



4. Using the diagram from question 3, which of these mathematical statements is correct?

A $\frac{y}{x} = \frac{m}{n}$

B $\frac{y}{x} = \frac{n}{m}$

For the following questions use $\frac{y}{x} = 10$

5. A farmer uses this 'Rule of Thumb' technique to find the distance he is from a distant fence. He aligned his thumb with one fence post using his left eye and another fence post using his right eye. He knew the fence posts were 120 metres apart. How far away is the fence?
6. In a game of Australian Rules Football a footballer used the 'Rule of Thumb' technique to calculate how far from goal he was standing. Using one eye his thumb aligned with one goal post and using the other eye his thumb aligned with the other goal post. The goal posts are seven metres apart. How far from the goal posts is the footballer?
7. In the school yard, on a sporting ground, in the classroom or choose somewhere else to use the 'Rule of Thumb' to calculate the distance to an object. Use a measuring tape to measure the actual distance and compare this to the calculated measurement.

PUZZLE

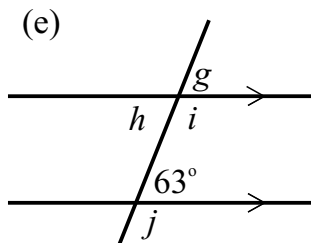
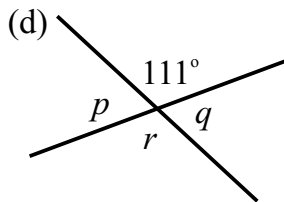
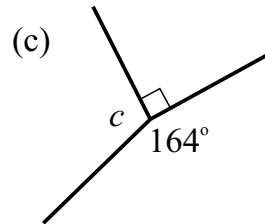
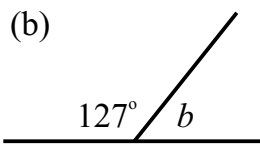
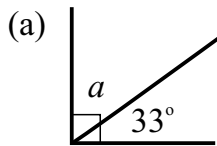
To complete this puzzle each 3×3 box and each row and column must contain all the letters:

TRIANGLES

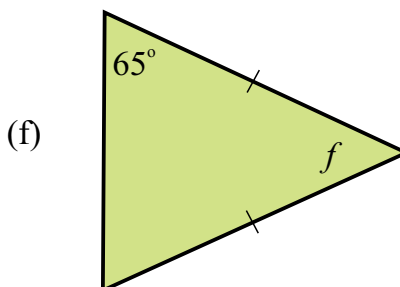
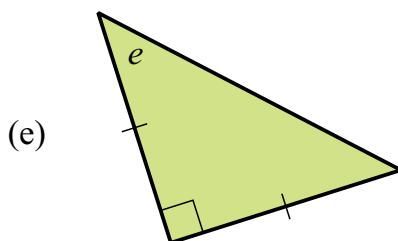
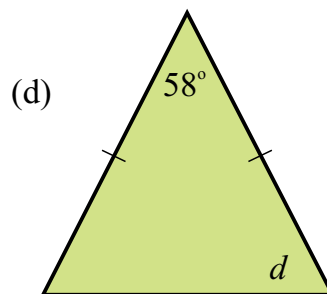
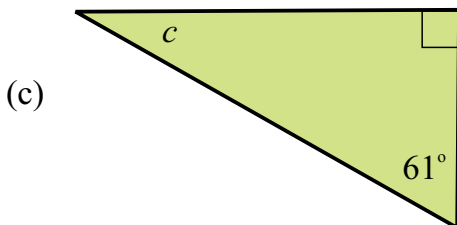
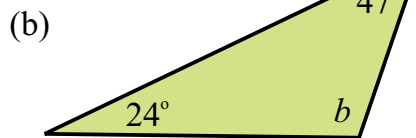
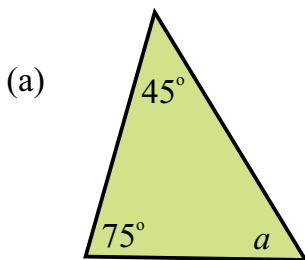
T	R	I	A	N	G	L	E	S
		G						N
N			T	L		A		
		R	L		E			A
	L	T			N		I	
E		A		S				
G				E	A	T		R
		S	R	I		N	G	L
		N		G		E	A	

CHAPTER REVIEW

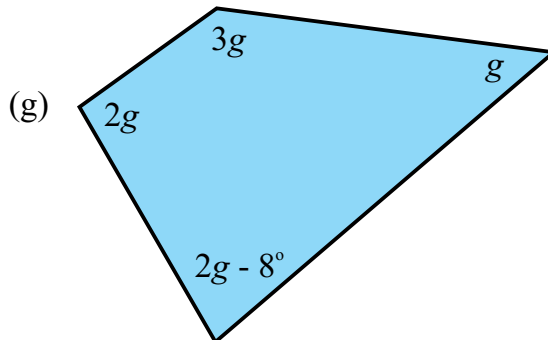
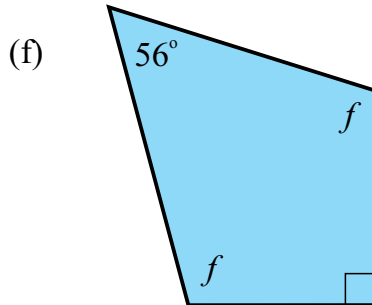
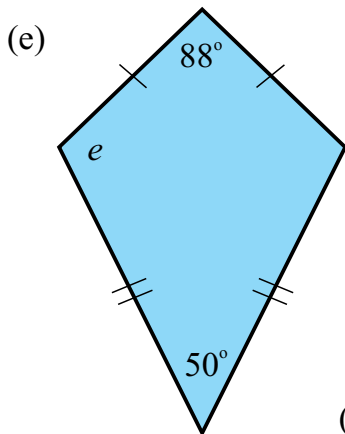
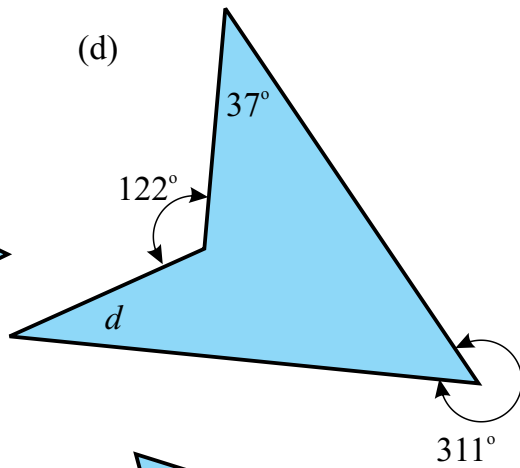
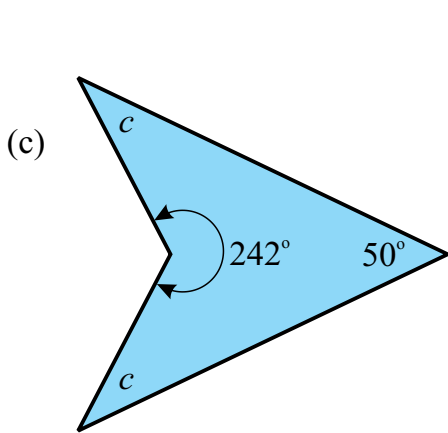
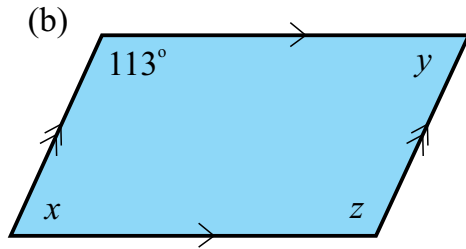
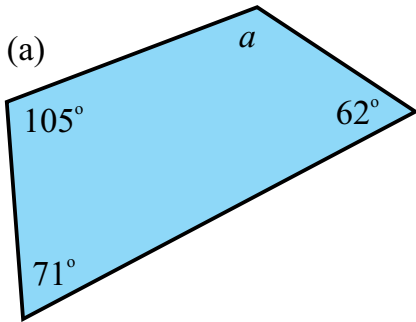
1. Find the unknown angles.



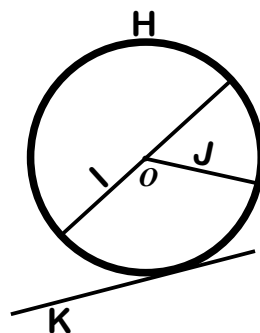
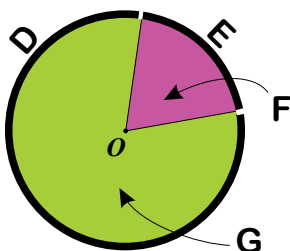
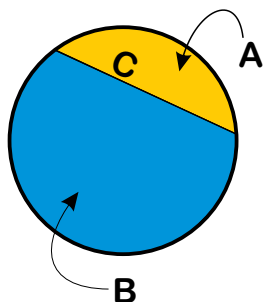
2. Find the unknown angles in the following triangles



3. Find the unknown angles in the following quadrilaterals.



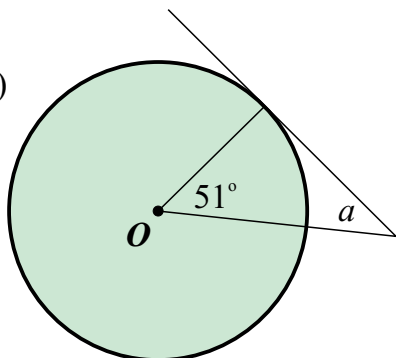
4. Match the names with the parts of the circles.



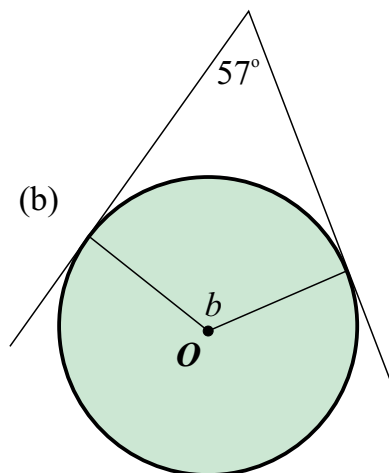
<i>Major Sector</i>	<i>Radius</i>	<i>Minor Segment</i>	<i>Tangent</i>
<i>Minor Sector</i>	<i>Major Segment</i>	<i>Major Arc</i>	<i>Diameter</i>
<i>Minor Arc</i>	<i>Circumference</i>	<i>Chord</i>	

5. Find the unknown angles

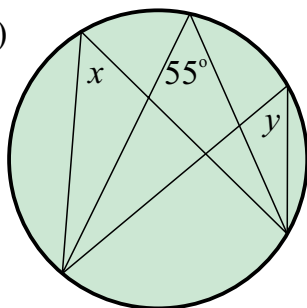
(a)



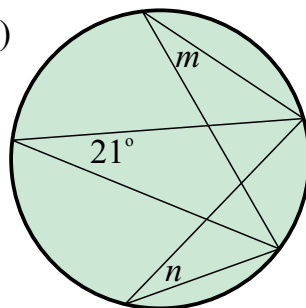
(b)

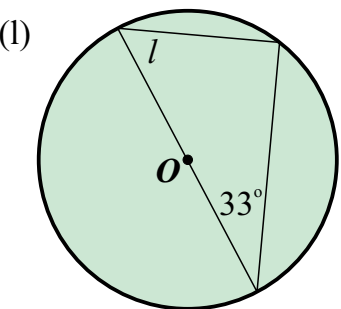
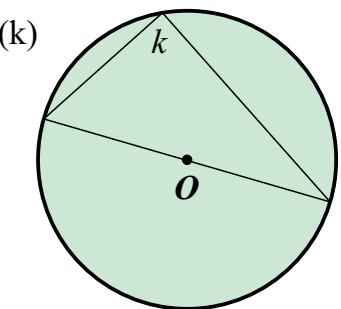
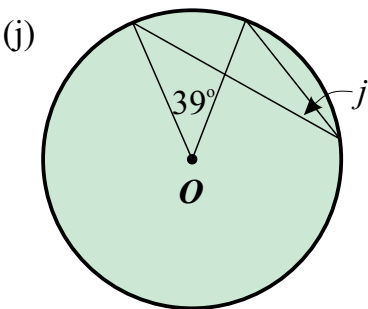
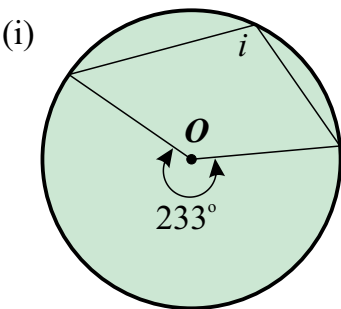
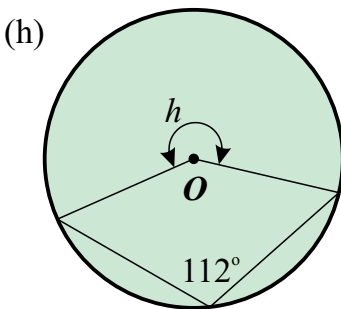
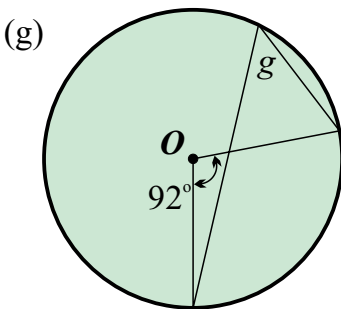
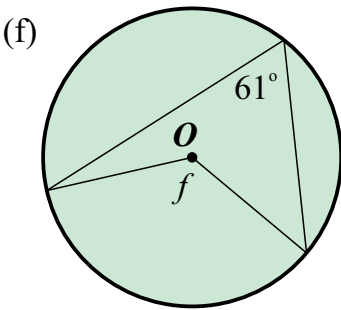
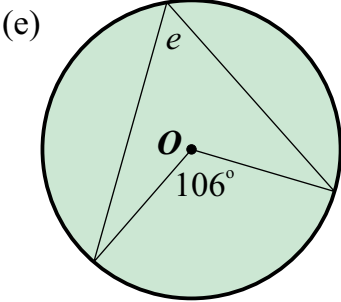


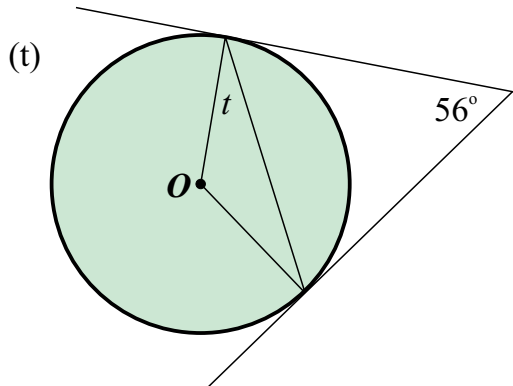
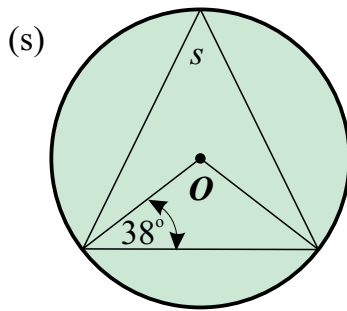
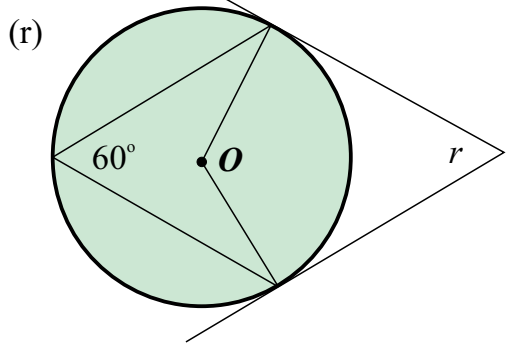
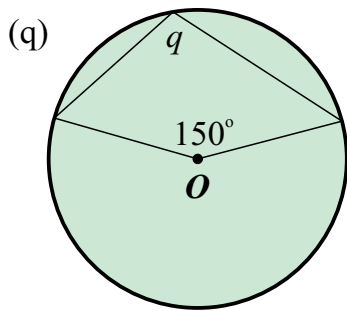
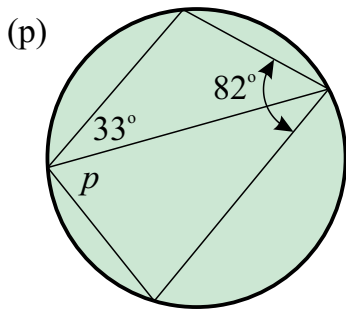
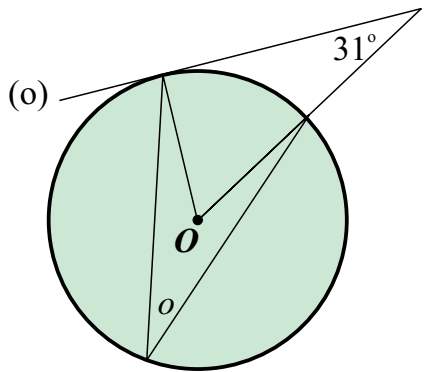
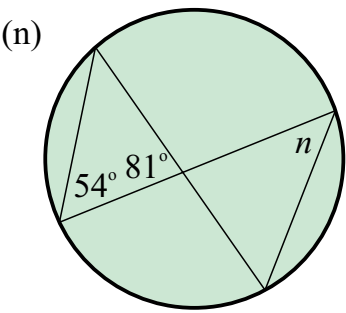
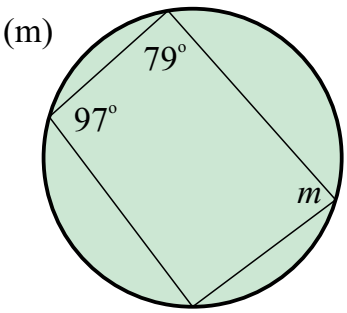
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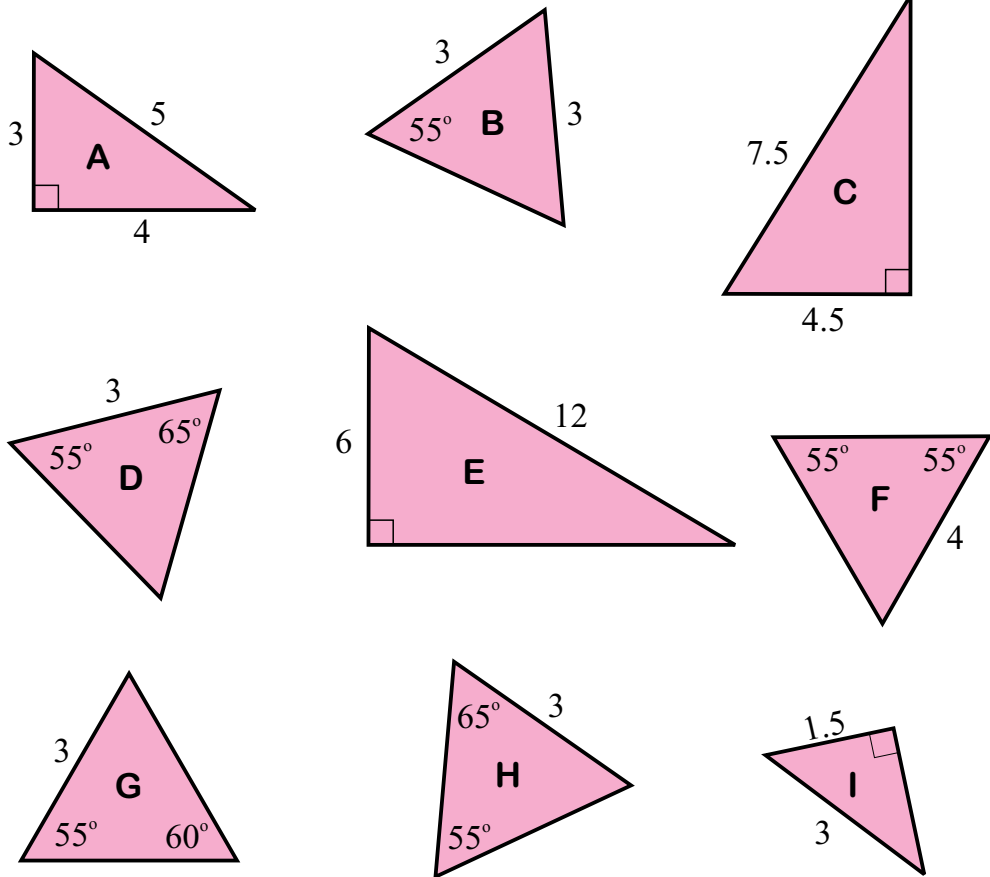
(d)



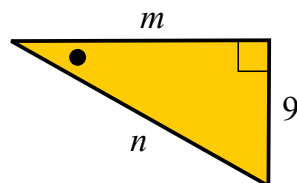
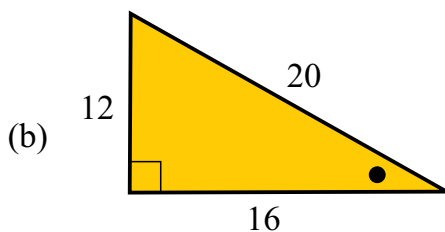
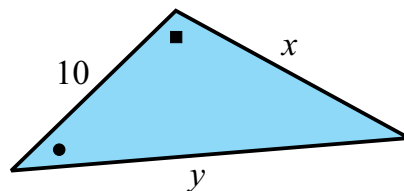
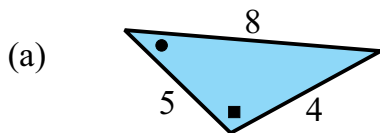




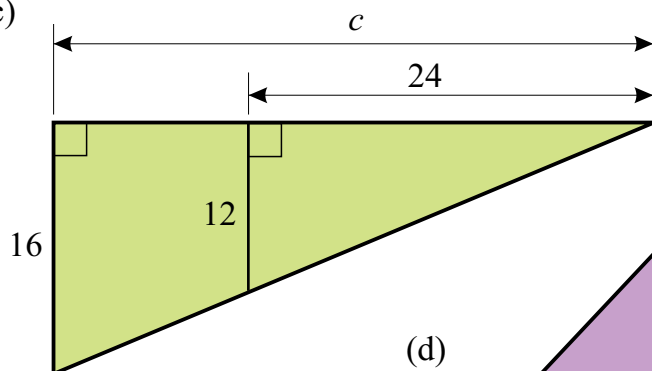
6. (a) State which of the following triangles are **similar triangles**.
 (b) State which of the following triangles are **congruent triangles**.



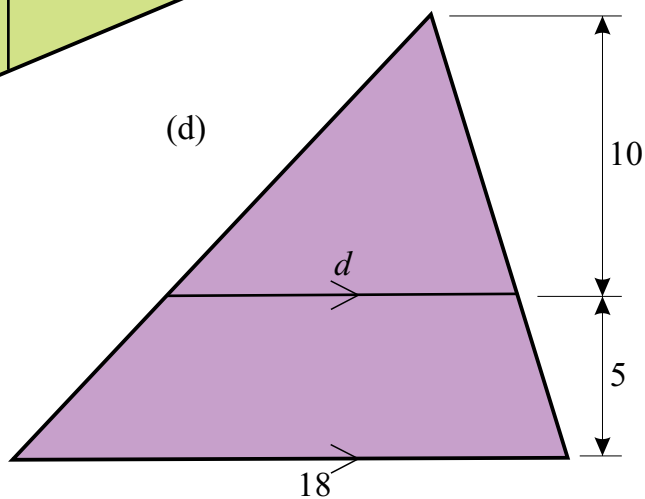
7. Find the unknown lengths.



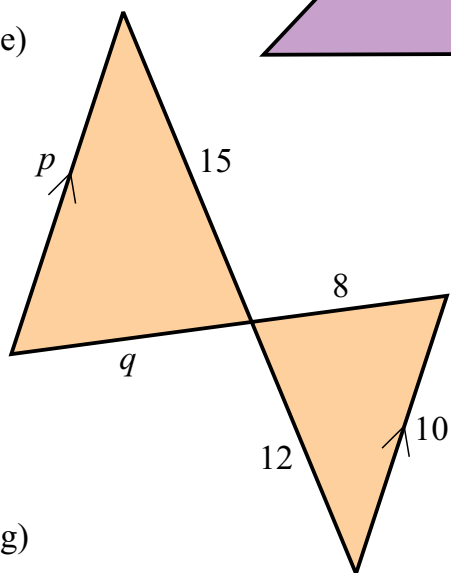
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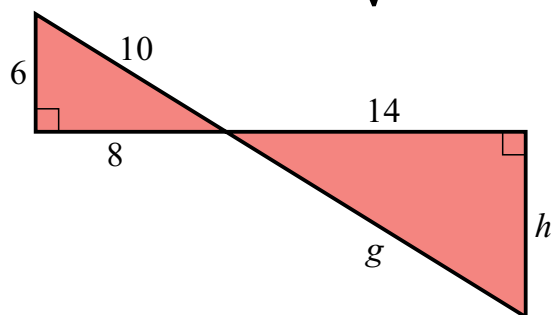
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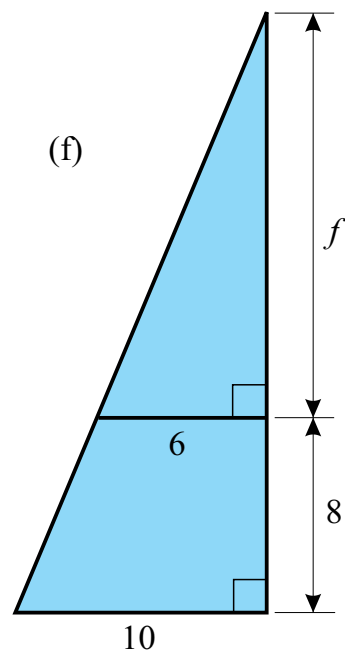
(e)



(g)



(f)



8. Alex is 1.32 metres tall and he measured his shadow one day to be 0.86 metres long. The length of a shadow cast by a goal post was 4.73 metres long. How tall is the goal post?
9. Louise was standing 5.5 metres from a camp fire and noticed her shadow from the fire was the same height as a nearby tree. Louise is 1.65 metres tall and she was standing 15.5 metres from the tree. How tall is the tree?



Trigonometry

Degrees, Minutes and Seconds

A common unit for measuring angles is the **degree** (symbol $^{\circ}$).

There are 360 degrees (360°) in one revolution.

Common units for measuring parts of a degree are **minutes** and **seconds**.

The symbol for minutes is $'$

The symbol for seconds is $''$

There are 60 minutes in one degree. ($60' = 1^{\circ}$)

There are 60 seconds in one minute. ($60'' = 1'$)

Convert the following degrees to **degrees, minutes and seconds**.

Examples

1. 0.5°

$$= \frac{1^{\circ}}{2}$$

$$= 30'$$

2. 3.5°

$$= 3^{\circ} \text{ and } 30'$$

$$= 3^{\circ}30'$$

3. 0.8°

$$= 0.8 \times 60'$$

$$= 48'$$

4. 43.8°

$$= 43^{\circ}48'$$

5. 27.3°

$$= 27^{\circ} \text{ and } 0.3 \times 60'$$

$$= 27^{\circ}18'$$

6. 59.95°

$$= 59^{\circ} \text{ and } 0.95 \times 60'$$

$$= 59^{\circ}57'$$

7. 0.32°

$$= 0.32 \times 60'$$

$$= 19.2'$$

$$= 19' \text{ and } 0.2 \times 60''$$

$$= 19' \text{ and } 12''$$

$$= 19^{\circ}12''$$

8. 17.43°

$$= 17^{\circ} \text{ and } 0.43 \times 60'$$

$$= 17^{\circ}25.8'$$

$$= 17^{\circ}25' \text{ and } 0.8 \times 60''$$

$$= 17^{\circ}25'48''$$

9. 34.21°

$$= 34^{\circ} \text{ and } 0.21 \times 60'$$

$$= 34^{\circ}12.6'$$

$$= 34^{\circ}12' \text{ and } 0.6 \times 60''$$

$$= 34^{\circ}12'36''$$

10. 68.351°

$$= 68^{\circ} \text{ and } 0.351 \times 60'$$

$$= 68^{\circ}21.06'$$

$$= 68^{\circ}21' \text{ and } 0.06 \times 60''$$

$$= 68^{\circ}21'3.6''$$

EXERCISE 14A

1. Convert the following degrees to *degrees, minutes and seconds*.

- (a) 72.3° (b) 40.75° (c) 9.1° (d) 19.35°
 (e) 125.15° (f) 68.45° (g) 0.65° (h) 320.85°

2. Convert the following degrees to *degrees, minutes and seconds*.

- (a) 62.17° (b) 48.93° (c) 7.52° (d) 87.09°
 (e) 132.81° (f) 52.64° (g) 37.125° (h) 26.355°

3. Convert the following degrees to *degrees, minutes and seconds*.

- (a) 17.561° (b) 4.773° (c) 49.218° (d) 122.309°
 (e) 57.117° (f) 32.797° (g) 0.096° (h) 200.002°

EXERCISE 14B

Convert the following angles to *decimal degrees*.

Examples

1. $34^\circ 24'$

$$24' = \frac{24}{60}^\circ$$

$$= 0.4^\circ$$

$$34^\circ 24' = 34.4^\circ$$

2. $56^\circ 19' 15.6''$

$$15.6'' = \frac{15.6}{60}'$$

$$= 0.26'$$

$$19.26' = \frac{19.26}{60}^\circ$$

$$= 0.321^\circ$$

$$56^\circ 19' 15.6'' = 56.321^\circ$$

1. Convert the following angles to *decimal degrees*.

- (a) $63^\circ 30'$ (b) $25^\circ 6'$ (c) $119^\circ 21'$ (d) $81^\circ 54'$

2. Convert the following angles to *decimal degrees*.

- (a) $18^\circ 14' 6''$ (b) $85^\circ 32' 42''$ (c) $7^\circ 39' 18''$
 (d) $92^\circ 51' 36''$ (e) $23^\circ 32' 31.2''$ (f) $60^\circ 15' 57.6''$
 (g) $111^\circ 28' 40.8''$ (h) $34^\circ 13' 19.2''$ (i) $237^\circ 54' 3.6''$

EXERCISE 14C

Most scientific and graphic calculators can convert between degrees, minutes, seconds and decimal degrees.

Familiarise yourself with these functions of your calculator.

Use the questions from Exercises 14A and 14B to practice these skills using your calculator:

1. Converting from decimal degrees to degrees, minutes and seconds (DMS).
2. Entering an angle in degrees, minutes and seconds into your calculator.
3. Converting from an angle in degrees, minutes and seconds (DMS) to a decimal angle.

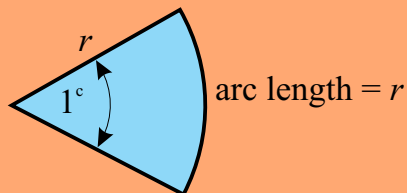
Radians

Another unit used in measuring angles is called the **radian**.

Radians are often used in mathematics, science and engineering.

The symbol for radians is $^{\text{c}}$

One radian (1^{c}) is the angle formed by an arc that has the same length as the radius of the arc.



$$1^{\text{c}} \approx 57.296^{\circ}$$

$$\begin{aligned}\pi^{\text{c}} &= 180^{\circ} \\ 180^{\circ} &= \pi^{\text{c}} \\ &\approx 3.1416^{\text{c}}\end{aligned}$$

$$\begin{aligned}2\pi^{\text{c}} &= 360^{\circ} \\ 360^{\circ} &= 2\pi^{\text{c}} \\ &= 2 \times \pi^{\text{c}} \\ &\approx 6.2832^{\text{c}}\end{aligned}$$

EXERCISE 14D

1. Convert the following angles from degrees to radians.

To convert from degrees to radians multiply by $\frac{\pi}{180}$.

- (i) Write answer as a fraction in its simplest form.
 (ii) Write answer as a decimal correct to two decimal places.

Example 56°

$$\begin{aligned} \text{(i)} \quad 56 &\times \frac{\pi}{180} \\ &= \frac{56\pi}{180} \\ &= \frac{14\pi}{45} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad &\frac{56\pi}{180} \\ &= \frac{56 \times \pi}{180} \\ &= \mathbf{0.98^\circ} \text{ (two dec. pl.)} \end{aligned}$$

- (a) 90° (b) 60° (c) 45° (d) 200° (e) 270°
 (f) 68° (g) 150° (h) 121° (i) 148° (j) 54°

2. Convert the following angles from radians to degrees.

To convert from radians to degrees multiply by $\frac{180}{\pi}$.

- (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$
 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{18}$
 (e) $\frac{7\pi}{6}$ (f) $\frac{7\pi}{9}$
 (g) $\frac{7\pi}{10}$ (h) $\frac{4\pi}{5}$
 (i) $\frac{5\pi}{12}$ (j) $\frac{7\pi}{15}$

Example

$$\begin{aligned} &\frac{3\pi}{5} \\ &= \frac{3\pi}{5} \times \frac{180^\circ}{\pi} \\ &= \frac{3\cancel{\pi}}{\cancel{5}_1} \times \frac{\cancel{180}^{36}}{\cancel{\pi}} \\ &= \frac{108}{1} \\ &= \mathbf{108^\circ} \end{aligned}$$

3. Convert the following angles from radians to degrees.

Give answers correct to two decimal places.

- (a) 0.6° (b) 2.1°
 (c) 0.85° (d) 1.62°
 (e) 0.14° (f) 1.9°
 (g) 3.33° (h) 4.6°

Example 1.35°

$$= 1.35 \times \frac{180^\circ}{\pi}$$

$$= 77.35^\circ \text{ (two dec. pl.)}$$

Trigonometric Functions

EXERCISE 14E

Make sure the calculator is in the correct **MODE**
DEGREES or **RADIANS**
 Check the angle unit

1. Use a calculator to find the following values correct to four decimal places. (Check calculator is in **DEGREE** mode)

- (a) $\sin 32^\circ$ (b) $\cos 58^\circ$ (c) $\tan 7^\circ$ (d) $\cos 45^\circ$
 (e) $\sin 23.8^\circ$ (f) $\tan 69.6^\circ$ (g) $\cos 15.4^\circ$ (h) $\sin 75.8^\circ$

2. Use a calculator to find the following values correct to four decimal places. (Check calculator is in **DEGREE** mode)

- (a) $\sin 42^\circ 16'$ (b) $\cos 17^\circ 28'$ (c) $\tan 8^\circ 42'$
 (d) $\cos 45^\circ 27' 15''$ (e) $\sin 29^\circ 39' 51''$ (f) $\tan 51^\circ 35' 29''$
 (g) $\cos 15^\circ 2' 5''$ (h) $\sin 75^\circ 0' 52''$ (i) $\tan 7'$

3. Use a calculator to find the following values correct to four decimal places. (Check calculator is in **RADIAN** mode)

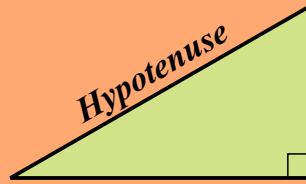
- (a) $\sin 0.6^\circ$ (b) $\cos 1.3^\circ$ (c) $\tan 0.45^\circ$ (d) $\cos 1.27^\circ$
 (e) $\sin 1.08^\circ$ (f) $\tan 0.97^\circ$ (g) $\cos 1.4^\circ$ (h) $\sin 0.22^\circ$

4. Use a calculator to find the following values correct to four decimal places. (Check calculator is in the correct mode)

- (a) $\sin 0.3^\circ$ (b) $\cos 13^\circ$ (c) $\tan 0.85^\circ$ (d) $\cos 16^\circ 21'$

SOHCAHTOA

The ***hypotenuse*** is the longest side of a ***right-angled triangle***.
It is always opposite the right-angle.



The two other sides of a right-angled triangle are labelled ***opposite*** and ***adjacent*** depending on the angle.

Example

For angle θ :

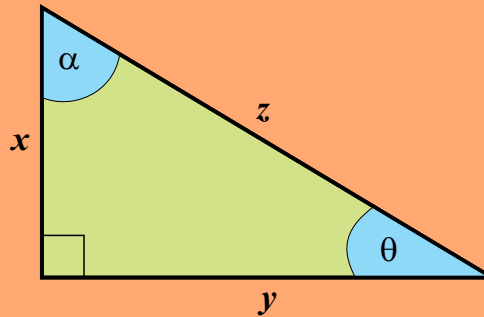
x is the ***opposite*** side

y is the ***adjacent*** side

For angle α :

y is the ***opposite*** side

x is the ***adjacent*** side

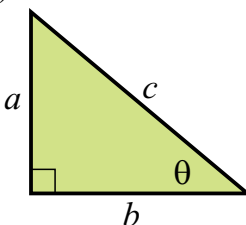


EXERCISE 14F

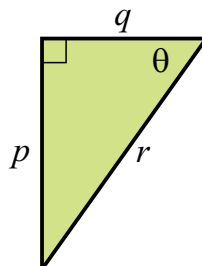
1. For the right-angled triangles below state:

- which side is the hypotenuse
- which side is opposite angle θ
- which side is adjacent to angle θ

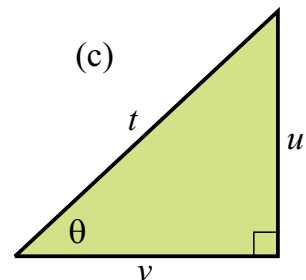
(a)



(b)

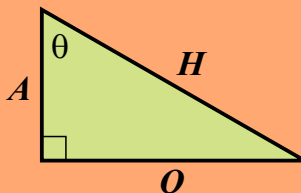


(c)

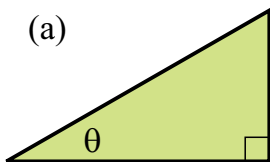


2. Copy these triangles and label the sides **H** (**H**ypotenuse), **O** (**O**pposite) and **A** (**A**djacent) for the angle shown.

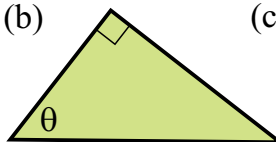
Example



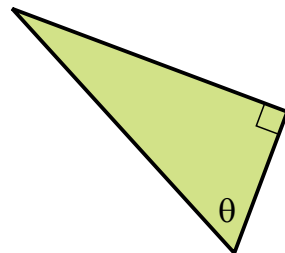
(a)



(b)



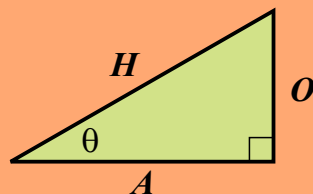
(c)



The **sine** of an angle is defined as the ratio of the side opposite the angle to the hypotenuse.

The sine of θ is abbreviated to **sin** θ .

$$\text{sine of } \theta = \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{O}{H}$$



The **cosine** of an angle is defined as the ratio of the side adjacent to the angle to the hypotenuse.

The cosine of θ is abbreviated to **cos** θ .

$$\text{cosine of } \theta = \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A}{H}$$

The **tangent** of an angle is defined as the ratio of the side opposite the angle to the side adjacent to the angle.

The tangent of θ is abbreviated to **tan** θ .

$$\text{tangent of } \theta = \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{O}{A}$$

SOHCAHTOA

$$\sin \theta = \frac{O}{H}$$

$$\cos \theta = \frac{A}{H}$$

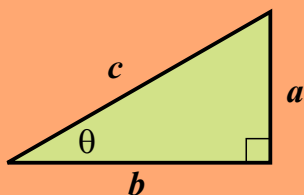
$$\tan \theta = \frac{O}{A}$$

EXERCISE 14G

1. For the following triangles find:

(i) $\sin \theta$ (ii) $\cos \theta$ (iii) $\tan \theta$

Example

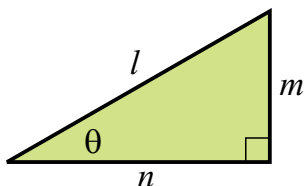


$$\sin \theta = \frac{a}{c}$$

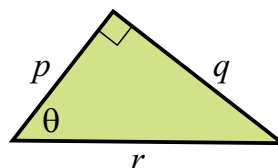
$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

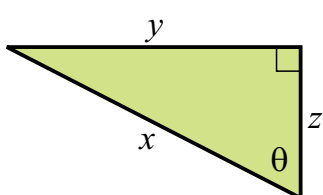
(a)



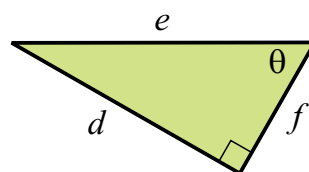
(b)



(c)

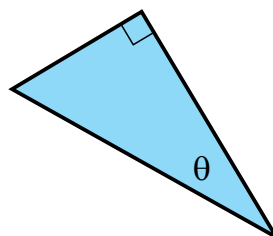


(d)



2. Redraw the triangle shown here labelling the sides, given that:

$$\sin \theta = \frac{U}{V} \quad \text{and} \quad \cos \theta = \frac{W}{V}$$



3. For the following triangles find:

(i) $\sin \theta$ (ii) $\cos \theta$ (iii) $\tan \theta$

Give answers correct to four decimal places.

Example

$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{9}{41}$$

$$\sin \theta = \mathbf{0.2195} \text{ (4 dec. pl.)}$$

$$\cos \theta = \frac{A}{H}$$

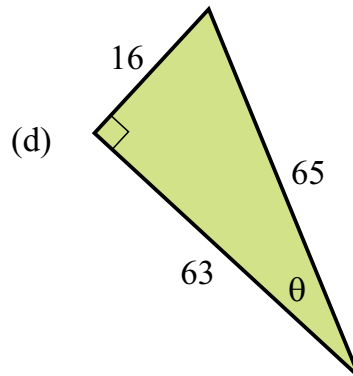
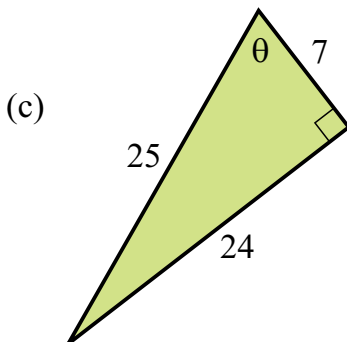
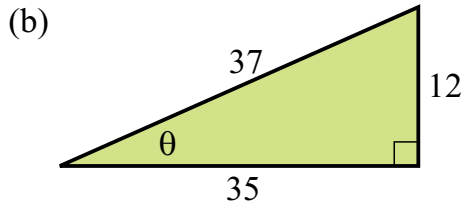
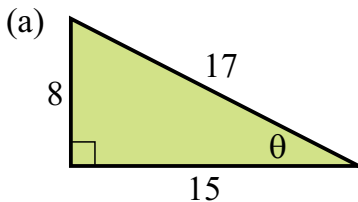
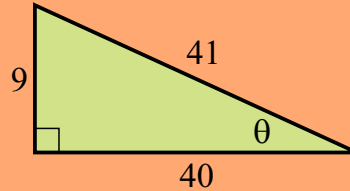
$$\cos \theta = \frac{40}{41}$$

$$\cos \theta = \mathbf{0.9756} \text{ (4 dec. pl.)}$$

$$\tan \theta = \frac{O}{A}$$

$$\tan \theta = \frac{9}{40}$$

$$\tan \theta = \mathbf{0.2250} \text{ (4 dec. pl.)}$$



Finding Side Lengths

To calculate an unknown side length of a right-angled triangle using trigonometry follow the steps below.

Step 1 Label sides *O*, *A* and *H* for the known angle.

Step 2 Decide which trigonometric function (sin, cos or tan) to use.

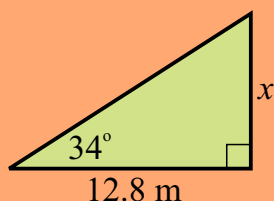
Step 3 Write the rule for the function.

Step 4 Substitute values.

Step 5 Solve for the unknown length.

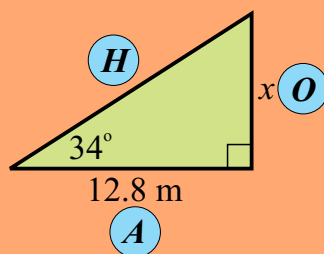
Example 1

Find the unknown length in the following triangle.



Step 1

Label the sides *O*, *A* and *H* for the given angle of 34°



Step 2 The unknown length (x) is the opposite side (*O*) and the known length (12.8 m) is the adjacent side (*A*).

We therefore need to use **tangent**.

Step 3 $\tan \theta = \frac{O}{A}$

Step 4 $\tan 34^\circ = \frac{x}{12.8}$

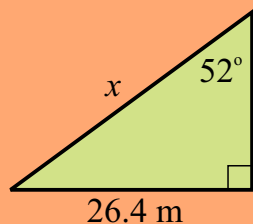
Step 5 $12.8 \times \tan 34^\circ = x$

$$x = 12.8 \times \tan 34^\circ$$

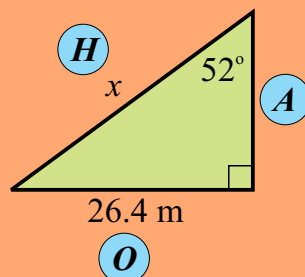
$$x = 8.6 \text{ m (one dec. place)}$$

Example 2

Find the unknown length in the following triangle.

**Step 1**

Label the sides
 O , A and H for
the given angle
of 52°



Step 2 The unknown length (x) is the hypotenuse (H) and the known length (26.4 m) is the opposite side (O).
We therefore need to use *sine*.

Step 3 $\sin \theta = \frac{O}{H}$

Step 4 $\sin 52^\circ = \frac{26.4}{x}$

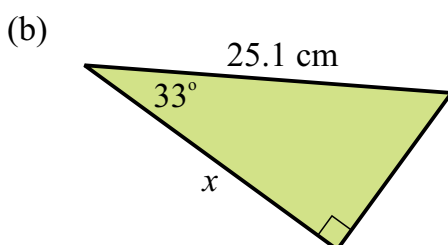
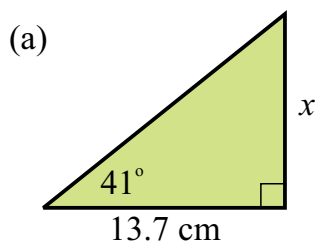
Step 5 $x \times \sin 52^\circ = 26.4$
 $x = \frac{26.4}{\sin 52^\circ}$

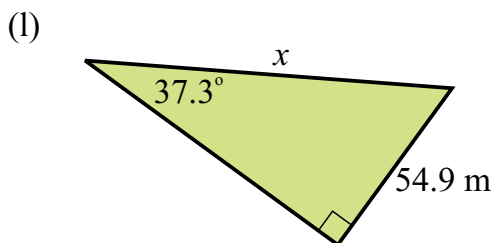
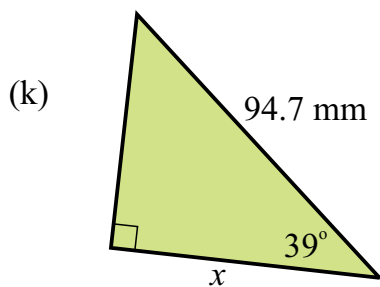
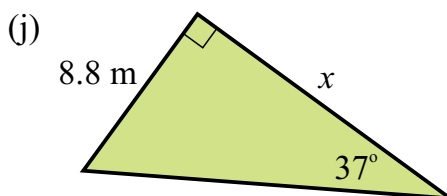
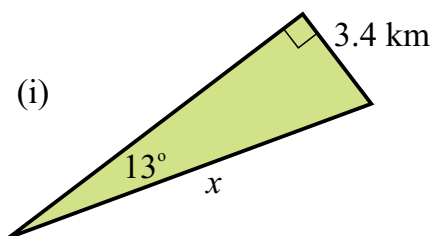
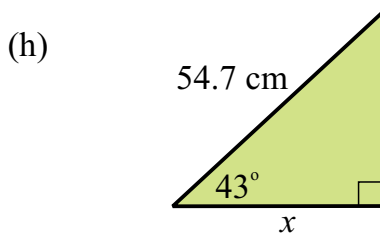
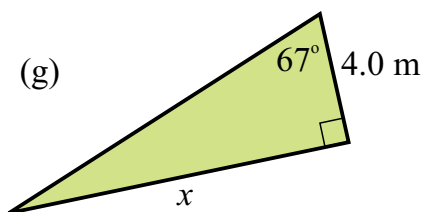
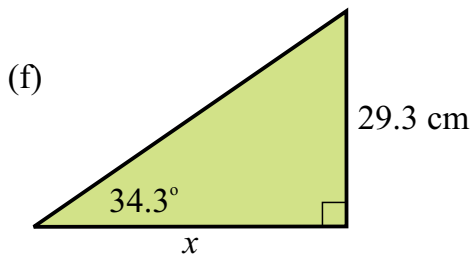
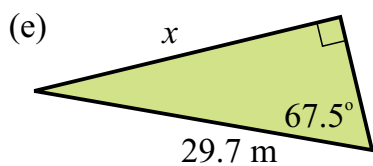
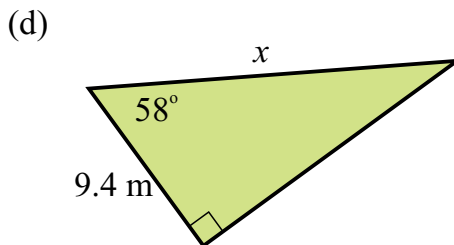
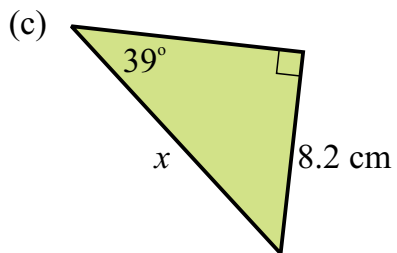
$x = 33.5$ m (one dec. place)

EXERCISE 14H

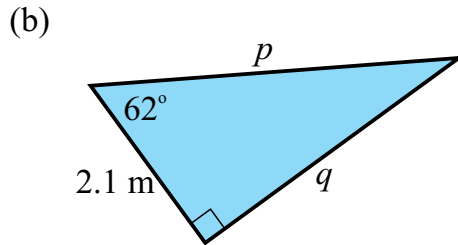
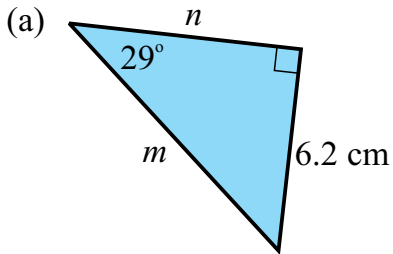
Check calculator is in **DEGREE** mode

1. Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

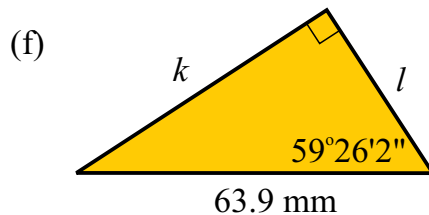
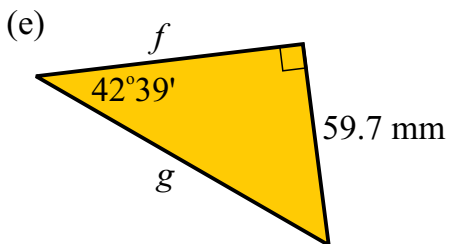
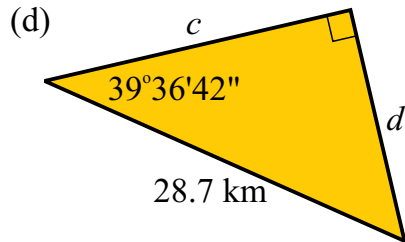
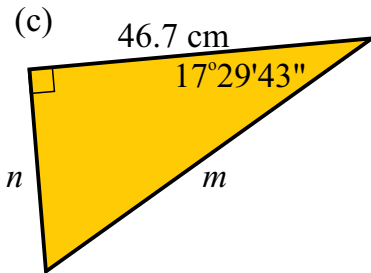
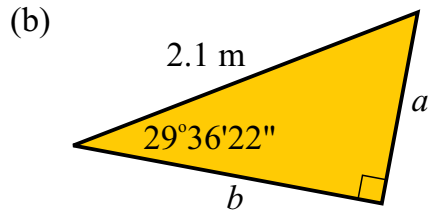
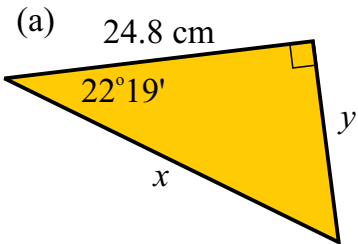




2. Find both unknown side lengths in these triangles.
Give answers correct to one decimal place.



3. Find both unknown side lengths in these triangles.
Give answers correct to one decimal place.



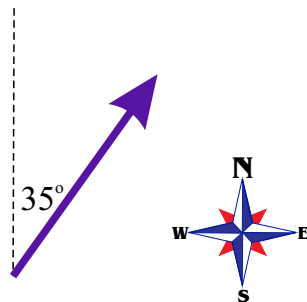
Applications

EXERCISE 14I

*Always draw a clear diagram.
Give answers correct to one decimal place.*

1. A yacht sails 46.8 km in a direction N 35° E.

- (a) How far east has the yacht sailed?
(b) How far north has the yacht sailed?



2. An aeroplane from the Royal Flying Doctors leaves its base and flies 456 km in a direction of S 28° E to visit a patient.

- (a) How far south of its base is it?
(b) How far east of its base is it?

The RFD then has to fly 348 km in a direction of S 68° E from its first stop.

- (c) How far then is it south of its base?
(d) How far east of its base is it?

3. The RFD aeroplane has the following flights to make. At the completion of its stops find:

- (i) how far north or south it is from its base.
(ii) how far east or west it is from its base.

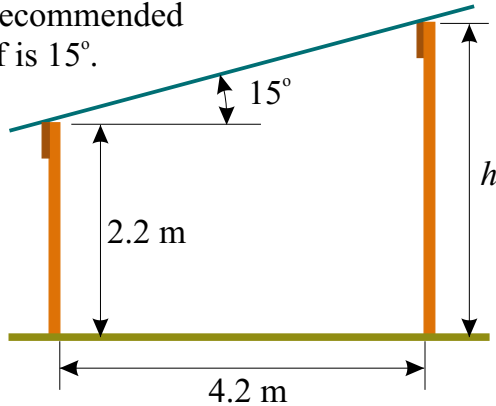
- (a) Two stops:

- the first is 278 km in a direction of N 39° W from its base
- the second is 368 km in a direction of S 21° W from its first stop

- (b) Three stops:

- the first stop is 321 km in a direction of S 78° W from base
- the second is 258 km in a direction of N 26° E from the first stop
- the third stop is 149 km in a direction of S 65° E from the second stop

4. Bob is building a carport. It is recommended that the angle (pitch) of the roof is 15° . The carport is to be 4.2 m wide and the lower side is to be 2.2 m high. Find the height (h) of the other side of the carport.



5. An aeroplane takes off at a speed of 80 metres per second. Its angle of ascent is 20° for one minute then 10° for one minute.
- Calculate the altitude of the aeroplane two minutes after takeoff.
 - Calculate how far horizontally the aeroplane is from the point of takeoff.
6. Dorothy is standing on the ground 150 metres from the base of a vertical cliff. She notices two people abseiling down the cliff. She has a telescopic camera on a stand with an angle adjustment. She moves the camera to an angle of 26° from the horizontal and can see the abseilers.
- How high are the abseilers above the height of the camera?
 - One hour later Dorothy moves her camera to an angle of 12° to see the abseilers. How far have they descended?



Calculating Angles

To find an angle in a right-angled triangle knowing two sides follow these steps:

Step 1 Label the sides (**O**, **A** or **H**) based on the angle to be found.

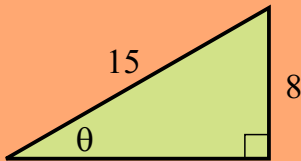
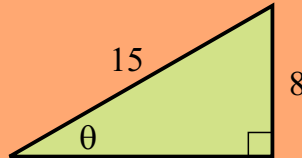
Step 2 Determine which function (sine, cosine or tangent) to use.

Step 3 Write the rule for the function.

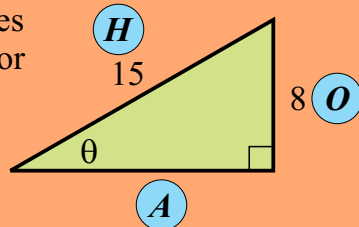
Step 4 Find the angle by calculating the inverse of the angle using a calculator: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

Example

Find the angle (θ) shown in this triangle.



Label the sides **O**, **A** and **H** for angle θ .



The two known sides are the opposite side (**O**) and the hypotenuse (**H**). We therefore need to use **sine**.

$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{8}{15}$$

$$\theta = \sin^{-1}\left(\frac{8}{15}\right)$$

$$\theta = 32.2^\circ \text{ (1 decimal place)}$$

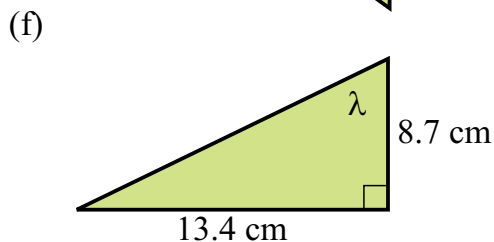
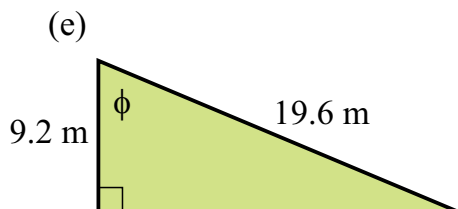
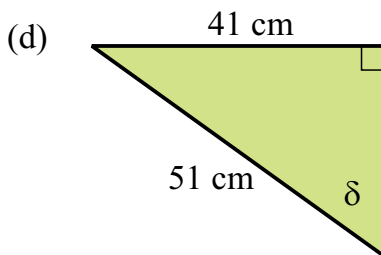
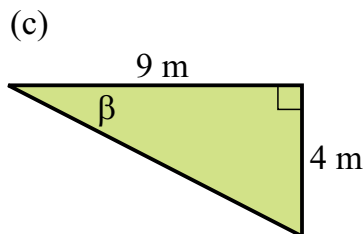
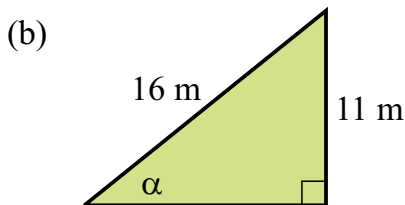
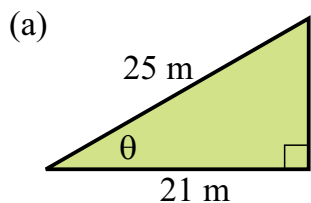
$$\theta = 32^\circ 13' 51.429'' \text{ (DMS)}$$

EXERCISE 14J

1. Find the unknown angles in the following triangles.

(i) Give answers correct to one decimal place.

(ii) Convert angles to degrees, minutes and seconds (DMS).

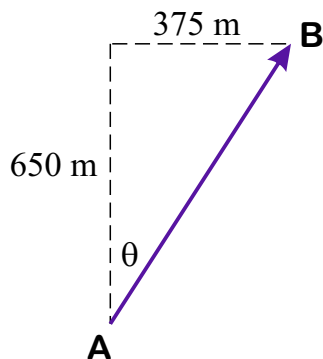


2. An orienteering course required competitors to run in a straight line between points **A** and **B** through bush.

Point **B** is 650 metres north and 375 metres east of point **A**.

At what angle (θ) should the runners set their compasses?

Give answer correct to the nearest degree.



3. Samantha wanted to calculate the change in the angle of the Sun's rays over a month. She measured the shadow cast by a 2.15 m high post at midday on July 1 to be 3.48 m long. At midday on August 1 the shadow was 2.52 m long.

By how many degrees did the angle of the Sun's rays change?

(a) Give answer as a decimal to two decimal places.

(b) Give answer in DMS.

4. The ramp on a building site is 4.2 m long. The end is 46 cm above the floor.

(a) Find the angle, θ , the ramp makes with the floor.

Give answer as a decimal correct to one decimal place.



(b) The maximum angle allowed for safety reasons is 5° .

What is the minimum length of the ramp for the angle to be less than 5° . Give answer to the nearest cm.

5. A yacht sails 8.5 km east from its port then 10.2 km north.

In what direction should the yacht travel to return to port?

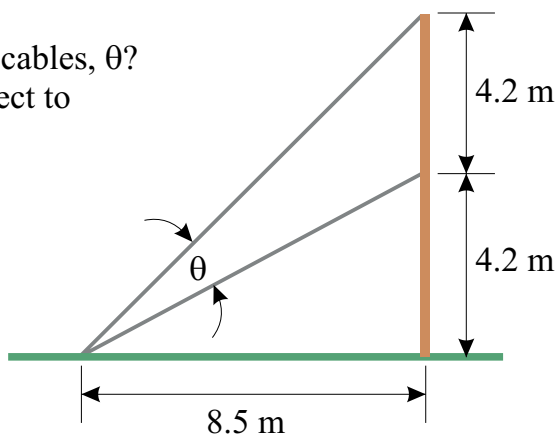
Give answer in DMS.

6. Two hikers walk at a steady pace for 3 hours east from their camp then 2 hours south. In what direction should they walk to return to their camp? Give answer in DMS.

7. Two cables are used to secure a pole as shown in this diagram.

What is the angle between the cables, θ ?

Give answer as a decimal correct to one decimal place.

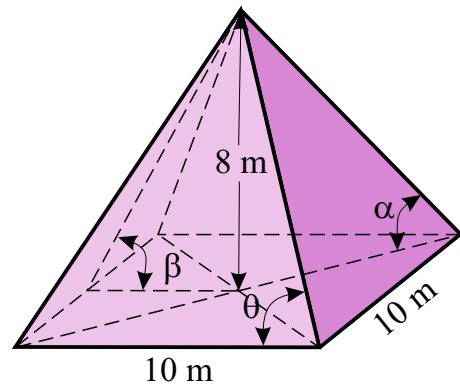


Applications

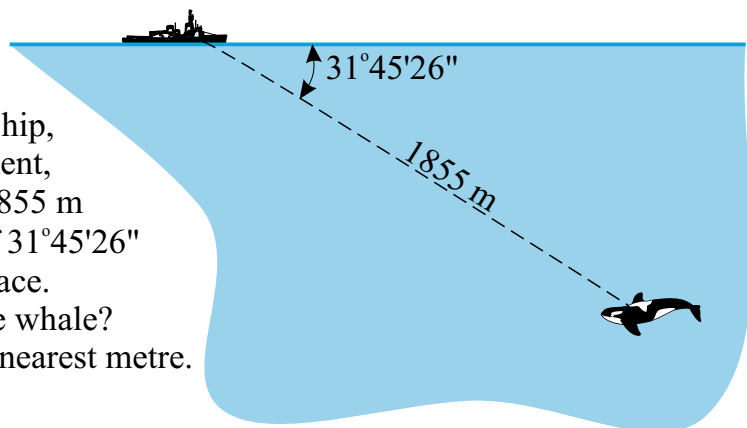
EXERCISE 14K

Pythagoras' Theorem may help to solve some problems

1. An aeroplane is flying at an altitude of 10 000 m. It is approaching an airport to land. The landing instructions are to descend at an angle of 12° to the horizontal till it reaches an altitude of 6000 m then to descend at an angle of 5° to the horizontal till it lands. How far from the airport should the pilot commence descent? Give answer to the nearest 100 m.
2. The square-based pyramid shown here has a base length of 10 m and a height of 8 m. Find the following angles. Give answers in degrees, minutes and seconds (DMS).
 - (a) The angle, β , between a face and the base.
 - (b) The angle, α , between a sloping edge and the base.
 - (c) The angle, θ , between a sloping edge and a base edge.



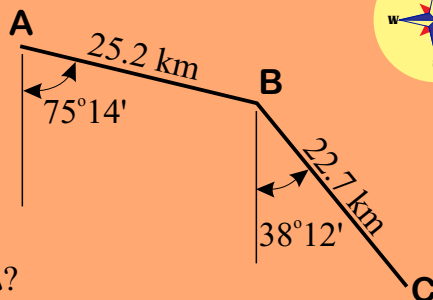
3. A marine research ship, using sonar equipment, observed a whale 1855 m away at an angle of $31^\circ 45' 26''$ down from the surface. At what depth is the whale? Give answer to the nearest metre.



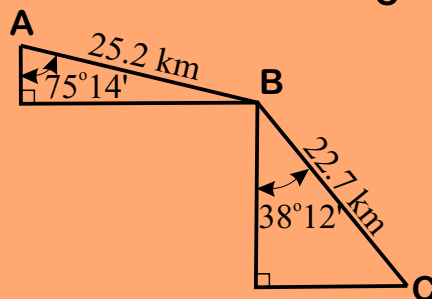
Example

A boat travelled 25.2 km in a direction of S $75^{\circ}14'$ E from **A** to **B** then 22.7 km in a direction of S $38^{\circ}12'$ E from **B** to **C**.

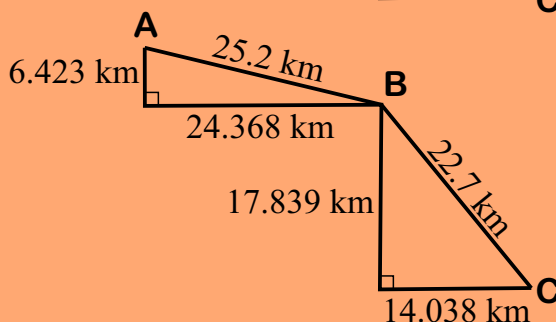
1. What is the distance in a straight line from **C** to **A**?
2. What is the direction from **C** to **A**?



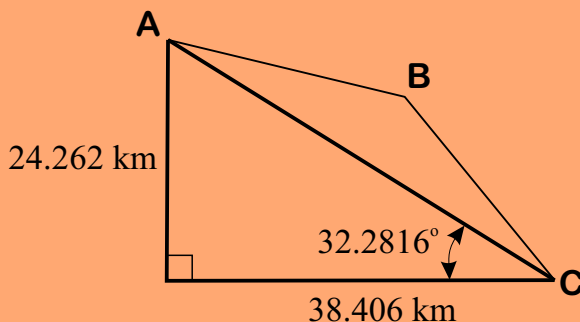
Step 1 Draw right-angled triangles with the short sides along North-South and West-East lines.



Step 2 Find the other side lengths using trigonometry. (Don't round answers until final answer is calculated)



Step 3 Draw right-angled triangle with **AC** as the hypotenuse. Side lengths will be the sum of the side lengths from step 2. Calculate angle at **C** using trigonometry.



Step 4 Use Pythagoras' Theorem or trigonometry to find the distance from **C** to **A** = **45.4 km** (one decimal place)
 The angle from north is $90 - 32.2816 = 57.7184^{\circ}$
 Direction from **C** to **A** = **N $57^{\circ}43'6''$ W** (nearest second)

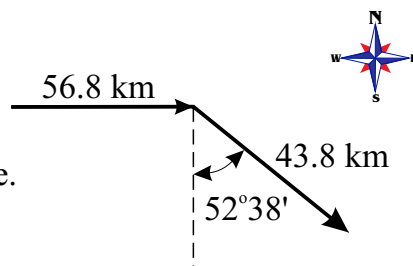
4. A yacht sails 56.8 km east from its dock then 43.8 km in a direction S $52^{\circ}38'$ E.

(a) How far, in a straight line, is the yacht from its dock?

Give answer in km to one decimal place.

(b) In what direction should the yacht sail to return to dock?

Give answer in DMS to the nearest second.



5. An aeroplane delivering mail in outback Australia flies 260 km in a direction N $48^{\circ}23'$ W to its first delivery. It then flies 185 km in a direction S $38^{\circ}49'$ W to its next delivery.

(a) How far, in a straight line, is the plane from its base?

Give answer to the nearest km.

(b) In what direction should the pilot set the course to fly straight back to base?

Give answer in DMS to the nearest second.

6. A boat is sinking 148 km in a direction of N $28^{\circ}38'$ E from Cairns. A rescue boat is already at sea, 56 km in a direction of S $85^{\circ}23'$ E from Cairns. If the rescue boat is to travel in a straight line to the sinking boat:

(a) in what direction should it travel? Give answer in DMS to the nearest second.

