

MASTER MATHS

STEVE PEScott



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Dedication

I would like to dedicate this book to my late mother
for her constant support and encouragement.

Noela Florence Pescott (20/4/1933 - 18/4/2010)

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Numbers

Numbers

EXERCISE 1A

1. Write the following numbers in words.

Example 817 = **eight hundred and seventeen**

- | | | | |
|-------------|-------------|---------------|----------------|
| (a) 651 | (b) 918 | (c) 6005 | (d) 4283 |
| (e) 19 047 | (f) 50 040 | (g) 75 806 | (h) 103 967 |
| (i) 700 700 | (j) 823 009 | (k) 3 057 041 | (l) 11 821 615 |

2. Write the following numbers in numerical form.

- (a) six hundred and fifty-seven
- (b) three thousand and ninety-five
- (c) twenty-four thousand, eight hundred and one
- (d) seven hundred thousand, four hundred and sixty-seven
- (e) two million, eighty-four thousand and seventy-nine
- (f) three hundred and ninety-four million, six hundred and fifty thousand, eight hundred and twenty

3. Without using a calculator, **add** 100 to each of the following numbers.

- | | | | |
|------------|----------|------------|----------------|
| (a) 6 | (b) 791 | (c) 956 | (d) 8071 |
| (e) 63 467 | (f) 4952 | (g) 59 901 | (h) 35 999 999 |

4. Without using a calculator, **subtract** 1000 from each of the following numbers.

- | | | | |
|------------|---------------|---------------|---------------|
| (a) 8567 | (b) 64 200 | (c) 10 003 | (d) 700 508 |
| (e) 50 971 | (f) 2 300 157 | (g) 8 040 222 | (h) 5 000 000 |

5. Which number is represented by the following words.

- | | | |
|-----------|-------------------|--------------|
| (a) dozen | (b) baker's dozen | (c) century |
| (d) gross | (e) kilo | (f) googol |
| (g) score | (h) billion | (i) trillion |

6. Round the following numbers to the nearest 10.

- (a) 23 (b) 78 (c) 7 (d) 104
 (e) 229 (f) 396 (g) 40 997 (h) 129 996

7. Round the following numbers to the nearest 100.

- (a) 68 (b) 139 (c) 28 (d) 752
 (e) 1041 (f) 3260 (g) 18 963 (h) 189 980

8. Round the following numbers to the nearest 1000.

- (a) 6780 (b) 2399 (c) 640 (d) 11 482
 (e) 33 097 (f) 106 841 (g) 99 560 (h) 1 929 656

9. (a) List the first 20 prime numbers.

(b) What is the largest *two digit* prime number?

(c) What is the largest *three digit* prime number?

(d) Twin primes are prime numbers that differ by 2.

Examples of twin primes are 3 and 5, 5 and 7, 11 and 13.

What is the largest two digit set of twin prime numbers?

Operations

EXERCISE 1B

1. Without using a calculator solve the following problems.

- (a) 8×9 (b) 7×6 (c) 5×12 (d) 11×11
 (e) $49 \div 7$ (f) $56 \div 8$ (g) $36 \div 12$ (h) $90 \div 3$
 (i) 3×23 (j) 5×31 (k) 20×20 (l) 6×72
 (m) $18 + 33$ (n) $67 + 93$ (o) $136 + 52$ (p) $216 + 357$
 (q) $37 - 14$ (r) $156 - 89$ (s) $320 - 256$ (t) $567 - 389$

2. Copy and complete this multiplication table.

×	3						
2							12
		50					60
	24				72		
4			28				
			63	99			
7						84	
				66			

3. Without using a calculator solve the following problems.
Remember to use BODMAS.

BRACKETS
OF
{ DIVISION
{ MULTIPLICATION
{ ADDITION
{ SUBTRACTION

Examples

$$\begin{aligned} 1. \quad & 8 + 3 \times 4 \\ & = 8 + 12 \\ & = \mathbf{20} \end{aligned}$$

$$\begin{aligned} 2. \quad & 3 \times (2 + 4) - \frac{1}{2} \text{ of } 8 \\ & = 3 \times 6 - \frac{1}{2} \text{ of } 8 \\ & = 3 \times 6 - 4 \\ & = 18 - 4 \\ & = \mathbf{14} \end{aligned}$$

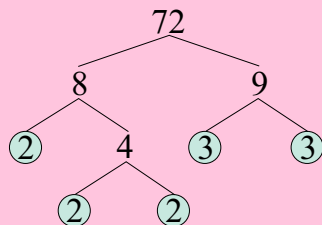
$$\begin{aligned} 3. \quad & 4 \times 4 \div 2 \times 3 \div 12 \\ & = 16 \div 2 \times 3 \div 12 \\ & = 8 \times 3 \div 12 \\ & = 24 \div 12 \\ & = \mathbf{2} \end{aligned}$$

- (a) $4 + 5 \times 2$ (b) $6 \times 3 - 2 \times 4$ (c) $\frac{1}{2} \text{ of } 8 - 4$
 (d) $12 \div 6 - 4 \div 2$ (e) $6 \times (3 + \frac{1}{2} \text{ of } 8)$ (f) $8 - \frac{1}{2} \text{ of } (12 - 4 \times 2)$
 (g) $4 \times (2 + 7) \div \frac{1}{2} \text{ of } 24$ (h) $8 + (12 - 14 \div 2) - 10 \div 5$
 (i) $(6 + 20 \div 10) \times (15 \div \frac{1}{2} \text{ of } 10)$
 (j) $6 \times 3 \div 9 \times 10 \div 2 \times 4 \div 8 \times 6 \div 2$
 (k) $5 + (\frac{1}{4} \text{ of } 12) \times (6 - 12 \div 6) - 8 \div (20 \div 2 - \frac{1}{2} \text{ of } 12)$

4. Write the following numbers as products of their prime factors in index form.

- (a) 16 (b) 36
 (c) 200 (d) 144
 (e) 300 (f) 78
 (g) 1000 (h) 437
 (i) 936 (j) 4096
 (k) 529 (l) 111

Example 72



$$\begin{aligned} 72 &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= \mathbf{2^3 \times 3^2} \end{aligned}$$

Roman Numerals

EXERCISE 1C

I = 1 V = 5 X = 10 L = 50 C = 100 D = 500 M = 1000

Examples

36 = XXXVI

462 = CDLXII

1263 = MCCLXIII

2594 = MMDXCIV

1. Convert the following roman numerals to number form.

(a) XXIV

(b) LVII

(c) XCIX

(d) MDCLXVI

(e) LXXIX

(f) CCXCV

(g) CMLIV

(h) MMMCCXXII

(i) CLIV

(j) CMI

(k) MMX

(l) CDXLIV

2. Convert the following numbers to roman numerals.

(a) 36

(b) 84

(c) 250

(d) 793

(e) 3408

(f) 1111

(g) 2449

(h) 938

(i) 333

(j) 3877

(k) 2020

(l) 672

3. The equation shown here of roman numerals is made with matches and is incorrect. Move **two** matches to make the equation correct.



Square Roots and Powers

EXERCISE 1D

1. Without using a calculator find the following square roots.

(a) $\sqrt{16}$

(b) $\sqrt{(200 \div 2)}$

(c) $\sqrt{9 \times 4}$

(d) $\sqrt{81} + \sqrt{9}$

(e) $\sqrt{(16 + 9)}$

(f) $\sqrt{(32 \times 2)}$

(g) $\sqrt{7^2}$

(h) $\sqrt{1}$

(i) $\sqrt{16^2}$

(j) $\sqrt{49^2}$

(k) $\sqrt{23^2}$

(l) $\sqrt{625^2}$

2. Without using a calculator find the following.

(a) 3^2

(b) 5^2

(c) 4^2

(d) $(12 - 2)^2$

(e) 2^3

(f) 10^3

(g) 7^2

(h) $1^2 + 2^2 + 3^2$

(i) 2^5

(j) 10^5

(k) $(6 + 5)^2$

(l) $(\sqrt{7})^2$

Rounding

EXERCISE 1E

1. Without using a calculator solve the following problems.

- (a) 10×20 (b) 30×100 (c) 200×100 (d) 30×20
 (e) 50×60 (f) 700×200 (g) 90×1000 (h) 60×8000
 (i) $200 \div 10$ (j) $500 \div 100$ (k) $6000 \div 10$ (l) $7000 \div 100$
 (m) $1000 \div 20$ (n) $8000 \div 200$ (o) $600 \div 30$ (p) $15\,000 \div 500$

2. In the following problems find approximate answers by rounding the numbers to the nearest 10 before evaluating.

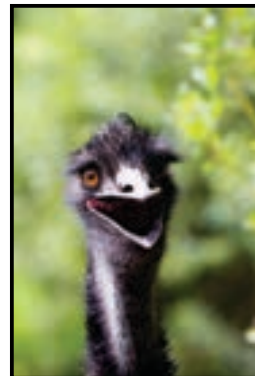
Do not use a calculator.

Example 42×88
 $\approx 40 \times 90$ (after rounding)
 $= \mathbf{3600}$ (approximate answer)

- (a) 68×21 (b) 56×99 (c) 222×13 (d) 67×112
 (e) $87 \div 33$ (f) $724 \div 93$ (g) $641 \div 84$ (h) $537 \div 57$

3. (i) Find approximate answers to the following problems by rounding the numbers before evaluating without using a calculator.
 (ii) Evaluate the exact answer by using a calculator.

- (a) Rangers at the Karimba State Park wanted to calculate the total number of emus in the park.
 They counted 38 emus in one square kilometre.
 The park is 122 square kilometres.
 Assume that the emus are distributed evenly throughout the park to calculate how many emus are in the park.



- (b) The students from a number of schools travelled by buses to a combined athletics competition. There were 33 large buses that each carried 47 students and 19 small buses that each carried 13 students.
 How many students were at the athletics competition?

Calculator Problems

EXERCISE 1F

Use a calculator to solve the following problems.

$$1. \frac{3843 \times 4}{118\,584 \div 54}$$

$$2. \frac{427 \times (458 - 572 + 123) \times \sqrt{(10\,464 \div 654)}}{(55\,489 + 63\,095) \div (5741 - 5687)}$$

$$3. \frac{\frac{8602}{23} - \frac{6764}{19}}{\sqrt{(342 - 333)}} + \frac{\frac{\sqrt{432} - \sqrt{1024}}{2739} - \frac{4446}{57}}{33}$$

$$4. \frac{1}{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}} \div \frac{3^5 - 183}{117\,696 - 7^6}$$

$$5. \frac{\frac{(1107 - 325)}{\sqrt{529}} - \frac{(1082 - 549)}{\sqrt{1681}}}{\frac{(574 + 596)}{\sqrt{13\,689}} - \frac{(3638 + 247)}{\sqrt{308\,025}}}$$

$$6. \frac{7237 - 658 - 9^4}{\sqrt{\frac{5112}{568}}} - \left[\frac{1560}{8^5 - 32\,703} - \left(\frac{4^4 - 43}{3^6 - 721} - \frac{\sqrt{6084} - 9}{65^2 - 4217} \right) \right]$$

$$7. 8 \div \left\{ \frac{1242}{54} - \left(\frac{675}{25} - \frac{704}{32} \right) \times \left[\frac{432}{8} - \left(\frac{5372}{79} - \frac{153}{9} \right) \right] \right\}$$

Brain Teasers

EXERCISE 1G

- Find this four digit number:
 - its digits add to 13
 - it has two more hundreds than it has thousands
 - it has four times as many units as it has thousands
- Sally is six years older than her sister Fiona. The sum of their ages is 30. How old are the girls?
- Greta's calculator had the symbols on the four operation buttons (+, -, ×, ÷) worn off. She wrote the letters **A**, **B**, **C** and **D** on the buttons but did not know which button was which. Greta performed the following calculation.

$$12 \text{ A } 4 \text{ B } 2 \text{ C } 3 = 7$$

- Find which letter (**A**, **B**, **C**, **D**) corresponds to each operation (+, -, ×, ÷).
 - Find the answer to each of the following calculations.
 - $6 \text{ A } 8 \text{ D } 4 \text{ C } 2$
 - $10 \text{ A } 3 \text{ B } 5 \text{ A } 16 \text{ D } 2$
- Use the numbers 1, 2, 3 and 4 and any of the operations (+, -, ×, ÷) and brackets if needed to form calculations that equal all the numbers from 0 to 20. All the numbers 1, 2, 3 and 4 must be used and only once.

Examples

$$4 + 1 - 2 - 3 = 0$$

$$(4 \div 2) + 1 - 3 = 0$$

$$3 + 2 - 4 - 1 = 0$$

$$4 \times 3 - 2 \div 1 = 10$$

$$4 \times 2 - 1 + 3 = 10$$

$$4 \times (3 - 1) + 2 = 10$$

5. Copy this addition problem into your book.

Use the numbers 0, 1, 2, 3, 4, 5 in the spaces shown to make the addition correct.

$$\begin{array}{r}
 \square 7 9 8 \\
 + 6 \square 1 \square \\
 \hline
 \square \square \square 1 3
 \end{array}$$

6. In this addition problem each of the letters represents one of the numbers 0, 1, 2, 3, 4, 5, 6. Find which number each letter represents. (*Hint: E = 0*)

$$\begin{array}{r}
 \mathbf{M} \mathbf{A} \mathbf{S} \mathbf{T} \mathbf{E} \mathbf{R} \\
 + \mathbf{M} \mathbf{A} \mathbf{T} \mathbf{H} \mathbf{S} \\
 \hline
 7 0 7 4 5 4
 \end{array}$$

7. Copy this subtraction problem into your books. Fill in the spaces to make the problem correct.

$$\begin{array}{r}
 1 \square 0 \square 1 \square \\
 - 9 \square 2 \square 4 \\
 \hline
 3 3 3 3 3
 \end{array}$$

8. If Arthur's and his wife Martha's ages are added the result is 91. Arthur is now twice as old as Martha was when Arthur was as old as Martha is now. How old are Arthur and Martha?

9. Kim has got a number of 35c, 45c and 55c stamps.

Find how many of each stamp is required to make the following totals.

(a) \$1.45 (b) \$1.80 (c) \$2.35 (d) \$3.80 (e) \$2.95 (f) \$4.35

PROBLEM SOLVING

The town of St George has 4500 people. There are four reservoirs that supply water to St George. The reservoirs are called Kangaroo, Currawong, Bilby and Kookaburra.

The tables shown below represent the following quantities.

Table 1 - (a) The amount of water (in litres) in each reservoir at the start of a particular year and

(b) The amount of water collected in each reservoir (in litres) for every millimetre of rain

Table 2 - The average monthly rainfall (in mm) for St George

Table 1

<i>Reservoir</i>	<i>Litres at start of year</i>	<i>Litres collected per mm of rain</i>
Kangaroo	14 500 000	500 000
Currawong	8 700 000	200 000
Bilby	12 300 000	400 000
Kookaburra	6 400 000	150 000

Table 2

Average monthly rainfall (in mm) in St George

J	F	M	A	M	J	J	A	S	O	N	D
28	31	25	22	36	40	53	46	38	29	30	27

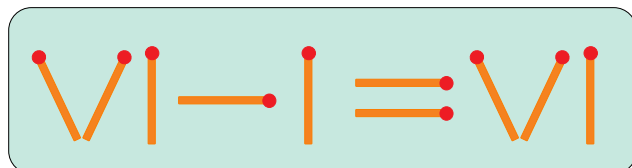
The average amount of water used by each person per day is 200 litres.

Use the above information to calculate the following quantities:

1. The total amount of water in the four reservoirs at the start of the year.
2. The total amount of water in the four reservoirs at the end of the year assuming average rainfall for the year and average daily water usage of 200 litres per person. (It is not a leap year)

PUZZLE

Move **two** matches to make this equation correct.



CHAPTER REVIEW

1. (a) Write 604 295 in words.
 (b) Write in numeral form:
 one hundred and fifty-six thousand, seven hundred and eight
2. Round to the nearest 100:
 (a) 672 (b) 1918 (c) 23 956 (d) 18
3. Without using a calculator evaluate the following problems.
 (a) 6×9 (b) 7×12 (c) $81 \div 9$ (d) $56 \div 8$
 (e) $17 + 91$ (f) $72 + 139$ (g) $173 - 51$ (h) $674 - 496$
4. Without using a calculator evaluate the following problems.
 (a) $6 + 3 \times 2 - 4 \div 2$ (b) $8 + \frac{1}{2}$ of $(8 + 4) \times 3$ (c) $4 \times 2 + 6 \div 2 + 1$
5. Write 540 as a product of its prime factors in index form.
6. Write the following numbers in roman numerals.
 (a) 78 (b) 394 (c) 489 (d) 2624
7. Without using a calculator evaluate the following problems.
 (a) $\sqrt{144}$ (b) $\sqrt{64 + 36}$ (c) $\sqrt{64} + \sqrt{36}$ (d) $\sqrt{9^2}$
 (e) 3^2 (f) 10^4 (g) $(3 + 4)^2$ (h) $(\sqrt{11})^2$
8. Without using a calculator evaluate the following problems.
 (a) 30×30 (b) 200×40 (c) $4000 \div 200$ (d) $6000 \div 30$
9. Round the numbers to the nearest 10 before evaluating the approximate answers to the following problems.
 (a) 27×33 (b) 83×47 (c) $403 \div 18$ (d) $638 \div 82$
10. Use a calculator to evaluate this problem.

$$\frac{(1027 - 547)}{(1560 \div 65)} - \frac{(660 - 564)}{(756 \div 63)}$$

$$\sqrt{(503 - 359)}$$
11. Find the next three numbers in this series.
 12, 12, 24, 72, 288, __, __, __.

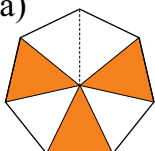
Fractions

Fractions

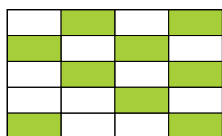
EXERCISE 2A

1. What fraction of each of the following shapes is shaded.

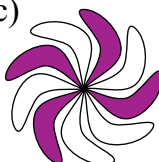
(a)



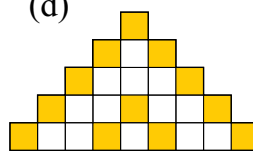
(b)



(c)



(d)



2. What fraction of each of the shapes in question 1 is **not** shaded.

3. Write the following fractions in their simplest form.

Example

$$\frac{25}{30} \xrightarrow{+5} \frac{5}{6} \xrightarrow{-5} \frac{25}{30}$$

$$\frac{25}{30} = \frac{5}{6}$$

(a) $\frac{10}{12}$

(b) $\frac{20}{40}$

(c) $\frac{24}{36}$

(d) $\frac{56}{64}$

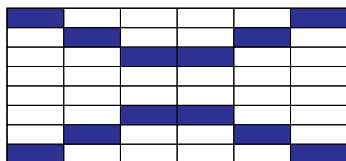
(e) $\frac{72}{90}$

(f) $\frac{82}{96}$

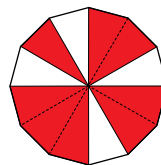
4. What fraction of each of the following shapes is shaded.

Give answers in simplest form.

(a)

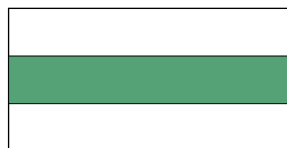
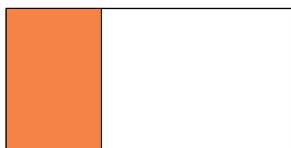


(b)



5. Draw 6 rectangles that are about 6 cm × 3 cm. Colour in one third of each rectangle so that each rectangle is different.

Two examples are shown here.



6. Complete the following equivalent fractions.

Example $\frac{2}{3} = \frac{\square}{18}$ $\frac{2}{3} = \frac{\square}{18}$ $\frac{2}{3} \xrightarrow{\times 6} \frac{12}{18}$ $\frac{2}{3} = \frac{12}{18}$

(a) $\frac{4}{5} = \frac{\square}{20}$

(b) $\frac{2}{7} = \frac{8}{\square}$

(c) $\frac{7}{8} = \frac{\square}{64}$

(d) $\frac{5}{9} = \frac{45}{\square}$

(e) $\frac{3}{4} = \frac{\square}{16} = \frac{15}{\square} = \frac{\square}{28} = \frac{33}{\square} = \frac{\square}{36} = \frac{\square}{80}$

7. Change the following fractions to have the same denominator and hence arrange them in order from the smallest to the largest.

Example $\frac{3}{4}, \frac{5}{8}, \frac{3}{5}$ Lowest common denominator = 40

$\frac{3}{4} = \frac{30}{40}$ $\frac{5}{8} = \frac{25}{40}$ $\frac{3}{5} = \frac{24}{40}$

In order from smallest to largest: $\frac{3}{5}, \frac{5}{8}, \frac{3}{4}$

(a) $\frac{3}{4}, \frac{5}{7}$

(b) $\frac{5}{12}, \frac{3}{8}, \frac{1}{3}$

(c) $\frac{5}{9}, \frac{3}{4}, \frac{7}{12}$

(d) $\frac{2}{3}, \frac{5}{8}, \frac{7}{9}$

8. Write the first quantity as a fraction of the second and write in its simplest form. Ensure that the two quantities are in the same units before writing as a fraction.

Example $60\text{c} : \$2$
 $= 60\text{c} : 200\text{c}$
 $= \frac{60}{200}$
 $= \frac{3}{10}$

(a) 10 cm : 2 m

(b) 8 mm : 4 cm

(c) 200 kg : 3 tonnes

(d) 60c : \$5

(e) 400 g : 6 kg

(f) 50 m : 2 km

(g) 25 mm : 5 cm

(h) 520 g : 8 kg

(i) 90c : \$3.60

9. Change the following mixed numbers to improper fractions.

Example $4\frac{3}{4} = \frac{19}{4}$

(a) $3\frac{1}{2}$

(b) $2\frac{3}{5}$

(c) $5\frac{2}{3}$

(d) $6\frac{1}{4}$

(e) $4\frac{5}{6}$

(f) $8\frac{7}{8}$

(g) $9\frac{3}{7}$

(h) $7\frac{7}{12}$

10. Change the following improper fractions to mixed numbers.

- (a) $\frac{9}{2}$ (b) $\frac{13}{3}$ (c) $\frac{27}{5}$ (d) $\frac{63}{10}$ (e) $\frac{33}{4}$
 (f) $\frac{41}{6}$ (g) $\frac{76}{9}$ (h) $\frac{53}{8}$ (i) $\frac{97}{7}$ (j) $\frac{145}{6}$

Operations

EXERCISE 2B

1. Find: (a) $\frac{1}{3}$ of 12 (b) $\frac{2}{3}$ of 12
 2. Find: (a) $\frac{1}{5}$ of 20 (b) $\frac{2}{5}$ of 20 (c) $\frac{3}{5}$ of 20 (d) $\frac{4}{5}$ of 20
 3. Find the following amounts.
 (a) $\frac{1}{4}$ of 80 kg (b) $\frac{3}{5}$ of \$50 (c) $\frac{5}{6}$ of 36 m
 (d) $\frac{3}{4}$ of 200 L (e) $\frac{7}{10}$ of \$300 (f) $\frac{5}{8}$ of 48 cm
 (g) $\frac{2}{3}$ of 360 kg (h) $\frac{3}{4}$ of \$120 (i) $\frac{2}{5}$ of 250 g
 4. Solve the following problems.

Examples	1. $\frac{3}{7} + \frac{2}{7}$ $= \frac{5}{7}$	2. $\frac{5}{6} + \frac{3}{8}$ $= \frac{20}{24} + \frac{9}{24}$ $= \frac{29}{24}$ $= 1\frac{5}{24}$	3. $2\frac{3}{4} - 1\frac{5}{6}$ $= \frac{11}{4} - \frac{11}{6}$ $= \frac{33}{12} - \frac{22}{12}$ $= \frac{11}{12}$
-----------------	---	--	---

- (a) $\frac{5}{9} + \frac{2}{9}$ (b) $\frac{7}{11} - \frac{4}{11}$ (c) $\frac{2}{3} + \frac{1}{5}$
 (d) $\frac{3}{4} - \frac{1}{8}$ (e) $\frac{3}{8} + \frac{1}{3}$ (f) $\frac{7}{12} - \frac{1}{3}$
 (g) $\frac{5}{6} + \frac{3}{10} + \frac{2}{3}$ (h) $\frac{7}{8} + \frac{5}{9} - \frac{5}{12}$ (i) $3\frac{1}{2} + 2\frac{3}{5}$
 (j) $4\frac{3}{4} - 2\frac{5}{6}$ (k) $3\frac{2}{3} + 4\frac{1}{6}$ (l) $5\frac{7}{10} - 2\frac{3}{4}$
 (m) $8\frac{2}{5} + 6\frac{3}{8}$ (n) $5\frac{2}{7} - 4\frac{5}{8}$ (o) $2\frac{1}{2} + 3\frac{3}{4} - 4\frac{7}{8}$

5. Solve the following problems.

Examples

$$1. \quad \frac{3}{7} \times \frac{2}{5}$$

$$= \frac{6}{35}$$

$$2. \quad \frac{2}{3} \text{ of } 14$$

$$= \frac{2}{3} \times \frac{14}{1}$$

$$= \frac{28}{3}$$

$$= 9\frac{1}{3}$$

$$3. \quad \frac{8}{9} \times \frac{3}{16}$$

$$= \frac{\overset{1}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{3}{\cancel{3}}}{\underset{2}{\cancel{16}}}$$

$$= \frac{1}{6}$$

$$4. \quad 3\frac{3}{4} \times 2\frac{2}{3}$$

$$= \frac{15}{4} \times \frac{8}{3}$$

$$= \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{8}{\cancel{8}^2}}{\underset{3}{\cancel{3}}_1}$$

$$= \frac{10}{1}$$

$$= 10$$

(a) $\frac{4}{5} \times \frac{6}{7}$

(b) $\frac{5}{8} \times \frac{4}{5}$

(c) $\frac{9}{16} \times \frac{8}{27}$

(d) $\frac{3}{5} \text{ of } 11$

(e) $\frac{2}{7} \text{ of } 20$

(f) $\frac{3}{8} \text{ of } 30$

(g) $3\frac{3}{5} \times 2\frac{1}{12}$

(h) $3\frac{3}{8} \times 3\frac{1}{3}$

(i) $1\frac{7}{8} \times 1\frac{1}{3}$

(j) $7\frac{7}{8} \times 4\frac{8}{9}$

(k) $10\frac{5}{6} \times 5\frac{2}{5}$

(l) $2\frac{7}{16} \times 4\frac{12}{13}$

6. Solve the following problems.

Examples

$$1. \quad 3 \div \frac{5}{6}$$

$$= \frac{3}{1} \div \frac{5}{6}$$

$$= \frac{3}{1} \times \frac{6}{5}$$

$$= \frac{18}{5}$$

$$= 3\frac{3}{5}$$

$$2. \quad \frac{2}{3} \div 4$$

$$= \frac{2}{3} \div \frac{4}{1}$$

$$= \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{3}}} \times \frac{1}{\underset{4}{\cancel{4}_2}}$$

$$= \frac{1}{6}$$

$$3. \quad \frac{4}{5} \div \frac{8}{15}$$

$$= \frac{\overset{1}{\cancel{4}}}{\underset{5}{\cancel{5}}} \times \frac{\overset{15}{\cancel{15}^3}}{\underset{2}{\cancel{8}}_2}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

$$4. \quad 3\frac{3}{4} \div \frac{3}{16}$$

$$= \frac{15}{4} \div \frac{3}{16}$$

$$= \frac{15}{4} \times \frac{16}{3}$$

$$= \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{16}{\cancel{16}^4}}{\underset{3}{\cancel{3}}_1}$$

$$= \frac{20}{1}$$

$$= 20$$

(a) $8 \div \frac{4}{5}$

(b) $\frac{3}{8} \div 9$

(c) $\frac{4}{7} \div 14$

(d) $\frac{5}{8} \div \frac{3}{4}$

(e) $\frac{15}{16} \div \frac{5}{8}$

(f) $\frac{3}{7} \div \frac{9}{14}$

(g) $\frac{8}{15} \div 6\frac{2}{5}$

(h) $3\frac{3}{8} \div 2\frac{1}{4}$

(i) $6\frac{2}{9} \div 2\frac{2}{3}$

7. (a) How many quarters are in $5\frac{1}{2}$?
 (b) How many halves are in $11\frac{1}{2}$?
 (c) How many thirds are in $9\frac{2}{3}$?
 (d) How many fifths are in 20?
 (e) How many $\frac{2}{3}$'s are in 10?
 (f) How many 5's are in 12?
 (g) How many 8's are in 5?

Using a Calculator

EXERCISE 2C

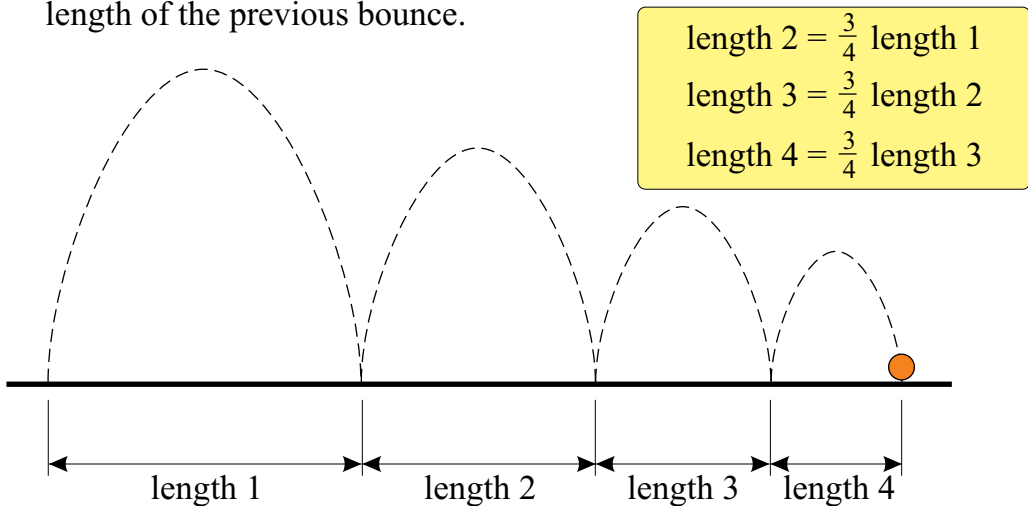
Use a calculator to solve the following problems.

- $8\frac{2}{3} \times (4\frac{1}{2} + 3\frac{3}{4})$
- $\frac{3\frac{3}{5} + 2\frac{1}{16}}{4\frac{2}{5} - 2\frac{3}{15}}$
- Find $\frac{2}{3}$ of \$3621
- Find $\frac{3}{7}$ of \$53 844
- Find $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{5}{8}$ of 48
- $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$
- $1\frac{2}{3} \times 4\frac{5}{6} \times 7\frac{8}{9}$
- $7\frac{1}{2} + \frac{(6\frac{2}{5} + 3\frac{3}{4})}{(5\frac{1}{6} - 2\frac{3}{4})}$
- How many $5\frac{2}{3}$'s are in $6\frac{7}{10}$?

Practical Problems

EXERCISE 2D

1. Skye had \$180 to spend on a five day holiday. She decided to spend one third of her money on Monday, one quarter on Tuesday, one fifth on Wednesday, one sixth on Thursday and the remaining money on Friday.
How much money did she spend on Friday?
2. John's car has a petrol tank that has a capacity of 60 litres. John fills up his petrol tank and then uses a third of the petrol driving to visit his brother.
He then uses a quarter of *the remaining petrol* visiting a friend.
He then uses a half of *the remaining petrol* driving to work.
He then uses a third of *the remaining petrol* on a weekend.
How many litres of petrol were left in the tank?
3. Bob ran three laps of an athletics track. Bob was getting slower each lap so that each lap took one and half times the time of the previous lap. His *third* lap took 135 seconds.
 - (a) How long did it take Bob to run the second lap?
 - (b) How long did it take Bob to run the three laps?
4. A ball bounces so that each bounce length is three-quarters of the length of the previous bounce.



Length 3 = 36 cm

Find length 1, length 2 and length 4.

5. A mix of bird seed contained one part sunflower seeds and two parts corn.
What fraction of the bird seed is corn?
6. Elaine breeds mice. Two-thirds of her mice are black and the remainder are white. She has 40 more black mice than white ones.
How many mice does she have?
7. A group of students start a ballroom dancing club.
Three-quarters of the group are girls.
What fraction of the girls will have to dance with another girl?
8. A number of students decided to play a year 9 versus year 10 soccer game.
Three-fifths of the students were from year 10.
What fraction of the year 10 students would need to play in the year 9 team to make the teams equal in size?
9. It took Tim $4\frac{1}{2}$ minutes to prune a rose bush.
(a) How long will it take Tim to prune 80 rose bushes?
(b) How many rose bushes could Tim prune in $2\frac{1}{4}$ hours?
10. Mrs. Smith has an apple orchard. From one year's crop one-tenth of the apples were too small to sell. Of the remaining apples, one-tenth had grubs and couldn't be sold.
(a) What fraction of the apples picked could be sold?
(b) If 3000 kg of apples were picked, how many kg could be sold?
11. To calculate the amount of GST (Goods and Services Tax) on a product, one-tenth of the original price is added on to find the new selling price.
What fraction of the new selling price is GST?
12. A water tank is two-thirds full. After 200 litres is used from the tank it is one-quarter full.
How many litres does the tank hold?

13. The population of koalas in a national park was increasing by one-fifth every year (i.e. the population at the end of a year was one-fifth more than it was at the start of that year).

There are 2000 koalas at the start of a particular year.

How many koalas would there be in the park three years later?



14. A class of students is divided into four groups to sing a musical round. They sing a song that has four equal parts.

The first group begins singing.

The second group begins singing when the first group has finished the first part of the song.

The third group begins singing when the second group has finished the first part of the song.

The fourth group begins singing when the third group has finished the first part of the song.

The continue singing until each group has completed the song *twice*.

(a) For what fraction of the singing time were all four groups singing?

(b) For what fraction of the singing time was one group singing?

15. A music class consisted of some students who played just piano, some who played just guitar, some who played piano and guitar and some who played just the violin.

One-third of the class played guitar and one-half of these also played piano.

One-half of the class played piano and one-third of these also played guitar.

There were eight students who played violin.

How many students were in the class?

PROBLEM SOLVING

A science student wanted to make three beakers of hydrochloric acid (HCl) mixed with water (H₂O). He wanted each beaker to have a different concentration of HCl in the water.

The three beakers each held two litres and he labelled them beaker A, beaker B and beaker C.

He started with the beakers empty and poured half a litre of HCl into beaker A and filled it with water mixing the liquids.

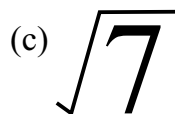
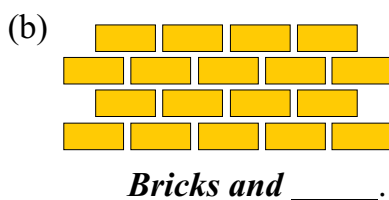
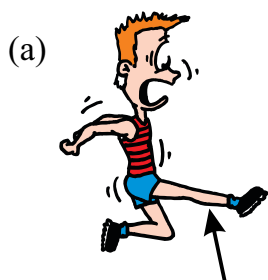
He then poured half of this mixture into beaker B and filled both with water.

He then poured one quarter of the mixture in beaker B into beaker C and filled both with water.

What fraction of the liquid in each beaker is hydrochloric acid?

PUZZLES

1. Find the fractions that rhyme with the following clues.



2. Rearrange the following letters to form fractions.

Example the thin sign = **eight-ninths**

(a) Finnish vet

(b) highest tent

(c) tennis thigh

(d) thin neon

(e) the snow vest

(f) white ghost

(g) huge fish rot

(h) hot surf inn

(i) festive high

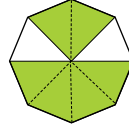
(j) even honest

(k) no queer rat

(l) one white tent

CHAPTER REVIEW

1. What fraction of this shape is shaded?
Give answer in its simplest form.



2. Complete the following equivalent fractions. $\frac{3}{5} = \frac{\square}{15} = \frac{6}{\square} = \frac{\square}{30} = \frac{36}{\square} = \frac{\square}{80}$

3. Change the following fractions to have the same denominator and hence arrange them in order from the smallest to the largest.

$$\frac{2}{3}, \frac{3}{5}, \frac{1}{2}$$

4. Write the first quantity as a fraction of the second in its simplest form.
(a) 8 cm : 2 m (b) 50c : \$3 (c) 600 kg : 3 tonnes

5. Change $7\frac{3}{8}$ to an improper fraction.

6. Change $\frac{79}{7}$ to a mixed number.

7. Solve the following problems.

(a) $\frac{2}{3}$ of \$60

(b) $\frac{3}{5}$ of 200 kg

(c) $\frac{3}{5} + \frac{2}{3}$

(d) $\frac{5}{9} - \frac{3}{8}$

(e) $3\frac{1}{2} - 1\frac{5}{8} + 2\frac{3}{4}$

(f) $\frac{6}{7} \times \frac{5}{11}$

(g) $\frac{3}{8} \times \frac{10}{33}$

(h) $3\frac{3}{5} \times 3\frac{1}{3}$

(i) $4\frac{1}{6} \div \frac{5}{12}$

8. How many fifths are in $7\frac{3}{5}$?

9. Use a calculator to solve: $3\frac{1}{4} \times (4\frac{2}{7} + 3\frac{5}{14})$

10. 4000 people were asked if they wanted a pool constructed in their town.

Three-fifths of the people wanted to build the pool and three-quarters of these people wanted it to be an indoor pool.

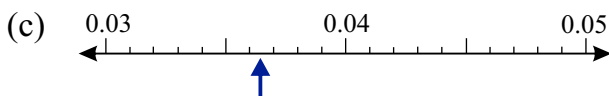
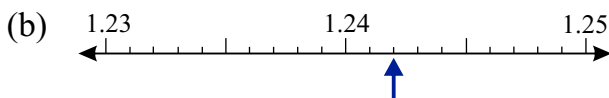
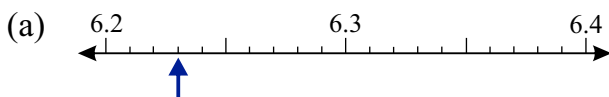
- (a) What fraction of the people wanted an indoor pool?
(b) How many people did not want the pool?

Decimals

Decimal Numbers

EXERCISE 3A

- Find the following numbers.
 - This three digit number is between 1 and 2.
 - It has five tenths.
 - The sum of the three digits is 10.
 - This five digit number is between 11 and 12.
 - It has four hundredths.
 - It has twice as many tenths as hundredths.
 - It has twice as many hundredths as thousandths.
- How many tenths is seventeen-hundredths?
- Place the decimal point in the following number so that it has six thousandths: 123456789
- What number is mid way between 3.8 and 3.94?
- What number is mid way between 17.81 and 17.86?
- Read the number indicated on each of the following scales.



- Add three thousandths to 8.8888.
- Is this statement true or false: 6 millionths = 0.0000006
- Subtract five hundredths from 34.8279
- Complete this pattern: 200, 20, 4, 1.2, 0.48, __, __.
- Write as a decimal number: eighty-nine thousandths
- Write in words: 0.0202

Fractions and Decimals

EXERCISE 3B

1. Copy and complete the following table showing conversions between fractions and decimals.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$		$\frac{1}{10}$	$\frac{3}{10}$		$\frac{1}{100}$		$\frac{23}{100}$		$\frac{1}{1000}$
Decimal			0.75			0.7		0.09		0.53	

2. Convert the following fractions to decimal numbers.

(a) $\frac{7}{100}$ (b) $\frac{9}{1000}$ (c) $\frac{17}{100}$ (d) $\frac{11}{1000}$ (e) $6\frac{9}{10}$
 (f) $12\frac{29}{1000}$ (g) $3\frac{157}{1000}$ (h) $\frac{21}{10\ 000}$ (i) $\frac{9}{10\ 000}$ (j) $23\frac{579}{10\ 000}$

3. Convert the following decimal numbers to fractions.

(a) 0.003 (b) 0.039 (c) 0.2357 (d) 0.0591 (e) 4.31
 (f) 0.00007 (g) 8.0092 (h) 5.2001 (i) 0.00059 (j) 9.043431

4. Convert the following fractions to decimal numbers.

Example $\frac{5}{8} = 5 \div 8 = 0.625$

(a) $\frac{3}{8}$ (b) $7\frac{1}{8}$ (c) $\frac{3}{5}$ (d) $11\frac{4}{5}$ (e) $\frac{11}{20}$
 (f) $\frac{9}{16}$ (g) $6\frac{3}{16}$ (h) $11\frac{36}{50}$ (i) $9\frac{39}{200}$ (j) $2\frac{413}{500}$

5. Convert the following fractions to decimal numbers (recurring decimals).

Examples

1. $\frac{2}{3} = 2 \div 3 = 0.666\ldots = 0.\dot{6}$ 2. $\frac{5}{12} = 5 \div 12 = 0.41666\ldots = 0.41\dot{6}$ 3. $\frac{3}{7} = 3 \div 7 = 0.4285742857\ldots = 0.\dot{4}2857$

(a) $\frac{1}{3}$ (b) $2\frac{5}{6}$ (c) $\frac{2}{9}$ (d) $13\frac{3}{11}$ (e) $\frac{13}{15}$
 (f) $\frac{11}{13}$ (g) $5\frac{6}{7}$ (h) $9\frac{19}{21}$ (i) $3\frac{528}{999}$ (j) $2\frac{529}{600}$

Rounding Decimal Numbers

To round a decimal number off to a certain digit follow the steps below.

Step 1 Identify the digit being rounded off to.

Step 2 Look at the digit *after* this digit.

Step 3 - If this digit is *less than 5* round down to the existing number.

- If this digit is *greater than or equal to 5* round up to the next number.

Examples

1. Round 59.7634 to *the nearest tenth*.

Step 1 Identify the digit being rounded off to (in this case the *tenth* - 7).

59.7634

Step 2 The next digit is 6.

59.7**6**34

Step 3 This is greater than 5 so round *up* to the next number.

Answer: 59.8

2. Round 8.03421 to *the nearest hundredth*.

Step 1 Identify the digit being rounded off to (in this case the *hundredth* - 3).

8.03421

Step 2 The next digit is 4.

8.03**4**21

Step 3 This is less than 5 so round *down* to the existing number.

Answer: 8.03

Examples continued

3. Round 19.03658 to **three decimal places**.

Step 1 Three decimal places means the three digits after the decimal point.

19.03658

Step 2 The next digit is 5.

19.03658

Step 3 When the next digit is 5 round **up** to the next number.

Answer: 19.037

4. Round 27.59982 to **three decimal places**.

Step 1 Three decimal places means the three digits after the decimal point.

27.59982

Step 2 The next digit is 8.

27.59982

Step 3 This is greater than 5 so round **up** to the next number.

Answer: 27.600

EXERCISE 3C

1. Round the following numbers to the nearest tenth.

- | | | | |
|-------------|------------|------------|--------------|
| (a) 0.3561 | (b) 6.719 | (c) 0.0831 | (d) 12.0095 |
| (e) 13.8791 | (f) 0.5965 | (g) 0.9567 | (h) 15.96711 |

2. Round the following numbers to the nearest hundredth.

- | | | | |
|--------------|-------------|--------------|---------------|
| (a) 0.7862 | (b) 10.5109 | (c) 9.5682 | (d) 90.909090 |
| (e) 124.5677 | (f) 6.59615 | (g) 129.9967 | (h) 67.9084 |

3. Round the following numbers to three decimal places.

- | | | | |
|-------------|-------------|-------------|---------------|
| (a) 0.44567 | (b) 9.60532 | (c) 0.06782 | (d) 100.80349 |
| (e) 8.10568 | (f) 45.0196 | (g) 7.9897 | (h) 0.099999 |

Scientific Notation (Standard Form)

Very large and very small numbers, particularly in science, are often expressed in a form called *scientific notation* or *standard form*.

To express a number in scientific notation move the decimal point to after the first digit and write the powers of ten in index form.

Examples

$$\begin{aligned} 1. \quad 6000 \\ &= 6 \times 1000 \\ &= 6 \times 10^3 \end{aligned}$$

$$\begin{aligned} 2. \quad 935\,000 \\ &= 9.35 \times 100\,000 \\ &= 9.35 \times 10^5 \end{aligned}$$

$$\begin{aligned} 3. \quad 0.000\,0079 \\ &= 7.9 \times 10^{-6} \end{aligned}$$

EXERCISE 3D

1. Write the following numbers in scientific notation.

- | | | |
|--------------------|-------------------|-----------------|
| (a) 5 430 000 | (b) 60 901 000 | (c) 230 000 000 |
| (d) 4100 | (e) 7 000 000 000 | (f) 0.0006 |
| (g) 0.000 067 54 | (h) 0.000 784 5 | (i) 0.000 780 4 |
| (j) 56 000 000 000 | (k) 0.000 000 077 | (l) 0.000 897 1 |

2. Write the following quantities in scientific notation.

- The distance from Earth to the Sun is 149 600 000 km.
- The distance from Mars to the Sun is 227 900 000 km.
- The distance between Earth and Mars when they are at their closest.
- Most bacteria are about 0.000 002 m long.
- A human egg is about 0.000 15 m in diameter.
- The heaviest animal was a blue whale that weighed 190 000 kg.
- The core temperature of the Sun is about 15 000 000°C.

Multiplying and Dividing by Powers of 10

EXERCISE 3E

Solve the following problems without using a calculator.

Examples	1. 4.56×1000 $= 4560$	When multiplying by a power of 10 move the decimal point to the right the same number of places as the number of zeros.
	2. $3113.6 \div 100$ $= 31.136$	When dividing by a power of 10 move the decimal point to the left the same number of places as the number of zeros.

- | | |
|----------------------------|------------------------------|
| 1. 8.7942×1000 | 2. $56.6 \times 10\ 000$ |
| 3. 0.0006×1000 | 4. 0.00785×1000 |
| 5. $0.089 \times 100\ 000$ | 6. 45×1000 |
| 7. $45.2 \div 1000$ | 8. $4504.6 \div 100$ |
| 9. $30\ 000 \div 100\ 000$ | 10. $10\ 402.6 \div 10\ 000$ |
| 11. $56.78 \div 100\ 000$ | 12. $0.05 \div 100$ |

Practical Problems

EXERCISE 3F

- Sam Speed took 11.6 seconds to run the 100 metres sprint and win the race. The second placed runner, Fleet O'Foot was three-tenths of a second behind Sam. The third placed runner, Dan Dash was half a second behind Fleet.
What was the time run by Fleet and Dan?
- The school record for the 200 metres sprint was 27.78 seconds. In the school athletics competition Jodie won the race but missed breaking the record by seven-hundredths of a second. Jessie came second, 0.12 seconds behind Jodie. Jacqui came third, two-tenths of a second behind Jessie.
Find the times run by each Jodie, Jessie and Jacqui.

3. In the 4×50 metres relay race in a swimming competition the average time swum by each of the four swimmers was 30.65 seconds. The first three swimmers swam 31.26 seconds, 30.52 seconds and 30.78 seconds respectively.

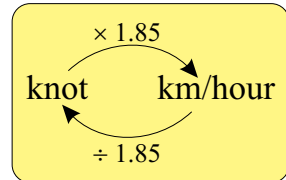
What time was swum by the fourth swimmer?

4. Jack sawed a 3.9 metre length of timber into two pieces. One piece was 0.4 metres longer than the other.

How long are the two pieces of timber?

5. A **knot** is a unit used when stating the speed of marine vessels.

$$1 \text{ knot} \approx 1.85 \text{ km/hour}$$



- (a) Use this conversion to change the following speeds to km/hour.

(i) 15 knots (ii) 8.5 knots (iii) 28 knots

- (b) Convert the following speeds to knots.

(Give answers correct to one decimal place)

(i) 20 km/hour (ii) 35 km/hour (iii) 46 km/hour

- (c) Yoti wanted to sail his yacht from Melbourne to Hobart - a distance of about 1000 kilometres. Yoti hopes to average about 20 knots for the trip.

How long (to the nearest hour) will Yoti's trip take?

6. (a) A piece of licorice weighed 0.025 kg. How many pieces would be in a one kilogram bag of licorice?

- (b) A 500 gram block of chocolate was divided into 12 equal pieces.

What did each piece weigh? Give answer correct to one decimal place.

7. When middle C is played on a stringed musical instrument the string vibrates 256 times every second.

What is the time of one vibration?

Give answer correct to three decimal places.

8. An engine is running at 2600 revolutions per minute (r.p.m.).

- (a) How many revolutions occur every second?

Give answer correct to one decimal place.

- (b) Use the **exact** value from part (a) to calculate the time (in seconds) of each revolution. Give answer correct to four decimal places.

9. The bearings of a wheel on a go-cart wear down by five-thousandths of a millimetre every kilometre travelled. The bearings need to be replaced after they have worn down three millimetres.

How many kilometres need to be travelled before the bearings need to be replaced?

10. Three friends each played different sports and they wanted to calculate who was the fastest. They each timed how long it took to travel a set distance and used these figures to calculate speed with the formula shown.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

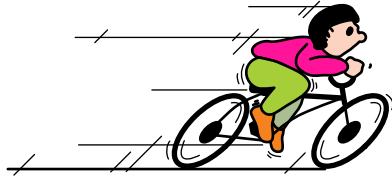
Kate skated 175 metres in 21.25 seconds.

Mike biked 250 metres in 30.35 seconds.

Skye skied 400 metres in 48.55 seconds.

Calculate their speeds correct to four decimal places.

Arrange them in order from fastest to slowest.



11. Mick has two drink containers (**A** and **B**) that each hold 1.5 litres. The containers have ten equally spaced marks on their sides.

Container **A** has pineapple juice up to mark 7.

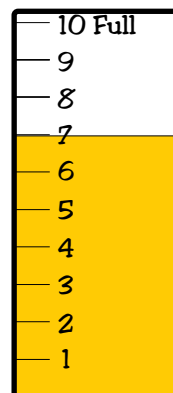
Container **B** has coconut milk up to mark 5.

Mick pours the pineapple juice from container **A** into the

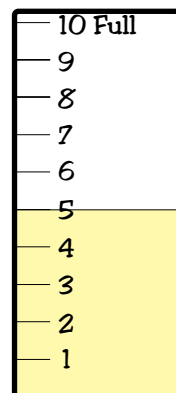
coconut milk until container **B** is full and stirs.

He then pours from container **B** into container **A** until both containers have the same amount of drink.

- How many litres of drink are now in each container?
- How many litres of pineapple juice are now in each container?
- How many litres of coconut milk are now in each container?



A



B

12. A tap is dripping once every second.

Each drop has a volume of 4.5×10^{-5} litres.

How many litres would be 'wasted' each day?

Give answer correct to one decimal place.

13. Atoms consist of neutrons, protons and electrons.

The mass of a neutron is 1.67×10^{-27} kg

The mass of a proton is 1.67×10^{-27} kg

The mass of an electron is 9.1×10^{-31} kg

(a) Find the mass of the following atoms.

(i) Carbon - 6 neutrons, 6 protons and 6 electrons

(ii) Oxygen - 8 neutrons, 8 protons and 8 electrons

(iii) Hydrogen - 1 proton and 1 electron

(b) How many carbon atoms would be in one kilogram of carbon?

14. Sparkey wanted to calculate how much it cost to run electrical appliances in his house. The formula below can be used to do this.

$$C = P \times t \times 16.75$$

where C = the cost to run the appliance (in cents)

P = the power used by the appliance (in kilowatts - kW)

t = the time the appliance is running (in hours)

Example A radio that uses 0.03 kW is run for 8 hours

$$C = P \times t \times 16.75$$

$$= 0.03 \times 8 \times 16.75$$

$$= \mathbf{4.02 \text{ cents}}$$

Use the above formula to calculate the cost to run the following appliances for the times shown.

Give answers correct to two decimal places.

(a) Television (0.12 kW) - 5 hours

(b) Heater (1.5 kW) - 6 hours

(c) Computer (0.08 kW) - 4 hours

(d) Iron (1.6 kW) - 30 minutes

(e) Toaster (0.8 kW) - 10 minutes

PROBLEM SOLVING

Our Solar System is part of a galaxy called the Milky Way Galaxy. The Milky Way Galaxy is in the shape of an ellipse that is 9.47×10^{27} km across. It is estimated that there are 2.5×10^{11} stars in the Milky Way Galaxy.

Sandie wanted to calculate how large a pile of sand would need to be to contain the same number of grains as there are stars in the Milky Way Galaxy.

A grain of sand has a volume of approximately $4 \times 10^{-11} \text{ m}^3$ (cubic metres).

Find the volume of sand (in m^3) to answer Sandie's question.

PUZZLES

- Find the values for **A**, **B** and **C** to make the following equation correct. (There are two answers)

$$\mathbf{A + B.C = A \times B.C}$$

- Polly had three dogs - Deefa, Bitza and Kevin. She needed to weigh them to give them tablets. The problem was they wouldn't stand on the scales by themselves - she had to weigh them two at a time.
Deefa and Bitza together weighed 7.9 kg.
Deefa and Kevin together weighed 9.2 kg.
Bitza and Kevin together weighed 10.3 kg.
Find the mass (in kg) of each dog.

CHAPTER REVIEW

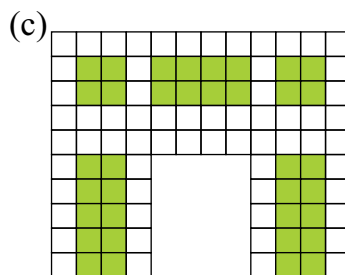
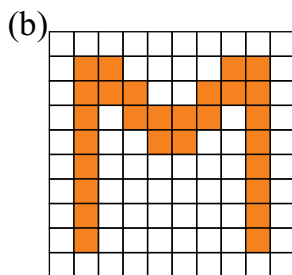
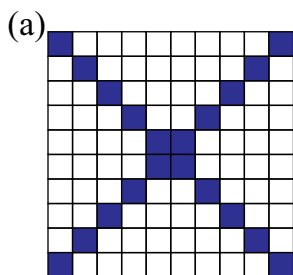
1. Write the decimal number that has six units, seven hundredths, one tenth and three thousandths.
2. Convert the following fractions to decimal numbers.
(a) $\frac{1}{4}$ (b) $\frac{7}{8}$ (c) $5\frac{6}{10}$ (d) $\frac{9}{1000}$ (e) $17\frac{19}{100}$
3. Convert the following decimal numbers to fractions.
(a) 0.4 (b) 6.09 (c) 3.0197 (d) 15.217 (e) 8.00007
4. Convert the following fractions to decimal numbers.
(a) $\frac{2}{3}$ (b) $3\frac{1}{6}$ (c) $7\frac{7}{9}$ (d) $3\frac{3}{7}$ (e) $\frac{1}{15}$
5. Round the following numbers to the nearest hundredth.
(a) 6.2439 (b) 0.0972 (c) 18.3961 (d) 4.789 (e) 29.9876
6. Round the following numbers to three decimals places.
(a) 4.31718 (b) 1.09439 (c) 11.8997 (d) 83.21783
7. Write the following numbers in scientific notation.
(a) 78 200 (b) 300 000 (c) 0.00172 (d) 0.0000042
8. Solve the following problems without using a calculator.
(a) 3.49×1000 (b) 0.0124×100 (c) $0.00056245 \times 100\ 000$
(d) $6200 \div 1000$ (e) $834.1 \div 100\ 000$ (f) $0.0973 \div 100$
9. How many cups with volume 0.02 litres could be taken from a 5 litre container?
10. How thick is a pile of 500 sheets of paper if each sheet is 0.012 cm thick?
11. (a) One acre is 0.405 hectares. How many acres are in one hectare?
(Give answer correct to three decimal places)
(b) How many hectares are in 30 acres?