(b) how far does it have to travel? Give answer to the nearest km.

(c) how long will it take if it travels at 80 km/h? Give answer to the nearest minute?

7. There are three islands Syne, Coz and Tann. Coz is 57.9 km in a direction of N $59^{\circ}21'15''$ E from Syne. Coz is 88.9 km in a direction of N $35^{\circ}27'16''$ W from Tann.

(a) How far is Syne from Tann in a straight line?

Give answer correct to one decimal place.

(b) What is the direction of Syne from Tann?

Give answer in DMS to the nearest second?

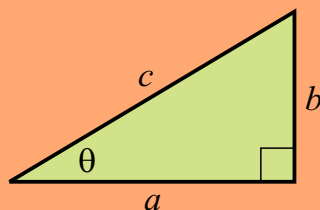


The Unit Circle

Consider the triangle shown here.

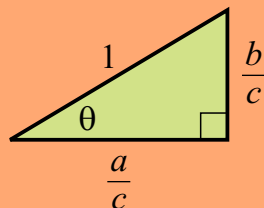
$$\sin \theta = \frac{b}{c}$$

$$\cos \theta = \frac{a}{c}$$



If each side length is divided by c , the resultant triangle is shown here.

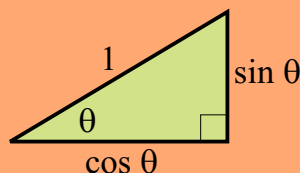
The length of the hypotenuse = 1



The length of the side opposite $\theta = \frac{b}{c} = \sin \theta$ (from original triangle)

The length of the side adjacent to $\theta = \frac{a}{c} = \cos \theta$ (from original triangle)

Therefore this triangle could be drawn with the side lengths as shown here.



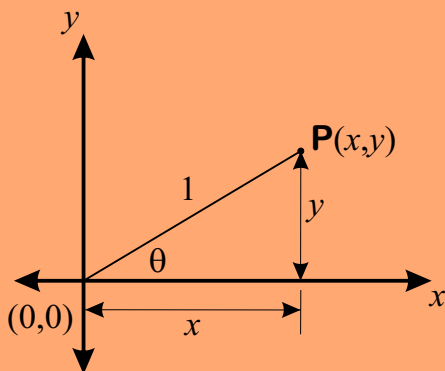
Locate a point **P**, with coordinates (x,y) , on coordinate axes a distance of 1 unit from the origin $(0,0)$ making an angle θ with the x -axis.

See diagram.

Comparing this diagram with the triangle above it can be seen that:

$$x = \cos \theta$$

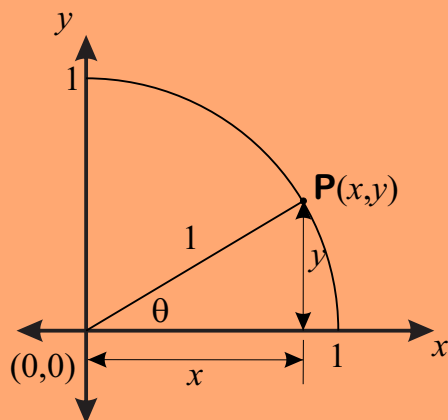
$$y = \sin \theta$$



These relationships would apply to all the points on the arc with radius 1 unit - all angles between 0° and 90° .

$$x = \cos \theta$$

$$y = \sin \theta$$



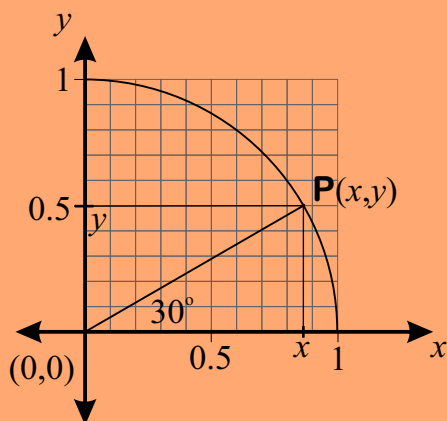
Example 1

Use the diagram shown here to find an approximate value for $\cos 30^\circ$ and $\sin 30^\circ$.

Each division on the 10×10 grid = 0.1

$$x = \cos 30^\circ \approx 0.86$$

$$y = \sin 30^\circ \approx 0.5$$



The actual values can be found using a calculator:

$$\cos 30^\circ = 0.8660 \text{ (four decimal places)}$$

$$\sin 30^\circ = 0.5$$

Example 2

Use the diagram shown here to find an approximate value for $\cos 64^\circ$ and $\sin 64^\circ$.

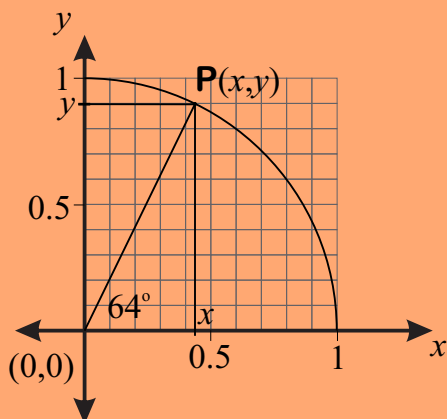
$$x = \cos 64^\circ \approx 0.43$$

$$y = \sin 64^\circ \approx 0.89$$

The actual values:

$$\cos 64^\circ = 0.4384 \text{ (4 dec. pl.)}$$

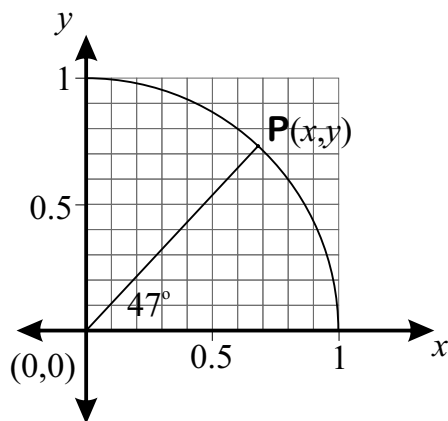
$$\sin 64^\circ = 0.8988 \text{ (4 dec. pl.)}$$



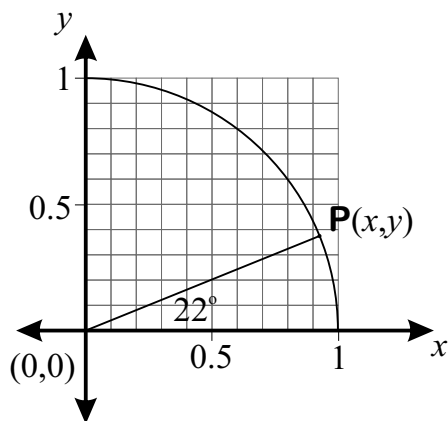
EXERCISE 14L

1. (i) Use the coordinate axes to find approximate values for $\cos \theta$ and $\sin \theta$ for the following angles.
 (ii) Use a calculator to find the exact values correct to four decimal places.

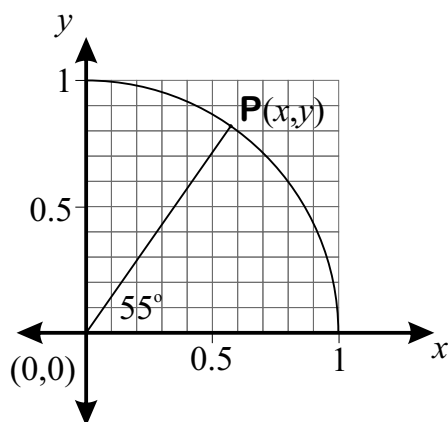
(a) $\theta = 47^\circ$



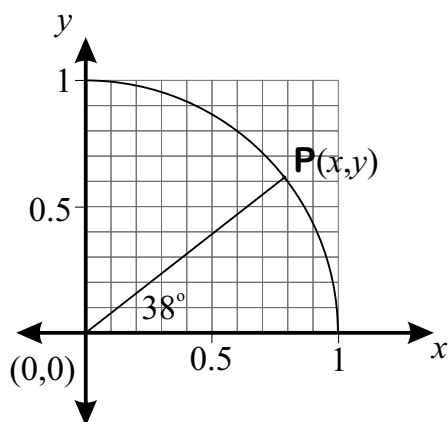
(b) $\theta = 22^\circ$



(c) $\theta = 55^\circ$



(d) $\theta = 38^\circ$



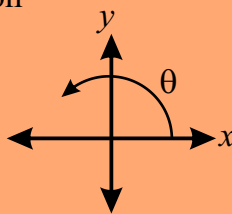
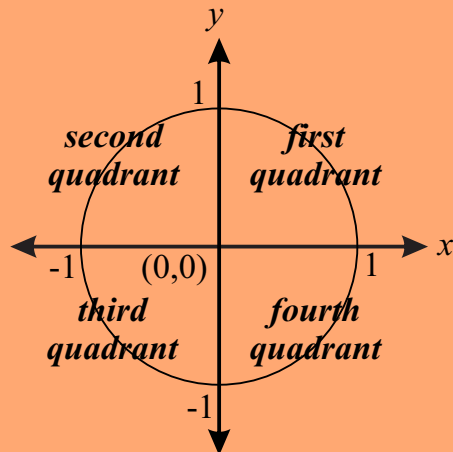
2. Use the coordinate axes to find the following values.

(a) $\cos 0^\circ$ (b) $\sin 0^\circ$ (c) $\cos 90^\circ$ (d) $\sin 90^\circ$

This 90° arc with radius 1 unit can be used to find the value of $\cos \theta$ and $\sin \theta$ for all angles between, and including, 0° and 90° . This is called the **first quadrant** of a set of coordinate axes.

If this 90° arc with radius 1 unit is extended to form a complete circle (called a **unit circle**) then it can be used to find the values of $\cos \theta$ and $\sin \theta$ for angles greater than 90° - angles in the **second, third and fourth** quadrants.

Angles are always measured in an anticlockwise direction from the positive x -axis.



Example 1

Find approximate values for $\cos \theta$ and $\sin \theta$ for $\theta = 130^\circ$

Using the graph:

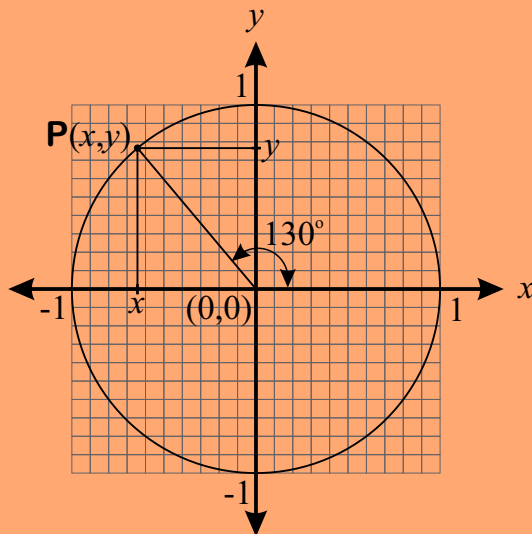
$$x = \cos 130^\circ \approx -0.64$$

$$y = \sin 130^\circ \approx 0.78$$

Using a calculator the exact values to four decimal places are:

$$\cos 130^\circ = -0.6428$$

$$\sin 130^\circ = 0.7660$$



Example 2

Find approximate values for $\cos \theta$ and $\sin \theta$ for $\theta = 304^\circ$

Using the graph:

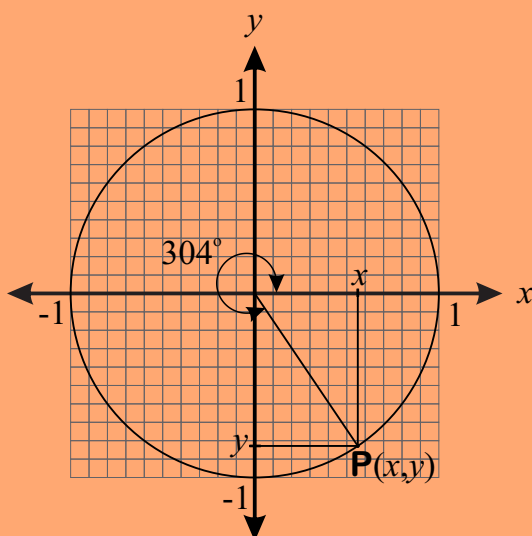
$$x = \cos 304^\circ \approx 0.56$$

$$y = \sin 304^\circ \approx -0.82$$

Using a calculator the exact values to four decimal places are:

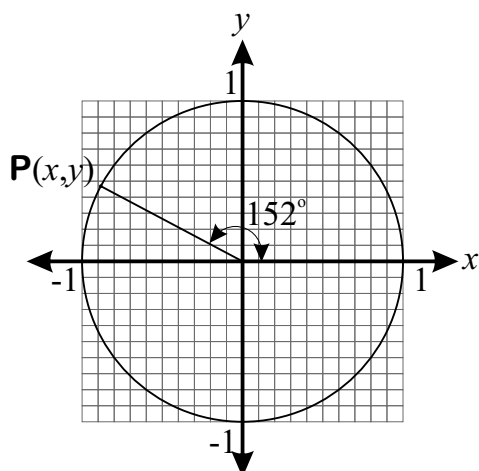
$$\cos 304^\circ = 0.5592$$

$$\sin 304^\circ = -0.8290$$

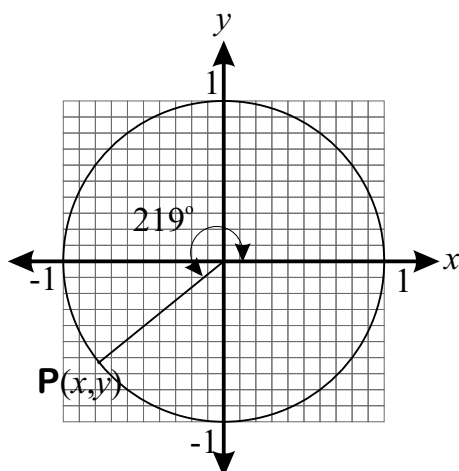
**EXERCISE 14M**

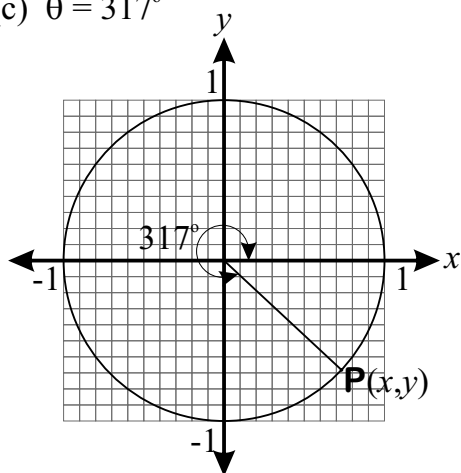
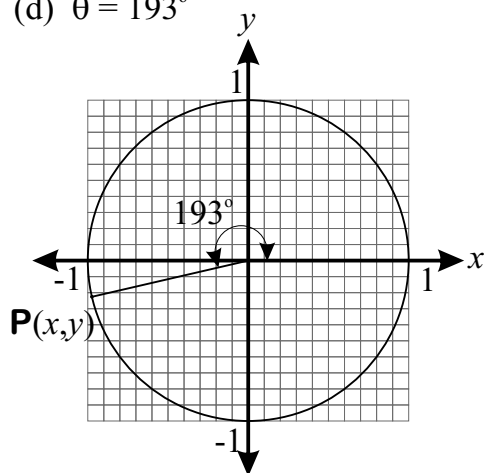
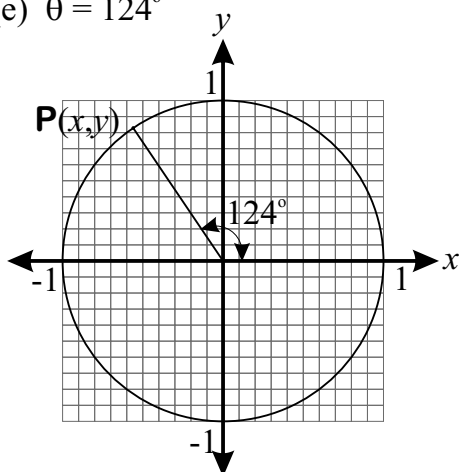
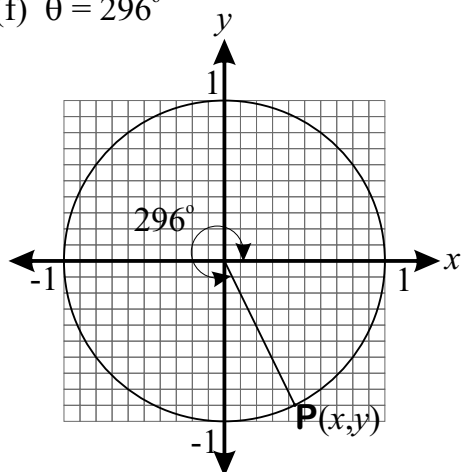
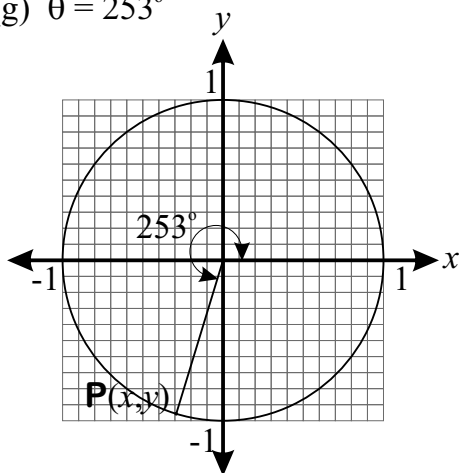
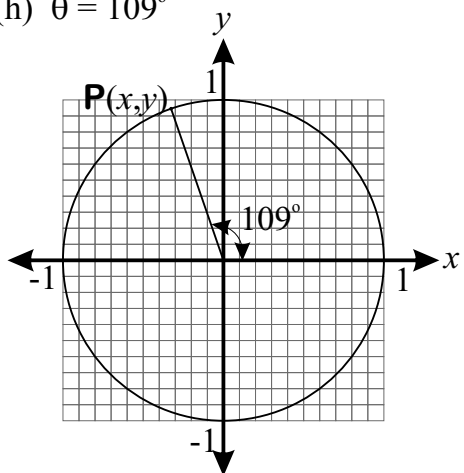
1. (i) Use the coordinate axes to find approximate values for $\cos \theta$ and $\sin \theta$ for the following angles.
- (ii) Use a calculator to find the exact values correct to four decimal places.

(a) $\theta = 152^\circ$



(b) $\theta = 219^\circ$

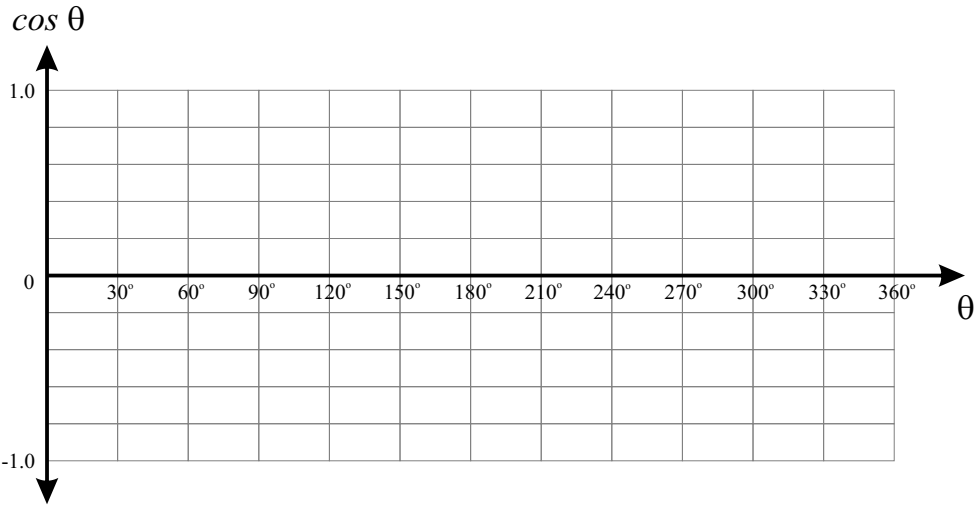
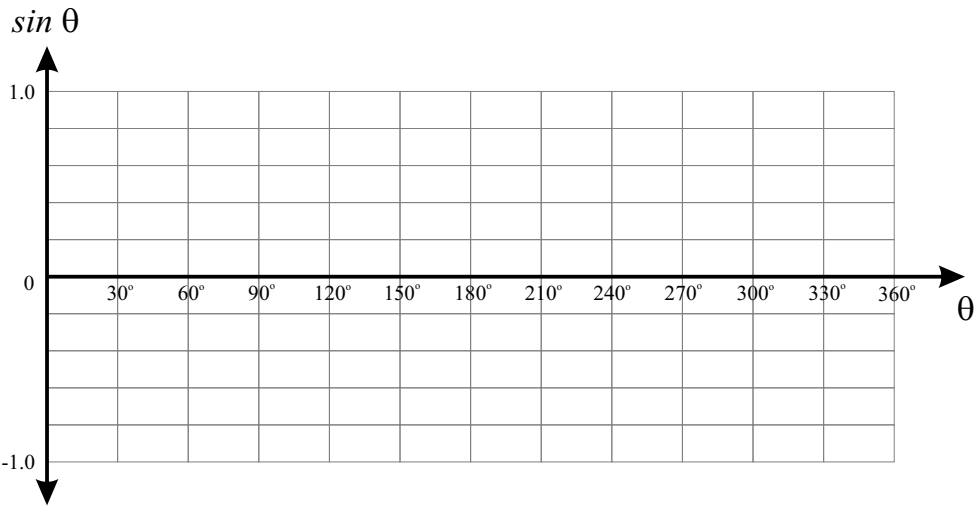


(c) $\theta = 317^\circ$ (d) $\theta = 193^\circ$ (e) $\theta = 124^\circ$ (f) $\theta = 296^\circ$ (g) $\theta = 253^\circ$ (h) $\theta = 109^\circ$ 

- [illegible]

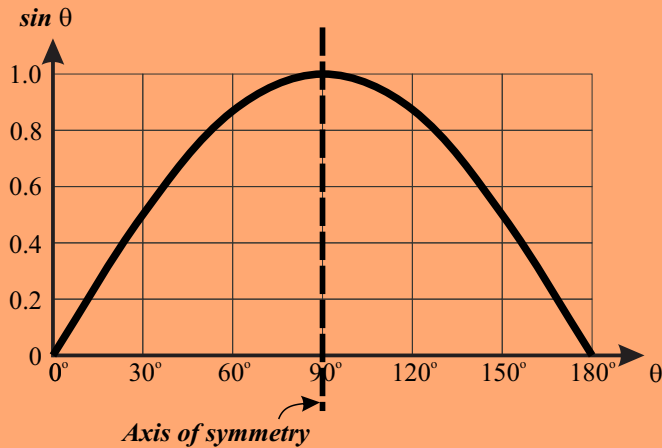
[illegible]

6. Using graph paper accurately plot the points from the two tables in question 5. Connect the points with smooth curves. Use a different graph for each set of values. The diagram below shows how the axes should be drawn.



It can be seen from these graphs that sections of each graph are *symmetrical*.

For example the section of the $\sin \theta$ graph between 0° and 180° is symmetrical with the axis of symmetry at $\theta = 90^\circ$.

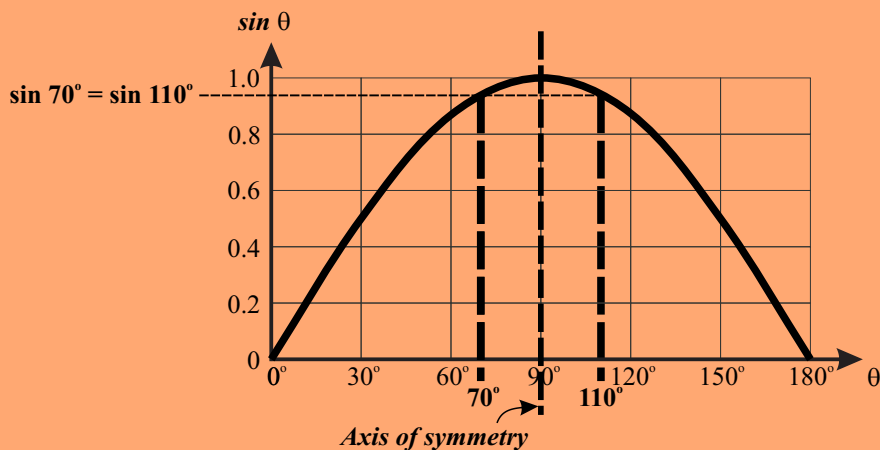


Therefore all the values on one side of the axis of symmetry will have corresponding values on the other side of the axis of symmetry that are equal. The two equal values will be equidistant from the axis of symmetry.

Example $\sin 70^\circ = \sin 110^\circ$

70° is 20° to the **left** of the axis of symmetry

110° is 20° to the **right** of the axis of symmetry



EXERCISE 14N

1. Use the section of the $\sin \theta$ graph between $\theta = 0^\circ$ and $\theta = 180^\circ$ to answer the following questions.

- | | |
|--|--|
| (a) $\sin 80^\circ = \sin$ <input type="text"/> | (b) $\sin 130^\circ = \sin$ <input type="text"/> |
| (c) $\sin 10^\circ = \sin$ <input type="text"/> | (d) $\sin 140^\circ = \sin$ <input type="text"/> |
| (e) $\sin 175^\circ = \sin$ <input type="text"/> | (f) $\sin 15^\circ = \sin$ <input type="text"/> |
| (g) $\sin 35^\circ = \sin$ <input type="text"/> | (h) $\sin 75^\circ = \sin$ <input type="text"/> |
| (i) $\sin 52^\circ = \sin$ <input type="text"/> | (j) $\sin 98^\circ = \sin$ <input type="text"/> |
| (k) $\sin 161^\circ = \sin$ <input type="text"/> | (l) $\sin 47^\circ = \sin$ <input type="text"/> |

2. From the following alternatives (**A - C**) choose the correct answer to complete the sentences below.

A 90° **B** 270° **C** 360°

- (a) The $\sin \theta$ graph is symmetrical between $\theta = 180^\circ$ and $\theta =$.
- (b) The axis of symmetry for this section of the $\sin \theta$ graph is $\theta =$.

3. Use this section of the $\sin \theta$ graph to answer the following questions.

- | | |
|--|--|
| (a) $\sin 260^\circ = \sin$ <input type="text"/> | (b) $\sin 300^\circ = \sin$ <input type="text"/> |
| (c) $\sin 350^\circ = \sin$ <input type="text"/> | (d) $\sin 200^\circ = \sin$ <input type="text"/> |
| (e) $\sin 275^\circ = \sin$ <input type="text"/> | (f) $\sin 185^\circ = \sin$ <input type="text"/> |
| (g) $\sin 325^\circ = \sin$ <input type="text"/> | (h) $\sin 305^\circ = \sin$ <input type="text"/> |
| (i) $\sin 352^\circ = \sin$ <input type="text"/> | (j) $\sin 198^\circ = \sin$ <input type="text"/> |
| (k) $\sin 269^\circ = \sin$ <input type="text"/> | (l) $\sin 347^\circ = \sin$ <input type="text"/> |

4. From the following alternatives (**A - C**) choose the correct answer to complete the sentences below.

A 180° **B** 270° **C** 360°

- (a) The $\cos \theta$ graph is symmetrical between $\theta = 90^\circ$ and $\theta = \boxed{}$.
 (b) The axis of symmetry for this section of the $\cos \theta$ graph is $\theta = \boxed{}$.

5. Use this section of the $\cos \theta$ graph to answer the following questions.

- | | |
|---|---|
| (a) $\cos 100^\circ = \cos \boxed{}$ | (b) $\cos 240^\circ = \cos \boxed{}$ |
| (c) $\cos 170^\circ = \cos \boxed{}$ | (d) $\cos 200^\circ = \cos \boxed{}$ |
| (e) $\cos 95^\circ = \cos \boxed{}$ | (f) $\cos 225^\circ = \cos \boxed{}$ |
| (g) $\cos 125^\circ = \cos \boxed{}$ | (h) $\cos 185^\circ = \cos \boxed{}$ |
| (i) $\cos 133^\circ = \cos \boxed{}$ | (j) $\cos 238^\circ = \cos \boxed{}$ |
| (k) $\cos 169^\circ = \cos \boxed{}$ | (l) $\cos 267^\circ = \cos \boxed{}$ |

6. From the following alternatives (**A - D**) choose the correct answer to complete the sentences below.

A 90° **B** 180° **C** 270° **D** 360°

- (a) The $\cos \theta$ graph is symmetrical between $\theta = 0^\circ$ and $\theta = \boxed{}$.
 (b) The axis of symmetry for this section of the $\cos \theta$ graph is $\theta = \boxed{}$.

7. Use this section of the $\cos \theta$ graph to answer the following questions.

- | | |
|---|---|
| (a) $\cos 10^\circ = \cos \boxed{}$ | (b) $\cos 280^\circ = \cos \boxed{}$ |
| (c) $\cos 300^\circ = \cos \boxed{}$ | (d) $\cos 50^\circ = \cos \boxed{}$ |
| (e) $\cos 305^\circ = \cos \boxed{}$ | (f) $\cos 25^\circ = \cos \boxed{}$ |
| (g) $\cos 75^\circ = \cos \boxed{}$ | (h) $\cos 355^\circ = \cos \boxed{}$ |
| (i) $\cos 73^\circ = \cos \boxed{}$ | (j) $\cos 298^\circ = \cos \boxed{}$ |
| (k) $\cos 329^\circ = \cos \boxed{}$ | (l) $\cos 71^\circ = \cos \boxed{}$ |

Trigonometric Identities

Remember Pythagoras' Theorem for right-angled triangles:

$$a^2 + b^2 = c^2$$

The terms from the triangle shown here (from earlier in the chapter) can be substituted into Pythagoras' Theorem:

$$(\sin \theta)^2 + (\cos \theta)^2 = 1^2$$

$$(\sin \theta)^2 + (\cos \theta)^2 = 1$$

This is written as:

$$\sin^2 \theta + \cos^2 \theta = 1$$

This is called the ***fundamental trigonometric identity***. A mathematical identity is a mathematical statement that is true for all values.

This identity can be transposed to form the following statements:

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

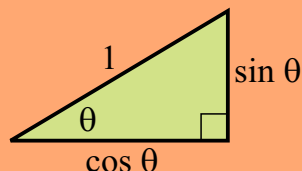
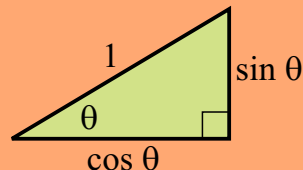
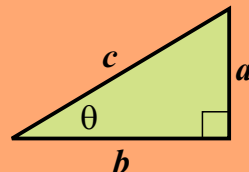
$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

The other useful relationship that can be formed from this triangle is:

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$



EXERCISE 14P

All angles in this exercise are in the first quadrant

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Examples

1. Find $\sin \theta$ if $\cos \theta = 0.4351$

$$\begin{aligned}\sin \theta &= \sqrt{1 - \cos^2 \theta} \\ &= \sqrt{1 - (0.4351)^2}\end{aligned}$$

$$\sin \theta = \mathbf{0.9004} \text{ (4 dec. pl.)}$$

2. Find $\tan \theta$ if $\sin \theta = 0.7771$

$$\text{and } \cos \theta = 0.6293$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{0.7771}{0.6293}$$

$$\tan \theta = \mathbf{1.2349} \text{ (4 dec.pl.)}$$

1. Find $\sin \theta$ for the following values of $\cos \theta$.

Give answers correct to four decimal places.

(a) $\cos \theta = 0.2345$

(b) $\cos \theta = 0.4268$

(c) $\cos \theta = 0.1903$

(d) $\cos \theta = 0.8300$

(e) $\cos \theta = 0.3099$

(f) $\cos \theta = 0.0317$

2. Find $\cos \theta$ for the following values of $\sin \theta$.

Give answers correct to four decimal places.

(a) $\sin \theta = 0.8903$

(b) $\sin \theta = 0.3363$

(c) $\sin \theta = 0.2222$

(d) $\sin \theta = 0.2714$

(e) $\sin \theta = 0.9912$

(f) $\sin \theta = 0.7565$

3. Find $\sin \theta$ for the following values of $\cos \theta$.

Give answers as fractions.

(a) $\cos \theta = \frac{3}{5}$

(b) $\cos \theta = \frac{12}{13}$

(c) $\cos \theta = \frac{9}{41}$

(d) $\cos \theta = \frac{15}{17}$

4. Find $\tan \theta$ for the following values of $\sin \theta$ and $\cos \theta$.

Give answers correct to four decimal places.

(a) $\sin \theta = 0.4384, \cos \theta = 0.8988$

(b) $\sin \theta = 0.7071, \cos \theta = 0.7071$

(c) $\sin \theta = 0.7660, \cos \theta = 0.6428$

(d) $\sin \theta = 0.9877, \cos \theta = 0.1564$

The Sine Rule

The **sine rule** can be used to find an unknown side length or angle size in a triangle that is not a right-angled triangle.

Before using the **sine rule** it is important to label the sides and angles correctly:

Label the angles **A**, **B** and **C**

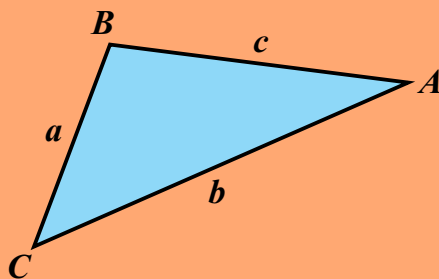
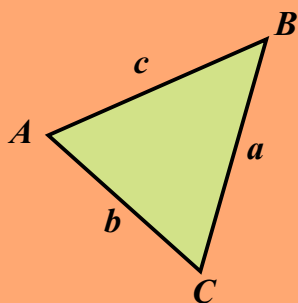
Label the sides **a**, **b** and **c**

Side **a** must be **opposite** angle **A**

Side **b** must be **opposite** angle **B**

Side **c** must be **opposite** angle **C**

These two triangles are labelled correctly.



The **sine rule** states:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The sine rule can be used in two types of situations:

1. **One side and two angles are known.**

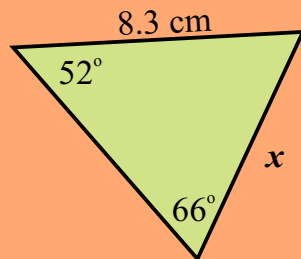
OR

2. **Two sides are known and an angle opposite one of the two known sides.**

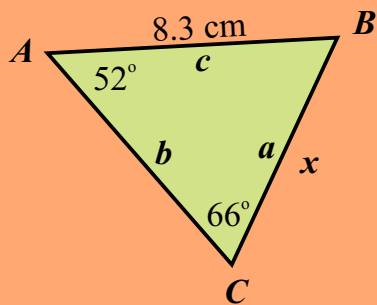
Example 1

Find the unknown side length, x , in the following triangle.
Give answer correct to one decimal place.

Step 1 Check that the *sine rule* can be used.
In this example one side length is known and two angles are known so the sine rule can be used.



Step 2 Label the angles and sides.
It doesn't matter which angle is A , B or C but the sides opposite the angles must be the correct labels.



Step 3 Identify which pairs of sides and angles can be used.
In this example angle A is known and side a is the unknown side. Angle C and side c are known.

Step 4 Write the sine rule for these angles and sides.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Step 5 Substitute the known values into the sine rule.

$$a = x, A = 52^\circ, c = 8.3 \text{ cm}, C = 66^\circ$$

$$\frac{x}{\sin 52} = \frac{8.3}{\sin 66}$$

Step 6 Solve for the unknown.

$$\frac{x}{\sin 52} = \frac{8.3}{\sin 66}$$

$$x = \frac{8.3 \times \sin 52}{\sin 66}$$

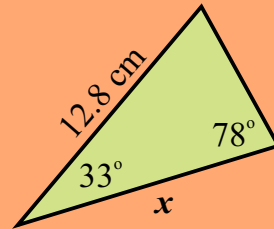
$$x = 7.159\dots$$

$$x = 7.2 \text{ cm (one decimal place)}$$

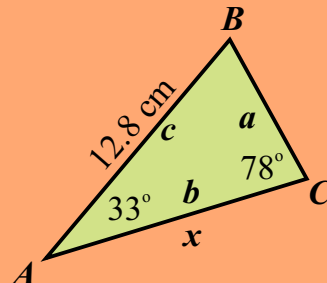
Example 2

Find the unknown side length, x , in the following triangle.
Give answer correct to one decimal place.

Step 1 Check that the *sine rule* can be used.
In this example one side length is known and two angles are known so the sine rule can be used.



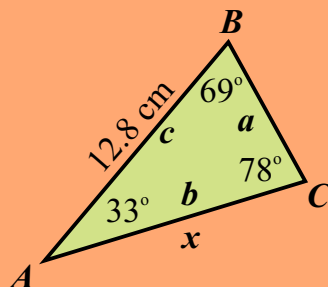
Step 2 Label the angles and sides.



Step 3 Identify which pairs of sides and angles can be used.

In this example angle C and side c are known. Side b is the unknown so we need to know angle B because two pairs are needed to use the sine rule. Angle B can easily be found by using the fact that the three angles in a triangle add to 180° .

$$\begin{aligned} A + B + C &= 180 \\ 33 + B + 78 &= 180 \\ B &= 69^\circ \end{aligned}$$



Step 4 Write the sine rule for these angles and sides.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Step 5 Substitute the known values into the sine rule.

$$b = x, B = 69^\circ, c = 12.8 \text{ cm}, C = 78^\circ$$

$$\frac{x}{\sin 69} = \frac{12.8}{\sin 78}$$

Step 6 Solve for the unknown.

$$\begin{aligned} \frac{x}{\sin 69} &= \frac{12.8}{\sin 78} \\ x &= \frac{12.8 \times \sin 69}{\sin 78} \end{aligned}$$

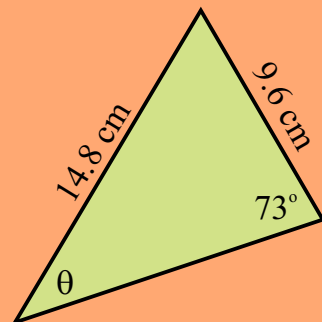
$$x = 12.216\dots$$

$$x = \mathbf{12.2 \text{ cm}} \text{ (one decimal place)}$$

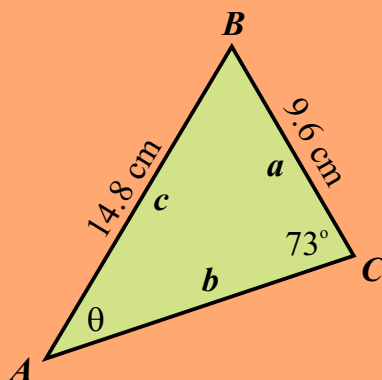
Example 3

Find the unknown angle, θ , in the following triangle.
Give answer correct to the nearest degree.

Step 1 Check that the sine rule can be used.
In this example two side lengths are known and the angle opposite one of the known sides so the sine rule can be used.



Step 2 Label the angles and sides.
It doesn't matter which angle is A , B or C but the sides opposite the angles must be the correct labels.



Step 3 Identify which pairs of sides and angles can be used.
In this example side a is known and angle A is the unknown angle. Angle C and side c are known.

Step 4 Write the sine rule for these angles and sides.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Step 5 Substitute the known values into the sine rule.

$$a = 9.6 \text{ cm}, A = \theta, c = 14.8 \text{ cm}, C = 73^\circ$$

$$\frac{9.6}{\sin \theta} = \frac{14.8}{\sin 73^\circ}$$

Step 6 Solve for the unknown angle.

$$\frac{9.6}{\sin \theta} = \frac{14.8}{\sin 73^\circ}$$

$$\frac{\sin \theta}{9.6} = \frac{\sin 73^\circ}{14.8} \quad \begin{array}{l} \text{(Invert both fractions so the} \\ \text{unknown angle is on the} \\ \text{numerator)} \end{array}$$

$$\sin \theta = \frac{\sin 73^\circ}{14.8} \times 9.6$$

$$\sin \theta = 0.6203....$$

$$\theta = \sin^{-1} 0.6203....$$

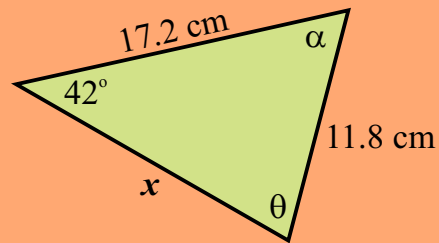
$$\theta = 38.33^\circ$$

$$\theta = \mathbf{38^\circ} \text{ (nearest degree)}$$

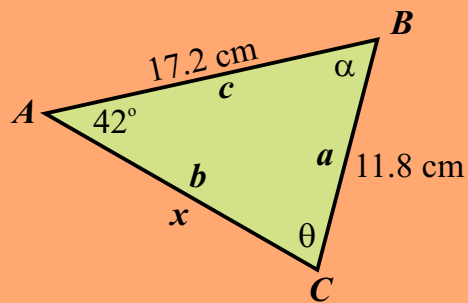
Example 4

Find all the unknown values in the following triangle.
Give unknown length correct to one decimal place and
unknown angles correct to nearest degree.

Step 1 Check that the sine rule can be used. Two sides are known and an angle opposite one of these sides so the sine rule can be used.



Step 2 Label the angles and sides.



Step 3 Identify two pairs of sides and angles that can be used to solve for one of the unknown values.
In this example angle A and side a are known and side c is known so angle C can be found first.

Step 4 Write the sine rule for these angles and sides.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Step 5 Substitute the known values into the sine rule.

$$a = 11.8 \text{ cm}, A = 42^\circ, c = 17.2 \text{ cm}, C = \theta$$

$$\frac{11.8}{\sin 42^\circ} = \frac{17.2}{\sin \theta}$$

Step 6 Solve for θ .

$$\frac{11.8}{\sin 42^\circ} = \frac{17.2}{\sin \theta}$$

$$\frac{\sin 42^\circ}{11.8} = \frac{\sin \theta}{17.2} \quad (\text{Invert both fractions so the unknown angle is on the numerator})$$

$$\frac{\sin \theta}{17.2} = \frac{\sin 42^\circ}{11.8} \quad (\text{Swap sides so the unknown angle is on the left side of the equation})$$

$$\sin \theta = \frac{\sin 42^\circ}{11.8} \times 17.2$$

$$\sin \theta = 0.9753\dots$$

$$\theta = 77.25^\circ$$

$$\theta = 77^\circ \text{ (nearest degree)}$$

Step 7 Solve for α .

All the angles add to 180° .

$$A + B + C = 180^\circ$$

$$42 + \alpha + 77 = 180^\circ$$

$$\alpha = 61^\circ$$

Step 8 Solve for x . Either of the following equations can be used because all of the other values are known.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Using the following equation will be more accurate because the side length a was given in the question.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = 11.8 \text{ cm}, A = 42^\circ, b = x, B = 61^\circ$$

$$\frac{11.8}{\sin 42} = \frac{x}{\sin 61}$$

$$\frac{x}{\sin 61} = \frac{11.8}{\sin 42}$$

$$x = \frac{11.8}{\sin 42} \times \sin 61$$

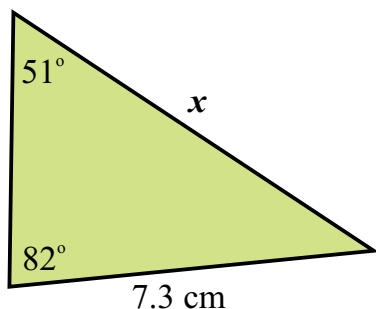
$$x = 15.42\dots$$

$$x = \mathbf{15.4 \text{ cm}} \text{ (one decimal place)}$$

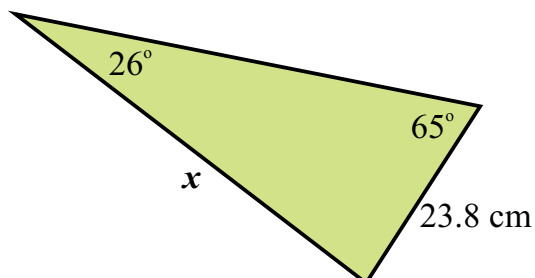
EXERCISE 14Q

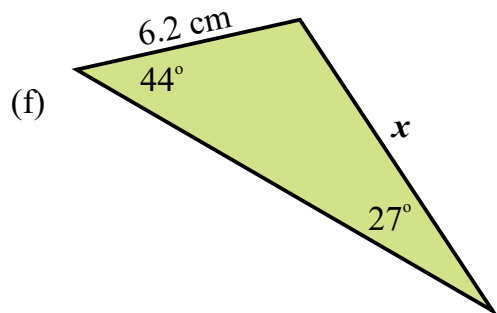
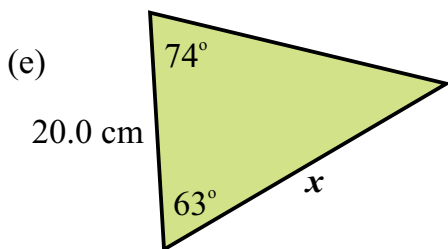
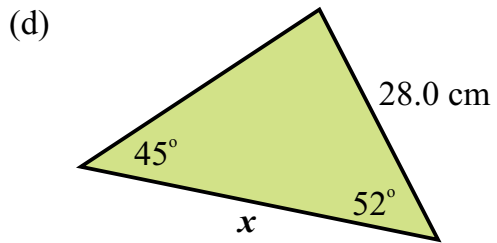
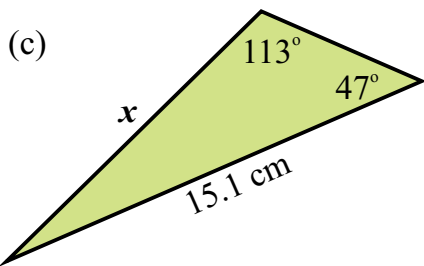
1. Find the unknown side length in each of the following triangles.
Give answers correct to one decimal place.

(a)

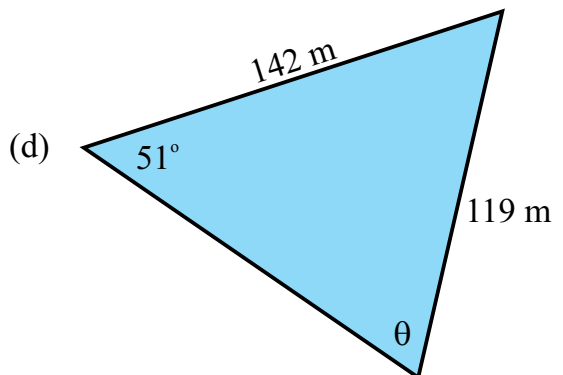
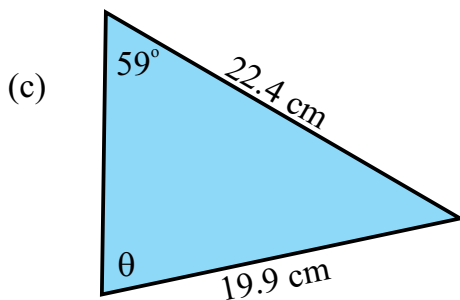
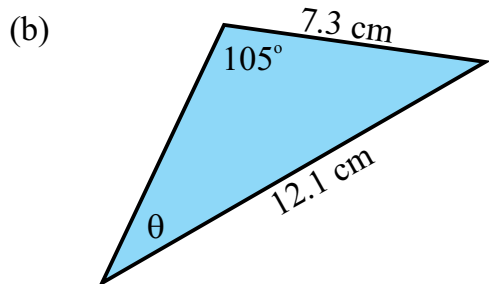
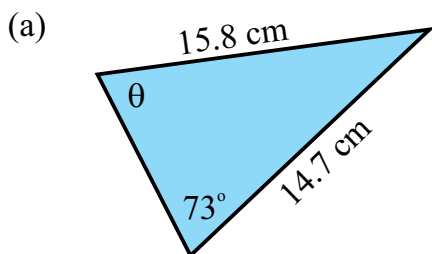


(b)



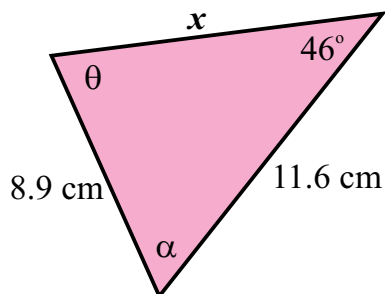


2. Find the unknown angle in each of the following triangles.
Give answers correct to the nearest degree.

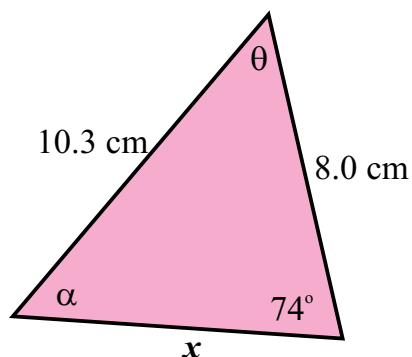


3. Find all the unknown values in each of the following triangles.
 Give lengths correct to one decimal place.
 Give angles correct to the nearest degree.

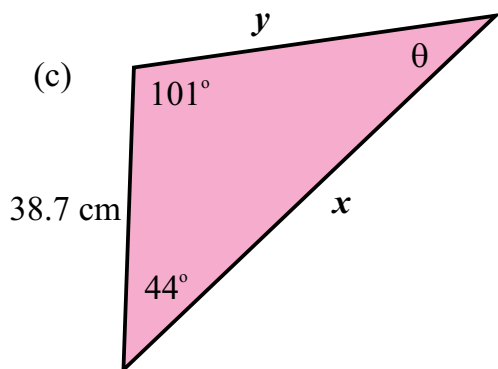
(a)



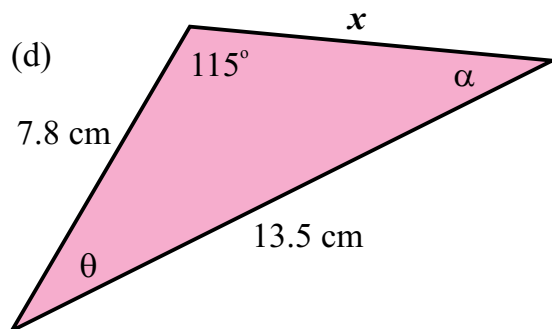
(b)



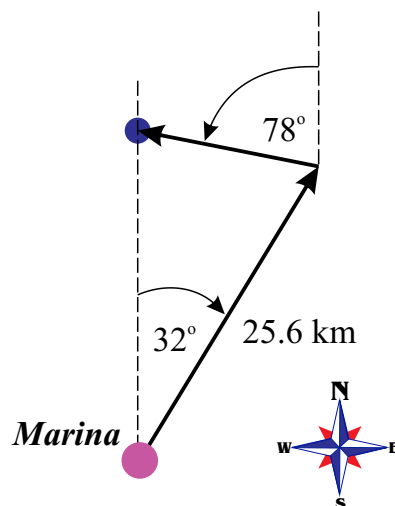
(c)



(d)



4. A yacht sails 25.6 km in a direction of N 32° E of a marina. It then sails in a direction of N 78° W until it is due north of the marina.
 How far is the yacht from the marina?
 Give answer correct to one decimal place.



The Cosine Rule

The **cosine rule** is another rule that can be used to find an unknown side length or angle size in a triangle that is not a right-angled triangle. Before using the **cosine rule** it is important to label the sides and angles in the same way as for the **sine rule**.

Label the angles **A**, **B** and **C**

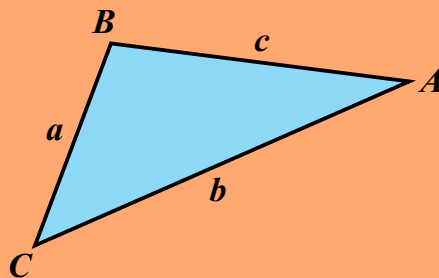
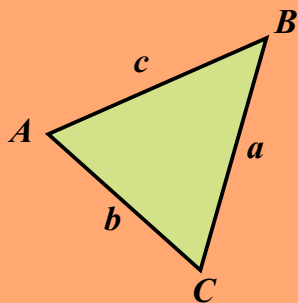
Label the sides **a**, **b** and **c**

Side **a** must be **opposite** angle **A**

Side **b** must be **opposite** angle **B**

Side **c** must be **opposite** angle **C**

These two triangles are labelled correctly.



The **cosine rule** states:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The cosine rule can be used in two types of situations:

1. **Two side lengths and the included angle are known.**

OR

2. **The three side lengths are known.**

The form of the cosine rule will depend on the known and unknown sides and angles.

If sides a and b and angle C are known then the cosine rule can be used to find side c and it will be written:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

If sides b and c and angle A are known then the cosine rule can be used to find side a and it will be written:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

If sides a and c and angle B are known then the cosine rule can be used to find side b and it will be written:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

To find an angle using the cosine rule the three sides need to be known and the cosine rule needs to be transposed to make the angle the subject:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

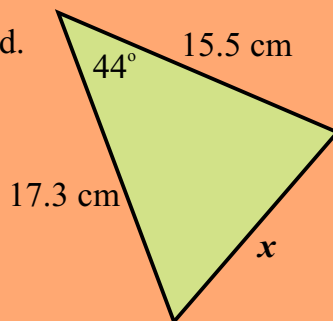
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 1

Find the unknown length, x , in the following triangle.
Give answer correct to one decimal place.

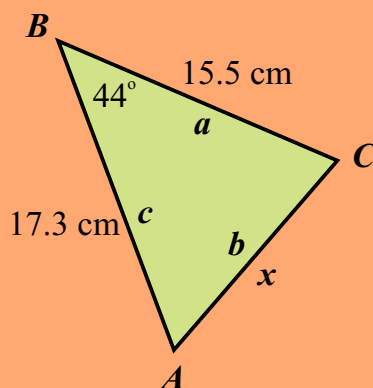
Step 1 Check that the cosine rule can be used.

In this example two sides and the included angle are known so the cosine rule can be used.



Step 2 Label the sides and angles.

Remember side a must be opposite angle A , side b must be opposite angle B and side c must be opposite angle C .



Step 3 Identify which sides and angle can be used.

In this example sides a and c and angle B are known.
Side b is the unknown side x .

Step 4 Write the appropriate form of the cosine rule.

In this example:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Step 5 Substitute the known values into the cosine rule and solve.

$$a = 15.5, b = x, c = 17.3, B = 44^\circ$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\begin{aligned} x^2 &= 15.5^2 + 17.3^2 - 2 \times 15.5 \times 17.3 \times \cos 44^\circ \\ &= 153.758... \end{aligned}$$

$$x = \sqrt{153.758...}$$

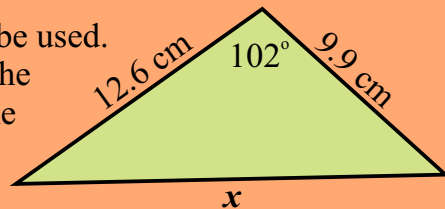
$$x = 12.399...$$

$$x = \mathbf{12.4 \text{ cm}} \text{ (one decimal place)}$$

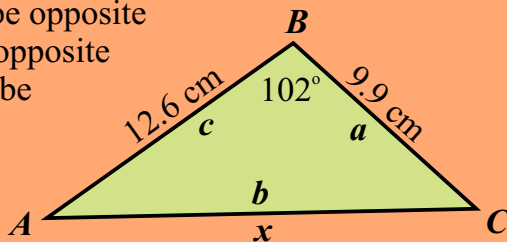
Example 2

Find the unknown length, x , in the following triangle.
Give answer correct to one decimal place.

Step 1 Check that the cosine rule can be used.
In this example two sides and the included angle are known so the cosine rule can be used.



Step 2 Label the sides and angles.
Remember side a must be opposite angle A , side b must be opposite angle B and side c must be opposite angle C .



Step 3 Identify which sides and angle can be used.

In this example sides ***a*** and ***c*** and angle ***B*** are known.
Side ***b*** is the unknown side ***x***.

Step 4 Write the appropriate form of the cosine rule.

In this example:

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Step 5 Substitute the known values into the cosine rule and solve.

$$a = 9.9, b = x, c = 12.6, B = 102^\circ$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\begin{aligned} x^2 &= 9.9^2 + 12.6^2 - 2 \times 9.9 \times 12.6 \times \cos 102^\circ \\ &= 98.01 + 158.76 - 2 \times 9.9 \times 12.6 \times (-.2079...) \\ &= 308.639... \end{aligned}$$

$$x = \sqrt{308.639...}$$

$$x = 17.568...$$

$$x = 17.6 \text{ cm (one decimal place)}$$

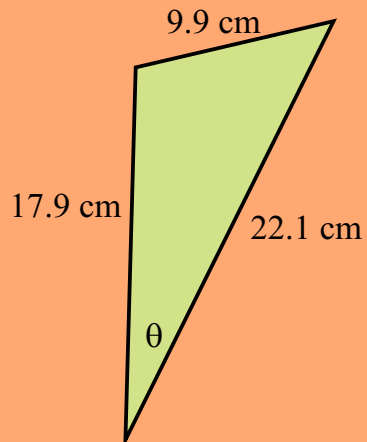
Note the cosine of an angle between 90° and 180° is negative

Example 3

Find the unknown angle, θ , in the following triangle.
Give answer correct to the nearest degree.

Step 1 Check that the cosine rule can be used.

In this example the three sides are known and an angle is unknown so the cosine rule can be used.

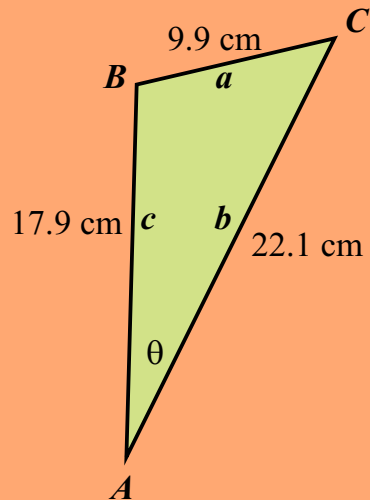


Step 2 Label the sides and angles.

Step 3 Identify which is the unknown angle.
In this example the unknown angle is A .

Step 4 Write the appropriate form of the cosine rule.
In this example:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



Step 5 Substitute the known values into the cosine rule and solve.

$$a = 9.9, b = 22.1, c = 17.9$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{22.1^2 + 17.9^2 - 9.9^2}{2 \times 22.1 \times 17.9}$$

$$\cos A = 0.8984...$$

$$A = \cos^{-1} 0.8984...$$

$$A = 26.049...$$

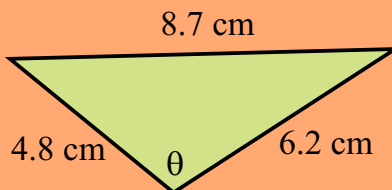
$$A = 26^\circ \text{ (nearest degree)}$$

Example 4

Find the unknown angle, θ , in the following triangle.
Give answer correct to the nearest degree.

Step 1 Check that the cosine rule can be used.

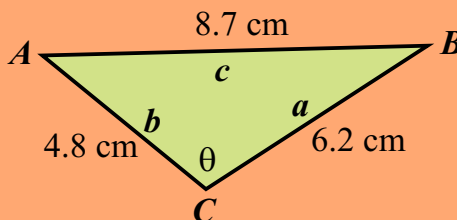
In this example the three sides are known and an angle is unknown so the cosine rule can be used.



Step 2 Label the sides and angles.

Step 3 Identify which is the unknown angle.

In this example the unknown angle is C .



Step 4 Write the appropriate form of the cosine rule.

In this example:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Step 5 Substitute the known values into the cosine rule and solve.

$$a = 6.2, b = 4.8, c = 8.7$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{6.2^2 + 4.8^2 - 8.7^2}{2 \times 6.2 \times 4.8}$$

$$\cos C = -0.2387...$$

$$C = \cos^{-1}(-0.2387...)$$

$$C = 103.812...$$

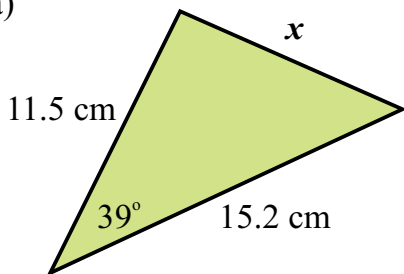
$$C = 104^\circ \text{ (nearest degree)}$$

Note the cosine of an angle between 90° and 180° is negative

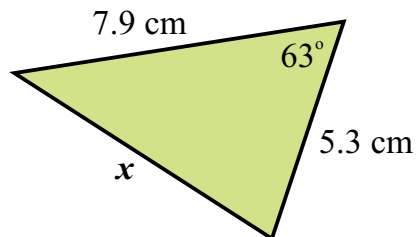
EXERCISE 14R

1. Find the unknown length, x , in each of the following triangles.
Give answers correct to one decimal place.

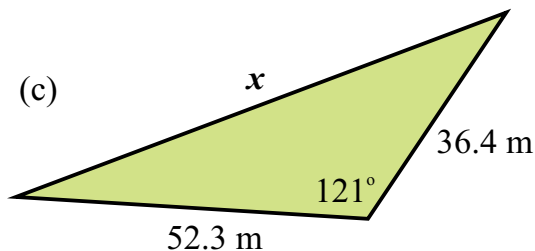
(a)



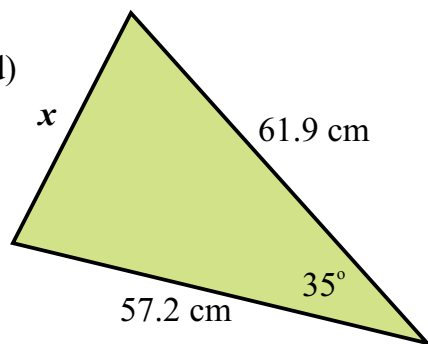
(b)



(c)

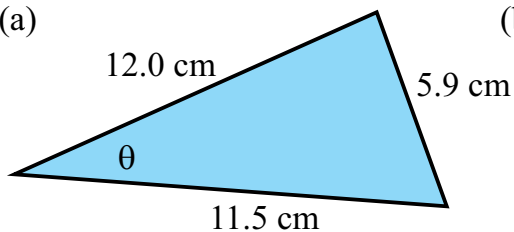


(d)

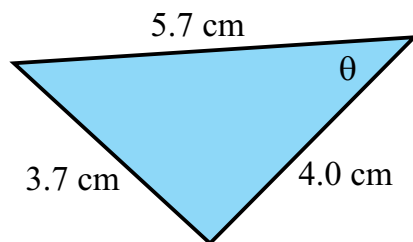


2. Find the unknown angle, θ , in each of the following triangles.
Give answer correct to the nearest degree.

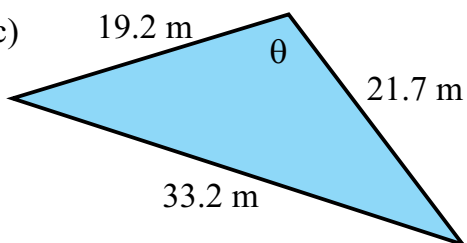
(a)



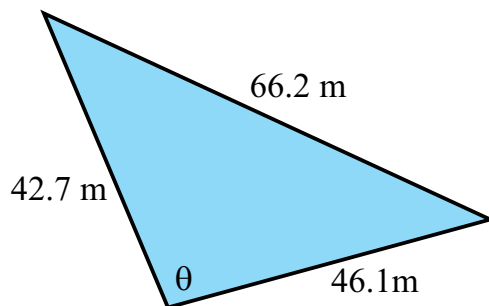
(b)



(c)



(d)



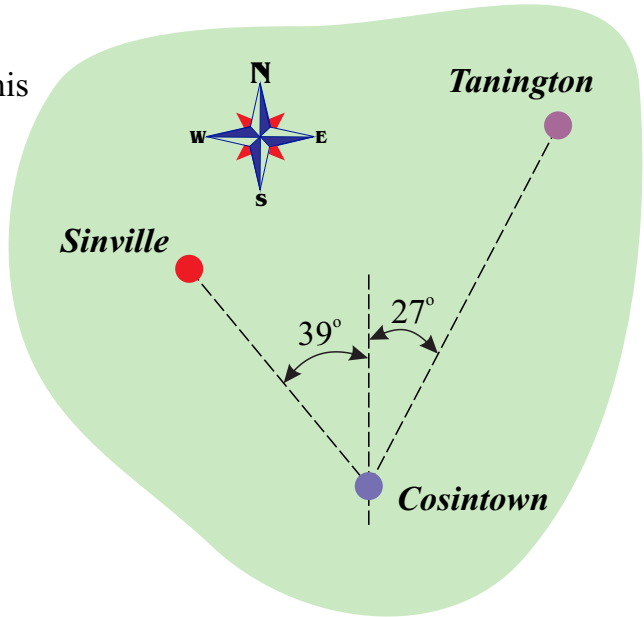
3. Three towns are shown on this map.

Tanington is 87 km N 27° E from Cosintown.

Sinville is 59 km N 39° W from Cosintown.

How far is Tanington from Sinville?

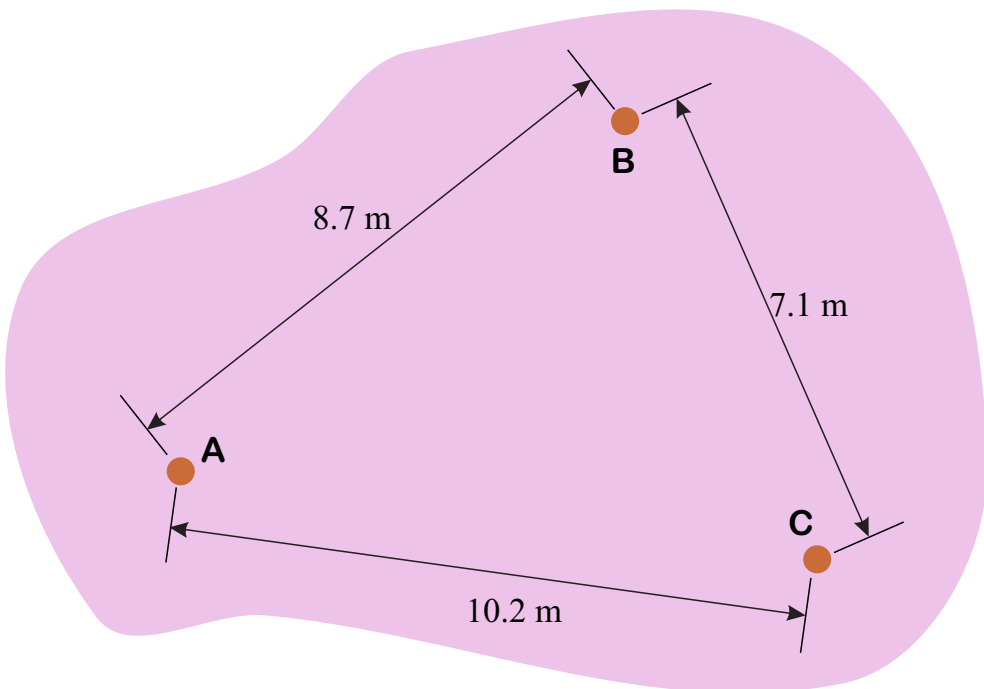
Give answer to nearest km.



4. Amity wants to make a shade cloth for her back courtyard. The courtyard is in the shape of a triangle and the distances between the corner posts are shown on the diagram below.

What is the angle at each of the corners - **A**, **B** and **C**?

Give angles correct to the nearest degree.