Percentages

Percentages

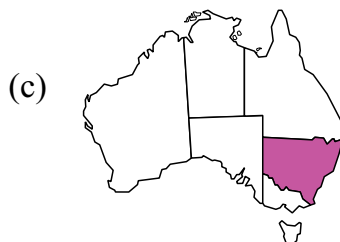
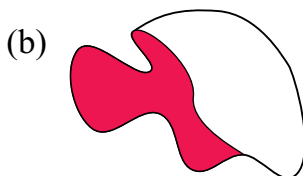
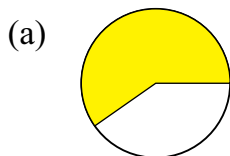
EXERCISE 4A

1. What percentage of each of the following shapes is shaded?



2. What percentage of each shape in question 1 is **not** shaded?

3. Choose which is the best estimate of the percentage of each of the following shapes that is shaded.



A 20% **B** 40%
C 60% **D** 80%

A 20% **B** 40%
C 60% **D** 80%

A 10% **B** 20%
C 30% **D** 40%

4. Kirk and Tenelle ran for the position of school captain.

Kirk got 54.5% of the votes.

(a) What percentage of the votes did Tenelle get?

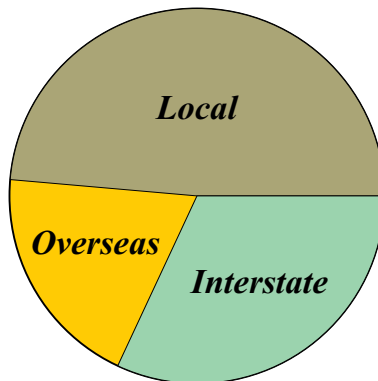
(b) Who will be the school captain?

5. The movement of a footballer during a game was analysed.

The percentage of game time that he spent sprinting, jogging, walking and standing were calculated. From the figures below find the percentage of the game that the footballer spent standing.

Sprinting - 23%, Jogging - 46%, Walking - 18%

6. The lithosphere is the rocky layer about the Earth that extends to a depth of about 100 km. The main elements that make up the lithosphere are as follows:
Oxygen (46.60%), Silicon (27.72%), Aluminium (8.13%),
Iron (5.00%), Calcium (3.63%), Sodium (2.83%),
Potassium (2.59%), Magnesium (2.09%).
What percentage of the lithosphere is made up of elements other than the ones listed above?
7. Aaron, Clara, Barton and Felicity ran a stall at the local market. During one market Aaron worked for one hour, Clara worked for two hours, Barton worked for three hours and Felicity worked for four hours.
They divided the profits according to the hours they each worked.
What percentage of the profits should they each receive?
8. At a game between the Marla Munchers and the Baram Bashers there were 30% more of the crowd that barracked for the Munchers than for the Bashers.
What percentage of the crowd barracked for each team?
9. A producer of olive oil used a blend of local, interstate and overseas olives.
This pie chart shows the proportion of the different types of olives used.
From the list below choose which percentage represents each type of olive.



11% 18% 23% 33% 35% 45% 49% 53%

Percentages and Fractions

EXERCISE 4B

1. Convert the following fractions to percentages.

Examples

To convert a fraction to a percentage:
- multiply by 100.

$$\begin{aligned} 1. \quad & \frac{3}{5} \\ &= \frac{3}{5} \times 100 \\ &= \mathbf{60\%} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{7}{8} \\ &= \frac{7}{8} \times 100 \\ &= \mathbf{87.5\%} \end{aligned}$$

- (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$ (e) $\frac{1}{5}$
 (f) $\frac{3}{4}$ (g) $\frac{7}{20}$ (h) $\frac{13}{50}$ (i) $\frac{12}{25}$ (j) $\frac{1}{8}$
 (k) $\frac{7}{16}$ (l) $1\frac{3}{16}$ (m) $2\frac{17}{20}$ (n) $1\frac{3}{8}$ (o) $3\frac{32}{125}$

2. Convert the following percentages to fractions in their simplest form.

Examples

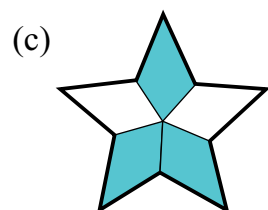
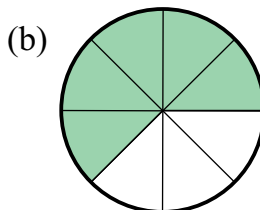
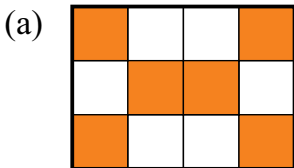
To convert a percentage to a fraction:
- divide by 100.

$$\begin{aligned} 1. \quad & 58\% \\ &= \frac{58}{100} \\ &= \frac{\mathbf{29}}{\mathbf{50}} \end{aligned}$$

$$\begin{aligned} 2. \quad & 45.5\% \\ &= \frac{45.5}{100} \\ &= \frac{\mathbf{91}}{\mathbf{200}} \end{aligned}$$

- (a) 22% (b) 95% (c) 12% (d) 120% (e) 2%
 (f) 15.5% (g) 62.5% (h) 20.75% (i) 0.2% (j) 5.3%

3. What percentage of each of the following shapes is shaded?



Finding Percentages

EXERCISE 4C

1. Write the first quantity as a percentage of the second.

Examples

Make sure the units are the same

1. 2 kg, 10 kg

$$= \frac{2}{10} \times 100$$

$$= 20\%$$

2. 60 cents, \$2

$$= 60 \text{ cents, } 200 \text{ cents}$$

$$= \frac{60}{200} \times 100$$

$$= 30\%$$

3. 85 cm, 5 m

$$= 85 \text{ cm, } 500 \text{ cm}$$

$$= \frac{85}{500} \times 100$$

$$= 17\%$$

(a) 8 kg, 20 kg

(b) 75 mm, 300 mm

(c) \$45, \$500

(d) 40 cents, \$8

(e) 400 mm, 2 m

(f) 150 g, 3 kg

(g) 36 sec, 1 min

(h) 12 min, 2 hours

(i) 50 mm, 20 cm

(j) 240 mL, 3 L

(k) 380 kg, 2 tonnes

(l) 81 cm, 3 m

2. Charles achieved the following marks for several mathematics tests. Convert these marks to percentages correct to one decimal place.

Percentages $\frac{78}{90}$

Algebra $\frac{66}{85}$

Graphs $\frac{48}{60}$

3. In a school of 950 students it was found that 795 of the students were right-handed, 136 were left-handed and the remainder were ambidextrous.
Find what percentage (correct to one decimal place) were in each category.
4. The total area of Australia is 7 682 300 km². Ownership of Australian land is divided into three categories:

Private land:

4 819 600 km²

Aboriginal and Torres Strait Islander Land:

1 094 800 km²

Public Land:

1 767 900 km²

Calculate the percentage (correct to one decimal place) of land that is in each category.

Percentages and Decimals

EXERCISE 4D

1. Convert the following decimals to percentages.

Example $0.21 \quad 0.21 \times 100$
 $= 0.21 \rightarrow$
 $= 21\%$

To convert a decimal to a percentage - multiply by 100. Move the decimal point two places to the right.

- (a) 0.33 (b) 0.19 (c) 0.87 (d) 0.06 (e) 0.4
 (f) 0.295 (g) 0.056 (h) 0.008 (i) 2 (j) 0.1057

2. Convert the following percentages to decimals.

Example $37\% \quad 37 \div 100$
 $= \overleftarrow{37}.$
 $= 0.37$

To convert a percentage to a decimal - divide by 100. Move the decimal point two places to the left.

- (a) 59% (b) 24% (c) 76% (d) 30% (e) 8%
 (f) 10% (g) 100% (h) 65.4% (i) 5.8% (j) 0.25%

3. Copy and complete this table.

Write all fractions in their simplest form.
 One line has been completed as an example.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
10%		
	$\frac{1}{4}$	
		0.2
	$\frac{9}{10}$	
66%		
	$\frac{9}{50}$	
		0.32
	$\frac{17}{20}$	
9%		
	$\frac{3}{100}$	
		0.04
	$\frac{1}{3}$	
	$\frac{2}{3}$	

Percentages of Quantities

EXERCISE 4E

1. Without using a calculator find the following quantities.

Example 50% of \$20
 $= \frac{1}{2}$ of \$20
 $= \text{\$10}$

- | | | |
|-------------------------------|-------------------------------|---------------------------------|
| (a) 50% of 8 m | (b) 25% of \$40 | (c) 10% of \$50 |
| (d) $33\frac{1}{3}\%$ of 9 kg | (e) $66\frac{2}{3}\%$ of 9 kg | (f) 75% of 12 m |
| (g) 50% of \$120 | (h) 25% of \$400 | (i) $33\frac{1}{3}\%$ of 30 km |
| (j) 10% of 250 kg | (k) 50% of \$8000 | (l) 20% of 500 kg |
| (m) 50% of \$5 | (n) 10% of \$3 | (o) $66\frac{2}{3}\%$ of \$3.90 |

2. Use a calculator to find the following quantities.

Example 43% of \$350
 $= \frac{43}{100} \times \$350$
 $= \text{\$150.50}$

- | | | |
|-------------------|----------------------|------------------------|
| (a) 62% of 450 kg | (b) 19% of 850 m | (c) 23% of \$2580 |
| (d) 47% of \$2400 | (e) 82% of 3675 m | (f) 11% of 3.5 m |
| (g) 6.5% of \$750 | (h) 8.5% of \$42 000 | (i) 12.75% of \$84 700 |

3. Tahlia had \$3500 in a bank account that earned 4% interest per year. How much interest would Tahlia receive after one year?

4. The population of a town was increasing at the rate of 3.6% each year. The population at the start of a year was 34 500.

- (a) By how many people would the population have increased by the end of the year?
 (b) What would be the population at the end of the year?

5. Yuri is a tennis player. He has calculated that he hits into play 55% of his first serves. If Yuri has 60 first serves in a match, how many would he expect to hit into play?



6. Scott and Lucien play in the same basketball team. In one season Scott had 112 foul shots and scored 78 of these. Lucien had 92 foul shots and scored 67 of these.
- Calculate the percentage of foul shots each player scored from the shots taken. (Give answers correct to one decimal place)
 - Who was the more accurate foul shooter?
 - If they were each to shoot 60 foul shots, how many would each player expect to score from?

7. The population of sugar gliders in a state park was increasing by 2.5% each year. At the start of a year there were 2600 sugar gliders.
- How many sugar gliders would there be at the end of that year?
 - How many sugar gliders would there be at the end of the next year?



8. The capacity of the water reservoir for a town was 1 950 000 litres. At the start of January one year the reservoir was 57.4% full. How many litres were in the reservoir?
9. The population of Sydbourne was about 2.4 million. It was estimated that 26% of the population attended at least one music concert a year. Of the people that attend music concerts, 30% were between the ages of 14 and 24. How many people between the ages of 14 and 24 attended at least one music concert a year?
10. The 180 year 9 students at a school were asked what they intended to do when they finish secondary college.
 72 students wanted to do further training (apprenticeship, etc)
 81 students wanted to enter tertiary education
 27 students wanted to get a job
- What percentage of year 9 students were in each category?
 - If there were 980 students at the school how many students would you expect to be in each category?

Finding 100%

EXERCISE 4F

To find 100% of a quantity knowing a percentage of it - first find 1% then find 100%

Examples

1. There are 170 students in year 9. This is 20% of the school.
How many students are in the school?

Find 1%:

$$170 \div 20 = 8.5$$

Find 100%:

$$8.5 \times 100 = \mathbf{850 \text{ students}}$$

2. A petrol tank is 35% full. There are 14 litres in the tank.
How many litres does the tank hold?

Find 1%:

$$14 \div 35 = 0.4$$

Find 100%:

$$0.4 \times 100 = \mathbf{40 \text{ litres}}$$

- Find 100% of the following quantities.

(a) 15% is \$75	(b) 20% is 45 kg
(c) 24% is \$300	(d) 72% is 450 g
(e) 5% is \$350	(f) 10.5% is \$84
- The unemployment rate in a city was 6%. There were 7500 people unemployed. How many employable people were in the city?
- A nurse had to make an antiseptic solution using antiseptic and water. The solution was to be 5% antiseptic and the remainder water. The nurse used 20 mL of antiseptic.
 - What was the volume of the solution made?
 - What volume of water was required?
- Rae was downloading information onto her computer. 16% of the information had downloaded after 8 seconds.
How long would it take to download all the information?

Discounts

EXERCISE 4G

Example

A shop is offering a 20% discount off a \$35 CD. What price will you pay for the discounted CD?

The discount = 20% of 35

$$= 20 \div 100 \times 35$$

$$= \$7$$

$$\text{Discounted price} = 35 - 7 = \mathbf{\$28}$$

1. Find the discounted prices for the following items.

- (a) A \$75 basketball discounted by 20%.
- (b) A \$400 lawnmower discounted by 15%.
- (c) A \$14 000 car discounted by 4%.
- (d) A \$2500 computer discounted by 25%.
- (e) A \$38 toaster discounted by 15%.
- (f) A \$26 book discounted by 8%.
- (g) A \$110 radio discounted by 35%.

Example

A skirt is selling for \$68 after a discount of 15% .

What was the original price of the skirt?

If it discounted by 15% then \$68 is 85% of the original price.

To find 100% use steps from Exercise 4F:

Find 1%:

$$68 \div 85 = 0.8$$

Find 100%:

$$0.8 \times 100 = \mathbf{\$80}$$

2. Find the original price of the following items.

- (a) A guitar is selling for \$120 after a discount of 20%.
- (b) A calculator is selling for \$45 after a discount of 10%.
- (c) A bicycle is selling for \$440 after a discount of 12%.
- (d) A roast chicken is selling for \$2.70 after a discount of 40%.
- (e) A horse saddle is selling for \$187.50 after a discount of 25%.
- (f) A parachute jump is selling for \$238 after a discount of 15%.
- (g) A television is selling for \$336 after a discount of 30%.

Profit

EXERCISE 4H

Example

A clothes shop owner buys jeans for \$40 (cost price) and sells them for \$70 (selling price).

Find the following quantities:

1. The profit
2. The profit as a percentage of cost price
3. The profit as a percentage of selling price

$$\begin{aligned} 1. \text{ Profit} &= \text{selling price} - \text{cost price} \\ &= 70 - 40 \end{aligned}$$

$$\text{Profit} = \$30$$

$$\begin{aligned} 2. \text{ Profit as a percentage of cost price} &= \frac{\text{profit}}{\text{cost price}} \times 100 \\ &= \frac{30}{40} \times 100 \end{aligned}$$

$$\text{Profit as a percentage of cost price} = 75\%$$

$$\begin{aligned} 3. \text{ Profit as a percentage of selling price} &= \frac{\text{profit}}{\text{selling price}} \times 100 \\ &= \frac{30}{70} \times 100 \end{aligned}$$

$$\text{Profit as a percentage of selling price} = 42.9\% \text{ (one dec. pl.)}$$

For the following items find:

- (a) the profit
 - (b) the profit as a percentage of cost price
 - (c) the profit as a percentage of selling price
- (Give answers correct to one decimal place)

Item	Cost price	Selling price
1. Skateboard	\$120	\$150
2. Wetsuit	\$100	\$200
3. Bed	\$350	\$550
4. Painting	\$140	\$200

Commission

EXERCISE 4I

Some salespeople earn money by receiving a commission on what they sell. The commission is a percentage based on the amount sold.

Examples

1. Serena sells linen and receives a commission of 20% of her sales. At one linen party she sells three doonas that each cost \$150. What commission does Serena receive?

$$\text{Sales} = 3 \times \$150 = \$450$$

$$\text{Serena's commission} = 20\% \text{ of her sales}$$

$$= 20\% \text{ of } \$450$$

$$= 20 \div 100 \times \$450$$

$$\text{Serena's commission} = \mathbf{\$90}$$

2. Brett sells computers. He gets paid \$200 per week plus a commission of 5% of his sales.

What will Brett earn in a week if he sells \$3000 worth of computers?

$$\text{Brett's commission} = 5\% \text{ of } \$3000$$

$$= 5 \div 100 \times 3000$$

$$= \$150$$

$$\text{Brett's total earnings} = \$200 + \$150$$

$$= \mathbf{\$350}$$

1. Brooke sells skin care products and earns a commission of 15% of her sales. What commission would she earn if she sold \$800 worth of the products?
2. Campbell sells cars and earns a commission of 3.5% of the value of the cars he sells. What commission would he earn if he sold a \$14 560 car?

3. Leteisha is a real estate salesperson. She receives a commission of 1.5% of the value of the properties she sells. Find her commission if she sells the following properties:

- (a) a house worth \$165 000.
- (b) a farm worth \$545 000
- (c) two blocks of land - one worth \$24 000, the other \$33 000.

4. Liam sells parasails. The parasails sell for \$2450 each.

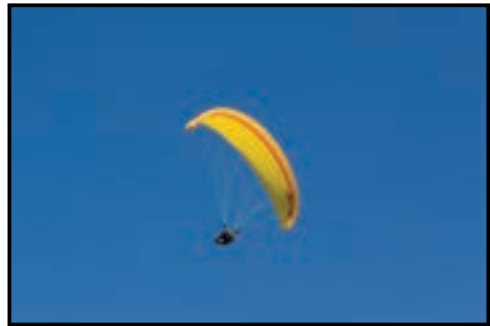
He has two options for his pay.

Option A: A commission of 15% of his sales

Option B: A weekly salary of \$250 + a commission of 10% of his sales

Calculate his weekly income for each option if he sells the following number of parasails in a week.

- (a) 1 parasail
- (b) 2 parasails
- (c) 3 parasails



5. Martine sells kitchenware. She earns a commission of 35% of her sales. The table below shows her sales at one kitchenware show. Calculate Martine's commission from this show.

<i>Item</i>	<i>Cost per item</i>	<i>Number of items sold</i>
Casserole Dish	\$35	2
Large Stainless Pot	\$100	3
Medium Stainless Pot	\$80	3
Small Stainless Pot	\$60	3
Baking Dish	\$25	2
Stainless Grater	\$12	5
Frypan	\$45	2
Wok	\$55	1
Chopping Board	\$32	4
Knife Set	\$120	2

Simple Interest

Interest is extra money earned if money is invested or extra money paid if money is borrowed.

When interest is calculated on the amount invested or borrowed and not added onto the original amount it is called **simple interest**.

To calculate the amount of simple interest the following formula can be used.

$$SI = \frac{PRT}{100}$$

Where SI = the amount of simple interest

P = the original amount or principal

R = the interest rate per year (or per annum p.a.)

T = the number of years or term of the loan or the investment

Example

Naomi invests \$5000 for 3 years at 5.5% p.a. simple interest. Calculate the amount of interest she will earn.

$$P = 5000, R = 5.5, T = 3$$

$$\begin{aligned} SI &= \frac{PRT}{100} \\ &= \frac{5000 \times 5.5 \times 3}{100} \\ &= \mathbf{\$825} \end{aligned}$$

EXERCISE 4J

1. Calculate the amount of simple interest that would be earned on the following investments.
- \$3500 invested for 2 years at 6% p.a.
 - \$600 invested for 3 years at 4.5% p.a.
 - \$20 000 invested for 1 year at 11% p.a.
 - \$6750 invested for 5 years at 8.5% p.a.
 - \$1200 invested for $2\frac{1}{2}$ years at 9% p.a.
 - \$8000 invested for 6 months at 10% p.a.

Example

Gerard borrows \$2000 for 2 years at a simple interest rate of 8% p.a.

- Calculate the amount of interest he will need to pay.
- Calculate the total amount of money he needs to repay.

$$P = 2000, R = 8, T = 2$$

$$\begin{aligned}
 \text{(a) } SI &= \frac{PRT}{100} \\
 &= \frac{2000 \times 8 \times 2}{100} \\
 &= \text{\$320}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) Total amount to repay} &= 2000 + 320 \\
 &= \text{\$2320}
 \end{aligned}$$

2. For the following loans calculate:
- the amount of simple interest that will be paid
 - the total amount of money that needs to be repaid
- \$4000 is borrowed for 2 years at 7.5% p.a.
 - \$12 000 is borrowed for 1 year at 6% p.a.
 - \$2500 is borrowed for 5 years at 12.5% p.a.
 - \$1850 is borrowed for 10 years at 12% p.a.
 - \$20 000 is borrowed for $3\frac{1}{2}$ years at 7.5% p.a.
 - \$6500 is borrowed for 6 months at 8.25% p.a.

PROBLEM SOLVING

1. Meredith has a photo that is $10\text{ cm} \times 6\text{ cm}$. She wants to enlarge it so that the dimensions are 25% larger.
 - (a) Find the dimensions of the enlarged photo.
 - (b) What is the percentage increase in the area of the photo?
(Remember: $\text{area} = \text{length} \times \text{width}$)
2. Stefan owns a clothing shop. He sells his jeans for \$80 each and sells, on average, 10 jeans per week.
He is advised that if he reduces his prices by 10% he will increase the number of jeans sold by 10%.
Should Stefan follow this advice?

PUZZLE

Decrypt the clues below to form a sentence relating to this chapter.



=

C



CHAPTER REVIEW

1. 70.8% of the Earth's surface is water. What percentage is land?

2. Copy and complete this conversion table.

<i>Fraction</i>	$\frac{1}{2}$				$\frac{5}{8}$
<i>Decimal</i>		0.2		0.125	
<i>Percentage</i>			25%		

3. 3500 people went to see a concert. 2200 of these people were under 20 years old. What percentage of the audience were under 20 years old? (Give answer correct to one decimal place)

4. Find the following quantities.

(a) 35% of \$4500 (b) 64% of 18.5 m

5. 65% of the students at a school had pets. If 689 students had pets how many students were at the school?

6. A \$90 pair of shoes is discounted by 20%. What is the discounted price?

7. A set of drums is selling for \$382.50 after being discounted by 15%. What was the original price of the set of drums?

8. Duncan buys a car for \$1500 and after a few minor repairs sells it for \$1950. What is the profit as a percentage of the cost?

9. William sells mobile phones for \$60 each. If he gets a commission of 12% of his sales, what will he earn if he sells 25 phones?

10. Calculate the simple interest earned if \$3750 is invested at 6% p.a. for 4 years.

Direct Proportion

Rates

EXERCISE 5A

1. Yvonne wanted to buy some cherries. The cherries cost \$12 per kg.
 - (a) Find the cost of the following quantities of cherries:
(i) 3 kg (ii) 4.5 kg (iii) 2.4 kg (iv) 600 g
 - (b) How many kilograms of cherries could be bought for the following amounts of money?
(i) \$24 (ii) \$18 (iii) \$14.40 (iv) \$4.80
2. Robyn loves chocolate. Her favourite chocolate costs \$1.20 per 100 g.
 - (a) Find the cost of the following amounts of chocolate:
(i) 300 g (ii) 50 g (iii) 1 kg (iv) 1.6 kg
 - (b) How many grams of chocolate could be bought for the following amounts of money?
(Give answers to the nearest whole number)
(i) \$2.40 (ii) \$2 (iii) \$5 (iv) \$10.50
3. Kim bought 5 kg of fishing bait for \$6.50.
 - (a) Find the cost of the following amounts of bait:
(i) 1 kg (ii) 3 kg (iii) 500 g (iv) 200 g
 - (b) How much bait could be bought for the following amounts of money?
(Give answer to two decimal places)
(i) \$1 (ii) \$1.60 (iii) \$2.50 (iv) \$12
4. Russell had to give some medicine to his dogs. The medicine had to be given at the rate of four tablets per kg.
Find how many tablets have to be given to each of his dogs.
 - (a) Yoda - 5 kg (b) Hubbsie - 3.5 kg (c) Pixie - 1.25 kg
5. Rashid has bought some water-cleaner for his fish tank. The water-cleaner has to be added to the fish tank at the rate of 5 mL per 100 litres. Rashid's fish tank is 550 litres.
How many mL of the water-cleaner should Rashid use?

6. Iona was on holidays overseas and was charged 40 cents per minute for her phone calls back home.
- (a) Find the cost of the following phone calls:
 (i) 3 minutes (ii) 40 minutes (iii) 8.5 minutes
- (b) Find the length of the calls that cost the following amounts:
 (i) \$2 (ii) \$1.28 (iii) \$5.04
7. Yasmin was downloading some information from the internet.
 The download rate was 2.5 MB per second.
 How long will it take Yasmin to download 600 MB?
8. Table 1 below shows how much energy (in kilojoules - kJ) is used in one hour of participation in certain activities.
 Table 2 shows the approximate energy (kJ per 100 g) provided by certain foods.

Table 1

<i>Activity</i>	<i>kJ per hour</i>
Running	2800
Walking, easy	700
Walking, brisk	1200
Tennis	2000
Swimming	3000

Table 2

<i>Food</i>	<i>Energy (kJ) per 100 g</i>
Bread	1000
Peanuts	2500
Porridge	1500
Bananas	350
Chocolate	2100

Use this information to answer the questions below.

- (a) How much energy (kJ) would be used in the following activities?
 (i) half an hour of running
 (ii) three hours of easy walking
 (iii) 15 minutes of tennis
 (iv) 40 minutes of swimming
- (b) How many grams of bread would be needed to provide the energy for the following activities?
 (i) one hour of swimming
 (ii) two hours of tennis
 (iii) five hours of easy walking
 (iv) two hours of brisk walking

(c) Find the amount of energy (kJ) in the following amounts of food:

- (i) 200 g of porridge
- (ii) 50 g of chocolate
- (iii) 20 g of peanuts
- (iv) 150 g of bread
- (v) 120 g of bananas

(d) How many minutes of tennis would 100 grams of porridge provide the energy for?

(e) How many grams of bananas would provide enough energy to run for half an hour?

(f) How many grams of bread provide the same amount of energy as 20 grams of peanuts?

9. Kylie owned a car that had a fuel consumption of 8 litres per 100 kilometres.

(a) How many litres would the car use travelling the following distances?

- (i) 200 kilometres
- (ii) 50 kilometres
- (iii) 75 kilometres
- (iv) 320 kilometres

(b) How far could be driven on the following amounts of petrol?

- (i) 2 litres
- (ii) 24 litres
- (iii) 20 litres
- (iv) 50 litres

10. David and Ben are bricklayers. David can lay 120 bricks in an hour. Ben can lay 90 bricks in an hour.

(a) How many hours would it take David to lay 360 bricks?

(b) How many hours would it take Ben to lay 360 bricks?

(c) How many minutes would it take David to lay 100 bricks?

(d) How many minutes would it take Ben to lay 100 bricks?

(e) How many bricks would David lay in 20 minutes?

(f) How long would it take David and Ben working together to lay 3150 bricks?

Speed

The formulae below can be used to solve problems related to speed.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$\text{Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$$

$$\text{Distance travelled} = \text{Speed} \times \text{Time taken}$$

Note the following abbreviations:

hour (**h**), minutes (**min**), seconds (**s**), metres per second (**m/s**),
kilometres per hour (**km/h**)

- Examples**
1. What is the speed of someone who runs 200 metres in 25 seconds?
 2. If a plane is flying at 600 km/h, how long will it take to fly 900 km?
 3. If a car drives at 85 km/h for 3 hours, how far will it travel?

Answers

1. $\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$

$$\text{Speed} = \frac{200}{25}$$

$$\text{Speed} = 8 \text{ m/s}$$

2. $\text{Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$

$$\text{Time taken} = \frac{900}{600}$$

$$\text{Time taken} = 1.5 \text{ hours}$$

3. $\text{Distance travelled} = \text{Speed} \times \text{Time taken}$
 $= 85 \times 3$

$$\text{Distance travelled} = 255 \text{ km}$$

EXERCISE 5B

1. James ran 225 metres in 50 seconds.
At what speed (in m/s) was James running?
2. Katrina rode her bike at 15 m/s to her friend's house, 3.15 km away.
 - (a) How many metres did Katrina ride?
 - (b) How many seconds did it take Katrina to get to her friend's house?
 - (c) How many minutes did it take Katrina to get to her friend's house?
3. A marathon is approximately 42 km. Michael ran a marathon in $2\frac{1}{2}$ hours. (Give the following answers to one decimal place)
Find Michael's speed in:
(a) km/h (b) m/s (c) m/min
4. The record for running a marathon backwards (!!) is 3 hr 53 min 17 sec.
At what speed (m/s) was this person running?
(Give answer to the nearest whole number)
5. The fastest animal is a cheetah that can run at about 108 km/h.
 - (a) Convert 108 km/h to m/s.
 - (b) How long (in seconds) would it take a cheetah to run 300 metres?
 - (c) How far would a cheetah run in 12 seconds?
6. The Earth's orbit about the Sun is approximately 940 000 000 km.
The Earth takes 365.25 days to travel once around the Sun.
 - (a) How many seconds does it take the Earth to travel once around the Sun?
 - (b) Find the Earth's speed in km/s.
(Give answer to one decimal place)



7. Pedro lives 1200 metres from school. He walks to school each day at a speed of 1 m/s.
Iain lives 4.5 kilometres from school. He rides his bike to school each day at a speed of 15 m/s.
- (a) If they both arrive at school at 8:30 a.m., at what time did they each leave home?
 - (b) Pedro and Iain live 4.8 kilometres apart. They planned to meet each other one Saturday morning and both left their homes at 9.00 a.m.. If Pedro walked and Iain rode, at what time would they meet?
8. In baseball the batter stands about 18 m from the pitcher.
The fastest pitch in baseball is about 160 km/h.
At this speed how many seconds would it take the baseball to reach the batter?
(Give answer to three decimal places)



Direct Proportion

If two quantities are in **direct proportion**, as one increases the other increases at the same rate. These two quantities are said to be in **direct proportion** or **directly proportional** to each other.

Another term used is that there is a **direct variation** between these two terms.

The ratio between any corresponding terms is equal.

Symbols used:

If x and y are in direct proportion then:

$$y \propto x$$

$\propto$ is the symbol for **directly proportional**

and

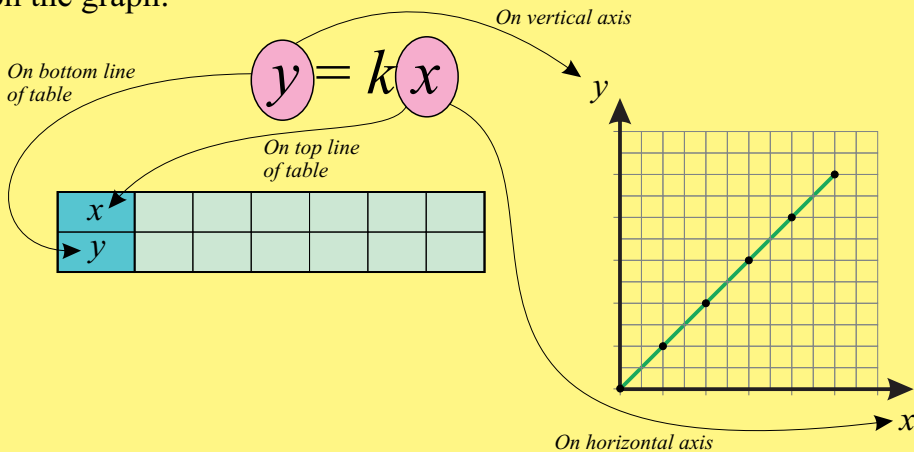
$$y = kx$$

k is called the **constant of proportionality**

There are two ways of checking if two quantities are directly proportional:

1. The ratio $\frac{y}{x}$ for all pairs of values (except 0,0) needs to be calculated and these must all be the same.
The value of this ratio is equal to the constant of proportionality.
2. A graph could be drawn displaying the information about the quantities.
This must be a straight line passing through the origin.

Note the correct way to write the equation and display the information on the graph:



Example 1

1. State if x and y , from the table below, are directly proportional.
2. Find the constant of proportionality.
3. Write the equation relating the two terms.

x	0	2	4	6	8	10
y	0	6	12	18	24	30

Answers

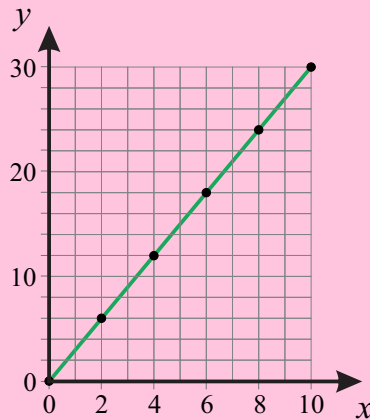
1. To find if y is directly proportional to x the ratio $\frac{y}{x}$ must be the same for all pairs of values (except 0,0) and the graph must be a straight line.

Checking this ratio: $\frac{6}{2} = 3$, $\frac{12}{4} = 3$, $\frac{18}{6} = 3$, $\frac{24}{8} = 3$, $\frac{30}{10} = 3$

The ratio is the same so y is directly proportional to x .

The graph could also have been drawn.

It can be seen that it is a straight line verifying that y is directly proportional to x .



2. The constant of proportionality (k) is equal to the ratio $\frac{y}{x} = 3$.

3. $y = kx$

therefore

$$y = 3x$$

Example 2

State if **A** and **B**, from the table below, are directly proportional.

A	0	5	10	15	20	25
B	0	20	40	50	60	100

Calculate the ratio, $\frac{B}{A}$, for each pair of values (except 0,0):

$$\frac{20}{5} = 4, \quad \frac{40}{10} = 4, \quad \frac{50}{15} = \frac{10}{3}, \quad \frac{60}{20} = 3, \quad \frac{100}{25} = 4$$

The values of this ratio are **NOT** all the same so **B** is **NOT** directly proportional to **A**.

Example 3

State if **P** and **Q**, from the table below, are directly proportional and if they are write the equation relating them.

P	0	4	10	12	18	30
Q	0	18	45	54	81	135

Calculate the ratio, $\frac{Q}{P}$, for each pair of values (except 0,0):

$$\frac{18}{4} = 4.5, \quad \frac{45}{10} = 4.5, \quad \frac{54}{12} = 4.5, \quad \frac{81}{18} = 4.5, \quad \frac{135}{30} = 4.5$$

The values of this ratio are the same so **Q** is directly proportional to **P**.

$$Q = kP$$

$$k = 4.5 \text{ or } \frac{9}{2}$$

$$Q = 4.5P \quad \text{or} \quad Q = \frac{9}{2}P$$

Example 4

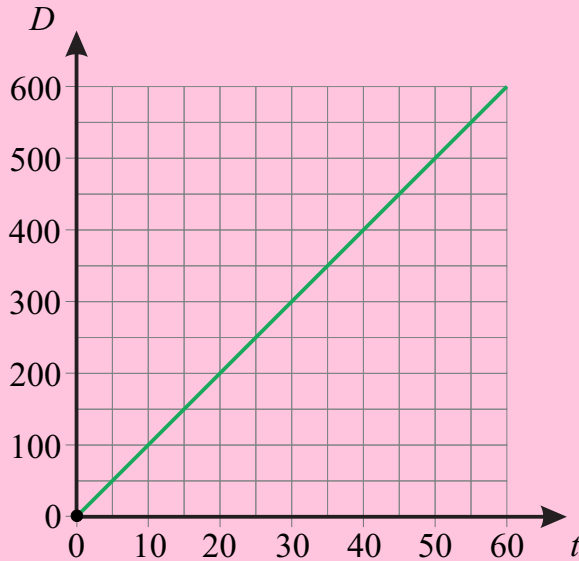
Find the equation relating D and t from the graph below.

The graph is a straight line so D is directly proportional to t .

$$D = kt$$

k = the ratio $\frac{D}{t}$

Any point on the graph can be used to find k because D is directly proportional to t .



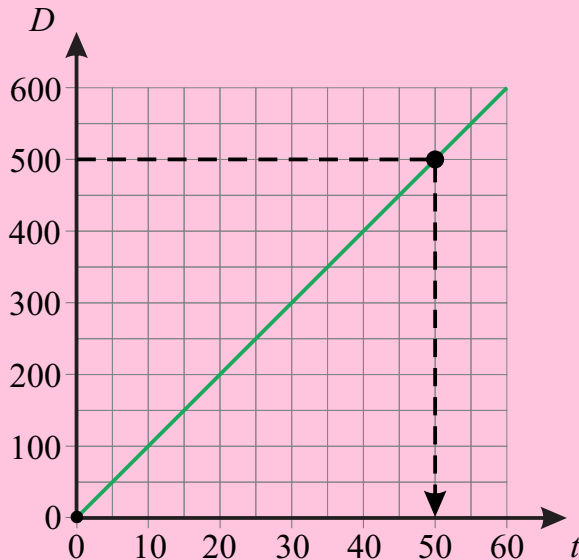
When $D = 500$, $t = 50$
(see below)

$$k = \frac{500}{50}$$

$$k = 10$$

$$D = kt$$

$$D = 10t$$



Example 5

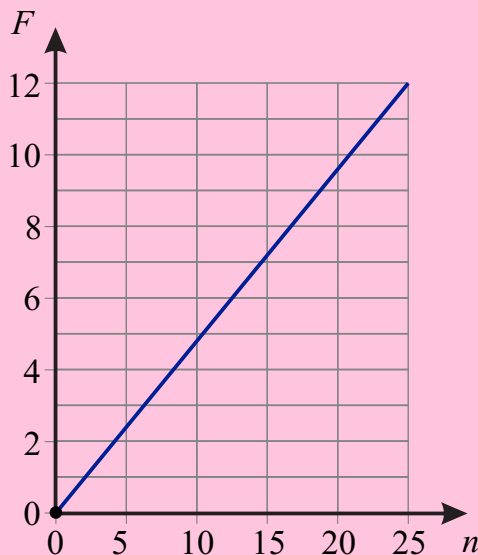
Find the equation relating F and n from the graph below.

The graph is a straight line so F is directly proportional to n .

$$F = kn$$

k = the ratio $\frac{F}{n}$

Any point on the graph can be used to find k because F is directly proportional to n . A point needs to be found that lies on the line.



When $F = 12$, $n = 25$
(see below)

$$k = \frac{12}{25}$$

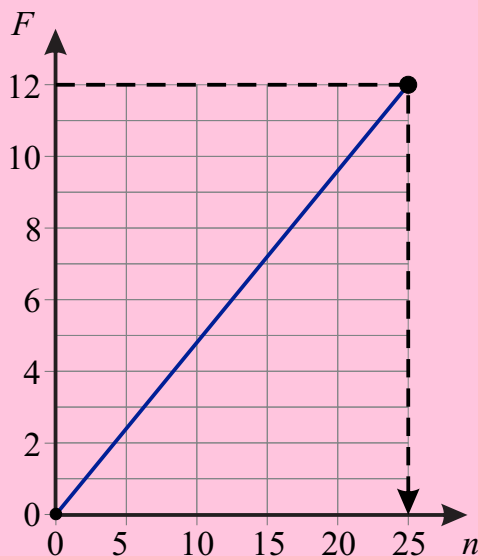
$$k = 0.48 \text{ or } \frac{12}{25}$$

$$F = kn$$

$$F = 0.48n$$

or

$$F = \frac{12}{25}n$$



Example 6

Charles used a hose to put water into a fish tank. There were marks on the side of the tank every 20 litres. Charles timed how long it took for the water level to reach each 20 litre mark. The table below shows these times.

Time (seconds)	0	10	20	30	40	50
Volume (litres)	0	20	40	60	80	100

1. State if the volume of water in the tank is directly proportional to the time taken.
Answer the following questions if Volume is directly proportional to Time.
2. Find the relationship between Volume and Time.
3. What is the volume of water in the tank after 25 seconds?
4. How long would it take 110 litres of water to be put into the tank?
5. What is the volume of water in the tank after 38 seconds?
6. How long did it take 62 litres of water to be put in the tank?

Answers

Let V = volume of water and t = time taken

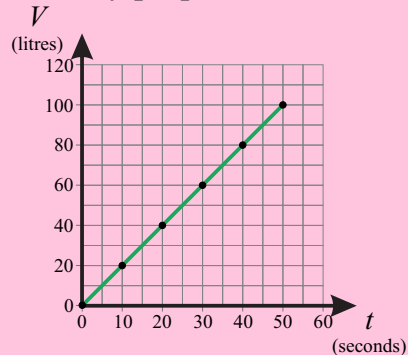
1. There are two ways to determine if V is directly proportional to t .

(a) Check the ratio $\frac{V}{t}$ for all the values except 0,0:

$$\frac{20}{10} = 2, \quad \frac{40}{20} = 2, \quad \frac{60}{30} = 2, \quad \frac{80}{40} = 2, \quad \frac{100}{50} = 2$$

All the ratios are the same so V is directly proportional to t .

- (b) Draw the graph.
The straight line confirms that V is directly proportional to t .



2. V is directly proportional to t so the equation between V and t will be:

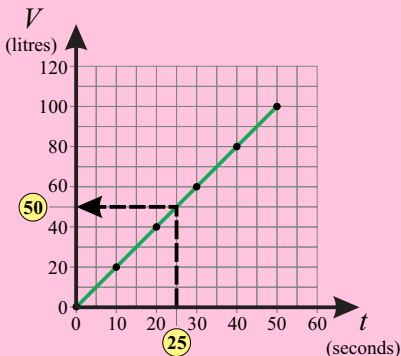
$$V = kt$$

From part 1 $k = 2$ (the ratio $\frac{V}{t}$) so:

$$V = 2t$$

3. $t = 25$
 $V = 2t$
 $V = 2 \times 25$
 $V = 50 \text{ litres}$

This could also have been found from the graph.



4. To find the time for a certain volume of water to be poured into the tank the equation needs to be transposed to make t the subject.

$$V = 2t$$

$$\frac{V}{2} = t$$

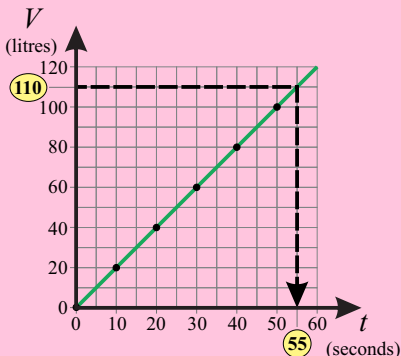
$$t = \frac{V}{2}$$

$$V = 110$$

$$t = \frac{110}{2}$$

$$t = 55 \text{ seconds}$$

This could also have been found from the graph. Note the line needs to be extended to find this point.



5. To find the volume of water in the tank after 38 seconds the equation relating V and t will need to be used. The graph would need to be drawn in clear detail to achieve an accurate answer.

$$V = 2t$$

$$V = 2 \times 38$$

$$V = 76 \text{ litres}$$

6. As with the answer to question 5 the equation needs to be used.

$$t = \frac{V}{2}$$

$$t = \frac{V}{2}$$

$$V = 62$$

$$t = \frac{62}{2}$$

$$t = 31 \text{ seconds}$$

EXERCISE 5C

1. State if the quantities in the tables below are in direct proportion (**DP**) or not in direct proportion (**NDP**).

(a)

x	0	5	10	15	20	25
y	0	20	40	60	80	100

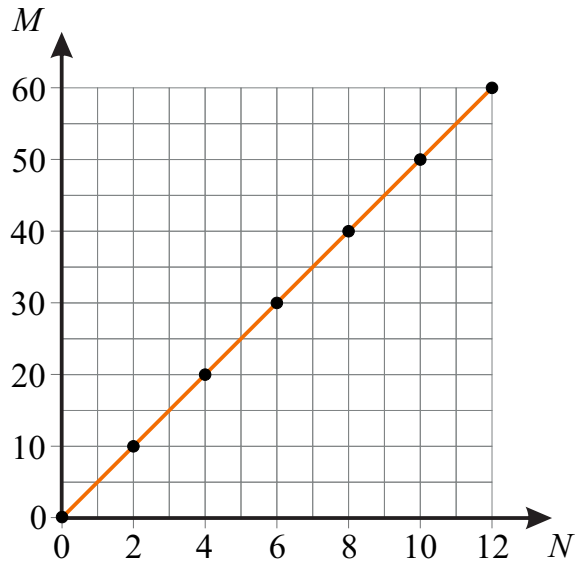
(b)

A	0	2	5	8	15	18
B	0	10	25	40	75	90

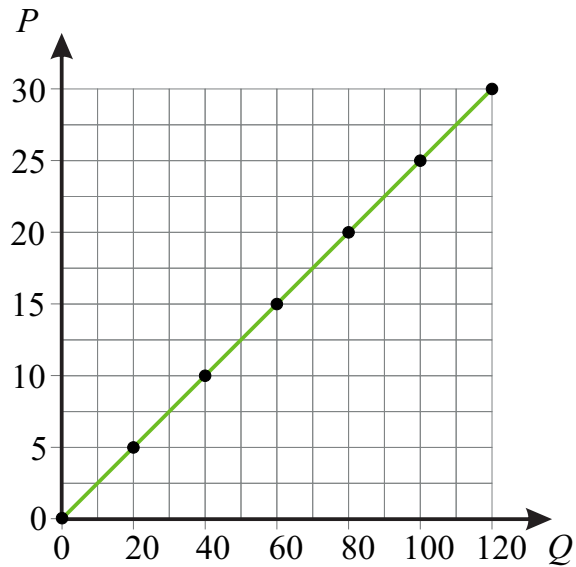
(c)

M	2	6	8	10	22	50
P	1	3	4	5	11	25

2. Find the equation relating the two quantities displayed on the graph below.



3. Find the equation relating the two quantities displayed on the graph below.



4. For the tables of data shown below:

- (i) state if the two quantities are directly proportional.
- (ii) if they are directly proportional find the equation relating the two quantities.

(a)

<i>a</i>	0	2	4	6	8	10
<i>b</i>	0	12	24	36	48	60

(b)

<i>T</i>	0	6	15	18	21	30
<i>W</i>	0	2	5	6	7	10

(c)

<i>K</i>	0	2	4	6	8	10
<i>H</i>	0	12	24	30	40	60

(d)

<i>g</i>	3	8	15	25	28	45
<i>B</i>	27	72	135	225	252	405

(e)

<i>Q</i>	0	8	15	25	30	52
<i>R</i>	0	4.8	9	15	18	31.2

(f)

<i>V</i>	3	8	12	18	24	38
<i>c</i>	4.5	12	18	30	40	45

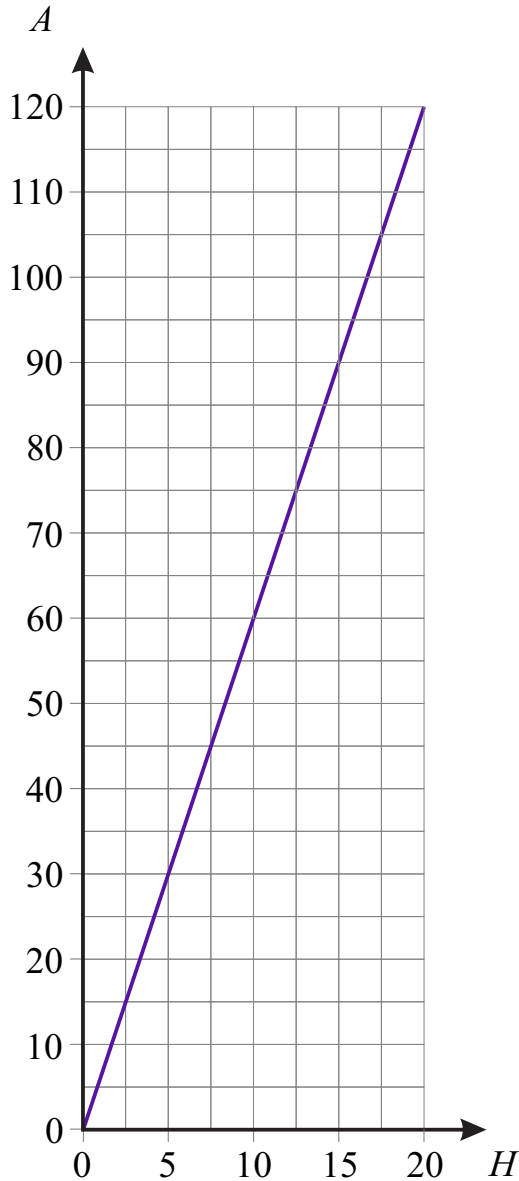
(g)

<i>I</i>	0	4	12	20	32	44
<i>H</i>	0	10	30	50	80	110

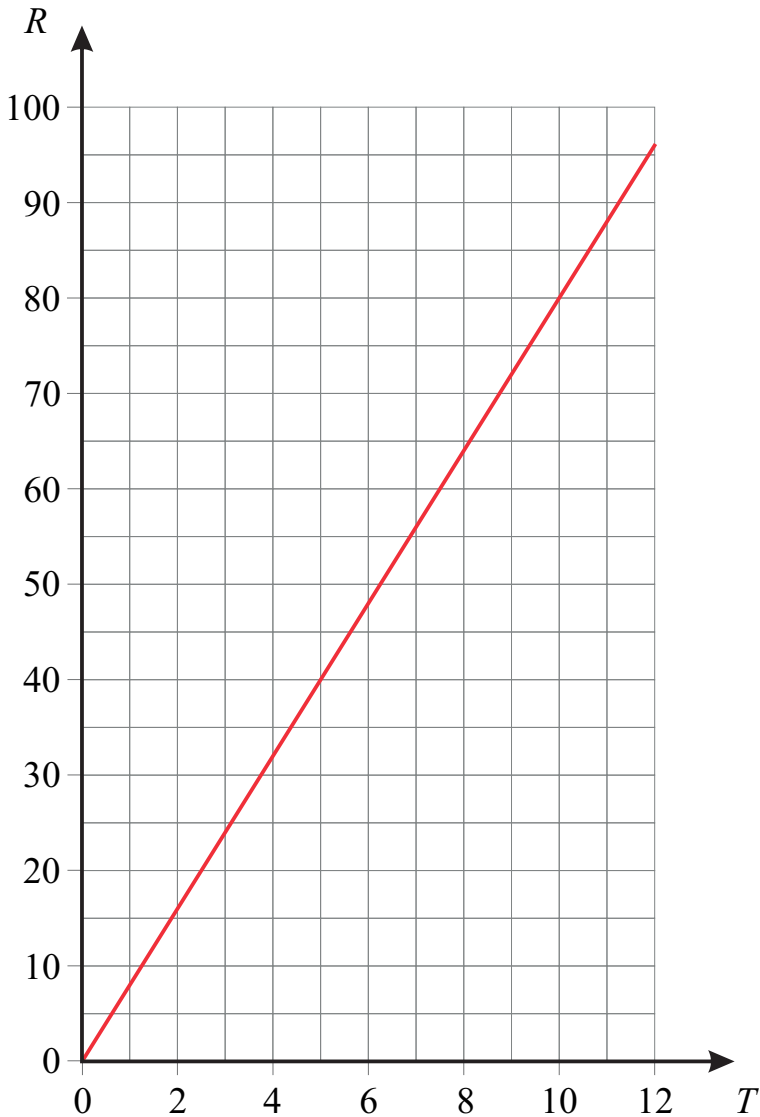
(h)

<i>j</i>	3	10	18	30	52	70
<i>Y</i>	0.9	3	5.4	9	15.6	21

5. (a) Find the equation relating the two quantities displayed on the graph below.
- (b) Find A for the following values of H :
(i) 5 (ii) 20 (iii) 8 (iv) 45 (v) 2.5 (vi) 15.2
- (c) Find H for the following values of A :
(i) 60 (ii) 90 (iii) 24 (iv) 180 (v) 75.6 (vi) 215.4



6. (a) Find the equation relating the two quantities displayed on the graph below.
- (b) Find R for the following values of T :
(i) 5 (ii) 3 (iii) 9 (iv) 15 (v) 24.5 (vi) 52.6
- (c) Find T for the following values of R :
(i) 80 (ii) 60 (iii) 8 (iv) 96 (v) 71.2 (vi) 122.4



7. (a) State if the two quantities displayed in the table below are directly proportional.

If they are directly proportional answer the following questions.

(b) Find the equation relating Y and P .

(c) Find P for the following values of Y .

(i) 3 (ii) 10 (iii) 35 (iv) 5.5 (v) 32.7

Y	0	2	6	9	16	24
P	0	4.8	14.4	21.6	38.4	57.6

8. Copy and complete the table of values for each of the following relationships.

(a) $M = 3N$

N	0	5		8.5		20.5
M			21		37.8	

(b) $Q = \frac{4}{5}K$

K	5		24		36.5	
Q		12		25.6		33.2

(c) $c = 4.5a$

a	0.4		7.4		12.5	
c		27		39.6		68.85

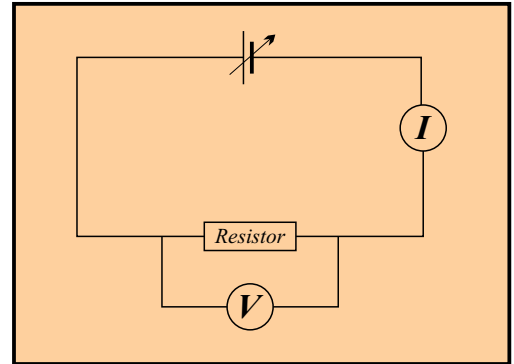
(d) $T = 0.7b$

b		5.3		12		20.6
T	1.75		6.23		10.57	

9. Students in a year 9 science class were studying electric circuits. They set up a simple electric circuit and measured the voltage (V) across a resistor for different currents (I). The measurements they recorded are shown in the table below.

Voltage (V) was measured in volts (V).

Current (I) is measured in amps (A).



I (A)	2	5	8.5	12	15.5	25
V (V)	24	60	102	144	186	300

- Is the voltage directly proportional to the current?
- Write an equation relating voltage and current.
- Find the voltage for the following currents:
 - 8 amps
 - 9.5 amps
 - 17.2 amps
- Find the current for the following voltages:
 - 36 volts
 - 174 volts
 - 296.4 volts

10. Harriet had a revolutions counter on one wheel of her bicycle. It measured the number of revolutions of the wheel as she rode. Harriet rode around a 400 metre long athletics track and the counter indicated her wheel had turned 182 revolutions. Harriet assumed the number of revolutions (N) was directly proportional to the distance travelled (D).

- Complete the following equation relating D and N . Write the constant of proportionality as a decimal correct to **one** decimal place.

$$D = kN$$

k is the constant of proportionality

- Harriet rode from her house to a nearby beach and the counter indicated 1595 revolutions. How far is the beach from Harriet's house?
- Harriet was going to compete in a 10 km bicycle race. How many revolutions would each wheel of her bicycle complete in this race?