Mixed Problems

SUMMARY OF SINE RULE AND COSINE RULE

SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The *sine rule* can be used when:

1. *One side and two angles are known.*

OR

2. *Two sides are known and an angle opposite one of the two known sides.*

COSINE RULE

$$a^2 = b^2 + c^2 - 2bc \cos A$$

The *cosine rule* can be used when:

1. *Two side lengths and the included angle are known.*

OR

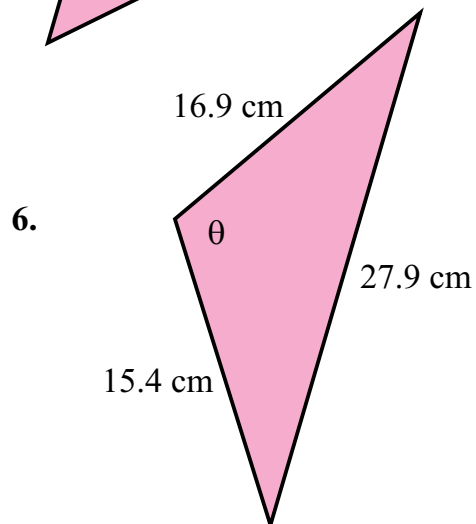
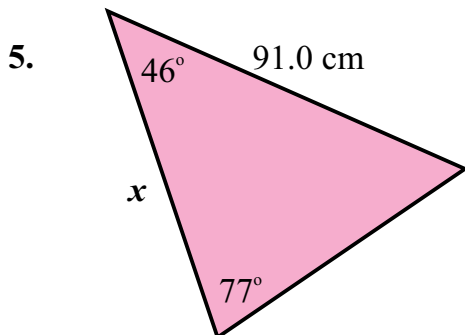
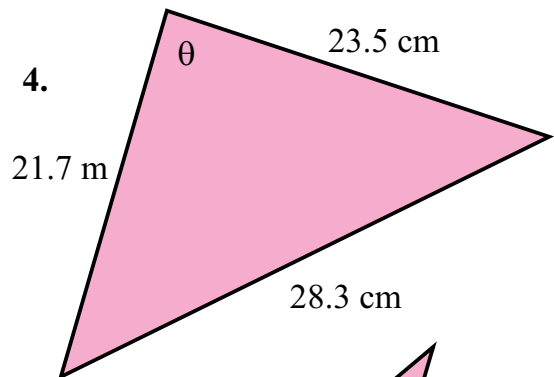
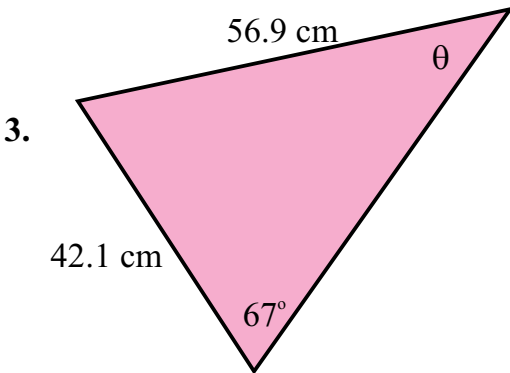
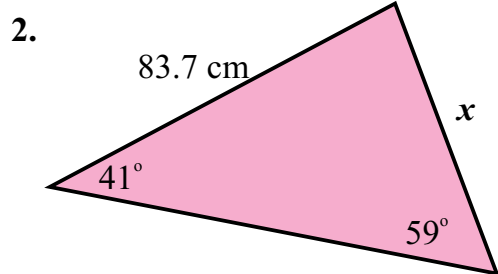
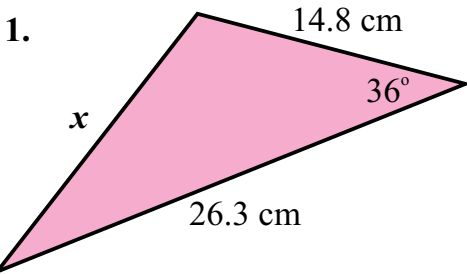
2. *The three side lengths are known.*

EXERCISE 14S

For each of the following triangles:

- (a) state which of the rules (*sine rule* or *cosine rule*) could be used to find the unknown value.
(b) find the unknown value.

Give lengths correct to one decimal place and angles correct to the nearest degree.

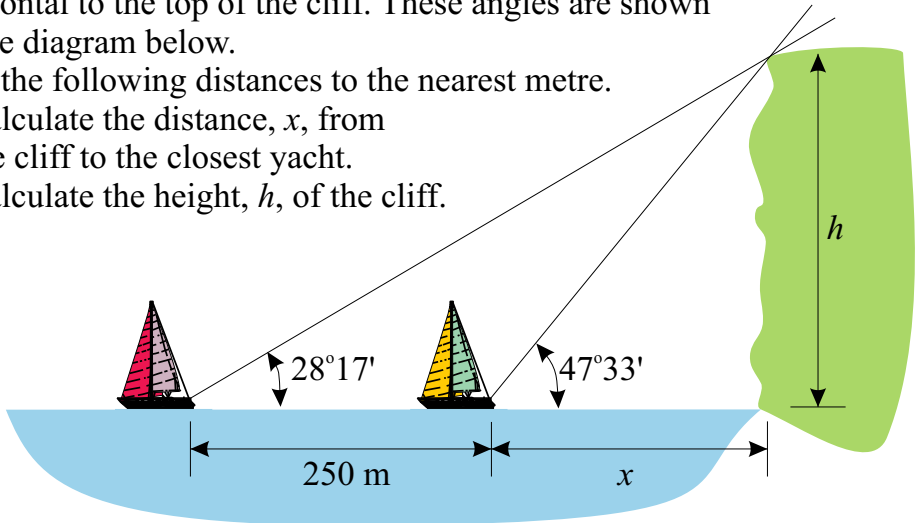


PROBLEM SOLVING

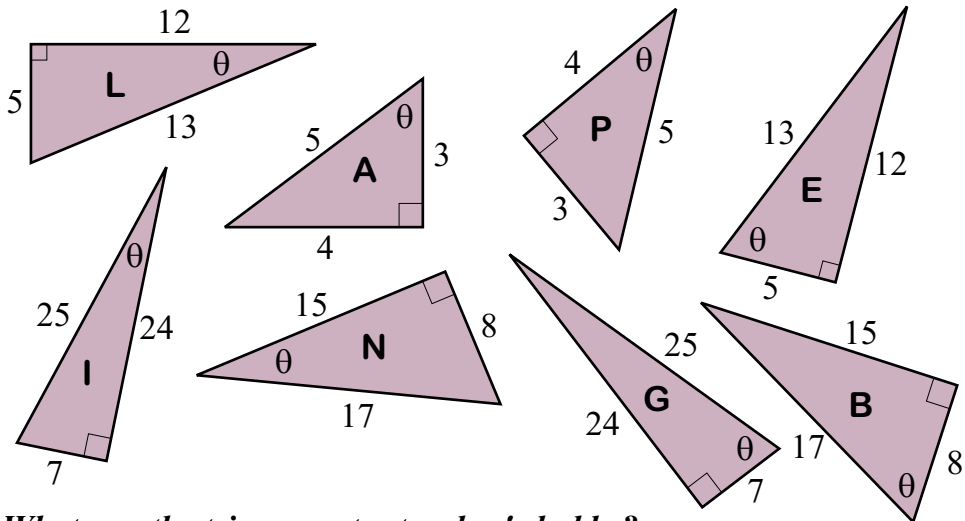
Two yachts are sailing near a cliff. The yachts are 250 m apart. The people on each yacht measure the angle above the horizontal to the top of the cliff. These angles are shown on the diagram below.

Find the following distances to the nearest metre.

1. Calculate the distance, x , from the cliff to the closest yacht.
2. Calculate the height, h , of the cliff.



PUZZLE



What was the trigonometry teacher's hobby?



$\tan \theta = \frac{4}{3}$
 $\sin \theta = \frac{8}{17}$
 $\cos \theta = \frac{7}{25}$
 $\tan \theta = \frac{5}{12}$
 $\cos \theta = \frac{24}{25}$
 $\tan \theta = \frac{8}{15}$
 $\sin \theta = \frac{24}{25}$

CHAPTER REVIEW

1. Convert the following degrees to degrees, minutes and seconds.

- (a) 23.2° (b) 68.35° (c) 8.46° (d) 50.82°
 (e) 8.105° (f) 72.518° (g) 105.473° (h) 67.211°

2. Convert the following angles to decimal degrees.

Give answers correct to two decimal places.

- (a) $25^\circ 25'$ (b) $4^\circ 17' 25''$ (c) $62^\circ 8' 27''$ (d) $35^\circ 27' 46.8''$

3. Convert the following angles from degrees to radians.

(i) Write answer as a fraction in its simplest form.

(ii) Write answer as a decimal correct to two decimal places

- (a) 30° (b) 20° (c) 125° (d) 234°

4. Convert the following angles from radians to degrees.

- (a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{5}$ (c) $\frac{5\pi}{9}$ (d) $\frac{11\pi}{12}$

5. Convert the following angles from radians to degrees.

Give answers correct to two decimal places.

- (a) 0.5° (b) 2.3° (c) 1.21° (d) 0.045°

6. Use a calculator to find the following values correct to four decimal places. (Check calculator is in **DEGREE** mode)

- (a) $\sin 48.2^\circ$ (b) $\cos 26.3^\circ$ (c) $\tan 58.37^\circ$
 (d) $\cos 21^\circ 43'$ (e) $\sin 49^\circ 37' 21''$ (f) $\tan 71^\circ 8' 39''$

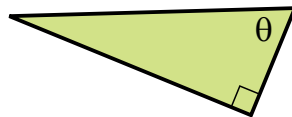
7. Use a calculator to find the following values correct to four decimal places. (Check calculator is in **RADIAN** mode)

- (a) $\sin 1.3^\circ$ (b) $\cos 0.37^\circ$ (c) $\tan 0.225^\circ$

8. (a) For what are O , A and H the abbreviations?

(b) Copy this triangle and label the sides O , A and H for the angle θ .

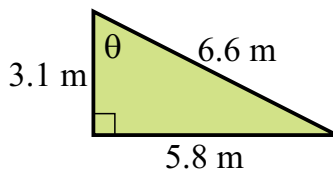
(c) Use the abbreviations O , A and H to complete the following.



$\sin \theta = \text{ ——— }$ $\cos \theta = \text{ ——— }$ $\tan \theta = \text{ ——— }$

9. For this triangle find the following values correct to four decimal places.

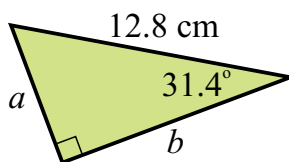
(a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$



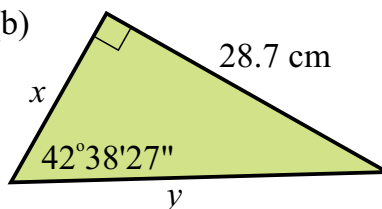
10. Check calculator is in **DEGREE** mode.

Find the unknown side lengths in each of the following triangles. Give answers correct to one decimal place.

(a)



(b)



11. A yacht sails 56 km in a direction S $45^{\circ}17'$ W.

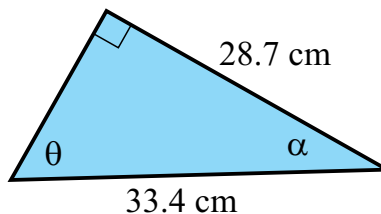
Give the following answers correct to one decimal place.

- (a) How far west is it from its starting point?
(b) How far south is it from its starting point?

12. An aeroplane takes off at a speed of 25 m/s at an angle of 15° to the horizontal until it reaches an altitude of 4000 m. How long did it take to reach this altitude? Give answer in seconds to the nearest second.

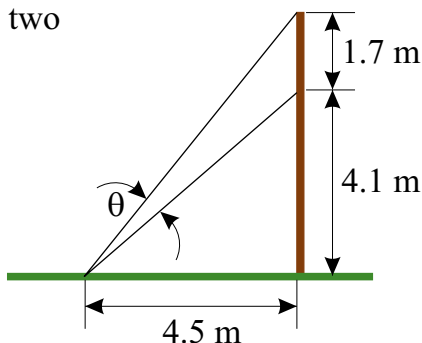
13. Find the two unknown angles in this triangle.

- (a) Give answers as decimals correct to two decimal places.
(b) Give answers in DMS correct to the nearest second.

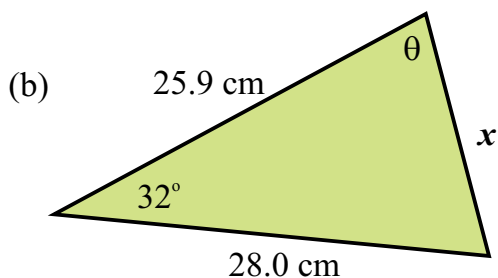
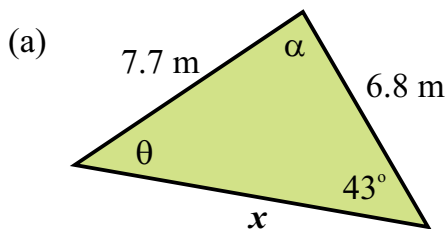


14. The pole shown here is secured by two cables. The distances to the fixed points are indicated.

Find the angle, θ , between the two cables. Give answer in DMS correct to the nearest second.

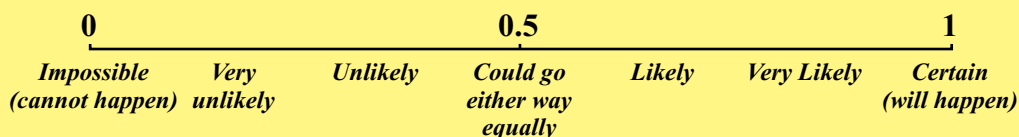


15. A plane leaves Alice Springs and flies 250 km in a direction S $43^{\circ}27'$ W then 325 km in a direction N $62^{\circ}38'$ W. They then want to fly straight back to Alice Springs.
- (a) How far will they need to fly? Give answer correct to one decimal place.
- (b) In which direction do they need to fly? Give answer in DMS correct to the nearest second.
16. In which quadrant, *first*, *second*, *third* or *fourth*, would the following angles be located?
- (a) 135° (b) 275° (c) 287° (d) 345° (e) 81° (f) 197°
17. Would the following values be *positive* or *negative*?
- (a) $\sin 128^{\circ}$ (b) $\cos 257^{\circ}$ (c) $\sin 300^{\circ}$ (d) $\cos 320^{\circ}$
 (e) $\cos 238^{\circ}$ (f) $\cos 87^{\circ}$ (g) $\sin 137^{\circ}$ (h) $\sin 201^{\circ}$
18. (a) Write the *fundamental trigonometric identity*.
 (b) Complete the following transposed versions of this identity.
- (i) $\sin \theta = \sqrt{\quad}$ (ii) $\cos \theta = \sqrt{\quad}$
19. Use the terms $\sin \theta$ and $\cos \theta$ to write an expression for $\tan \theta$.
20. Find $\sin \theta$ if $\cos \theta = 0.4561$. Give answer correct to four decimal places.
21. Find $\cos \theta$ if $\sin \theta = 0.2069$. Give answer correct to four decimal places.
22. Find $\tan \theta$ if $\sin \theta = 0.3846$ and $\cos \theta = 0.9231$.
23. Use the *sine rule* or *cosine rule* to find the unknown values each of the following triangles. Give lengths correct to one decimal place and angles correct to the nearest degree.



Probability

1. Rate the probability of the following events occurring according to the scale below. For several of the events the answers will vary depending on circumstances. Answers will be decimal numbers.



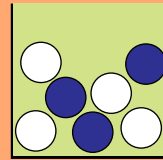
- (a) It will rain tomorrow.
 - (b) The maximum temperature will be more than 20°C tomorrow.
 - (c) The maximum temperature will be less than 20°C tomorrow.
 - (d) Tomorrow will be Sunday.
 - (e) If you toss a coin it will show heads.
 - (f) A WA team will win the AFL grand final next year.
 - (g) A NSW team will win the NRL grand final next year.
 - (h) A new born baby will be a boy.
 - (i) A pregnant woman is having twins.
 - (j) Someone else in your class will have their birthday in the same month as yours.
 - (k) Someone else in your class will have their birthday on the same day as yours.
 - (l) Your favourite song is the same as your parent's.
 - (m) You would get a 'strike' at ten-pin bowling.
 - (n) You would hit a fault when serving while playing tennis.
 - (o) You will finish all of your homework tonight.
 - (p) Christmas Day will be the 25th of December.
 - (q) If you dropped a glass jar it would break.
 - (r) If you dropped a glass jar it would not break.
2. List two events that would have the following probabilities of occurring: (a) 0 (b) 0.1 - 0.4 (c) 0.5 (d) 0.6 - 0.9 (e) 1

The probability of an event occurring (success) can be calculated by finding the ratio of the number of successful outcomes to the total number of outcomes.

$$P(\text{success}) = \frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$$

Example 1

Find the probability of randomly choosing a white ball from this box.



$$P(\text{choosing a white ball}) = \frac{\text{Number of white balls}}{\text{Total number of balls}}$$

$$P(\text{choosing a white ball}) = \frac{4}{7}$$

Note: Probabilities can be written as decimals, fractions or percentages

Example 2

Find the probability of tossing a coin and heads resulting.

$$P(\text{heads}) = \frac{1}{2} = 0.5 = 50\%$$

If A represents an event occurring then A' represents the event A not occurring. A' is called the **complement** of A .

$$P(A) + P(A') = 1$$

or

$$P(A') = 1 - P(A)$$

Example 3

$$\text{Find } P(A') \text{ if } P(A) = \frac{3}{5}$$

$$P(A') = 1 - \frac{3}{5} = \frac{2}{5}$$

Example 4

The probability of a certain brand of water bed leaking was 0.15.
What is the probability that a bed will not leak?

Let the probability of a bed leaking = $P(L)$

$$P(L) = 0.15$$

The probability of a bed not leaking = $P(L')$

$$P(L') = 1 - P(L)$$

$$= 1 - 0.15$$

$$P(L') = \mathbf{0.85}$$

The sum of the probabilities of all the possible outcomes = 1

Example 5

A bag contains chocolates with strawberry, peppermint or caramel centres. You are told that the probability of choosing a strawberry, $P(S)$, is 0.35 and the probability of choosing a peppermint, $P(P)$, is 0.42. Find the probability of choosing a caramel, $P(C)$.

$$P(S) + P(P) + P(C) = 1$$

$$0.35 + 0.42 + P(C) = 1$$

$$0.77 + P(C) = 1$$

$$P(C) = 1 - 0.77$$

$$P(C) = \mathbf{0.23}$$

3. If one number is chosen randomly from the numbers shown below find the following probabilities.

Write probabilities as fractions in their simplest form.

- $P(\text{the number is even})$
- $P(\text{the number is greater than } 20)$
- $P(\text{the number is odd and less than } 20)$

25	17	36	22	11	29	32	7	34
	14	18	41	20	13	40	33	

4. Kelsey bought 20 tickets in a raffle that had 500 tickets. What is the probability that Kelsey had the winning ticket?
Give answer as a decimal.

5. In the game of **SCRABBLE** there are tiles with the letters of the alphabet. The number of tiles of each letter is shown below.

A - 9, **B** - 2, **C** - 2, **D** - 4, **E** - 12, **F** - 2, **G** - 3, **H** - 2, **I** - 9, **J** - 1,
K - 1, **L** - 4, **M** - 2, **N** - 6, **O** - 8, **P** - 2, **Q** - 1, **R** - 6, **S** - 4, **T** - 6,
U - 4, **V** - 2, **W** - 2, **X** - 1, **Y** - 2, **Z** - 1

Find the following probabilities if the letters are randomly selected from a **full** box of tiles.

Give answers as fractions in their simplest form.

- (a) P(selecting an **E**) (b) P(selecting a vowel)
(c) P(selecting a **D**, **M** or **T**)

Find the probabilities below if the following letters have been used:

A - 2, **B** - 1, **E** - 4, **H** - 2, **I** - 4, **N** - 1, **O** - 3, **S** - 3, **T** - 1, **Y** - 2

- (d) P(selecting an **E**) (e) P(selecting a vowel)
(f) P(selecting a **B**, **C** or **T**) (g) P(selecting a **Y**)

6. There are 64 girls and 72 boys in year 10 at a school where there are 480 girls and 462 boys. **One** student is chosen at random from the school student population to represent the school at a council meeting. Find the following probabilities.

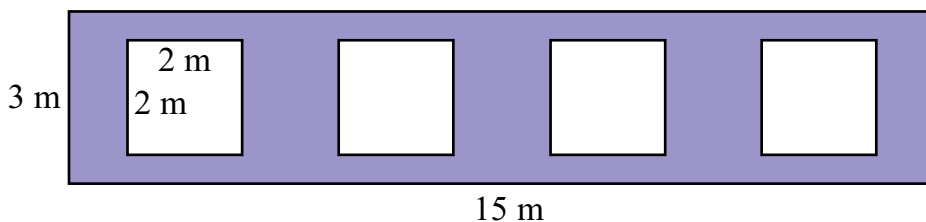
Give answers as fractions in their simplest form.

- (a) P(the student is a girl) (b) P(the student is from year 10)
(c) P(the student is a boy from year 10)

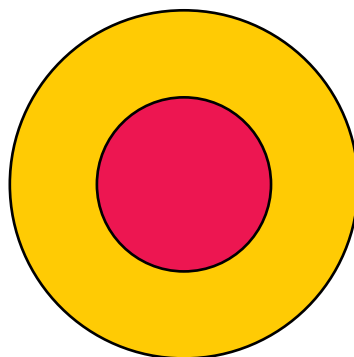
7. The table here shows the total number of people in a city competing in certain sports and the total number of serious injuries in each sport. Based on this data, find the probability of being injured in each sport and list the sports in order from the most 'dangerous' to the least 'dangerous'. Give probabilities as decimals correct to three decimal places.

<i>Sport</i>	<i>Number Competing</i>	<i>Number of Injuries</i>
Football	3460	124
Rugby	875	72
Netball	3288	83
Basketball	4120	89
Soccer	1586	36

8. In a lucky dip the probability of picking a prize was $\frac{2}{9}$.
What is probability of not picking a prize?
9. In a raffle the probability of winning a mountain bike was 0.008, the probability of winning a helmet was 0.122 and the probability of winning one of the other prizes was 0.138.
What is the probability of not winning a prize?
10. A box contained blue, red and yellow balls. The probability of picking a red ball was three times the probability of picking a blue ball. The probability picking a yellow ball was twice that of picking a red ball. Find the probability of picking each coloured ball.
Give answers as decimal numbers.
11. A coin is tossed four times and heads comes up each time. If the coin is tossed again, what is the probability that heads will come up?
12. Jasper hit a cricket ball that struck a wall that was 15 metres long and 3 metres high with four square windows each with side length of 2 metres.
What is the probability that the ball will hit a window?
Give answer as a decimal correct to three decimal places.



13. The archery target shown here has a central circle that has a radius that is half the radius of the outer circle.
An archer fires an arrow that randomly hits the target.
What is the probability that it will hit the central circle?
Give answer as a decimal.



14. A deck of playing cards contains four suits - hearts♥, diamonds♦, clubs♣ and spades♠. Each suit has 13 cards - an ace, cards numbered 2 - 10, a jack, queen and king.

Hearts and diamonds are *red* and clubs and spades are *black*.

Give the following answers as fractions in their simplest form.

(a) A card is drawn randomly from a full deck. Find the following probabilities.

- (i) $P(\text{drawing a club})$
- (ii) $P(\text{drawing a red card})$
- (iii) $P(\text{drawing a jack, queen or king})$
- (iv) $P(\text{drawing a black jack})$
- (v) $P(\text{drawing the ace of hearts})$
- (vi) $P(\text{not drawing an ace})$

(b) Three aces are removed from the deck. What is the probability of randomly drawing the remaining ace from the deck?

(c) The following cards have been used in a game:

5♠, 10♣, jack♦, ace♣, 6♥, jack♠, 3♠, jack♥, queen♦, jack♣

A card is drawn from the remainder of the deck. Find the following probabilities.

- (i) $P(\text{drawing an ace})$
- (ii) $P(\text{drawing a jack})$
- (iii) $P(\text{drawing a black card})$

15. Michael is a specialist three-point shooter in the school basketball team. The table below shows the number of attempts at goal and the number of goals he scored in eight games.

Based on this data, what is the probability that Michael scores a goal?

Give answer as decimal correct to three decimal places.

Attempts	Goals
8	6
9	7
11	8
6	2
5	4
12	9
13	8
10	6



Predicting Outcomes

EXERCISE 15B

Example

Kirby wanted to find out the probability of winning his favourite card game of patience.

He played the game 500 times and won 100.

The probability of winning = $\frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$

$$P(\text{winning}) = \frac{100}{500} = \frac{1}{5} = 0.2 = 20\%$$

If Kirby played the game 20 times, how many could he expect to win?

$$\text{Number of successful outcomes} = P(\text{success}) \times \text{Total number of outcomes}$$

$$\begin{aligned} \text{Number of successful outcomes} &= \frac{1}{5} \times 20 \\ &= 4 \end{aligned}$$

1. (a) What is the probability of rolling a 6 on a standard die?
Give answer as a fraction.
- (b) If a die is rolled 120 times, how many times would you expect a 6 to be rolled?
2. A gardener experimented with grafting cuttings onto plants. She grafted 30 cuttings and 12 grew.
 - (a) Based on this data, what is the probability that a grafted cutting will grow? Give answer as a fraction in its simplest form.
 - (b) If the gardener was to graft 160 cuttings, how many could she expect to grow?
 - (c) If she wanted 100 grafted cuttings to grow, how many cuttings should she graft?

3. The full forward in an AFL football team in a season had 148 attempts at kicking a goal. He kicked 86 goals, 55 behinds and had 7 misses.

(a) Based on these figures, calculate the following probabilities **as percentages** correct to one decimal place.

(i) $P(\text{goal})$ (ii) $P(\text{behind})$ (iii) $P(\text{miss})$

(b) If, in a game, the full forward has 16 attempts at goal find the following. Round answers to the nearest whole number.

(i) How many goals could he expect to kick?

(ii) How many behinds could he expect to kick?

4. The probability of a green sea turtle surviving to be a fully grown adult is 0.01%. Scientists recorded 10 000 baby turtles hatched on Heron Island one year. How many of these will survive to be adults?

(This is why the green sea turtle is listed as endangered)



5. The scores of a batter in cricket over a season are shown below.

5, 87, 23, 6, 8, 0, 95, 2, 119, 4, 0, 75, 11, 138, 107, 12, 90, 2, 5

Based on these scores, find the following probabilities as percentages correct to one decimal place.

(a) What is the probability of the cricketer scoring more than 20 runs in an innings?

(b) If the batter gets to 20, what is the probability he scores more than 50 runs?

(c) If the batter gets to 20, what is the probability he scores more than 100 runs?

6. 1 in 12 males and 1 in 250 females are colour-blind.

(a) What is the probability of a male being colour-blind?

Write answer as a decimal correct to three decimal places.

(b) What is the probability of a female being colour-blind?

Write answer as a decimal correct to three decimal places.

(c) There are 480 boys and 500 girls at a school.

(i) How many boys would be expected to be colour-blind?

(ii) How many girls would be expected to be colour-blind?

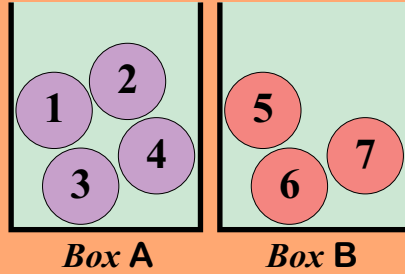
Listing Possible Outcomes for Multiple Events

There are two convenient ways of displaying all the possible outcomes for multiple events - *tree diagrams* and *two-way tables*.

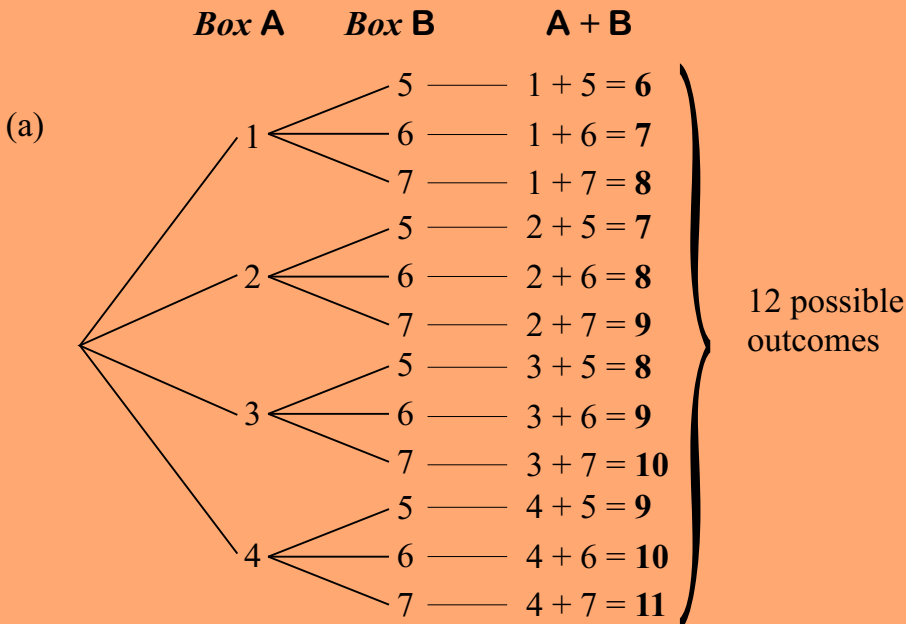
Tree Diagrams

Example 1

A ball is taken randomly from each of the two boxes shown - Box **A** and Box **B**. The numbers on the two balls are added.



- Construct a tree diagram to show all the possible outcomes.
- Find the probability that the sum of the two numbers is less than 10, $P(A + B < 10)$.

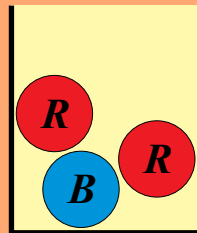


- (b) There are nine of the possible outcomes that have a sum less than 10.

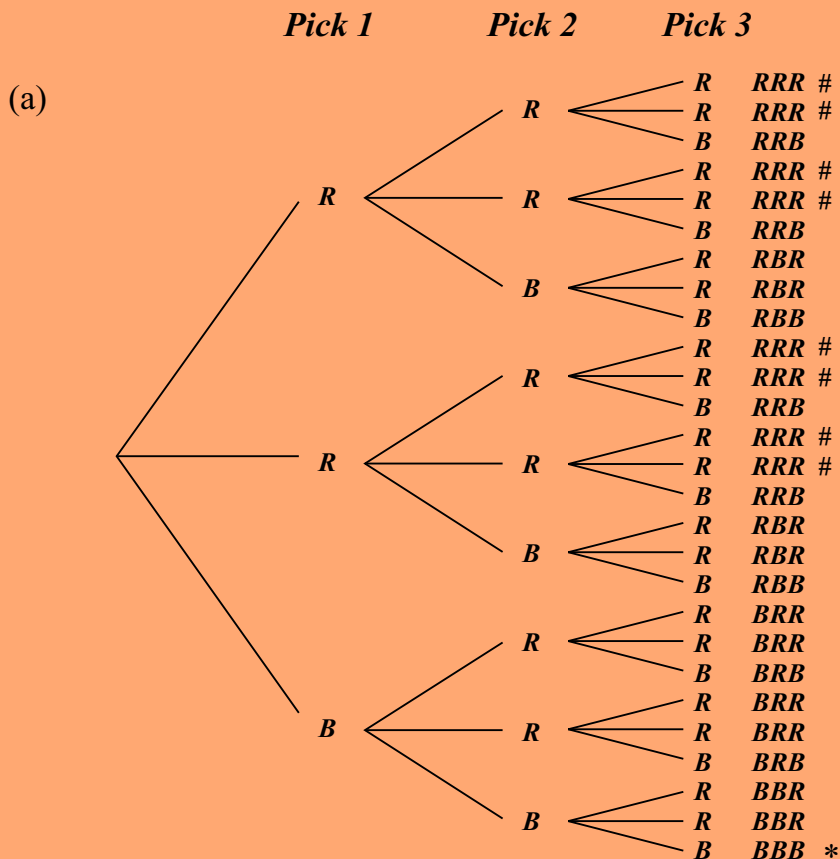
$$P(A + B < 10) = \frac{9}{12} = \frac{3}{4}$$

Example 2

There are three balls in a box - two red (**R**) and one blue (**B**).
One ball is taken randomly from the box, is replaced, another is taken randomly, is replaced and another ball is randomly taken out.



- (a) Construct a tree diagram to show all the possible outcomes.
(b) How many possible outcomes are there?
(c) Find the following probabilities:
(i) P(the three balls were blue) (ii) P(the three balls were red)

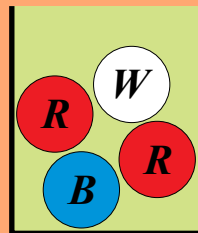


Example 3

There are four balls in a box - two red (*R*), one blue (*B*) and one white (*W*).

One ball is taken randomly from the box. Without replacing the first ball another is taken randomly.

Without replacing either of the removed balls another ball is randomly taken out.



(a) Construct a tree diagram to show all the possible outcomes.

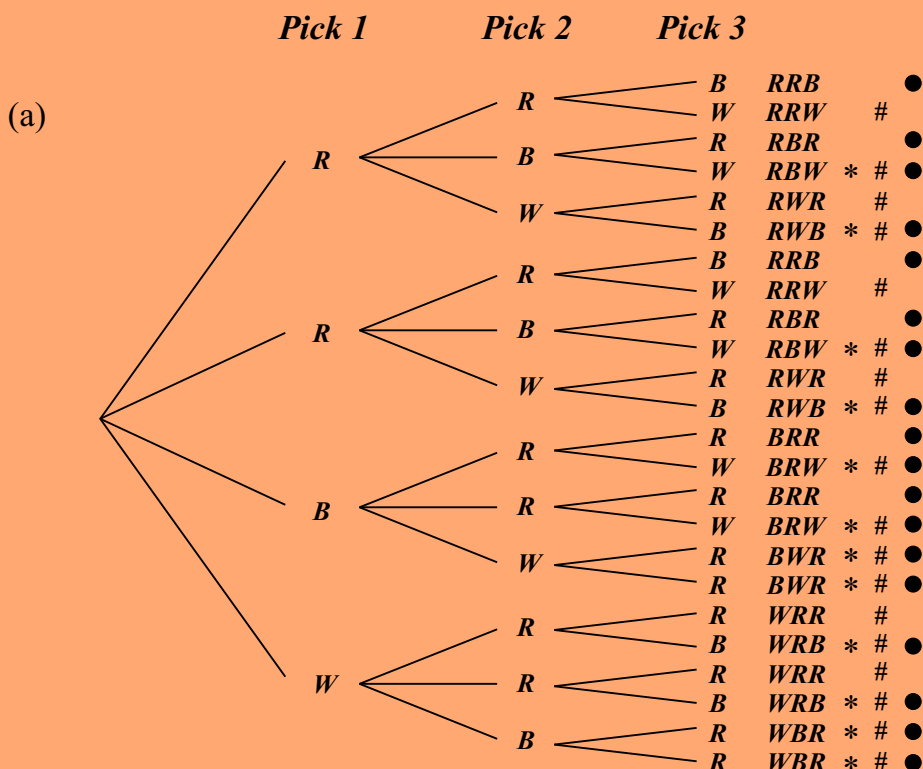
(b) How many possible outcomes are there?

(c) Find the following probabilities:

(i) P(the three balls were all different)

(ii) P(the white and at least one red ball were taken)

(iii) P(the blue ball was taken)



(b) there are 24 possible outcomes

(c)(i) $\frac{12}{24} = \frac{1}{2}$ (*) (ii) $\frac{18}{24} = \frac{3}{4}$ (#) (iii) $\frac{18}{24} = \frac{3}{4}$ (●)

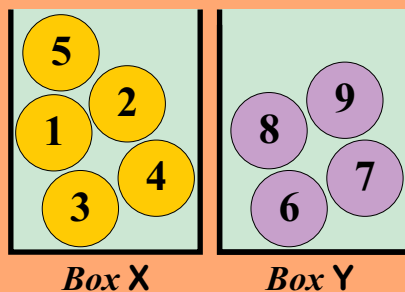
Two-Way Tables

Two-way tables can be used to display all the possible outcomes when there are two events and are more convenient than tree diagrams when there is a large number of outcomes.

Example

A ball is taken randomly from each of the two boxes shown - Box X and Box Y.

The numbers on the two balls are multiplied.



- Construct a two-way table to show all the possible outcomes.
- Find the probability that the product of the two numbers is less than 30, $P(X \times Y < 30)$.

(a)

		Box X				
		1	2	3	4	5
Box Y	6	(6)	(12)	(18)	(24)	30
	7	(7)	(14)	(21)	(28)	35
	8	(8)	(16)	(24)	32	40
	9	(9)	(18)	(27)	36	45

- There are 20 possible outcomes. The products that are less than 30 are circled. There are 14 of these.

$$P(X \times Y < 30) = \frac{14}{20} = \frac{7}{10}$$

EXERCISE 15C

1. Two dice are rolled and the two numbers are added.
 - (a) Construct a tree diagram and two-way table to display all possible outcomes.
 - (b) Find the following probabilities writing the answers as fractions in their simplest form.

(i) $P(\text{the sum} = 7)$	(ii) $P(\text{the sum} > 9)$
(iii) $P(\text{the sum} < 8)$	(iv) $P(\text{the sum is } 15)$
(v) $P(\text{the sum is an odd number})$	

2. Two dice are rolled and the two numbers are multiplied.
 - (a) Construct a tree diagram and two-way table to display all possible outcomes.
 - (b) Find the following probabilities writing the answers as fractions in their simplest form.

(i) $P(\text{the product} = 24)$	(ii) $P(\text{the product} > 30)$
(iii) $P(\text{the product} < 100)$	(iv) $P(\text{the product} = 1)$

3. Two boxes contain coloured balls.
 Box **A** contains two red balls, three blue balls and one yellow ball.
 Box **B** contains three red balls, one blue ball and four yellow balls.
 One ball is taken from each box.
 - (a) Using the abbreviations red (**R**), blue (**B**) and yellow (**Y**), construct a two-way table to display all possible outcomes.
 - (b) Find the following probabilities and write as fractions in their simplest form. Ignore the order of the balls (**BR** = **RB**)

(i) $P(\mathbf{RR})$	(ii) $P(\mathbf{BR})$	(iii) $P(\mathbf{YY})$	(iv) $P(\mathbf{RY})$
(v) $P(\text{the two balls are different colours})$			
(vi) $P(\text{the two balls are the same colour})$			

4. Joel's pen box contained three blue pens, two green pens and one purple pen. He randomly chose one pen to do some work, replaced it, then randomly chose another pen to do some more work.
 - (a) Construct a two-way table to display all possible outcomes.
 - (b) Find the following probabilities and write as fractions in their simplest form.

(i) $P(\text{both pens were blue})$	(ii) $P(\text{both pens were purple})$
(iii) $P(\text{the pens were different colours})$	

5. Breanne has five pens in her pen box - two black, two green and one red. She chooses one pen randomly, it runs out of ink so she chooses another one randomly.
- (a) Construct a tree diagram to display all the possible outcomes.
 - (b) Find the following probabilities and write as fractions in their simplest form.
 - (i) $P(\text{both are black})$
 - (ii) $P(\text{both are the same colour})$
 - (iii) $P(\text{one is red})$
6. Five students nominate for two positions on school council. The students names are Sophie, Jack, Nicole, Mia and Dylan. Two of these students are selected randomly.
- (a) Construct a tree diagram to display all the possible outcomes.
 - (b) Find the following probabilities and write as fractions in their simplest form.
 - (i) $P(\text{both are girls})$
 - (ii) $P(\text{both are boys})$
 - (iii) $P(\text{a boy and a girl})$
7. Brianna and Ahmed each work one evening a week in a restaurant. The night each of them works is chosen randomly and could be any night from Monday to Sunday.
- (a) Construct a two-way table showing all possible outcomes of the nights when Brianna and Ahmed work.
 - (b) Find the following probabilities and write as fractions in their simplest form.
 - (i) $P(\text{both work on a weekend})$
 - (ii) $P(\text{both work on a week day: Monday - Friday})$
 - (iii) $P(\text{they both work on the same night})$
 - (iv) $P(\text{they work on consecutive nights})$



8. Julia has four shirts (green, red, blue and yellow), two pairs of shorts (yellow and blue) and three pairs of shoes (green, blue and yellow). One morning she grabs one of each item of clothing without looking at the colours.
- (a) Construct a tree diagram to display all possible combinations.
 - (b) How many combinations are there?
 - (c) What is the probability that she chooses three items that are the same colour?
 - (d) What is the probability that all the items are a different colour?
 - (e) What is the probability she chose at least one yellow item?
9. Lou owns a caravan park that has four cabins (**A**, **B**, **C** and **D**). She lost the tags for the keys and when four families came in to stay in the cabins she gave them the keys randomly.
- (a) Construct a tree diagram to show all the possible orders that she could have given out the keys.
 - (b) How many combinations are there?
 - (c) What is the probability that all four families received the correct key for their cabin?
 - (d) What is the probability that at least two families received the correct key?
10. Elke had two brothers and two sisters. For Christmas she bought a green scarf for each of her brothers and a pink scarf for each of her sisters. After wrapping them she forgot which scarf was in each parcel so she handed them out randomly.
- (a) Construct a tree diagram to show all the possible ways that the presents could be handed out?
 - (b) How many different ways could the presents be handed out?
 - (c) What is the probability that they all received the coloured scarf that Elke wanted them to receive?
 - (d) What is the probability that at least one of her sisters received the correctly coloured scarf?

Venn Diagrams

Example

20 people were asked if they played sport or a musical instrument. 13 people played sport, 11 played a musical instrument, 5 played sport and a musical instrument.

This Venn diagram displays this information.

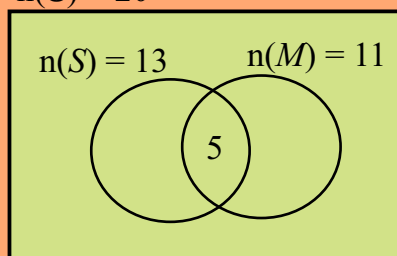
$\mathcal{E}$ is the universal set (all the people surveyed).

$n(\mathcal{E})$ = the total number of people

$n(S)$ = number of people playing sport

$n(M)$ = number of people playing a musical instrument

$$n(\mathcal{E}) = 20$$



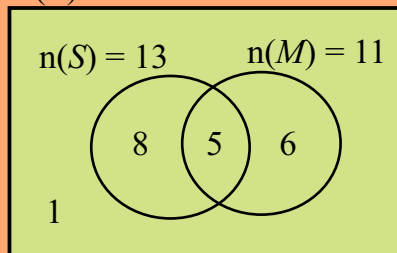
All areas of the Venn diagram can be calculated.

There are 8 people who only play sport ($13 - 5$)

There are 6 people who only play a musical instrument ($11 - 5$)

There is one person who does not play sport or a musical instrument ($20 - 8 - 5 - 6$).

$$n(\mathcal{E}) = 20$$



The probability of randomly selecting someone from any group can be found.

The probability that the person plays sport = $P(S)$

$$P(S) = \frac{\text{the number of people who play sport}}{\text{total number of people}} = \frac{n(S)}{n(\mathcal{E})} = \frac{13}{20}$$

The probability that the person plays a musical instrument = $P(M) = \frac{11}{20}$

Definitions

S' = complement of S (all elements **not** in S)

$S \cap M$ = intersection of S and M (elements common to both sets)

$S \cup M$ = union of S and M (combined elements in S and M)

$$P(S') = \frac{7}{20} \text{ (not playing sport)}$$

$$P(M') = \frac{9}{20} \text{ (not playing a musical instrument)}$$

$$P(S \text{ only}) = \frac{8}{20} = \frac{2}{5}$$

$$P(M \text{ only}) = \frac{6}{20} = \frac{3}{10}$$

$$P(S \cap M) = \frac{5}{20} = \frac{1}{4} \text{ (playing sport and a musical instrument)}$$

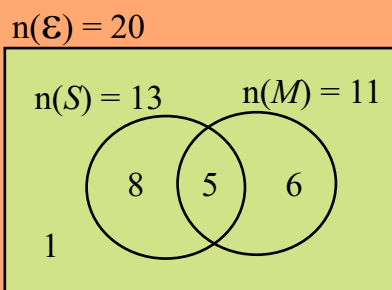
$$P(S \cup M) = \frac{19}{20} \text{ (playing sport and/or a musical instrument)}$$

$$P(S' \cap M') = \frac{1}{20}$$

$$P(S' \cup M') = \frac{15}{20} = \frac{3}{4}$$

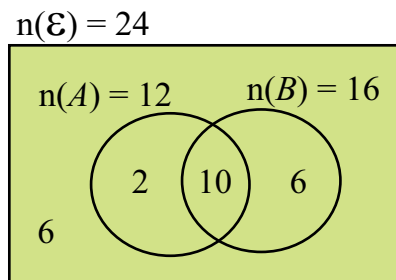
$$P(S' \cap M) = \frac{6}{20} = \frac{3}{10}$$

$$P(S \cap M') = \frac{8}{20} = \frac{2}{5}$$

**EXERCISE 15D**

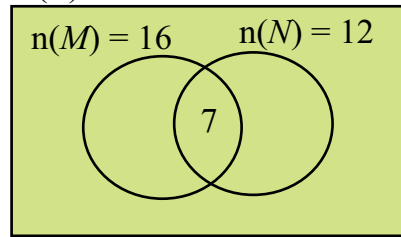
1. For the Venn diagram shown here find the following probabilities if an element is randomly chosen. Give answers as fractions in their simplest form.

- | | |
|---------------------|---------------------|
| (a) $P(A)$ | (b) $P(B)$ |
| (c) $P(A')$ | (d) $P(B')$ |
| (e) $P(A \cap B)$ | (f) $P(A \cup B)$ |
| (g) $P(A' \cap B)$ | (h) $P(A \cap B')$ |
| (i) $P(A' \cap B')$ | (j) $P(A' \cup B')$ |



2. (a) Copy and complete this Venn diagram by calculating the number of elements that are in each region.
- (b) Find the following probabilities if an element is chosen randomly. Write fractions in their simplest form.

$$n(\mathcal{E}) = 30$$



- (i) $P(M)$ (ii) $P(N)$ (iii) $P(M \cap N)$ (iv) $P(M \cup N)$
 (v) $P(M')$ (vi) $P(N')$ (vii) $P(M' \cap N)$ (viii) $P(M' \cup N')$

3. 100 people were asked if they used a tram and/or a bus to travel in a major city. 65 people used a tram (T) and 14 used a bus (B). 10 people used both forms of public transport.

- (a) Construct a Venn diagram to display this information showing the number in each section.
- (b) If one person is chosen randomly from this group find the following probabilities. Write fractions in their simplest form.
- (i) $P(T)$ (ii) $P(B)$ (iii) $P(T \text{ only})$ (iv) $P(B \text{ only})$
 (v) $P(T')$ (vi) $P(B')$ (vii) $P(T \cap B)$ (viii) $P(T \cup B)$

4. There were 106 students in year 11 at a college. 23 students have decided to study physics in year 12 and 19 have decided to study chemistry. Seven students will be studying physics but not chemistry.

- (a) Construct a Venn diagram to display this information showing the number in each section.
- (b) If one student is chosen randomly from year 11 find the following probabilities. Write fractions in their simplest form.
- (i) $P(\text{they will be studying physics})$
 (ii) $P(\text{they will be studying both physics and chemistry})$
 (iii) $P(\text{they will not be studying physics})$
 (iv) $P(\text{they will not be studying physics or chemistry})$

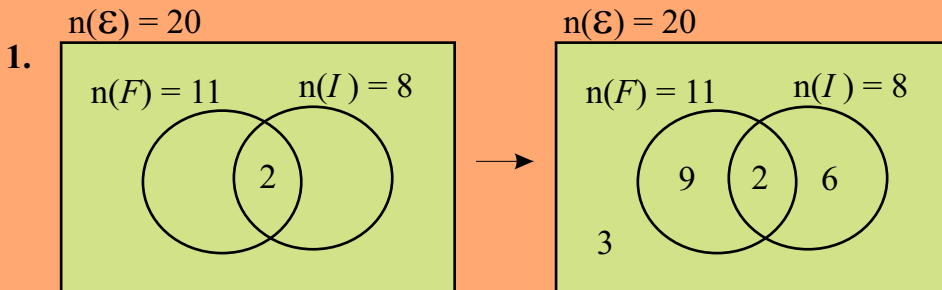
Karnaugh Maps

Karnaugh maps can be used to represent the information displayed on a Venn diagram.

Example

20 people were studying languages. 11 were studying French, 8 were studying Indonesian and 2 were studying both.

1. Represent this information on a Venn diagram showing the number in all regions.
2. Display the information on a Karnaugh map.
3. Find the probability that someone chosen randomly from the group:
 - (a) is studying French
 - (b) is studying French and Indonesian
 - (c) is studying Indonesian but not French
 - (d) is studying a language other than French or Indonesian



2. The information shown on a Karnaugh map is described here.

	F	F'	total	
I	$F \cap I$	$F' \cap I$		those studying French and Indonesian
I'	$F \cap I'$	$F' \cap I'$		those not studying French but studying Indonesian
total				

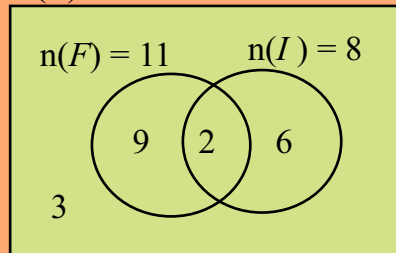
those studying French but not Indonesian

those not studying French or Indonesian

The number in each section can be shown on a Karnaugh map.

	F	F'	total
I	2	6	8
I'	9	3	12
total	11	9	20

$$n(\mathcal{E}) = 20$$



The probability of someone being in each section can also be shown on a Karnaugh map.

	F	F'	total
I	$\frac{2}{20} = \frac{1}{10}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{8}{20} = \frac{2}{5}$
I'	$\frac{9}{20}$	$\frac{3}{20}$	$\frac{12}{20} = \frac{3}{5}$
total	$\frac{11}{20}$	$\frac{9}{20}$	1

← will always be 1

3. (a) $P(\text{someone is studying French}) = \frac{11}{20}$

(b) $P(\text{someone is studying French and Indonesian}) = \frac{1}{10}$

(c) $P(\text{someone is studying Indonesian but not French}) = \frac{3}{10}$

(c) $P(\text{someone is studying a language other than French or Indonesian}) = \frac{3}{20}$

EXERCISE 15E

1. From a group of 30 young adults 26 had their car licence, 7 had their motorcycle licence and 5 had both.
 - (a) Draw a Venn diagram to show this information.
 - (b) Draw a Karnaugh map to display the information.
 - (c) If one person is chosen at random from this group, find the following probabilities. Fractions in their simplest form.
 - (i) $P(\text{they have a car licence})$
 - (ii) $P(\text{they have a motorcycle licence but not a car licence})$
 - (iii) $P(\text{they don't have either licence})$

2. 60 people were asked if they use the internet to buy and/or sell items. 36 had bought items and 25 had sold items using the internet. 15 had used the internet to buy and sell items.
 - (a) Draw a Venn diagram to show this information.
 - (b) Draw a Karnaugh map to display the information.
 - (c) If one person is chosen at random from this group, find the following probabilities. Fractions in their simplest form.
 - (i) $P(\text{they have bought but not sold items using the internet})$
 - (ii) $P(\text{they have sold items using the internet})$
 - (iii) $P(\text{they haven't used the internet to buy or sell items})$

3. 100 people were surveyed to find out if they get their news from the television or the internet. 57 said the television, 53 said the internet and 11 said they got their news from another source.
 - (a) Draw a Venn diagram to show this information.
 - (b) Draw a Karnaugh map to display the information.
 - (c) If one person is chosen at random from this group, find the following probabilities. Fractions in their simplest form.
 - (i) $P(\text{they get their news from television but not internet})$
 - (ii) $P(\text{they get their news from television and internet})$
 - (iii) $P(\text{get their news from the internet})$

P(*A* or *B*)

The symbol that represents the probability that event *A* **or** event *B* occurs is:

$$P(A \cup B)$$

In general, if it is required to find the probability that event *A* **or** event *B* occurs the following formula is used.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 1

A die is rolled. Find the probability of rolling an even number or a number less than 3.

Let: A = set of even numbers = {2, 4, 6}
 B = set of numbers less than 3 = {1, 2}
 $A \cap B = \{2\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(A \cap B) = \frac{1}{6}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3}$$

When the elements of the sets are known, as in this example, $P(A \cup B)$ can alternatively be found by finding the set $A \cup B$.

$$A \cup B = \{1, 2, 4, 6\}$$

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3}$$

Example 2

45 students were asked if they played tennis and/or golf.

15 played tennis, 9 played golf and 3 played both.

Find the probability that if a student was chosen randomly from this group they would play tennis or golf.

$$P(T \cup G) = P(T) + P(G) - P(T \cap G)$$

$$P(T) = \frac{15}{45} = \frac{1}{3}$$

$$P(G) = \frac{9}{45} = \frac{1}{5}$$

$$P(T \cap G) = \frac{3}{45} = \frac{1}{15}$$

$$P(T \cup G) = P(T) + P(G) - P(T \cap G)$$

$$= \frac{1}{3} + \frac{1}{5} - \frac{1}{15}$$

$$= \frac{5}{15} + \frac{3}{15} - \frac{1}{15}$$

$$P(T \cup G) = \frac{7}{15}$$

In this type of example a Venn diagram could have been used.

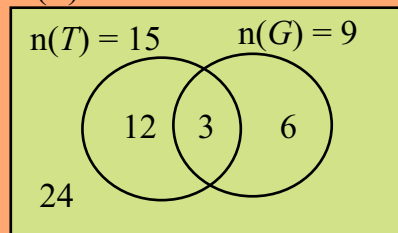
When the number in each region is calculated it can be seen that:

$$n(T \cup G) = 21$$

Therefore:

$$P(T \cup G) = \frac{21}{45} = \frac{7}{15}$$

$$n(\mathcal{E}) = 45$$



Example 3

A bag contains eight balls - four blue, two green and two red. If a ball is randomly picked from the bag find the probability it is blue or red.

$$P(B \cup R) = P(B) + P(R) - P(B \cap R)$$

$$P(B) = \frac{4}{8} = \frac{1}{2}$$

$$P(R) = \frac{2}{8} = \frac{1}{4}$$

$P(B \cap R) = 0$ (a blue ball and red ball **cannot** be picked at the same time)

$$P(B \cup R) = P(B) + P(R) - P(B \cap R)$$

$$= \frac{1}{2} + \frac{1}{4} - 0$$

$$P(B \cup R) = \frac{3}{4}$$

In this example it was seen that $P(B \cap R) = 0$. There are many situations where the two (or more) events cannot occur at the same time. Any events that cannot occur at the same time are called **mutually exclusive**.

For events A and B that are mutually exclusive:

$$P(A \cup B) = P(A) + P(B)$$

EXERCISE 15F

1. State which of the following pairs of events are *mutually exclusive*.
 - (a) Event A - Rolling an even number on a die.
Event B - Rolling a number bigger than 2.
 - (b) Event C - Drawing a red card from a deck of cards.
Event D - Drawing a club from a deck of cards.
 - (c) Event E - Randomly choosing a girl from a group of people.
Event F - Randomly choosing a left handed person from the group.
 - (d) Event G - Randomly selecting a vowel from the alphabet.
Event H - Randomly selecting a **B, K, L** or **G**.
 - (e) Event I - Drawing a red card from a deck of cards.
Event J - Drawing a queen from a deck of cards.

2. A number from 1 to 25 (inclusive) is picked randomly. Find the following probabilities as fractions in their simplest form.
 - (a) $P(\text{even or a multiple of 5})$
 - (b) $P(\text{a multiple of 7 or a multiple of 9})$
 - (c) $P(\text{less than 20 or greater than 10})$
 - (d) $P(\text{a multiple of 6 or a number greater than 20})$
 - (e) $P(\text{a multiple of 4 or an odd number})$

3. The probability that someone bought bread from a shop was 0.25.
The probability that someone bought milk from the shop was 0.15.
The probability that someone bought bread and milk was 0.12.
Find the probability that someone entering the shop would buy bread or milk?

4. A letter is randomly chosen from the alphabet. Find the following probabilities. Write answers as fractions in their simplest form.
 - (a) $P(\text{the letter is a vowel or one of the letters from the word **PROBLEM**})$
 - (b) $P(\text{the letter is a consonant or one of the letters from the word **PROBLEM**})$
 - (c) $P(\text{the letter is a vowel or one of the letters from the word **HYMN**})$
 - (d) $P(\text{the letter is one of the letters of the word **NUMBER** or one of the letters of the word **PROBLEM**})$

5. A deck of playing cards contains four suits - hearts♥, diamonds♦, clubs♣ and spades♠. Each suit has 13 cards - an ace, cards numbered 2 - 10, a jack, queen and king.

Hearts and diamonds are *red* and clubs and spades are *black*.

A card is drawn randomly. Find the following probabilities giving answers as fractions in their simplest form.

- (a) $P(\text{a club or an ace})$
- (b) $P(\text{a black card or a jack})$
- (c) $P(\text{an ace or a queen})$
- (d) $P(\text{a black queen or a spade})$
- (e) $P(6♥, 8♣, \text{jack}♥, \text{king}♠, 10♠, \text{ace}♦ \text{ or a red card})$
- (f) $P(\text{numbered card greater than 8 or a heart})$
- (g) $P(\text{black jack or the queen}♥)$
- (h) $P(\text{red queen or queen}♥)$

6. The householders in a town were asked to complete a survey and state:

A - if their houses were insulated

B - if they used low energy light globes

84% of the houses were insulated, 76% of the houses used low energy light globes and 68% of the houses were insulated and used low energy light globes.

If a house was chosen at random, what is the probability that it was insulated or had low energy light globes?

7. Two sets of numbers are shown below.

$$A = \{1, 4, 6, 8, 9, 13, 15, 18\}$$

$$B = \{2, 4, 7, 9, 15, 16, 20\}$$

- (a) Draw a Venn diagram including all the numbers from 1 to 20 inclusive showing the numbers in sets A and B .
- (b) If a number is chosen randomly from 1 to 20, what is the probability it is from set A or set B ?

8. A refrigerator repairer has found that the two main problems that occur in a refrigerator are the compressor or thermostat.

For a refrigerator that is not working, the probability that the compressor needs repairing is $\frac{7}{10}$ and the probability that the thermostat needs repairing is $\frac{1}{4}$. One in 20 refrigerators needs both compressor and thermostat repaired.

Find the probability that a refrigerator that is not working has a fault that is *not* the compressor or thermostat.

P(*A* and *B*) - Independent Events

The symbol that represents the probability that event *A* **and** event *B* occur is:

$$P(A \cap B)$$

If event *A* does not affect event *B* then these two events are called **independent** and:

$$P(A \cap B) = P(A) \times P(B)$$

Example 1

A die is rolled and coloured ball is taken from a bag that contains six red and eight black balls.

Find the probability that the result is a six on the die **and** a red ball.

These events are independent so:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A) = P(6 \text{ on die}) = \frac{1}{6}$$

$$P(B) = P(\text{red ball}) = \frac{6}{14} = \frac{3}{7}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{6} \times \frac{3}{7}$$

$$P(A \cap B) = \frac{1}{14}$$

Example 2

A bag contains three black balls and five green balls. A ball is taken from the bag, replaced and another ball taken. Find the probability that both balls are black.

These events are independent because the first ball is replaced.

$$P(\text{black ball}) = \frac{3}{8}$$

$$P(A \cap B) = P(A) \times P(B) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$$

EXERCISE 15G

Give all probabilities in this exercise as fractions in their simplest form.

1. Find the probability of rolling a six on a die and tossing heads on a coin.
2. A bag contains six red balls, three black balls and one white ball.
 - (a) Find the probability of randomly picking the white ball, replacing it and then randomly picking the white ball again.
 - (b) Find the probability of randomly picking a black ball, replacing it and then randomly picking a red ball.
3. Lindy is in a class of 24 students. Her twin sister is in a class of 20 students. One student is chosen randomly from each class. What is the probability that both sisters are chosen?
4. Find the probability that the following two cards are chosen from a deck of playing cards given that the second card is chosen after the first one is replaced in the deck.
 - (a) $P(\text{a heart then a black card})$
 - (b) $P(\text{an ace then another ace})$
 - (c) $P(\text{the ace of spades then the ace of diamonds})$
 - (d) $P(\text{a red jack then a queen})$
5. The Raiders netball team has five boys and three girls. The Dreamers netball team has four boys and four girls. One player is chosen from each team to umpire a game.
 - (a) What is the probability that both umpires will be boys?
 - (b) What is the probability that both umpires will be girls?
 - (c) What is the probability that there will be a boy and girl umpire?
6. Zeta sat a multiple choice answer exam. Each question had five alternatives. She ran out of time and guessed the last two questions.
 - (a) What is the probability that Zeta got both these questions correct?
 - (b) What is the probability that Zeta got both these questions incorrect?
7. In the town of Dampagan in June there is, on average, 22 days of rain. What is the probability that it will **not** rain on a particular weekend in Dampagan in June?

This same rule applies to more than two independent events.

Example

Find the probability of tossing four consecutive heads on a coin.

$$\begin{aligned}
 P(\text{four heads}) &= P(\text{heads and heads and heads and heads}) \\
 &= P(\text{heads}) \times P(\text{heads}) \times P(\text{heads}) \times P(\text{heads}) \\
 &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\
 &= \frac{1}{16}
 \end{aligned}$$

8. What is the probability of rolling three consecutive sixes on a die?
9. Zac sat a multiple choice answer test. Each question had four alternatives. Zac guessed five questions. What is the probability of Zac answering all of these five questions correctly?
10. Robin is an archer. He knows that he can hit the centre of a target with an arrow six out of ten times. What is the probability of Robin hitting the centre of a target with three consecutive arrows?
11. Li is a basketball player who, on average, scores 80% of the free throws he attempts. If he had four free throws, find the probability of the following events occurring.
 - (a) Li scores all four free throws.
 - (b) Li misses all four free throws.
 - (c) Li misses the first and scores the next three.
12. The management at a park want to carry out a fuel reduction burn in September. To do so they will need four consecutive days when the wind is at a safe level. The bureau of meteorology has forecast four consecutive days when there is a 90% chance of a safe wind level on each day.
 - (a) What is the probability of the wind being at a safe level for the four days?
 - (b) What is the probability that the wind will be safe for the first three days and unsafe on the fourth day?



P(A and B) - Dependent Events

If event A **does** affect event B then these two events are called **dependent** and the probability of event A **and** event B occurring is given by:

$$P(A \cap B) = P(A) \times P(B^*)$$

Where $P(B^*)$ represents the probability of event B occurring after the affect of event A has been considered.

Example 1

A bag that contains six red and four black balls.

Find the probability that a red ball is removed then another red ball is removed without replacing the first one.

It can be seen that after the first red ball is removed the number of balls in the bag is changed so the probability of removing a second red ball needs to take this into account.

$$P(A) = P(\text{first ball is red}) = \frac{6}{10} = \frac{3}{5}$$

$$P(B^*) = P(\text{second ball is red}) = \frac{5}{9}$$

$\swarrow$ five red balls remaining
 $\swarrow$ total of nine balls remaining

$$P(\text{red ball then another red ball}) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$$

Example 2

Three cards are taken from a deck of playing cards. Find the probability that all three are hearts. (There are 13 hearts in a deck of 52 cards)

$$P(A) = P(\text{first card is a heart}) = \frac{13}{52} = \frac{1}{4}$$

$$P(B^*) = P(\text{second card is a heart}) = \frac{12}{51}$$

$\swarrow$ 12 hearts remaining
 $\swarrow$ 51 cards remaining

$$P(C^*) = P(\text{third card is a heart}) = \frac{11}{50}$$

$\swarrow$ 11 hearts remaining
 $\swarrow$ 50 cards remaining

$$P(\text{all are hearts}) = \frac{1}{4} \times \frac{12}{51} \times \frac{11}{50} = \frac{132}{10\,200} = 0.0129 \text{ (4 dec. pl.)}$$

EXERCISE 15H

Give all probabilities in this exercise as fractions in their simplest form.

1. A bag contains five blue balls and four green balls.
 - (a) Find the probability of removing two blue balls one at a time without replacing them.
 - (b) Find the probability of removing three green balls one at a time without replacing them.
 - (c) Find the probability of removing a blue, then green, then blue and then another green one at a time without replacing them.
2. Herman and Sherman are on a seven person committee. A president and secretary are selected randomly. What is the probability that Herman is the president and Sherman is the secretary?
3. Jocelyn bought 20 tickets in a raffle that had 100 tickets. What is the probability that she wins the first, second and third prizes?
4. In a tattslotto draw six numbers are drawn from 45 numbers. Kumar has one ticket of six numbers.
 - (a) What is the probability that the first number drawn is one of Kumar's six?
 - (b) What is the probability that Kumar's six numbers are the six winning numbers? Give answer as a fraction in its simplest form and a decimal to eight decimal places.
5. Five people had their eyes tested and needed glasses. The optometrist prepared the glasses but lost his glasses so he mixed up the names on the boxes of where the glasses were to be sent.
What is the probability that all five people receive the correct pair of glasses?

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Multiple Events

Example

The probability of Lauren scoring a goal from a free throw in basketball was 0.7. Lauren had three free throws.

1. Draw a tree diagram to show the probability of all possible outcomes.
2. Find the probability that she scores:
 - (a) three goals
 - (b) one goal
 - (c) at least one goal

Answers

1. Include on the tree diagram the probabilities.

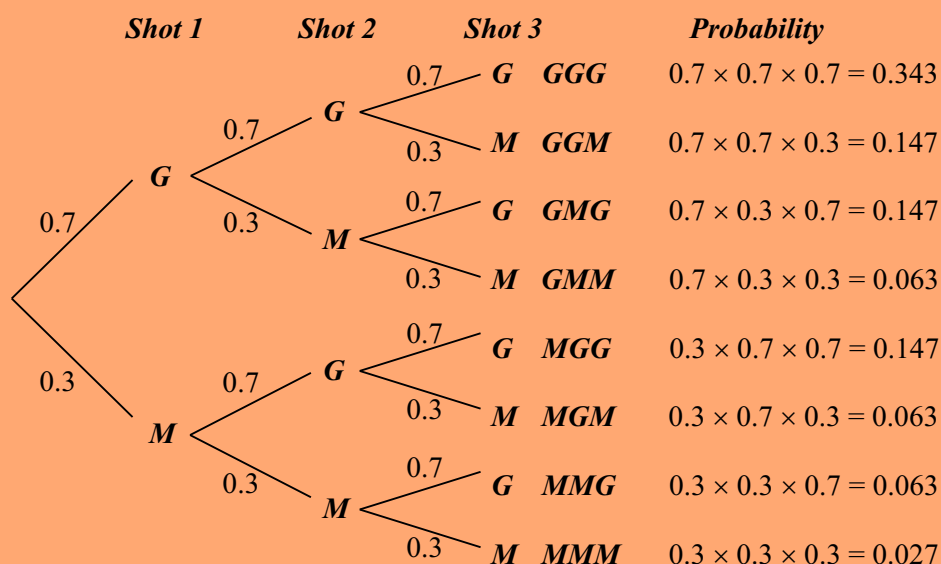
The probability of a goal, $P(G)$, is 0.7.

The probability of missing a goal, $P(M)$, is $1 - 0.7 = 0.3$.

Remember - to find the probability of one event **and** other events occurring the probabilities need to be **multiplied**.

$$P(GGG) = P(G \text{ from shot 1}) \text{ and } P(G \text{ from shot 2}) \text{ and } P(G \text{ from shot 3})$$

$$= 0.7 \times 0.7 \times 0.7 = \mathbf{0.343}$$



Remember - the sum of all the probabilities should equal 1

$$0.343 + 0.147 + 0.147 + 0.063 + 0.147 + 0.063 + 0.063 + 0.027 = 1$$

2. (a) $P(GGG) = 0.343$

(b) **Remember** - to find the probability that one event *or* other events occur the probabilities must be **added**.

There are three ways that just one goal is scored:

GMM, MGM, MMG .

These three probabilities must be added:

$$\begin{aligned} P(\text{one goal}) &= P(GMM) + P(MGM) + P(MMG) \\ &= 0.063 + 0.063 + 0.063 \\ &= \mathbf{0.189} \end{aligned}$$

(c) $P(\text{at least one goal}) =$ the sum of all the probabilities of one, two or three goals

$$\begin{aligned} &= P(GMM) + P(MGM) + P(MMG) + P(GGM) \\ &\quad + P(GMG) + P(MGG) + P(GGG) \\ &= 0.063 + 0.063 + 0.063 + 0.147 + 0.147 + 0.147 + 0.343 \\ &= \mathbf{0.973} \end{aligned}$$

Note - $P(\text{at least one goal}) = 1 - P(\text{all misses})$

Remember - $P(A \text{ and } B) = P(A) \times P(B)$

$P(A \text{ or } B) = P(A) + P(B)$

The sum of all probabilities is 1.

EXERCISE 15I

Use decimal probabilities in this exercise

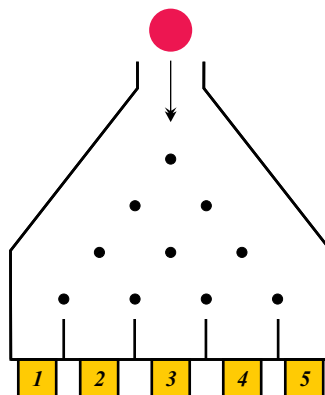
1. The probability of rain on a day in July in Bucket Downs was 0.8. Tilly stayed in Bucket Downs for three days in July.
 - (a) Draw a tree diagram to show all possible outcomes of it raining (R) or being dry (D) on the three days. Show also the probabilities of each possible outcome.
 - (b) What is the probability of it raining on all three days?
 - (c) What is the probability of it **not** raining on any of the three days?
 - (d) What is the probability of it raining on at least two days?

2. A student's pen box contains three black pens and two green pens. Each class the student randomly takes a pen from the box to use and then replaces it at the end of the class. The student has three classes in a day.
- What is the probability of randomly choosing a black pen from the pen box?
 - Draw a tree diagram to show the probabilities of all the possible combinations of pens used in the day.
 - Find the probability that all pens used were black.
 - Find the probability that the same colour pen was used for the three classes.
3. Two out of every ten people in a town contracted the flu one winter. There are three people who work at a shop.
- Draw a tree diagram to show the probabilities of the three people contracting the flu.
 - What is the probability that all three will contract the flu?
 - What is the probability that none of the three will contract the flu?
 - What is the probability that at least one will contract the flu?
4. There were three year 10 classes at a school.
- In **10A** there were 15 boys and 10 girls.
 In **10B** there were 12 boys and 12 girls.
 In **10C** there were 12 boys and 4 girls.
 One student is randomly chosen from each class.
- Draw a tree diagram to show the probabilities of the selected students being boys or girls.
 - What is the probability that all three were boys?
 - What is the probability that all three were girls?
 - What is the probability that there was at least one girl chosen?
5. Jonah went on a whale watching trip. The trip visited three bays. There was a 70% chance of seeing a whale at Pilot Bay. There was a 40% chance of seeing a whale at Wright Bay. There was a 60% chance of seeing a whale at Blue Bay. Draw a tree diagram and find the probability that Jonah will see at least one whale.

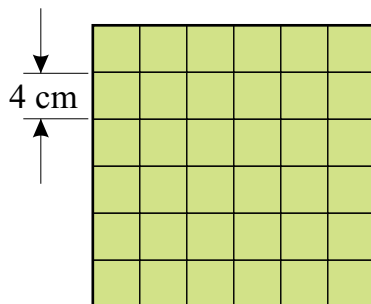


PROBLEM SOLVING

1. A game is shown here where a ball is dropped into the top tube and falls around the pegs and into one of the numbered boxes at the bottom. Each time it hits a peg the ball has an equal chance of falling each way. Find the probability that the ball falls into each of the numbered boxes.



2. Another game involves throwing a 20 cent piece (3 cm diameter) onto a grid with 4 cm squares. What is the probability that the 20 cent piece will touch one of the lines?



3. A computer game involves trying to escape from a dungeon. The following instructions are given at the start of the game. What is the probability of escaping on the first attempt?

*As you enter the cave,
Two doors there will be.
Behind each door,
Are tunnels one, two and three.
At the end of each tunnel,
Four ladders stand tall.
Climb one of these ladders,
But do not fall.
At the top of each ladder,
Five keys you will see,
Only one of these keys,
Will set you free.*

PUZZLE

Change **ROLL** into **DICE** in *four* steps by changing one letter at a time to form a new word.

Example
CAT can be changed
to **DOG** in three steps

CAT
COT
DOT
DOG

CHAPTER REVIEW

1. A bag contains three black balls, five red balls and two purple balls.

(a) A ball is randomly taken from the bag.

- (i) What is the probability it is a purple ball?
- (ii) What is the probability it is not a purple ball?
- (iii) What is the probability it is a green ball?

(b) A ball is taken randomly, replaced and another is taken.

What is the probability both balls are red?

2. Rex enjoys fly-fishing. The number of fish he caught on 15 fly-fishing trips were: 0, 2, 3, 0, 4, 1, 1, 0, 1, 0, 5, 2, 0, 3, 2.

Based on these figures:

(a) what is the probability that Rex will catch a fish on a fishing trip?

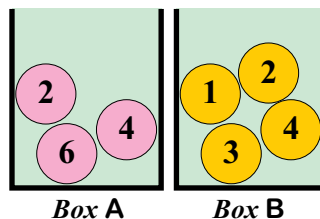
(b) what is the probability that Rex will catch more than two fish?

3. A ball is taken randomly from Box **A** and Box **B** and the numbers are multiplied.

(a) Draw a tree diagram to show all possible outcomes.

(b) What is the probability that the product is greater than 10?

(c) What is the probability that the product is an even number?



4. Leah, Kane, Polly and Mia apply for the three jobs of serving, cooking and washing at a local restaurant. The jobs are filled randomly from the four applicants.

(a) Draw a tree diagram to show all possible outcomes.

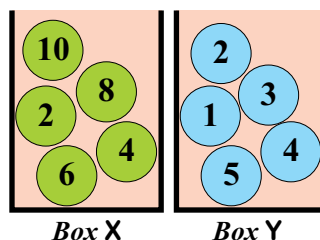
(b) What is the probability that Kane is cooking?

(c) What is the probability that Leah does not get a job?

5. A ball is taken randomly from Box **X** and Box **Y** and the numbers added.

(a) Construct a two-way table to show all possible outcomes.

(b) What is the probability that the sum of the numbers is an odd number?



6. 50 people who had walked in a park were asked if they had seen a koala and/or a snake. 36 had seen a koala, 14 had seen a snake and 8 had seen both.
- (a) Draw a Venn diagram to display this information.
 - (b) Draw a Karnaugh map to display this information.
 - (c) Based on these figures, find the following probabilities if someone walked in the park.
 - (i) $P(\text{they would see a snake but not a koala})$
 - (ii) $P(\text{they would see a koala or a snake})$
 - (iii) $P(\text{they would not see a koala or a snake})$
7. In the same park it was found that the probability of seeing a kangaroo was 0.62 and the probability of seeing an eagle was 0.27. The probability of seeing both was 0.18.
Find the probability of seeing a kangaroo or an eagle.
8. The probability of it raining on a day in August was 0.65. Soibhan had two cross-country runs in August.
- (a) What is the probability that it will rain on both days?
 - (b) What is the probability that both runs will be held on dry days?
9. A code for access to a computer is to be a letter (**A-Z**) and a number (**0-9**). What is the probability that someone guesses a code?
10. Another computer code is to be two different letters. What is the probability of guessing one of these codes?
11. A glass blower knew that the probability of a particular vase breaking was 0.1. She made three of these vases.
- (a) Draw a tree diagram to show the probability of all possible outcomes.
 - (b) What is the probability that all three vases broke?
 - (c) What is the probability that none of the three broke?
 - (d) What is the probability that she made at least two vases that did not break?

Data

Data

Data is collected for many reasons.

- A clothes shop may want to find the types of clothes their customers would like stocked.
- A local government may want to find if a pedestrian crossing is needed in a certain street.
- A farmer may need to find whether to water more regularly.
- A company producing string may need to find the strength of their product.

Many organisations or groups collect data. For example:

- All levels of government
- Retailers
- Companies
- Scientific and research organisations
- Educational organisations
- Community groups
- Sporting and recreational organisations
- Workplaces

When collecting data the following steps need to be followed:

- What data is needed?
- What is the best way to collect the data?
- What is the best way to present the data?
- How is the data to be interpreted?

The Australian census is conducted every five years and is completed by every Australian. People answer questions about age, where they live, religion, etc. This information is available from the Australian Bureau of Statistics (ABS) and is used for many reasons such as planning for the future.

Collecting Data

There are several ways that data can be collected.

- From the many ***data collecting organisations*** (for example: the Australian Bureau of Statistics).
- ***Observation*** (for example: standing near a road and counting the number of cars)
- ***Questionnaire*** (for example: asking a number of people to answer a set of questions) - this can be achieved face-to-face, or by mail, email, telephone, etc.
- ***Experiment*** (for example: testing a number of car windows to see if they break under certain conditions)

Collecting data can be a time consuming and expensive process so it is important that sufficient planning be put into preparing for the collection of data.

If a ***questionnaire*** is being designed it is important that the questions are simple, clear and relevant. It is also important that the information can be easily collated such as ***tick the box*** or a ***rating scale***.

Examples

Do you think the main street needs more rubbish bins? ***Yes*** ***No***
☐ ☐

How is the quality of your local street? ***poor*** ***good*** ***excellent***
☐ ☐ ☐

How well is the local radio station meeting the needs of the community? Use the rating scale of 0 - 10 (0 for not meeting the needs to 10 for meeting all the needs).

Where would you like to take your next holiday?

beach ***snow*** ***outback*** ***city*** ***overseas*** ***other***
☐ ☐ ☐ ☐ ☐ ☐

EXERCISE 16A

1. Choose which method (*observation* or *questionnaire*) would be best to use for the collection of the following data.
 - (a) How many students at a secondary school want to proceed to tertiary studies?
 - (b) How many students at a school eat at least two pieces of fruit each day?
 - (c) How many cars are in a supermarket car park?
 - (d) A local government wants to find out if the residents want a swimming pool?
 - (e) How many houses in a city have solar water heaters?
 - (f) How many families in a city own more than one television?
 - (g) How many supporters at a sporting event are wearing team colours?
 - (h) How many people in a city follow the local sporting team?
 - (i) How many people in a small town do their shopping in the town and how many shop in the major city nearby?
 - (j) How many families have more than one car?
 - (k) How many people have been overseas on a holiday?
 2. Think of an experiment that could be used in the following situations.
 - (a) A producer of light globes wants to find the average life of their light globes.
 - (b) The manufacturer of a hay baling machine wants to find the average mass of the hay bales.
 - (c) A producer of balloons wants to find out the average diameter of the balloons before they burst.
- A photograph of a young boy with light brown hair, wearing a blue short-sleeved shirt, blowing into a large, inflated white balloon. He is standing outdoors in front of a dense green bush. The balloon is held in front of his face, and his mouth is pressed against it.
3. For the following situations of data collection, suggest questions for a questionnaire.
 - (a) A local council is considering whether to build a swimming pool.
 - (b) A radio station wants to find the best music to play.

Samples and Population

When collecting data, it is often not possible to include every person or object that is affected. In these situations a smaller representative sample is used.

In statistics, the total number of people or objects affected is called the **population**. A smaller representative group is called a **sample**.

For example, a local council wants to find out if the residents want a library or a swimming pool. 500 residents are surveyed to get an idea of what they prefer.

In this example all the residents in the area would be the **population**. The 500 that were surveyed is the **sample**.

To make the results of a survey as accurate as possible:

1. The sample must be as large as possible.
2. The people or items surveyed must be a true representation of the population.

It has been found that the minimum acceptable size for the sample is the square root of the population.

$$\text{sample} = \sqrt{\text{population}}$$

Example

For a town that has 4500 people, an acceptable sample to survey would be $\sqrt{4500} = 67$ (Round to the nearest whole number)

The data found from the sample can then be converted to percentages and applied to the entire population.

Example

There were 850 students at a school. 60 students were asked if they played a musical instrument. To get a true representation it is important that a cross-section of the student population is involved in the sample (boys/girls, students from each year level). If 45 out of the 60 students played a musical instrument this is 75%. It can be assumed that 75% of the students in the school play musical instruments.

EXERCISE 16B

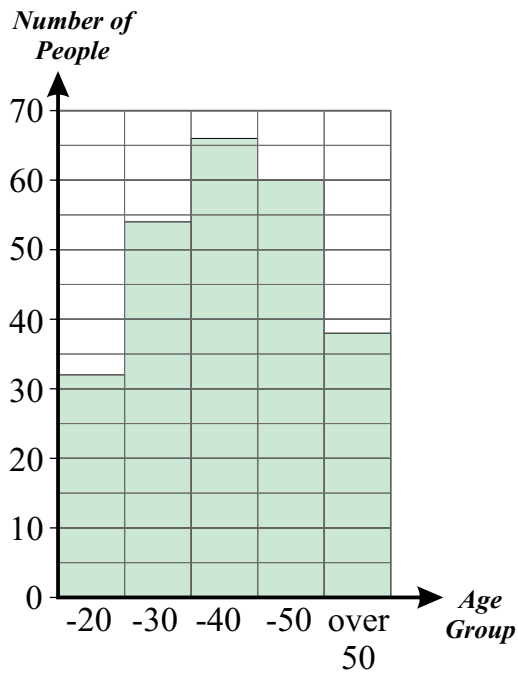
1. What is the minimum sample size that should be surveyed for the following populations. Round to the nearest whole number.
 - (a) A school of 880 students.
 - (b) A city with 260 000 people.
 - (c) A state with 2 500 000 people.
 - (d) A penguin colony of 8000.

 2. In a school of 900 students a survey was to be conducted using a questionnaire. All classes in the school had 25 students. It was decided to give the questionnaire to two students in each class. Is this an adequate number of students for the questionnaire to be considered acceptable?

 3. A farmer had 3500 sheep. He wanted to weigh a sample number of the sheep to check their weight.
 - (a) What is the minimum number of sheep he should sample?
 - (b) He decided to weigh 80 sheep and 12 were underweight.
What percentage of the sample was underweight?
 - (c) Based on this sample, how many of the total flock would the farmer expect to be underweight?
-
4. A football club had 25 400 members. The club wanted to survey the members to see how many wanted a monthly newsletter.
 - (a) What is the minimum number of members that should be surveyed?
 - (b) The club surveyed 400 members and 328 wanted a newsletter.
What percentage of the members surveyed wanted a newsletter?
 - (c) Based on this sample, how many of the members wanted a newsletter?

 5. 6000 cars used a major road between 8 am and 9 am on a weekday. The number of people in 200 of the cars was noted. 30 had four people, 50 had three people, 75 had two people and the remainder had one person. Based on these figures, how many people used the road between 8 am and 9 am?

6. 5500 people attended a music festival. 250 of them were asked their age and the results are shown in this column graph.
- (a) Based on these figures, what percentage of the people at the festival were in each age group? Give answers correct to one decimal place.
- (b) Based on these figures, how many people at the festival were in each age group?



7. The tally sheet below shows the results of observing children as they left a show and recording how many show bags they had bought.

Number of Show Bags	Tally
0	
1	
2	
3	
4	
5	
6	

- There were 8500 children who visited the show. Based on the above sample, find the following. Round answers to the nearest whole number.
- (a) How many children who visited the show bought:
- (i) no show bags?
 - (ii) less than three show bags?
 - (iii) more than four show bags?
- (b) What was the total number of show bags purchased?

Types of Data

There are three main types of data:

1. **Continuous numerical** - the data is numerical and can take any value (within a practical range).
2. **Discrete numerical** - the data is numerical but can take only certain values.
3. **Categorical** - the data is not numerical but falls into different categories.

1. **Examples of continuous numerical data:**

- height of people
- length of someone's hair

2. **Examples of discrete numerical data:**

- number of people at a theatre
- goals kicked by a footballer

3. **Examples of categorical data:**

- colour of people's eyes
- student's favourite sport

EXERCISE 16C

Classify the following data as **continuous numerical**, **discrete numerical** or **categorical**.

1. Number of people in the audience of a concert.
2. Favourite holiday destinations.
3. Volume of water in a reservoir.
4. The number of bricks needed to build a house.
5. Price of a car.
6. Time taken to swim 100 metres.
7. Height jumped by a pole vaulter.
8. Level of satisfaction with the performance of a car - poor, good, very good, excellent.
9. Number of passengers on an aeroplane.
10. Number of points scored by a football team in a season.
11. Amount of ink remaining in an ink cartridge.
12. Length of a telephone conversation.

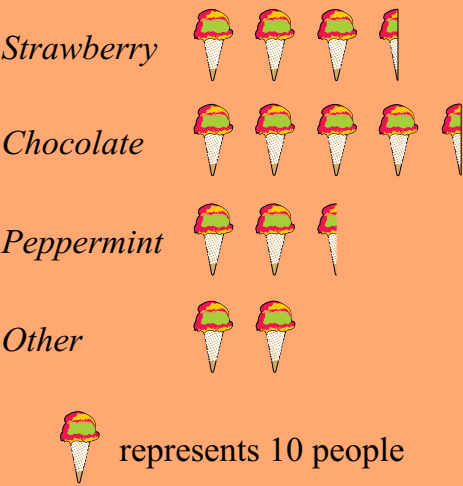
Presenting Data

Popular methods of presenting discrete numerical and categorical data are:
Pictographs, Column Graphs, Bar Graphs and Pie Charts

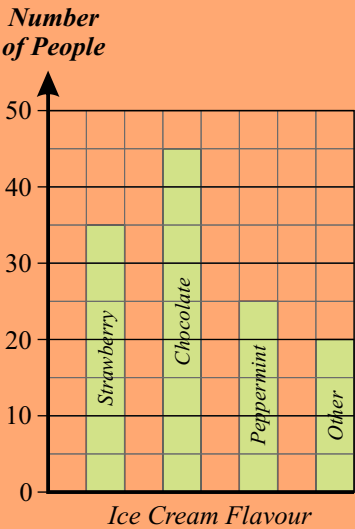
Example

<i>Favourite Ice Cream Flavour</i>	<i>Number of People</i>
Strawberry	35
Chocolate	45
Peppermint	25
Other	20

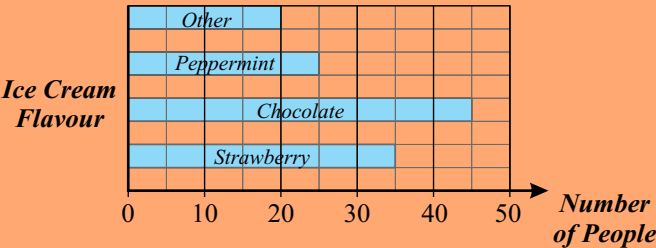
Pictograph



Column Graph



Bar Graph



Pie Chart

The angles in a pie chart can be calculated by using one of these formulae.

$$\frac{\text{number}}{\text{total number}} \times 360$$

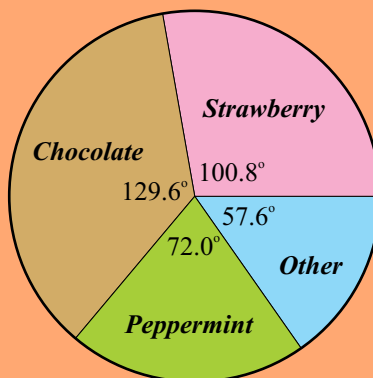
or

$$\frac{\text{percentage}}{100} \times 360$$

Example the angle for strawberry = $\frac{35}{125} \times 360$
 $= 100.8^\circ$

<i>Favourite Ice Cream Flavour</i>	<i>Number of People</i>	<i>Angle</i>
Strawberry	35	100.8°
Chocolate	45	129.6°
Peppermint	25	72.0°
Other	20	57.6°
total = 125		

The pie chart can be drawn accurately using the calculated angles.



EXERCISE 16D

- A survey was conducted to find what colour car people would prefer. The results were:
white - 452, **red** - 224, **blue** - 315, **green** - 96, **black** - 62, **other** - 89
 What would be the angle for each colour on a pie chart?
 Give answers correct to one decimal place.

2. The students in year 10 at a school were asked what they planned to do at the end of secondary school. Their responses are shown in the tally sheet below.

<i>Plans for year after secondary school</i>	<i>Tally</i>
Tertiary Studies	
TAFE	
Apprenticeship	
Work	
Travel	
Other	

- (a) How many students are in year 10 at the school?
 (b) Construct a column graph displaying this data.
 (c) If a pie chart was constructed to display this data, what would be the angle for each of the activities?
 Give answers correct to one decimal place)

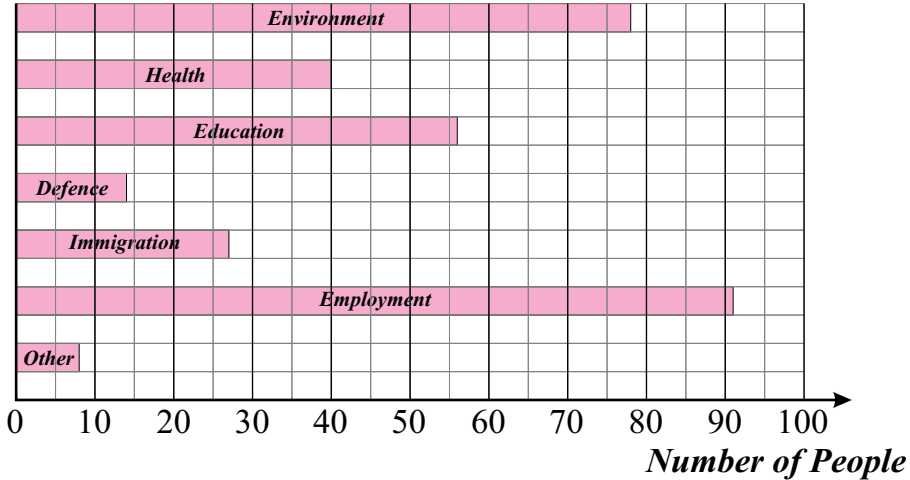
3. This pictograph shows the results of a survey conducted to find the activity preferred by a number of people to keep fit.

Running	    
Swimming	  
Walking	       
Dancing	  
Gym	   
Bicycle riding	 
Team Sport	      
Other	 

- (a) How many people were surveyed?
 (b) What percentage of the people surveyed participated in each of the activities?
 Give answers to correct to one decimal place.
 (c) Based on these figures, how many people in a town of 6000 would choose swimming?

 represents 10 people

4. The Federal Government conducted a survey to find out which portfolio people considered to be the most important. The results are shown on the bar graph below.

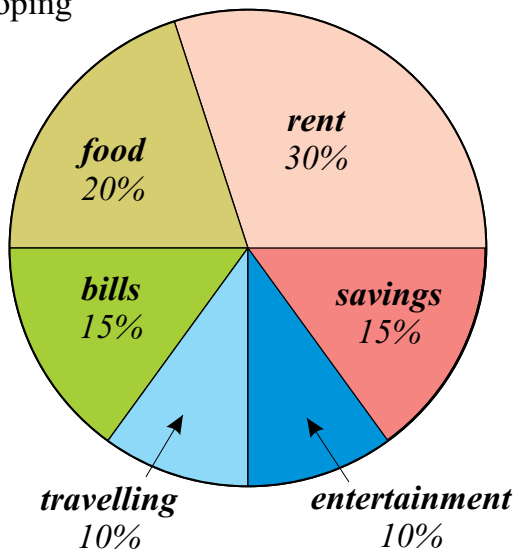


- (a) Based on these figures, what percentage of the people selected each of the portfolios as the most important?
Give answers correct to one decimal place.
- (b) If this data was to be displayed on a pie chart, what would be the angle for each portfolio?
Give answers correct to one decimal place.

5. The pie chart shown here represents the percentage of weekly income that Louis and Sam were hoping to spend on their weekly expenses.

Their combined nett income each week is \$1300.

How much should they spend on each of their weekly expenses?

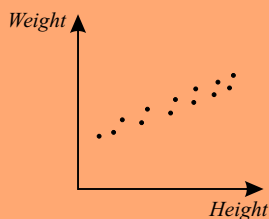


Bivariate Data

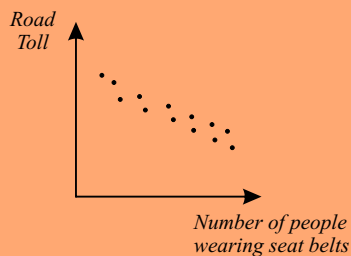
The shape of a bivariate data graph gives an indication of the relationship between the two variables.

Bivariate means two variables.

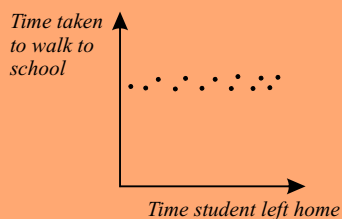
Examples



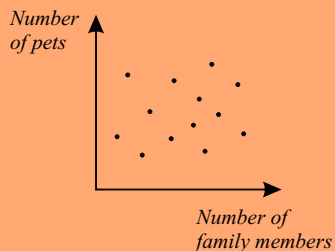
This graph indicates that as the height ***increases*** the weight also ***increases***.



This graph indicates that as the number of people wearing seat belts ***increases*** the road toll ***decreases***.



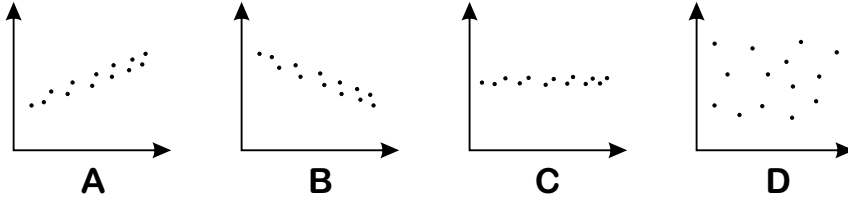
This graph indicates that the time that this student leaves home does not affect the time taken to walk to school. The time taken to walk to school is always about the same.



This graph indicates that the number of pets in a family is not affected by the number of family members.

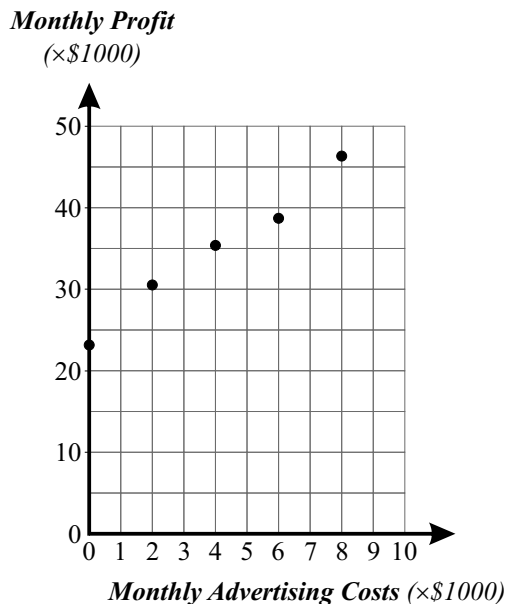
EXERCISE 16E

1. Select which of these four graphs would best illustrate the shape of the graph resulting from plotting data obtained from each of the bivariate studies below.



- The hours spent on homework versus the results achieved on tests.
- The number of police patrol cars versus the number of car accidents.
- The age of a person versus their body temperature.
- The price of a item versus the number sold.
- The time spent watching television versus exam results.
- The time taken to run 100 metres versus mark on mathematics exam.
- Age of a car versus its value.
- The number of spectators attending a football game versus the price of petrol.
- The time taken for a particular student to walk to school versus the daily temperature.

2. The **monthly profit** of a company versus the **monthly advertising costs** is shown on this graph.
- From this graph does increasing advertising costs **increase** or **decrease** the profit.
 - Based on this graph, what would be the approximate expected profit if the following amounts were spent on advertising?
 - \$5000
 - \$10 000



3. The **Sticky B** honey company found that changing the temperature of the honey in a vat changed the time it took to fill a jar. The data below shows the time to fill a honey jar from a vat for different temperatures of the honey.

Temperature (°C)	6	8	10	12	14	16	18	20	22
Time to fill jar (sec)	9.8	9.2	8.2	7.5	7.2	6.2	5.6	4.7	4.1

- (a) Draw a graph displaying this information using scales of **Temperature** (0°C - 26°C) on the horizontal axis and **Time to fill jar** (0 - 12 seconds) on the vertical axis.

- (b) Does increasing the temperature **increase** or **decrease** the time taken to fill a honey jar?

- (c) Based on this information, approximately how long would it take to fill a honey jar at the following temperatures? A straight line of best fit through the points will help.

(i) 15°C (ii) 25°C (iii) 3°C



4. The amount of fuel needed by a ferry to travel across a bay was dependant on the number of cars it was carrying. The volume of fuel used (litres) and the number of cars on board were recorded on eight days. The results are shown below. The weather conditions were similar on the eight days.

Number of cars	5	8	6	2	12	7	10	9
Volume of fuel (L)	38.1	42.4	39.6	33.0	50.2	41.6	46.3	45.9

- (a) Draw a graph displaying this information using scales of **Number of cars** (0 - 15) on the horizontal axis and **Volume of fuel** (30 - 56 L) on the vertical axis.

- (b) Based on this information, find the approximate amount of fuel used with the following number of cars on board.

A straight line of best fit through the points will help.

(i) no cars (ii) 3 cars (iii) 15 cars

Mean, Mode and Median - Discrete Ungrouped Data

The **mean**, **mode** and **median** are statistical measures used to summarise data and give a measure of the **central tendency** of a set of numerical data.

The **mean** of a set of ungrouped values can be found by adding all the values and dividing by the total number of values.

$\bar{x}$ is the mathematical symbol for the mean of the x values

Σ is the Greek letter sigma which is used in mathematics to represent **the sum of**. Therefore:

Σx represents the sum of the x values

N represents the total number of x values

$$\text{mean} = \bar{x} = \frac{\text{sum of all the values}}{\text{number of values}}$$

$$\bar{x} = \frac{\Sigma x}{N}$$

mode = the value that occurs the most (there may be no mode, one mode or more than one mode)

median = the middle value when the values are arranged in numerical order (if there is an even number of values the median is the average of the two middle values)

Example 1 Find the mean, mode and median of the following set of values:

6 8 8 15 15 15 19 29 38

$$\text{mean} = \bar{x} = \frac{\Sigma x}{N} = \frac{6+8+8+15+15+15+19+29+38}{9} = 17$$

mode = 15 (the value that occurs the most)

median = 15 (middle value: 6 8 8 15 (15) 15 19 29 38)

Example 2 Find the mean, mode and median of the following set of values:

8 24 19 11 3 37 15

$$\text{mean} = \bar{x} = \frac{\Sigma x}{N} = \frac{8+24+19+11+3+37+15}{7} = 16.7 \text{ (one dec. pl.)}$$

There is no mode (only one of each value)

To find the median the values need to be arranged in numerical order:

3 8 11 15 19 24 37

median = 15 (middle value: 3 8 11 (15) 19 24 37)

Example 3 Find the mean, mode and median of the following set of values:

5 17 11 16 17 16 21 14 14 19 16 14 8 11

$$\begin{aligned} \text{mean} = \bar{x} &= \frac{\Sigma x}{N} \\ &= \frac{5+17+11+16+17+16+21+14+14+19+16+14+8+11}{14} \\ &= 14.2 \text{ (one decimal place)} \end{aligned}$$

Arranging the values in numerical order will make it easier to find the mode and the median.

5 8 11 11 14 14 14 16 16 16 17 17 21 19

mode = 14 and 16

There is an even number of values therefore the median is the average of the middle two values.

5 8 11 11 14 14 (14) (16) 16 16 17 17 21 19

$$\text{median} = 15 \left(\frac{14+16}{2} \right)$$

EXERCISE 16F

1. For each of the following sets of values find the:

(i) mean (ii) mode (iii) median

Give answers correct to one decimal place where appropriate.

(a) 6 9 10 10 12 14 17 19 29

(b) 7 8 12 16 23 28

(c) 25 28 33 37

(d) 28 17 21 15 14 32 21

(e) 8 10 9 9 11 12 11 13 9 10 14 10

(f) 24 36 18 44 56 21 38 58

2. The ages of 11 people at a library are shown below.

28, 38, 43, 61, 55, 29, 11, 39, 50, 62, 44

(a) What is the mean age of the people in the library?

Give answer correct to one decimal place.

(b) What is the median age of the people in the library?

3. This table shows the scores *for* and *against* a football team for the first eight rounds of a competition.

Find the mean and median scores for and against the team for these eight rounds.

Give answers correct to one decimal place.

<i>Round</i>	<i>score for</i>	<i>score against</i>
1	128	113
2	97	86
3	133	90
4	102	122
5	87	75
6	73	98
7	88	87
8	110	62

4. The prices of six houses sold in a suburb are shown below.

\$200 000, \$156 000, \$278 000, \$223 000, \$1 504 000, \$189 000

(a) Find the mean and median price of the houses sold.

(b) Which of these values give a better indication of the value of the houses in this suburb? Why?

Mean, Mode and Median - Discrete Grouped Data

When there is a large amount of data with similar values it is convenient to group the data and display it in a frequency table.

Example 1

15 16 15 14 18 15 16 14 15 13 17 18 13
14 13 14 16 18 17 16 14 15 14 13 14 15

To group the data arrange it in numerical order or construct a tally sheet.

13 13 13 13 14 14 14 14 14 14 14 15 15
15 15 15 15 16 16 16 16 17 17 18 18 18

Value	Tally
13	
14	
15	
16	
17	
18	

Value x	Frequency f
13	4
14	7
15	6
16	4
17	2
18	3
total (N)	26

A frequency table can then be drawn.

The frequency is how many times each value occurs.

$$\begin{aligned} \text{mean} &= \bar{x} = \frac{\Sigma(x \times f)}{N} \\ &= \frac{(13 \times 4) + (14 \times 7) + (15 \times 6) + (16 \times 4) + (17 \times 2) + (18 \times 3)}{26} \end{aligned}$$

mean = 15.1 (one decimal place)

mode = 14

median = 15 (the two middle numbers are 15 and 15)

Example 2

Find the mean, mode and median of the data represented in this frequency table.

Note: $N = \Sigma f = 41$

<i>Value x</i>	<i>Frequency f</i>
25	6
26	8
27	10
28	10
29	7

$$\begin{aligned} \text{mean} = \bar{x} &= \frac{\Sigma (x \times f)}{N} \\ &= \frac{(25 \times 6) + (26 \times 8) + (27 \times 10) + (28 \times 10) + (29 \times 7)}{41} \end{aligned}$$

mean = 27.1 (one decimal place)

mode = 27 and 28

median = 27 (there are 41 values therefore the median will be the 21st value)

$$\text{median} = \text{value number} \left(\frac{N+1}{2} \right)$$

To find the median in a frequency table use the 'running total' of the frequency called the **cumulative frequency**.

<i>Value x</i>	<i>Frequency f</i>	<i>Cumulative Frequency</i>
25	6	6
26	8	14
27	10	24
28	10	34
29	7	41

← 21st value
in this group

EXERCISE 16G

1. Construct a frequency table for the following sets of data.

- (a) 8 8 9 9 9 9 10 10 10 11 11 11 11 12 12 12 13 14
 (b) 23 24 21 20 21 22 24 25 21 23 20 20 22 24 23 23 22
 (c) 18 16 17 19 20 16 17 18 16 15 16 17 18 20 17 17 16
 18 17 15 16 20 18 17 15 17 19 20 19 17 16 18 15 20

2. Find the mean, mode and median for the sets the data in question 1.
 Give answers to one decimal place where appropriate.

3. Jayme plays for the Lizards futsal team. She kicked the following number of goals in each of the games in one season.

3, 6, 4, 2, 3, 4, 5, 1, 2, 4, 3, 3, 3, 2, 5, 7, 2, 3, 1, 0, 5, 2, 4, 3

- (a) Construct a frequency table for this information.
 (b) Calculate the mean number of goals kicked by Jayme each game.
 Give answer to one decimal place.
 (c) Find the median number of goals kicked by Jayme each game.



4. Tamazyn is the quality controller at a biscuit company. She wanted to check how many biscuits were in the packets of **Fruity Bites**. The company states there is an average of at least 35 biscuits in each packet. Tamazyn counted the number of biscuits in 40 packets and the data is given below.

**36 35 34 37 35 34 38 33 35 36 37 35 34 36 37 38 32 35 37 36
 35 36 37 38 33 34 35 36 36 37 37 36 35 34 35 36 33 34 35 36**

- (a) Construct a frequency table to display this information.
 (b) Calculate the mean number of biscuits per packet.
 Give answer correct to one decimal place.
 (c) Based on this data, is the company's claim correct?

Mean, Mode and Median - Continuous Data

Continuous numerical data is often grouped in tables such as the one shown here representing the height of a number of people.

The class interval of 140-<150 represents all heights between 140 cm and 150 cm, including 140 cm but less than 150 cm.

Height of people

Class Interval (cm)	Frequency f
140-<150	8
150-<160	14
160-<170	27
170-<180	35
180-<190	25

The **mean**, $\bar{x}$, can be calculated by using the midpoints of the class intervals as the x values.

Height of people

Class Interval (cm)	Midpoint x	Frequency f
140-<150	145	8
150-<160	155	14
160-<170	165	27
170-<180	175	35
180-<190	185	25
total (N)		109

$$\begin{aligned}
 \text{mean} = \bar{x} &= \frac{\Sigma(x \times f)}{N} \\
 &= \frac{(145 \times 8) + (155 \times 14) + (165 \times 27) + (175 \times 35) + (185 \times 25)}{109}
 \end{aligned}$$

mean **170.0 cm** (one decimal place)

EXERCISE 16H

1. Calculate the mean of the following sets of data.

Give answers correct to one decimal place.

(a) *Mass*

<i>Class Interval (kg)</i>	<i>Frequency f</i>
50-<60	13
60-<70	19
70-<80	33
80-<90	28
90-<100	17

(b) *Length*

<i>Class Interval (cm)</i>	<i>Frequency f</i>
40-<60	20
60-<80	33
80-<100	57
100-<120	44
120-<140	23

(c) *Height*

<i>Class Interval (cm)</i>	<i>Frequency f</i>
0-<100	22
100-<200	57
200-<300	143
300-<400	127
400-<500	98

(d) *Age*

<i>Class Interval (years)</i>	<i>Frequency f</i>
0-<5	43
5-<10	50
10-<15	59
15-<20	68
20-<25	30

2. A number of seals were caught, tagged and released. Their masses, in kg, are shown below.

27 38 16 42 57 9 35 47 31 29 42 55 28 30 45 19 61 58
29 37 33 41 18 60 34 23 19 8 46 59 62 41 27 37 26 32

- Construct a frequency table to display this information using class intervals 0-<10, 10-<20, 20-<30, etc.
- Use the frequency table to calculate the mean mass of the seals. Give answer correct to one decimal place.
- Calculate the mean mass of the seals using the individual masses of the seals. Give answer correct to one decimal place.

The **median** of continuous numerical data can be found by constructing a **cumulative frequency graph**. This can be done after completing the **cumulative frequency** ('running total') column in the frequency table. See example below.

Height of people

<i>Class Interval (cm)</i>	<i>Frequency f</i>	<i>Cumulative Frequency</i>
140-<150	8	8
150-<160	14	22
160-<170	27	49
170-<180	35	84
180-<190	25	109

The median is the item number ($\frac{N+1}{2}$).

In this example $N = 109$ therefore the median will be item number 55 ($\frac{109+1}{2}$).

To find the median follow these steps:

Step 1 - Draw the cumulative frequency graph.

Step 2 - On the cumulative frequency axis locate ($\frac{N+1}{2}$), 55 in this example.

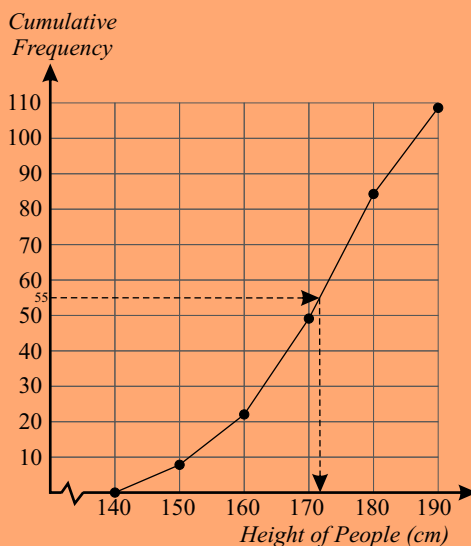
Step 3 - Draw a horizontal line across to the curve.

Step 4 - Draw a vertical line from this point to the data axis. This point will be the approximate value of the median.

In this example:

median ≈ 172 cm

Cumulative Frequency Graph



EXERCISE 16I

1. For each of the following sets of data draw a cumulative frequency graph and hence find an approximate value for the median.

(a) *Mass*

<i>Class Interval (kg)</i>	<i>Frequency f</i>
50-<60	5
60-<70	12
70-<80	18
80-<90	13
90-<100	7

(b) *Length*

<i>Class Interval (cm)</i>	<i>Frequency f</i>
40-<60	12
60-<80	23
80-<100	28
100-<120	29
120-<140	20

(c) *Height*

<i>Class Interval (cm)</i>	<i>Frequency f</i>
0-<100	11
100-<200	26
200-<300	39
300-<400	19
400-<500	7

(d) *Age*

<i>Class Interval (years)</i>	<i>Frequency f</i>
0-<5	22
5-<10	37
10-<15	47
15-<20	42
20-<25	29

2. The ages of a number of people who attended a festival are shown below.

27 21 38 45 16 28 42 10 57 51 9 27 35 47 31 29 42 55
 28 30 45 19 61 58 29 22 9 37 41 33 41 18 60 34 23 19
 8 46 35 59 19 62 16 41 27 27 37 52 26 50 32 29 15 48

- Construct a frequency table to display this information using class intervals 0-<10, 10-<20, 20-<30, etc.
- Use the frequency table to draw a cumulative frequency graph.
- Use this graph to find the approximate value for the median age.
- Arrange the ages in numerical order and find the exact value of the median.

The mean, mode (modal class) and median can be found from a frequency table of continuous numerical data by including the following columns to the table:

midpoint (x), ($x \times f$), cumulative frequency

Example

Length

Class Interval (cm)	Frequency f
0-<20	8
20-<40	15
40-<60	23
60-<80	19
80-<100	11

Include *midpoint (x), ($x \times f$), cumulative frequency* columns.

Length

Class Interval (cm)	Midpoint x	Frequency f	$x \times f$	Cumulative Frequency
0-<20	10	8	80	8
20-<40	30	15	450	23
40-<60	50	23	1150	46
60-<80	70	19	1330	65
80-<100	90	11	990	76
			4000	

$\Sigma(x \times f)$

N

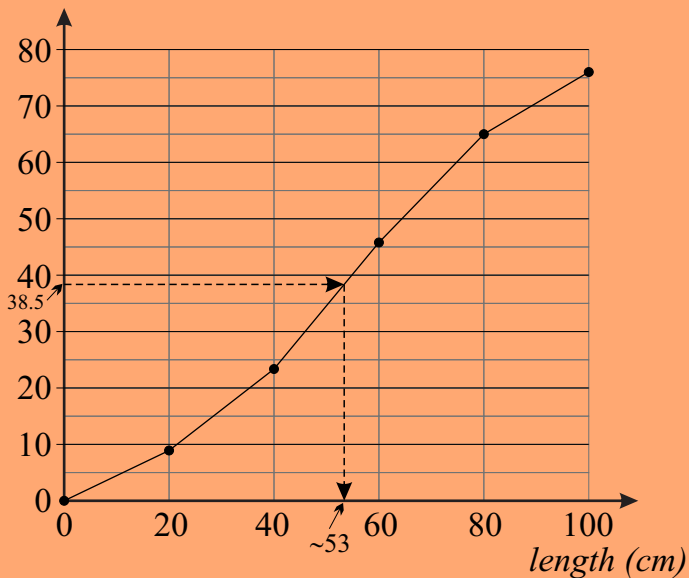
$$\begin{aligned}
 \text{mean} &= \frac{\Sigma(x \times f)}{N} \\
 &= \frac{4000}{76}
 \end{aligned}$$

mean = 52.6 cm (one decimal place)

The median can be found by drawing the cumulative frequency curve. Find $(\frac{N+1}{2})$ on the cumulative frequency axis.

$$\frac{N+1}{2} = \frac{76+1}{2} = \frac{77}{2} = 38.5$$

*cumulative
frequency*



***median* $\approx$ 53 cm**

The ***modal class*** is the class that has the highest frequency = **40 cm - <60 cm**

Answers:

***mean* = 52.6 cm (one decimal place)**

***modal class* = 40 cm - <60 cm**

***median* $\approx$ 53 cm**

EXERCISE 16J

1. For the following sets of data:

- redraw the frequency table including *midpoint*, $(x \times f)$ and *cumulative frequency* columns.
- calculate the mean (correct to one decimal place)
- find the modal class
- draw a cumulative frequency curve
- find an approximate value of the median

(a) *Height*

<i>Class Interval (m)</i>	<i>Frequency f</i>
0-<10	12
10-<20	23
20-<30	45
30-<40	32
40-<50	19

(b) *Time*

<i>Class Interval (min)</i>	<i>Frequency f</i>
30-<35	15
35-<40	23
40-<45	28
45-<50	33
50-<55	27

2. The times taken (in seconds) for a number of students to solve a puzzle are shown below.

35 78 48 83 51 50 42 66 72 61 58 67 62 80 71 56
64 73 68 60 59 44 69 52 38 47 56 63 71 82 65 47
56 37 80 77 63 57 43 48 58 62 59 66 73 47 64 61

- Using class intervals of 30-<40, 40-<50, etc construct a frequency table that includes *class interval*, *midpoint*, *frequency*, $(x \times f)$ and *cumulative frequency*. A tally sheet may assist in finding the frequency in each class interval.
- Calculate the mean from this frequency table (one decimal place).
- Find the modal class.
- Draw a cumulative frequency graph.
- Find the approximate value for the median from this graph.
- Use the raw data to calculate the mean and median.
- See how long it takes you to solve the puzzle:

Tom, Tim and Tam are brothers. Tom, the eldest, is ten years older than Tim, the youngest. The mean of their ages is 10 and the median age is 8. Find the ages of the brothers.

Stemplots (Stem-and-Leaf Plots)

EXERCISE 16K

Example

1. Construct a non-ordered stemplot for the numbers below.
2. Construct an ordered stemplot for the numbers below.
3. Find the median.

24, 35, 26, 48, 51, 17, 34, 26, 28, 40, 20, 19, 22, 37, 41, 49, 51, 53

1. non-ordered stemplot

Stem	Leaf
1	7 9
2	4 6 6 8 0 2
3	5 4 7
4	8 0 1 9
5	1 1 3

$$\overline{1|3} = 13$$

2. ordered stemplot

Stem	Leaf
1	7 9
2	0 2 4 6 6 8
3	4 5 7
4	0 1 8 9
5	1 1 3

$$\overline{1|3} = 13$$

3. The **median** is item number $(\frac{N+1}{2})$.

$N = 18$ so the median will be item number 9.5 - average of items 9 and 10 (34 and 35).

median = 34.5

Stem	Leaf
1	7 9
2	0 2 4 6 6 8
3	④ ⑤ 7
4	0 1 8 9
5	1 1 3

$$\overline{1|3} = 13$$

1. For the following sets of data:

- (i) construct a non-ordered stemplot
- (ii) construct an ordered stemplot
- (iii) find the median

(a) 23, 36, 67, 35, 37, 49, 51, 28, 33, 36, 40, 38, 57, 52, 29, 37

(b) 8, 16, 7, 32, 25, 26, 19, 18, 16, 18, 25, 33, 42, 19, 41, 39, 30

2. Two athletes each threw a javelin 10 times and the distances, to the nearest metre, are shown below.

Spike: 36, 44, 52, 46, 36, 28, 33, 37, 40, 48

Lance: 36, 35, 59, 33, 29, 32, 34, 37, 41, 36

For these athlete's results complete the following.

- Construct non-ordered stemplots.
- Construct ordered stemplots.
- Find the furthest distance each athlete threw the javelin.
- Find the median of each set of throws.
- Find the mean distance thrown by each athlete.



3. Jacqui, a forestry researcher, wanted to investigate how the population of honeyeaters was recovering in a national park a year after a bush fire. The national park was 5000 hectares. Each day she covered an area of 10 hectares counting the number of honeyeaters. She did this for twenty-four days and the number of honeyeaters she counted each day are shown below.

**58, 36, 27, 98, 26, 38, 50, 42, 62, 36, 48, 50,
25, 16, 30, 42, 38, 55, 46, 61, 45, 74, 39, 40**

- Construct a non-ordered stemplot.
- Construct an ordered stemplot.
- What is the median number of honeyeaters in a 10 hectare area?
- What is the mean number of honeyeaters in a 10 hectare area?

Give answer correct to one decimal place.

- Based on these figures, calculate the number of honeyeaters in the national park using:
 - the median
 - the mean (nearest whole number)



Boxplots (Box-and-Whisker Plots)

Examples For each of the following sets of data:
 (i) find the *interquartile range*.
 (ii) construct a *boxplot*.

1.

19, 18, 12, 28, 26, 13, 15, 24

Step 1 Arrange in numerical order from smallest to largest.

12 13 15 18 19 24 26 28

Step 2 Find the median.

12 13 15 18 19 24 26 28

Median = 18.5

Step 3 Find the median of the bottom half of the numbers.
 This is called the *lower quartile* (Q_1).

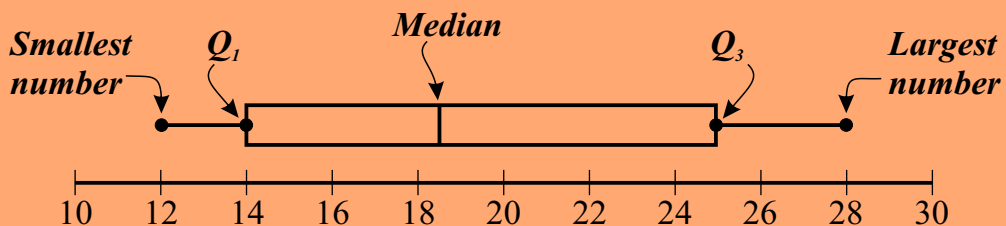
12 13 15 18 19 24 26 28
 $Q_1 = 14$

Step 4 Find the median of the top half of the numbers.
 This is called the *upper quartile* (Q_3).

12 13 15 18 19 24 26 28
 $Q_3 = 25$

Step 5 *Interquartile range* = $Q_3 - Q_1$
 = $25 - 14$
 = 11

Step 6 Draw the boxplot.



2.

40, 33, 27, 25, 34, 29, 35, 31, 37, 23, 38

Step 1 23 25 27 29 31 33 34 35 37 38 40

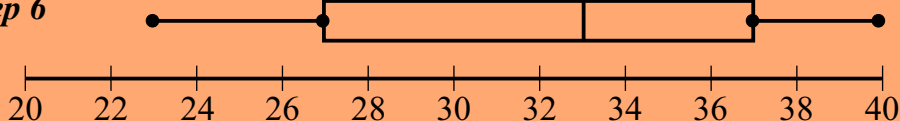
Step 2 23 25 27 29 31 (33) 34 35 37 38 40 **Median** = 33

Step 3 23 25 (27) 29 31 (33) 34 35 37 38 40 **Q_1** = 27

Step 4 23 25 (27) 29 31 (33) 34 35 (37) 38 40 **Q_3** = 37

Step 5 **Interquartile range** = $37 - 27 = 10$

Step 6



EXERCISE 16L

1. For each of the following sets of data:

(i) find the interquartile range.

(ii) construct a boxplot.

(a) 9, 12, 13, 10, 8, 17, 15

(b) 17, 21, 28, 37, 40, 13, 42, 33, 24, 15

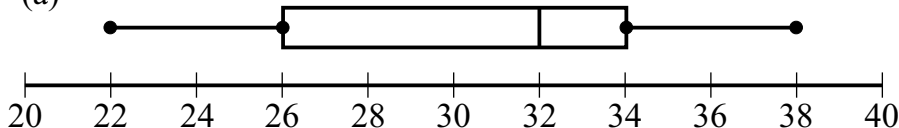
(c) 40, 38, 47, 51, 55, 43, 36, 40, 37

(d) 12, 19, 18, 28, 29, 30, 27, 12

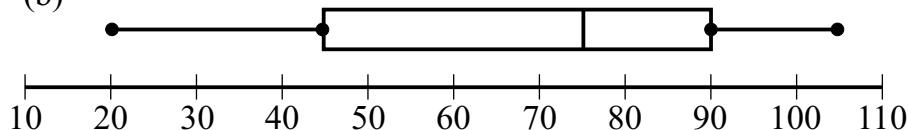
2. For the following boxplots find the:

(i) median (ii) interquartile range

(a)



(b)



Outliers

As described earlier, the *mean*, *mode* and *median* are statistical measures that are used to summarise data and give a measure of the '*centre*' of a set of data.

Sometimes the *mean* can be affected disproportionately by items of data that are considerably larger or smaller than the rest of the data. These items of data are called *outliers*.

Outliers must be included when calculating the mean, mode and median. They will not affect the mode or median but could distort the mean of a set of data.

Examples

Calculate the mean, mode and median of the following sets of data.

1. 5, 56, 58, 60, 61, 61, 61, 62, 62

mean = 54, *mode* = 61, *median* = 61

2. 25, 27, 27, 28, 28, 29, 29, 30, 32, 157

mean = 41.2, *mode* = 27, 28, 29, *median* = 28.5

In both these examples it can be seen that the mean has been affected by the extremely low or high values.

When analysing data, outliers need to be recognised as factors that could distort the summary of the data particularly the mean. Sometimes statistics are deliberately distorted by outliers.

Example

Six houses sell in a suburb. The prices are:

\$200 000, \$178 000, \$210 000, \$951 000, \$206 000, \$235 000

The *median* price = **\$208 000**

The *mean* price = **\$330 000**

The price of \$951 000 has shifted the mean house price to well above the price of the other five houses. This may have been an unusual house not representative of the area. The median price is probably a better indication of the house prices in this suburb. A real estate agent in the area could use the mean price as a way of increasing price expectations.

An outlier is defined as any item of numerical data that is:

- more than $1.5 \times$ the interquartile range (IQR) above the upper quartile value (Q_3) or
- more than $1.5 \times$ the interquartile range (IQR) below the lower quartile value (Q_1)

Outliers are shown as dots on a boxplot but not included in the 'whiskers'.

Example

Draw a boxplot of the following data indicating all outliers as dots.

45, 21, 33, 78, 53, 36, 29, 3, 31, 40

Step 1 Arrange data in ascending numerical order.

3, 21, 29, 31, 33, 36, 40, 45, 53, 78

Step 2 Find the median

3, 21, 29, 31, 33, 36, 40, 45, 53, 78

median = 34.5

Step 3 Find Q_1 and Q_3

3, 21, (29), 31, 33, 36, 40, (45), 53, 78

Q_1

median = 34.5

Q_3

Step 4 Find the interquartile range (IQR)

$$\text{IQR} = Q_3 - Q_1 = 45 - 29 = 16$$

Step 5 Find $1.5 \times \text{IQR} = 1.5 \times 16 = 24$

Step 6 Find $Q_1 - (1.5 \times \text{IQR})$ and $Q_3 + (1.5 \times \text{IQR})$

$$Q_1 - 24 = 29 - 24 = 5$$

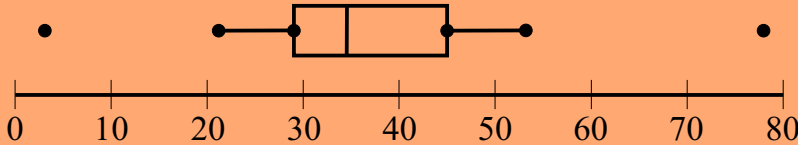
$$Q_3 + 24 = 45 + 24 = 69$$

Therefore any values less than 5 or greater than 69 are classified as outliers.

(3), 21, 29, 31, 33, 36, 40, 45, 53, (78)

outliers

Step 7 Draw the boxplot



EXERCISE 16M

- Construct boxplots for the following sets of data, clearly showing any outliers.
 - 29, 15, 25, 48, 69, 95, 46, 13, 18, 28, 33
 - 50, 43, 53, 57, 63, 48, 11, 91, 37
 - 55, 52, 35, 39, 40, 10, 71, 53
- The scores made by two batters in cricket for 10 matches are shown below.

Edgey Ed: 10, 6, 0, 21, 389, 3, 8, 0, 79, 0

Driver Dan: 45, 36, 58, 55, 87, 22, 48, 55, 40, 60

- Find the median score by each batter.
- Find the mean score by each batter.
- Who is the more consistent batter?
- Does the median or the mean give a better indication of who is the more consistent batter?
- Find the mean of each batter's scores if the top score for each is ignored (correct to one decimal place).



- The time taken for a tram to complete a city route was recorded each day for 10 week days. These times (to the nearest minute) are shown here.

43, 39, 41, 44, 40, 67, 42, 43, 39, 40

- Find the median time.
- Calculate the mean time.
- Calculate the mean time if outliers are ignored (correct to one decimal place).

Normal Distribution and Standard Deviation

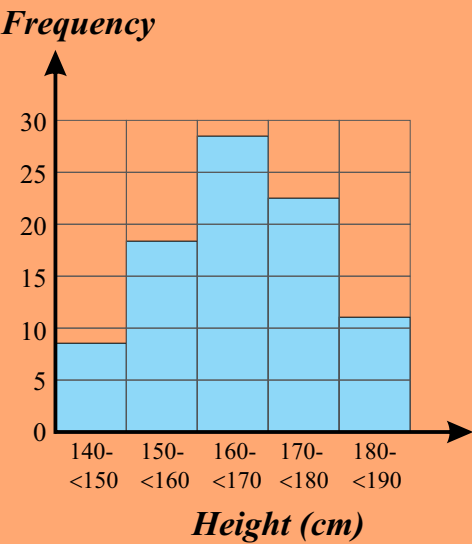
If the frequency of numerical data is displayed on a graph with the columns joined this is called a histogram.

Example

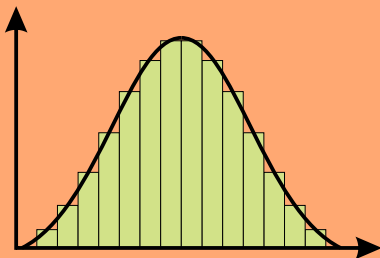
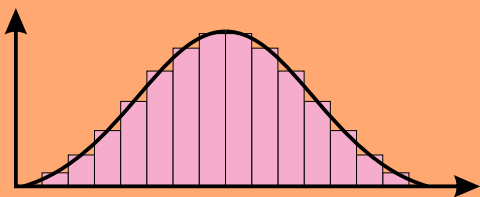
The height of a number of students is shown in the frequency table and frequency graph (histogram) below.

Height of students

<i>Height Interval (cm)</i>	<i>Frequency f</i>
140-<150	8
150-<160	19
160-<170	28
170-<180	22
180-<190	11



The shape of this histogram is very similar to the shape of histograms formed by displaying the frequency of many types of data: height, weight, growth rates, times taken, heart rates, etc. The more data that is collected, in many cases, the closer the histogram will look like the graphs shown here.



This curve is normal for many types of 'real life' data and is called a ***Normal Distribution***.

The curve is also called a ***Bell Curve*** because of its shape.

The 'perfect' Bell Curve is symmetrical.

The centre of the perfect curve is the mean and median and more data is in this section of the graph. The number of items of data decreases the further they are from the centre.

This is to be expected. For example the height of people. There is an average (mean) height and there less people the further you move from this mean.

One measure of the spread of data in a Normal Distribution curve is the ***Standard Deviation***. It is a measurement that indicates the deviation from the standard measurement (mean).

The symbol for standard deviation is σ (the Greek letter sigma).

We will study how to calculate standard deviation later but in terms of a normal distribution graph the standard deviation is a measure of the spread of the data from the mean.

68% of the data will lie between one standard deviation either side of the mean.

95% of the data will lie between two standard deviations either side of the mean.

99.7% of the data will lie between three standard deviations either side of the mean.

Remember the symbol for mean is $\bar{x}$.

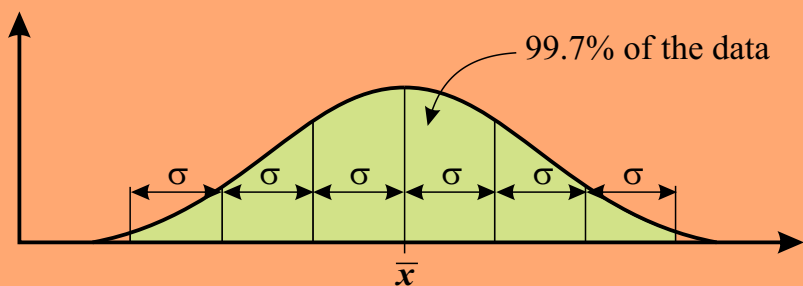
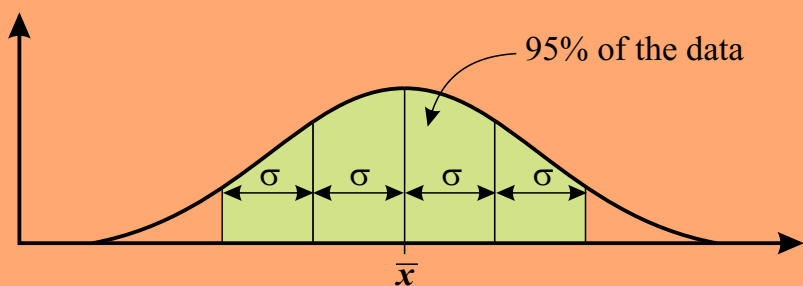
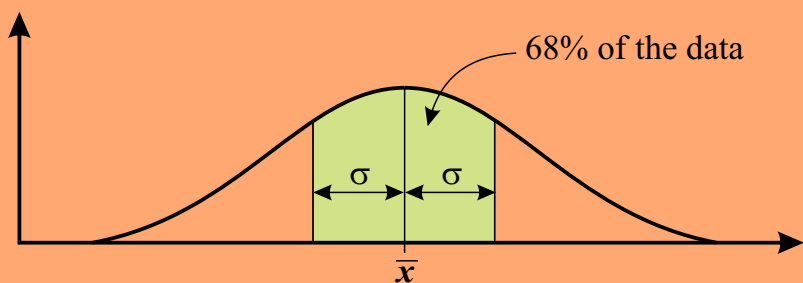
Using symbols:

$\bar{x} \pm \sigma$ represents 68% of the data.

$\bar{x} \pm 2\sigma$ represents 95% of the data.

$\bar{x} \pm 3\sigma$ represents 99.7% of the data.

Looking at the normal distribution graph:



Example 1

A number of people donated money to a charity.

The mean amount donated was \$30 per person and the standard deviation of the amounts donated was \$5 per person.

This means that:

68% of the people donated between \$25 and \$35 (one standard deviation either side of the mean)

95% of the people donated between \$20 and \$40 (two standard deviations either side of the mean)

99.7% of the people donated between \$15 and \$45 (three standard deviations either side of the mean)

Example 2

A group of students were given a spelling test of 45 words.

The mean number of words spelt correctly was 32.

68% of the students spelt between 28 and 36 words correctly.

1. Calculate the standard deviation of the results.
2. Complete the following sentence.

95% of the students correctly spelt between and words.

Answers

1. 68% of the students is one standard deviation either side of the mean.
The mean is 32 which is midway between 28 and 36.
The standard deviation is therefore **4**.
2. 95% of the students is two standard deviations either side of the mean. Two standard deviations = 8.

$$\begin{aligned}\bar{x} \pm 2\sigma \\ &= 32 \pm 8 \\ &= 24, 40\end{aligned}$$

95% of the students correctly spelt between 24 and 40 words.

Example 3

Sunny and Manger are mango farmers. They each harvest 1000 mango trees.

The average (mean) yield of mangoes per tree for each farmer was 80 kg.

The standard deviation for Sunny's yield was 10 kg.

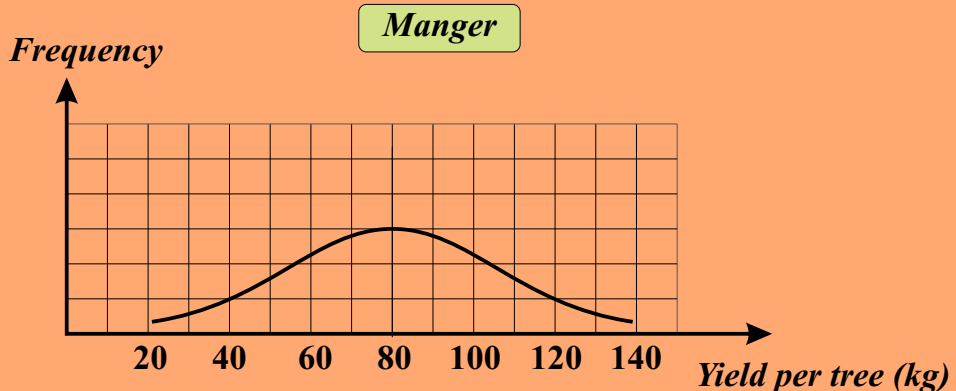
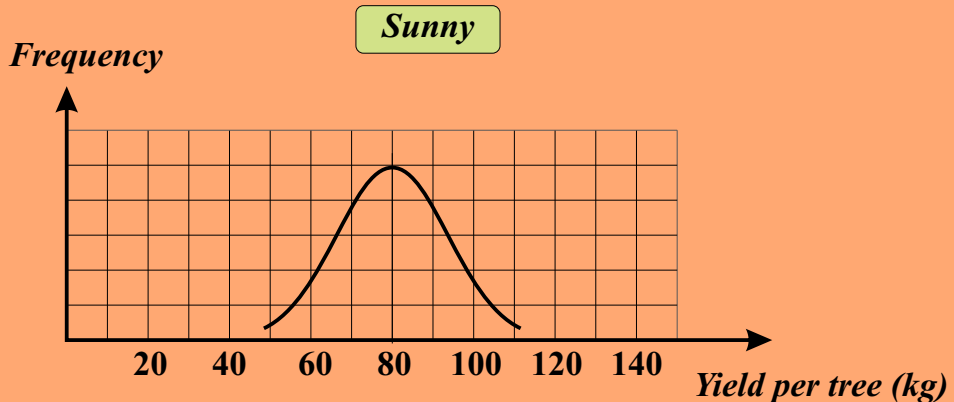
The standard deviation for Manger's yield was 20 kg.

Assume the data is a normal distribution.

1. Draw a curve showing the shape of the frequency distribution histogram for each, showing the main differences.
2. What effect does the size of the standard deviation have on the frequency distribution histogram and normal distribution curve.
3. Describe the differences between Sunny's and Manger's mango trees.

Answers

1.



2. A larger standard deviation means a larger spread (variation) of the data.

For **Sunny's farm** the mean is 80 and standard deviation is 10. This means 68% of the trees will be between 70 and 90 (one standard deviation either side of the mean).

95% of the trees will be between 60 and 100 ($\bar{x} \pm 2\sigma$).

99.7% of the trees will be between 50 and 110 ($\bar{x} \pm 3\sigma$).

For **Manger's farm** the mean is 80 and standard deviation is 20.

68% of the trees will be between 60 and 100 ($\bar{x} \pm \sigma$).

95% of the trees will be between 40 and 120 ($\bar{x} \pm 2\sigma$).

99.7% of the trees will be between 20 and 140 ($\bar{x} \pm 3\sigma$).

In this example because the overall number of trees is the same for both farmers the number of trees around the mean on Manger's farm will be less because there are more 'further away' from the mean.

3. There is a larger variation in the yield of Manger's mangoes. Sunny's farm has more trees that produce around the average (mean) yield.