11. A team of marine scientists were studying a coral reef. They used a specially equipped submarine to dive and study the marine life near the reef.

On one particular dive they were descending at a constant rate. After 2 minutes they had descended to a depth of 15 metres.

- (a) Assuming they kept diving at the same rate, complete the following equation relating depth (D), in metres, to time (t), in minutes.

$$D = kt$$

k is the constant of proportionality

- (b) At what depth would they be after the following times?
(i) 10 minutes (ii) 25 minutes (iii) 36 minutes
- (c) How long would it take them to dive to the following depths?
(i) 45 metres (ii) 112.5 metres (iii) 195 metres
- (d) The submarine can safely dive to a depth of 300 metres. How long would it take them to reach this depth?
- (e) After taking several measurements and samples at a depth of 300 m the scientists started to ascend back to the surface. They ascended at a constant rate and after 5 minutes had reached a depth of 260 m. How long did it take them to reach the surface from their dive depth of 300 m?



Photo Courtesy Triton Submarines

12. Henry is a farmer who is considering planting a canola crop. He has a 200 hectare farm. The agriculture recommendations were that he would need 500 kg of canola seed to plant and he could expect a yield of 300 tonnes of canola seed from his 200 hectare farm.

- (a) Assuming that the amount of seed to plant is directly proportional to land area, complete this equation relating the amount of seed to plant (P), in kg, to land area (A), in hectares.

$$P = kA \quad k \text{ is the constant of proportionality}$$

- (b) Assuming that the yield is directly proportional to land area, complete this equation relating yield (Y), in tonnes, to land area (A), in hectares.

$$Y = cA \quad c \text{ is the constant of proportionality}$$

- (c) Pam is also a farmer who is considering planting canola. She owns a 3000 hectare farm. Use the above equations to find the following:

- (i) the amount of seed she will need to plant all of her farm with canola.
 - (ii) the yield she could expect from her crop.
- (d) If the cost of planting canola seed is \$35/kg and the price expected on yield is \$400 per tonne find:
- (i) the amount Henry will spend on seed.
 - (ii) the amount Pam will spend on seed.
 - (iii) the amount Henry will receive for his crop.
 - (iv) the amount Pam will receive for his crop.



Inverse Proportion

Previously in this chapter the relationship between two quantities that vary at the same rate has been studied. As one quantity increased, the other also increased at the same rate. As stated this relationship is called ***direct proportion*** and the relationship is represented mathematically by the following equation:

$$y = kx \quad \text{where } k \text{ is the constant of proportionality}$$

Another type of relationship is when one quantity ***decreases*** at the same rate as the other quantity ***increases***.

For example, the speed of an object and the time taken to travel a certain distance. If the speed is ***doubled***, the time taken is ***halved***. These types of relationships are called ***inverse proportionality***. In the example of speed and time taken:

speed (s) is inversely proportional to the time taken (t)

As a mathematical equation this is written as:

$$s \propto \frac{1}{t}$$

or

$$s = \frac{k}{t}$$

where k is the constant of proportionality

Example

The time taken, t (in seconds), for a cyclist to ride around a track is recorded for different speeds, s (in m/s), and is shown in the table below.

t (s)	5	6	10	12	15
s (m/s)	12	10	6	5	4

1. State if t and s are inversely proportional.
2. Find the constant of proportionality.
3. Write the equation relating t and s .
4. Find the speed if the cyclist took 20 seconds to ride around the track.
5. How long would it take a cyclist riding at 2 m/s to ride around the track?
6. Draw the graph displaying this information.

Answers

1. If t and s are inversely proportional then:

$$s = \frac{k}{t}$$

where k is the constant of proportionality

This equation can be transposed:

$$st = k$$

This means that $s \times t$ should be a constant for all pairs of values.

To check this multiply s and t for all pairs of values:

$$5 \times 12 = \mathbf{60}$$

$$6 \times 10 = \mathbf{60}$$

$$10 \times 6 = \mathbf{60}$$

$$12 \times 5 = \mathbf{60}$$

$$15 \times 4 = \mathbf{60}$$

Because the product of s and t ($s \times t$) is the same for all pairs of values then s is inversely proportional to t . The value of the product is equal to k , the constant of proportionality.

2. $k = 60$

3.

$$s = \frac{k}{t}$$

$$s = \frac{60}{t}$$

4. $t = 20$ Substitute $t = 20$ into the equation:

$$s = \frac{k}{t}$$

$$s = \frac{60}{20}$$

$$s = 3 \text{ m/s}$$

5. $s = 2 \text{ m/s}$

$$s = \frac{k}{t}$$

$$2 = \frac{60}{t}$$

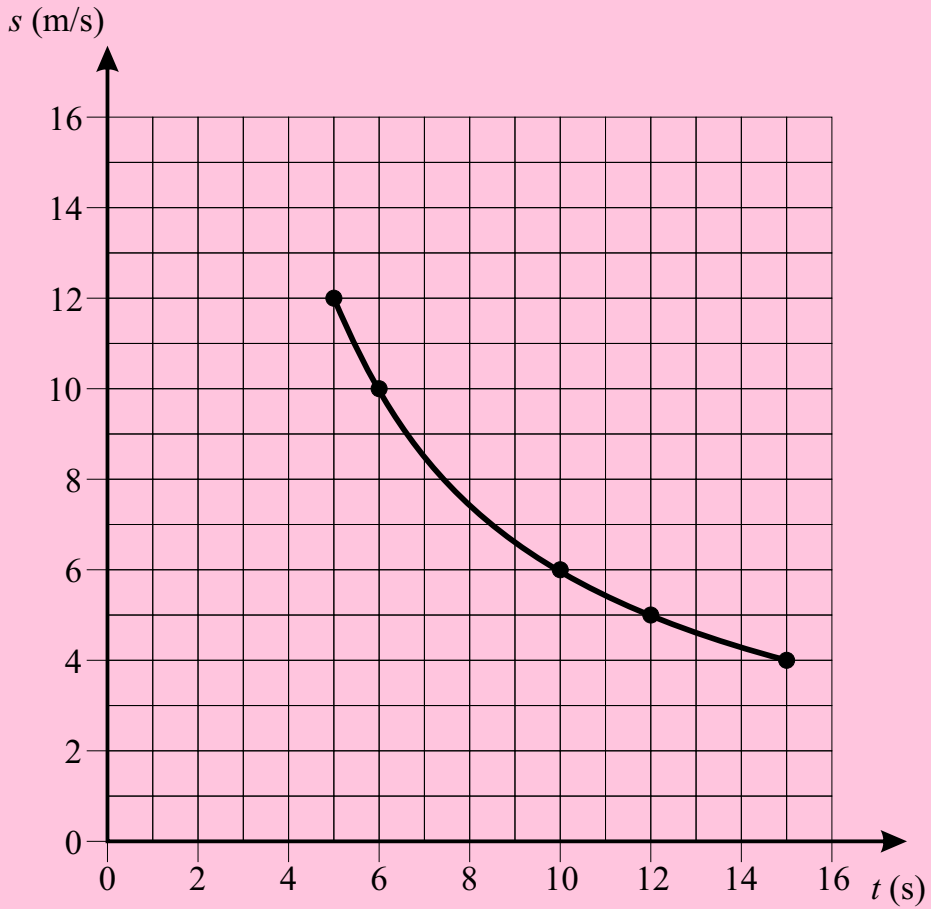
This equation needs to be transposed to solve for t .

$$2t = 60$$

$$t = \frac{60}{2}$$

$$t = 30 \text{ s}$$

6.



The graphs of all inverse proportional relationships are this shape.
The name given to a curve with this shape is a *hyperbola*.

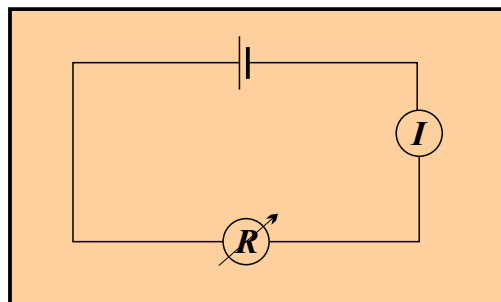
EXERCISE 5D

1. The time taken, t (in seconds), for a cyclist to ride around a track is recorded for different speeds, s (in m/s), and is shown in the table below.

t (s)	2	3	4	6	8
s (m/s)	24	16	12	8	6

- State if t and s are inversely proportional.
- Find the constant of proportionality.
- Write the equation relating t and s ($s = \dots$).
- Find the speed if the cyclist took 16 seconds.
- How long would it take a cyclist riding at 1 m/s to ride around the track?
- Draw the graph displaying this information.

2. An experiment was conducted to find the relationship between resistance, R (measured in ohms), and current, I (measured in amps), in an electrical circuit. The size of the resistor was changed and the current measured for each resistor. A table showing the size of the resistor and the current measured is shown below.



R (ohms)	5	20	60	90	120
I (amps)	144	36	12	8	6

- State if R and I are inversely proportional.
- Find the constant of proportionality.
- Write the equation relating R and I ($I = \dots$).
- Find the current when the resistance is:
 - 2 ohms
 - 40 ohms
 - 360 ohms
- What resistance would result in the following currents?
 - 1 amp
 - 30 amps
 - 180 amps
- Draw the graph displaying this information.

3. Clementine had a large tree in her yard that she was going to decorate with lights at Christmas. She was going to power the lights by batteries that were charged by solar panels. She noted that the number of lights on the tree determined how long the lights remained on before the batteries went flat. The table below shows the time the lights remain on, T (in minutes), for different numbers of lights, N .



N	2000	3000	4000	12 000
T (minutes)	240	160	120	40

- State if N and T are inversely proportional.
 - Find the constant of proportionality.
 - Write the equation relating N and T ($T = \dots$).
 - Find the time that the following number of lights would remain on:
 - 6000
 - 5000
 - 20 000
 - Clementine wanted to turn the lights on at 8:30 pm and have them turn off at 11.00 pm. How many lights should she put on the tree?
4. Science students wanted to investigate the relationship between pressure and volume in a gas. They connected a pressure gauge to a bicycle pump and recorded the pressure, P (in kilopascals - kPa), and volume, V (in cm^3), as they compressed the pump. Their measurements are shown in the table below.



$V (\text{cm}^3)$	100	90	80	60
P (kPa)	72	80	90	120

- State if V and P are inversely proportional.
- Find the constant of proportionality.
- Write the equation relating V and P ($P = \dots$).
- Find the pressure for the following volumes:
 - 30 cm^3
 - 50 cm^3
 - 120 cm^3
- Find the volume that will create the following pressures:
 - 100 kPa
 - 150 kPa
 - 180 kPa

EXERCISE 5E

State if the relationship between the following quantities would be a direct proportionality (**D**) or inverse proportionality (**I**).

1. The size of a house to be painted and the time taken for one painter to paint the house.
2. The number of painters painting a house and the time taken to paint the house.
3. The size of a wheat farm and the time taken for a harvester to harvest the wheat.
4. The number of fruit pickers employed to pick all the fruit in an orchard and the time taken to pick the crop.
5. The time taken for a water tank to fill and the amount of water needed to fill the tank.
6. The time taken for a packet of bird seed to be eaten and the number of birds in the aviary.
7. The amount of food needed for the fish in an aquarium and the number of fish in the aquarium.



PROBLEM SOLVING

1. Michel and Angelo are painters. They have a job to paint three identical rooms in a house. Michel takes three hours to paint one of the rooms and Angelo takes six hours to paint another one of the rooms. They decide to work together to paint the third room. How long should it take them to paint the third room if they are working together?

2. Adelaide and Matilda decide to run around a park. They start at the same point and at the same time but run in opposite directions. Adelaide runs at 5 m/s. Matilda runs at 4 m/s. They meet 11 minutes later.
 - (a) What is the distance around the park?
 - (b) Who will return to their starting point first?
 - (c) How many seconds later will the other runner return to their starting point?

PUZZLES

1. Change **SLOW** into **FAST** in six steps by changing one letter at a time to form a new word. (Two letters of the middle word are included)

Example

CAT can be changed to **DOG** in three steps

CAT
COT
DOT
DOG

SLOW
L _ _ T
FAST

2. Rearrange the letters of the following words and phrases to form words from this chapter.

(a) MITE	(b) NEAT DISC	(c) STARE	(d) NO POOR TRIP
(e) TEAR	(f) CREDIT	(g) NEVER IS	(h) LOONY AIRPORT PIT

CHAPTER REVIEW

- The cost of rolled oats at a bulk store was \$4.00 for 5 kg.
 - Find the cost of the following amounts of rolled oats:
 - 2 kg
 - 7 kg
 - 8.5 kg
 - How many kilograms of rolled oats could be bought for the following amounts of money?
 - \$8
 - \$10
 - \$3
- Margie was swimming in a pool that was 25 metres long. The average time it takes her to swim a lap is 40 seconds.
Give the answers to parts (a) and (b) below in minutes and seconds.
 - How long will it take Margie to swim 20 laps?
 - How long will it take Margie to swim 1 kilometre?
 - How many metres will she swim in 30 minutes?
- Kyle took 40 minutes to ride 20 km on his bicycle. At what speed, in km/h, was Kyle riding?
- Lena jogged 600 metres in 3 minutes. How long would it take her to jog 5 km?
- State if the quantities in the tables below are in direct proportion (**DP**) or not in direct proportion (**NDP**).

(a)

<i>M</i>	0	2	4	6	8	10
<i>N</i>	0	12	20	36	32	80

(b)

<i>g</i>	0	2	6	10	12	15
<i>h</i>	0	8	24	40	48	60

(c)

<i>y</i>	3	9	12	21	27	39
<i>z</i>	1	3	4	7	9	13

- Write the equation relating the pairs of quantities from question 5 that are directly proportional.

7. (a) Find the equation relating the two quantities displayed on this graph.

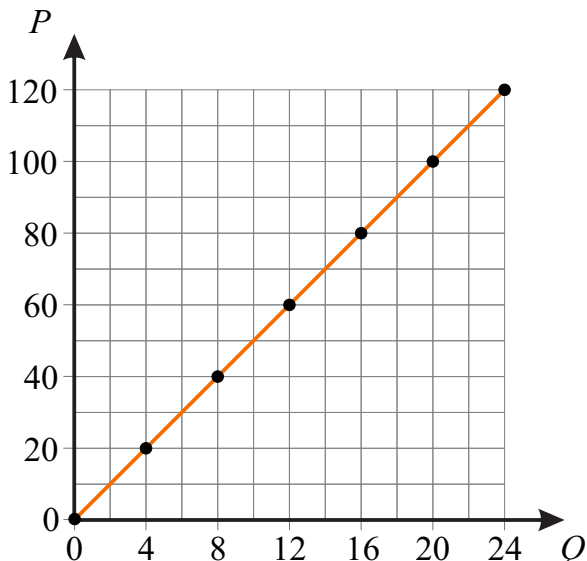
(b) Find P for the following values of Q .

(i) 6 (ii) 19 (iii) 52

(c) Find Q for the following values of P :

(i) 70 (ii) 130 (iii) 168

(d) What is the constant of proportionality in this equation?



8. (a) Find the equation relating the quantities shown in the table of values below.

K	0	2	4	6	8	10
L	0	3	6	9	12	15

(b) Find L for the following values of K :

(i) 18 (ii) 25 (iii) 9.6 (iv) 37.8

(c) Find K for the following values of L :

(i) 24 (ii) 51 (iii) 28.5 (iv) 97.95

9. Copy and complete the table of values for each of the following relationships.

(a) $M = 4N$

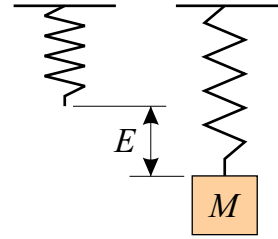
N	0	5		9.5		20.5
M			24		50	

(b) $Q = 2.4K$

K	2		21		30.5	
Q		9.6		61.2		96.48

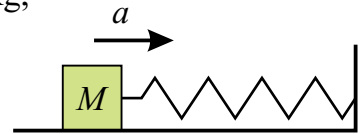
10. A vertical spring is extended by adding masses. The extension, E , in cm, for different masses, M , in kg, is recorded in the table below.

M (kg)	0.2	0.5	1.2	1.5	2.2
E (cm)	0.6	1.5	3.6	4.5	6.6



- (a) Is the extension directly proportional to the mass added to the spring?
 (b) Write an equation relating mass and extension.
 (c) Find the extension for the following masses added to the spring:
 (i) 0.4 kg (ii) 1.3 kg (iii) 2.7 kg
 (d) What mass would result in the following extensions?
 (i) 0.3 cm (ii) 4.2 cm (iii) 5.4 cm
11. A mass is attached to the end of a horizontal spring on a table. The spring is extended and the mass released. An experiment was conducted to find the relationship between the mass, M in kg, and its acceleration, a in m/s^2 . The results are shown in the table below.

M (kg)	0.2	0.4	1.0	1.5	2.0
a (m/s^2)	30	15	6	4	3



- (a) State if M and a are directly or inversely proportional.
 (b) Find the constant of proportionality.
 (c) Write the equation relating M and a .
 (d) Find the acceleration for the following masses:
 (i) 0.5 kg (ii) 1.2 kg (iii) 2.4 kg
 (e) What mass would cause an acceleration of 0.8 m/s^2 ?

Index Laws

Index Laws

Example

Base number Index, Power or Exponent Basic numeral

$$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$$

Exponent form
or
Index form Factor form

***Note:** the plural of *index* is *indices**

EXERCISE 6A

1. Write the following in factor form.

Example $4^8 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

- (a) 2^4 (b) 6^7 (c) 10^2 (d) 9^8 (e) 8^{11}

2. Write the following in index form.

Example $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

- (a) $6 \times 6 \times 6 \times 6$ (b) $9 \times 9 \times 9 \times 9 \times 9 \times 9$
 (c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (d) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

3. Write the following in factor form then combine into index form.

Example

$$\begin{array}{c}
 5^3 \times 5^4 \\
 \swarrow \quad \searrow \\
 \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5 \times 5} \\
 = 5^7
 \end{array}$$

(a) $3^2 \times 3^4$

(b) $2^3 \times 2^2$

(c) $7^5 \times 7^3$

(d) $6^3 \times 6^5$

(e) $5^3 \times 5^3$

(f) $10^2 \times 10^6$

(g) $9^4 \times 9^3$

(h) $8^6 \times 8^5$

4. Can you think of a way of finding the answers in question 3 without writing the factors?

Index Law 1

Index Law 1 states:

If terms with the **same base number** are multiplied, the indices can be **added**.

Note: It is not necessary to show an index of 1.

$$8 = 8^1, \quad 4 = 4^1, \quad 2 = 2^1, \quad 12 = 12^1, \text{ etc}$$

Examples

$$\begin{aligned}
 1. \quad & 4^6 \times 4^7 \\
 & = 4^{(6+7)} \\
 & = 4^{13}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 7^8 \times 7^6 \\
 & = 7^{(8+6)} \\
 & = 7^{14}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & 6^4 \times 6 \\
 & = 6^4 \times 6^1 \\
 & = 6^{(4+1)} \\
 & = 6^5
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & 9 \times 9^{14} \\
 & = 9^1 \times 9^{14} \\
 & = 9^{(1+14)} \\
 & = 9^{15}
 \end{aligned}$$

5. Use Index Law 1 to find the answers to the following in index form.

(a) $6^3 \times 6^7$

(b) $5^4 \times 5^8$

(c) $4^9 \times 4^2$

(d) $8^6 \times 8$

(e) $2^{10} \times 2^5$

(f) $13^6 \times 13^8$

(g) 3×3^8

(h) $15^7 \times 15^4$

(i) $9^{14} \times 9^{13}$

(j) $17^8 \times 17^{14}$

(k) 11×11^{12}

(l) $26^{15} \times 26$

EXERCISE 6B

1. Write the following in factor form then simplify and write the answer in index form.

Example

$$\begin{aligned}
 & \frac{6^5}{6^2} \\
 = & \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6} \\
 = & \frac{6 \times 6 \times 6 \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6}} \\
 = & \frac{6 \times 6 \times 6}{1} \\
 = & 6 \times 6 \times 6 \\
 = & 6^3
 \end{aligned}$$

The two 6's on the denominator (bottom line) can be cancelled with two 6's on the numerator (top line).

(a) $\frac{4^5}{4^2}$

(b) $\frac{2^6}{2^4}$

(c) $\frac{7^7}{7^3}$

(d) $\frac{9^6}{9^2}$

(e) $\frac{5^8}{5^5}$

(f) $\frac{10^7}{10}$

(g) $\frac{3^6}{3^3}$

(h) $\frac{6^9}{6^2}$

(i) $\frac{8^8}{8^6}$

(j) $\frac{11^9}{11^5}$

(k) $\frac{2^5}{2}$

(l) $\frac{13^4}{13^3}$

(m) $\frac{3^5}{3^4}$

(n) $\frac{15^7}{15^5}$

(o) $\frac{20^4}{20}$

(p) $\frac{7^9}{7^7}$

2. Can you think of a way of finding the answers in question 1 without writing the factors?

Index Law 2

Index Law 2 states:

If terms with the **same base number** are divided, the index on the denominator (bottom line) is **subtracted from** the index on the numerator (top line).

Examples

$$\begin{aligned} 1. \quad & \frac{7^8}{7^6} \\ &= 7^{(8-6)} \\ &= 7^2 \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{9^{12}}{9^5} \\ &= 9^{(12-5)} \\ &= 9^7 \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{8^7}{8^6} \\ &= 8^{(7-6)} \\ &= 8^1 \\ &= 8 \end{aligned}$$

$$\begin{aligned} 4. \quad & \frac{3^7}{3} \\ &= \frac{3^7}{3^1} \\ &= 3^{(7-1)} \\ &= 3^6 \end{aligned}$$

3. Use Index Law 2 to find the answers to the following in index form.

(a) $\frac{5^7}{5^2}$

(b) $\frac{3^9}{3^5}$

(c) $\frac{4^8}{4^3}$

(d) $\frac{7^6}{7^4}$

(e) $\frac{6^{13}}{6^5}$

(f) $\frac{12^{15}}{12^9}$

(g) $\frac{2^8}{2}$

(h) $\frac{8^9}{8^8}$

(i) $\frac{9^{12}}{9^{11}}$

(j) $\frac{15^9}{15}$

(k) $\frac{9^5}{9}$

(l) $\frac{17^{19}}{17^{13}}$

4. Use Index Law 2 to find the answers to the following in index form.

(a) $9^8 \div 9^5$

(b) $12^9 \div 12^3$

(c) $7^{10} \div 7^7$

(d) $8^{13} \div 8^{11}$

(e) $10^8 \div 10^7$

(f) $6^8 \div 6$

(g) $5^{15} \div 5^9$

(h) $2^{21} \div 2^{13}$

(i) $3^9 \div 3$

(j) $20^{17} \div 20^{16}$

(k) $4^{13} \div 4^{12}$

(l) $17^{16} \div 17^5$

Example

$$\begin{aligned} & 6^7 \div 6^4 \\ &= \frac{6^7}{6^4} \\ &= 6^{(7-4)} \\ &= 6^3 \end{aligned}$$

EXERCISE 6C

1. Write the following in factor form then simplify by cancelling.

(a) $\frac{5^4}{5^4}$ (b) $\frac{3^7}{3^7}$ (c) $\frac{4^3}{4^3}$ (d) $\frac{7^5}{7^5}$

2. Use Index Law 2 to find the answers to the problems in question 1.

3. Use the answers from questions 1 and 2 to complete the following:

(a) $5^0 = \underline{\hspace{1cm}}$ (b) $3^0 = \underline{\hspace{1cm}}$ (c) $4^0 = \underline{\hspace{1cm}}$ (d) $7^0 = \underline{\hspace{1cm}}$

Index Law 3

Index Law 3 states:

If a term has an index of **0** then the answer is **1**.

$$7^0 = 1, \quad 3^0 = 1, \quad 27^0 = 1, \quad 123^0 = 1, \text{ etc}$$

4. Evaluate the following.

Examples

$$\begin{aligned} 1. \quad 6^0 + 8 \\ &= 1 + 8 \\ &= \mathbf{9} \end{aligned}$$

$$\begin{aligned} 2. \quad 7^0 + 2^2 - 8^0 \\ &= 1 + 4 - 1 \\ &= \mathbf{4} \end{aligned}$$

$$\begin{aligned} 3. \quad (7 + 5)^0 \\ &= 12^0 \\ &= \mathbf{1} \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{5^0 + 17}{2^3 + 9^0} \\ &= \frac{1 + 17}{8 + 1} \\ &= \frac{18}{9} \\ &= \mathbf{2} \end{aligned}$$

(a) $7 - 8^0$

(b) $12 + 7^0$

(c) $3^0 + 3^1 + 3^2$

(d) $(7 + 4)^0$

(e) $(4 + 8^0)^2$

(f) 5×4^0

(g) $(124 - 87)^0$

(h) $(7^5 + 9^4)^0$

(i) $\frac{9 + 18}{3^0}$

(j) $\frac{4^2 - 7^0}{2^2 - 2^0}$

(k) $\frac{(4 + 2)^2}{8^0 + 11}$

(l) $\frac{(6^2 + 3^2) \times 7^0}{(5 - 4^0)^2 - 6^0}$

EXERCISE 6D

1. Write the following in factor form then combine into index form.

Example

$$\begin{aligned}
 & (5^3)^4 \\
 &= 5^3 \times 5^3 \times 5^3 \times 5^3 \\
 &= \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5} \times \underbrace{5 \times 5 \times 5} \\
 &= 5^{12}
 \end{aligned}$$

(a) $(4^2)^4$

(b) $(8^3)^2$

(c) $(6^5)^2$

(d) $(3^2)^5$

(e) $(2^4)^4$

(f) $(9^2)^2$

(g) $(5^4)^3$

(h) $(10^3)^5$

(i) $(12^2)^6$

(j) $(7^7)^2$

(k) $(3^2)^7$

(l) $(2^3)^6$

2. Can you think of a way to find the answers in question 1 without writing the factors?

Index Law 4

Index Law 4 states:

If a term with an index is raised to an index, the indices are **multiplied**.

Examples

$$\begin{aligned}
 1. \quad & (9^5)^6 \\
 &= 9^{(5 \times 6)} \\
 &= 9^{30}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (3^7)^5 \\
 &= 3^{(7 \times 5)} \\
 &= 3^{35}
 \end{aligned}$$

3. Use Index Law 4 to find the answers to the following in index form.

(a) $(5^6)^4$

(b) $(6^5)^5$

(c) $(2^7)^4$

(d) $(9^7)^8$

(e) $(7^8)^5$

(f) $(8^4)^9$

(g) $(10^5)^{10}$

(h) $(11^9)^6$

(i) $(13^7)^6$

(j) $(9^{12})^6$

(k) $(3^9)^7$

(l) $(8^{20})^8$

Index Law 5**Index Law 5** states:

If any number of terms that are multiplied or divided are all raised to the same index, each term is raised to that index.