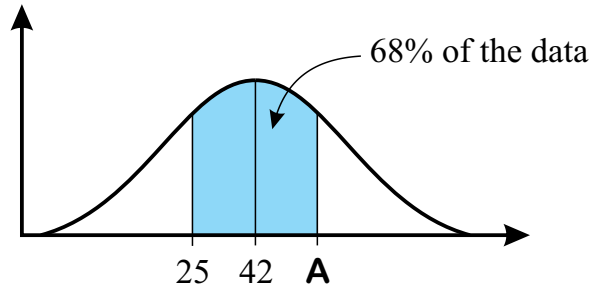


Assume all the questions in this exercise follow a normal distribution curve.

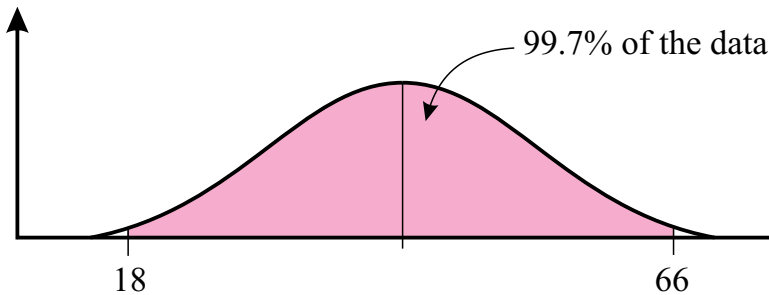
EXERCISE 16N

1. The **mean** of a set of data is 50 and the **standard deviation** is 6.
Complete the following sentences.
 - (a) 68% of the data will occur between and .
 - (b) 95% of the data will occur between and .
 - (c) 99.7% of the data will occur between and .
2.
 - (a) The central 68% of a set of data lies between 72 and 88.
 - (i) What is the standard deviation of this set of data?
 - (ii) What is the mean of this set of data?
 - (b) The central 95% of a set of data lies between 96 and 156.
 - (i) What is the standard deviation of this set of data?
 - (ii) What is the mean of this set of data?
 - (c) The central 99.7% of a set of data lies between 45 and 117.
 - (i) What is the standard deviation of this set of data?
 - (ii) What is the mean of this set of data?
 - (d) The central 68% of a set of data lies between 17.2 and 34.6.
 - (i) What is the standard deviation of this set of data?
 - (ii) What is the mean of this set of data?
3. The mean of a set of data is 84 and 34% of the data lies between 68 and 84.
Complete the following sentences.
 - (a) 68% of the data will occur between and .
 - (b) 95% of the data will occur between and .
 - (c) 99.7% of the data will occur between and .
4. The mean of a set of data is 56 and the standard deviation is 12.
 - (a) What percentage of the data is greater than 68?
 - (b) What percentage of the data is less than 32?
 - (c) What percentage of the data is greater than 44?
 - (d) What percentage of the data is less than 92?

5. For the data shown on this graph find:
 (a) the standard deviation.
 (b) **A**.



6. For the data shown on the graph below find:
 (a) the standard deviation
 (b) the mean



7. Scientists measured and recorded the age of a number of koalas in a state park. They found that 68% of the koalas were between 5 and 9 years old.
 (a) What is the mean age of the koalas?
 (b) What percentage of the koalas would be older than 3?
 (c) What percentage of the koalas would be older than 13?



8. Mason is a cricketer who calculated the mean of his batting scores after 100 innings to be 43 and 68% of his scores were between 31 and 55.
(a) What is the standard deviation of his scores?
(b) What percentage of his scores were over 67?
9. The average (mean) rainfall in July for a town was 83 mm and 16% of rainfall totals in July were more than 98 mm.
What is the standard deviation of the rainfall totals?
10. Records showed that 95% of the maximum temperatures in January in an outback town were between 28°C and 42°C .
(a) What is the mean maximum temperature?
(b) What is the standard deviation of the maximum temperatures?
11. Dahna and Leena were discus throwers. They recorded the distances of 120 of each of their throws.
The average (mean) distance thrown by Dahna was 45 m.
The standard deviation of Dahna's throws was 2 m.
The average of Leena's throws was 42 m and the standard deviation was 4 m.
(a) Draw a curve showing the shape of the frequency distribution histogram for each of the discus throwers.
(b) Describe the differences in the distances thrown by the two throwers.



Calculating Standard Deviation

The value of the standard deviation for a set of data can be calculated using most calculators.

However it is worthwhile to know the formula used to calculate standard deviation.

There are two versions of the formula:

- one is used when calculating the standard deviation of a **population**.
- the other is used when calculating the standard deviation of a **sample**.

For a **population** the standard deviation is calculated by using this formula:

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$$

$\bar{x}$ is the mean

N is the number in the population

$\Sigma(x - \bar{x})^2$ means the sum of all the $(x - \bar{x})^2$ values.

For a **sample** the standard deviation is calculated by using this formula:

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}}$$

$\bar{x}$ is the mean

N is the number in the sample

$\Sigma(x - \bar{x})^2$ means the sum of all the $(x - \bar{x})^2$ values.

The easiest way to calculate standard deviation using these formulae is to set up a table of values as shown in the following examples. A spreadsheet could be used.

Example 1

The ages of the 12 teachers at a small school are shown below.
Calculate the mean and standard deviation of the ages.
Give answers correct to one decimal place.

28, 45, 31, 24, 33, 29, 25, 36, 30, 27, 26, 31

With this example we are dealing with all the teachers at the school so the **population** version of the standard deviation formula is used.

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$$

Step 1 Calculate the mean ($\bar{x}$).

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{x} = \mathbf{30.4} \text{ (one decimal place)}$$

Step 2 Draw the following table.

x	$x - \bar{x}$	$(x - \bar{x})^2$
28		
45		
31		
24		
33		
29		
25		
36		
30		
27		
26		
31		

Step 3 Complete the table.

x	$x - \bar{x}$	$(x - \bar{x})^2$
28	-2.4	5.76
45	14.6	213.16
31	-0.6	0.36
24	-6.4	40.96
33	2.6	6.76
29	-1.4	1.96
25	-5.4	29.16
36	5.6	31.36
30	-0.4	0.16
27	-3.4	11.56
26	-4.4	19.36
31	0.6	0.36
Total		360.92

Step 4 Calculate the standard deviation.

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N}}$$

$$\Sigma(x - \bar{x})^2 = 360.92$$

$$N = 12$$

$$\sigma = \sqrt{\frac{360.92}{12}}$$

$$= \sqrt{30.0766..}$$

$$\sigma = \mathbf{5.5} \text{ (one decimal place)}$$

Example 2

Ten bilbies were tagged in a national park and each fitted with a GPS to study their movement.

The distance travelled by each bilby, in metres, in one night was recorded and these distances are shown below.

68, 130, 96, 121, 87, 118, 104, 92, 88, 107

1. Calculate the mean and standard deviation of these distances.
Give answers correct to one decimal place.
2. Calculate the distances between which 68% of these bilbies travelled each night.

With this example we are dealing with a sample of the bilbies so the **sample** version of the standard deviation formula is used.

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}}$$

Answers

1.

Step 1 Calculate the mean ($\bar{x}$).

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{x} = 101.1 \text{ (one decimal place)}$$

Step 2 Draw the following table.

x	$x - \bar{x}$	$(x - \bar{x})^2$
68		
130		
96		
121		
87		
118		
104		
92		
88		
107		

Step 3 Complete the table.

x	$x - \bar{x}$	$(x - \bar{x})^2$
68	-33.1	1095.61
130	28.9	835.21
96	-5.1	26.01
121	19.9	396.01
87	-14.1	198.81
118	16.9	285.61
104	2.9	8.41
92	-9.1	82.81
88	-13.1	171.61
107	5.9	34.81
Total		3134.90

Step 4 Calculate the standard deviation.

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}}$$

$$\Sigma(x - \bar{x})^2 = 3134.90$$

$$N - 1 = 9$$

$$\sigma = \sqrt{\frac{3134.90}{9}}$$

$$= \sqrt{348.322..}$$

$$\sigma = \mathbf{18.7} \text{ (one decimal place)}$$

2. 68% of the bilbies travelled a distance of $\bar{x} \pm \sigma$.

$$= 101.1 \pm 18.7$$

$$= \mathbf{82.4 \text{ m and } 119.8 \text{ m}}$$

EXERCISE 160

1. There were 14 students in a Year 10 mathematics class.
For a test on statistics the marks achieved, as a percentage, by the students are shown below.
Calculate the mean and standard deviation of these marks.
Give answers correct to one decimal place.

68, 79, 93, 80, 82, 77, 95, 69, 73, 80, 88, 90, 66, 100

2. Marine scientists were studying a certain type of starfish in an area of the Great Barrier Reef. They divided this small area of the reef into 12 squares each with an area of 200 m^2 and counted the number of starfish in each square.
The number of starfish in each 200 m^2 area is shown below.

50, 28, 22, 62, 33, 45, 27, 36, 31, 39, 28, 41

- (a) Calculate the mean and standard deviation for this data.
Give answers correct to one decimal place.
(b) Between which two amounts would 95% of the data lie?



PROBLEM SOLVING

1. A die was rolled five times. The mode of the numbers that appeared on the top face was one more than the median which was one more than the mean.
What were the five numbers that were rolled?



2. A basketball team is about to play in the 22nd game of the season. It needs to score 111 points to have an average of 100 points per game.
What is the average points per game before this game?
Give answer to two decimal places.



3. Zac has sat four tests each with 10 questions. The median of his test results is six correct answers and the mean is five correct answers. Zac completes another 10 question test and the median of his five results is seven correct answers and the mean is six.
Each time Zac sat the test he answered more questions correctly than he did on the previous test.
How many questions did he answer correctly on each of the five tests?

PUZZLE

The answers to this puzzle are the same horizontally and vertically down.

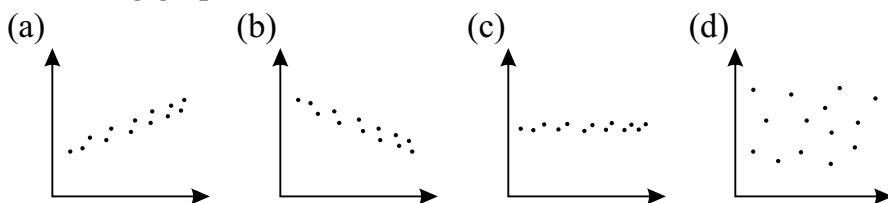
- 1. Unit of mass
- 2. Uncommon
- 3. Region
- 4. Average

1.	2.	3.	4.
2.			
3.			
4.			

CHAPTER REVIEW

1. (a) There are 32 840 spectators at a game between the Rebels and Falcons. What is the minimum number of people that should be surveyed to find out who they barrack for?
 (b) 200 people are surveyed and 125 barrack for the Rebels. Based on these figures, how many of the spectators barrack for the Rebels?
2. 400 people were asked what they ate for breakfast. The results were: cereal - 178, toast - 115, eggs - 59, fruit/yoghurt - 28, other - 20. If this data was displayed on a pie chart what would be the angle for each type of breakfast?

3. Describe the relationship between the data represented on the following graphs.



4. Find the mean, mode and median of the following sets of data.

- (a) 6, 15, 7, 8, 11, 10, 11, 12, 13, 11, 10
 (b) 25, 19, 24, 27, 31, 30

5. Find the mean, mode and median of the data displayed in this frequency table.
 Give the mean correct to one decimal place.

<i>Value</i> <i>x</i>	<i>Frequency</i> <i>f</i>
30	6
31	9
32	13
33	15
34	4

6. Construct a frequency table for the times (in seconds) given below and find the mean and median times.

56 58 65 56 57 58 63 62 61 59 58 60 64 63 61 66
 59 60 61 63 64 59 58 59 60 61 62 61 60 59 56 59

7. For the following set of data:

- calculate the mean (correct to one decimal place)
- find the modal class
- draw a cumulative frequency curve
- find the median

Height

<i>Class Interval (m)</i>	<i>Frequency f</i>
0-<10	12
10-<20	23
20-<30	45
30-<40	32
40-<50	19

8. For the masses (in kg) below:

- construct a non-ordered stemplot
- construct an ordered stemplot
- find the median
- construct a frequency table that includes *class interval* (30-<40, 40-<50, etc), *midpoint*, *frequency*, $(x \times f)$ and *cumulative frequency*.
- calculate the mean from this table correct to one decimal place
- draw a cumulative frequency graph
- find the approximate value of the median from this graph

45 68 48 83 51 50 42 66 72 61 58 67 62 70 71 56
64 73 68 60 59 44 59 52 38 47 56 63 71 82 65 47
56 37 80 77 63 57 43 48 58 62 59 66 73 47 64 61

9. Construct a boxplot for the following set of data clearly showing any outliers.

69 71 53 11 45 115 58 39 56

10. Robin is an archer who calculated the mean of his archery scores after 100 events to be 152 and 68% of his scores were between 125 and 179.

- What is the standard deviation of his scores?
- What percentage of his scores were over 125?

Answers

Chapter 1

Exercise 1A

- (a) T (b) F (c) F (d) F (e) F
(f) T (g) F (h) T (i) T (j) F
(k) T (l) T (m) T (n) T (o) F
(p) F (q) F (r) T (s) T (t) T
- (a) $\{5, 12\} \subset N$
(b) $\{-3, 4, 2\frac{4}{7}, 8.9\} \subset Q$
(c) $\{10, 0, 7\} \subset W$
(d) $\{-8, 0, 13\} \subset Z$
(e) $\{7\pi, 6.9, -5, 0, \frac{8}{9}, -11.8\} \subset R$
(f) $\{\sqrt{7}, 2\pi, \sqrt{13}\} \subset I$

Exercise 1B

- (a) 7836 (b) 93 982
- (a) 108 (b) 173 (c) 705 (d) 1026
(e) 5114 (f) 7018 (g) 15 093
(h) 600 099
- (a) 1804 (b) 75 (c) 19 732
(d) 539 036 (e) 199 917
(f) 2 999 000 (g) 1 233 567
(h) 71 999 295
- (a) 60 (b) 90 (c) 200 (d) 300
(e) 790 (f) 2800 (g) 9080
(h) 13 000
- (a) 5000 (b) 1000 (c) 0
(d) 15 000 (e) 30 000 (f) 320 000
(g) 208 000 (h) 100 000
- | | | | | | | | | |
|----------|----|----|----|----|----|-----|----|----|
| $\times$ | 2 | 8 | 7 | 9 | 6 | 12 | 5 | 3 |
| 4 | 8 | 32 | 28 | 36 | 24 | 48 | 20 | 12 |
| 7 | 14 | 56 | 49 | 63 | 42 | 84 | 35 | 21 |
| 5 | 10 | 40 | 35 | 45 | 30 | 60 | 25 | 15 |
| 9 | 18 | 72 | 63 | 81 | 54 | 108 | 45 | 27 |
| 6 | 12 | 48 | 42 | 54 | 36 | 72 | 30 | 18 |
| 10 | 20 | 80 | 70 | 90 | 60 | 120 | 50 | 30 |
| 11 | 22 | 88 | 77 | 99 | 66 | 132 | 55 | 33 |
| 8 | 16 | 64 | 56 | 72 | 48 | 96 | 40 | 24 |

- (a) 14 (b) 10 (c) 0 (d) 0 (e) 42
(f) 6 (g) 3 (h) 11 (i) 24 (j) 15
(k) 15

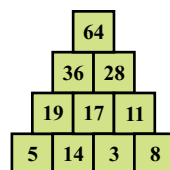
- (a) 2^6 (b) $2^2 \times 3^2$ (c) $2^3 \times 5^2$
(d) $2^4 \times 3^2$ (e) $2^2 \times 3 \times 5^2$
(f) $2 \times 3 \times 13$ (g) $2^3 \times 5^3$
(h) 19×23 (i) $2^3 \times 3^2 \times 13$
(j) 2^{12} (k) 23^2 (l) 3×37

9. 20

$$\begin{array}{r} 8 \quad 7 \quad 9 \quad 1 \quad 6 \\ + \quad 5 \quad 0 \quad 4 \quad 7 \quad 5 \\ \hline 1 \quad 3 \quad 8 \quad 3 \quad 9 \quad 1 \end{array}$$

$$\begin{array}{r} 3 \quad 2 \quad 1 \quad 5 \quad 4 \\ - \quad 2 \quad 8 \quad 6 \quad 4 \quad 9 \\ \hline 3 \quad 5 \quad 0 \quad 5 \end{array}$$

11.



Exercise 1C

- (a) 24 (b) 57 (c) 99 (d) 1666
(e) 79 (f) 295 (g) 954 (h) 3222
(i) 154 (j) 901 (k) 2010 (l) 444
- (a) XXXVI (b) LXXXIV
(c) CCL (d) DCCXCIII
(e) MMMCDVIII (f) MCXI
(g) MMCDXLIX
(h) CMXXXVIII (i) CCCXXXIII
(j) MMMDCCCLXXVII
(k) MMXX (l) DCLXXII

Exercise 1D

- (a) $\frac{5}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{8}$
(e) $\frac{4}{5}$ (f) $\frac{41}{48}$
- (a) $\frac{16}{20}$ (b) $\frac{8}{28}$ (c) $\frac{56}{64}$ (d) $\frac{45}{81}$
(e) $\frac{12}{16} = \frac{15}{20} = \frac{21}{28} = \frac{33}{44} = \frac{27}{36} = \frac{60}{80}$
(f) $\frac{32}{48} = \frac{48}{72} = \frac{4}{6} = \frac{2}{3} = \frac{12}{18} = \frac{14}{21} = \frac{44}{66} = \frac{56}{84}$

3. (a) $\frac{5}{7} (\frac{20}{28})$ $\frac{3}{4} (\frac{21}{28})$
 (b) $\frac{1}{3} (\frac{8}{24})$ $\frac{3}{8} (\frac{9}{24})$ $\frac{5}{12} (\frac{10}{24})$
 (c) $\frac{5}{9} (\frac{20}{36})$ $\frac{7}{12} (\frac{21}{36})$ $\frac{3}{4} (\frac{27}{36})$
 (d) $\frac{5}{8} (\frac{45}{72})$ $\frac{2}{3} (\frac{48}{72})$ $\frac{7}{9} (\frac{56}{72})$
4. (a) Sue ($\frac{8}{21}$) (b) Russell ($\frac{6}{21}$)
5. (a) $\frac{1}{20}$ (b) $\frac{1}{5}$ (c) $\frac{1}{15}$ (d) $\frac{3}{25}$
 (e) $\frac{1}{15}$ (f) $\frac{1}{40}$ (g) $\frac{1}{2}$ (h) $\frac{13}{200}$
 (i) $\frac{1}{4}$
6. (a) $\frac{7}{2}$ (b) $\frac{13}{5}$ (c) $\frac{17}{3}$ (d) $\frac{25}{4}$
 (e) $\frac{29}{6}$ (f) $\frac{71}{8}$ (g) $\frac{66}{7}$ (h) $\frac{91}{12}$
7. (a) $4\frac{1}{2}$ (b) $4\frac{1}{3}$ (c) $5\frac{2}{5}$ (d) $6\frac{3}{10}$
 (e) $8\frac{1}{4}$ (f) $6\frac{5}{6}$ (g) $8\frac{4}{9}$ (h) $6\frac{5}{8}$
 (i) $13\frac{6}{7}$ (j) $24\frac{1}{6}$
8. (a) 4 (b) 8
9. (a) 4 (b) 8 (c) 12 (d) 16
10. (a) 20 kg (b) \$30 (c) 30 m
 (d) 150 L (e) \$210 (f) 30 cm
 (g) 240 kg (h) \$90 (i) 100 g
 (j) \$650 (k) 720 kg (l) 1408 km
11. (a) $\frac{7}{9}$ (b) $\frac{3}{11}$ (c) $\frac{13}{15}$ (d) $\frac{5}{8}$
 (e) $\frac{17}{24}$ (f) $\frac{1}{4}$ (g) $1\frac{4}{5}$ (h) $1\frac{1}{72}$
 (i) $6\frac{1}{10}$ (j) $1\frac{11}{12}$ (k) $7\frac{5}{6}$ (l) $2\frac{19}{20}$
 (m) $14\frac{31}{40}$ (n) $\frac{37}{56}$ (o) $1\frac{3}{8}$
12. (a) $\frac{24}{35}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $6\frac{3}{5}$
 (e) $5\frac{5}{7}$ (f) $11\frac{1}{4}$ (g) $7\frac{1}{2}$ (h) $11\frac{1}{4}$
 (i) $2\frac{1}{2}$ (j) $38\frac{1}{2}$ (k) $58\frac{1}{2}$ (l) 12
13. (a) 10 (b) $\frac{1}{24}$ (c) $\frac{2}{49}$ (d) $\frac{5}{6}$
 (e) $1\frac{1}{2}$ (f) $\frac{2}{3}$ (g) $\frac{1}{12}$ (h) $1\frac{1}{2}$
 (i) $2\frac{1}{3}$
14. (a) 22 (b) 206 (c) 20 (d) $2\frac{4}{5}$ (e) $\frac{3}{8}$

15. (a) $71\frac{1}{2}$ (b) $2\frac{101}{176}$ (c) \$2414
 (d) $1\frac{201}{280}$ (e) $63\frac{89}{162}$ (f) $11\frac{7}{10}$

16. (a) 80 mL (b) 2000 km
 (c) 3600 bricks (d) 5 hours
 (e) 12 minutes (f) 55 seconds
 (g) 5 kg (h) \$2.80

17. (a) \$150 (b) $\frac{5}{12}$
 18. (a) 1000 litres (b) $\frac{1}{3}$

19. $\frac{3}{5}$

20. \$6000

21. Manager - \$1200

Foreman - \$900

Storeman - \$675

22. 96 grams

23. (a) 75 minutes (b) 2:50

Exercise 1E

1. (a) 3.52 (b) 15.421
 2. 1.2
 3. 1234.56789
 4. 3.89
 5. 17.865
 6. (a) 6.26 (b) 1.244 (c) 0.0375
 7. 8.8938
 8. true
 9. 34.7879
 10. 200, 40, 8, 1.6, 0.32, **0.064**, **0.0128**
 11. 0.089
 12. three-hundred and fifty-seven ten-thousandths

13.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{1}{100}$	$\frac{9}{100}$	$\frac{32}{100}$	$\frac{53}{100}$	$\frac{1}{1000}$
Decimal	0.5	0.25	0.75	0.1	0.3	0.7	0.01	0.09	0.32	0.53	0.001

14. (a) 0.07 (b) 0.009 (c) 0.17
 (d) 0.011 (e) 6.9 (f) 12.029
 (g) 3.157 (h) 0.0021 (i) 0.0009
 (j) 23.0579
15. (a) $\frac{3}{1000}$ (b) $\frac{39}{1000}$ (c) $\frac{2357}{10\ 000}$ (d) $\frac{591}{10\ 000}$
 (e) $4\frac{31}{100}$ (f) $\frac{7}{100\ 000}$ (g) $8\frac{92}{10\ 000}$
 (h) $5\frac{2001}{10\ 000}$ (i) $\frac{59}{100\ 000}$ (j) $9\frac{43\ 431}{1\ 000\ 000}$

16. (a) 0.375 (b) 7.125 (c) 0.6
(d) 11.8 (e) 0.55 (f) 0.5625
(g) 6.1875 (h) 11.72 (i) 9.195
(j) 2.826
17. (a) $0.\dot{3}$ (b) $2.8\dot{3}$ (c) $0.\dot{2}$ (d) $13.\overline{27}$
(e) $0.8\dot{6}$ (f) $0.84615\overline{3}$ (g) $5.85714\overline{2}$
(h) $9.90476\overline{1}$ (i) $3.5\overline{28}$ (j) $2.881\dot{6}$
18. (a) $\frac{4}{9}$ (b) $\frac{7}{9}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$
(e) $\frac{5}{11}$ (f) $\frac{4}{15}$ (g) $\frac{4}{11}$ (h) $\frac{5}{37}$ (i) $\frac{29}{55}$
(j) $\frac{757}{1980}$ (k) $\frac{208}{999}$ (l) $\frac{5}{7}$ (m) $\frac{7}{12}$ (n) $\frac{20}{21}$
(o) $\frac{469}{1110}$ (p) $\frac{1525}{3333}$
19. (a) 0.4 (b) 6.7 (c) 0.1 (d) 12.0
(e) 13.9 (f) 0.6 (g) 1.0 (h) 16.0
20. (a) 0.79 (b) 10.51 (c) 9.57
(d) 90.91 (e) 124.57 (f) 6.60
(g) 130.00 (h) 67.91
21. (a) 0.446 (b) 9.605 (c) 0.068
(d) 100.803 (e) 8.106 (f) 45.020
(g) 7.990 (h) 0.100 (i) 56.230
(j) 7.800 (k) 23.402 (l) 2.000
(m) 4.700 (n) 30.000 (o) 25.001
(p) 5.801
22. (a) 5.43×10^6 (b) 6.0901×10^7
(c) 2.3×10^8 (d) 4.1×10^3
(e) 7×10^9 (f) 6×10^{-4}
(g) 6.754×10^{-5} (h) 7.845×10^{-4}
(i) 4.1×10^{-8} (j) 5.6×10^{10}
(k) 7.7×10^{-8} (l) 8.971×10^{-4}
23. (a) 1.9891×10^{30} kg
(b) 1.813×10^8 km²
(c) 4.4493×10^7 km²
(d) 9.109×10^{31} kg
(e) 1.672×10^{-27} kg
24. (a) 8794.2 (b) 566 000 (c) 0.6
(d) 7.85 (e) 8900 (f) 45 000
(g) 0.0452 (h) 45.046 (i) 0.3
(j) 1.040 26 (k) 0.000 567 8
(l) 0.0005
25. (a) $\frac{2}{3}$ (b) $\frac{4}{7}$ (c) $\frac{5}{9}$ (d) $\frac{2}{11}$
(e) $\frac{75}{121}$ (f) $\frac{1}{49}$ (g) $\frac{491}{773}$ (h) $\frac{3}{95}$ (i) $\frac{713}{2161}$

26. (a) $\frac{5}{9}$ (b) $\frac{4}{9}$

27. $\frac{5}{11}$

28. 0.359 litres

29. 58.032 seconds

30. (a) 0.714 seconds (b) 48 flashes

31. 200 000 years

32. 10.25 mm

33. (a)(i) 0.075 mm (ii) 0.9 mm
(iii) 0.015 mm

(b)(i) 8 turns (ii) 0.04 turns
(iii) 0.6 turns

Exercise 1F

1. (a) -2 (b) 3 (c) -11 (d) -8 (e) -12
(f) -2 (g) -3 (h) 17 (i) -4 (j) -21
(k) 9 (l) 10 (m) -2 (n) 13 (o) 36
(p) 2
2. (a) -12 (b) -24 (c) 18 (d) -20
(e) -42 (f) -36 (g) 18 (h) -55
(i) -6 (j) -3 (k) 5 (l) -4 (m) 9
(n) -4 (o) -3 (p) -4
3. (a)

+	6	-2	-9	-13	15	27
-8	-2	-10	-17	-21	7	19
11	17	9	2	-2	26	38
-17	-11	-19	-26	-30	-2	10
22	28	20	13	9	37	49
-30	-24	-32	-39	-43	-15	-3
16	22	14	7	3	31	43

(b)

×	5	-8	3	-6	9	-12
-2	-10	16	-6	12	-18	24
-7	-35	56	-21	42	-63	84
4	20	-32	12	-24	36	-48
-11	-55	88	-33	66	-99	132
8	40	-64	24	-48	72	-96
5	25	-40	15	-30	45	-60

4. (a) 13 (b) 5 (c) -14 (d) 18
(e) -25 (f) 14 (g) -1 (h) -1
(i) -6 (j) -5 (k) -3 (l) -5 (m) -1
(n) -8 (o) 9 (p) 10 000 (q) -4
(r) -7

5. (a) -3 (b) -3 (c) -5 (d) 4 (e) -10
(f) -8 (g) -8 (h) -5 (i) -12 (j) 9
(k) -2 (l) 10 (m) -2 (n) -4 (o) 1
(p) -2 (q) -1 (r) -8
6. (a) -4 (b) -9 (c) -3 (d) 3 (e) -4
(f) -9 (g) -2 (h) -2.5
7. (a) -0.8 (b) -1.5 (c) -6.8 (d) -0.5
(e) 0.8 (f) -2.4 (g) -8 (h) -9.5
(i) 3.6 (j) -4.1 (k) -1.7 (l) -4.7
(m) -7.4 (n) -1.3 (o) 3.6
8. (a) -7 (b) -10 (c) 9 (d) 1 (e) -12
(f) 10 (g) 120 (h) 2 (i) 144
(j) 4 (k) -11 (l) 64 (m) -3 (n) 0
(o) 5

9.

1	-4	19	-2
13	4	-5	2
-12	9	6	11
12	5	-6	3

10. $A = -4$, $B = -8$, $C = 9$

11. 95 km south

Exercise 1G

1. 12 laps
2. 12 kg
3. Pixie - 2.3 kg
Hubbsie - 5.6 kg
Footsie - 4.7 kg
4. (a) 3.4 kg (b) 100 small bricks
5. 44.1 sec
6. 3.2 tonnes
7. $\frac{4}{5}$
8. 5.4 m
9. 7.6 MW
10. 1.8 tonnes
11. (a) $\frac{35}{47}$ (b) 35 weeks
12. Pole 1 - 9.139 m
Pole 2 - 9.494 m
Pole 3 - 6.727 m
Pole 4 - 7.795 m
Pole 5 - 11.145 m

Chapter Review

1. (a) T (b) F (c) T (d) T (e) T
(f) T (g) F (h) T
2. (a) $\{5, -7, \sqrt{8}, \frac{9}{7}, -\frac{5}{3}, 7\pi, 8.2, -5.6\} \subset R$
(b) $\{5, -7, \frac{9}{7}, -\frac{5}{3}, 8.2, -5.6\} \subset Q$
(c) $\{5\} \subset N$
(d) $\{5\} \subset W$
(e) $\{5, -7\} \subset Z$
(f) $\{\sqrt{8}, 7\pi\} \subset I$
3. (a) 1056 (b) 9254 (c) 20 943
(d) 24 600 104
4. (a) 7731 (b) 945 (c) 120 619
(d) 499 903
5. (a) 32 (b) 12 (c) 56 (d) 26
(e) -3 (f) 1
6. $2^3 \times 3^2 \times 5^2$
7. (a) 121 (b) 464 (c) 247 (d) 2764
8. (a) CCLVI (b) DCCLXXXIV
(c) MCDXLII (d) MMDCCCXXXIV
9. $\frac{28}{36} = \frac{14}{18} = \frac{7}{9} = \frac{35}{45} = \frac{21}{27} = \frac{63}{81} = \frac{42}{54}$
10. $\frac{3}{8} = \frac{27}{72}$ $\frac{4}{9} = \frac{32}{72}$ $\frac{5}{6} = \frac{60}{72}$
11. (a) $\frac{1}{25}$ (b) $\frac{13}{500}$ (c) $\frac{7}{17}$
12. $\frac{27}{4}$
13. $8\frac{3}{7}$
14. (a) 36 kg (b) \$1550 (c) 287.42 m
15. (a) $\frac{8}{11}$ (b) $\frac{1}{4}$ (c) $\frac{35}{36}$ (d) $5\frac{17}{20}$
(e) $1\frac{24}{35}$ (f) $5\frac{7}{12}$ (g) $\frac{8}{35}$ (h) $\frac{3}{10}$
(i) 15 (j) 12 (k) $\frac{7}{10}$ (l) $\frac{34}{35}$
16. $1\frac{4991}{5800}$
17. 1000 mL
18. 18 km
19. (a) 3.9195 (b) 6.941 (c) $2\frac{673}{10\,000}$
(d) 6.878 (e) 4.6 (f) 6.875
20. 4.285714
21. (a) $\frac{38}{99}$ (b) $\frac{41}{90}$
22. (a) 4.57 (b) 6.01 (c) 0.90
(d) 34.29
23. (a) 89.1 (b) 7.9 (c) 12.4 (d) 1.0

24. (a) 3.45×10^7 (b) 8.9×10^6
 (c) 6.7×10^{-5} (d) 2×10^{-8}
 25. (a) 678 (b) 0.056
 26. (a) 56.718 sec (b) 1.892 L
 27. (a) -4 (b) -3 (c) 13 (d) -18
 (e) -5 (f) -3.6 (g) 1.1 (h) 5.5
 (i) 48 (j) -4 (k) 14 (l) -8
 28. (a) -24 (b) 2 (c) -8
 29. 17 minutes

Chapter 2

Exercise 2A

1. (a) **C** (b) **D** (c) **C**
 2. (a) 96.62% (b) 6.8%
 3. 39.2%
 4. Romance (12%), Sci-Fi (22%),
 Comedy (28%), Action (38%)

Exercise 2B

1. (a) 10% (b) 30% (c) 25% (d) 50%
 (e) 20% (f) 75% (g) 35% (h) 26%
 (i) 48% (j) 12.5% (k) 43.75%
 (l) 118.75% (m) 285% (n) 137.5%
 (o) 325.6%
 2. (a) $\frac{11}{50}$ (b) $\frac{19}{20}$ (c) $\frac{3}{25}$ (d) $1\frac{1}{5}$ (e) $\frac{1}{50}$
 (f) $\frac{31}{200}$ (g) $\frac{5}{8}$ (h) $\frac{83}{400}$ (i) $\frac{1}{500}$ (j) $\frac{53}{1000}$
 3. (a) 45% (b) 56.25% (c) 15.625%
 4. (a) 33% (b) 19% (c) 87% (d) 6%
 (e) 40% (f) 29.5% (g) 5.6%
 (h) 0.8% (i) 200% (j) 10.57%
 5. (a) 0.59 (b) 0.24 (c) 0.762 (d) 0.3
 (e) 0.08 (f) 0.0125 (g) 1 (h) 0.654
 (i) 0.058 (j) 0.0025

6.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
10%	$\frac{1}{10}$	0.1
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
90%	$\frac{9}{10}$	0.9
66%	$\frac{2}{3}$	0.66
18%	$\frac{9}{50}$	0.18
32%	$\frac{8}{25}$	0.32
85%	$\frac{17}{20}$	0.85
9%	$\frac{9}{100}$	0.09
3%	$\frac{3}{100}$	0.03
4.5%	$\frac{9}{200}$	0.045
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\bar{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\bar{6}$
18.75%	$\frac{3}{16}$	0.1875
68.75%	$\frac{11}{16}$	0.6875
55.5%	$\frac{111}{200}$	0.555
0.22%	$\frac{11}{5000}$	0.0022

Exercise 2C

1. (a) 40% (b) 25% (c) 9% (d) 5%
 (e) 20% (f) 5% (g) 60% (h) 10%
 (i) 25% (j) 8% (k) 19% (l) 27%
 (m) 128% (n) 130% (o) 120%
 (p) 144% (q) 200% (r) 175%
 2. (a) 66.4% (b) 25.6% (c) 8.0%
 (d) 75.7% (e) 35.0% (f) 17.5%
 (g) 25.7% (h) 427.0% (i) 223.8%
 (j) 90.7% (k) 269.2% (l) 66.7%
 3. (a) Sample **A** - 0.207%
 Sample **B** - 0.202%
 (b) Sample **A**
 4. 11.8%
 5. (a) 74.5% (b) 70.8%
 (c) 0-4 (6.3%), 5-14 (13.5%),
 15-24 (13.6%), 25-54 (42.2%),
 55-64 (11%), 65- (13.4%)
 6. (a) 93.2% (b) 57.3%
 7. (a) 4.8% (b) 33.3%
 8. (a) 7.6%
 (b) (i) 6.2% (ii) 20.8% (iii) 31.5%
 (c) 15.2 days

Exercise 2D

1. (a) 4 m (b) \$10 (c) \$5 (d) 3 kg
 (e) 6 kg (f) 9 m (g) \$60 (h) \$100
 (i) 10 km (j) 25 kg (k) \$4000
 (l) 100 kg (m) \$2.50 (n) 30c
 (o) \$2.60

2. (a) \$279 (b) 161.5 m (c) \$593.40
(d) \$1128 (e) 3013.5 m
(f) 0.385 m (g) \$48.75 (h) \$3570
(i) \$10 799.25

3. \$210

4. \$49 536

5. 875 hectares

6. 144 648.6 hectares

7. 50 625 m²

8. Republican 136°

Democratic 156°

Undecided 68°

9. (a) \$14.95 (b) \$14.59 (c) \$15.29

10. (a)(i) 70% (ii) 85% (iii) 92%

(iv) 72.5% (v) 93.6%

(b)(i) 120% (ii) 117% (iii) 106%

(iv) 145.6% (v) 103.8%

11. (a) \$212.50 (b) \$700 (c) \$66

(d) \$48.75

12. 66.3 cm³

13. 3003 kg

14. 15.66 km/L

15. (a) 56% (b) 33% (c) 58.5%

(d) 96% (e) 23.4% (f) 96.32%

(g) 3.68% (h) 3.25% (i) 21.6%

(j) 36.4% (k) 38.665% (l) 85.8%

16. (a) \$49 (b) 416 kg (c) 2618 L

(d) \$11 088 (e) 525 kg

(f) \$69 596

17. \$288

18. (a) no (b) 44%

19. (a) 84.8% (b) \$12 720

20. 1 728 000 kg

21. No. There is only 99% of the
amount of biscuits in each packet.

22. The overall income will be the
same.

Exercise 2E

1. (a) \$500 (b) 225 kg (c) \$1250
(d) 625 g (e) \$7000 (f) \$800

2. 12 000 litres

3. 500 seconds

4. \$2200

5. 30 000 kg

6. 2575 seats

7. 35 hours

Exercise 2F

1. (a) \$420 (b) \$81 (c) \$2200

(d) \$2868.75 (e) \$270 (f) \$400

2. (a)(i) \$600 (ii) \$4600

(b)(i) \$720 (ii) \$12 720

(c)(i) \$1562.50 (ii) \$4062.50

(d)(i) \$2220 (ii) \$4070

(e)(i) \$5250 (ii) \$25 250

(f)(i) \$268.13 (ii) \$6768.13

Exercise 2G

1. (a)(i) \$4410 (ii) \$410

(b)(i) \$2519.42 (ii) \$519.42

(c)(i) \$14 907.56 (ii) \$2907.56

(d)(i) \$62 940.41 (ii) \$17 940.41

3. \$1852.92

4. (a) \$605.20 (b) \$1224.91

(c) \$2042.36

5. (a) \$7346.64 (b) \$7429.74

(c) \$7449.23 (d) \$7458.80

6. The shorter the compounding time
the larger the final amount.

7. \$11 646.42

8. (a) \$3155.39 (b) \$3177.29

(c) \$3195.36 (d) \$3187.70

Exercise 2H

1. (a) 8.2% (b) 4.8% (c) 3.7%

(d) 17.6% (e) 3.0%

2. 4.9%

3. 18.7%

4. 9.6%

Chapter Review

1. 7.5%

2. 21.6%

3. (a)(i) 80% (ii) 62.5%

(b)(i) 76.5% (ii) 2.5%

(c)(i) $\frac{9}{20}$ (ii) $\frac{41}{500}$ (iii) $\frac{63}{400}$

(d)(i) 0.068 (ii) 0.841

4. (a) 6.25% (b) 20% (c) 64%
 5. (a) 56.5% (b) 14.1% (c) 113.2%
 6. 53.4%
 7. 13.1%
 8. (a) \$210 (b) 64 kg (c) 32.4 m
 (d) 100 m (e) \$32 (f) 42 kg
 9. (a) 124.8 kg (b) \$292.50
 (c) \$2064
 10. \$46 740
 11. (a) \$545.60 (b) \$3465
 12. 6514
 13. (a) 12% (b) 114 students
 14. 800 litres
 15. 15 000 tonnes
 16. \$700
 17. \$12 029.25
 18. \$1262.39
 19. 15.0%

Chapter 3

Exercise 3A

1. (a) 9.76, 0.5, $\sqrt{25}$, $0.\dot{6}$, $\frac{7}{17}$, $0.75\overline{32}$,
 -3.7311 , $\sqrt{529}$
 (b) $\frac{\pi}{2}$, $\sqrt{11}$
 2. (a) 1.41421 (b) 2.23607
 (c) 5.38516 (d) 12.49000
 (e) 2.55391 (f) 1.45547
 (g) 7.12504 (h) 8.45556
 3. (a) 13 (b) 6.5 (c) 15.6 (d) 58 (e) $\frac{29}{37}$
 4. (a) 7 (b) 10 (c) 8 (d) 6 (e) 2
 (f) 5 (g) 9 (h) 12 (i) 11 (j) 4
 (k) 20 (l) $\frac{4}{5}$ (m) $\frac{7}{8}$ (n) $\frac{9}{10}$ (o) $\frac{6}{13}$
 5. (a) 5.5 (b) 9.5 (c) 2.2 (d) 10.2
 (e) 11.6 (f) 7.5 (g) 8.8 (h) 3.7
 (i) 4.9 (j) 6.6

Exercise 3B

1. (a) 56 (b) 50 (c) 72 (d) 18 (e) 33
 (f) 20 (g) 50 (h) 90 (i) 200 (j) 132
 (k) 900 (l) 2304
 2. (a) $\frac{7}{8}$ (b) $\frac{3}{10}$ (c) $\frac{2}{11}$ (d) $\frac{6}{13}$

3. (a) $2\sqrt{5}$ (b) $3\sqrt{3}$ (c) $10\sqrt{2}$ (d) $3\sqrt{5}$
 (e) $5\sqrt{2}$ (f) $4\sqrt{3}$ (g) $2\sqrt{14}$ (h) $7\sqrt{2}$
 (i) $5\sqrt{3}$ (j) $4\sqrt{5}$ (k) $5\sqrt{5}$ (l) $6\sqrt{6}$
 (m) $6\sqrt{5}$ (n) $9\sqrt{5}$ (o) $9\sqrt{6}$ (p) $10\sqrt{7}$
 (q) $6\sqrt{10}$ (r) $30\sqrt{10}$ (s) $20\sqrt{2}$
 (t) $40\sqrt{3}$ (u) $21\sqrt{5}$ (v) $48\sqrt{7}$
 (w) $60\sqrt{13}$ (x) $126\sqrt{3}$
 4. (a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{500}$ (d) $\sqrt{150}$
 (e) $\sqrt{40}$ (f) $\sqrt{294}$ (g) $\sqrt{99}$ (h) $\sqrt{72}$
 (i) $\sqrt{112}$ (j) $\sqrt{128}$ (k) $\sqrt{432}$ (l) $\sqrt{175}$
 (m) $\sqrt{68}$ (n) $\sqrt{270}$ (o) $\sqrt{176}$
 (p) $\sqrt{243}$
 5. (a) $\sqrt{15}$ (b) $\sqrt{30}$ (c) $3\sqrt{10}$
 (d) $5\sqrt{70}$ (e) 5 (f) 36 (g) $15\sqrt{21}$
 (h) $12\sqrt{33}$ (i) $2\sqrt{3}$ (j) $8\sqrt{5}$
 (k) $30\sqrt{2}$ (l) $3\sqrt{5}$ (m) $18\sqrt{7}$
 (n) $30\sqrt{2}$ (o) 105 (p) $10\sqrt{3}$
 (q) $42\sqrt{2}$ (r) 60
 6. (a) $24\sqrt{15}$ (b) $180\sqrt{6}$ (c) $240\sqrt{5}$
 (d) $40\sqrt{15}$ (e) 2160 (f) $900\sqrt{30}$
 (g) $54\sqrt{55}$ (h) $360\sqrt{30}$ (i) $640\sqrt{10}$
 (j) $100\sqrt{30}$ (k) $1350\sqrt{6}$ (l) 3080
 7. (a) 2 (b) 10 (c) $\frac{2\sqrt{2}}{\sqrt{5}}$ (d) $4\sqrt{17}$
 (e) $5\sqrt{5}$ (f) 2 (g) 9 (h) $\frac{\sqrt{5}}{\sqrt{2}}$
 8. (a) $15\sqrt{3}$ (b) $6\sqrt{5}$ (c) $3\sqrt{7}$
 (d) $-2\sqrt{7}$ (e) $2\sqrt{11}$ (f) $10\sqrt{5}$
 (g) $3\sqrt{6}$ (h) $4\sqrt{10}$ (i) 0 (j) $-2\sqrt{3}$
 (k) 0 (l) $-2\sqrt{3}$ (m) $5\sqrt{3} + 3\sqrt{5}$
 (n) $\sqrt{7} + 9\sqrt{3}$ (o) $4\sqrt{5} + 3\sqrt{3}$
 (p) $\sqrt{11} + 7\sqrt{7}$ (q) $3\sqrt{2} + 9\sqrt{5}$
 (r) $3\sqrt{3} - \sqrt{7}$ (s) $5\sqrt{5}$ (t) $3\sqrt{3}$
 (u) $-\sqrt{7}$ (v) 0 (w) $2\sqrt{2}$ (x) $5\sqrt{5}$
 (y) $-\sqrt{11} + 3\sqrt{7}$ (z) $-5\sqrt{7}$
 9. (a) $16\sqrt{7}$ (b) 105
 10. (a) $16\sqrt{3} + 16\sqrt{5}$ (b) $43\sqrt{15}$

Exercise 3C

1. (a) $2\sqrt{5} + 2\sqrt{3}$ (b) $3\sqrt{7} - 3\sqrt{2}$
 (c) $\sqrt{6} - \sqrt{33}$ (d) $\sqrt{10} + \sqrt{14}$
 (e) $\sqrt{30} + 5$ (f) $2\sqrt{3} - 3\sqrt{2}$
 (g) $4\sqrt{3} + 3$ (h) $5\sqrt{2} - 10$
 (i) $4 - 2\sqrt{11}$ (j) $2\sqrt{3} + 6\sqrt{5}$
 (k) $9 + \sqrt{15}$ (l) $2\sqrt{35} + \sqrt{15}$
 (m) $2\sqrt{21} + 7\sqrt{2}$ (n) $20 + 6\sqrt{3}$
 (o) $4\sqrt{15} - 3\sqrt{35}$ (p) $6\sqrt{5} + 10\sqrt{3}$
 (q) $3\sqrt{10} + 3\sqrt{14}$ (r) $12 + 8\sqrt{15}$
 (s) $-10\sqrt{7} - 105$ (t) $-14\sqrt{15} + 42$
 (u) $30 - 9\sqrt{35}$ (v) $14\sqrt{6}$
 (w) $15\sqrt{6} + 15\sqrt{15}$ (x) $42\sqrt{2} - 84$
2. (a) $\sqrt{15} + \sqrt{6} + \sqrt{10} + 2$ (b) $17 + 7\sqrt{5}$
 (c) $\sqrt{14} + 7 + \sqrt{6} + \sqrt{21}$ (d) $8\sqrt{3} + 15$
 (e) $17 - \sqrt{3}$ (f) $3\sqrt{3} - 3\sqrt{2} + \sqrt{6} - 2$
 (g) $\sqrt{6} + \sqrt{15} - 2 - \sqrt{10}$
 (h) $\sqrt{6} - 2 - \sqrt{15} + \sqrt{10}$
 (i) $\sqrt{21} - \sqrt{6} - 7 + \sqrt{14}$ (j) $5\sqrt{5} - 11$
 (k) 33 (l) 2
 (m) $30 + 2\sqrt{10} + 3\sqrt{15} + \sqrt{6}$
 (n) $20 + 5\sqrt{6}$
 (o) $30 - 6\sqrt{5} + 2\sqrt{35} - 2\sqrt{7}$
 (p) $2\sqrt{5} - 22$ (q) $18 - 5\sqrt{10}$
 (r) $72 - 13\sqrt{35}$ (s) $-34 - 14\sqrt{21}$
 (t) $40 - 11\sqrt{10}$ (u) -33 (v) -13
 (w) -27 (x) 397

Exercise 3D

1. (a) $\frac{3\sqrt{7}}{7}$ (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{4\sqrt{5}}{5}$
 (d) $\frac{5\sqrt{3}}{3}$ (e) $\frac{2\sqrt{11}}{11}$ (f) $\frac{2\sqrt{35}}{5}$
 (g) $\frac{3\sqrt{10}}{2}$ (h) $\frac{5\sqrt{6}}{3}$ (i) $\frac{4\sqrt{30}}{5}$
 (j) $\sqrt{15}$
2. (a) $\frac{3\sqrt{2}}{2}$ (b) $\sqrt{3}$ (c) $\frac{3\sqrt{2}}{2}$
 (d) $\frac{2\sqrt{15}}{3}$ (e) $3\sqrt{6}$

3. (a) $\sqrt{10}$ (b) $\frac{3\sqrt{35}}{10}$ (c) $\frac{2\sqrt{30}}{9}$
 (d) $\frac{5\sqrt{21}}{9}$ (e) $\frac{10}{3}$ (f) $\frac{2\sqrt{21}}{5}$
 (g) $\frac{4\sqrt{10}}{5}$ (h) $\frac{\sqrt{21}}{7}$ (i) $\frac{2\sqrt{15}}{5}$
 (j) $\frac{7}{2}$
4. (a) $\frac{6 + \sqrt{15}}{9}$ (b) $\frac{5\sqrt{7} - 3\sqrt{14}}{14}$
 (c) $\frac{5\sqrt{77} + \sqrt{33}}{55}$ (d) $\frac{5\sqrt{10} - 6}{14}$
 (e) $\frac{6 + 7\sqrt{22}}{6}$ (f) $\frac{5\sqrt{35} - 15}{10}$
 (g) $\frac{8\sqrt{6} + 3\sqrt{15}}{12}$ (h) $\frac{4\sqrt{35} - 21}{14}$
 (i) $\frac{7\sqrt{5} + 6}{2}$
5. (a) $\frac{25 - 5\sqrt{3}}{22}$ (b) $\frac{2\sqrt{7} + 4}{3}$
 (c) $\frac{36 - 4\sqrt{2}}{79}$ (d) $\frac{5\sqrt{7} - \sqrt{14}}{23}$
 (e) $\frac{3\sqrt{10} + \sqrt{20}}{7}$ (f) $\frac{7\sqrt{11} - \sqrt{55}}{44}$
 (g) $\frac{5\sqrt{7} + 5\sqrt{3}}{4}$ (h) $\frac{4\sqrt{11} - 4\sqrt{5}}{3}$
 (i) $\frac{\sqrt{13} + \sqrt{7}}{6}$ (j) $\frac{2\sqrt{5} + 2\sqrt{3} - \sqrt{15} - 3}{2}$
 (k) $\frac{\sqrt{21} - \sqrt{6} + 4\sqrt{7} - 4\sqrt{2}}{5}$
 (l) $\frac{8\sqrt{11} - 8\sqrt{5} - \sqrt{77} + \sqrt{35}}{6}$
 (m) $\frac{\sqrt{35} + \sqrt{21} + 5 + \sqrt{15}}{2}$
 (n) $\frac{\sqrt{14} - \sqrt{10} - \sqrt{21} + \sqrt{15}}{2}$
 (o) $\frac{3\sqrt{33} + 3\sqrt{21} - \sqrt{55} - \sqrt{35}}{4}$
 (p) $\frac{2\sqrt{14} + 2\sqrt{6} + 3\sqrt{21} + 9}{4}$
 (q) $\frac{7\sqrt{10} + 23}{3}$ (r) $\frac{15\sqrt{15} - 63}{2}$

6. (a) $\frac{9\sqrt{5} - 6\sqrt{2} + 15 - 2\sqrt{10}}{37}$
 (b) $\frac{42\sqrt{2} + 18\sqrt{3} - 14 - 3\sqrt{6}}{71}$
 (c) $\frac{15 - 2\sqrt{15} + 27\sqrt{5} - 18\sqrt{3}}{33}$
 (d) $\frac{5\sqrt{10} + 2\sqrt{6} - 5\sqrt{15} - 6}{113}$
 (e) $\frac{2\sqrt{21} - 6 - 2\sqrt{35} - 2\sqrt{15}}{16}$
 (f) $\frac{1}{2}$
 (g) $\frac{6\sqrt{35} + 4\sqrt{21} + 45 + 6\sqrt{15}}{33}$
 (h) $\frac{19\sqrt{6} - 57}{57}$ (i) $\frac{2\sqrt{15} - 31}{17}$
 (j) $\frac{6\sqrt{14} + 8\sqrt{6} + 9\sqrt{21} + 36}{15}$
 (k) $\frac{25\sqrt{10} + 84}{62}$ (l) $\frac{93\sqrt{15} - 450}{245}$

Chapter Review

1. $\sqrt{89}$ $\frac{\pi}{5}$ $\sqrt{124}$
 2. (a) $5\sqrt{2}$ (b) $4\sqrt{6}$ (c) $9\sqrt{3}$ (d) $21\sqrt{5}$
 3. (a) $\sqrt{125}$ (b) $\sqrt{54}$ (c) $\sqrt{300}$ (d) $\sqrt{112}$
 4. (a) $3\sqrt{30}$ (b) $12\sqrt{15}$ (c) $30\sqrt{10}$
 (d) $36\sqrt{10}$ (e) 6 (f) $\frac{10\sqrt{3}}{\sqrt{7}}$
 5. (a) $5\sqrt{5}$ (b) $3\sqrt{7}$ (c) $-3\sqrt{2}$
 (d) $3\sqrt{3} + 6\sqrt{7}$ (e) $-\sqrt{3}$ (f) $-2\sqrt{5}$
 6. (a) $5\sqrt{3} + 5\sqrt{5}$ (b) $5 - \sqrt{10}$
 (c) $2\sqrt{15} + 9$ (d) $6\sqrt{35} - 45$
 (e) $\sqrt{6} - \sqrt{15} + 2 - \sqrt{10}$ (f) $12 - \sqrt{14}$
 7. (a) $\frac{2\sqrt{3}}{3}$ (b) $\frac{3\sqrt{14}}{7}$ (c) $\frac{5\sqrt{77}}{33}$
 (d) $\frac{\sqrt{10} - 3}{5}$ (e) $\frac{6 + 2\sqrt{22}}{10}$
 (f) $\frac{12\sqrt{5} - 15}{11}$
 (g) $\frac{2\sqrt{14} - 2\sqrt{10} + 3\sqrt{35} - 15}{2}$
 (h) $\frac{8\sqrt{35} + 20\sqrt{5} - 42 - 15\sqrt{7}}{3}$
 (i) $\frac{26\sqrt{10} - 75}{5}$

Chapter 4

Exercise 4A

1. (a) 7^4 (b) 9^6 (c) 2^5 (d) 12^5
 (e) $(-3)^4$ (f) $(-8)^5$ (g) y^3 (h) a^9
 (i) m^2 (j) $(-p)^6$ (k) 5^7 (l) 7^4
 (m) 10^{15} (n) k^{12} (o) a^{10} (p) x^3
 2. (a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$
 (b) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
 (c) $10 \times 10 \times 10 \times 10$
 (d) $8 \times 8 \times 8$ (e) $a \times a \times a \times a$
 (f) $x \times x \times x \times x \times x \times x \times x$
 (g) $y \times y \times y \times y \times y \times y$
 (h) $b \times b \times b \times b \times b \times b \times b \times b \times b \times b$
 (i) $7 \times a \times a \times b \times b \times b \times b \times b \times b$
 (j) $3 \times x \times x \times x \times x \times x \times y \times y \times y \times y$
 (k) $2 \times m \times m \times n$
 (l) $3 \times a \times a \times b \times b \times b \times b \times c \times c \times c \times c$
 3. (a) $4a^2b^3$ (b) $15x^4y^3$ (c) $16m^4n^5$
 (d) $30a^3b^3c^5$ (e) $120x^3y^2$
 4. (a) 10 000 (b) 512 (c) 16 807
 (d) 625 (e) -7776 (f) -512
 (g) 671 (h) 37 (i) 584 (j) 0
 (k) -4160 (l) -867 074
 5. (a) 3989.5 (b) -9.6 (c) 122.4
 (d) 320.5 (e) -1.0 (f) 0.9
 6. (a) 4 (b) 7 (c) 4 (d) 3 (e) 8 (f) 6

Exercise 4B

1. (a) a^8 (b) n^8 (c) y^{11} (d) b^7 (e) m^9
 (f) x^{10} (g) a^{14} (h) x^{16} (i) y^9
 2. (a) $18a^8$ (b) $8x^{11}$ (c) $6m^{10}$
 (d) $12n^{18}$ (e) $2a^5b^7$ (f) $6x^4y^7$
 (g) $12a^7y^7$ (h) $14a^7b^9c^2$
 (i) $12m^8n^8$ (j) $12x^9y^6z^8$
 (k) $24a^{10}b^4c^8$ (l) $120m^{17}n^8$

Exercise 4C

1. (a) x^3 (b) a^3 (c) n^6 (d) m^7 (e) b^3
 (f) y^5 (g) p^6 (h) w (i) m^7 (j) x^9
 (k) n^{14} (l) m

2. (a) $2x^3y^5$ (b) $4m^3n^6$ (c) $3a^3b^3$
 (d) $\frac{4}{3}m^2n^3$ (e) $\frac{2}{3}x^3y^3$ (f) $\frac{4}{3}a^3b^3$
 (g) $\frac{3}{2}x^2y^5$ (h) $\frac{5}{6}m^7n$ (i) $\frac{5}{4}a^3b^{11}$
 (j) $2x^3y$ (k) $\frac{6}{5}m^7n^{11}$ (l) $\frac{6}{7}ab^9$
 (m) $3m^2n^6$ (n) $\frac{2}{3}x^6y$ (o) $3mn$
 (p) $\frac{2}{3}hi^5$ (q) $\frac{1}{2}a^6$ (r) $\frac{4}{3}ac$

Exercise 4D

1. (a) m^{12} (b) x^{15} (c) a^{24} (d) c^{18}
 (e) p^{30} (f) b^{21} (g) n^{36} (h) y^{20}
 2. (a) $9x^4$ (b) $16m^{12}$ (c) $27m^{15}$
 (d) $16n^{12}$ (e) $36c^{14}$ (f) $-32n^{20}$
 (g) $-27a^{12}$ (h) $-81u^{24}$ (i) $216a^9$
 (j) $-125y^{15}$ (k) $-64c^8$ (l) $1024y^{15}$

Exercise 4E

1. (a) a^8b^{12} (b) x^8y^{14} (c) p^8q^{36}
 (d) $u^{30}v^{18}$ (e) $m^{25}n^{40}o^{30}$
 (f) $a^{18}b^9c^{27}$ (g) $9x^6y^{10}$ (h) $16a^6b^2$
 (i) $-125m^3n^{18}$ (j) $64a^{10}b^6$
 (k) $64a^{24}b^6c^{18}$ (l) $81p^8q^{14}r^6$
 (m) $-8m^9n^{18}$ (n) $-81x^{16}y^{20}$
 (o) $-125u^{18}v^{21}$ (p) a^9b^{12}
 (q) $-256m^{24}n^{12}$ (r) $128a^{21}b^{14}$
 (s) $216x^{12}y^3z^{15}$ (t) $2401a^{16}b^{12}c^4$
 (u) $3125p^{15}q^{35}r^{40}$
 2. (a) a^{15}/b^6 (b) m^{14}/n^6 (c) x^{21}/y^{15}
 (d) m^{10}/n^8 (e) a^{18}/b^{21} (f) e^{20}/r^{10}
 (g) $m^{12}n^{16}/p^{16}q^{12}$ (h) $a^{12}b^6/x^4y^{16}$
 (i) $m^{30}n^{18}/n^{24}q^{12}$ (j) $8a^9b^{12}/m^6n^{15}$
 (k) $x^{12}/27y^9z^{15}$ (l) $16m^4n^{10}/p^{10}q^8$
 (m) $64m^9n^{18}/27a^6b^3$ (n) $25x^6y^8/36z^{14}$
 (o) $9a^8b^{18}/49x^{10}y^{14}$ (p) $8a^3b^3c^9/27x^6y^9z^{12}$
 (q) $625/m^8n^8p^8$ (r) $343m^3n^{21}/1000p^{12}$

Exercise 4F

1. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 1
 (g) 1 (h) 1
 2. (a) 5 (b) 3 (c) 4 (d) $2a$ (e) 2
 (f) 3 (g) 9 (h) 5 (i) 3 (j) $7x^2$
 (k) 4 (l) 7

3. (a) 1 (b) z^2 (c) $2m^3$ (d) $4b^6/5$
 (e) 2 (f) $2m/3$ (g) $3y^2z/4$ (h) $2ab$

Exercise 4G

1. (a) $1/x^8$ (b) $1/3^4$ (c) $1/a^2$ (d) $1/e^{10}$
 (e) $1/c$ (f) $3/m^4$ (g) x/y^2 (h) $4m/n^7$
 (i) $6x^2/y^3$ (j) $8b^9/a^2$
 2. (a) $3m^2/n^6$ (b) $7a^3/b^5$ (c) a^4b^3/c^6
 (d) $6y^3/x^5$ (e) $3m^2/4n^3$ (f) $5a^3/7b^4$
 (g) $y^3/2x^4$ (h) $x^3y/3z^2$ (i) $4a^5c/9b^3$
 (j) $2p^4/7q^3r^2$ (k) $3/8a^2b^3c^2$
 (l) $w^2/5x^7v^3$

Exercise 4H

1. (a) x^8 (b) m^7 (c) c^3 (d) a^5b^5
 (e) m^5n^8 (f) $3x^4$ (g) $2m^2$ (h) $9x^9$
 (i) $3c^3/2$ (j) $b^2/4$ (k) $5x^2/3$ (l) 6
 (m) $8x^3$ (n) $3m^3/2$ (o) $8x^{15}/5$
 2. (a) x^5y^{10} (b) m (c) $a^{13}b^9$ (d) $4x^{10}y^9$
 (e) $4mn^2/3$ (f) $6x^5y^7$ (g) $x^9y^{16}/6$
 (h) $3n^4/2$ (i) $2ab^3/3$ (j) $4x^4y^4$
 (k) $3m^7n^2/8$ (l) 2
 3. (a) $5x^3/7$ (b) $6c^4$ (c) $8a^3/3$ (d) $6a^5$
 (e) $x^6y^2/6$ (f) 1 (g) $2xy^2$ (h) $2x^9y^{13}$
 (i) $8a^3$ (j) $3m^6n^5/4$

Exercise 4I

1. (a) $x^{\frac{1}{5}}$ (b) $a^{\frac{1}{7}}$ (c) $m^{\frac{1}{8}}$ (d) $z^{\frac{1}{12}}$
 (e) $n^{\frac{1}{a}}$ (f) $x^{\frac{1}{y}}$
 2. (a) $\sqrt[4]{x}$ (b) $\sqrt[9]{y}$ (c) $\sqrt[10]{m}$ (d) $\sqrt[13]{a}$
 (e) $\sqrt[3]{n}$ (f) $\sqrt[4]{z}$

Exercise 4J

1. (a) 2 (b) 3 (c) 4 (d) 10 (e) 2
 2. (a) 3 (b) 4 (c) 8 (d) 9
 3. (a) 23 (b) 9 (c) 3 (d) 17 (e) 32
 (f) 2 (g) 7 (h) 6
 4. (a) 2.28 (b) 1191.58 (c) 1.77
 (d) 65.41 (e) 11.62
 5. (a) 7.48 (b) 3.30 (c) 33.63
 (d) 16.70 (e) 77.37

Exercise 4K

- (a) $x^{\frac{1}{2}}$ (b) $x^{\frac{3}{5}}$ (c) $a^{\frac{5}{7}}$ (d) n (e) $x^{\frac{1}{2}}$
 (f) $a^{\frac{13}{20}}$ (g) $n^{\frac{7}{6}}$ (h) $m^{\frac{35}{12}}$ (i) $6x^{\frac{31}{15}}$
 (j) $20n^{\frac{29}{21}}$ (k) $2a^{\frac{49}{10}}$ (l) $\frac{3}{5}y^{\frac{43}{20}}$
- (a) $x^{\frac{1}{2}}$ (b) $x^{\frac{2}{7}}$ (c) $a^{\frac{1}{2}}$ (d) $n^{\frac{1}{6}}$ (e) x^2
 (f) $a^{\frac{17}{6}}$ (g) $4n^{\frac{1}{6}}$ (h) $4m^{\frac{5}{12}}$ (i) $\frac{4}{3}x^{\frac{7}{30}}$
 (j) $\frac{2}{3}x^2$ (k) $\frac{2}{3}n^{\frac{29}{6}}$ (l) $\frac{3}{2}x^{\frac{31}{28}}$
- (a) x^3 (b) $x^{\frac{8}{7}}$ (c) $a^{\frac{5}{2}}$ (d) $n^{\frac{15}{2}}$ (e) x^4
 (f) $a^{\frac{1}{2}}$ (g) n^6 (h) $m^{\frac{4}{5}}$
- (a) x^3y^{10} (b) $x^{\frac{5}{4}}y^{\frac{1}{3}}$ (c) $a^{18}b^{28}$
 (d) $n^{\frac{15}{2}}m^{24}$ (e) $x^5y^{\frac{4}{3}}$ (f) $a^{\frac{8}{3}}b^{\frac{3}{11}}$
 (g) $\frac{n^{\frac{8}{5}}}{m^{\frac{27}{5}}}$ (h) $\frac{x^{14}}{y^6}$
- (a) 1 (b) 1 (c) 1 (d) n^6
- (a) $\frac{1}{x^{\frac{3}{4}}}$ (b) $\frac{y^{\frac{5}{6}}}{x^{\frac{4}{7}}}$ (c) $\frac{a^{\frac{5}{7}}}{b^{\frac{2}{3}}}$ (d) $\frac{1}{m^{\frac{6}{5}}n^{\frac{3}{4}}}$
- (a) x (b) 1 (c) $c^{\frac{1}{4}}$ (d) $a^{\frac{9}{4}}b^{\frac{11}{4}}$ (e) $mn^{\frac{17}{2}}$
 (f) $3x^{\frac{1}{2}}$ (g) $3m^{\frac{1}{2}}$ (h) $2x^{\frac{4}{3}}$ (i) $\frac{3}{2}c^{\frac{5}{12}}$
 (j) $2x^{\frac{9}{4}}$ (k) $4m^{\frac{9}{4}}$ (l) $9x^{\frac{27}{16}}$ (m) $x^{\frac{5}{3}}y^4$
 (n) $m^{\frac{1}{6}}n^{\frac{1}{10}}$ (o) $a^{\frac{41}{12}}b^{\frac{44}{3}}$ (p) $4y^{\frac{31}{6}}$
 (q) $\frac{4}{3}m^{-\frac{1}{2}}n^{\frac{5}{4}}$ (r) $3x^{\frac{15}{2}}y^{\frac{19}{6}}$ (s) $\frac{1}{6}x^{\frac{3}{2}}y^{-\frac{1}{2}}$
 (t) $\frac{3}{2}m^{-\frac{7}{6}}n^{\frac{23}{20}}$

Exercise 4L

- (a) 9×10^3 (b) 5×10^4 (c) 7×10^8
 (d) 2.75×10^5 (e) 3.2×10^5
 (f) 8.21×10^7 (g) 3.6547×10^8
 (h) 2.3×10^{11} (i) 5.0001×10^4
 (j) 6.587×10^3 (k) 2.5434×10^2
 (l) 1.5378×10^3
- (a) 5×10^{-4} (b) 9×10^{-6}
 (c) 4×10^{-9} (d) 1.3×10^{-4}
 (e) 6.92×10^{-3} (f) 5.79×10^{-5}
 (g) 2.045×10^{-3} (h) 6×10^{-6}
 (i) 6.81×10^{-8} (j) 2.0204×10^{-5}
 (k) 2×10^{-10} (l) 5.7×10^{-9}

- (a) 36 000 (b) 8 000 000
 (c) 671 000 (d) 245 500 000
 (e) 72 010 000 (f) 382 000 000 000
 (g) 0.0027 (h) 0.000 07
 (i) 0.000 93 (j) 0.000 000 483
 (k) 0.000 000 005 917
 (l) 0.000 000 000 100 8
- (a) 4.1×10^8 (b) 9.3×10^{18}
 (c) 5.9×10^{12} (d) 5.8×10^{-6}
 (e) 5.7×10^{50} (f) 1.2
- (a) 3.1536×10^7 (b) 9.4608×10^{15}
 (c) 3×10^{16}
- (a) 9.109×10^{-31} kg
 (b) 1.6726×10^{-27} kg
 (c) 1.6749×10^{-27} kg
 (d) 1.0715×10^{-25} kg
 (e) 9.3327×10^{24}
- 7.8540×10^5
- (a) 3.58×10^{22} N (b) 1.99×10^{20} N
- (a) 11 164 m/s (b) 2375 m/s

Exercise 4M

- 8711
- 2446
- \$518 142
- 1189
- \$8857
- 409 600
- \$6375
- 2560
- (a) 6.25 kg (b) 1 tonne
- (a)(i) 6.0×10^3 (ii) 5.4×10^4
 (iii) 5.6×10^{14}
 (b) 7 hours
- (a) 80 (b) 324 (c) 7 years
 (d) 15%
- (a)(i) 4000 (ii) 2560 (iii) 24
 (b) 20% (c) 15 hours
- (a)(i) 45.9 kg (ii) 42.0 kg
 (iii) 8.8 kg
 (b) 40 years

14. (a) 128 (b) 1023 (c) 9.22×10^{18}
 (d) $G = 2^{(N-1)}$
 15. (a)(i) 1.6 mm (ii) 25.6 mm
 (b) A (d) $T = 0.1 \times 2^F$
 (e) 42

Chapter Review

1. (a) 6^7 (b) $6a^5b^2$
 2. (a) 27 (b) 41 (c) 343
 3. (a) $1/x^6$ (b) $8m^2/n^3$ (c) $2x^5/5y^2z^4$
 4. (a) x^9 (b) $12m^{11}n^6$ (c) a^{12}
 (d) $32m^{15}n^{10}$ (e) $-a^6b^9$ (f) $-81p^8q^{12}r^8$
 (g) 1 (h) 7 (i) $3a^{10}$ (j) x^2y^{16}
 (k) $3mn$
 5. $11^{7/6}$
 6. $\sqrt[7]{x}$
 7. 2
 8. 13
 9. 795.43
 10. (a) $2x^{3/2}$ (b) $8m^2$ (c) $16x^{7/2}y^{7/3}$
 11. (a) 8.62×10^9 (b) 5.72×10^{-8}
 12. (a) 67 100 (b) 0.000 003 01
 13. 1.96×10^{16}
 14. 146

Chapter 5

Exercise 5A

1. (a) $\log_3 81 = 4$ (b) $\log_5 25 = 2$
 (c) $\log_{10} 100 = 2$ (d) $\log_8 4096 = 4$
 (e) $\log_7 1 = 0$ (f) $\log_4 1024 = 5$
 (g) $\log_9 9 = 1$ (h) $\log_{20} 8000 = 3$
 2. (a) $2^5 = 32$ (b) $4^3 = 64$ (c) $6^5 = 7776$
 (d) $8^3 = 512$ (e) $7^3 = 343$
 (f) $10^4 = 10\,000$ (g) $3^3 = 27$
 (h) $9^0 = 1$ (i) $25^4 = 390\,625$
 3. (a) $x = 6$ (b) $x = 243$ (c) $x = 6$
 (d) $x = 2$ (e) $x = 144$ (f) $x = 4$
 (g) $x = 4$ (h) $x = 5$

Exercise 5B

1. (a) 6 (b) 5 (c) 4 (d) 13
 2. (a) 7 (b) 5 (c) 6 (d) 5

Exercise 5C

1. (a) $\log_5 42$ (b) $\log_2 27$ (c) $\log_7 40$
 (d) $\log_n 46$ (e) $\log_6 ac$ (f) $\log_x 12pq$
 (g) $\log_3 x^2$ (h) $\log_{10} 15m^2$ (i) $\log_x 18a^4b^6$
 2. (a) $x = 4$ (b) $x = 8$ (c) $x = 5$ (d) $x = 2$
 (e) $x = 5$ (f) $x = b$ (g) $x = 13$ (h) $x = 24$

Exercise 5D

1. (a) $\log_4 \left(\frac{5}{9}\right)$ (b) $\log_3 \left(\frac{8}{5}\right)$ (c) $\log_{10} \left(\frac{5}{3}\right)$
 (d) $\log_5 \left(\frac{3}{4}\right)$ (e) $\log_6 4$ (f) $\log_8 8$
 (g) $\log_3 \left(\frac{3}{4}\right)$ (h) $\log_2 \left(\frac{n}{2}\right)$ (i) $\log_x \left(\frac{2a^2}{3b^2}\right)$
 2. (a) $x = 24$ (b) $x = 30$ (c) $x = 15$
 (d) $x = 36b$ (e) $x = 108mn^2$
 (f) $x = 162a^3$

Exercise 5E

1. (a) $7\log_4 3$ (b) $3\log_3 5$ (c) $c\log_a n$
 (d) $48\log_5 7$ (e) $4\log_2 7$
 (f) $5d \log_x 8 = 15d \log_x 2$ (g) $3np\log_{10} 2$
 (h) $32\log_6 2$ (i) $40\log_4 9 = 80\log_4 3$
 2. (a) $x = 1000$ (b) $x = 81$ (c) $x = b^a$
 (d) $x = 8$ (e) $x = 9$ (f) $x = 243$

Exercise 5F

1. 1 2. 3 3. x 4. a

Exercise 5G

1. 0 2. 0 3. 0 4. 0

Exercise 5H

1. (a) $\frac{\log_6 3}{\log_6 4}$ (b) $\frac{\log_6 12}{\log_6 7}$ (c) $\frac{\log_6 n}{\log_6 a}$
 2. (a) $\frac{\log_n 15}{\log_n 5}$ (b) $\frac{\log_n 9}{\log_n 10}$ (c) $\frac{\log_n m}{\log_n x}$

Exercise 5I

1. 5 2. $\frac{5}{3}$ 3. $\frac{3}{2}$ 4. $\frac{3}{5}$ 5. $3 - n$

6. $m - 9$ 7. $5 + x$ 8. $2(a + 2b)$

9. $\log_5 4$ $(2\log_5 2)$ 10. 0 11. 0

12. 1 13. 0 14. 1

Exercise 5J

1. 1 2. 2 3. 1 4. $\log_5\left(\frac{3}{2}\right)$

5. $\log_5\left(\frac{x}{3}\right)$ 6. $\log_4\left(\frac{4}{y}\right)$ 7. $\log_3\left(\frac{2x}{3}\right)$

8. $\log_5 3a$ 9. $\log_4\left(\frac{n^2}{2}\right)$ 10. $4\log_{10} x$

11. 1 12. 0 13. $\log_4\left(\frac{2}{y}\right)$

14. $\log_3\left(\frac{a}{3}\right)$ 15. $\log_2 4x^2$ $(2\log_2 2x)$

16. 0 17. 1 18. $\log_2\left(\frac{1}{2x^3y^3}\right)$

Exercise 5K

1. $x = 2$ 2. $x = 9$ 3. $x = \frac{1}{2}$

4. $x = 2$ 5. $x = 24$ 6. $x = 2$

7. $x = 1$ 8. $x = \frac{7}{9}$ 9. $x = \frac{1}{8}$

10. $x = 9$

Exercise 5L

1. (a) $\log(9.85 \times 10^{13}) = 13.99$

(b) $\log(1.62 \times 10^{21}) = 21.21$

(c) $\log(3.64 \times 10^{17}) = 17.56$

(d) $\log(8.5711 \times 10^{22}) = 22.93$

(e) $\log(8 \times 10^{13}) = -12.10$

(f) $\log(4.3 \times 10^{-19}) = -18.37$

(g) $\log(7.568 \times 10^{-10}) = -9.12$

(h) $\log(2.22 \times 10^{-13}) = -12.65$

2. (a) $\log(7.2 \times 10^{15}) = 15.86$

(b) $\log(3.09 \times 10^{-17}) = -16.51$

(c) $\log(1.984 \times 10^{25}) = 25.30$

(d) $\log(5.89 \times 10^{-12}) = -11.23$

(e) $\log(4.321 \times 10^{35}) = 35.64$

(f) $\log(9.76 \times 10^{-14}) = -13.01$

3.

x	x (Scientific Notation)	$\log x$
1400	1.4×10^3	3.15
14 000	1.4×10^4	4.15
140 000	1.4×10^5	5.15
1 400 000	1.4×10^6	6.15
14 000 000	1.4×10^7	7.15
140 000 000	1.4×10^8	8.15
1 400 000 000	1.4×10^9	9.15

4. (a) The power of 10 increases by 1.
(b) $\log x$ increases by 1.

5. (a) 4.51 (b) 5.51 (c) 6.51

6. (a) 7.45 (b) 8.45 (c) 12.45

7. (a) 9.1×10^{12} (b) 9.1×10^{13}
(c) 9.1×10^{14} (d) 9.1×10^{17}
(e) 9.1×10^{20} (f) 9.1×10^{24}

8. (a) 5.89×10^{15} (b) 6.61×10^{12}
(c) 1.15×10^{24} (d) 2.45×10^{17}
(e) 3.09×10^{35} (f) 3.55×10^{-10}
(g) 4.68×10^{-8} (h) 6.17×10^{-16}
(i) 4.57×10^{-20}

Exercise 5M

1. (a) 2.8 (b) 3.7 (c) 5.9 (d) 4.5 (e) 7.9
(f) 8.5

2. (a) 5.0×10^3 (b) 39.8 (c) 1.6×10^5
(d) 2.0×10^8 (e) 4.0×10^4

3. (a) 2.5×10^2 (b) 2.5×10^3
(c) 2.5×10^4 (d) 2.5×10^5
(e) 2.5×10^6

4. (a) A increases by a factor of 10.
(b) 2.5×10^7

5. (a) Increased by 2.
(b) Increased by 3.
(c) Increased by 4.
(d) Increased by 5.
(e) Increased by 6.

6. 7.5

7. Because of the large range of intensities ($100 - 10^9$) the logarithmic scale makes it easier to compare the magnitude of earthquakes.

Exercise 5N

- (a) 7 dB (b) 85 dB
- (a) 10 dB (b) 75 dB
- (a) 9 dB (b) 54 dB
- 75%
- 60%
- Intensity after hearing aid = 251% of intensity before the aid.
Increase of 151%.

Exercise 5O

- (a) 2.3 (b) 1.1 (c) 5.0 (d) 5.1 (e) 10.4 (f) 3.2
- (a) 6.3×10^{-9} moles per litre
(b) 5.0×10^{-13} moles per litre
(c) 2.0×10^{-3} moles per litre
(d) 7.9×10^{-8} moles per litre
- (a) 1.0×10^{-7} moles per litre
(b) **A, B, D**

Chapter Review

- $\log_3 243 = 5$
- $2^4 = 16$
- $x = 64$
- 3
- (a) 6 (b) 4 (c) 0 (d) $\log_2 4x$
(e) $\log_4 2x^3 y^2$ (f) 0
- (a) $x = 1$ (b) $x = \frac{3}{8}$
- (a) 7.92 (b) -7.22 (c) 7.95 (d) -11.47
- 10.57
- 5.6
- 60%
- 3.3

Chapter 6**Exercise 6A**

- (a) xz (b) $G + H$ (c) $p - 2$
(d) $d + 7$ (e) $w + y$ (f) $4t + 9w$
(g) $m - 7$ (h) $\frac{H}{3}$ (i) $t(5 - q)$
(j) $\$15b$ (k) $(a + b)(c + d)$
- (i) (a)(d)(e)(g) (ii) (b)(f)(h)
- (a)(i) 7 (ii) a (b)(i) 34 (ii) abc^2
(c)(i) -2 (ii) p (d)(i) 1 (ii) F
(e)(i) $\frac{3}{5}$ (ii) $m^2 n^3$ (f)(i) $-\frac{2}{7}$ (ii) $w^2 z^5$
- $(2m, -4m)$ $(4n^2, \frac{1}{4}n^2)$ $(6mn, \frac{1}{4}mn)$
 $(n, 4n)$ $(6m^2, 3m^2)$ $(4m^2 n, -3nm^2)$
- (a) $7y$ (b) $3b$ (c) $8k$ (d) $9z$ (e) $11ab$
(f) $3xy$ (g) $5mn$ (h) $8xyz$ (i) $-2d$
(j) $-3pq^2$ (k) $3x^2$ (l) $-3y^2$ (m) $-a$
(n) $-3mn$ (o) 0 (p) $2y$ (q) $-6xy$
(r) 0
- (a) $9x + 8y$ (b) $7a + 9b$ (c) $7r + 9t$
(d) $13u + 2w$ (e) $-2y + 6z$ (f) $-5k$
(g) $7w + 6h$ (h) $-14a$ (i) $-10x$
(j) 0 (k) $k^2 l - 3kl^2$ (l) $5ab^2 + 2a^2 b$
(m) xyz (n) $2x^2 y - 5xy^2$
(o) $a^2 b + 4a^2 + 4b$ (p) 0
- (a) $22a$ (b) $2p + 6q$ (c) $4x + 10y$
- (a) $5x$ (b) $35x$
- (a) $10ab$ (b) $8abc$ (c) $6xyz$ (d) $8d^2$
(e) $30pq$ (f) $18tuv$ (g) $18qrs$
(h) $24a^2 b^2$ (i) $30x^2 y^2 z$ (j) $60a^2 b^2 c^2$
(k) $252defg$ (l) $24lm^2 n^2$
- (a) $24n^2$ (b) $6xy$ (c) $21ab$
- (a) $4a$ (b) ay (c) $5az$

Exercise 6B

1. (a) $3y + 6$ (b) $2x - 8$ (c) $4m - 24$
 (d) $7a - 21$ (e) $12 + 15m$
 (f) $16 + 6d$ (g) $3m - 3n$
 (h) $18p + 6q$ (i) $14t + 21u$
 (j) $-3x - 9$ (k) $-5a - 35$ (l) $-32 - 8p$
 (m) $-5a - 5b$ (n) $-12 - 6z$
 (o) $-28y - 35z$ (p) $-3x + 15$
 (q) $-4a + 24$ (r) $-6y + 9$
 (s) $-6a + 10b$ (t) $24m - 18n$
 (u) $10a - 15b + 20c$ (v) $-28x + 8y$
 (w) $-15g - 30h$ (x) $-12x + 8y + 20z$
2. (a) $2a^2 + 3a$ (b) $2mn + 5mp$
 (c) $3xy - 2xz$ (d) $6pq - 14p$
 (e) $6x^2 - 10xy$ (f) $18ab + 21ac$
 (g) $15m^2 - 10mn$ (h) $2b^2 - 14bc$
 (i) $6t^2 - 20tu$ (j) $-2x^2 - xy$
 (k) $-6a^2 - 9ac$ (l) $-6ax - 4bx$
 (m) $-8ab + 10a$ (n) $-6mp + 9mq$
 (o) $-10vw + 25vz$
 (p) $-24mn + 16mp$ (q) $-8ad - 12dx$
 (r) $-6a^2 + 15ab + 6ac$
 (s) $-20a^2 + 10a^2b$ (t) $-12nt + 16t^2$
 (u) $-10x^2 + 4xy - 6xz$
 (v) $6x^2y - 9xyz$ (w) $-6x^2y - 9xy^2$
 (x) $6m^2n - 4m^2 - 10mn$
3. (a) $5a + 17$ (b) $5x + 12$ (c) $9y + 2$
 (d) $7m + 5n$ (e) $5x - 13$ (f) $y - 6$
 (g) $2n + 2$ (h) $2p + 11$ (i) $11y^2 - 9y$
 (j) $8a^2 + 5ab$ (k) $8x^2 - x$
 (l) $14m - 27mn$ (m) $-2a^2 + 43a$
 (n) $24x^2 - 16xy$ (o) $9m - 16m^2$
 (p) $-2c^2 + 15cd$ (q) $33xy + xz$
 (r) $2mp$ (s) $-6a^2 + 4b^2$
 (t) $-34xy + 15x^2 + 18y^2$
4. (a) $4x + 8$ (b) $4a + 6$ (c) $6n + 18$
5. $5x^2 + 8x$
6. (a) $x^2 + 5x + 6$ (b) $y^2 + 6y + 5$
 (c) $m^2 - 8m + 12$ (d) $y^2 - 11y + 24$
 (e) $n^2 + 4n - 21$ (f) $x^2 + 15x + 50$
 (g) $w^2 - 2w - 8$ (h) $p^2 + 3p - 18$
 (i) $y^2 - 7y + 12$ (j) $y^2 + 2y - 3$
 (k) $m^2 + m - 12$ (l) $n^2 - 17n + 70$
 (m) $y^2 - 9$ (n) $m^2 - 1$ (o) $p^2 - 36$
7. (a) $6x^2 + 5x + 1$ (b) $6m^2 + 13m + 6$
 (c) $6x^2 + x - 15$ (d) $10p^2 - 17p + 3$
 (e) $5c^2 + 17c + 6$ (f) $6m^2 + 11m - 10$
 (g) $6m^2 - 13m + 5$ (h) $4n^2 - 29n - 24$
 (i) $6c^2 + 11c - 10$ (j) $10d^2 + 3d - 18$
 (k) $6y^2 - 19y + 10$ (l) $8a^2 - 42a + 49$
8. (a) $6x^2 + 11xy + 3y^2$ (b) $6a^2 + 7ab + 2b^2$
 (c) $2x^2 + xy - y^2$ (d) $a^2 - b^2$
 (e) $9m^2 - 9mn - 10n^2$ (f) $8p^2 - 22pq + 15q^2$
 (g) $9w^2 - 4z^2$ (h) $15n^2 - 19np - 10p^2$
 (i) $12x^2 - 23xy + 10y^2$ (j) $6x^2 - 4xy - 42y^2$
 (k) $9x^2y^2 - 4a^2b^2$ (l) $100a^2 - 49b^2$
9. (a) $x^2 + 10x + 25$ (b) $y^2 + 14y + 49$
 (c) $a^2 - 4a + 4$ (d) $m^2 - 8m + 16$
 (e) $4n^2 + 12n + 9$ (f) $9a^2 + 30a + 25$
 (g) $25t^2 - 20t + 4$ (h) $16d^2 - 56d + 49$
 (i) $64n^2 - 48n + 9$ (j) $a^2 + 2ab + b^2$
 (k) $n^2 - 2mn + m^2$ (l) $x^2 + 2xy + y^2$
 (m) $4x^2 + 12xy + 9y^2$
 (n) $16m^2 - 24mn + 9n^2$
 (o) $4p^2 + 20pq + 25q^2$
 (p) $49a^2 - 42ab + 9b^2$
 (q) $64x^2 + 48xy + 9y^2$
 (r) $100p^2 - 60pq + 9q^2$
10. (a) $8a^3 + 6ab^2 + 4a^2b + 3b^3$
 (b) $3x^3 - 4xy^2 - 15x^2y + 20y^3$
 (c) $15m^3 - 20mn^2 + 6m^2n^2 - 8n^4$
 (d) $3m^2n + 2m^2n^2 + 6m^3 + 4m^3n$
 (e) $9x^3y^2 + 15x^2y^2 - 6x^3y^3 - 10x^2y^3$
 (f) $a^3b^2 - 3a^4b^2 + 2a^2b^3 - 6a^3b^3$
 (g) $12m^3n^2 + 8m^4n^2 - 15m^2n^3 - 10m^3n^3$
 (h) $8x^4y^3 - 6x^3y^4 + 12x^3y^3 - 9x^2y^4$
11. (a) $5a^2 - 30a + 40$ (b) $2m^2 + 14m + 24$
 (c) $6x^2 + 24x - 126$ (d) $3y^2 - 108$
 (e) $12n^2 + 14mn - 6m^2$
 (f) $12c^2 - 36cd + 27d^2$
 (g) $6x^3 + 8x^2y - 8xy^2$
 (h) $18m^3 + 33m^2n + 9mn^2$
 (i) $12a^3 - 38a^2b + 30b^2$
 (j) $30x^3 + 35x^2y - 100xy^2$
 (k) $8x^4y + 30x^3y^2 - 8x^5$
 (l) $24m^3n^2 + 12m^4n + 18m^2n^4 + 9m^3n^3$

12. (a) $2x^2 + 20x + 50$

(b) $3y^2 - 12y + 12$

(c) $4a^2 + 8a + 4$

(d) $2m^2 - 24m + 72$

(e) $a^3 + 2a^2b + ab^2$

(f) $4m^3 - 12m^2n + 9mn^2$

(g) $18m^3 + 24m^2 + 8m$

(h) $12x^3 - 36x^2y + 27xy^2$

(i) $108n - 72n^2 + 12n^3$

(j) $20a^3 + 60a^2b + 45ab^2$

13. (a) $x^3 + 4x^2 + 8x + 5$

(b) $a^3 + 6a^2 + 11a + 6$

(c) $n^3 + 2n^2 - 9n - 18$

(d) $y^3 - y^2 - 2y + 8$

(e) $m^3 - 4m^2 + 2m + 1$

(f) $c^3 + 3c^2 - 11c - 28$

(g) $2x^3 + 17x^2 + 15x - 9$

(h) $3a^3 - 13a^2 + 27a - 20$

(i) $4m^3 + 14m^2 + 8m - 8$

(j) $6n^3 + 11n^2 - 31n + 14$

(k) $x^3 + 9x^2 + 13x - 30$

(l) $n^3 + 2n^2 - 19n + 30$

(m) $a^3 - 2a^2 - 31a + 20$

(n) $m^3 - 4m^2 - m + 12$

(o) $3x^3 + 17x^2 + 38x + 30$

(p) $2c^3 - 19c^2 + 19c - 5$

(q) $6m^3 + 17m^2 + 15m + 4$

(r) $9x^3 - 9x^2 + 17x - 10$

(s) $10a^3 + 13a^2 - 36a - 15$

(t) $20n^3 + 2n^2 - 19n + 6$

14. (a) $x^3 + 6x^2 + 11x + 6$

(b) $x^3 + 5x^2 + 2x - 8$

(c) $a^3 - 6a^2 - a + 30$

(d) $m^3 - 19m + 30$

(e) $n^3 - 5n^2 - 18n + 72$

(f) $c^3 - 10c^2 + 31c - 30$

(g) $x^3 + x^2 - 44x - 84$

(h) $a^3 - a^2 - 82a - 80$

(i) $m^3 + 7m^2 + 16m + 12$

(j) $n^3 + 2n^2 - 15n - 36$

(k) $p^3 - 8p^2 + 21p - 18$

(l) $r^3 - 12r + 16$

(m) $a^3 + a^2 - 8a - 12$

(n) $n^3 - 2n^2 - 15n + 36$

(o) $4a^3 - 4a^2 - 21a - 9$

(p) $6x^3 - 23x^2 + 12x + 20$

(q) $12n^3 + 16n^2 - 31n + 10$

(r) $30n^3 - 17n^2 - 58n + 24$

(s) $18a^3 + 69a^2 + 68a + 20$

(t) $80x^3 - 72x^2 + 21x - 2$

(u) $x^3 + 6x^2 + 12x + 8$

(v) $n^3 - 9n^2 + 27n - 27$

(w) $8m^3 - 60m^2 + 150m - 125$

(x) $27a^3 + 108a^2 + 144a + 64$

Exercise 6C

1. (a) 1, 2, 5, 10 (b) 1, 2, 3, 6, x

(c) 1, 2, 3, 6, 9, 18, a

(d) 1, 2, 3, 4, 6, 9, 12, 18, 36, x, x^2

(e) 1, 2, 3, 4, 6, 8, 12, 24, p, q, q^2

(f) -1, 1, 2, 3, 6, 9, 18, m

(g) 1, 2, 3, 5, 6, 10, 15, 30, x, x^2, y, z

(h) -1, 1, 2, 4, 7, 14, 28, c, c^2

(i) -1, 1, 3, 9, 27, p, q

(j) -1, 1, 2, 3, 4, 6, 8, 12, 16, 24,

$48, m, m^2, n, n^2$

2. (a) 4 (b) 9 (c) 10 (d) x (e) 2

(f) 4 (g) -4 (h) $2x$ (i) x (j) b

(k) $4m$ (l) $-5x$ (m) $2x$ (n) $4a$

(o) xy (p) $4a$ (q) $-2x$ (r) $6m$

(s) $-q$ (t) $-3m$ (u) $16g$ (v) $18ab^2$

(w) $-8a^2b$

3. (a) $2(4p + 5)$ (b) $2(3c - 5)$

(c) $4(3x + 4)$ (d) $2(2a - 7)$

(e) $6(3 + 2m)$ (f) $6(p + 2q)$

(g) $5(m - 3n)$ (h) $9(2q + 3r)$

4. (a) $4p(2q - 3)$ (b) $4y(3x + 4)$

(c) $3c(d + 2)$ (d) $4m(4n - 5)$

(e) $2a(b + 12)$ (f) $4a(1 + 7b)$

(g) $6x(1 - 3y)$ (h) $5b(5a - 3)$

(i) $2ab(3c + 4)$ (j) $5yz(x + 2)$

(k) $8mp(2n + 3)$

5. (a) $-3(3p + 4q)$ (b) $-2(2m + 3n)$
 (c) $-3(a + 4b)$ (d) $-4(a + 2)$
 (e) $-2(3x + 5y)$ (f) $-2(5b + 3c)$
 (g) $-4(5p + 6q)$ (h) $-2(4x + 9)$
 (i) $-12(2 + m)$ (j) $-3(2x + 1)$
 (k) $-18(2a + b)$ (l) $-8(4x + 3y)$
6. (a) $2n(3n + 4)$ (b) $2a(a - 2)$
 (c) $2p(3p - 10)$ (d) $2p(5p + 7q)$
 (e) $2r(3r - 1)$ (f) $4m(2m + 1)$
 (g) $4m(3n + 4m)$ (h) $-5x(2x + 3)$
 (i) $-6b(1 + 3b)$ (j) $-2x(x + 8)$
 (k) $-2m(3m + 5n)$ (l) $-7xy(2x + 1)$

Exercise 6D

1. (a) $(a + 4)(2 + b)$ (b) $(b - 2)(4 - c)$
 (c) $(d + 3)(5 - c)$ (d) $(x - y)(3 + z)$
 (e) $(m + 4)(7 - n)$ (f) $(p - 1)(6 + q)$
2. (a) $(a + 3)(2 + b)$ (b) $(m - 2)(3 + n)$
 (c) $(p + 2)(5 + q)$ (d) $(x + 3)(4 + y)$
 (e) $(2a + 3)(3 + b)$ (f) $(p + 1)(6 + q)$
3. (a) $(3 + z)(y + 5)$ (b) $(b + 7)(2 + c)$
 (c) $(m - 3)(5 + n)$ (d) $(c + 5)(4 + d)$
 (e) $(n - 1)(3 + m)$ (f) $(y - 3)(4 + x)$
 (g) $(a + 3)(b - 2)$ (h) $(m + 5)(n + 4)$
 (i) $(x + 3)(x + y)$ (j) $(b + 3)(a + 7)$
4. (a) $(m + 2)(4 - n)$ (b) $(b + 5)(3 - c)$
 (c) $(x + 4)(2 - y)$ (d) $(n + 3)(m - 3)$
 (e) $(b + 5)(a - 4)$ (f) $(h + 3)(g - 7)$
 (g) $(d + 4)(c - 6)$ (h) $(p + 5)(4 - q)$
 (i) $(x + 2)(x - y)$ (j) $(a + b)(a - 3)$
 (k) $(n + 5)(n - p)$ (l) $(6 + w)(w - y)$
5. (a) $(a - 3)(3 - b)$ (b) $(m - 5)(2 - n)$
 (c) $(p - 1)(4 - q)$ (d) $(x - 4)(3 - y)$
 (e) $(y - 2)(2 - z)$ (f) $(b - 3)(5 - c)$
 (g) $(m - 5)(6 - n)$ (h) $(a - 3)(4 - b)$
 (i) $(a - 5)(3 - b)$ (j) $(n - 3)(6 - p)$
 (k) $(x - 3)(x - y)$ (l) $(a - 4)(a - b)$
6. (a) $(n + 3)(m + 3)$ (b) $(b - 2)(a - 7)$
 (c) $(a + 6)(3 - c)$ (d) $(q + 7)(p - 8)$
 (e) $(b + c)(a + d)$ (f) $(n - 10)(m - 5)$
 (g) $(3 - a)(5 - b)$ (h) $(b + 2c)(3a - 7)$
 (i) $(x + 5)(x - y)$ (j) $(a - b)(a - 3)$
 (k) $(p - 5)(q - 2)$ (l) $(w - y)(z - 6)$
 (m) $(b - c)(b + d)$ (n) $(d - 5)(c - 6)$
- (o) $(2y + 3x)(x - z)$
 (p) $(2a - 5)(3a + 2b)$
 (q) $(q - 2p)(4p + 5)$
 (r) $(mp + ab)(a + b)$

Exercise 6E

1. (a) $(m - n)(m + n)$ (b) $(A - B)(A + B)$
 (c) $(v - w)(v + w)$ (d) $(y - 3)(y + 3)$
 (e) $(5 - c)(5 + c)$ (f) $(x - 7)(x + 7)$
 (g) $(p - 10)(p + 10)$ (h) $(g - 4)(g + 4)$
 (i) $(b - 1)(b + 1)$
2. (a) $(2a - 3b)(2a + 3b)$
 (b) $(4p - 7q)(4p + 7q)$
 (c) $(10x - 9y)(10x + 9y)$
 (d) $(3n - m)(3n + m)$
 (e) $(8g - 11h)(8g + 11h)$
 (f) $(7a - 8b)(7a + 8b)$
 (g) $(5x - y)(5x + y)$
 (h) $(6a - b)(6a + b)$
 (i) $(13n - 15m)(13n + 15m)$
3. (a) $3(3n - m)(3n + m)$
 (b) $2(4a - b)(4a + b)$
 (c) $6(2x - 3y)(2x + 3y)$
 (d) $2(2a - 3b)(2a + 3b)$
 (e) $2(5g - h)(5g + h)$
 (f) $2(4p - 7q)(4p + 7q)$
 (g) $5(2x - y)(2x + y)$
 (h) $8(5a - b)(5a + b)$
 (i) $5(3n - 2m)(3n + 2m)$
4. (a) $(x - 5)(x - 1)$ (b) $(a - 3)(a + 5)$
 (c) $(n - 8)(n + 2)$ (d) $(m + 5)(m + 7)$
 (e) $(y - 10)y$ (f) $x(x + 18)$
 (g) $(c - 17)(c + 3)$ (h) $(n - 3)(n + 11)$
 (i) $(y - 16)y$
5. (a) $(2x + 5)$ (b) $7(2a + 3)$ (c) $-2(2n - 6)$
 (d) $-2(2m + 12)$ (e) $3(2c - 9)$
 (f) $5(2x - 11)$ (g) $4y$ (h) $-4ab$
6. (a) $(x - 1)(5x + 7)$ (b) $(a - 22)(9a + 2)$
 (c) $(n + 3)(13n - 45)$
 (d) $(-6m + 51)(12m - 39)$
 (e) $(-2y + 60)(18y + 20)$
 (f) $(4x - 8)(20x + 56)$
 (g) $(-7a + 67)(15a - 43)$
 (h) $(-2n - 22)(28n - 82)$

7. (a) $(x - \sqrt{5})(x + \sqrt{5})$
 (b) $(a - \sqrt{13})(a + \sqrt{13})$
 (c) $(c - \sqrt{2})(c + \sqrt{2})$
 (d) $(n - \sqrt{23})(n + \sqrt{23})$

8. (a) $2(x - \sqrt{7})(x + \sqrt{7})$
 (b) $5(a - \sqrt{6})(a + \sqrt{6})$
 (c) $3(n - \sqrt{3})(n + \sqrt{3})$
 (d) $7(m - \sqrt{10})(m + \sqrt{10})$

9. (a) $(2x - \sqrt{7})(2x + \sqrt{7})$
 (b) $(5a - \sqrt{21})(5a + \sqrt{21})$
 (c) $2(3x - \sqrt{5})(3x + \sqrt{5})$
 (d) $(n - \sqrt{3}m)(n + \sqrt{3}m)$
 (e) $(2n - \sqrt{5}m)(2n + \sqrt{5}m)$
 (f) $(\sqrt{7}p - q)(\sqrt{7}p + q)$
 (g) $(\sqrt{3}x - \sqrt{5}y)(\sqrt{3}x + \sqrt{5}y)$
 (h) $8(a - \sqrt{3}b)(a + \sqrt{3}b)$
 (i) $3(x - \sqrt{15}y)(x + \sqrt{15}y)$

10. (a) $2(a - 3)(a + 3)$
 (b) $(5a - 11b)(5a + 11b)$
 (c) $(ab - xy)(ab + xy)$
 (d) $\left(\frac{x}{2} - \frac{y}{4}\right)\left(\frac{x}{2} + \frac{y}{4}\right)$
 (e) $\left(5c - \frac{2}{d}\right)\left(5c + \frac{2}{d}\right)$
 (f) $\left(\frac{6x}{7} - \frac{8}{3y}\right)\left(\frac{6x}{7} + \frac{8}{3y}\right)$
 (g) $\left(\frac{x}{2} - \sqrt{3}\right)\left(\frac{x}{2} + \sqrt{3}\right)$
 (h) $\left(\frac{m}{\sqrt{7}} - \frac{n}{\sqrt{13}}\right)\left(\frac{m}{\sqrt{7}} + \frac{n}{\sqrt{13}}\right)$
 (i) $\left(\frac{m}{\sqrt{7}n} - \frac{\sqrt{5}}{a}\right)\left(\frac{m}{\sqrt{7}n} + \frac{\sqrt{5}}{a}\right)$
 (j) $\left(\frac{x+3}{5} - \frac{y-4}{7}\right)\left(\frac{x+3}{5} + \frac{y-4}{7}\right)$
 (k) $\left[\frac{3(x-5)}{5} - \frac{2(y+7)}{9}\right]\left[\frac{3(x-5)}{5} + \frac{2(y+7)}{9}\right]$
 (l) $\left(\frac{\sqrt{3}x}{\sqrt{11}} - \frac{\sqrt{7}y}{\sqrt{51}}\right)\left(\frac{\sqrt{3}x}{\sqrt{11}} + \frac{\sqrt{7}y}{\sqrt{51}}\right)$
 (m) $3(n - 15)(n + 5)$
 (n) $5(b - 2)(b + 6)$
 (o) $2(\sqrt{11} - \sqrt{23}m)(\sqrt{11} + \sqrt{23}m)$

- (p) $(x^2 - y^2)(x^2 + y^2)$
 $= (x - y)(x + y)(x^2 + y^2)$
 (q) $(\theta - \phi)(\theta + \phi)$
 (r) $21(x - \sqrt{3}y)(x + \sqrt{3}y)$

Exercise 6F

1. (a) $(x + 2)(x + 3)$ (b) $(x + 3)(x + 4)$
 (c) $(n + 2)(n + 7)$ (d) $(a + 2)(a + 8)$
 (e) $(p + 2)(p + 9)$ (f) $(a + 8)(a + 6)$
 (g) $(y + 4)(y + 4)$ (h) $(x + 3)(x + 8)$
 (i) $(n + 4)(n + 6)$ (j) $(x + 2)(x + 12)$
 (k) $(a + 1)(a + 10)$ (l) $(y + 1)(y + 8)$
2. (a) $(x - 3)(x - 4)$ (b) $(m - 4)(m - 4)$
 (c) $(a - 3)(a - 8)$ (d) $(c - 5)(c - 6)$
 (e) $(z - 4)(z - 8)$ (f) $(x - 2)(x - 15)$
 (g) $(n - 3)(n - 12)$ (h) $(y - 2)(y - 14)$
3. (a) $(x - 3)(x + 8)$ (b) $(a - 3)(a + 10)$
 (c) $(n + 3)(n - 10)$ (d) $(c + 2)(c - 10)$
 (e) $(m - 2)(m + 8)$ (f) $(y - 2)(y + 6)$
 (g) $(x - 2)(x + 9)$ (h) $(a + 3)(a - 8)$
 (i) $(y - 4)(y + 9)$ (j) $(x + 5)(x - 8)$
 (k) $(n - 6)(n + 7)$ (l) $(a - 3)(a + 4)$
 (m) $(x + 4)(x - 5)$ (n) $(g + 5)(g - 6)$
4. (a) $(x + 6)(x + 9)$ (b) $(c - 5)(c - 8)$
 (c) $(z + 2)(z - 9)$ (d) $(a - 5)(a - 7)$
 (e) $(x + 6)(x + 10)$ (f) $(n - 7)(n - 8)$
 (g) $(y + 6)(y - 11)$ (h) $(b + 8)(b - 9)$
 (i) $(x - 5)(x - 20)$ (j) $(m - 9)(m - 11)$
 (k) $(a - 7)(a + 9)$ (l) $(x - 11)(x - 12)$
 (m) $(y - 2)(y + 20)$ (n) $(r + 4)(r + 8)$
 (o) $(a + 3)(a + 4)$ (p) $(m + 3)(m - 6)$

Exercise 6G

1. (a) $(2x + 3)(x + 2)$ (b) $(2x + 5)(x + 1)$
 (c) $(5x + 4)(x + 9)$ (d) $(3x - 2)(x - 2)$
 (e) $(5x - 3)(x - 1)$ (f) $(3x - 1)(x - 4)$
 (g) $(2x + 7)(3x + 2)$ (h) $(8x - 3)(x - 2)$
 (i) $(2x + 5)(5x + 4)$ (j) $(3x + 2)(5x + 6)$

2. (a) $(3x - 2)(x + 5)$
 (b) $(2x - 3)(2x + 5)$
 (c) $(2x + 3)(3x - 4)$
 (d) $(2x + 5)(3x - 4)$
 (e) $(2x + 5)(7x - 2)$
 (f) $(2x + 1)(3x - 4)$
 (g) $(4x + 3)(x - 4)$
 (h) $(3x - 2)(4x + 5)$
 (i) $(2x + 3)(x - 2)$
 (j) $(2x + 1)(3x - 5)$
3. (a) $(3x - 4)(2x - 7)$
 (b) $(x - 2)(6x + 5)$
 (c) $(3x + 4)(3x - 2)$
 (d) $(12x - 35)(x - 1)$
 (e) $(5x - 12)(2x - 3)$
 (f) $(2x + 1)(12x - 15)$
 (g) $(3x + 10)(5x - 1)$
 (h) $(12x - 1)(x - 12)$

Exercise 6H

1. (a) $(x + 1)(x + 1) = (x + 1)^2$
 (b) $(x + 2)^2$ (c) $(x - 1)^2$
 (d) $(x + 3)^2$ (e) $(x + 5)^2$
 (f) $(x - 5)^2$ (g) $(x - 9)^2$
 (h) $(x + 10)^2$
2. (a) $x^2 + 12x + 36 = (x + 6)^2$
 (b) $x^2 - 14x + 49 = (x - 7)^2$
 (c) $x^2 + 8x + 16 = (x + 4)^2$
 (d) $x^2 - 16x + 64 = (x - 8)^2$

Exercise 6I

1. (a) $(x + 1 - \sqrt{2})(x + 1 + \sqrt{2})$
 (b) $(x - 4 - \sqrt{7})(x - 4 + \sqrt{7})$
 (c) $(x - 5 - \sqrt{29})(x - 5 + \sqrt{29})$
 (d) $(x + 2 - \sqrt{10})(x + 2 + \sqrt{10})$
 (e) $(x + 3 - \sqrt{6})(x + 3 + \sqrt{6})$
 (f) $(x - 7 - \sqrt{57})(x - 7 + \sqrt{57})$
 (g) $(x - 10 - \sqrt{77})(x - 10 + \sqrt{77})$
 (h) $(x + 8 - \sqrt{33})(x + 8 + \sqrt{33})$
 (i) $(x + 20 - \sqrt{277})(x + 20 + \sqrt{277})$
 (j) $(x - 16 - \sqrt{267})(x - 16 + \sqrt{267})$
2. (a) $(x + 9)(x + 13)$ (b) $(x + 7)(x + 13)$
 (c) $(x - 16)(x - 8)$ (d) $(x - 18)(x - 2)$
 (e) $(x + 13)(x + 15)$ (f) $(x - 15)(x - 11)$

Exercise 6J

1. (a) $\frac{3x}{4}$ (b) $\frac{5a}{6}$ (c) $\frac{m}{3}$ (d) $\frac{4}{7y}$
 (e) $\frac{4}{5n}$ (f) $\frac{3}{10}$ (g) $\frac{3}{4}$ (h) $\frac{3}{4}$
 (i) $\frac{7}{9}$ (j) $\frac{1}{4}$ (k) $\frac{4}{9x}$ (l) $3y$
 (m) $\frac{1}{2a}$ (n) $\frac{4}{5m}$ (o) $\frac{3}{7m}$
2. (a) $\frac{x-2}{5}$ (b) $\frac{y+3}{2}$ (c) $\frac{2(y-3)}{3}$
 (d) $\frac{3}{a+4}$ (e) $\frac{3(b-5)}{7}$ (f) $\frac{8}{5(p+3)}$
 (g) $(x + 1)$ (h) $\frac{1}{y-5}$ (i) $2(a - 3)$
 (j) $\frac{2}{2-m}$ (k) $\frac{5}{x+1}$ (l) $\frac{3}{y-4}$
 (m) $\frac{m-5}{3}$ (n) $\frac{2a-3}{4}$ (o) $\frac{2}{1+2n}$
 (p) $\frac{2m}{3}$ (q) $\frac{3}{2a}$ (r) $\frac{3}{4}$
 (s) $\frac{2}{5}$ (t) $2n$
3. (a) $\frac{4}{x+5}$ (b) $\frac{5}{n+4}$ (c) $\frac{2m}{m+2}$
 (d) $\frac{6n}{n-7}$ (e) $\frac{4}{y-1}$ (f) $\frac{c+6}{5}$
 (g) $\frac{n-5}{n-6}$ (h) $\frac{a+2}{a+4}$ (i) $\frac{p+4}{p+6}$
 (j) $\frac{x+7}{x-5}$ (k) $\frac{x+3}{x+4}$ (l) $\frac{x+7}{x+2}$
 (m) $\frac{x+7}{x-5}$ (n) $\frac{a-6}{a+3}$ (o) $\frac{x+6}{x+10}$
4. (a) $\frac{7x}{5}$ (b) $\frac{7a}{3}$ (c) $\frac{3m}{7}$ (d) $\frac{6c}{11}$
 (e) $\frac{3x}{2}$ (f) $\frac{5a}{2}$ (g) m (h) $2i$
5. (a) $\frac{11x}{15}$ (b) $\frac{11y}{9}$ (c) $\frac{7a}{10}$ (d) $\frac{n}{3}$
 (e) $\frac{a}{8}$ (f) $\frac{x}{3}$ (g) $\frac{n}{2}$ (h) $\frac{3u}{14}$
 (i) $\frac{17x}{12}$ (j) $\frac{27a}{20}$ (k) $\frac{5d}{6}$ (l) $\frac{7m}{12}$
 (m) $\frac{2c}{15}$ (n) $\frac{31x}{24}$ (o) $\frac{13n}{28}$ (p) $\frac{37c}{40}$

6. (a) $\frac{4x+1}{3}$ (b) $\frac{5n+2}{2}$ (c) $\frac{2c+3}{5}$

(d) $\frac{3x+2}{2}$ (e) $\frac{3(a+1)}{2}$ (f) $3x+2$

7. (a) $\frac{5x+7}{4}$ (b) $\frac{8n+5}{6}$ (c) $\frac{x-1}{6}$

(d) $\frac{a-2}{3}$ (e) $\frac{4x-3}{12}$ (f) $\frac{7a-6}{15}$

(g) $\frac{27x+5}{6}$ (h) $\frac{16n+11}{12}$ (i) $\frac{16x-1}{10}$

(j) $\frac{11x+2}{15}$ (k) $\frac{7n-7}{6}$ (l) $\frac{2a-3}{12}$

(m) $\frac{2x+1}{20}$ (n) $\frac{x+10}{6}$ (o) $\frac{26a+21}{30}$

(p) $\frac{13x+11}{28}$ (q) $\frac{57a-28}{24}$ (r) $\frac{51x-9}{35}$

(s) $\frac{3x-1}{63}$ (t) $\frac{3x+2}{24}$ (u) $\frac{30a+83}{72}$

8. (a) $\frac{3}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{12}{5}$

(e) $\frac{16}{5}$ (f) $\frac{9}{49}$ (g) $\frac{1}{2}$ (h) $\frac{1}{5}$

(i) $\frac{2}{5}$ (j) $\frac{3}{2}$ (k) $\frac{3}{2}$ (l) $\frac{2}{3}$

9. (a) $\frac{2}{3}$ (b) $\frac{3}{10}$ (c) $\frac{1}{4}$ (d) $\frac{2}{9}$

(e) $\frac{2}{15}$ (f) $\frac{8}{27}$ (g) $\frac{9}{2}$ (h) $\frac{3x}{x+3}$

(i) $\frac{3}{n+5}$ (j) $\frac{6}{5(x-5)}$ (k) $\frac{5}{6a}$ (l) $\frac{3n}{n+5}$

(m) 1

10. (a) $\frac{3}{2}$ (b) $\frac{9}{8}$ (c) $\frac{5}{2}$ (d) $\frac{5}{8}$

(e) $\frac{x}{2(x+4)}$ (f) $\frac{3a}{a+4}$ (g) $\frac{x+3}{2}$

(h) $\frac{4n}{3(n-5)}$ (i) $\frac{2(x-2)}{(x+5)}$ (j) $\frac{3}{x-3}$

(k) 1 (l) 1

Exercise 6K

1. $C = A + B$

2. $g = hd$

3. $L = \frac{M}{N}$

4. $S = A(E + F)$

5. $Z = \frac{3P}{(Q - T)}$

6. $b = c^2 + n^2$

7. $A = LW$

8. $A = \pi r^2$

9. $P = VI$

10. $F = ma$

11. $F = BIl$

12. $s = \frac{d}{t}$

13. $E = mc^2$

14. $F = m \frac{v^2}{R}$

15. $T = 2\pi \sqrt{\frac{l}{g}}$

16. $v = \sqrt{\frac{2GM}{R}}$

Exercise 6L

1. $x = A - y$

3. $z = c + B$

5. $a = \frac{C + b}{3}$

7. $t = h - uc$

9. $m = \frac{F}{a}$

11. $m = \frac{H}{EG}$

13. $u = v - at$

15. $t = \frac{s - am}{u}$

17. $a = \frac{C}{3} - b$

19. $T = \frac{L + RA}{G}$

2. $p = \frac{q}{5}$

4. $x = Rm$

6. $m = \frac{P - n}{a}$

8. $Y = X - A$

10. $a = \frac{F}{m}$

12. $B = \frac{F}{Il}$

14. $a = \frac{v - u}{t}$

16. $v = t - u$

18. $m = \frac{G - k^2}{3p}$

20. $x = \sqrt{P}$

21. $y = \sqrt{Z - t}$ 22. $h = \sqrt{MN + 2pQ}$

23. $A = FD - C$ 24. $m = F(F + C)$

25. $C = \frac{5}{9}(F - 32)$ 26. $R = \sqrt{\frac{3K}{4h}}$

27. $a = \frac{2A}{h} - b$ 28. $R = \sqrt{\frac{GM}{F}}$

Exercise 6M

1. $y = 22$ 2. $v = 65$ 3. $F = 57.2$

4. $s = 16$ 5. $h = 113$ 6. $E = 490$

7. $P = 400$ 8. $v = 5$ 9. $F = 32$

10. $x = 5$ 11. $I = 4448$

Exercise 6N

1. (a) 110 m (b) $a = \frac{2(d - ut)}{t^2}$
(c) 2 m/s^2

2. (a) 1.3 sec (b) 3.4 sec

(c) $l = g\left(\frac{T}{2\pi}\right)^2$ (d) 1.82 m

3. (a)(i) 3.2 m/s (ii) 9.7 m/s
(iii) 2.0 m/s (iv) 4.1 m/s

(b) $h = \frac{v^2}{2g}$ (c) 1.5 m

4. (a)(i) 5Ω (ii) 4Ω (iii) 3.75Ω
(b)(i) 4Ω (ii) 60Ω

Chapter Review

1. (a) $5x - 15$ (b) $-2a + 6b$

(c) $10xy - 15xz$

2. (a) $7m - 4$ (b) $4x - 8$

(c) $x^2 + 2x - 15$ (d) $8y^2 - 2y - 21$

(e) $6n^2 + 19mn + 10m^2$

(f) $9a^2 - 4b^2$ (g) $4x^2 + 12x + 9$

(h) $9p^2 - 24pq + 16q^2$

(i) $12a^3b^3 - 6a^4b + 20ab^4 - 10a^2b^2$

(j) $24x^3 - 28x^2y - 80xy^2$

(k) $18m^3 + 48m^2n + 32mn^2$

(l) $2a^3 + 5a^2 - 7a + 20$

(m) $x^3 - 4x^2 - 11x + 30$

(n) $12x^3 + 16x^2 - 33x - 45$

3. (a) $2y(5x - 6)$ (b) $-3(2a + 3b)$

(c) $(a - 4)(3 + b)$

4. (a) $(3x + 2)(5 + y)$

(b) $(3b - 4)(2a + 3)$

(c) $(a + 5)(b + 2)$

(d) $(a + 2)(a - b)$

5. (a) $(a - b)(a + b)$ (b) $(x - 4)(x + 4)$

(c) $(5m - 7n)(5m + 7n)$

(d) $(m - 5)(m + 7)$ (e) $7(2x - 3)$

(f) $(-c - 42)(17c - 6)$ (g) $(x - \sqrt{7})(x + \sqrt{7})$

(h) $(4n - \sqrt{13})(4n + \sqrt{13})$

(i) $3(2y - 6 - \sqrt{11})(2y - 6 + \sqrt{11})$

6. (a) $(a + 2)(a + 9)$ (b) $(n + 2)(n + 12)$

(c) $(p - 3)(p - 8)$ (d) $(x - 4)(x - 7)$

(e) $(a + 9)(a - 4)$ (f) $(n - 4)(n + 3)$

(g) $(2x - 3)(x + 4)$ (h) $(3x + 8)(4x + 1)$

(i) $(3x + 10)(5x - 1)$ (j) $(x - 8)^2$

(k) $(x + 12)^2$

7. (a) $(x + 2 - \sqrt{10})(x + 2 + \sqrt{10})$

(b) $(x - 3 - \sqrt{2})(x - 3 + \sqrt{2})$

(c) $(x - 1)(x + 11)$

8. (a) $\frac{4}{5}$ (b) $\frac{1}{2 - m}$ (c) $2n$ (d) $\frac{6x}{x - 7}$

(e) $\frac{9x}{5}$ (f) $\frac{11x - 18}{12}$ (g) $\frac{9}{2}$ (h) $\frac{3n}{n + 5}$

(i) $\frac{3a}{a + 4}$ (j) 1

9. (a) $Y = A + B$ (b) $P = mc^2$

(c) $V = Al$

10. (a) $m = p - n$ (b) $a = \frac{B}{5}$

(c) $r = \frac{h + k}{4}$ (d) $y = 5(A - x)$

(e) $y = \frac{z}{2} - x$ (f) $c = \sqrt{N + G}$

(g) $T = \frac{WD - V}{3}$

(h) $F = B(P - 3) - 2A^2$

11. (a) $a = 7$ (b) 23 (c) 78 (d) 72

(e) 10 (f) 16

Chapter 7

Exercise 7A

- (a) $x = 6$ (b) $a = 8$ (c) $c = 25$
(d) $n = 18$ (e) $m = 18$ (f) $b = 7$
(g) $y = 22$ (h) $a = -8$ (i) $d = -2$
(j) $y = -5$ (k) $a = -35$ (l) $p = 5$
(m) $a = 2$ (n) $a = 6$ (o) $d = 5$
(p) $x = 7$ (q) $m = 8$ (r) $c = 9$
(s) $m = -4$ (t) $n = -6$ (u) $t = -2$
(v) $a = 5$ (w) $x = -3$ (x) $y = -4$
(y) $a = 16$ (z) $n = -12$
- (a) $n = 7$ (b) $a = 11$ (c) $m = 10$
(d) $x = 15$ (e) $y = 21$ (f) $w = 30$
- (a) $n = 12$ (b) $a = 10$ (c) $x = 14$
(d) $m = 16$ (e) $c = -20$ (f) $n = -27$
(g) $n = 6$ (h) $a = 12$ (i) $b = 32$
(j) $m = 36$ (k) $d = -36$ (l) $v = -5$
(m) $n = 17$ (n) $a = 8$ (o) $b = 13$
(p) $m = 44$ (q) $x = 30$ (r) $c = -5$
(s) $e = 2$ (t) $f = -62$ (u) $s = -67$
- (a) $n = 13$ (b) $a = 6$ (c) $m = 10$
(d) $x = 7$ (e) $x = -1$ (f) $a = -3$
(g) $n = -10$ (h) $m = 0$
- (a) $n = 3/2$ (b) $m = 6/5$
(c) $n = 7/5$ (d) $k = -5/2$
- (a) $x = 3$ (b) $a = 2$ (c) $m = 9$
(d) $y = 3$ (e) $n = -6$ (f) $m = -2$
(g) $g = 6$ (h) $d = -3$ (i) $u = 4$
- (a) $a = 2$ (b) $m = 2$ (c) $d = 2$
(d) $a = 3$ (e) $x = 2$ (f) $y = -1$
(g) $n = -3$ (h) $a = -3$ (i) $r = -1$
(j) $b = 2$ (k) $a = -2$ (l) $x = 3$
(m) $n = 2$ (n) $a = 3$ (o) $m = 3$
(p) $x = 7$ (q) $m = 2$ (r) $x = 5$
- (a) $x = 5$ (b) $x = 1$ (c) $a = 2$
(d) $n = 4$ (e) $m = 2$ (f) $y = 3$
(g) $m = 3$ (h) $t = -3$ (i) $a = 7$
(j) $c = 3$ (k) $a = 5$ (l) $m = 3$
(m) $y = -1$ (n) $d = -1$ (o) $x = -4$
(p) $n = -2$ (q) $x = -1$ (r) $y = 2$
(s) $w = -3$ (t) $b = -5$ (u) $x = -1$
(v) $n = 5$ (w) $m = 7$ (x) $y = 11$
(y) $k = 5/4$ (z) $n = 33/8$

- (a) $x = 8$ (b) $a = 2$ (c) $y = -12$
(d) $a = 1/2$ (e) $m = -2/3$
- (a) $x = 1/4$ (b) $m = 3/5$
(c) $n = 11/4$ (d) $y = -5/3$
(e) $n = 4/3$ (f) $x = -2/5$
(g) $a = -5/2$ (h) $x = -7/3$
(i) $n = -5/3$ (j) $n = 35$ (k) $x = 6/5$
(l) $a = 13/2$ (m) $m = 35/2$
(n) $x = 2/3$ (o) $u = -8/5$
(p) $n = 2/15$ (q) $x = 13/8$
(r) $m = -11/15$ (s) $x = 2/15$
(t) $a = 47/4$ (u) $v = 29/10$
(v) $x = 21/5$ (w) $n = 35/2$
(x) $n = 58/15$
- (a) $a = 2/3$ (b) $x = 3$ (c) $y = 5/21$
(d) $a = 13/30$ (e) $x = 4/39$
(f) $y = 5/4$ (g) $x = 5$ (h) $x = 9$
(i) $x = 5/3$

Exercise 7B

- (a) $2n + 5 = 13$, $n = 4$
(b) $4n - 7 = 25$, $n = 8$
(c) $n/4 + 4 = 9$, $n = 20$
(d) $3(n + 6) = 33$, $n = 5$
(e) $7(n - 9) = 21$, $n = 12$
(f) $5(3n + 8) = 55$, $n = 1$
(g) $3(4n - 6) = 18$, $n = 3$
(h) $3n + 7 = 4n + 2$, $n = 5$
(i) $5n - 6 = 3n + 8$, $n = 7$
- length = 7 km, width = 5 km
- (a) $x + 5$ (b) $2x + 5$
(c) Shaheen = 12, Charmaine = 17
- 15 years old
- A** = 0.8 m, **B** = 2.0 m, **C** = 2.6 m
- \$22
- \$240
- 60 litres
- (a) 18 (b) 28×2 kg, 14×5 kg
(c) 23×2 kg, 16×5 kg
- 48 kg
- 1.6 litres

Exercise 7C

- $x = 0$ or 6
 - $x = 0$ or -7
 - $x = 0$ or -1
 - $x = 0$ or 3
 - $x = 0$ or -10
 - $x = 0$ or 1
 - $x = 0$ or $7/2$
 - $x = 0$ or $-8/3$
 - $x = 0$ or $9/7$
 - $x = 0$ or -3
 - $x = 0$ or $-7/3$
 - $x = 0$ or $-4/3$
- $x = 4$ or -4
 - $x = 10$ or -10
 - $x = 5/2$ or $-5/2$
 - $x = 8/7$ or $-8/7$
 - $x = 2$ or -2
 - $x = 5/2$ or $-5/2$
 - $x = 0$ or -6
 - $x = 2$ or 4
 - $x = 17/4$ or $7/4$
 - $x = -41/9$ or $-49/9$
 - $x = 3/2$
 - $x = 12$ or $8/3$
 - $x = \sqrt{11}$ or $-\sqrt{11}$
 - $x = \sqrt{7}/2$ or $-\sqrt{7}/2$
 - $x = \sqrt{3/2}$ or $-\sqrt{3/2}$
 - $x = \sqrt{10/3}$ or $-\sqrt{10/3}$
- $x = -7$ or -2
 - $x = -3$ or -8
 - $x = 3$ or 12
 - $x = 2$ or -6
 - $x = -3$ or 10
 - $x = -5$ or 8
 - $x = -4$ or 5
 - $x = 2$ or 14
 - $x = -4/5$ or -9
 - $x = 2/3$ or 2
 - $x = 2/3$ or -5
 - $x = -5/2$ or $3/2$
 - $x = 4/3$ or $-3/2$
 - $x = -5/2$ or $4/3$
 - $x = -5/2$ or $2/7$
 - $x = -1/2$ or $4/3$
 - $x = 4$ or $-3/4$
 - $x = -5/4$ or $2/3$
 - $x = 1$
 - $x = -3$
 - $x = -5$
 - $x = 5$
 - $x = 9$
 - $x = -10$
- $x = -1$ or -3
 - $x = -5$ or -2
 - $x = -2$ or -8
 - $x = -4$ or 12
 - $x = 5$ or $-3/2$
 - $x = -5$ or $4/3$
 - $x = 3/2$ or $4/3$
 - $x = -5/2$ or $7/4$
 - $x = 5/3$
 - $x = 7/2$
 - $x = -11$
 - $x = 5/2$ or $-5/2$
 - $$(m) x = \frac{-3 + \sqrt{29}}{2} \text{ or } \frac{-3 - \sqrt{29}}{2}$$

$$(n) x = \frac{-1 + \sqrt{89}}{4} \text{ or } \frac{-1 - \sqrt{89}}{4}$$

$$(o) x = \frac{11 + \sqrt{229}}{6} \text{ or } \frac{11 - \sqrt{229}}{6}$$

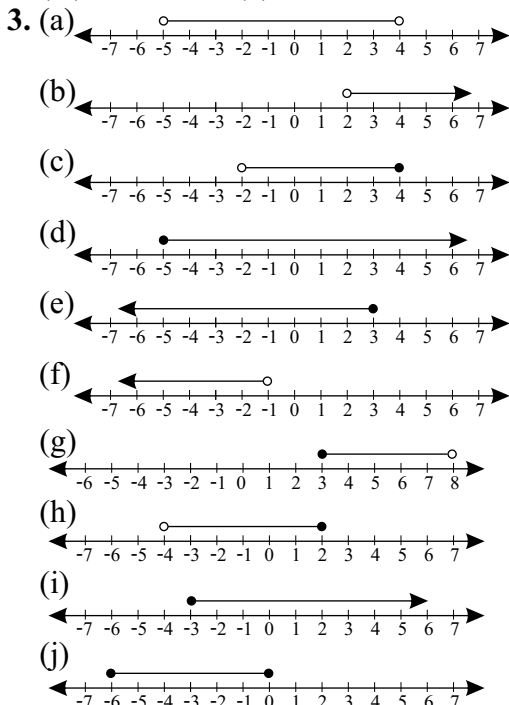
$$(p) x = \frac{11 + \sqrt{41}}{10} \text{ or } \frac{11 - \sqrt{41}}{10}$$

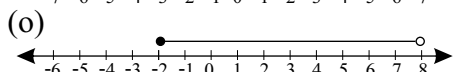
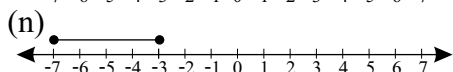
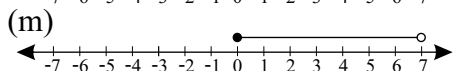
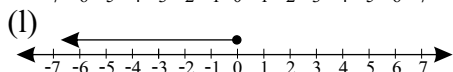
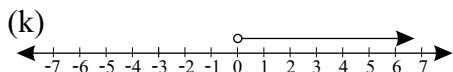
Exercise 7D

- \$1.80
- 4 km/h and 12 km/h
- 320 cm^2
- \$20 000 in the high risk,
\$60 000 in the low risk
- daughter is 8, father is 56
- 68
- (a) 1.5 m/s (b) 13.65 km
- 12 min
- 12 hours
- 1.2 litres
- 8 km

Exercise 7E

- B**
 - C**
 - A**
 - D**
- $-2 < n \leq 1$
 - $n \geq 0$
 - $n < -1$
 - $-3 \leq n < 2$
 - $-3 \leq n \leq 2$
 - $n > 1$
 - $n < 1$
 - $-2 < n \leq 3$
 - $n \geq -1$
 - $-3 < n < 2$
 - $-5 \leq n < 6$
 - $-7 < n < 5$
 - $-6 \leq n \leq 7$
 - $n < 6$





4. (a) $n > 14$ (b) $n < -4$ (c) $n \geq -4$
 (d) $n \leq 8$ (e) $n \geq 3$ (f) $n \leq -2$
 (g) $n < 9$ (h) $n > -4$ (i) $n \leq -1$
 (j) $n \geq -18$ (k) $n < -16$ (l) $n < -48$
 (m) $n \leq 11/2$ (n) $n > -19/2$
 (o) $n \geq 21$ (p) $n < -18$

5. (a) $n < 4$ (b) $n > 1$ (c) $n \leq 5$
 (d) $n \geq -6$ (e) $n \leq 2$ (f) $n \geq -2$
 (g) $n > -12$ (h) $n > 24$ (i) $n \geq -7$
 (j) $n \leq -10$ (k) $n > 12$ (l) $n \geq 3$
 (m) $n \leq 3$ (n) $n < -5$ (o) $n > -5$
 (p) $n \leq -4$ (q) $n < 5/6$ (r) $n \geq -67/8$

Exercise 7F

1. (a) $d \geq 5t$ (b) $d \geq 6000$ m
 (c) $t \leq 40$ minutes
 2. (a) $24 \text{ km/h} = 400 \text{ m/min}$
 $42 \text{ km/h} = 700 \text{ m/min}$
 (b) $\frac{d}{700} \leq t \leq \frac{d}{400}$
 (c) $40 \leq t \leq 70$
 3. (a) $A \geq 3000 + 250n$ (b) $w \leq 28$
 4. (a) $A \leq 4000 - 100w$ (b) $w \leq 40$
 5. (a) $2400 + 80n \leq 4000$ (b) $n \leq 20$
 6. (a) $A \geq 75n$ (b) $A \geq \$675$
 (c) $8(p - 15) \geq 720, p \geq \$105$
 7. (a) $2 + 0.5n \leq H \leq 2 + 2n$
 (b) $8 \leq H \leq 26$ (c) $14 \leq n \leq 56$

Chapter Review

1. (a) $x = -4$ (b) $n = 2$ (c) $a = 6$
 (d) $y = 20$ (e) $m = 3$ (f) $x = -8$
 (g) $x = -2$ (h) $a = -1/2$ (i) $m = -11/3$
 (j) $n = -1/6$ (k) $n = 11$ (l) $c = -3$
 (m) $m = 34$ (n) $x = 9$ (o) $n = 7$
 (p) $c = -8/3$ (q) $x = -1$ (r) $m = 45/4$
 (s) $m = 2$ (t) $n = 2$ (u) $a = 5$
 (v) $y = -1$ (w) $a = 1/2$ (x) $n = 58/15$
 (y) $x = 5/7$ (z) $y = 5/24$

2. (a) $2(5n + 3) = 26, n = 2$

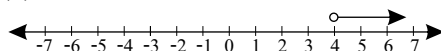
(b) 10 m, 15 m, 20 m

(c) 300 g of **A**, 700 g of **B**

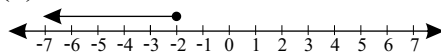
3. (a) $x = 0$ or -13 (b) $x = 0$ or $4/5$
 (c) $x = 8$ or -8 (d) $x = 1/2$ or $-5/2$
 (e) $x = \sqrt{15}$ or $-\sqrt{15}$
 (f) $x = \sqrt{3}/2$ or $-\sqrt{3}/2$ (g) $x = 4$ or 5
 (h) $x = 4$ or -6 (i) $x = -5/2$ or 6
 (j) $x = 5/4$ or $-4/3$ (k) $x = -8$
 (l) $x = -3$ or -6
 (m) $x = -2 + \sqrt{14}$ or $-2 - \sqrt{14}$

$$(n) x = \frac{7 + \sqrt{89}}{4} \text{ or } \frac{7 - \sqrt{89}}{4}$$

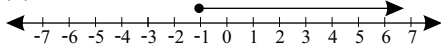
4. (a) $n > 4$



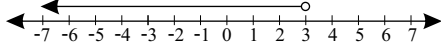
- (b) $n \leq -2$



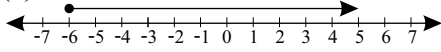
- (c) $n \geq -1$



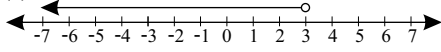
- (d) $n < 3$



- (e) $n \geq -6$



- (f) $n < 3$



5. (a) $40 + 0.2k \leq 250$ (b) $k \leq 1050$ km

Chapter 8

Exercise 8A

1. A, B, C, F, H

2. (a)(i) $x^4 + 6x^3 + 4x - 7$
(ii) Degree 4
- (b)(i) $-6m^5 + 5m^4 + 2m^2 + 9$
(ii) Degree 5
- (c)(i) $-5a^6 + 3a^4 - 4a^3 + 2a^2 + a$
(ii) Degree 6
- (d)(i) $-3x^7 + 5x^5 + \frac{7}{3}x^4 + x^3 - 2x^2 + 9$
(ii) Degree 7

Exercise 8B

1. $10x^2 - 7x - 12$ Degree 2
2. $-3a^2 + 17a - 20$ Degree 2
3. $8a^3 + 10a^2 - 3a$ Degree 3
4. $2n^3 + 17n^2 + 30n$ Degree 3
5. $-9x^3 + 57x^2 - 60x$ Degree 3
6. $12x^3 - 26x^2 - 56x$ Degree 3
7. $n^3 + 2n^2 - 11n - 12$ Degree 3
8. $-2a^3 + 5a^2 + 37a - 60$ Degree 3
9. $-12x^3 + 28x^2 + 9x - 10$ Degree 3
10. $18x^3 - 45x^2 - 32x + 80$ Degree 3
11. $12x^3 + 16x^2 - 75x - 100$ Degree 3
12. $60x^3 - 173x^2 + 135x - 28$ Degree 3
13. $m^4 - 6m^3 + 5m^2 + 12m$ Degree 4
14. $-2m^4 + 14m^3 - 32m^2 + 24m$
Degree 4
15. $12n^4 - 62n^3 + 6n^2 + 20n$ Degree 4
16. $-12x^4 + 86x^3 - 172x^2 + 80x$
Degree 4
17. $6x^4 - 11x^3 - 93x^2 + 176x - 48$
Degree 4
18. $24x^4 + 34x^3 - 331x^2 + 459x - 180$
Degree 4
19. $24x^4 - 122x^3 + 179x^2 - 92x + 15$
Degree 4
20. $15x^4 - 134x^3 + 185x^2 + 534x - 360$
Degree 4