Examples

$$\begin{aligned} 1. \quad & (3^6 \times 5^2)^4 \\ &= (3^6)^4 \times (5^2)^4 \\ &= 3^{24} \times 5^8 \end{aligned}$$

$$\begin{aligned} 2. \quad & \left(\frac{4^2}{8^9} \right)^3 \\ &= \frac{(4^2)^3}{(8^9)^3} \\ &= \frac{4^6}{8^{27}} \end{aligned}$$

$$\begin{aligned} 3. \quad & (7^3 \div 9^5)^6 \\ &= \left(\frac{7^3}{9^5} \right)^6 \\ &= \frac{(7^3)^6}{(9^5)^6} \\ &= \frac{7^{18}}{9^{30}} \end{aligned}$$

EXERCISE 6E

1. Expand the following terms.

(a) $(3^5 \times 4^2)^6$

(b) $(7^4 \times 2^8)^3$

(c) $(6^7 \times 9^4)^7$

(d) $(5^8 \times 8^9)^6$

Example $(5^3 \times 7^6)^2$
 $= 5^6 \times 7^{12}$

2. Expand the following terms.

(a) $\left(\frac{3^6}{8^3} \right)^7$

(b) $\left(\frac{5^9}{6^5} \right)^4$

(c) $(4^3 \div 9^{11})^5$

(d) $(2^6 \div 7^8)^{12}$

Examples

$$\begin{aligned} 1. \quad & \left(\frac{7^5}{2^4} \right)^6 \\ &= \frac{7^{30}}{2^{24}} \end{aligned}$$

$$\begin{aligned} 2. \quad & (9^7 \div 5^8)^2 \\ &= \left(\frac{9^7}{5^8} \right)^2 \\ &= \frac{9^{14}}{5^{16}} \end{aligned}$$

ExampleSimplify $\frac{6^3}{6^5}$

This can be done two ways.

1. Use Index Law 2.

$$\begin{aligned}\frac{6^3}{6^5} \\ &= 6^{(3-5)} \\ &= 6^{-2}\end{aligned}$$

2. Write in factor form and cancel.

$$\begin{aligned}\frac{6^3}{6^5} \\ &= \frac{6 \times 6 \times 6}{6 \times 6 \times 6 \times 6 \times 6} \\ &= \frac{\cancel{6} \times \cancel{6} \times \cancel{6}}{6 \times 6 \times \cancel{6} \times \cancel{6} \times \cancel{6}} \\ &= \frac{1}{6 \times 6} \\ &= \frac{1}{6^2}\end{aligned}$$

It can be seen from these two answers that:

$$6^{-2} = \frac{1}{6^2}$$

This is Index Law 6

Index Law 6**Index Law 6** states:

A term with a negative index can be written on the denominator with a positive index.

Example $6^{-2} = \frac{1}{6^2}$

EXERCISE 6F

1. Write the following with positive indices.

(a) 6^{-6}

(b) 8^{-9}

(c) 4^{-13}

(d) 5^{-7}

(e) 12^{-8}

2. Write the following terms with positive indices.

Examples

1. 3×7^{-9}

$$= 3 \times \frac{1}{7^9}$$

$$= \frac{3}{7^9}$$

2. $6^4 \times 5^{-3}$

$$= 6^4 \times \frac{1}{5^3}$$

$$= \frac{6^4}{5^3}$$

(a) 3×7^{-9}

(b) 8×2^{-5}

(c) $9^{-4} \times 3$

(d) $4^{-6} \times 2$

(e) $6^2 \times 4^{-8}$

(f) $12^3 \times 3^{-8}$

(g) $7^{-2} \times 4^6$

(h) $8^{-10} \times 7^{12}$

3. Write the following terms with positive indices.

A term in the denominator with a negative index is the same as that term in the numerator with a positive index.

Examples

1. $\frac{1}{4^{-7}}$

$$= 4^7$$

2. $\frac{7}{2^{-8}}$

$$= 7 \times 2^8$$

3. $\frac{9}{4 \times 7^{-2}}$

$$= \frac{9 \times 7^2}{4}$$

4. $\frac{6 \times 4^{-3}}{2^{-9} \times 5}$

$$= \frac{6 \times 2^9}{5 \times 4^3}$$

(a) $\frac{1}{8^{-3}}$

(b) $\frac{1}{5^{-7}}$

(c) $\frac{4}{2^{-9}}$

(d) $\frac{3}{6^{-2}}$

(e) $\frac{4}{9 \times 8^{-4}}$

(f) $\frac{7}{3 \times 2^{-9}}$

(g) $\frac{5}{7^{-3} \times 6}$

(h) $\frac{8}{4^{-5} \times 11}$

(i) $\frac{7^{-2}}{6 \times 5^{-8}}$

(j) $\frac{5 \times 7^{-2}}{4 \times 6^{-3}}$

(k) $\frac{3^5 \times 5^{-4}}{9^8 \times 7^{-2}}$

(l) $\frac{4^{-7} \times 2^6}{3^2 \times 9^{-8}}$

EXERCISE 6G

Use the Index Laws to simplify the following.
Write the answers in index form with positive indices.

Examples

$$\begin{aligned} 1. \quad & \frac{(2^7 \times 2^5)^4}{(2^6 \times 2^9)^3} \\ &= \frac{(2^{12})^4}{(2^{15})^3} \\ &= \frac{2^{48}}{2^{45}} \\ &= 2^3 \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{(6^7 \times 6^{-9})^{-4}}{(6^{-3})^5} \\ &= \frac{(6^{-2})^{-4}}{6^{-15}} \\ &= \frac{6^8}{6^{-15}} \\ &= 6^{23} \end{aligned}$$

$$1. \quad \frac{(4^5 \times 4^6)^3}{(4^7 \times 4^3)^2}$$

$$2. \quad \frac{(7^{-3} \times 7^6)^5}{(7^7 \times 7^0)^2}$$

$$3. \quad \frac{8^{13}}{8^7} \times \frac{8^{19}}{8^{17}}$$

$$4. \quad \frac{(3^7 \times 3^9)^{-3}}{(3^{14} \times 3^{10})^{-2}}$$

$$5. \quad \frac{(5^{-8} \times 5^{-6})^{-4}}{(5^{11} \times 5^{16})^0}$$

$$6. \quad \frac{8^{13} \times 8^{-6}}{(8^{-7})^2} \times \frac{8^{13}}{8^{29}}$$

$$7. \quad \frac{(9^{-7})^8}{(9^{12} \times 9^3)^3} \times \frac{(9^{20})^5}{(9^{-7} \times 9^5)^9}$$

$$8. \quad \frac{(6^7 \times 6^{13})^{-3}}{(6^4 \times 6^{10})^{-4}} \times \frac{(6^{16} \times 6)^5}{(6 \times 6^{10})^7}$$

$$9. \quad \frac{(9^{12} \times 9^{-3})^3 \times 9^{-5}}{(9^3)^{-4}}$$

$$10. \quad \frac{(8^{-7} \times 8^{-3})^{-2}}{(8^4 \times 8^{-7})^{-3}}$$

$$11. \quad (5^{-3} \times 5^{-4})^2 \times (5^{-4})^3$$

$$12. \quad \left(\frac{7^5}{7^2}\right)^6 \times \left(\frac{7^{-8}}{7^3}\right)^{-7}$$

$$13. \quad \left(\frac{8^5}{8^5}\right)^3 \times \left(\frac{8^{-7}}{8^{-7}}\right)^{-4}$$

$$14. \quad \left(\frac{3^{-5}}{3^2}\right)^3 \times \left(\frac{3^5}{3^{-3}}\right)^{-2}$$

$$15. \quad \left(\frac{9^{-5}}{9^{-2}}\right)^3 \times \left(\frac{9^{-4}}{9^6}\right)^3$$

$$16. \quad \left(\frac{2^{-5}}{2^7}\right)^{-3} \times \left(\frac{2^{-8}}{2^5}\right)^0$$

$$17. \quad \left(\frac{4^6}{4^2}\right)^3 \times \left(\frac{4^{-3}}{4^{-15}}\right)^4$$

$$18. \quad 4^{12} \times 4^{-3} \times 4^{-8} \times 4^{-8}$$

$$19. \quad \frac{6^{-2}}{6^4} \times \frac{6^{-1}}{6^{-5}} \times \frac{6^8}{6^{-2}}$$

$$20. \quad \left(\frac{2^{-6}}{2^{-2}} \times \frac{2^{-9}}{2^{-12}}\right)^{-3}$$

EXERCISE 6H

1. Evaluate the following *without* using a calculator.

Examples

1. 5^3

$= 5 \times 5 \times 5$

$= 125$

2. 4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{16}$

(a) 2^2

(b) 2^3

(c) 2^{-4}

(d) 2^5

(e) 2^6

(f) 2^{-7}

(g) 2^8

(h) 2^{-9}

(i) 2^{10}

(j) 2^{11}

(k) 3^2

(l) 3^3

(m) 3^{-4}

(n) 5^2

(o) 5^3

(p) 10^2

(q) 10^{-3}

(r) 10^4

(s) 10^{-5}

(t) 10^6

(u) 8^3

(v) 9^{-2}

(w) 6^{-3}

(x) 4^{-4}

(y) 7^4

2. Use the Index Laws to simplify the following then evaluate the answer *without* using a calculator.

Examples

1. $2^3 \times 2^4$

$= 2^7$

$= 128$

2. $2^5 \times 2^{-8}$

$= 2^{-3}$

$= \frac{1}{2^3}$

$= \frac{1}{8}$

(a) $10^{-8} \times 10^5$

(b) $8^3 \times 8^0$

(c) $3^{-3} \times 3^3$

(d) $\frac{9^{13}}{9^{11}}$

(e) $\frac{(7^4)^{-5}}{(7^3)^{-6}}$

(f) $\frac{(6^7 \times 6^{-4})^2}{(6^{-5} \times 6^7)^2}$

(g) $\frac{2^{13}}{2^{11}} \times \frac{3^{-14}}{3^{-15}}$

(h) $\frac{5^{14}}{5^{12}} \times \frac{2^{-10}}{2^{-11}}$

(i) $\frac{(2^7 \times 2^{-8})^{-4}}{(2^{-5} \times 2^8)^{-2}}$

EXERCISE 6I

1. Use a calculator to evaluate the following problems.

(a) 15^4

(b) $13^5 - 21^4$

(c) $\frac{25^4 - 40^3}{5^3}$

(d) $(63^2 - 54^2)^2$

(e) $\left(\frac{72^3}{3^6}\right)^2$

(f) $\frac{93^2 - 15^2}{26}$

(g) $\frac{85^3 - 13^5}{2^4} - 23^3$

(h) $\left(\frac{474 - 118}{89}\right)^6$

(i) $\frac{2^{13}}{8^4} + \frac{3^{13}}{27^4}$

(j) $(5^2)^4 - (3^3)^3$

(k) $\frac{(7^6)^7}{(7^4)^9}$

(l) $(9^4)^2 - (17^2)^3 - (4^3)^4$

2. Use a calculator to evaluate the following problems.

Give answers as decimals correct to two decimal places.

(a) 0.41^{-3}

(b) $1.4^4 + 0.32^{-3}$

(c) $0.3^{-4} - 0.12^{-3}$

(d) $0.4^{-3} + 2.4^3$

(e) $\frac{5.3^{-4}}{3.6^{-6}}$

(f) $\frac{(2.3 \times 6.7)^{-2}}{5.18^{-4}}$

(g) $\frac{3.2^{-3} + 1.7^{-1}}{5.8^{-2} + 6.2^{-2}}$

(h) $\frac{(4.2^3 + 5.6^2)^{-2}}{(7.8^2 - 5.2^2)^{-3}}$

(i) $\left(\frac{628 - 5^4}{6^5 - 7780}\right)^{-3}$

Index Laws and Variables

The index laws also apply to variables as well as numbers.

Index Law 1

$$a^m \times a^n = a^{m+n}$$

EXERCISE 6J

Examples

$$\begin{aligned} 1. \quad x^4 \times x^3 \\ = x^{(4+3)} \\ = x^7 \end{aligned}$$

$$\begin{aligned} 2. \quad m^6 \times m \\ = m^6 \times m^1 \\ = m^{(6+1)} \\ = m^7 \end{aligned}$$

$$\begin{aligned} 3. \quad n^5 \times n \times n^7 \\ = n^5 \times n^1 \times n^7 \\ = n^{(5+1+7)} \\ = n^{13} \end{aligned}$$

$$\begin{aligned} 4. \quad 3a^3b^5 \times 2ab^4 \\ = 3 \times a^3 \times b^5 \times 2 \times a \times b^4 \\ = 3 \times 2 \times a^3 \times a \times b^5 \times b^4 \\ = 6a^4b^9 \end{aligned}$$

$$\begin{aligned} 5. \quad 5x^3y^2 \times x^5y^2 \times 4xy^3 \\ = 5 \times x^3 \times y^2 \times x^5 \times y^2 \times 4 \times x \times y^3 \\ = 5 \times 4 \times x^3 \times x^5 \times x \times y^2 \times y^2 \times y^3 \\ = 20x^9y^7 \end{aligned}$$

1. Simplify the following.

(a) $a^5 \times a^3$

(b) $n^4 \times n^4$

(c) $y^7 \times y^4$

(d) $b^6 \times b$

(e) $m^3 \times m^2 \times m^4$

(f) $x^4 \times x^5 \times x$

(g) $a \times a^6 \times a^7$

(h) $x^{10} \times x^5 \times x$

(i) $y^2 \times y \times y^5 \times y$

2. Simplify the following.

(a) $3a^5 \times 6a^3$

(b) $2x^4 \times 4x^7$

(c) $2m^3 \times 3m \times m^6$

(d) $3n^6 \times 2n^5 \times 2n^7$

(e) $2a^2b^2 \times a^3b^5$

(f) $xy \times 6x^3y^6$

(g) $4a^5y^6 \times 3a^2y$

(h) $2a^3b^4c \times 7a^4b^5c$

(i) $3m^5n^2 \times 2mn \times 2m^2n^5$

(j) $6x^5y^3z^4 \times 2x^3yz^2 \times xy^2z^2$

(k) $3a^4b^3c \times 8a^6bc^7$

(l) $5m^5n^3 \times 4m^6n^4 \times 2n \times 3m^6$

Index Law 2

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

EXERCISE 6K**Examples**

$$\begin{aligned} 1. \quad x^6 \div x^4 \\ &= x^{6-4} \\ &= x^2 \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{m^7}{m^4} \\ &= m^{7-4} \\ &= m^3 \end{aligned}$$

$$\begin{aligned} 3. \quad 8x^6y^7 \div 4x^5y \\ &= \frac{8}{4}x^{6-5}y^{7-1} \\ &= 2xy^6 \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{18m^8n^6}{12m^3n^4} \\ &= \frac{3}{2}m^{8-3}n^{6-4} \\ &= \frac{3}{2}m^5n^2 \end{aligned}$$

1. Simplify the following.

(a) $x^5 \div x^2$

(b) $a^8 \div a^5$

(c) $n^9 \div n^3$

(d) $m^{10} \div m^3$

(e) $b^{11} \div b^8$

(f) $y^6 \div y$

(g) $p^7 \div p$

(h) $w^4 \div w^3$

(i) $\frac{m^{12}}{m^5}$

(j) $\frac{x^{13}}{x^4}$

(k) $\frac{n^{15}}{n}$

(l) $\frac{m^{11}}{m^{10}}$

2. Simplify the following.

(a) $10x^6y^7 \div 5x^3y^2$

(b) $16m^8n^9 \div 4m^5n^3$

(c) $24a^7b^5 \div 8a^4b^2$

(d) $16m^7n^6 \div 12m^5n^3$

(e) $8x^{10}y^8 \div 12x^7y^5$

(f) $20a^{11}b^7 \div 15a^8b^4$

(g) $12x^6y^6 \div 8x^4y$

(h) $20m^8n^{11} \div 24mn^{10}$

(i) $30a^8b^{12} \div 24a^5b$

(j) $\frac{18x^8y^4}{9x^5y^3}$

(k) $\frac{24m^8n^{12}}{20mn}$

(l) $\frac{24a^{10}b^{10}}{28a^9b}$

(m) $\frac{36m^8n^{11}}{12m^6n^5}$

(n) $\frac{32x^{15}y^{10}}{48x^9y^9}$

(o) $\frac{42m^7n^9}{14m^6n^8}$

(p) $\frac{54h^7i^{14}}{81h^6i^9}$

(q) $\frac{27a^7b^5}{54ab^5}$

(r) $\frac{48a^8b^7c^6}{36a^7b^7c^5}$

Index Law 3

$$a^0 = 1$$

EXERCISE 6L**Examples**

$$\begin{aligned} 1. \quad 4^0 \\ = 1 \end{aligned}$$

$$\begin{aligned} 2. \quad (a^2b^3)^0 \\ = 1 \end{aligned}$$

$$\begin{aligned} 3. \quad 3x^0 \\ = 3 \times x^0 \\ = 3 \times 1 \\ = 3 \end{aligned}$$

1. Simplify the following.

(a) y^0

(b) 5^0

(c) p^0

(d) 68^0

(e) $(5m)^0$

(f) $(3x)^0$

(g) $(7ab)^0$

(h) $(6xyz)^0$

2. Simplify the following.

(a) $5m^0$

(b) $3a^0$

(c) $4(3c^3)^0$

(d) $2a(3b^2)^0$

(e) $3^0 + 8^0$

(f) $2x^0 + (4x)^0$

(g) $(3m^0)^2$

(h) $5(n^5)^0$

(i) $4x^0 - (2x)^0$

(j) $5x^2y^0 + 2(xy^0)^2$

(k) $4x^0y^0z^0$

(l) $(3c^0)^2 - 2c^0$

3. Simplify the following.

(a) $\frac{m^6}{m^6}$

(b) $\frac{x^2z^5}{x^2z^3}$

(c) $\frac{6m^5n^4}{3m^2n^4}$

(d) $\frac{8a^4b^7}{10a^4b}$

(e) $\frac{12u^3v^3}{6u^3v^3}$

(f) $\frac{8m^8n^4}{12m^7n^4}$

(g) $\frac{18x^4y^3z^5}{24x^4yz^4}$

(h) $\frac{20a^5b^4c}{10a^4b^3c}$

Example

$$\begin{aligned} & \frac{4m^5n^3}{3m^3n^3} \\ &= \frac{4}{3} m^{5-3} n^{3-3} \\ &= \frac{4}{3} m^2 n^0 \\ &= \frac{4}{3} m^2 \times 1 \\ &= \frac{4}{3} m^2 \end{aligned}$$

Index Law 4

$$(a^m)^n = a^{m \times n}$$

EXERCISE 6M**Examples**

$$\begin{aligned} 1. \quad (x^3)^4 \\ &= x^{3 \times 4} \\ &= x^{12} \end{aligned}$$

$$\begin{aligned} 2. \quad (2m^2)^3 \\ &= 2^3 m^{2 \times 3} \\ &= 8m^6 \end{aligned}$$

1. Simplify the following.

(a) $(m^2)^6$

(b) $(x^3)^5$

(c) $(a^4)^6$

(d) $(c^6)^3$

(e) $(p^5)^6$

(f) $(b^7)^3$

(g) $(n^9)^4$

(h) $(y^4)^5$

2. Expand the brackets in the following.

(a) $(3x^2)^2$

(b) $(2m^3)^4$

(c) $(3m^5)^3$

(d) $(4n^6)^2$

(e) $(-6c^7)^2$

(f) $(-2n^4)^5$

(g) $(-3a^4)^3$

(h) $(-3u^6)^4$

(i) $(6a^3)^3$

(j) $-(5y^5)^3$

(k) $-(8c^4)^2$

(l) $-(-4y^3)^5$

Index Law 5

$$\begin{aligned}
 (a \times b)^n &= (ab)^n = a^n b^n \\
 (a \div b)^n &= \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}
 \end{aligned}$$

EXERCISE 6N**Examples**

$$\begin{aligned}
 1. \quad (x^3 \times y^5)^4 &= (x^3 y^5)^4 \\
 &= (x^3)^4 (y^5)^4 \\
 &= x^{12} y^{20}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad (2m^2 n^3)^3 &= 2^3 (m^2)^3 (n^3)^3 \\
 &= 8m^6 n^{15}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (a^3 \div b^6)^5 &= \left(\frac{a^3}{b^6}\right)^5 \\
 &= \frac{(a^3)^5}{(b^6)^5} \\
 &= \frac{a^{15}}{b^{30}}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \left(\frac{3m^2 n^3}{5x^3 y^5}\right)^2 &= \frac{3^2 (m^2)^2 (n^3)^2}{5^2 (x^3)^2 (y^5)^2} \\
 &= \frac{9m^4 n^6}{25x^6 y^{10}}
 \end{aligned}$$

1. Expand the brackets and simplify the following.

(a) $(a^2 \times b^3)^4$

(b) $(x^4 \times y^7)^2$

(c) $(p^2 \times q^9)^4$

(d) $(u^5 v^3)^6$

(e) $(m^5 n^8 o^6)^5$

(f) $(a^6 b^3 c^9)^3$

(g) $(3x^3 y^5)^2$

(h) $(4a^3 b)^2$

(i) $(-5mn)^3$

(j) $(8a^5 b^3)^2$

(k) $(-2a^4 bc^3)^6$

(l) $(9p^4 q^7 r^3)^2$

(m) $-(2m^3 n^6)^3$

(n) $-(3x^4 y^5)^4$

(o) $-(5u^6 v^7)^3$

(p) $-(-a^3 b^4)^3$

(q) $-(-4m^6 n^3)^4$

(r) $-(-2a^3 b^2)^7$

(s) $(6x^4 yz^5)^3$

(t) $(7a^4 b^3 c)^4$

(u) $(5p^3 q^7 r^8)^5$

2. Expand the brackets and simplify the following.

(a) $(a^5 \div b^2)^3$

(b) $(m^7 \div n^3)^2$

(c) $(x^7 \div y^5)^3$

(d) $\left(\frac{m^5}{n^4}\right)^2$

(e) $\left(\frac{a^6}{b^7}\right)^3$

(f) $\left(\frac{e^4}{r^2}\right)^5$

(g) $\left(\frac{m^3 n^4}{p^4 q^3}\right)^4$

(h) $\left(\frac{a^6 b^3}{x^2 y^8}\right)^2$

(i) $\left(\frac{m^5 p^3}{n^4 q^2}\right)^6$

(j) $\left(\frac{2a^3 b^4}{m^2 n^5}\right)^3$

(k) $\left(\frac{x^4}{3y^3 z^5}\right)^3$

(l) $\left(\frac{4m^2 n^5}{p^5 q^4}\right)^2$

(m) $\left(\frac{4m^3 n^6}{3a^2 b}\right)^3$

(n) $\left(\frac{5x^3 y^4}{6z^7}\right)^2$

(o) $\left(\frac{3a^4 b^9}{7x^5 y^7}\right)^2$

(p) $\left(\frac{2abc^3}{3x^2 y^3 z^4}\right)^3$

(q) $\left(\frac{5}{m^2 n^3 p^2}\right)^4$

(r) $\left(\frac{7mn^7}{10p^4}\right)^3$

Index Law 6

$$a^{-m} = \frac{1}{a^m}$$

EXERCISE 6O**Examples**

$$\begin{aligned} 1. \quad x^{-5} \\ = \frac{1}{x^5} \end{aligned}$$

$$\begin{aligned} 2. \quad 5n^{-6} \\ = \frac{5}{n^6} \end{aligned}$$

$$\begin{aligned} 3. \quad 2m^4n^{-3} \\ = \frac{2m^4}{n^3} \end{aligned}$$

$$\begin{aligned} 4. \quad \frac{2}{3}x^2y^{-4} \\ = \frac{2x^2}{3y^4} \end{aligned}$$

$$\begin{aligned} 5. \quad \frac{1}{6}a^3b^{-5}c^{-2} \\ = \frac{a^3}{6b^5c^2} \end{aligned}$$

1. Write the following with positive indices.

(a) x^{-8}

(b) 3^{-4}

(c) a^{-2}

(d) e^{-10}

(e) c^{-1}

(f) $3m^{-4}$

(g) xy^{-2}

(h) $4mn^{-7}$

(i) $6x^2y^{-3}$

(j) $8a^{-2}b^9$

2. Write the following with positive indices.

(a) $3m^2n^{-6}$

(b) $7a^3b^{-5}$

(c) $a^4b^3c^{-6}$

(d) $6x^{-5}y^3$

(e) $\frac{3}{4}m^2n^{-3}$

(f) $\frac{5}{7}a^3b^{-4}$

(g) $\frac{1}{2}x^{-4}y^3$

(h) $\frac{1}{3}x^3yz^{-2}$

(i) $\frac{4}{9}a^5b^{-3}c$

(j) $\frac{2}{7}p^4q^{-3}r^{-2}$

(k) $\frac{3}{8}a^{-2}b^{-3}c^{-2}$

(l) $\frac{1}{5}v^{-3}w^2x^{-7}$

Summary of Index Laws

Index Law 1

$$a^m \times a^n = a^{m+n}$$

Index Law 2

$$a^m \div a^n = a^{m-n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

Index Law 3

$$a^0 = 1$$

Index Law 4

$$(a^m)^n = a^{m \times n}$$

Index Law 5

$$\begin{aligned} (a \times b)^n &= (ab)^n \\ &= a^n b^n \end{aligned} \quad \begin{aligned} (a \div b)^n &= \left(\frac{a}{b}\right)^n \\ &= \frac{a^n}{b^n} \end{aligned}$$

Index Law 6

$$a^{-m} = \frac{1}{a^m}$$

Using All The Index Laws

EXERCISE 6P

1. Simplify the following.

Examples

$$\begin{aligned} 1. \quad & \frac{x^3 \times x^6}{x^4} \\ &= \frac{x^9}{x^4} \\ &= x^5 \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{4m \times 6m^5}{12m^2} \\ &= \frac{24m^6}{12m^2} \\ &= 2m^4 \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{(2x^3)^3}{6x^4} \\ &= \frac{2^3 x^9}{6x^4} \\ &= \frac{8x^9}{6x^4} \\ &= \frac{4x^5}{3} \end{aligned}$$

$$(a) \quad \frac{x^5 \times x^6}{x^3}$$

$$(b) \quad \frac{m^6 \times m^2}{m}$$

$$(c) \quad \frac{c^7 \times c}{c^5}$$

$$(d) \quad \frac{a^2 b^3 \times a^6 b^4}{a^3 b^2}$$

$$(e) \quad \frac{m^5 n^3 \times m^4 n^6}{m^4 n}$$

$$(f) \quad \frac{6x^2 \times 2x^5}{4x^3}$$

$$(g) \quad \frac{5m^4 \times 4m^3}{10m^5}$$

$$(h) \quad \frac{3x^7 \times 6x^6}{2x^4}$$

$$(i) \quad \frac{8c^6 \times 3c}{16c^4}$$

$$(j) \quad \frac{b^3 \times 2b^4}{8b^5}$$

$$(k) \quad \frac{4x \times 5x^3}{12x^2}$$

$$(l) \quad \frac{9n^2 \times 4n^4}{6n^6}$$

$$(m) \quad \frac{(4x^3)^2}{2x^3}$$

$$(n) \quad \frac{(3m^4)^2}{6m^5}$$

$$(o) \quad \frac{(2x^4)^4}{10x}$$

2. Simplify the following.

Examples

$$\begin{aligned}
 1. \quad & \frac{(x^2y^3)^2 \times (x^3y)^3}{(x^3y^2)^2 \times x^4y^5} \\
 &= \frac{x^4y^6 \times x^9y^3}{x^6y^4 \times x^4y^5} \\
 &= \frac{x^{13}y^9}{x^{10}y^9} \\
 &= x^3y^0 \\
 &= x^3
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{3(x^4y^3)^3}{(2x^2y)^3} \times \frac{4(x^5y^2)^4}{(x^3y^2)^2} \\
 &= \frac{3x^{12}y^9}{8x^6y^3} \times \frac{4x^{20}y^8}{x^6y^4} \\
 &= \frac{12x^{32}y^{17}}{8x^{12}y^7} \\
 &= \frac{3}{2} x^{20}y^{10}
 \end{aligned}$$

$$(a) \quad \frac{(x^4y^2)^3 \times (xy^5)^3}{(x^2y)^4 \times x^2y^7}$$

$$(b) \quad \frac{m^5n^8 \times (m^4n^2)^{-3}}{(m^2n^4)^2 \times (mn^3)^2}$$

$$(c) \quad \frac{(a^5b^3)^4 \times (a^3b^2)^3}{(a^2b^3)^2 \times (a^4b)^3}$$

$$(d) \quad \frac{3(x^4y^3)^2 \times (4x^3y^6)^2}{2(xy^2)^2 \times 6x^2y^5}$$

$$(e) \quad \frac{(3m^5n^3)^2 \times 8(m^4n^2)^3}{2(m^3n^2)^2 \times (3m^5n^3)^3}$$

$$(f) \quad \frac{(3x^5y^{-3})^3 \times (4x^3y^6)^2}{2(x^6y^2)^2 \times (6x^2y^{-5})^2}$$

$$(g) \quad \frac{(x^2y^3)^{-4}}{(3xy^2)^2} \times \frac{6(x^3y^6)^3}{(2x^3y^5)^2}$$

$$(h) \quad \frac{(3m^6)^2}{(2m^3)^3} \times \frac{4(n^5)^2}{3(mn^2)^3}$$

$$(i) \quad \frac{6a^3b^5}{(2a^2b^3)^4} \times \frac{(4a^5b^6)^2}{(3a^2b)^2}$$

$$(j) \quad \frac{(10x^3)^2}{(4y^2)^3} \times \frac{(8y^5)^2}{(5x)^2}$$

$$(k) \quad \frac{4(m^3n^2)^4}{(3m^2n)^3} \times \frac{(9m^3n^6)^2}{(2mn^3)^5}$$

$$(l) \quad \frac{(2a^3b^4)^3}{4(a^5b^{10})^{-3}} \times \frac{(3a^2b^5)^4}{(9ab)^2}$$

3. Simplify the following.

Examples

$$\begin{aligned}
 1. \quad & \frac{3a^2}{5} \div \frac{9a}{10} \\
 &= \frac{3a^2}{5} \times \frac{10}{9a} \\
 &= \frac{30a^2}{45a} \\
 &= \frac{2a}{3}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{(2mn^4)^2}{(3m^2n^3)^2} \div \frac{(6m^2n^3)^2}{(3m^2n^4)^3} \\
 &= \frac{4m^2n^8}{9m^4n^6} \div \frac{36m^4n^6}{27m^6n^{12}} \\
 &= \frac{4m^2n^8}{9m^4n^6} \times \frac{27m^6n^{12}}{36m^4n^6} \\
 &= \frac{1\cancel{4}m^2n^8}{\cancel{9}m^4n^6} \times \frac{\overset{1}{\cancel{2}}7m^6n^{\overset{1}{\cancel{2}}12}}{\underset{3}{\cancel{3}}6m^4n^{\underset{3}{\cancel{6}}6}} \\
 &= \frac{1m^8n^{20}}{3m^8n^{12}} \\
 &= \frac{n^8}{3}
 \end{aligned}$$

$$(a) \quad \frac{4x^4}{3} \div \frac{28x}{15}$$

$$(b) \quad \frac{15c^7}{8} \div \frac{5c^3}{16}$$

$$(c) \quad \frac{20}{9a^3} \div \frac{15}{18a^6}$$

$$(d) \quad \frac{12a^5}{9b^3} \div \frac{4}{18b^3}$$

$$(e) \quad \frac{7x^7}{8y^3} \div \frac{21x}{4y^5}$$

$$(f) \quad \frac{30x^4}{25y^3} \div \frac{6x^4}{5y^3}$$

$$(g) \quad \frac{6x^3y^5}{(5x^2y^4)^2} \div \frac{3x^8y^5}{25(x^5y^5)^2}$$

$$(h) \quad \frac{(2x^2y^3)^5}{(3x^2y^4)^2} \div \frac{(4xy^2)^3}{(6x^3y^6)^2}$$

$$(i) \quad \frac{(8a^3b^7)^3}{(5a^4b^5)^3} \div \frac{(2a^2b^3)^9}{(10a^8b^7)^3}$$

$$(j) \quad \frac{(3m^2n^3)^4}{(2mn^3)^6} \div \frac{(3m^2n^3)^3}{(4m^5n^{10})^2}$$

Scientific Notation (Standard Form)

Scientific notation, or **standard form**, is a convenient way of writing very large or very small numbers. It is often used in science because of the large numbers (for example the distance between planets) or small numbers (for example the mass of an atom). Scientific notation involves moving the decimal point to after the first non-zero digit and using powers of 10.

EXERCISE 6Q

1. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

$$\begin{aligned} 1. \quad 6000 \\ &= 6 \times 1000 \\ &= 6 \times 10^3 \end{aligned}$$

$$\begin{aligned} 2. \quad 935\,000 \\ &= 9.35 \times 100\,000 \\ &= 9.35 \times 10^5 \end{aligned}$$

(a) 9000

(b) 50 000

(c) 700 000 000

(d) 275 000

(e) 320 000

(f) 82 100 000

(g) 365 470 000

(h) 230 000 000 000

(i) 50 001

(j) 6587

(k) 254.34

(l) 1537.8

2. Convert the following numbers into scientific notation.

Move the decimal point to after the first digit.

Examples

$$\begin{aligned} 1. \quad 0.0076 \\ &= 7.6 \div 1000 \\ &= 7.6 \times \frac{1}{1000} \\ &= 7.6 \times 10^{-3} \end{aligned}$$

$$\begin{aligned} 2. \quad 0.000\,000\,582 \\ &= 5.82 \div 10\,000\,000 \\ &= 5.82 \times \frac{1}{10\,000\,000} \\ &= 5.82 \times 10^{-7} \end{aligned}$$

(a) 0.0005

(b) 0.000 009

(c) 0.000 000 004

(d) 0.000 13

(e) 0.006 92

(f) 0.000 057 9

(g) 0.002 045

(h) 0.000 006

(i) 0.000 000 068 1

(j) 0.000 020 204

(k) 0.000 000 000 2

(l) 0.000 000 005 7

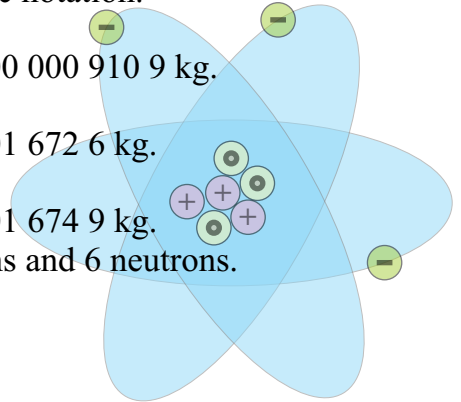
3. Convert the following numbers into numeral form.

- | | | |
|---------------------------|----------------------------|-----------------------------|
| (a) 3.6×10^4 | (b) 8×10^6 | (c) 6.71×10^5 |
| (d) 2.455×10^8 | (e) 7.201×10^7 | (f) 3.82×10^{11} |
| (g) 2.7×10^{-3} | (h) 7×10^{-5} | (i) 9.3×10^{-4} |
| (j) 4.83×10^{-7} | (k) 5.917×10^{-9} | (l) 1.008×10^{-10} |

4. Atoms consist of three particles - electrons, protons and neutrons.

Write the following dimensions in scientific notation.

- (a) The mass of an electron is
0.000 000 000 000 000 000 000 000 000 000 910 9 kg.
- (b) The mass of a proton is
0.000 000 000 000 000 000 000 000 000 001 672 6 kg.
- (c) The mass of a neutron is
0.000 000 000 000 000 000 000 000 000 001 674 9 kg.
- (d) A carbon atom has 6 electrons, 6 protons and 6 neutrons.
Find the mass of a carbon atom.



5. Write the following quantities in scientific notation.

- (a) The mass of the Sun is:
1 990 000 000 000 000 000 000 000 000 000 kg.
- (b) The temperature at the core of the Sun is:
15 000 000°C.

6. (a) Write the following planet masses in scientific notation.

Mercury - 330 000 000 000 000 000 000 000 kg
Venus - 4 870 000 000 000 000 000 000 000 kg
Earth - 5 980 000 000 000 000 000 000 000 kg
Mars - 642 000 000 000 000 000 000 000 kg
Jupiter - 1 900 000 000 000 000 000 000 000 000 kg
Saturn - 569 000 000 000 000 000 000 000 000 kg
Uranus - 86 900 000 000 000 000 000 000 000 kg
Neptune - 102 000 000 000 000 000 000 000 000 kg
Pluto - 12 000 000 000 000 000 000 000 000 kg

- (b) List the planets in order from the lightest to the heaviest.
 Pluto is now classified as a dwarf planet.

7. There are 400 billion stars in the milky way galaxy.
Write this in scientific notation (one billion in this case is one thousand million).



8. Convert the following measurements to scientific notation.
- (a) The width of the smallest atom (hydrogen) is 0.000 000 000 1 m.
 - (b) The width of a human egg is 0.000 12 m.
 - (c) The width of an E. coli bacterium is 0.000 002 m.
 - (d) The width of a human red blood cell is 0.000 007 m
 - (e) The width of a human hair is 0.0001 m.



9. (a) The speed of light is approximately 3×10^8 metres per sec (m/s).
Convert this speed to kilometres per second (km/s).
- (b) The distance from the Earth to the Sun is 150 million kilometres.
Convert this distance into scientific notation.
- (c) By using the formula for time taken shown here calculate the time it takes for light to travel from the Sun to the Earth.
- $$\text{time taken} = \frac{\text{distance}}{\text{speed}}$$
- Give answer in:
- (i) seconds (ii) minutes and seconds
- (d) The distance from the Earth to the Moon is 384 400 km. Find the time it takes light to travel from the Earth to the Moon.
Give answer in seconds correct to two decimal places.

10. Communications satellites are used to relay and amplify telecommunications signals around the Earth. If a satellite is in a geosynchronous orbit it is 36 000 km above the Earth.

Telecommunications signals travel at the speed of light.

How long would it take a signal to travel from the Earth to the satellite and back to the Earth?



11. Another way for telecommunications signals to be transmitted is through optic fibres or via telecommunications relays. These signals also travel at the speed of light.
Find the time it would take a signal to travel the following distances.
Give answers in seconds in scientific notation.
- (a) Sydney to Perth - 3300 km.
(b) Brisbane to Ballina - 210 km
(c) 900 m
(d) 60 cm
(e) 30 mm

12. A unit of time often used when dealing with very small durations of time is the nanosecond (ns).

1 nanosecond = 10^{-9} seconds

Convert the times in question 11 to nanoseconds.

PROBLEM SOLVING

1. The speed of light is 3×10^8 metres per second.
 - (a) How far would light travel in:
 - (i) 2 seconds (ii) 1 minute (iii) 1 hour
 - (b) A **light year** is the distance that light travels in a year.
Calculate how many kilometres are in one light year.
 - (c) The milky way galaxy is 100 000 light years across.
How many kilometres is this?
 - (d) Find the name of the nearest star to our solar system and how far it is from Earth in light years and kilometres.

2. Scientists were studying the growth of bacteria on food after it was taken out of a refrigerator.
They found that the number of bacteria on an item of food could be calculated by using the following formula:

$$n = 200 \times 5^t$$

where n = the number of bacteria
and t = the number of hours after the food was taken out of the refrigerator

Example

Find the number of bacteria on the food after 2 hours:

$$\begin{aligned}
 t &= 2 & n &= 200 \times 5^2 \\
 & & &= 200 \times 25 \\
 n &= \mathbf{5000 \text{ bacteria}}
 \end{aligned}$$

- (a) Find the number of bacteria after: (i) 3 hours (ii) 5 hours
- (b) After how many hours are there 7.8125×10^7 bacteria?

PUZZLE

Rearrange the letters in the following phrases to make words or phrases from this chapter.

1. INFECTION IS INTO CAT
3. TRANSFORM DAD
5. IDIOTIC EVE SPINS

2. AXLES WIND
4. BARE BUSMEN
6. CUBAN REALISM

CHAPTER REVIEW

1. Write $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ in index form.

2. Simplify the following.

(a) $3^5 \times 3^4$

(b) $\frac{6^8}{6^3}$

(c) $4^9 \div 4$

(d) $3^2 + 2^0$

(e) $(7^5)^4$

(f) $(5^3 \times 5^7)^2$

(g) $\left(\frac{9^6}{5^3}\right)^4$

(h) $(4^5 \div 7^8)^4$

3. Write the following with positive indices.

(a) 6^{-5}

(b) 5×3^{-8}

(c) $\frac{4}{8 \times 7^{-3}}$

(d) $\frac{2^9 \times 7^{-6}}{5^{-8} \times 4^3}$

4. Simplify the following.

(a) $x^5 \times x^4$

(b) $3m^6n^2 \times 4m^5n^4$

(c) $(a^3)^4$

(d) $(2m^3n^2)^5$

(e) $(-a^2b^3)^3$

(f) $-(3p^2q^3r^2)^4$

(g) $(3m)^0$

(h) $6a^0 + (2b)^0$

(i) $\frac{a^6 \times 6a^7}{2a^3}$

(j) $\frac{(3x^2y^5)^4}{(9x^3y^2)^2}$

(k) $\frac{(2m^3n^5)^4}{(3mn^3)^3} \times \frac{(9m^5n^6)^2}{(4m^9n^{11})^2}$

(l) $\frac{(5x^3y^5)^2}{(6x^4y^6)^2} \div \frac{(10x^4y^3)^2}{(4x^5y^4)^3}$

5. Write the following with positive indices.

(a) x^{-6}

(b) $8m^2n^{-3}$

(c) $\frac{2}{5} x^5y^{-2}z^{-4}$

6. Convert the following numbers into scientific notation.

(a) 8 620 000 000

(b) 0.000 000 057 2

7. Convert the following numbers into numeral form.

(a) 6.71×10^4

(b) 3.01×10^{-6}

8. Use a calculator to evaluate the following.

(a) $4^6 - 6^4$

(b) $\left(\frac{8^6}{2^{17}}\right)^6$

(c) $\frac{2^{-10}}{8^{-4}} + \frac{3^{-12}}{9^{-6}}$

Algebra

Algebraic Symbols

EXERCISE 7A

Examples

Write the following statements as algebraic expressions or terms.

1. The sum of x and y .
2. a more than 5.
3. The product of M and N .
4. The total cost of n items that each cost \$8.
5. One quarter of Z .
6. The sum of $5a$ and $6b$.
7. 5 times the sum of c and d .
8. The product of k and $l - 6$.

Answers

1. $x + y$
2. $5 + a$
3. $M \times N$ or MN
4. $8n$
5. $Z/4$
6. $5a + 6b$
7. $5(c + d)$
8. $k(l - 6)$

1. Write the following statements as algebraic expressions or terms.
 - (a) The product of x and z .
 - (b) The sum of G and H .
 - (c) 2 less than p .
 - (d) 7 more than d .
 - (e) y more than w .
 - (f) The sum of $4t$ and $9w$.
 - (g) 7 less than m .
 - (h) One third of H .
 - (i) The product of t and $5 - q$.
 - (j) The total cost of b books that cost \$15 each.

All algebraic **terms** consist of a **coefficient** (number) and **pronumerals** (symbols).

Examples $6x$: 6 is the coefficient
 x is the pronumeral

$-\frac{3}{8}abc^3$: $-\frac{3}{8}$ is the coefficient
 abc^3 are the pronumerals

pq : 1 is the coefficient
 pq are the pronumerals

Like terms are terms that have the same pronumerals.

Example Group the **like terms** from the following list.

$2a$ $3b$ $6a^2$ b $-\frac{1}{2}a$ $-ab$ $12a^2$ $12ba$ $6a$ $2b$

The groups of **like terms** are:

$2a, -\frac{1}{2}a, 6a$

$3b, b, 2b$

$6a^2, 12a^2$

$-ab, 12ba$

Note: the pronumerals can
 be in any order.
 These are **like terms**.

Only **like terms** can be added or subtracted.

Examples 1. $5x + 3y + 8x - y$

$$= \textcircled{5x} + \textcircled{3y} + \textcircled{8x} - \textcircled{y}$$

$$= 13x + 2y$$

2. $5x - 7 - 1 - 8x$

$$= \textcircled{5x} - \textcircled{7} - \textcircled{1} - \textcircled{8x}$$

$$= -3x - 8$$

Algebraic **expressions** are the addition or subtraction of more than one algebraic term.

Examples $4x - 5y$ $3x^2 + 6y^3$ $A + 2C - 8DF^2$

2. Which of the following are: (i) terms? (ii) expressions?

- (a) $4x$ (b) $5p + 2q$ (c) $8(h - g)$ (d) $5mn^6$ (e) $-\frac{3}{4}v^2w^3y^7$
 (f) $5xy - 2z + 4ab$ (g) $5AB^7C^3D^2$ (h) $6 + 5x - 7z$

3. For each of the following terms state the:

(i) coefficient (ii) pronumerals

- (a) $7a$ (b) $34abc^2$ (c) $-2p$ (d) F (e) $\frac{3}{5}m^2n^3$ (f) $-\frac{2}{7}w^2z^5$

4. Match all the like terms in the following list of terms.

$2m$ $4n^2$ $6mn$ n $-4m$ $6m^2$ $\frac{1}{4}n^2$ $4n$ $\frac{1}{4}mn$ $4m^2n$ $3m^2$ $-3m^2n$

5. Simplify the following expressions.

- (a) $3y + 4y$ (b) $8b - 5b$
 (c) $7k + k$ (d) $6z + 5z - 2z$
 (e) $4ab + 7ab$ (f) $5xy - 2xy$
 (g) $9mn - mn - 3mn$ (h) $6xyz + xyz + xyz$
 (i) $6d - 8d$ (j) $2pq^2 + 4pq^2 - pq^2$
 (k) $x^2 + x^2 + x^2$ (l) $4y^2 - 8y^2 + y^2$
 (m) $8a - 10a$ (n) $5mn - 2mn - 6nm$
 (o) $-5t - 3t$ (p) $-6y + 9y$
 (q) $-4xy + 5yx - 7xy$ (r) $-cd - 4cd + 9dc - 7dc - 3cd + cd + 5dc$

Examples

1. $6a + 3a = 9a$
2. $5p - p + 2p = 6p$
3. $6k - 9k = -3k$
4. $-2ab + 5ba = 3ab$

6. Simplify the following expressions by collecting all the like terms.

Examples

1. $3a + 4b + 5a + 3b$
 $= 8a + 7b$

2. $-5x - 7y + 3x - 2y$
 $= -2x - 9y$

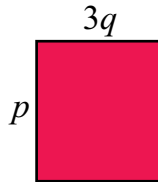
- (a) $7x + 5y + 2x + 3y$ (b) $4a + 3b + 6b + 3a$
 (c) $4r + 3t + 2r + 6t + r$ (d) $8u + w + 5u + w$
 (e) $6y + 3z + y - z$ (f) $10k + 7m - 7k - 4m$
 (g) $6w + 13h + 5w - 7h - 4w$ (h) $4a + 5b - 4a - 5b$
 (i) $-2x - 3x - 5y - y$ (j) $-3m - 4m + 10m - 3m$
 (k) $3k^2l - 2lk^2 + 2kl^2 - 5kl^2$ (l) $-ab^2 + 4ba^2 - 2a^2b + 6b^2a$
 (m) $-2xyz + 5xzy - 6yxz + 4zxy$ (n) $2x^2y - 3y^2x + 3xy^2 - 5xy^2$
 (o) $3a^2b + 5a^2 - 2b - 2ba^2 + 6b - a^2$
 (p) $-3m^2n + 5n^2m + 2nm^2 - 3mn^2 - 2n^2m + m^2n$

7. Find the perimeter of the following shapes.

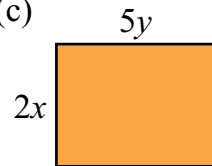
(a)



(b)



(c)



8. Darnelle earned x dollars per hour. How much would he earn if he worked the following times?

(a) 5 hours (b) 20 hours one week and 15 hours the next week

9. Simplify the following expressions.

Examples

$$1. \quad 3x \times 2y \\ = 6xy$$

$$2. \quad 5a \times 4a \\ = 20a^2$$

$$3. \quad 2p \times 6pq \times 3qr \\ = 36p^2q^2r$$

$$4. \quad 4c \times 2b \times 5ac \\ = 40abc^2$$

$$5. \quad 2y \times x \times 3z \times y \times x \times 2z \\ = 12x^2y^2z^2$$

Always write answers
in alphabetical order

(a) $5a \times 2b$

(b) $4c \times 2b \times a$

(c) $3y \times 2x \times z$

(d) $4d \times 2d$

(e) $5p \times 6q$

(f) $6u \times 3v \times t$

(g) $3r \times 2q \times 3s$

(h) $2ab \times 3a \times 4b$

(i) $2xy \times 3yx \times 5z$

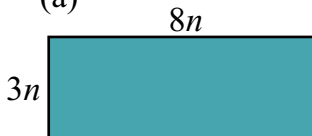
(j) $2bc \times 3ac \times 10ba$

(k) $2d \times 3f \times 6e \times 7g$

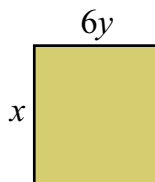
(l) $3mn \times 2mn \times 2 \times 2l$

10. Find the area of the following shapes.

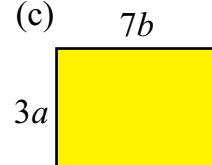
(a)



(b)



(c)



11. Charley could pick a apples per hour. How many apples would she pick in the following times?

(a) 4 hours (b) y hours (c) 5 hours a day for z days

Expanding Algebraic Expressions

EXERCISE 7B

1. Expand the following algebraic expressions.

Examples

Each term in the bracket is multiplied by the term outside the bracket.

$$\begin{aligned} 1. \quad & 3(x + 5) \\ &= 3 \times x + 3 \times 5 \\ &= \mathbf{3x + 15} \end{aligned}$$

$$\begin{aligned} 2. \quad & -4(2a - 5) \\ &= -4 \times 2a + -4 \times \textcolor{green}{-5} \\ &= \mathbf{-8a + 20} \end{aligned}$$

The minus sign makes the number negative.

(a) $3(y + 2)$

(b) $2(x - 4)$

(c) $4(m - 6)$

(d) $7(a - 3)$

(e) $3(4 + 5m)$

(f) $2(8 + 3d)$

(g) $3(m - n)$

(h) $6(3p + q)$

(i) $7(2t + 3u)$

(j) $-3(x + 3)$

(k) $-5(a + 7)$

(l) $-8(4 + p)$

(m) $-5(a + b)$

(n) $-2(6 + 3z)$

(o) $-7(4y + 5z)$

(p) $-3(x - 5)$

(q) $-4(a - 6)$

(r) $-3(2y - 3)$

(s) $-2(3a - 5b)$

(t) $6(4m - 3n)$

(u) $5(2a - 3b + 4c)$

(v) $-4(7x - 2y)$

(w) $-5(3g + 6h)$

(x) $-4(3x - 2y - 5z)$

2. Expand the following algebraic expressions.

Examples

$$\begin{aligned} 1. \quad & x(3x - 1) \\ &= x \times 3x + x \times -1 \\ &= \mathbf{3x^2 - x} \end{aligned}$$

$$\begin{aligned} 2. \quad & -3y(2x - 5y) \\ &= -3y \times 2x + -3y \times -5y \\ &= \mathbf{-6xy + 15y^2} \end{aligned}$$

(a) $a(2a + 3)$

(b) $m(2n + 5p)$

(c) $x(3y - 2z)$

(d) $2p(3q - 7)$

(e) $2x(3x - 5y)$

(f) $3a(6b + 7c)$

(g) $5m(3m - 2n)$

(h) $2b(b - 7c)$

(i) $2t(3t - 10u)$

(j) $-x(2x + y)$

(k) $-3a(2a + 3c)$

(l) $-2x(3a + 2b)$

(m) $-2a(4b - 5)$

(n) $-3m(2p - 3q)$

(o) $-5v(2w - 5z)$

(p) $-8m(3n - 2p)$

(q) $-4d(2a + 3x)$

(r) $-3a(2a - 5b - 2c)$

(s) $-5a(4a - 2ab)$

(t) $-4t(3n - 4t)$

(u) $-2x(5x - 2y + 3z)$

(v) $3x(2xy - 3yz)$

(w) $-3xy(2x + 3y)$

(x) $2m(3nm - 2m - 5n)$

3. Expand the following algebraic expressions and simplify, where possible, by collecting like terms.

Examples

$$\begin{aligned} 1. \quad & 2(x + 5) + 3(x - 4) \\ &= 2x + 10 + 3x - 12 \\ &= 5x - 2 \end{aligned}$$

$$\begin{aligned} 2. \quad & a(2b + 3) + 2a(5 - 3b) \\ &= 2ab + 3a + 10a - 6ab \\ &= -4ab + 13a \\ &\text{(or } 13a - 4ab\text{)} \end{aligned}$$

$$\begin{aligned} 3. \quad & 3m(2m + 3n) - 2n(3m - n) \\ &= 6m^2 + 9mn - 6mn + 2n^2 \\ &= 6m^2 + 3mn + 2n^2 \end{aligned}$$

(a) $3(a + 5) + 2(a + 1)$

(c) $5(y + 2) + 4(y - 2)$

(e) $3(x - 5) + 2(x + 1)$

(g) $4(n + 3) - 2(n + 5)$

(i) $y(2y + 3) + 3y(3y - 4)$

(k) $2x(5x - 2) - x(2x - 3)$

(m) $5a(2a + 7) - 4a(3a - 2)$

(o) $2m(6 - 5m) - 3m(2m + 1)$

(q) $8x(3y + 2z) - 3x(5z - 3y)$

(s) $-3a(2a - 2b) - 2b(3a - 2b)$

(b) $2(x + 3) + 3(x + 2)$

(d) $3(m + n) + 2(2m + n)$

(f) $4(y - 3) + 3(2 - y)$

(h) $5(p + 4) - 3(p + 3)$

(j) $2a(a + 4b) + 3a(2a - b)$

(l) $4m(2 - 3n) - 3m(5n - 2)$

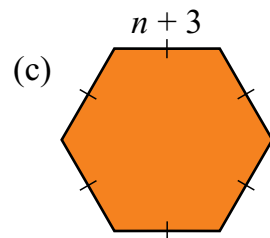
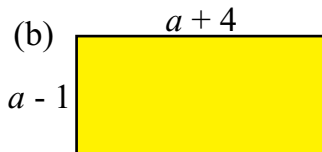
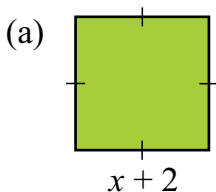
(n) $5x(3x - 2y) - 3x(2y - 3x)$

(p) $2c(2c + 3d) + 3c(3d - 2c)$

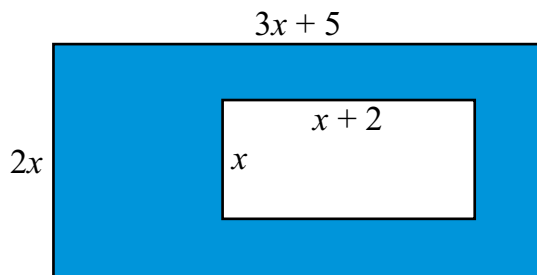
(r) $3m(4p - 2q) + 2m(3q - 5p)$

(t) $-5x(2y - 3x) - 6y(4x - 3y)$

4. Find the *perimeter* of these shapes.



5. Find the *area* of the shaded region in this shape.



Expand the following expressions and collect the like terms.

Examples

1. $(x + 3)(x + 5)$

When expanding two brackets, each term in the second bracket must be multiplied by each term in the first bracket.