Exercise 8C

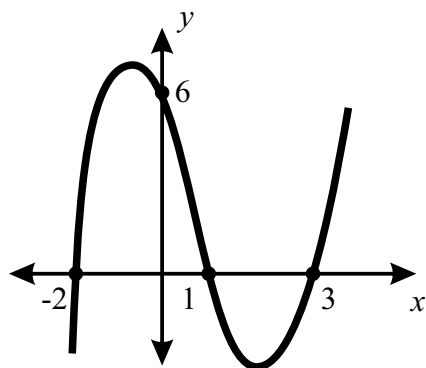
1. $(x - 1)(x + 2)(x + 4)$
2. $(x - 1)(x - 5)(x - 7)$
3. $(x + 1)(x - 3)(x + 12)$
4. $(x - 2)(x - 3)(x + 6)$
5. $(x - 2)(x + 2)(3x - 4)$
6. $(x - 3)(x + 4)(2x + 5)$
7. $(x - 2)(x - 4)(3x + 4)$
8. $(x + 2)(x - 4)(x - 4) = (x + 2)(x - 4)^2$
9. $(x + 1)(x + 3)(x - 4)$
10. $(x - 1)(x - 4)(x + 5)$
11. $x(x - 3)(x + 8)$
12. $4x(x - 2)(x + 4)$
13. $(x + 4)(2x + 1)(3x - 2)$
14. $(x - 5)(2x + 3)(3x - 2)$
15. $(x + 4)(2x + 5)(4x - 1)$
16. $(x - 6)(3x + 4)(3x - 2)$

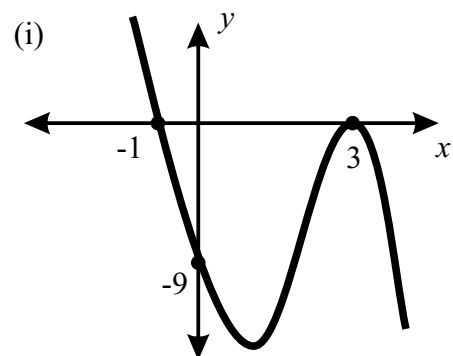
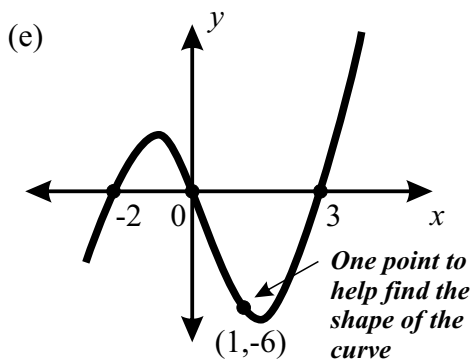
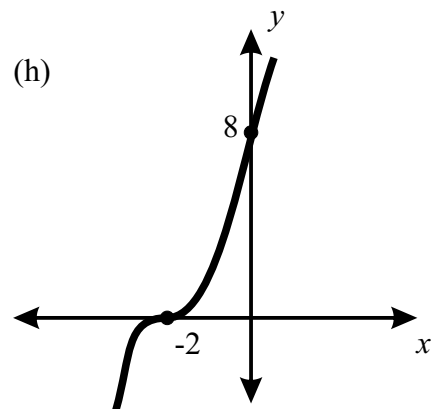
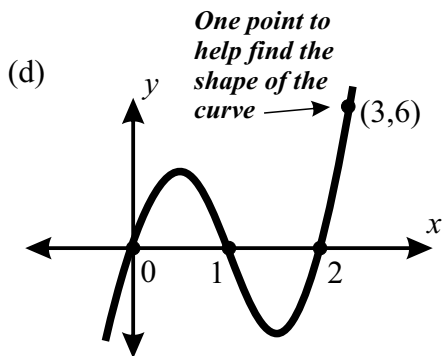
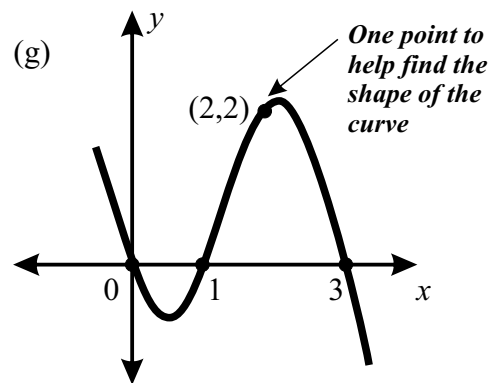
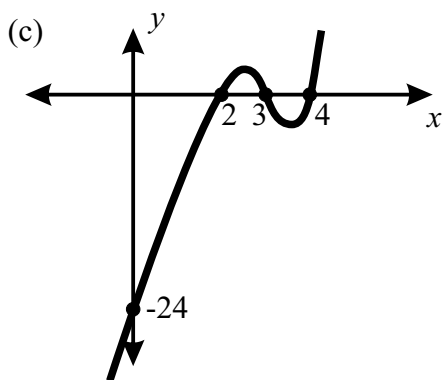
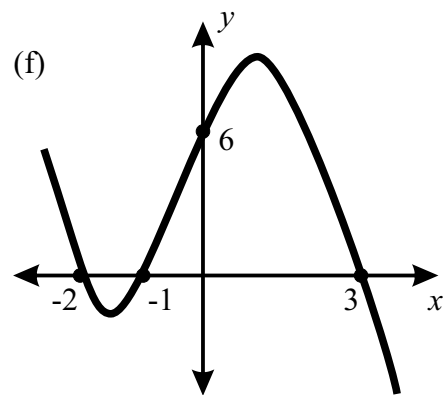
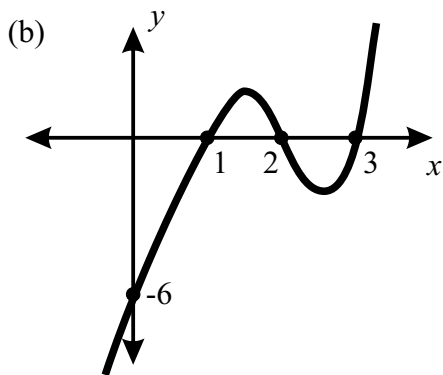
Exercise 8D

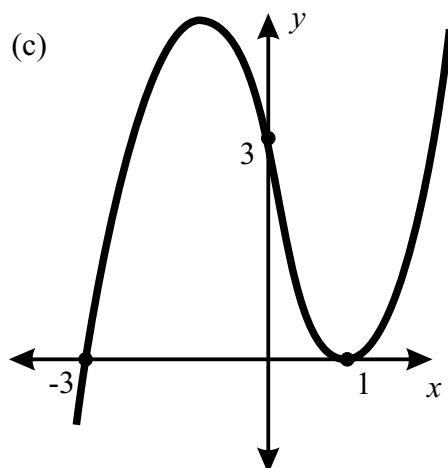
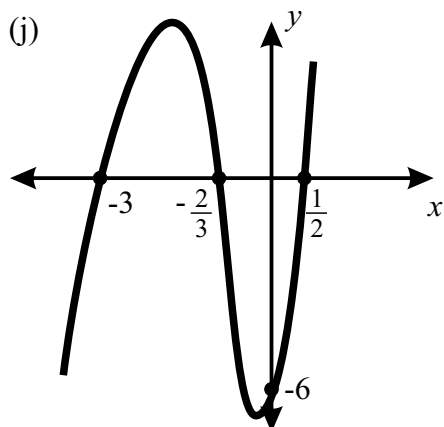
1. $x = 2, -3$ or 4
2. $x = -3$ or 2
3. $x = -7$
4. $x = \frac{3}{2}, -\frac{1}{3}$ or 5
5. $x = \frac{5}{4}, -\frac{1}{2}$

Exercise 8E

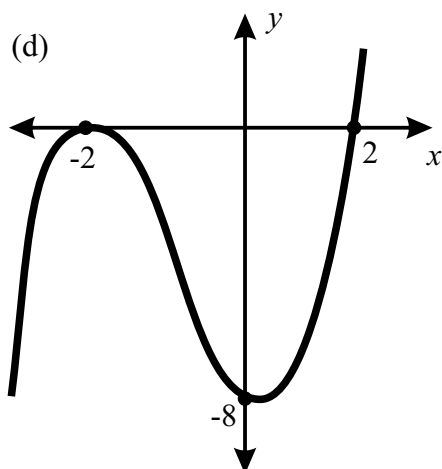
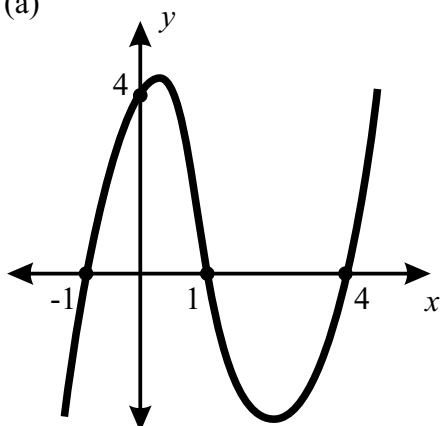
1. (a)



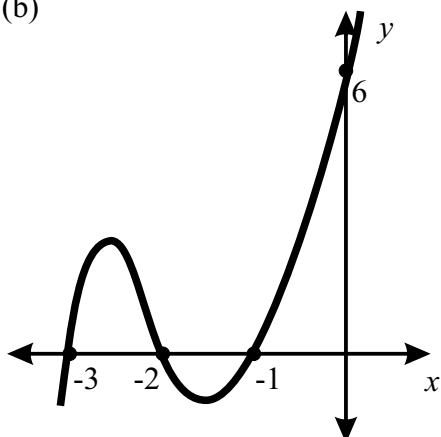




2. (a)



(b)



Exercise 8F

1. (a) Quotient $= x^2 + x - 5$
Remainder $= 5$

(b) Quotient $= 3x^2 - 2x + 3$
Remainder $= -13$

(c) Quotient $= 2x^2 + x + 3$
Remainder $= 15$

(d) Quotient $= 2x^2 - 7x + 28$
Remainder $= -121$

3. (a) 35 (b) -432 (c) 257 (d) 79
(e) 123 (f) 3511 (g) -17 817

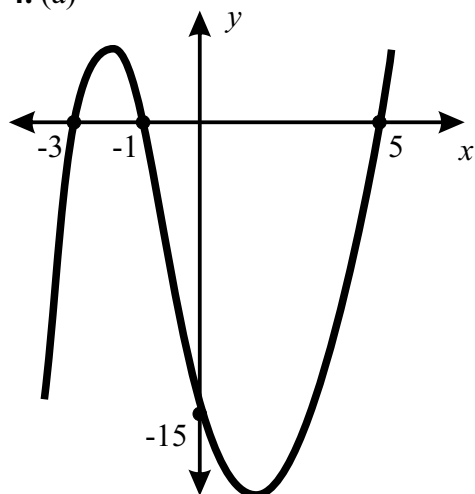
Chapter Review

1. A, B, D, E, F

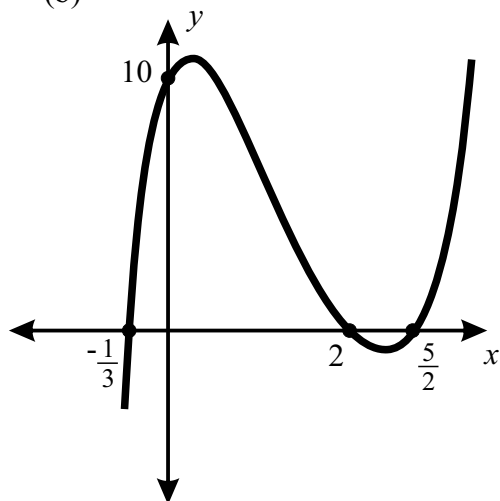
2. (a) 4 (b) 5

3. (a) $6x^2 - 13x - 28$ Degree 2(b) $-16a^2 + 46a - 15$ Degree 2(c) $3x^3 + 13x^2 - 10x$ Degree 3(d) $54m^3 - 6m$ Degree 3(e) $2x^3 - 7x^2 - 19x + 60$ Degree 3(f) $6x^4 + 11x^3 - 26x^2 - 15x$ Degree 4

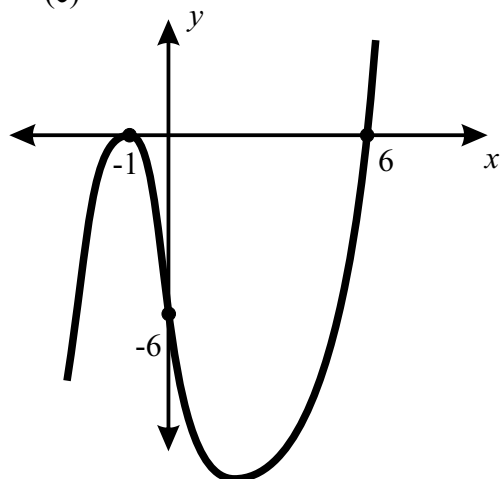
4. (a)



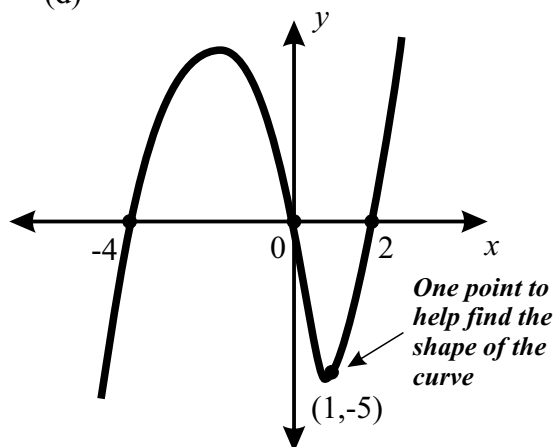
(b)



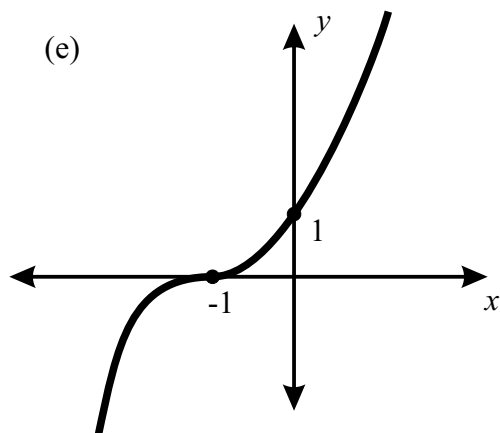
(c)



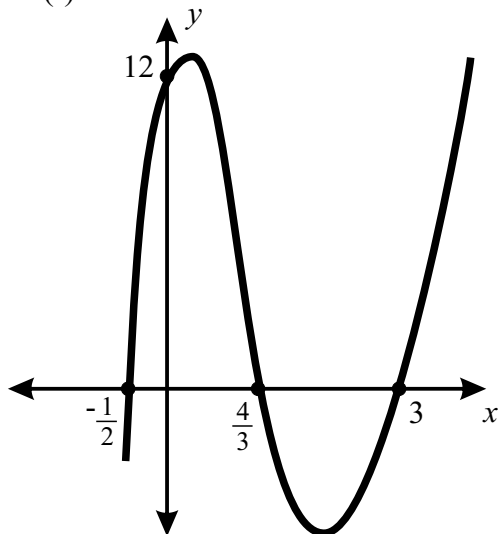
(d)



(e)



(f)



5. (a) Quotient = $x^2 - 4x - 23$
Remainder = -100
(b) Quotient = $4x^2 - 9x + 18$
Remainder = -27
6. (a) 75 (b) 24 (c) 32

Chapter 9

Exercise 9A

1. (a) $y = 11$ (b) $a = -1$ (c) $M = -14$
(d) $C = -6$
2. (a)

m	-3	-2	-1	0	1	2	3
n	-1	0	1	2	3	4	5

(b)

x	-3	-2	-1	0	1	2	3
y	-11	-9	-7	-5	-3	-1	1

(c)

N	-3	-2	-1	0	1	2	3
M	15	11	7	3	-1	-5	-9

3. (a) $y = 4x$ (b) $A = B + 3$
(c) $d = c - 2$ (d) $N = 2M - 1$
(e) $b = 3a + 1$ (f) $Q = -P$
(g) $y = 4x - 1$ (h) $n = 3 - m$
(i) $D = 2 - 2C$ (j) $y = 1 - 3x$
4. (a) $y = 3x - 1$ (b) $y = x + 8$
(c) $y = 2x - 6$ (d) $y = -x + 12$
(e) $y = -3x + 11$ (f) $y = 5x - 8$
(g) $y = \frac{5}{2}x + 3$ (h) $y = \frac{1}{3}x - 2$

5. (a)

A	0	5	12	22	38
B	11	36	71	121	201

- (b) $B = 5A + 11$ (c) 336 (d) 48
6. (a)

N	0	2	6	12	24
V	30 000	28 800	26 400	22 800	15 600

- (b) $V = 30\,000 - 600N$
(c) 9000 litres (d) 50 days
7. (a) $T = 24 - 0.005H$ (b) 18°C
(c) 4800 m
8. (a) $F = 10 + 2.5V$ (b) 40 mL/min

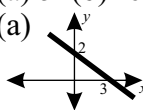
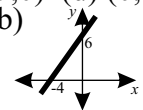
Exercise 9B

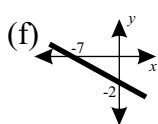
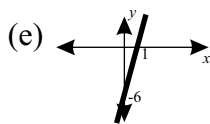
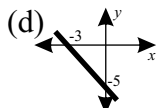
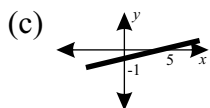
1. (a) 3 (b) 1 (c) 4
2. (a) 2 (b) 1 (c) 1 (d) 2 (e) 4
(f) 1 (g) 3 (h) 2
3. (a) $1/2$ (b) $1/2$ (c) $1/2$ (d) $1/3$
(e) $3/10$ (f) $2/3$
4. (a) 1 (b) 2 (c) 1 (d) $1/2$ (e) $4/7$
(f) $1/3$ (g) $6/5$ (h) 1 (i) $5/3$
(j) $3/7$
5. A, D, E, F, G
6. (a) -1 (b) -2 (c) -1 (d) + (e) $-1/2$
(f) $-3/5$ (g) $-1/2$ (h) +

Exercise 9C

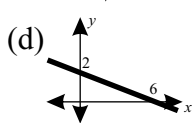
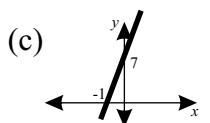
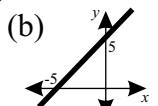
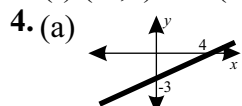
1. (a) 1 (b) 2 (c) 2 (d) $8/5$ (e) $6/5$
(f) $4/7$ (g) $3/5$ (h) $3/7$ (i) $-2/3$
(j) -2 (k) $1/7$ (l) $9/8$ (m) $-12/7$
(n) $-9/10$ (o) $-5/7$ (p) $-2/5$ (q) -2
(r) -6 (s) 0 (t) 0
2. (a) x-axis (b) 0
3. (a) E (b) D (c) B (d) C (e) F

Exercise 9D

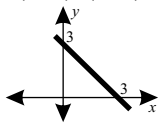
1. (a) 5 (b) -6 (c) (5,0) (d) (0,-6)
2. (a)  (b) 



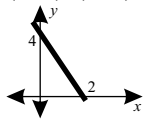
3. (a) $(3,0)$ and $(0,2)$
 (b) $(-4,0)$ and $(0,6)$
 (c) $(5,0)$ and $(0,-1)$
 (d) $(-3,0)$ and $(0,-5)$
 (e) $(1,0)$ and $(0,-6)$
 (f) $(-7,0)$ and $(0,-2)$



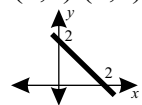
5. (a) $(3,0)$ $(0,3)$



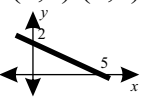
- (b) $(2,0)$ $(0,4)$



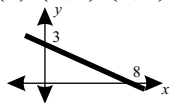
- (c) $(2,0)$ $(0,2)$



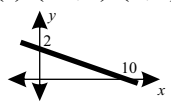
- (d) $(5,0)$ $(0,2)$



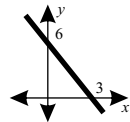
- (e) $(8,0)$ $(0,3)$



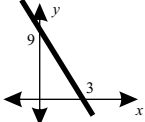
- (f) $(10,0)$ $(0,2)$



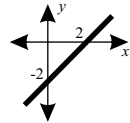
- (g) $(3,0)$ $(0,6)$



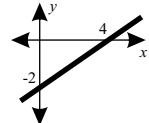
- (h) $(3,0)$ $(0,9)$



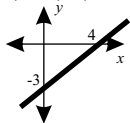
- (i) $(2,0)$ $(0,-2)$



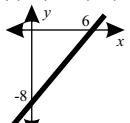
- (j) $(4,0)$ $(0,-2)$



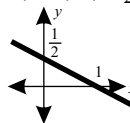
- (k) $(4,0)$ $(0,-3)$



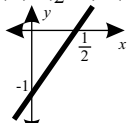
- (l) $(6,0)$ $(0,-8)$



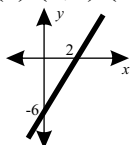
- (m) $(1,0)$ $(0,\frac{1}{2})$



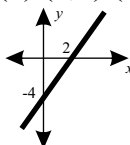
- (n) $(\frac{1}{2},0)$ $(0,-1)$



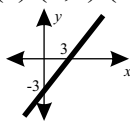
6. (a) $(2,0)$ $(0,-6)$



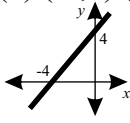
- (b) $(2,0)$ $(0,-4)$



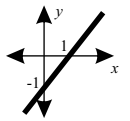
- (c) $(3,0)$ $(0,-3)$



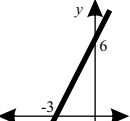
- (d) $(-4,0)$ $(0,4)$



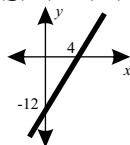
- (e) $(1,0)$ $(0,-1)$



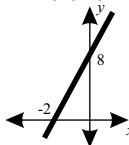
- (f) $(-3,0)$ $(0,6)$



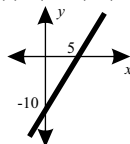
- (g) $(4,0)$ $(0,-12)$



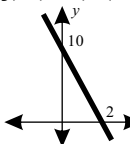
- (h) $(-2,0)$ $(0,8)$



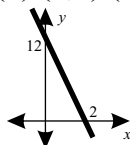
- (i) $(5,0)$ $(0,-10)$



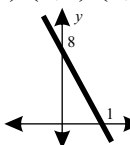
- (j) $(2,0)$ $(0,10)$



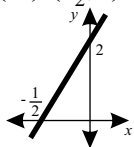
- (k) $(2,0)$ $(0,12)$



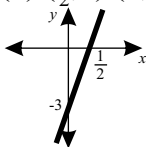
- (l) $(1,0)$ $(0,8)$



(m) $(-\frac{1}{2}, 0)$ $(0, 2)$



(n) $(\frac{1}{2}, 0)$ $(0, -3)$

**Exercise 9E**

- gradient = 3, y-intercept = 4
 - gradient = 2, y-intercept = 6
 - gradient = 1, y-intercept = 5
 - gradient = 1, y-intercept = -1
 - gradient = 2, y-intercept = -5
 - gradient = 6, y-intercept = -7
 - gradient = -3, y-intercept = 2
 - gradient = -4, y-intercept = -1
 - gradient = -1, y-intercept = -2
 - gradient = -5, y-intercept = 0
 - gradient = 2, y-intercept = $\frac{1}{2}$
 - gradient = -7, y-intercept = $-\frac{3}{5}$
 - gradient = $\frac{1}{3}$, y-intercept = 2
 - gradient = $-\frac{3}{4}$, y-intercept = $-\frac{1}{4}$
 - gradient = 2, y-intercept = 5
 - gradient = -1, y-intercept = 3
 - gradient = 3, y-intercept = 6
 - gradient = 8, y-intercept = 2
 - gradient = $\frac{1}{3}$, y-intercept = 1
 - gradient = $\frac{3}{5}$, y-intercept = -4
- $y = 5x + 3$
 - $y = 2x - 1$
 - $y = -4x + 5$
 - $y = -2x - 7$
 - $y = x - 2$
 - $y = 3x$
 - $y = \frac{1}{3}x - 2$
 - $y = 2x - \frac{2}{9}$
- Straight lines that are **parallel** have the same **gradient**.
- B, E, F, I, J**
- $y = 3x + 5$, $m = 3$, $c = 5$
 - $y = -2x - 6$, $m = -2$, $c = -6$
 - $y = x - 3$, $m = 1$, $c = -3$
 - $y = -x + 2$, $m = -1$, $c = 2$
 - $y = 3x + 4$, $m = 3$, $c = 4$
 - $y = x - 2$, $m = 1$, $c = -2$
 - $y = -x + 3$, $m = -1$, $c = 3$
 - $y = 2x + \frac{3}{5}$, $m = 2$, $c = \frac{3}{5}$
 - $y = -4x + \frac{5}{2}$, $m = -4$, $c = \frac{5}{2}$

(j) $y = \frac{5}{3}x - 4$, $m = \frac{5}{3}$, $c = -4$

(k) $y = \frac{1}{5}x + \frac{1}{5}$, $m = \frac{1}{5}$, $c = \frac{1}{5}$

(l) $y = -\frac{4}{3}x$, $m = -\frac{4}{3}$, $c = 0$

(m) $y = 2x - \frac{5}{2}$, $m = 2$, $c = -\frac{5}{2}$

(n) $y = \frac{2}{3}x - \frac{4}{3}$, $m = \frac{2}{3}$, $c = -\frac{4}{3}$

6. (a) **I** (b) **G** (c) **H** (d) **E** (e) **B**
(f) **D** (g) **C** (h) **F**

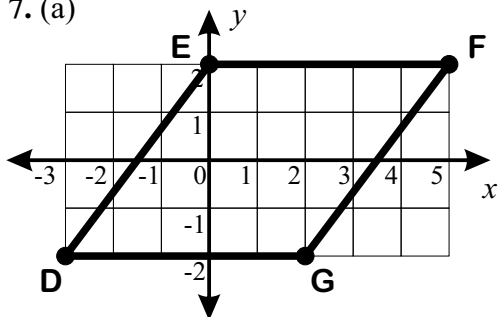
Exercise 9F

- $-\frac{1}{5}$
 - $\frac{1}{3}$
 - 4
 - 7
 - $\frac{2}{5}$
 - $-\frac{8}{3}$
- $-\frac{1}{4}$
 - 1
 - 1
 - $\frac{1}{7}$
 - $-\frac{4}{3}$
 - 7
 - $\frac{1}{2}$
 - $-\frac{1}{5}$
 - $-\frac{5}{7}$
 - $\frac{9}{2}$
- $-\frac{1}{3}$
 - $\frac{1}{4}$
 - 1
 - 1
 - 2
 - 4
 - $\frac{6}{7}$
 - $-\frac{8}{3}$
 - $\frac{12}{11}$
 - $-\frac{3}{8}$

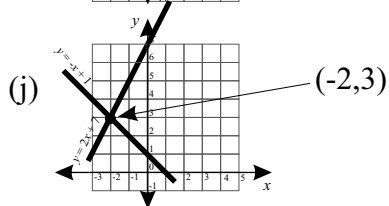
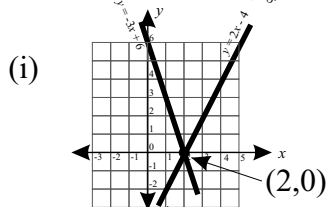
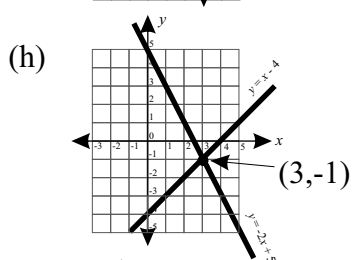
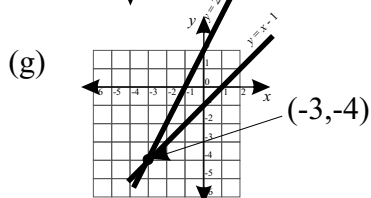
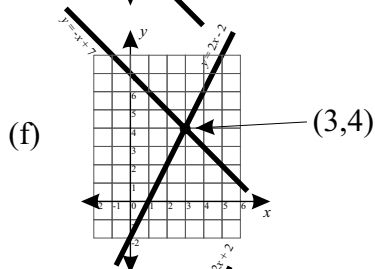
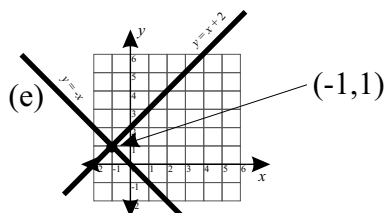
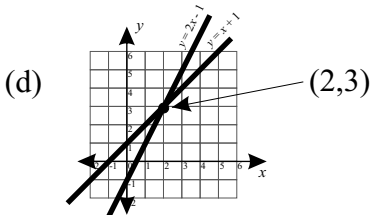
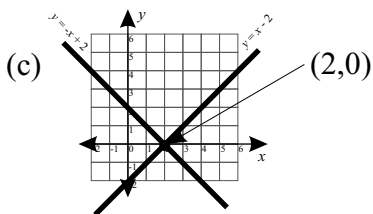
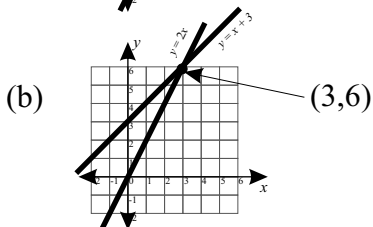
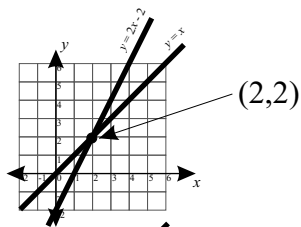
Exercise 9G

- $y = 2x + 3$
 - $y = 4x - 1$
 - $y = x - 2$
 - $y = -3x + 5$
 - $y = -x - 2$
 - $y = \frac{1}{2}x + 5$
 - $y = -\frac{1}{4}x - 3$
 - $y = \frac{2}{3}x + 7$
 - $y = 2x$
 - $y = -\frac{1}{5}x$
- $y = 2x - 8$
 - $y = -2x + 12$
 - $y = \frac{1}{2}x + 4$
 - $y = -\frac{2}{5}x - 4$
- x-intercept = $\frac{3}{2}$, y-intercept = -6
 - x-intercept = -2, y-intercept = -6
 - x-intercept = 0, y-intercept = 0
 - x-intercept = $-\frac{13}{4}$, y-intercept = 13
- $y = 3x - 2$
 - $y = 4x - 3$
 - $y = -2x$
 - $y = -5x - 32$
 - $y = 2x$
 - $y = \frac{2}{5}x - 7$
 - $y = -\frac{7}{3}x + 28$
- $y = -3x + 11$
 - $y = -\frac{1}{4}x + 5$
 - $y = -\frac{2}{5}x + 2$
 - $y = 5x - 17$
 - $y = \frac{7}{4}x - 7$
 - $y = -\frac{3}{8}x - \frac{9}{2}$
 - $y = -\frac{9}{4}x + \frac{29}{4}$
- Gradient of **AB** = 2
Gradient of **AC** = $-\frac{1}{2}$
The product of these two gradients is -1
so they must be perpendicular.
The right-angle is at point **A**.

7. (a)

(b) Gradient of $EG = -2$ Gradient of $DF = \frac{1}{2}$ (c) The product of these gradients is -1 so the diagonals are perpendicular.**Exercise 9H**

1. (a)



2. (a) (2,2) (b) (3,6) (c) (4,9)
 (d) (1,1) (e) (-2,-5) (f) (-2,-7)
 (g) (2,8) (h) (4,11)
3. (a) (3,2) (b) (2,1) (c) (-1,6)
 (d) (1,5) (e) (2,5) (f) (-1,-2)
 (g) (2,-3) (h) (-2,-2)
4. (a) (1,2) (b) (3,4) (c) (-1,3)
 (d) (-2,1) (e) (-1,-3) (f) (-2,-3)
 (g) (5,-2) (h) (-4,4)
5. (a) (1,1) (b) (2,2) (c) (3,2)
 (d) (3,-2) (e) (3,-2) (f) (-2,-3)
 (g) (0,-5) (h) (-3,-4)
6. (a) (4,2) (b) (-8,6) (c) (1,2)
 (d) (2,5) (e) (-3,3) (f) (2,-3)
 (g) (-3,-2) (h) (-5,0)
7. (a) (4,3) (b) (-1,4) (c) (1,-2)
 (d) (1,1) (e) (-2,-3) (f) (-2,6)
 (g) (-3,-3) (h) (0,4)
8. (a) (3,2) (b) (1,3) (c) (-1,4)
 (d) (-2,3) (e) (-3,-1) (f) (2,-6)
 (g) (-2,3) (h) (1,1) (i) (2,-4)
 (j) (-3,-2) (k) (-3,-2) (l) (2,2)
9. (a) (2,1) (b) (-2,-1) (c) (-3,2)
 (d) (4,-2) (e) (-2,-2) (f) (5,-2)
 (g) (-4,3) (h) (6,5) (i) (4,-4)
 (j) (6,-5) (k) (-3,8) (l) (-3,5)
10. (a) (2,4) (b) (1,3) (c) (1,-1)
 (d) (-2,2) (e) (-1,3) (f) (-4,3)
 (g) (-1,2) (h) (5,-3) (i) (-2,-3)
 (j) (-2,0) (k) (2,3) (l) (-1,-2)
 (m) (-3,-2) (n) (2,4) (o) (1,1)
 (p) (2,-1)

Exercise 9I

1. salad roll = \$4, sandwich = \$3
 2. shirt = \$35, jeans = \$85
 3. large = 0.75 kg, small = 0.25 kg
 4. Terry - 120, Damon - 80
 5. chimpanzee - 4, baboon - 6

Chapter Review

1. (a)

x	-3	-2	-1	0	1	2	3
y	-13	-10	-7	-4	-1	2	5

(b)

m	-3	-2	-1	0	1	2	3
n	9	7	5	3	1	-1	-3

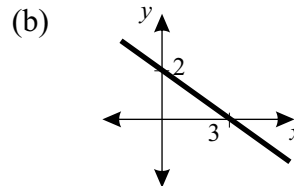
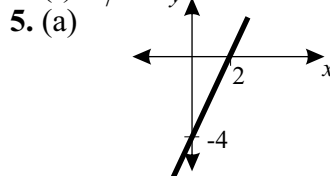
2. (a) $y = 2x + 3$ (b) $a = 3b + 5$

3. (a)

T	0	2	5	10	25
V	600	1000	1600	2600	5600

(b) $V = 600 + 200T$ (c) 37 minutes

4. (a) 5 (b) 3 (c) 4 (d) $\frac{1}{2}$ (e) -1
 (f) $-\frac{4}{7}$



6. (a) (2,0) and (0,-4)
 (b) (3,0) and (0,2)

7. (a) (5,0), (0,5) (b) (-3,0), (0,2)
 (c) (-1,0) (0,1) (d) (4,0) (0,-12)

8. (a) $m = 3, c = -5$
 (b) $m = -1, c = 2$
 (c) $m = \frac{2}{3}, c = -2$
 (d) $m = -4, c = -\frac{1}{2}$

9. (a) $y = 2x + 6$ (b) $y = -2x - 3$
 (c) $y = \frac{3}{4}x$

10. (a) $y = -5x + 3, m = -5, c = 3$
 (b) $y = \frac{3}{2}x - 3, m = \frac{3}{2}, c = -3$
 (c) $y = \frac{4}{3}x + 4, m = \frac{4}{3}, c = 4$

11. (a) $y = 2x + 4$ (b) $y = -3x + 6$

12. (a) $y = -2x + 6$ (b) $y = \frac{3}{4}x - 9$

13. x-intercept = 3, y-intercept = -12

14. (a) $y = -4x + 5$ (b) $y = -\frac{3}{2}x - 20$

15. (a) (5,1) (b) (-2,1) (c) (-2,4)
 (d) (-3,2)

16. maths - 0.8 kg, English - 0.6 kg

Chapter 10

Exercise 10A

1. B, C, F, H, J, K

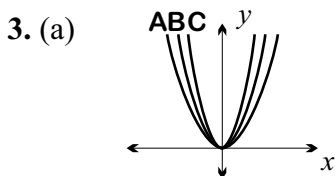
2. (a)

x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

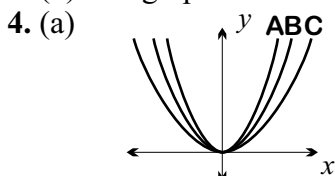


(b)

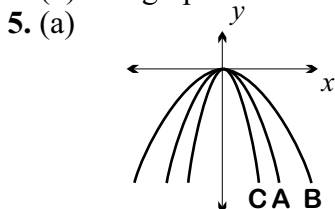
x	-5	-4	-3	-2	-1	0	1
y	6	1	-2	-3	-2	1	6

(b) The graph becomes thinner



(b) The graph becomes flatter



(b) The graph is inverted

6. (a) B, C, E

(b) A, D, F

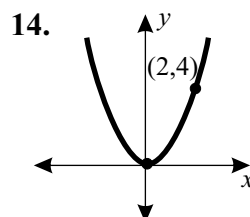
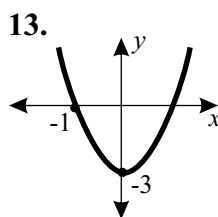
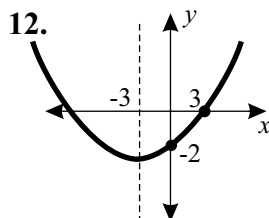
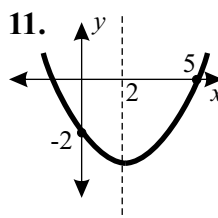
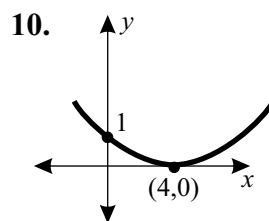
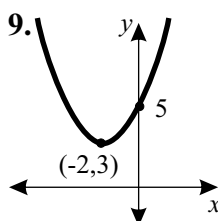
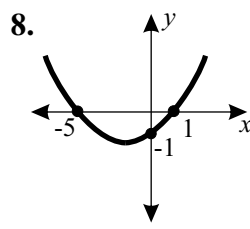
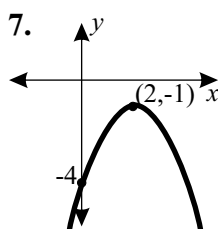
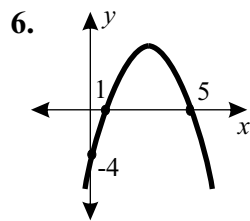
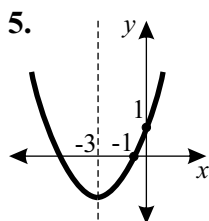
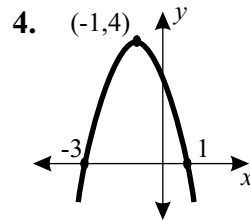
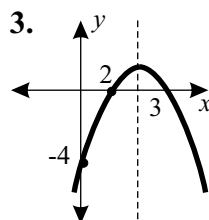
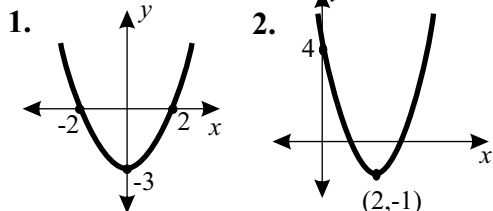
7. (a) A, B, D, E, H, I, J, N, O

(b) C, F, G, K, L, M, P

(c) A, C, E, F, K, L, M, P

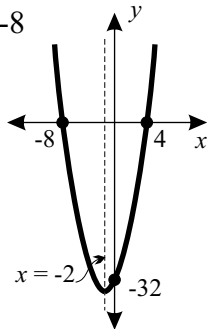
(d) D, G, H, J, N, O

Exercise 10B

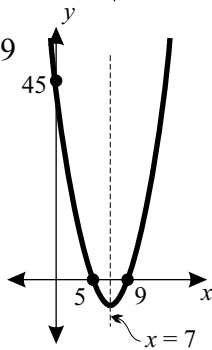


Exercise 10C

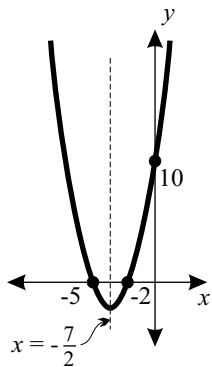
1. (a)
- x
- ints = 4 and -8

 y -int = -32axis of
symmetry $x = -2$ 

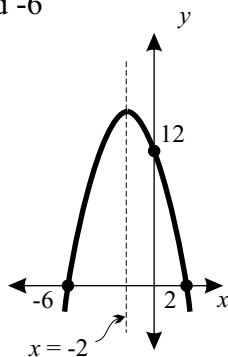
- (b)
- x
- ints = 5 and 9

 y -int = 45axis of
symmetry $x = 7$ 

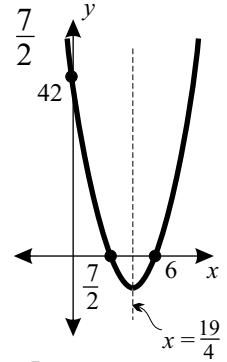
- (c)
- x
- ints = -2 and -5

 y -int = 10axis of
symmetry $x = -\frac{7}{2}$ 

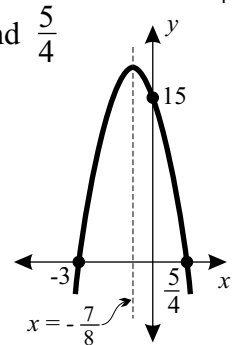
- (d)
- x
- ints = 2 and -6

 y -int = 12axis of
symmetry $x = -2$ 

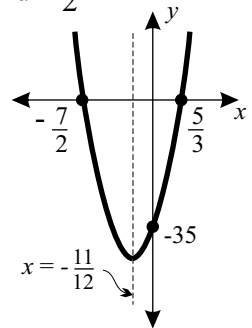
- (e)
- x
- ints = 6 and
- $\frac{7}{2}$

 y -int = 42axis of
symmetry $x = \frac{19}{4}$ 

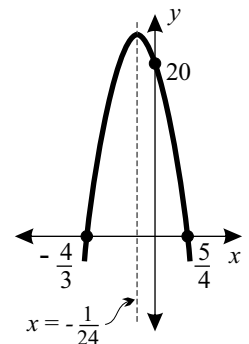
- (f)
- x
- ints = -3 and
- $\frac{5}{4}$

 y -int = 15axis of
symmetry $x = -\frac{7}{8}$ 

- (g)
- x
- ints =
- $\frac{5}{3}$
- and
- $-\frac{7}{2}$

 y -int = -35axis of
symmetry $x = -\frac{11}{12}$ 

- (h)
- x
- ints =
- $\frac{5}{4}$
- and
- $-\frac{4}{3}$

 y -int = 20axis of
symmetry $x = -\frac{1}{24}$ 

Exercise 10D

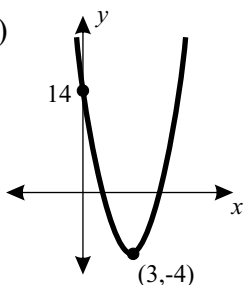
1. (a) $(-2, -36)$ (b) $(7, -4)$ (c) $(-\frac{7}{2}, -\frac{9}{4})$
 (d) $(-2, 16)$ (e) $(\frac{19}{4}, -\frac{25}{8})$
 (f) $(-\frac{7}{8}, \frac{289}{16})$ (g) $(-\frac{11}{12}, -\frac{961}{24})$
 (h) $(-\frac{1}{24}, \frac{961}{48})$

Exercise 10E

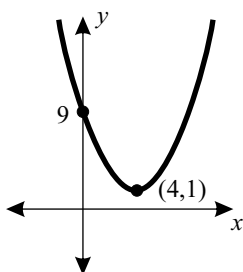
1. (a) $(5, -23)$ (b) $(8, -40)$ (c) $(-2, -7)$
 (d) $(-3, -4)$ (e) $(-1, -9)$ (f) $(7, -48)$
 (g) $(10, -100)$ (h) $(-9, -81)$
 2. (a) $(\frac{5}{2}, -\frac{13}{4})$ (b) $(\frac{1}{2}, \frac{15}{4})$
 (c) $(-\frac{3}{2}, -\frac{41}{4})$ (d) $(-\frac{9}{2}, -\frac{93}{4})$
 3. (a) $(2, -1)$ (b) $(-6, 33)$ (c) $(2, -8)$
 (d) $(-2, 27)$ (e) $(-\frac{5}{4}, \frac{89}{8})$
 (f) $(\frac{3}{4}, -\frac{39}{4})$ (g) $(\frac{3}{2}, \frac{5}{4})$
 (h) $(-\frac{2}{3}, \frac{53}{3})$

Exercise 10F

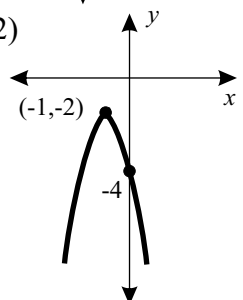
1. (a) T.P. = $(3, -4)$
 y -int = 14



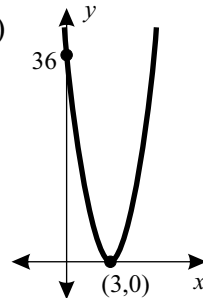
- (b) T.P. = $(4, 1)$
 y -int = 9



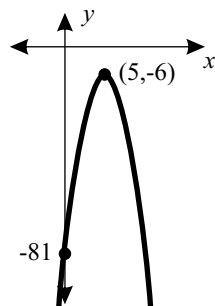
- (c) T.P. = $(-1, -2)$
 y -int = -4



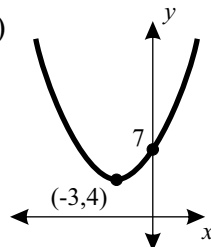
- (d) T.P. = $(3, 0)$
 y -int = 36



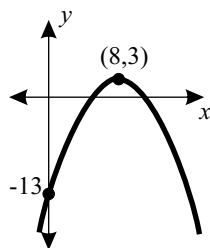
- (e) T.P. = $(5, -6)$
 y -int = -81



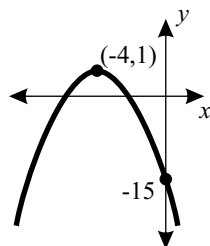
- (f) T.P. = $(-3, 4)$
 y -int = 7



- (g) T.P. = $(8, 3)$
 y -int = -13

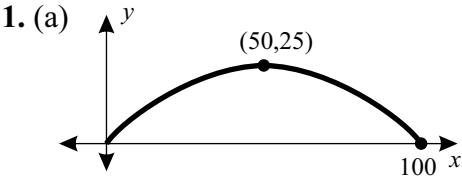


- (h) T.P. = $(-4, 1)$
 y -int = -15

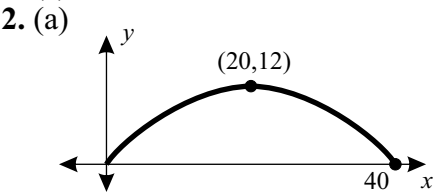


2. (a), (d) touches the x -axis once,
 (g), (h)

Exercise 10G



(b) $y = -\frac{1}{100}(x - 50)^2 + 25$
(c) 16 m



(b) $y = -\frac{3}{100}(x - 20)^2 + 12$
(c) 5.25 m

3. (a) **A** - 100 m, **B** - 120 m
(b) **A** - 40 m, **B** - 30 m
(c) **A** - 33.6 m, **B** - 22.5 m

4. (a) $y = \frac{1}{160}(x - 40)^2 + 30$
(b)

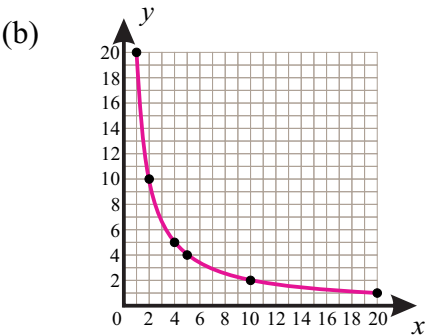
Distance from A (m)	0	10	20	30	40	50	60	70	80
Height above ground (m)	40	35.6	32.5	30.6	30	30.6	32.5	35.6	40

5. (a) **A** and **K** = 50 m
(b) **B** and **J** = 41 m, **C** and **I** = 34 m,
D and **H** = 29 m, **E** and **G** = 26 m,
F = 25 m

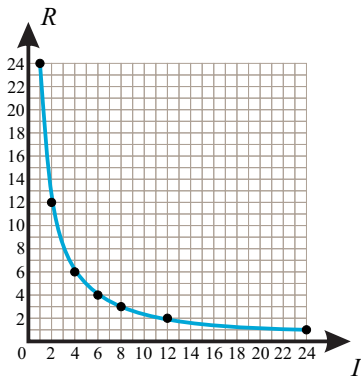
Exercise 10H

1. (a)

x	1	2	4	5	10	20
y	20	10	5	4	2	1

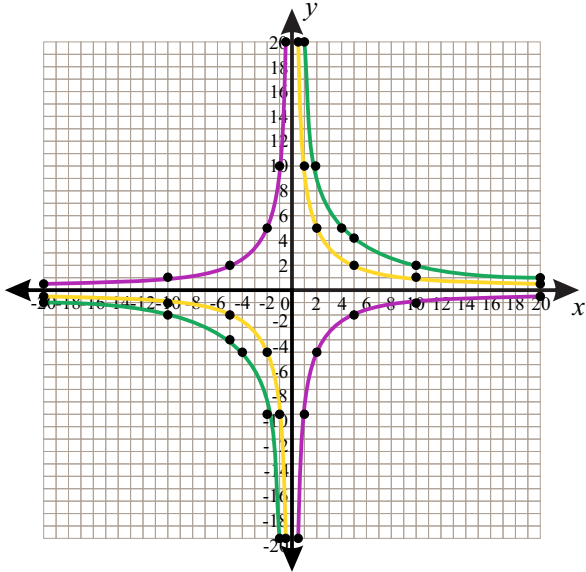


2. (a) 100
(b)(i) 4 (ii) 0.5 (iii) 200 (iv) 400
3. (a) 2.5
(b)(i) 0.05 (ii) 5 (iii) 25 (iv) 100
4. (a) 24
(b)(i) 240 (ii) 120 (iii) 20 (iv) 0.5
(v) 0.1
(c)



5. (a)

x	-20	-10	-5	-2	-1	-0.5	0.5	1	2	5	10	20
y	-0.5	-1	-2	-5	-10	-20	20	10	5	2	1	0.5

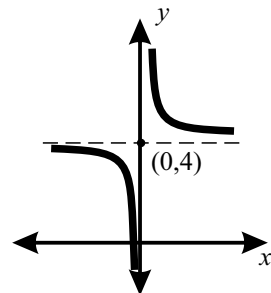


6. (a)

x	-20	-10	-5	-4	-2	-1	1	2	4	5	10	20
y	-1	-2	-4	-5	-10	-20	20	10	5	4	2	1

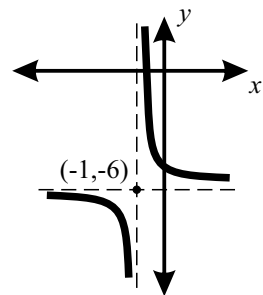
7. (a)

x	-20	-10	-5	-2	-1	-0.5	0.5	1	2	5	10	20
y	0.5	1	2	5	10	20	-20	-10	-5	-2	-1	-0.5

Exercise 10I1. (a) **D** (b) **A** (c) **C** (d) **E** (e) **B**2. (a) **J** (b) **F** (c) **I** (d) **K** (e) **H** (f) **G** (d) (0,4)**Exercise 10J**1. **D**2. **C**3. **C**4. **B**

5. (a) $y = \frac{1}{(x-3)} + 7$

(e) (-1,-6)



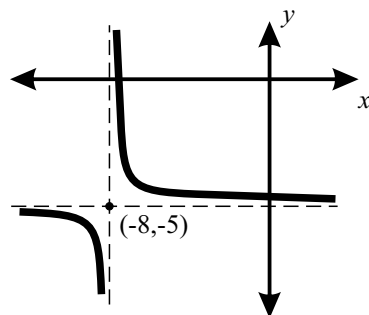
(b) $y = \frac{1}{(x+5)}$

(c) $y = \frac{1}{(x+1)} - 5$

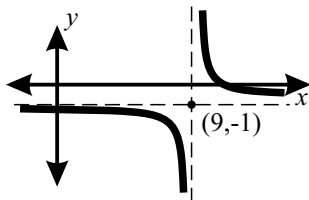
(d) $y = \frac{1}{x} + 8$

(e) $y = \frac{1}{(x-6)} - 5$

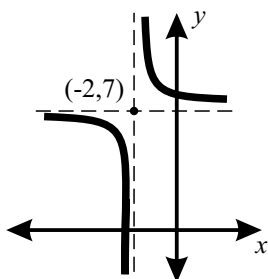
(f) (-8,-5)



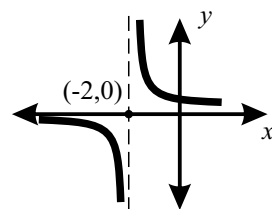
6. (a) (9,-1)



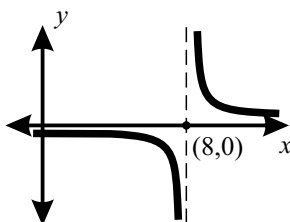
(b) (-2,7)



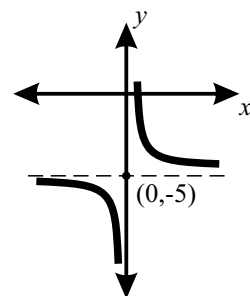
(g) (-2,0)



(c) (8,0)



(h) (0,-5)



Exercise 10K

1. (a) $x^2 + y^2 = 25$
(b) $x^2 + (y - 3)^2 = 16$
(c) $(x - 4)^2 + y^2 = 1$
(d) $(x + 5)^2 + y^2 = 9$
(e) $(x - 2)^2 + (y - 4)^2 = 36$
(f) $(x + 3)^2 + (y - 5)^2 = 16$
(g) $(x - 6)^2 + (y + 5)^2 = 49$
(h) $(x - 7)^2 + (y - 9)^2 = 144$
(i) $(x + 5)^2 + (y + 8)^2 = 100$
(j) $(x + 6)^2 + (y + 1)^2 = 81$
2. (a) centre (0,0), radius = 8
(b) centre (7,0), radius = 1
(c) centre (-3,5), radius = 7
(d) centre (0,2), radius = 9
(e) centre (6,1), radius = 6
(f) centre (-2,-9), radius = 10
(g) centre (12,-15), radius = 12
(h) centre (-13,11), radius = 13
3. (a) $x^2 + (y - 1)^2 = 16$
(b) $(x - 2)^2 + (y + 3)^2 = 9$
(c) $(x - 1)^2 + y^2 = 25$
(d) $(x + 3)^2 + (y + 5)^2 = 1$
4. $(x - 3)^2 + (y + 1)^2 = 25$

Exercise 10L

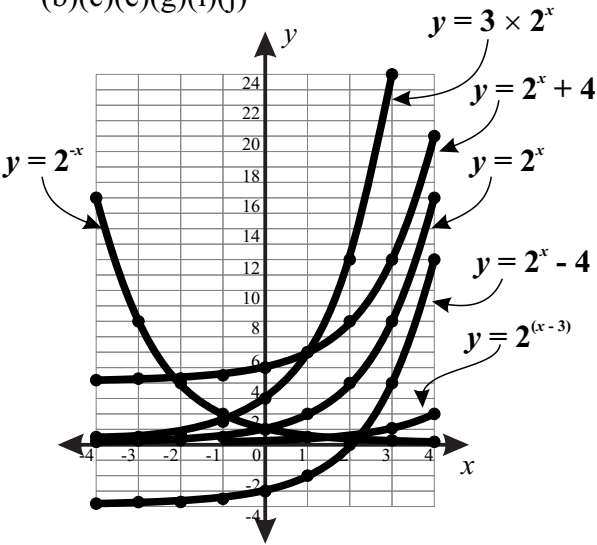
1. centre (1,5), radius = 6
2. centre (3,-10), radius = 10
3. centre (0,6), radius = 1
4. centre (-4,0), radius = 4
5. centre (6,-3), radius = 7
6. centre (12,-5), radius = 13
7. centre (-7,8), radius = 10
8. centre (10,11), radius = 15

Exercise 10M

1. (a)

x	-4	-3	-2	-1	0	1	2	3	4
y	0.0625	0.125	0.25	0.5	1	2	4	8	16

(b)(c)(e)(g)(i)(j)



(d)

x	-4	-3	-2	-1	0	1	2	3	4
y	16	8	4	2	1	0.5	0.25	0.125	0.0625

(f)

x	-4	-3	-2	-1	0	1	2	3
y	0.1875	0.375	0.75	1.5	3	6	12	24

(h)

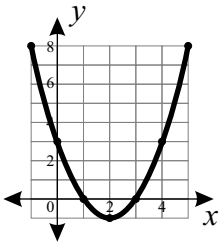
x	0	1	2	3	4
y	0.125	0.25	0.5	1	2

2. A $y = 3^{-x}$
B $y = 3^{(x+2)}$
C $y = 3^x + 2$
D $y = 3^x - 2$
E $y = 3^{(x-2)}$

Chapter Review

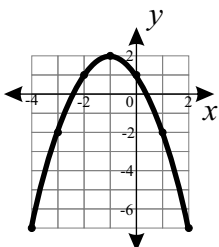
1. (a)

x	-1	0	1	2	3	4	5
y	8	3	0	-1	0	3	8

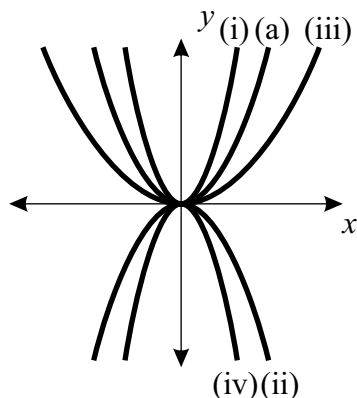


(b)

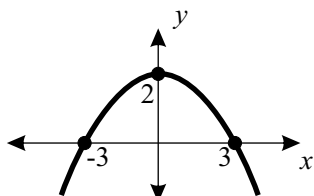
x	-4	-3	-2	-1	0	1	2
y	-7	-2	1	2	1	-2	-7



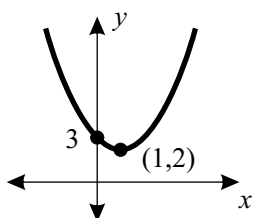
2.

3. (a) **A, C, D** (b) **B**

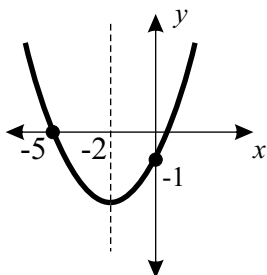
4. (a)



(b)



(c)

5. (a) x -ints = 3 and 5, y -int = 15

T.P. = (4, -1)

(b) x -ints = 4 and -4, y -int = -16

T.P. = (0, -16)

(c) x -ints = 2 and -8, y -int = -16

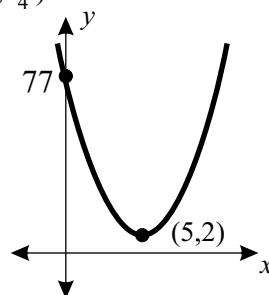
T.P. = (-3, -25)

(d) x -ints = 0 and 8, y -int = 0

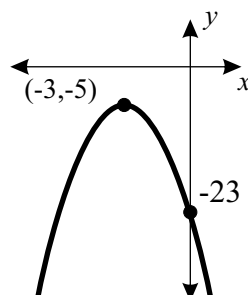
T.P. = (4, -16)

6. (a) $(-5, -34)$ (b) $(\frac{5}{2}, \frac{7}{4})$ (c) (3, 2)(d) $(-\frac{3}{4}, \frac{61}{4})$

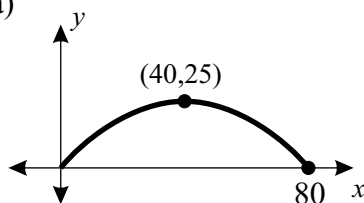
7. (a)



(b)



8. (a)

(b) $y = -\frac{1}{64}(x - 40)^2 + 25$

(c) 10.9 m

9. (a) $y = \frac{1}{250}(x - 50)^2 + 40$

(b)

Distance from A (m)	0	20	40	60	80	100
Height above ground (m)	50	43.6	40.4	40.4	43.6	50

10. (a) $y = \frac{5}{(x-7)} + 4$

(b) $y = \frac{5}{(x+3)} - 6$

11. $y = \frac{3}{(x+6)}$

12. (a) (5,4) (b) (0,-2)

13. (a) $(x-2)^2 + y^2 = 36$

(b) $(x-7)^2 + (y+5)^2 = 64$

14. (a) centre (-3,7), radius = 9

(b) centre (0,-4), radius = 1

15. $(x-3)^2 + (y+2)^2 = 25$

16. A $y = 4^{-x}$

B $y = 4^{(x+3)}$

C $y = 4^x + 3$

D $y = 4^x - 3$

E $y = 4^{(x-3)}$

Chapter 11

Exercise 11A

- (a) 17 000 km (b) 3.1 m
(c) 40 000 km (d) 490 km
(e) 36 000 km (f) 2.45 m
(g) 272 cm (h) 60 000 km
(i) 8.95 m (j) 132.6 m
(k) 11 300 m (l) 4.23 m
(m) 1800 km (n) 33.6 m
(o) 10 km (p) 10 m
- (a) 60 mm (b) 800 cm
(c) 20 000 m (d) 2360 mm
(e) 428.5 mm (f) 680 m
(g) 75 cm (h) 0.75 m
(i) 76.85 km (j) 0.001 m
(k) 798 cm (l) 0.0276 m
(m) 0.3 cm (n) 23.57 mm
(o) 578.1 m (p) 30 000 cm
(q) 430 000 mm (r) 1.7 m
(s) 0.065 mm (t) 0.0476 m
(u) 0.0006 cm (v) 0.00605
(w) 0.06 km (x) 0.05645 km

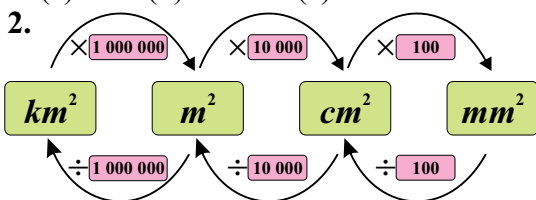
- (a) 2.11 m (b) 3.2 cm (c) 6.06 m
(d) 20.2 m (e) 5.02 km
(f) 18.05 m (g) 5.08 m
(h) 6.238 m (i) 4.007 m
(j) 20.4 cm (k) 3.882 m
(l) 10.057 m
- (a) 3 m (b) 66 m (c) 14 m
(d) 30 m (e) 16 m (f) 20 m
(g) 3 m (h) 3 m (i) 16 m (j) 4 m
(k) 8 m (l) 10 m (m) 11 m
(n) 6 m (o) 48 m
- (a) 3 cm (b) 3 cm (c) 16 cm
(d) 1 cm (e) 14 cm (f) 26 cm
(g) 1 cm (h) 7 cm (i) 20 cm
- (a) 750 mm (b) 56 000 strides
(c) 326 seconds
- (a) 307.32 m (b) 2.4 steps/sec

Exercise 11B

- (a) 26.4 cm (b) 14.4 m (c) 13.9 m
- (a) 49.3 m (b) 149.5 mm
- (a) 12.6 m (b) 71.4 mm
- (a) 26.2 m (b) 41.5 m (c) 28.7 m
- (a) 31.2 m (b) 12.2 m (c) 40.6 cm
(d) 84.6 m (e) 43.0 m (f) 194.0 cm
(g) 30.0 m (h) 234.9 cm (i) 366.7 m
- (a) 14.1 cm (b) 15.4 cm
(c) 19.4 cm (d) 21.2 cm
- (a) 17 052 315 revolutions
(b) 43 982 km
- 6856 mm
- (a) 18 301 mm (b) 183 sec

Exercise 11C

- (a) 100 (b) 10 000 (c) 1 000 000



- (a) 5 cm² (b) 20 mm² (c) 70m²
(d) 34 000 cm² (e) 1900 m² (f) 6 m²
(g) 310 cm² (h) 8300 m² (i) 70 km²
(j) 630 m²

4. (a) 6 ha (b) 23 ha (c) 0.4 ha
(d) 8.6 ha (e) 300 ha (f) 560 ha
(g) 7 ha (h) 32 000 ha
5. (a) 176 000 mm² (b) 1760 cm²
(c) 0.176 m²
6. 14.4 m²
7. 17.5 ha
8. 529 ha

Exercise 11D

1. (a) 159.6 cm² (b) 75.69 m²
(c) 56.42 cm² (d) 2031.02 mm²
(e) 103.95 mm² (f) 1066.02 cm²
(g) 28.35 m² (h) 138.765 m²
2. (a) 141.0 m² (b) 607.0 cm²
(c) 75.4 cm² (d) 2724.7 m²
(e) 86.0 cm² (f) 8499.8 mm²
(g) 121.0 m² (h) 2874.0 cm²

Exercise 11E

1. (a) 846.5 cm² (b) 2345.2 cm²
(c) 236.5 cm² (d) 383.7 cm²
(e) 141.2 cm² (f) 36.2 m²
2. (a) 1521.7 cm² (b) 16773.3 cm²

Exercise 11F

1. 36.1 m²
2. 182.9 cm²
3. 8.7 m²
4. 255.7 cm²

Exercise 11G

1. (a) 300 hectares (b) 20%
2. (a) 900 m² (b) 800 m² (c) 675 m²
(d) 576 m² (e) 500 m²
3. E
4. \$4160

Exercise 11H

1. (a) 108 m² (b) 579.42 cm²
(c) 8.1 m² (d) 35.02 m²
(e) 520 cm² (f) 1056 cm²
(g) 8500 cm² (h) 41.65 m²
2. (a) 1407.4 cm² (b) 88.0 m²
(c) 215.1 m² (d) 578.1 cm²
(e) 6141.8 cm² (f) 9.9 m²
(g) 201.1 cm² (h) 4071.5 cm²
(i) 7238.2 cm²
3. (a) 29 044.3 cm² (b) 63.4 m²
(c) 260.1 m² (d) 6144.0 m²
(e) 2259.7 cm² (f) 149.8 m²
(g) 7068.6 cm² (h) 110.5 m²
4. (a) 246 cm² (b) 408 cm²
(c) 2555.1 cm² (d) 5543.8 cm²
(e) 10204.9 cm² (f) 38.3 m²
5. \$2197.17
6. \$3187.46
7. (a) 1248 (b) 4992

Exercise 11I

1. (a) 50 L (b) 20 mm³ (c) 0.6 L
(d) 8100 L (e) 250 L (f) 890 cm³
(g) 0.3 m³ (h) 7000 mm³ (i) 8 mL
(j) 0.1 L (k) 2400 cm³ (l) 0.75 cm³
2. 10.5 L
3. 40 days
4. 6 hours

Exercise 11J

1. (a) 210 cm³ (b) 420 cm³ (c) 280 cm³
(d) 30 m³ (e) 6.4 m³ (f) 2.64 m³
2. (a) 226.2 m³ (b) 9.5 m³
(c) 62 831.9 cm³
3. (a) 4188.8 cm³ (b) 65.4 m³
(c) 16 210.2 cm³
4. (a) 3769.9 cm³ (b) 12.6 m³
(c) 80 424.8 cm³
5. (a) 1906.7 cm³ (b) 10.1 m³
6. (a) 136 350 cm³ (b) 36 285.4 cm³
(c) 21 700 cm³ (d) 40 780.0 cm³

7. (b) 7.2 L
 8. 17496 cm^3
 9. (a) 544 L (b) 896 L (c) 1080 L
 (d) 1120 L (e) 1040 L
 10. 1200 kg
 11. 125 mm
 12. 27.1%
 13. (a) 2604 (b) 141
 14. 120

Chapter Review

1. (a) 140 mm (b) 830 cm (c) 4000 m
 (d) 560 mm (e) 97.5 mm (f) 1260 m
 (g) 8.3 cm (h) 4.5 m (i) 3.45 km
 (j) 0.029 m (k) 55 mm (l) 35.6 m
 2. (a) 2.11 m (b) 3.2 cm (c) 6.06 m
 (d) 8.075 m
 3. (a) 5 m (b) 7 m (c) 3 m (d) 3 m
 (e) 785 m (f) 399 m
 4. 320
 5. (a) 21.4 m (b) 148.3 cm
 6. (a) 32.4 cm (b) 58.3 m
 (c) 15.0 m (d) 300.0 mm
 (e) 30.1 m (f) 194.9 cm
 7. 10 610
 8. (a) 8 cm^2 (b) 26 mm^2 (c) 15 m^2
 (d) 8400 cm^2 (e) 70 km^2
 (f) 8300 m^2
 9. (a) 15 ha (b) 700 m
 10. (a) 418 cm^2 (b) 87.33 cm^2
 (c) 8.4 m^2 (d) 6099.66 m^2
 11. (a) 22.9 m^2 (b) 149.6 cm^2
 (c) 482.2 cm^2 (d) $14\,015.9 \text{ mm}^2$
 12. 1603.5 m^2
 13. 57.3 cm^2
 14. 50.3 cm
 15. (a) 69.56 m^2 (b) 600 cm^2
 (c) $14\,325 \text{ cm}^2$
 16. (a) 37.7 m^2 (b) 777.5 cm^2
 (c) 394.1 cm^2
 17. (a) 54.2 m^2 (b) $26\,170.3 \text{ cm}^2$
 (c) 3.4 m^2 (d) 3760.5 cm^2

18. (a) 80 L (b) 35 mm^3 (c) 0.7 L
 (d) 3200 L (e) 35 mL (f) 90 cm^3
 (g) 0.8 m^3 (h) 700 mm^3 (i) 8 mL
 (j) 0.1 L (k) 2400 cm^3
 (l) 0.75 cm^3
 19. 45
 20. (a) 360 cm^3 (b) 864 cm^3
 (c) 21.85 m^3
 21. (a) 86.5 m^3 (b) $150\,532.6 \text{ cm}^3$
 (c) 9684.5 cm^3 (d) 2933.3 cm^3
 (e) 2278.7 cm^3
 22. 5440 cm^3
 23. 341 mm

Chapter 12

Exercise 12A

1. (a) 5 m (b) 13 cm (c) 39 m
 (d) 25 mm (e) 25 m (f) 37 cm
 (g) 89 m (h) 225 cm (i) 481 m
 2. (a) 7.8 m (b) 13.5 cm (c) 20.7 cm
 3. (a) $x = 21.8 \text{ cm}$ (b) $y = 28.3 \text{ mm}$
 (c) $a = 18.6 \text{ cm}$, $b = 20.7 \text{ cm}$
 (d) $m = 26.6 \text{ cm}$ (e) $c = 8.6 \text{ m}$
 (f) $s = 2.8 \text{ m}$ (g) $p = 29.1 \text{ cm}$
 (h) $k = 3.0 \text{ m}$

Exercise 12B

1. (a) 7 cm (b) 84 m (c) 30 mm
 (d) 18 m (e) 15 m (f) 21 mm
 (g) 55 m (h) 36 m (i) 60 m
 2. (a) 7.1 m (b) 13.9 cm (c) 13.0 cm
 3. (a) $x = 5.7 \text{ m}$
 (b) $c = 12.4 \text{ cm}$, $d = 10.8 \text{ cm}$
 (c) $k = 19.1 \text{ cm}$ (d) $t = 4.5 \text{ m}$
 (e) $w = 5.2 \text{ m}$ (f) $y = 23.7 \text{ m}$

Exercise 12C

1. 35.4 cm
 2. 105 cm
 3. (a) $a = 12.2 \text{ m}$ (b) $b = 13.8 \text{ m}$
 (c) $p = 36.1 \text{ m}$, $q = 17.3 \text{ m}$
 (d) $x = 23.9 \text{ cm}$, $y = 6.3 \text{ cm}$
 (e) $m = 6.3 \text{ m}$, $n = 16.1 \text{ m}$
 (f) $d = 14.9 \text{ cm}$

Exercise 12D

- (a) 151.7 km (b) 179.0 km
- (a) 5071 mm (b) to the right
- 4334 mm

Exercise 12E

- 128.8 cm 2. 5.4 m
- 23.7 cm 4. 4.1 m

Chapter Review

- $x = 15.1$ cm, $y = 5.1$ cm
- 19.5 km 3. 9.9 m 4. 9.9 m

Chapter 13**Exercise 13A**

- $a = 36^\circ$ 2. $b = 129^\circ$ 3. $c = 165^\circ$
- $d = 117^\circ$, $e = 63^\circ$, $f = 117^\circ$
- $g = 68^\circ$, $h = 68^\circ$, $i = 68^\circ$, $j = 112^\circ$

Exercise 13B

- (a) $a = 62^\circ$ (b) $b = 35^\circ$ (c) $c = 33^\circ$
(d) $d = 44^\circ$ (e) $e = 45^\circ$ (f) $f = 66^\circ$
(g) $g = 22.5^\circ$ (h) $h = 58^\circ$
- (a) $a = 120^\circ$ (b) $x = 116^\circ$, $y = 64^\circ$,
 $z = 116^\circ$ (c) $c = 34^\circ$ (d) $d = 27^\circ$
(e) $e = 107^\circ$ (f) $f = 104^\circ$ (g) $g = 28^\circ$

Exercise 13C

- 28 2. (a) 60° (b) 24°
- (a) major segment (b) diameter
(c) chord (d) minor sector
(e) tangent (f) minor segment
(g) major sector
- (a) 31° (b) 133°

Exercise 13D

- (a) $x = 50^\circ$, $y = 50^\circ$
(b) $m = 25^\circ$, $n = 25^\circ$
(c) $c = 56^\circ$ (d) $d = 21^\circ$
- (a) $a = 110^\circ$ (b) $b = 63^\circ$
(c) $c = 94^\circ$ (d) $d = 214^\circ$
(e) $e = 113^\circ$ (f) $f = 21.5^\circ$
- (a) $a = 90^\circ$ (b) $b = 58^\circ$

- (a) $x = 97^\circ$, $y = 86^\circ$
(b) $p = 117^\circ$, $q = 74^\circ$
- (a) $a = 45^\circ$ (b) $b = 55^\circ$
(c) $c = 64^\circ$ (d) $d = 22^\circ$
(e) $e = 31^\circ$ (f) $f = 59^\circ$
(g) $g = 146^\circ$ (h) $h = 120^\circ$
(i) $i = 37^\circ$ (j) $j = 58^\circ$
- (a) isosceles
(b) radii are the same length
(c) equal
- (a) $a = 50^\circ$ (b) $b = 55^\circ$
(c) $c = 106^\circ$ (d) $d = 136^\circ$
(e) $e = 29^\circ$ (f) $f = 27^\circ$
(g) $g = 66^\circ$ (h) $h = 59.5^\circ$
(i) $i = 61^\circ$ (j) $j = 58^\circ$
- (a) isosceles (b) x (c) $180 - 2x$
(d) $2x$ (e) isosceles (f) y
(g) $180 - 2y$ (h) $2y$ (i) $x + y$
(j) $2x + 2y$ (k) Theorem 2

Exercise 13E

A and **L**, **B** and **K**, **C** and **N**, **D** and **J**,
E and **P**, **F** and **H**, **G** and **I**, **M** and **O**

Exercise 13F

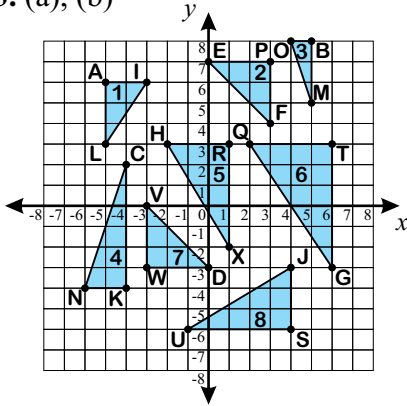
- (a) $a = 14$ (b) $b = 21$
(c) $x = 15$, $y = 24$
(d) $m = 7.5$, $n = 6$
(e) $g = 12$, $h = 10$
(f) $v = 10$, $w = 12.5$
(g) $g = 24$ (h) $h = 16.2$
(i) $i = 18$ (j) $j = 10.5$
(k) $a = 13.5$, $b = 10.5$
(l) $p = 44$, $q = 55$
(m) $a = 12.75$, $b = 7.5$
(n) $n = 20$ (o) $k = 8$
- 16 m 3. 3 m 4. 300 m 5. 280 m
- 108 m

Exercise 13G

A and **C**, **B** and **J**, **D** and **K**, **E** and **L**,
F and **I**, **G** and **H**

Exercise 13H

1. **A** and **E**, **F** and **H**
 2. **B** and **D**, **C** and **J**, **G** and **I**
 3. (a), (b)



- (c)(i) **1** and **6**, **3** and **4**
 (ii) **2** and **7**, **5** and **8**

Chapter Review

1. (a) $a = 57^\circ$ (b) $b = 53^\circ$
 (c) $c = 106^\circ$
 (d) $p = 69^\circ$, $q = 69^\circ$, $r = 111^\circ$
 (e) $g = 63^\circ$, $h = 63^\circ$, $i = 117^\circ$, $j = 117^\circ$
 2. (a) $a = 60^\circ$ (b) $b = 109^\circ$
 (c) $c = 29^\circ$ (d) $d = 61^\circ$
 (e) $e = 45^\circ$ (f) $f = 50^\circ$
 3. (a) $a = 122^\circ$
 (b) $x = 67^\circ$, $y = 67^\circ$, $z = 113^\circ$
 (c) $c = 34^\circ$ (d) $d = 36^\circ$
 (e) $e = 111^\circ$ (f) $f = 107^\circ$ (g) $g = 46^\circ$
 4. (a) **A** - minor segment
B - major segment
C - chord
D - major arc
E - minor arc
F - minor sector
G - major sector
H - circumference
I - diameter
J - radius
K - tangent

5. (a) $a = 39^\circ$ (b) $b = 123^\circ$
 (c) $x = 55^\circ$, $y = 55^\circ$
 (d) $m = 21^\circ$, $n = 21^\circ$
 (e) $e = 53^\circ$ (f) $f = 122^\circ$ (g) $g = 46^\circ$
 (h) $h = 224^\circ$ (i) $i = 116.5^\circ$
 (j) $j = 19.5^\circ$ (k) $k = 90^\circ$ (l) $l = 57^\circ$
 (m) $m = 83^\circ$ (n) $n = 45^\circ$
 (o) $o = 29.5^\circ$ (p) $p = 65^\circ$
 (q) $q = 105^\circ$ (r) $r = 60^\circ$ (s) $s = 52^\circ$
 (t) $t = 28^\circ$

6. (a) **A** and **C**, **B** and **F**, **E** and **I**,
D and **H**, **G** and **H**
 (b) **D** and **G**
 7. (a) $x = 8$, $y = 16$
 (b) $m = 12$, $n = 15$
 (c) $c = 32$ (d) $d = 12$
 (e) $p = 12.5$, $q = 10$ (f) $f = 12$
 (g) $g = 17.5$, $h = 10.5$

8. 7.26 m

9. 6.3 m

Chapter 14**Exercise 14A**

1. (a) $72^\circ 18'$ (b) $40^\circ 45'$ (c) $9^\circ 6'$
 (d) $19^\circ 21'$ (e) $125^\circ 9'$ (f) $68^\circ 27'$
 (g) $39'$ (h) $320^\circ 51'$
 2. (a) $62^\circ 10' 12''$ (b) $48^\circ 55' 48''$
 (c) $7^\circ 31' 12''$ (d) $87^\circ 5' 24''$
 (e) $132^\circ 48' 36''$ (f) $52^\circ 38' 24''$
 (g) $37^\circ 7' 30''$ (h) $26^\circ 21' 18''$
 3. (a) $17^\circ 33' 39.6''$ (b) $4^\circ 46' 22.8''$
 (c) $49^\circ 13' 4.8''$ (d) $122^\circ 18' 32.4''$
 (e) $57^\circ 7' 1.2''$ (f) $32^\circ 47' 49.2''$
 (g) $5' 45.6''$ (h) $200^\circ 0' 7.2''$

Exercise 14B

1. (a) 63.5° (b) 25.1° (c) 119.35°
 (d) 81.9°
 2. (a) 18.235° (b) 85.545° (c) 7.655°
 (d) 92.86° (e) 23.542° (f) 60.266°
 (g) 111.478° (h) 34.222° (i) 237.901°

Exercise 14D

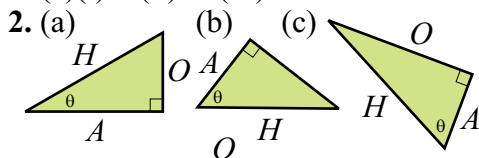
1. (a)(i) $\frac{\pi}{2}^\circ$ (ii) 1.57° (b)(i) $\frac{\pi}{3}^\circ$ (ii) 1.05°
 (c)(i) $\frac{\pi}{4}^\circ$ (ii) 0.79° (d)(i) $\frac{10\pi}{9}^\circ$ (ii) 3.49°
 (e)(i) $\frac{3\pi}{2}^\circ$ (ii) 4.71° (f)(i) $\frac{17\pi}{45}^\circ$ (ii) 1.19°
 (g)(i) $\frac{5\pi}{6}^\circ$ (ii) 2.62° (h)(i) $\frac{121\pi}{180}^\circ$ (ii) 2.11°
 (i)(i) $\frac{37\pi}{45}^\circ$ (ii) 2.58° (j)(i) $\frac{3\pi}{10}^\circ$ (ii) 0.94°
2. (a) 90° (b) 45° (c) 60° (d) 10°
 (e) 210° (f) 140° (g) 126°
 (h) 144° (i) 75° (j) 84°
3. (a) 34.38° (b) 120.32° (c) 48.70°
 (d) 92.82° (e) 8.02° (f) 108.86°
 (g) 190.79° (h) 263.56°

Exercise 14E

1. (a) 0.5299 (b) 0.5299 (c) 0.1228
 (d) 0.7071 (e) 0.4035 (f) 2.6889
 (g) 0.9641 (h) 0.9694
2. (a) 0.6726 (b) 0.9539 (c) 0.1530
 (d) 0.7015 (e) 0.4949 (f) 1.2613
 (g) 0.9658 (h) 0.9660 (i) 0.0020
3. (a) 0.5646 (b) 0.2675 (c) 0.4831
 (d) 0.2963 (e) 0.8820 (f) 1.4592
 (g) 0.1700 (h) 0.2182
4. (a) 0.2955 (b) 0.9744 (c) 1.1383
 (d) 0.9596

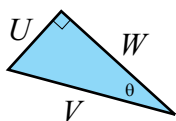
Exercise 14F

1. (a)(i) c (ii) a (iii) b
 (b)(i) r (ii) p (iii) q
 (c)(i) t (ii) u (iii) v

**Exercise 14G**

1. (a)(i) $\frac{m}{l}$ (ii) $\frac{n}{l}$ (iii) $\frac{m}{n}$
 (b)(i) $\frac{a}{r}$ (ii) $\frac{p}{r}$ (iii) $\frac{a}{p}$
 (c)(i) $\frac{y}{x}$ (ii) $\frac{z}{x}$ (iii) $\frac{y}{z}$
 (d)(i) $\frac{d}{e}$ (ii) $\frac{f}{e}$ (iii) $\frac{d}{f}$

2.



3. (a)(i) 0.4706 (ii) 0.8824 (iii) 0.5333
 (b)(i) 0.3243 (ii) 0.9459 (iii) 0.3429
 (c)(i) 0.9600 (ii) 0.2800 (iii) 3.4286
 (d)(i) 0.2462 (ii) 0.9692 (iii) 0.2540

Exercise 14H

1. (a) 11.9 cm (b) 21.1 cm
 (c) 13.0 cm (d) 17.7 m (e) 27.4 m
 (f) 43.0 cm (g) 9.4 m (h) 40.0 cm
 (i) 15.1 km (j) 11.7 m
 (k) 73.6 mm (l) 90.6 m
2. (a) $m = 12.8$ cm, $n = 11.2$ cm
 (b) $p = 4.5$ m, $q = 3.9$ m
3. (a) $x = 26.8$ cm, $y = 10.2$ cm
 (b) $a = 1.0$ m, $b = 1.8$ m
 (c) $m = 49.0$ cm, $n = 14.7$ cm
 (d) $c = 22.1$ km, $d = 18.3$ km
 (e) $f = 64.8$ mm, $g = 88.1$ mm
 (f) $k = 55.0$ mm, $l = 32.5$ mm

Exercise 14I

1. (a) 26.8 km (b) 38.3 km
2. (a) 402.6 km (b) 214.1 km
 (c) 533.0 km (d) 536.7 km
3. (a)(i) 127.5 km south
 (ii) 306.8 km west
 (b)(i) 102.2 km north
 (ii) 65.8 km west
4. 3.3 m
5. (a) 2475.2 m (b) 9237.6 m
6. (a) 73.2 m (b) 41.3 m

Exercise 14J

1. (a)(i) 32.9° (ii) $32^\circ 51' 35.569''$
 (b)(i) 43.4° (ii) $43^\circ 25' 57.132''$
 (c)(i) 24.0° (ii) $23^\circ 57' 44.96''$
 (d)(i) 53.5° (ii) $53^\circ 30' 22.44''$
 (e)(i) 62.0° (ii) $62^\circ 0' 19.578''$
 (f)(i) 57.0° (ii) $57^\circ 0' 22.324''$
2. 30°
3. (a) 8.76° (b) $8^\circ 45' 41.504''$
4. (a) 6.3° (b) 528 cm
5. S $39^\circ 48' 20.056''$ W
6. N $56^\circ 18' 35.757''$ W
7. 18.4°

Exercise 14K

- 87 400 m
- (a) $57^{\circ}59'40.62''$
(b) $48^{\circ}31'37.436''$
(c) $62^{\circ}4'35.248''$
- 976 m
- (a) 95.4 km (b) N $73^{\circ}49'8''$ W
- (a) 312 km (b) S $84^{\circ}44'49''$ E
- (a) $6^{\circ}24'36''$ (b) 135 km
(c) 101 minutes
- (a) 110.1 km (b) N $67^{\circ}3'45''$ W

Exercise 14L

- (a) $\cos 47^{\circ} = 0.6820$, $\sin 47^{\circ} = 0.7314$
(b) $\cos 22^{\circ} = 0.9272$, $\sin 22^{\circ} = 0.3746$
(c) $\cos 55^{\circ} = 0.5736$, $\sin 55^{\circ} = 0.8192$
(d) $\cos 38^{\circ} = 0.7880$, $\sin 38^{\circ} = 0.6157$
- (a) 1 (b) 0 (c) 0 (d) 1

Exercise 14M

- (a) $\cos 152^{\circ} = -0.8829$
 $\sin 152^{\circ} = 0.4695$
(b) $\cos 219^{\circ} = -0.7771$
 $\sin 219^{\circ} = -0.6293$
(c) $\cos 317^{\circ} = 0.7314$
 $\sin 317^{\circ} = -0.6820$
(d) $\cos 193^{\circ} = -0.9744$
 $\sin 193^{\circ} = -0.2250$
(e) $\cos 124^{\circ} = -0.5592$
 $\sin 124^{\circ} = 0.8290$
(f) $\cos 296^{\circ} = 0.4384$
 $\sin 296^{\circ} = -0.8988$
(g) $\cos 253^{\circ} = -0.2924$
 $\sin 253^{\circ} = -0.9563$
(h) $\cos 109^{\circ} = -0.3256$
 $\sin 109^{\circ} = 0.9455$
- (a) positive (b) positive
(c) positive (d) negative
(e) negative (f) negative
(g) negative (h) positive
- (a) third (b) second (c) first
(d) fourth (e) fourth (f) third

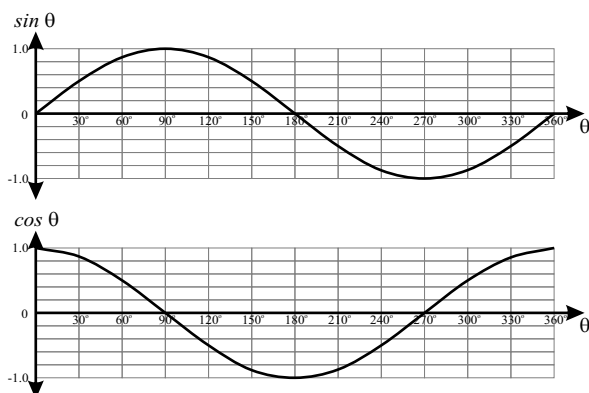
- (a) negative (b) negative
(c) positive (d) negative
(e) negative (f) positive
(g) positive (h) negative

5.

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin \theta$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\cos \theta$	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1

6.

**Exercise 14N**

- (a) $\sin 100^{\circ}$ (b) $\sin 50^{\circ}$ (c) $\sin 170^{\circ}$
(d) $\sin 40^{\circ}$ (e) $\sin 5^{\circ}$ (f) $\sin 165^{\circ}$
(g) $\sin 145^{\circ}$ (h) $\sin 105^{\circ}$ (i) $\sin 128^{\circ}$
(j) $\sin 82^{\circ}$ (k) $\sin 19^{\circ}$ (l) $\sin 133^{\circ}$
- (a) **C** (b) **B**
- (a) $\sin 280^{\circ}$ (b) $\sin 240^{\circ}$ (c) $\sin 190^{\circ}$
(d) $\sin 340^{\circ}$ (e) $\sin 265^{\circ}$ (f) $\sin 355^{\circ}$
(g) $\sin 215^{\circ}$ (h) $\sin 235^{\circ}$ (i) $\sin 188^{\circ}$
(j) $\sin 342^{\circ}$ (k) $\sin 271^{\circ}$ (l) $\sin 193^{\circ}$
- (a) **B** (b) **A**
- (a) $\cos 260^{\circ}$ (b) $\cos 120^{\circ}$ (c) $\cos 190^{\circ}$
(d) $\cos 160^{\circ}$ (e) $\cos 265^{\circ}$ (f) $\cos 135^{\circ}$
(g) $\cos 235^{\circ}$ (h) $\cos 175^{\circ}$ (i) $\cos 227^{\circ}$
(j) $\cos 122^{\circ}$ (k) $\cos 191^{\circ}$ (l) $\cos 93^{\circ}$
- (a) **D** (b) **B**
- (a) $\cos 350^{\circ}$ (b) $\cos 80^{\circ}$ (c) $\cos 60^{\circ}$
(d) $\cos 310^{\circ}$ (e) $\cos 55^{\circ}$ (f) $\cos 335^{\circ}$
(g) $\cos 285^{\circ}$ (h) $\cos 5^{\circ}$ (i) $\cos 287^{\circ}$
(j) $\cos 62^{\circ}$ (k) $\cos 31^{\circ}$ (l) $\cos 289^{\circ}$

Exercise 14O

1. 0° , 180° , 360° , 540° , **720°** , **900°** ,
 1080° , **1260°** , **1440°** , **1620°**
2. 90° , 270° , 450° , **630°** , **810°** , **990°** ,
 1170° , **1350°** , **1530°** , **1710°**

Exercise 14P

1. (a) 0.9721 (b) 0.9043 (c) 0.9817
(d) 0.5578 (e) 0.9508 (f) 0.9995
2. (a) 0.4554 (b) 0.9418 (c) 0.9750
(d) 0.9625 (e) 0.1324 (f) 0.6540
3. (a) $\frac{4}{5}$ (b) $\frac{5}{13}$ (c) $\frac{40}{41}$ (d) $\frac{8}{17}$
4. (a) 0.4878 (b) 1 (c) 1.1917
(d) 6.3152

Exercise 14Q

1. (a) 9.3 cm (b) 49.2 cm (c) 12.0 cm
(d) 39.3 cm (e) 28.2 cm (f) 9.5 cm
2. (a) 63° (b) 36° (c) 75° (d) 68°
3. (a) $\theta = 70^\circ$, $\alpha = 64^\circ$, $x = 11.2$ cm
(b) $\alpha = 48^\circ$, $\theta = 58^\circ$, $x = 9.1$ cm
(c) $\theta = 35^\circ$, $x = 66.2$ cm, $y = 46.9$ cm
(d) $\alpha = 32^\circ$, $\theta = 33^\circ$, $x = 8.2$ cm
4. 24.6 km

Exercise 14R

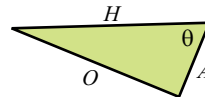
1. (a) 9.6 cm (b) 7.2 cm (c) 77.6 m
(d) 36.1 cm
2. (a) 29° (b) 40° (c) 108° (d) 96°
3. 83 km
4. **$A = 43^\circ$** , **$B = 80^\circ$** , **$C = 57^\circ$**

Exercise 14S

1. (a) cosine rule (b) 16.8 cm
2. (a) sine rule (b) $x = 64.1$ cm
3. (a) sine rule (b) $\theta = 43^\circ$
4. (a) cosine rule (b) $\theta = 77^\circ$
5. (a) sine rule (b) $x = 78.3$ cm
6. (a) cosine rule (b) $\theta = 119^\circ$

Chapter Review

1. (a) $23^\circ 12'$ (b) $68^\circ 21'$ (c) $8^\circ 27' 36''$
(d) $50^\circ 49' 12''$ (e) $8^\circ 6' 18''$
(f) $72^\circ 31' 4.8''$ (g) $105^\circ 28' 22.8''$
(h) $67^\circ 12' 39.6''$
2. (a) 25.42° (b) 4.29° (c) 62.14°
(d) 35.46°
3. (a)(i) $\frac{\pi}{6}^\circ$ (ii) 0.52° (b)(i) $\frac{\pi}{9}^\circ$ (ii) 0.35°
(c)(i) $\frac{25\pi}{36}^\circ$ (ii) 2.18° (d)(i) $\frac{13\pi}{10}^\circ$ (ii) 4.08°
4. (a) 120° (b) 144° (c) 100° (d) 165°
5. (a) 28.65° (b) 131.78° (c) 69.33°
(d) 2.58°
6. (a) 0.7455 (b) 0.8965 (c) 1.6236
(d) 0.9290 (e) 0.7618 (f) 2.9281
7. (a) 0.9636 (b) 0.9323 (c) 0.2289
8. (a) Opposite, Adjacent, Hypotenuse
(b)



$$(c) \sin \theta = \frac{O}{H}$$

$$\cos \theta = \frac{A}{H} \quad \tan \theta = \frac{O}{A}$$

9. (a) 0.8788 (b) 0.4697 (c) 1.8710
10. (a) $a = 6.7$ cm, $b = 10.9$ cm
(b) $x = 31.2$ cm, $y = 42.4$ cm
11. (a) 39.8 km (b) 39.4 km
12. 618 sec (10 min 18 sec)
13. (a) $\alpha = 30.76^\circ$, $\beta = 59.24^\circ$
(b) $\alpha = 30^\circ 45' 50''$, $\beta = 59^\circ 14' 10''$
14. $9^\circ 51' 23''$
15. (a) 461.7 km (b) N $86^\circ 0' 48''$ E
16. (a) second (b) fourth (c) fourth
(d) fourth (e) first (f) third
17. (a) positive (b) negative
(c) negative (d) positive
(e) negative (f) positive
(g) positive (h) negative
18. (a) $\sin^2 \theta + \cos^2 \theta = 1$
(b)(i) $\sin \theta = \sqrt{1 - \cos^2 \theta}$
(ii) $\cos \theta = \sqrt{1 - \sin^2 \theta}$
19. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
20. 0.8899

21. 0.9784
 22. 0.4166
 23. (a) $\theta = 37^\circ$, $\alpha = 100^\circ$, $x = 11.1$ m
 (b) $x = 15.0$ cm, $\theta = 82^\circ$

Chapter 15

Exercise 15A

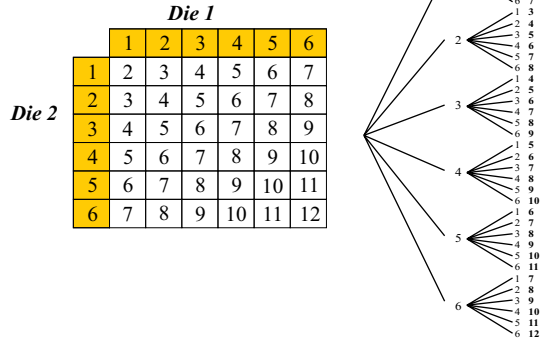
1. (e) 0.5 (h) 0.5 (p) 1
 3. (a) $\frac{1}{2}$ (b) $\frac{9}{16}$ (c) $\frac{1}{4}$
 4. 0.04
 5. (a) $\frac{6}{49}$ (b) $\frac{3}{7}$ (c) $\frac{6}{49}$ (d) $\frac{8}{75}$ (e) $\frac{29}{75}$
 (f) $\frac{8}{75}$ (g) 0
 6. (a) $\frac{80}{157}$ (b) $\frac{68}{471}$ (c) $\frac{12}{157}$
 7. Rugby (0.082), Football (0.036),
 Netball (0.025), Soccer (0.023),
 Basketball (0.022)
 8. $\frac{7}{9}$
 9. 0.732
 10. blue - 0.1, red - 0.3, yellow - 0.6
 11. 0.5
 12. 0.356
 13. 0.25
 14. (a)(i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{3}{13}$ (iv) $\frac{1}{26}$
 (v) $\frac{1}{52}$ (vi) $\frac{12}{13}$
 (b) $\frac{1}{49}$ (c)(i) $\frac{1}{14}$ (ii) 0 (iii) $\frac{10}{21}$
 15. 0.676

Exercise 15B

1. (a) $\frac{1}{6}$ (b) 20
 2. (a) $\frac{2}{5}$ (b) 64 (c) 250
 3. (a)(i) 58.1% (ii) 37.2% (iii) 4.7%
 (b)(i) 9 (ii) 6
 4. 1
 5. (a) 42.1% (b) 87.5% (c) 37.5%
 6. (a) 8.3% (b) 0.4% (c)(i) 40 (ii) 2

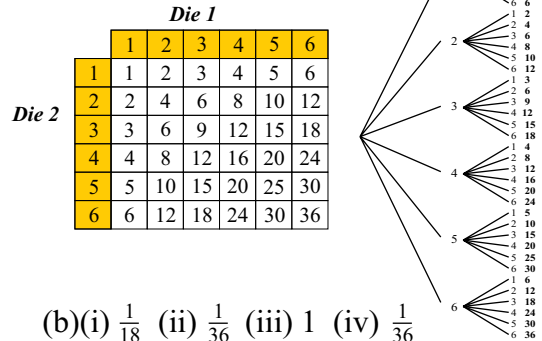
Exercise 15C

1. (a)



- (b)(i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{7}{12}$ (iv) 0
 (v) $\frac{1}{2}$

2. (a)



- (b)(i) $\frac{1}{18}$ (ii) $\frac{1}{36}$ (iii) 1 (iv) $\frac{1}{36}$

3. (a)

		Box A					
		R	R	B	B	B	Y
Box B	R	RR	RR	BR	BR	BR	YR
	R	RR	RR	BR	BR	BR	YR
	R	RR	RR	BR	BR	BR	YR
	B	RB	RB	BB	BB	BB	YB
	Y	RY	RY	BY	BY	BY	YY
	Y	RY	RY	BY	BY	BY	YY
	Y	RY	RY	BY	BY	BY	YY

- (b)(i) $\frac{1}{8}$ (ii) $\frac{11}{48}$ (iii) $\frac{1}{12}$ (iv) $\frac{11}{48}$
 (v) $\frac{35}{48}$ (vi) $\frac{13}{48}$

4. (a)

		Pen 1					
		B	B	B	G	G	P
Pen 2	B	BB	BB	BB	GB	GB	PB
	B	BB	BB	BB	GB	GB	PB
	B	BB	BB	BB	GB	GB	PB
	G	BG	BG	BG	GG	GG	PG
	G	BG	BG	BG	GG	GG	PG
	P	BP	BP	BP	GP	GP	PP

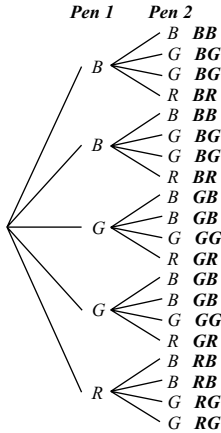
- (b)(i) $\frac{1}{4}$ (ii) $\frac{1}{36}$ (iii) $\frac{11}{18}$

5. (a)

(b)(i) $\frac{1}{10}$

(ii) $\frac{1}{5}$

(iii) $\frac{2}{5}$

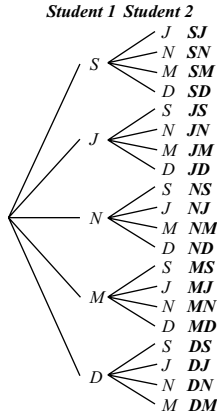


6. (a)

(b)(i) $\frac{3}{10}$

(ii) $\frac{1}{10}$

(iii) $\frac{3}{5}$



7. (a)

Brianna

Ahmed

	M	Tu	W	Th	F	Sa	Su
M	MM	TuM	WM	ThM	FM	SaM	SuM
Tu	MTu	TuTu	WTu	ThTu	FTu	SaTu	SuTu
W	MW	TuW	WW	ThW	FW	SaW	SuW
Th	MTh	TuTh	WTh	ThTh	FTh	SaTh	SuTh
F	MF	TuF	WF	ThF	FF	SaF	SuF
Sa	MSa	TuSa	WSa	ThSa	FSa	SaSa	SuSa
Su	MSu	TuSu	WSu	ThSu	FSu	SaSu	SuSu

(b)(i) $\frac{4}{49}$ (ii) $\frac{25}{49}$ (iii) $\frac{1}{7}$ (iv) $\frac{12}{49}$

8. (a)

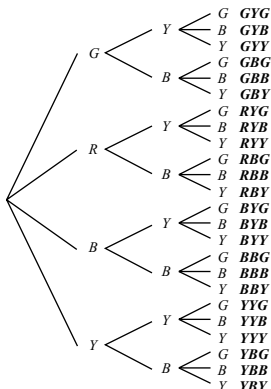
shirts shorts shoes

(b) 24

(c) $\frac{1}{12}$

(d) $\frac{1}{3}$

(e) $\frac{3}{4}$

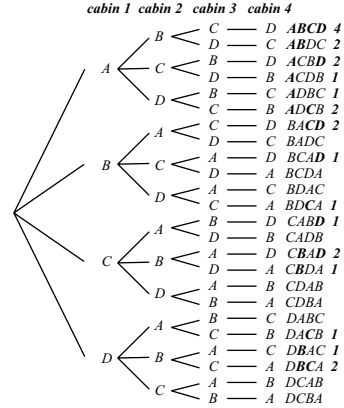


9. (a)

(b) 24

(c) $\frac{1}{24}$

(d) $\frac{7}{24}$

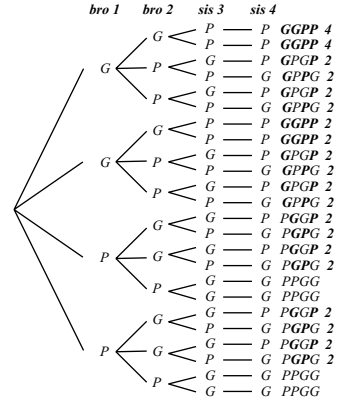


10. (a)

(b) 24

(c) $\frac{1}{12}$

(d) $\frac{5}{6}$



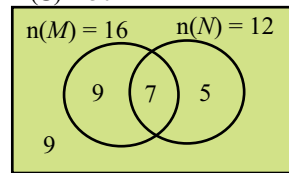
Exercise 15D

1. (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{5}{12}$

(f) $\frac{3}{4}$ (g) $\frac{1}{4}$ (h) $\frac{1}{12}$ (i) $\frac{1}{4}$ (j) $\frac{7}{12}$

2. (a)

$n(E) = 30$

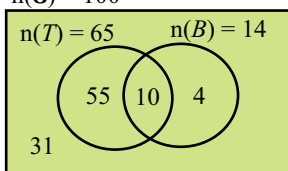


(b)(i) $\frac{8}{15}$ (ii) $\frac{2}{5}$ (iii) $\frac{7}{30}$ (iv) $\frac{7}{10}$

(v) $\frac{7}{15}$ (vi) $\frac{3}{5}$ (vii) $\frac{1}{6}$ (viii) $\frac{23}{30}$

3. (a)

$n(E) = 100$

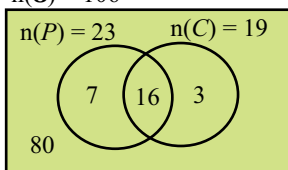


(b)(i) $\frac{13}{20}$ (ii) $\frac{7}{50}$ (iii) $\frac{11}{20}$ (iv) $\frac{1}{25}$

(v) $\frac{7}{20}$ (vi) $\frac{43}{50}$ (vii) $\frac{1}{10}$ (viii) $\frac{69}{100}$

4. (a)

$n(E) = 106$

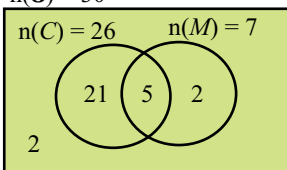


(b)(i) $\frac{23}{106}$ (ii) $\frac{8}{53}$ (iii) $\frac{83}{106}$ (iv) $\frac{40}{53}$

Exercise 15E

1. (a)

$n(E) = 30$



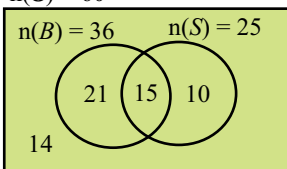
(b)

	C	C'	total
M	5	2	7
M'	21	2	23
total	26	4	30

(c)(i) $\frac{13}{15}$ (ii) $\frac{1}{15}$ (iii) $\frac{1}{15}$

2. (a)

$n(E) = 60$



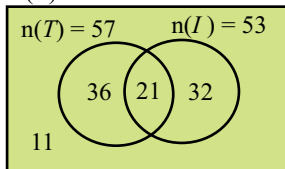
(b)

	B	B'	total
S	15	10	25
S'	21	14	35
total	36	24	60

(c)(i) $\frac{7}{20}$ (ii) $\frac{5}{12}$ (iii) $\frac{7}{30}$

3. (a)

$n(E) = 100$



(b)

	T	T'	total
I	21	32	53
I'	36	11	47
total	57	43	100

(c)(i) $\frac{9}{25}$ (ii) $\frac{21}{100}$ (iii) $\frac{53}{100}$

Exercise 15F1. (a) not ME (b) ME (c) not ME
(d) ME (e) not ME

2. (a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{8}{25}$ (e) $\frac{19}{25}$

3. 0.28

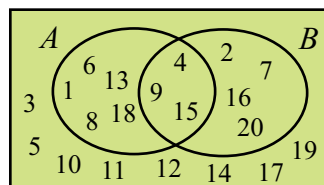
4. (a) $\frac{5}{13}$ (b) $\frac{23}{26}$ (c) $\frac{9}{26}$ (d) $\frac{9}{26}$

5. (a) $\frac{4}{13}$ (b) $\frac{7}{13}$ (c) $\frac{2}{13}$ (d) $\frac{7}{26}$ (e) $\frac{29}{52}$

(f) $\frac{19}{52}$ (g) $\frac{3}{52}$ (h) $\frac{1}{26}$

6. 92%

7. (a)



(b) $\frac{3}{5}$

8. $\frac{1}{10}$

Exercise 15G

1. $\frac{1}{12}$

2. (a) $\frac{1}{100}$ (b) $\frac{9}{50}$

3. $\frac{1}{480}$

4. (a) $\frac{1}{8}$ (b) $\frac{1}{169}$ (c) $\frac{1}{2704}$ (d) $\frac{1}{338}$

5. (a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$

6. (a) $\frac{1}{25}$ (b) $\frac{16}{25}$

7. $\frac{16}{225}$

8. $\frac{1}{216}$

9. $\frac{1}{1024}$

10. $\frac{27}{125}$

11. (a) $\frac{256}{625}$ (b) $\frac{1}{625}$ (c) $\frac{64}{625}$

12. (a) $\frac{6561}{10\,000}$ (b) $\frac{729}{10\,000}$

Exercise 15H

1. (a) $\frac{5}{18}$ (b) $\frac{1}{21}$ (c) $\frac{5}{63}$

2. $\frac{1}{42}$

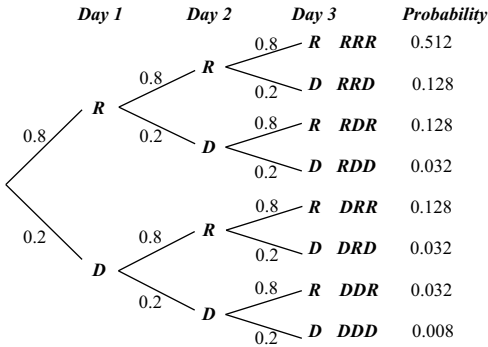
3. $\frac{19}{2695}$

4. (a) $\frac{6}{45}$ (b) $\frac{1}{8\,145\,060}$, 0.00000012

5. $\frac{1}{120}$

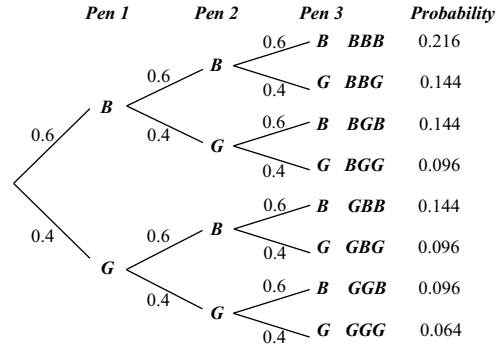
Exercise 15I

1. (a)



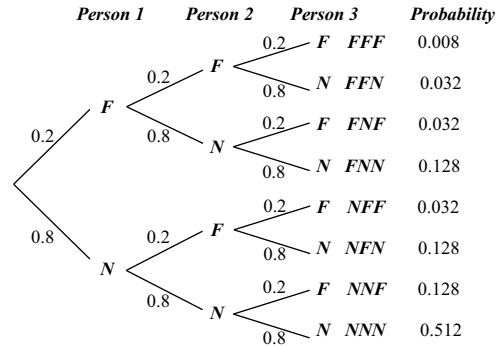
(b) 0.512 (c) 0.008 (d) 0.896

2. (a) 0.6
(b)



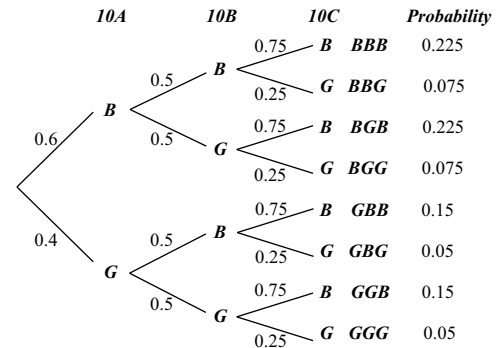
(c) 0.216 (d) 0.28

3. (a)



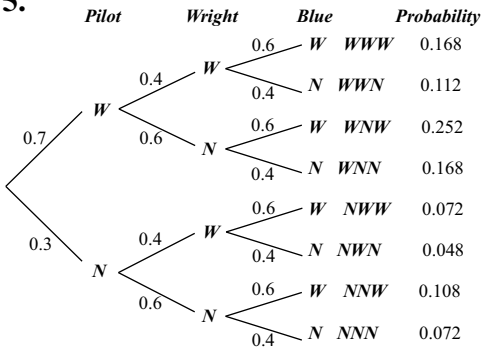
(b) 0.008 (c) 0.512 (d) 0.488

4. (a)



(b) 0.225 (c) 0.05 (d) 0.775

5.



Probability of seeing at least one whale = 0.928

Chapter Review

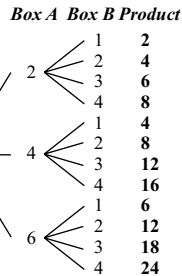
1. (a)(i) $\frac{1}{5}$ (ii) $\frac{4}{5}$ (iii) 0 (b) $\frac{1}{4}$

2. (a) $\frac{2}{3}$ (b) $\frac{4}{15}$

3. (a)

(b) $\frac{5}{12}$

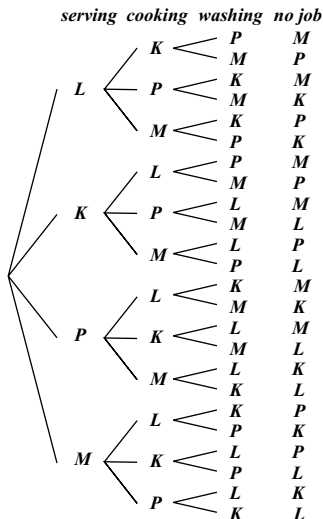
(c) 1



4. (a)

(b) $\frac{1}{4}$

(c) $\frac{1}{4}$

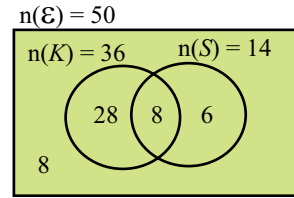


5. (a)

		Box X				
		2	4	6	8	10
Box Y	1	3	5	7	9	11
	2	4	6	8	10	12
	3	5	7	9	11	13
	4	6	8	10	12	14
	5	7	9	11	13	15

(b) $\frac{3}{5}$

6. (a)



(b)

	K	K'	total
S	8	6	14
S'	28	8	36
total	36	14	50

(c)(i) $\frac{3}{25}$ (ii) $\frac{21}{25}$ (iii) $\frac{4}{25}$

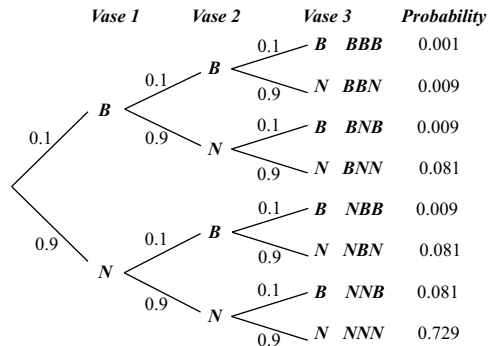
7. 0.71

8. (a) 0.4225 (b) 0.1225

9. $\frac{1}{260}$

10. $\frac{1}{676}$

11. (a)



(b) 0.001 (c) 0.729 (d) 0.972

Chapter 16

Exercise 16A

- (a) questionnaire (b) questionnaire (c) observation (d) questionnaire (e) questionnaire (f) questionnaire (g) observation (h) questionnaire (i) questionnaire (j) questionnaire (k) questionnaire
- (a) Leave a number of light globes switched on under different conditions and record when they fail
(b) Randomly select a number of bales, weigh them and find the average weight.
(c) Inflate a number of balloons, recording their diameter until they burst.
- (a) Questions need to be asked about how often people would use the pool.
(b) A combination of tick the box and suggestions about the kind of music people listen to.

Exercise 16B

- (a) 30 (b) 510 (c) 1581 (d) 89
- Yes (36 classes = 72 surveyed)
minimum sample size = 30
- (a) 59 (b) 15% (c) 525 sheep
- (a) 159 (b) 82% (c) 20 828
- 13 950 people

(a)	Age Group	Percentage	Number
(b)	-20	12.8%	704
	-30	21.6%	1188
	-40	26.4%	1452
	-50	24.0%	1320
	over 50	15.2%	836

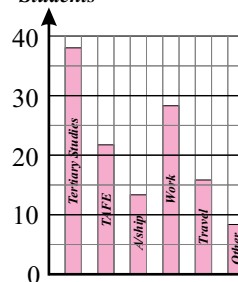
- (a)(i) 789 (ii) 5282 (iii) 607
(b) 19 489

Exercise 16C

- Discrete numerical
- Categorical
- Continuous numerical
- Discrete numerical
- Discrete numerical
- Continuous numerical
- Continuous numerical
- Categorical
- Discrete numerical
- Discrete numerical
- Continuous numerical
- Continuous numerical

Exercise 16D

- white - 131.4° , red - 65.1° , blue - 91.6° , green - 27.9° , black - 18.0° , other - 25.9°
- (a) 125
(b) *Number of Students*



- (c) Tertiary Studies - 109.4° , TAFE - 63.4° , A/ship - 37.4° , Work - 80.6° , Travel - 46.1° , Other - 23.0°
- (a) 300
(b) Running - 16.7%
Swimming - 8.3%
Walking - 21.7%
Dancing - 10%
Gym - 13.3%
Bicycle riding - 5%
Team sport - 18.3%
Other - 6.7%
(c) 500

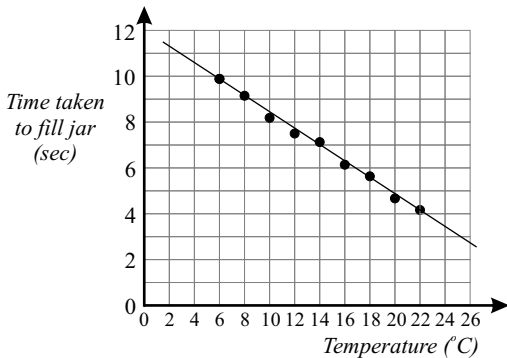
4. (a) Environment - 24.8%
Health - 12.7%
Education - 17.8%
Defence - 4.5%
Immigration - 8.6%
Employment - 29.0%
Other - 2.5%

- (b) Environment - 89.4°
Health - 45.9°
Education - 64.2°
Defence - 16.1°
Immigration - 31.0°
Employment - 104.3°
Other - 9.2°

5. Food - \$260, rent - \$390,
savings - \$195,
entertainment - \$130,
travelling - \$130, bills - \$195

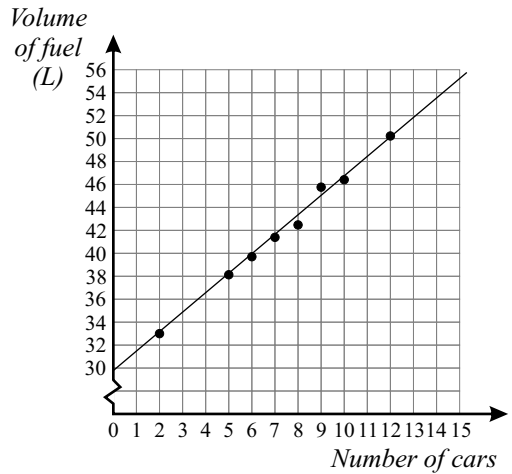
Exercise 16E

1. (a) **A** (b) **B** (c) **C** (d) **B** (e) **B**
(f) **D** (g) **B** (h) **C** (i) **C**
2. (a) increase
(b)(i) \$37 000 (ii) \$50 000
3. (a)



- (b)(i) 6.7 sec (ii) 3.1 sec
(iii) 10.9 sec

4. (a)



- (b)(i) 30 L (ii) 35 L (iii) 55 L

Exercise 16F

1. (a)(i) 14 (ii) 10 (iii) 12
(b)(i) 15.7 (ii) no mode (iii) 14
(c)(i) 30.8 (ii) no mode (iii) 30.5
(d)(i) 21.1 (ii) 21 (iii) 21
(e)(i) 10.5 (ii) 9, 10 (iii) 10
(f)(i) 36.9 (ii) no mode (iii) 37
2. (a) 41.8 (b) 43
3. mean score for = 102.3
median score for = 99.5
mean score against = 91.6
median score against = 88.5
4. (a) mean = \$425 000
median = \$211 500
(b) The median is a better indication of the prices because the mean has been distorted by \$1 504 000.

Exercise 16G

1. (a)

Value x	Frequency f
8	2
9	4
10	3
11	4
12	3
13	1
14	1

1. (b)

Value x	Frequency f
20	3
21	3
22	3
23	4
24	3
25	1

(c)

Value x	Frequency f
15	4
16	7
17	9
18	6
19	3
20	5

2. (a) mean = 10.5, median = 10.5

(b) mean = 22.2, median = 22

(c) mean = 17.4, median = 17

3. (a)

Number of Goals	Frequency f
0	1
1	2
2	5
3	7
4	4
5	3
6	1
7	1

(b) 3.2 (c) 3

4. (a)

Number of Biscuits	Frequency f
32	1
33	3
34	6
35	10
36	10
37	7
38	3

(b) 35.5 (c) yes

Exercise 16H

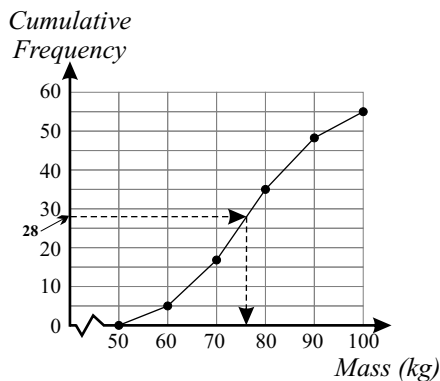
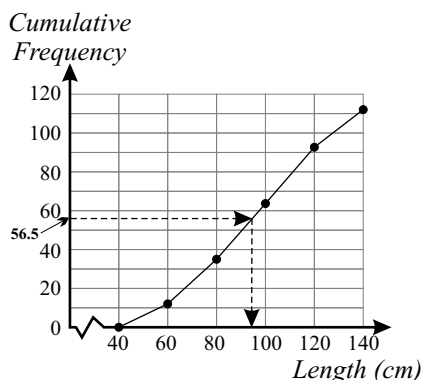
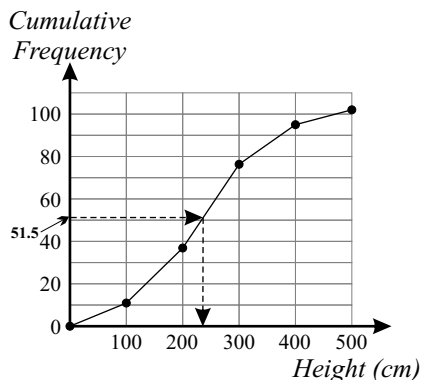
1. (a) 76.5 kg (b) 91.9 cm

(c) 299.7 cm (d) 12.3 years

2. (a)

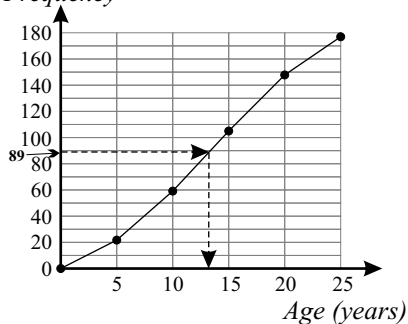
Mass	
Class Interval (kg)	Frequency f
0-<10	2
10-<20	4
20-<30	7
30-<40	9
40-<50	7
50-<60	4
60-<70	3

2. (b) 35.8 kg (c) 36.1 kg

Exercise 16I1. (a) median ≈ 76 kg(b) median ≈ 94 cm(c) median ≈ 235 cm

(d) median ≈ 13 years

Cumulative
Frequency

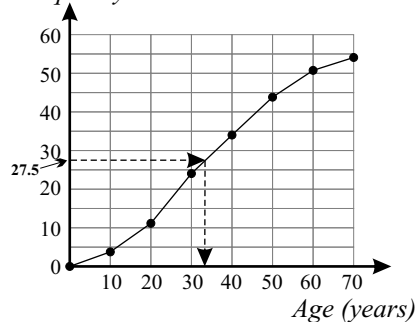


2. (a)

Age	
Class Interval (years)	Frequency f
0-<10	3
10-<20	8
20-<30	13
30-<40	10
40-<50	10
50-<60	7
60-<70	3

(b)

Cumulative
Frequency



(c) median ≈ 33 years

(d) 8 9 9 10 15 16 16 18 19 19 21
22 23 26 27 27 27 27 28 28 29 29 29
30 31 32 33 34 35 35 37 37 38 41 41
41 42 42 45 45 46 47 48 50 51 52 55
57 58 59 60 61 62

median = 32.5 years

Exercise 16J

1. (a)(i)

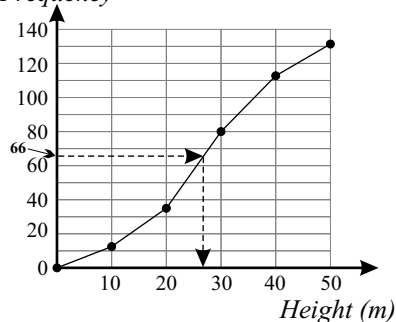
Height

Class Interval (m)	Midpoint x	Frequency f	$x \times f$	Cumulative Frequency
0-<10	5	12	60	12
10-<20	15	23	345	35
20-<30	25	45	1125	80
30-<40	35	32	1120	112
40-<50	45	19	855	131

(ii) 26.8 m (iii) 20-<30

(iv) see below (v) median ≈ 27 years

Cumulative
Frequency



(b)(i)

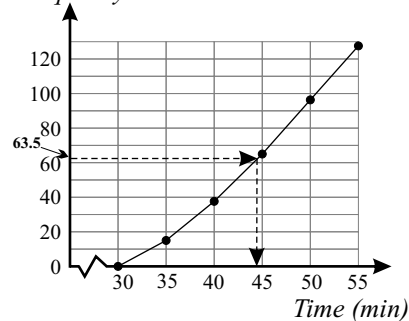
Time

Class Interval (min)	Midpoint x	Frequency f	$x \times f$	Cumulative Frequency
30-<35	32.5	15	487.5	15
35-<40	37.5	23	862.5	38
40-<45	42.5	28	1190	66
45-<50	47.5	33	1567.5	99
50-<55	52.5	27	1417.5	126

(ii) 43.8 min (iii) 45-<50

(iv) see below (v) median ≈ 44.5 min

Cumulative
Frequency



2. (a)

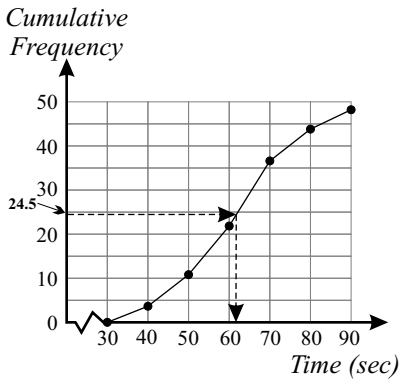
Time

Class Interval (sec)	Midpoint x	Frequency f	$x \times f$	Cumulative Frequency
30-<40	35	3	105	3
40-<50	45	8	360	11
50-<60	55	11	605	22
60-<70	65	15	975	37
70-<80	75	7	525	44
80-<90	85	4	340	48

(b) 60.6 sec (c) 60-<70 sec

(d) see below (e) median ≈ 62 sec

(f) mean = 60.2 sec, median = 61 sec



(g) Tim (6), Tam (8), Tom (16)

Exercise 16K

1. (a)(i)

Stem	Leaf
2	3 8 9
3	6 5 7 3 6 8 7
4	9 0
5	1 7 2
6	7

$$\overline{1|3} = 13$$

(ii)

Stem	Leaf
2	3 8 9
3	3 5 6 6 7 7 8
4	0 9
5	1 2 7
6	7

$$\overline{1|3} = 13$$

(iii) 37

(b)(i)

Stem	Leaf
0	8 7
1	6 9 8 6 8 9
2	5 6 5
3	2 3 9 0
4	2 1

$$\overline{1|3} = 13$$

(ii)

Stem	Leaf
0	7 8
1	6 6 8 8 9 9
2	5 5 6
3	0 2 3 9
4	1 2

$$\overline{1|3} = 13$$

(iii) 25

2. (a)

Spike

Lance

Stem	Leaf
2	8
3	6 6 3 7
4	4 6 0 8
5	2

Stem	Leaf
2	9
3	6 5 3 2 4 7 6
4	1
5	9

$$\overline{2|3} = 23$$

$$\overline{2|3} = 23$$

(b)

Spike

Lance

Stem	Leaf
2	8
3	3 6 6 7
4	0 4 6 8
5	2

Stem	Leaf
2	9
3	2 3 4 5 6 6 7
4	1
5	9

$$\overline{2|3} = 23$$

$$\overline{2|3} = 23$$

(c) Spike - 52 m, Lance - 59 m

(d) Spike - 38.5 m, Lance - 35.5 m

(e) Spike - 40.0 m, Lance - 37.2 m

3. (a)

Stem	Leaf
1	6
2	7 6 5
3	6 8 6 0 8 9
4	2 8 2 6 5 0
5	8 0 0 5
6	2 1
7	4
8	
9	8

$$2\overline{)3} = 23$$

(b)

Stem	Leaf
1	6
2	5 6 7
3	0 6 6 8 8 9
4	0 2 2 5 6 8
5	0 0 5 8
6	1 2
7	4
8	
9	8

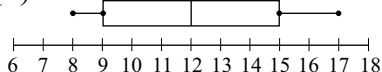
$$2\overline{)3} = 23$$

(c) 42 (d) 45.1

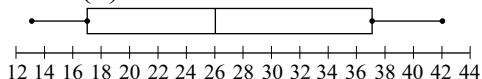
(e)(i) 21 000 (ii) 22 550

Exercise 16L1. (a)(i) $IQR = 15 - 9 = 6$

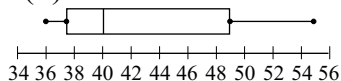
(ii)

(b)(i) $IQR = 37 - 17 = 20$

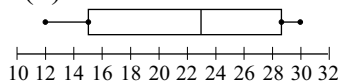
(ii)

(c)(i) $IQR = 49 - 37.5 = 11.5$

(ii)

(d)(i) $IQR = 28.5 - 15 = 13.5$

(ii)

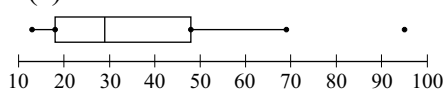


2. (a)(i) 32 (ii) 8

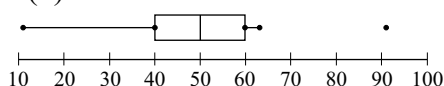
(b)(i) 75 (ii) 45

Exercise 16M

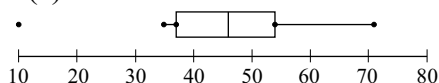
1. (a)



(b)



(c)



2. (a) Edgey Ed - 7, Driver Dan - 51.5

(b) Edgey Ed - 51.6, Driver Dan - 50.6

(c) Driver Dan

(d) Median

(e) Edgey Ed - 14.1, Driver Dan - 46.6

3. (a) 41.5 min (b) 43.8 min (c) 41.2 min

Exercise 16N1. (a) 68% of the data will occur between **44** and **56**.(b) 95% of the data will occur between **38** and **62**.(c) 99.7% of the data will occur between **32** and **68**.

2. (a)(i) 8 (ii) 80

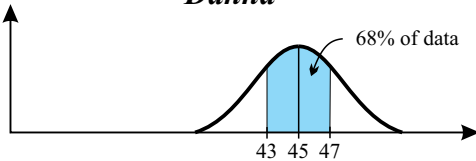
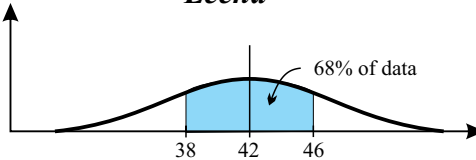
(b)(i) 15 (ii) 126

(c)(i) 12 (ii) 81

(d)(i) 8.7 (ii) 25.9

3. (a) 68% of the data will occur between **68** and **100**.(b) 95% of the data will occur between **52** and **116**.(c) 99.7% of the data will occur between **36** and **132**.

4. (a) 16% (b) 2.5% (c) 84%
 (d) 99.85%
 5. (a) 17 (b) 59
 6. (a) 8 (b) 42
 7. (a) 7 (b) 97.5% (c) 0.15%
 8. (a) 12 (b) 2.5%
 9. 15
 10. (a) 35°C (b) 3.5
 11. (a)

Dahna**Leena**

- (b) The standard deviation of Dahna's throws is less than Leena's so the spread of Dahna's throws is less. Dahna has more throws closer to the mean. Dahna is more consistent with the length of her throws.

Exercise 160**1.**

x	$x - \bar{x}$	$(x - \bar{x})^2$
68	-13.4	179.56
79	-2.4	5.76
93	11.6	134.56
80	-1.4	1.96
82	0.6	0.36
77	-4.4	19.36
95	13.6	184.96
69	-12.4	153.76
73	-8.4	70.56
80	-1.4	1.96
88	6.6	43.56
90	8.6	73.96
66	-15.4	237.16
100	18.6	345.96
		1453.44

$$\bar{x} = 81.4\%$$

$$\sigma = 10.2\%$$

2.

x	$x - \bar{x}$	$(x - \bar{x})^2$
50	13.2	174.24
28	-8.8	77.44
22	-14.8	219.04
62	25.2	635.04
33	-3.8	14.44
45	8.2	67.24
27	-9.8	96.04
36	-0.8	0.64
31	-5.8	33.64
39	2.2	4.84
28	-8.8	77.44
41	4.2	17.64
		1417.68

$$\bar{x} = 36.8$$

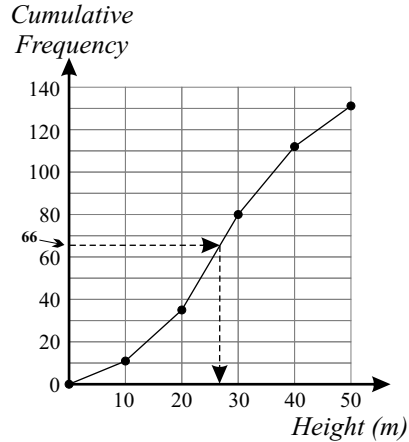
$$\sigma = 11.4$$

Chapter Review

- (a) 181 (b) 20 525
- cereal - 160.2° , toast - 103.5° ,
eggs - 53.1° , fruit/yoghurt - 25.2° ,
other - 18°
- (a) as one increases the other
increases
(b) as one increases the other
decreases
(c) one variable remains the same
independent of the other
(d) both variables can have a
number of values independent
of the other
- (a) mean = 10.4, mode = 11,
median = 11
(b) mean = 26, no mode,
median = 26
- mean = 32.0, mode = 33,
median = 32
- | Value
x | Frequency
f |
|--------------|------------------|
| 56 | 3 |
| 57 | 1 |
| 58 | 4 |
| 59 | 6 |
| 60 | 4 |
| 61 | 5 |
| 62 | 2 |
| 63 | 3 |
| 64 | 2 |
| 65 | 1 |
| 66 | 1 |

mean = 60.25, median = 60

- (a) 26.8 m (b) 20-<30 m
(c) see below (d) 26 m



- (a)

Stem	Leaf
3	8 7
4	5 8 2 4 7 7 3 8 7
5	1 0 8 6 9 9 2 6 6 7 8 9
6	8 6 1 7 2 4 8 0 3 5 3 2 6 4 1
7	2 0 1 3 1 7 3
8	3 2 0

$$\overline{1|3} = 13$$

- (b)

Stem	Leaf
3	7 8
4	2 3 4 5 7 7 7 8 8
5	0 1 2 6 6 6 7 8 8 9 9 9
6	0 1 1 2 2 3 3 4 4 5 6 6 7 8 8
7	0 1 1 2 3 3 7
8	0 2 3

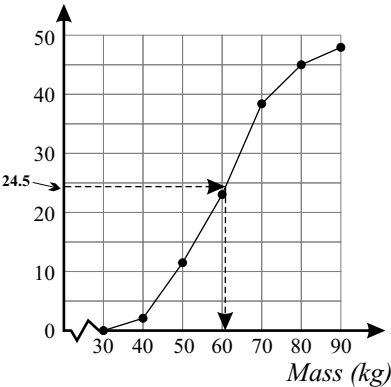
$$\overline{1|3} = 13$$

8. (c) 60.5 kg (d) see below
(e) 60.2 kg (f) see below
(g) 61 kg

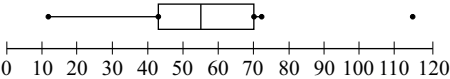
Mass

Class Interval (kg)	Midpoint x	Frequency f	$x \times f$	Cumulative Frequency
30-<40	35	2	70	2
40-<50	45	9	405	11
50-<60	55	12	660	23
60-<70	65	15	975	38
70-<80	75	7	525	45
80-<90	85	3	255	48

Cumulative Frequency



9.



10. (a) 27 (b) 84%