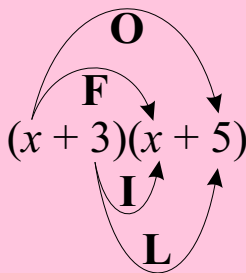
The acronym FOIL can be used to help.

First - multiply the *first* term in each bracket

Outer - multiply the *outer* two terms

Inner - multiply the *inner* two terms

Last - multiply the *last* term in each bracket



$$\begin{aligned}
 &(x + 3)(x + 5) \\
 &= x \times x + x \times 5 + 3 \times x + 3 \times 5 \\
 &= x^2 + 5x + 3x + 15 \\
 &= \mathbf{x^2 + 8x + 15}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad &(3y + 5)(2y - 4) \\
 &= 3y \times 2y + 3y \times -4 + 5 \times 2y + 5 \times -4 \\
 &= 6y^2 - 12y + 10y - 20 \\
 &= \mathbf{6y^2 - 2y - 20}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad &(2n - 5)(3n - 7) \\
 &= 2n \times 3n + 2n \times -7 + -5 \times 3n + -5 \times -7 \\
 &= 6n^2 - 14n - 15n + 35 \\
 &= \mathbf{6n^2 - 29n + 35}
 \end{aligned}$$

6. Expand and simplify the following expressions.

- | | | |
|----------------------|----------------------|-----------------------|
| (a) $(x + 2)(x + 3)$ | (b) $(y + 5)(y + 1)$ | (c) $(m + 6)(m + 2)$ |
| (d) $(c + 8)(c + 3)$ | (e) $(a + 2)(a + 1)$ | (f) $(x + 10)(x + 5)$ |
| (g) $(p + 7)(p + 9)$ | (h) $(p + 6)(p - 3)$ | (i) $(x + 5)(x - 2)$ |
| (j) $(y + 3)(y - 1)$ | (k) $(m + 4)(m - 3)$ | (l) $(n + 7)(n - 3)$ |
| (m) $(w + 2)(w - 4)$ | (n) $(y + 1)(y - 6)$ | (o) $(x - 2)(x + 4)$ |
| (p) $(y - 3)(y - 8)$ | (q) $(z - 7)(z - 2)$ | (r) $(b - 6)(b - 4)$ |
| (s) $(x - 2)(x - 3)$ | (t) $(y - 3)(y - 4)$ | (u) $(n - 7)(n - 10)$ |
| (v) $(n - 6)(n - 7)$ | (w) $(y - 3)(y + 3)$ | (x) $(x + 4)(x - 4)$ |
| (y) $(m - 1)(m + 1)$ | (z) $(p + 6)(p - 6)$ | |

7. Expand and simplify the following expressions.

- | | |
|------------------------|------------------------|
| (a) $(2x + 1)(3x + 1)$ | (b) $(3m + 2)(2m + 3)$ |
| (c) $(2p + 5)(3p + 2)$ | (d) $(4m + 3)(2m + 5)$ |
| (e) $(5c + 2)(c + 3)$ | (f) $(4y + 7)(y + 2)$ |
| (g) $(3m + 5)(2m - 1)$ | (h) $(4n + 3)(n - 8)$ |
| (i) $(2c + 5)(3c - 2)$ | (j) $(5d - 6)(2d + 3)$ |
| (k) $(3m - 2)(5 + 2m)$ | (l) $(2x - 3)(5 + 3x)$ |
| (m) $(2y - 5)(3y - 2)$ | (n) $(2p - 3)(5p - 1)$ |
| (o) $(4y - 1)(2y - 5)$ | (p) $(6y - 7)(3y - 4)$ |
| (q) $(2c - 9)(3c - 5)$ | (r) $(4a - 7)(2a - 7)$ |

8. Expand and simplify the following expressions.

Example

$$\begin{aligned}
 & (2x + 3y)(4x - 5y) \\
 &= 2x \times 4x + 2x \times -5y + 3y \times 4x + 3y \times -5y \\
 &= 8x^2 - 10xy + 12xy - 15y^2 \\
 &= \mathbf{8x^2 + 2xy - 15y^2}
 \end{aligned}$$

- | | |
|------------------------------|----------------------------|
| (a) $(2x + 3y)(3x + y)$ | (b) $(3a + 2b)(2a + b)$ |
| (c) $(x + y)(2x - y)$ | (d) $(a + b)(a - b)$ |
| (e) $(3m + 2n)(3m - 5n)$ | (f) $(2p - 3q)(4p - 5q)$ |
| (g) $(3w + 2z)(3w - 2z)$ | (h) $(5n + 2p)(3n - 5p)$ |
| (i) $(4x - 5y)(3x - 2y)$ | (j) $(3x + 7y)(2x - 6y)$ |
| (k) $(2v + 5w)(2v - 5w)$ | (l) $(3m - 2n)(3m - 2n)$ |
| (m) $(5p - 2q)(5p - 2q)$ | (n) $(10a + 7b)(10a - 7b)$ |
| (o) $(3xy - 2ab)(3xy + 2ab)$ | (p) $(5x + 6y)(5x + 6y)$ |

9. Expand and simplify the following expressions.

Examples

$$\begin{aligned} 1. \quad & (x + 3)^2 \\ &= (x + 3)(x + 3) \\ &= x^2 + 3x + 3x + 9 \\ &= x^2 + 6x + 9 \end{aligned}$$

$$\begin{aligned} 2. \quad & (3a - 4b)^2 \\ &= (3a - 4b)(3a - 4b) \\ &= 9a^2 - 12ab - 12ab + 16b^2 \\ &= 9a^2 - 24ab + 16b^2 \end{aligned}$$

(a) $(x + 5)^2$

(b) $(y + 7)^2$

(c) $(a - 2)^2$

(d) $(m - 4)^2$

(e) $(2n + 3)^2$

(f) $(3a + 5)^2$

(g) $(5t - 2)^2$

(h) $(4d - 7)^2$

(i) $(8n - 3)^2$

(j) $(a + b)^2$

(k) $(n - m)^2$

(l) $(x + y)^2$

(m) $(2x + 3y)^2$

(n) $(4m - 3n)^2$

(o) $(2p + 5q)^2$

(p) $(7a - 3b)^2$

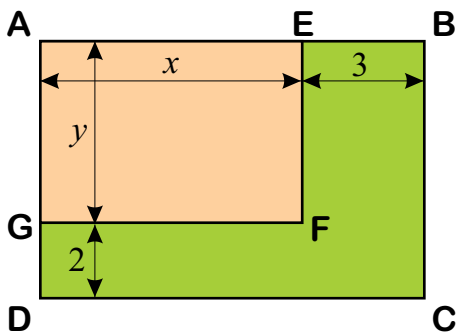
(q) $(8x + 3y)^2$

(r) $(10p - 3q)^2$

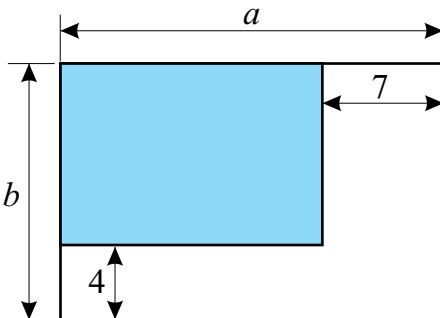
10. (a) In terms of x and y , write an expression for the **area** AEFG.

(b) In terms of x and y , write an expression, in its simplest form, for the **area** ABCD.

(c) In terms of x and y , write an expression, in its simplest form, for the difference between these two areas.



11. In terms of a and b , write an expression, in its simplest form, for the shaded region shown here.



Factorisation

EXERCISE 7C

1. List *all* the factors of the following terms.

Examples

1. 12

Factors: 1, 2, 3, 4, 6, 12

2. $15x$

Factors: 1, 3, 5, 15, x

3. $20ab^2$

Factors: 1, 2, 4, 5, 10, 20, a , b , b^2

4. $-6xy$

Factors: -1, 1, 2, 3, 6, x , y

(a) 10

(b) $6x$

(c) $18a$

(d) $36x^2$

(e) $24pq^2$

(f) $-18m$

(g) $30x^2yz$

(h) $-28c^2$

(i) $-27pq$

(j) $-48m^2n^2$

2. Find the highest common factor (HCF) in the following sets of terms.

Examples

1. 8, 12

The factors of 8 are: 1, 2, 4, 8

The factors of 12 are: 1, 2, 3, 4, 6, 12

The highest common factor = 4

2. $-9a$, $-12ab$

The factors of $-9a$ are: -1, 1, 3, 9, a

The factors of $-12ab$ are: -1, 1, 2, 3, 4, 6, 12, a , b

The highest common factor = $-3a$

3. $8x$, $10y$

HCF = 2

4. $3x^2y$, xy^2

HCF = xy

5. $12x^2$, $-36x$

HCF = $12x$

6. $-12ab$, $-18b$

HCF = $-6b$

(a) 16, 20

(b) 18, 27

(c) 20, 30

(d) x^2 , $3x$

(e) $6a$, $4b$

(f) $12m$, $-16n$

(g) -28, -16

(h) $6x$, $-8x$

(i) xy , $3x$

(j) ab , $4b$

(k) $12m$, $16mn$

(l) $-15x$, $-25x^2$

(m) $10x^2$, $-16x$

(n) $12a^2$, $20a$

(o) x^2y , $-2xy$

(p) $4a^2$, $8a$

(q) $-4x^2$, $-6x$

(r) $24m^2$, $30m$

(s) $-pq$, $-q^2$

(t) $-6m^2$, $-9m$

(u) $-32g^2$, $16g$

(v) $54ab^2$, $36a^2b^2$

(w) $-8a^2b$, $-24a^2b$

3. Factorise the following expressions by finding the highest common factor (HCF).

Factorising is the opposite to expanding

Examples 1. $6a + 8$ (HCF = 2)

$$\begin{aligned}
 &= 2 \times 3a + 2 \times 4 \\
 &= 2(3a + 4)
 \end{aligned}$$

2. $20x - 4$ (HCF = 4)

$$\begin{aligned}
 &= 4 \times 5x - 4 \times 1 \\
 &= 4(5x - 1)
 \end{aligned}$$

(a) $8p + 10$

(b) $6c - 10$

(c) $12x + 16$

(d) $4a - 14$

(e) $18 + 12m$

(f) $6p + 12$

(g) $5m - 15$

(h) $9q + 27$

(i) $15t - 25$

(j) $24 + 36n$

(k) $7a - 21$

(l) $9v + 45$

(m) $6 - 24y$

(n) $25y - 5$

(o) $18a + 6$

(p) $35x - 25$

4. Factorise the following expressions by finding the highest common factor (HCF).

Examples 1. $10a + 15b$ (HCF = 5)

$$\begin{aligned}
 &= 5 \times 2a + 5 \times 3b \\
 &= 5(2a + 3b)
 \end{aligned}$$

2. $12xy - 18x$ (HCF = $6x$)

$$\begin{aligned}
 &= 6x \times 2y - 6x \times 3 \\
 &= 6x(2y - 3)
 \end{aligned}$$

(a) $4p + 12q$

(b) $12a - 28b$

(c) $6m + 9n$

(d) $10ab + 14b$

(e) $8pq - 12p$

(f) $12xy + 16y$

(g) $3cd + 6c$

(h) $16mn - 20m$

(i) $2ab + 24a$

(j) $4a + 28ab$

(k) $6x - 18xy$

(l) $3ab - 15b$

(m) $6abc + 8ab$

(n) $5xyz + 10yz$

(o) $16mnp + 24mp$

5. Factorise the following expressions by finding the highest common factor (HCF).

Examples 1. $-6a - 8$ (HCF = -2)

$$\begin{aligned}
 &= -2 \times 3a - 2 \times 4 \\
 &= -2(3a + 4)
 \end{aligned}$$

2. $-10m - 20n$ (HCF = -10)

$$\begin{aligned}
 &= -10 \times m - 10 \times 2n \\
 &= -10(m + 2n)
 \end{aligned}$$

(a) $-9p - 12q$

(b) $-4m - 6n$

(c) $-3a - 12b$

(d) $-4a - 8$

(e) $-6x - 10y$

(f) $-10b - 6c$

(g) $-20p - 24q$

(h) $-8x - 18$

(i) $-24 - 12m$

(j) $-6x - 3$

(k) $-20p - 5$

(l) $-12x - 4$

(m) $-36p - 24q$

(n) $-40s - 8t$

(o) $-36a - 18b$

(p) $-32x - 24y$

6. Factorise the following expressions by finding the highest common factor (HCF).

Examples 1. $3a + 4a^2$ (HCF = a)
 $= a \times 3 + a \times 4a$
 $= a(3 + 4a)$

2. $-6m^2n - 18m$ (HCF = $-6m$)
 $= -6m \times mn + -6m \times 3$
 $= -6m(mn + 3)$

(a) $6n^2 + 8n$

(b) $2a^2 - 4a$

(c) $6p^2 - 20p$

(d) $10p^2 + 14pq$

(e) $6r^2 - 2r$

(f) $8m^2 + 4m$

(g) $12mn + 16m^2$

(h) $-10x^2 - 15x$

(i) $-6b - 18b^2$

(j) $-2x^2 - 16x$

(k) $-6m^2 - 10mn$

(l) $-14x^2y - 7xy$

Factorising by Grouping

EXERCISE 7D

1. Factorise the following expressions.

Example $2(x + 3) + y(x + 3)$ The common factor is $(x + 3)$
 $= (x + 3)(2 + y)$

(a) $2(a + 4) + b(a + 4)$

(b) $4(b - 2) + c(b - 2)$

(c) $5(d + 3) - c(d + 3)$

(d) $3(x - y) + z(x - y)$

(e) $7(m + 4) - n(m + 4)$

(f) $6(p - 1) + q(p - 1)$

(g) $9(a - 5) + b(a - 5)$

(h) $m(n + 2) + 3(n + 2)$

2. Factorise the following expressions.

Example $\underbrace{3x + 6}_{\text{HCF} = 3} + \underbrace{xy + 2y}_{\text{HCF} = y}$
 $= 3(x + 2) + y(x + 2)$ The common factor is $(x + 2)$
 $= (x + 2)(3 + y)$

(a) $2a + 6 + ab + 3b$

(b) $3m - 6 + mn - 2n$

(c) $5p + 10 + pq + 2q$

(d) $4x + 12 + xy + 3y$

(e) $3n - 15 + mn - 5m$

(f) $6p + 6 + pq + q$

(g) $6a + 9 + 2ab + 3b$

(h) $8x - 4 + 2xy - y$

(i) $15x + 10 + 3xy + 2y$

(j) $2pq + p + 8q + 4$

(k) $3ab - 5a + 18b - 30$

(l) $6uv - 8u + 9v - 12$

3. Factorise the following expressions by grouping the terms so the common factors can be found.

Example

$$4x + 3y + 12 + xy$$

These two pairs of terms
have common factors.
Rearrange the expression
so these two pairs are together.

$$\begin{aligned} &= 4x + 12 + 3y + xy \\ &= 4(x + 3) + y(3 + x) \quad \leftarrow (x + 3) = (3 + x) \\ &= (x + 3)(4 + y) \end{aligned}$$

(a) $3y + 5z + yz + 15$

(c) $5m - 3n - 15 + nm$

(e) $3n - m - 3 + mn$

(g) $ab - 6 + 3b - 2a$

(i) $pq + 10 + 5q + 2p$

(k) $x^2 + 3y + 3x + xy$

(m) $m^2 + n + mn + m$

(o) $pq - 5r - 5p + rq$

(b) $2b + 7c + 14 + cb$

(d) $4c + 5d + 20 + cd$

(f) $4y - 3x - 12 + xy$

(h) $mn + 20 + 5n + 4m$

(j) $ab + 21 + 3a + 7b$

(l) $ab - 2c - cb + 2a$

(n) $nm - 3p + pm - 3n$

(p) $cb + 7d + db + 7c$

4. Factorise the following expressions.

Example

$$\underbrace{3x + 12}_{\text{HCF} = 3} - \underbrace{xy - 4y}_{\text{HCF} = -y}$$

$$\begin{aligned} &= 3(x + 4) - y(x + 4) \quad \text{The common factor is } (x + 4) \\ &= (x + 4)(3 - y) \end{aligned}$$

(a) $4m + 8 - nm - 2n$

(c) $2x + 8 - xy - 4y$

(e) $ab + 5a - 4b - 20$

(g) $cd + 4c - 6d - 24$

(i) $x^2 + 2x - xy - 2y$

(k) $n^2 + 5n - pn - 5p$

(b) $3b + 15 - cb - 5c$

(d) $mn + 3m - 3n - 9$

(f) $gh + 3g - 7h - 21$

(h) $4p + 20 - qp - 5q$

(j) $a^2 + ab - 3a - 3b$

(l) $6w + w^2 - 6y - wy$

5. Factorise the following expressions.

Examples 1. $4x - 8 - xy + 2y$
 $= 4(x - 2) - y(x - 2)$
 $= (x - 2)(4 - y)$

2. $5m + 3n - 15 - mn$
 $= 5m - 15 + 3n - mn$
 $= 5(m - 3) - n(-3 + m)$
 $= 5(m - 3) - n(m - 3)$
 $= (m - 3)(5 - n)$

(a) $3a - 9 - ab + 3b$

(b) $2m - 10 - nm + 5n$

(c) $4p - 4 - pq + q$

(d) $3x - 12 - xy + 4y$

(e) $2y - 4 - yz + 2z$

(f) $5b - 15 - cb + 3c$

(g) $6m - 30 - mn + 5n$

(h) $4a - 12 - ab + 3b$

(i) $3a + 5b - 15 - ab$

(j) $6n + 3p - 18 - np$

(k) $4x + 7y - 28 - xy$

(l) $pq + 5r - 5p - rq$

(m) $x^2 + 3y - 3x - xy$

(n) $a^2 - ab - 4a + 4b$

6. Factorise the following expressions.

(a) $mn + 3m + 3n + 9$

(b) $ab - 2a - 7b + 14$

(c) $3a + 18 - 6c - ac$

(d) $pq - 56 + 7p - 8q$

(e) $ab + dc + ac + db$

(f) $nm - 10m + 50 - 5n$

(g) $15 - 5a - 3b + ab$

(h) $3ab + 6ac - 7b - 14c$

(i) $x^2 + 5x - xy - 5y$

(j) $a^2 + 3b - ab - 3a$

(k) $pq + 10 - 5q - 2p$

(l) $wz - yz + 6y - 6w$

(m) $b^2 - cb - cd + db$

(n) $cd - 5c - 6d + 30$

(o) $2xy + 3x^2 - 2yz - 3xz$

(p) $6a^2 - 15a - 10b + 4ab$

(q) $4pq - 8p^2 + 5q - 10p$

(r) $amp + a^2b + bmp + ab^2$

Factorising the Difference of Perfect Squares (DOPS)

EXERCISE 7E

The factors of algebraic expressions of the form $a^2 - b^2$ are $(a - b)$ and $(a + b)$.

$$a^2 - b^2 = (a - b)(a + b)$$

Examples

$$\begin{aligned} 1. \quad x^2 - y^2 \\ = (x - y)(x + y) \end{aligned}$$

$$\begin{aligned} 2. \quad 16 - a^2 \\ = 4^2 - a^2 \\ = (4 - a)(4 + a) \end{aligned}$$

1. Factorise the following expressions.

(a) $m^2 - n^2$

(b) $A^2 - B^2$

(c) $v^2 - w^2$

(d) $y^2 - 9$

(e) $25 - c^2$

(f) $x^2 - 49$

(g) $p^2 - 100$

(h) $g^2 - 16$

(i) $b^2 - 1$

2. Factorise the following expressions.

Example

$$\begin{aligned} 4x^2 - 25y^2 \\ = (2x)^2 - (5y)^2 \\ = (2x - 5y)(2x + 5y) \end{aligned}$$

(a) $4a^2 - 9b^2$

(b) $16p^2 - 49q^2$

(c) $100x^2 - 81y^2$

(d) $9n^2 - m^2$

(e) $64g^2 - 121h^2$

(f) $49a^2 - 64b^2$

(g) $25x^2 - y^2$

(h) $36a^2 - b^2$

(i) $169n^2 - 225m^2$

3. Factorise the following expressions.

Example

$$\begin{aligned} 8x^2 - 32y^2 \\ = 8(x^2 - 4y^2) \\ = 8[x^2 - (2y)^2] \\ = 8(x - 2y)(x + 2y) \end{aligned}$$

(a) $27n^2 - 3m^2$

(b) $32a^2 - 2b^2$

(c) $24x^2 - 54y^2$

(d) $8a^2 - 18b^2$

(e) $50g^2 - 2h^2$

(f) $32p^2 - 98q^2$

(g) $20x^2 - 5y^2$

(h) $200a^2 - 8b^2$

(i) $45n^2 - 20m^2$

Factorising Quadratic Trinomials

A **quadratic** is an algebraic expression that has the power of 2 as its highest power (x^2 , a^2 , etc).

A **trinomial** is an expression with three terms.

A **quadratic trinomial** is an algebraic expression that has three terms and has the power of 2 as its highest power.

Examples $x^2 + 3x - 5$ $4a^2 - 3a + 10$

The factors of a quadratic trinomial are the two expressions that multiply to equal that trinomial.

Example The factors of $x^2 + 7x + 10$ are $(x + 2)(x + 5)$
This can be shown by multiplying the brackets:

$$\begin{aligned}(x + 2)(x + 5) \\ &= x^2 + 5x + 2x + 10 \\ &= x^2 + 7x + 10\end{aligned}$$

To find the factors of this type of quadratic trinomial follow the steps below.

$$x^2 + 7x + 10$$

Step 1: Find two numbers that multiply to give the last number (**10**) and add to give the coefficient of x (**7**). In this example these numbers are **2 and 5**.

Step 2: Use these two numbers to form the factors.

$$(x + 2)(x + 5)$$

Examples

1. $x^2 + 8x + 15$

Step 1: Two numbers that multiply to give 15 and add to give 8:
3 and 5.

Step 2: The two factors are:

$$(x + 3)(x + 5)$$

2. $a^2 + 9a + 20$

Step 1: Two numbers that multiply to give 20 and add to give 9:
4 and 5.

Step 2: The two factors are:

$$(a + 4)(a + 5)$$

Examples continued

3. $x^2 + 8x + 12$

Step 1: Two numbers that multiply to give 12 and add to give 8:
2 and 6.

Step 2: The two factors are:

$$(x + 2)(x + 6)$$

4. $a^2 + 7a + 6$

Step 1: Two numbers that multiply to give 6 and add to give 7:
1 and 6.

Step 2: The two factors are:

$$(a + 1)(a + 6)$$

EXERCISE 7F

1. Factorise the following quadratic trinomials.

(a) $x^2 + 5x + 6$

(b) $x^2 + 7x + 12$

(c) $n^2 + 9n + 14$

(d) $a^2 + 10a + 16$

(e) $p^2 + 11p + 18$

(f) $a^2 + 13a + 22$

(g) $y^2 + 8y + 16$

(h) $x^2 + 11x + 24$

(i) $n^2 + 10n + 24$

(j) $x^2 + 14x + 24$

(k) $a^2 + 11a + 10$

(l) $y^2 + 9y + 8$

(m) $x^2 + 13x + 40$

(n) $a^2 + 14a + 48$

(o) $b^2 + 27b + 50$

(p) $x^2 + 16x + 63$

(q) $v^2 + 23v + 42$

(r) $x^2 + 14x + 45$

(s) $a^2 + 18a + 17$

(t) $m^2 + 33m + 90$

Example

Factorise the following quadratic trinomial.

$$x^2 - 8x + 15$$

The same technique as for question 1 is used to factorise this quadratic trinomial.

Step 1: Find two numbers that multiply to give the last number (15) and add to give the coefficient of x (-8).

These two numbers are **-3 and -5.**

Step 2: Use these numbers to form the factors:

$$(x - 3)(x - 5)$$

Examples continued

1. $x^2 - 12x + 20$

Step 1: Two numbers that multiply to give 20 and add to give -12:
-2 and -10.

Step 2: The two factors are:

$$(x - 2)(x - 10)$$

2. $a^2 - 12a + 27$

Step 1: Two numbers that multiply to give 27 and add to give -12:
-3 and -9.

Step 2: The two factors are:

$$(a - 3)(a - 9)$$

2. Factorise the following quadratic trinomials.

(a) $x^2 - 7x + 12$

(b) $m^2 - 8m + 16$

(c) $a^2 - 11a + 24$

(d) $c^2 - 11c + 30$

(e) $z^2 - 12z + 32$

(f) $x^2 - 17x + 30$

(g) $n^2 - 15n + 36$

(h) $y^2 - 16y + 28$

Example

Factorise the following quadratic trinomial.

$$x^2 + 2x - 15$$

The same technique as for question 1 is used to factorise this quadratic trinomial.

Step 1: Find two numbers that multiply to give the last number (**-15**) and add to give the coefficient of x (**2**).

These two numbers are **-3** and **5**.

Step 2: Use these numbers to form the factors:

$$(x - 3)(x + 5)$$

Examples

1. $x^2 + 3x - 10$

Step 1: Two numbers that multiply to give -10 and add to give 3:
-2 and 5.

Step 2: The two factors are:

$$(x - 2)(x + 5)$$

Examples continued

2. $x^2 + 4x - 21$

Step 1: Two numbers that multiply to give -21 and add to give 4:
-3 and 7.

Step 2: The two factors are:

$$(x - 3)(x + 7)$$

3. $a^2 - 4a - 12$

Step 1: Two numbers that multiply to give -12 and add to give -4:
2 and -6.

Step 2: The two factors are:

$$(a + 2)(a - 6)$$

3. Factorise the following quadratic trinomials.

(a) $x^2 + 5x - 24$

(b) $a^2 + 7a - 30$

(c) $n^2 - 7n - 30$

(d) $c^2 - 8c - 20$

(e) $m^2 + 6m - 16$

(f) $y^2 + 4y - 12$

(g) $x^2 + 7x - 18$

(h) $a^2 - 5a - 24$

(i) $y^2 + 5y - 36$

(j) $x^2 - 3x - 40$

(k) $n^2 + n - 42$

(l) $a^2 + a - 12$

(m) $x^2 - x - 20$

(n) $g^2 - g - 30$

4. Factorise the following quadratic trinomials.

(a) $x^2 + 15x + 54$

(b) $c^2 - 13c + 40$

(c) $z^2 - 7z - 18$

(d) $a^2 - 12a + 35$

(e) $x^2 + 16x + 60$

(f) $n^2 - 15n + 56$

(g) $y^2 - 5y - 66$

(h) $b^2 - b - 72$

(i) $x^2 - 25x + 100$

(j) $m^2 - 20m + 99$

(k) $a^2 + 2a - 63$

(l) $x^2 - 23x + 132$

(m) $y^2 + 18y - 40$

(n) $r^2 + 12r + 32$

(o) $a^2 + 7a + 12$

(p) $m^2 - 3m - 18$

(q) $p^2 - 10p + 24$

(r) $r^2 - 6r - 27$

(s) $y^2 - 15y + 44$

(t) $b^2 + b - 56$

(u) $n^2 - 13n + 36$

(v) $q^2 + 18q + 32$

5. Factorise the following quadratic trinomials.

(a) $a^2 + 8a + 16$

(b) $m^2 + 20m + 100$

(c) $n^2 + 18n + 81$

(d) $c^2 - 10c + 25$

(e) $y^2 + 14y + 49$

(f) $x^2 - 12x + 36$

(g) $y^2 + 16y + 64$

(h) $a^2 - 20a + 100$

(i) $u^2 + 22u + 121$

(j) $p^2 - 24p + 144$

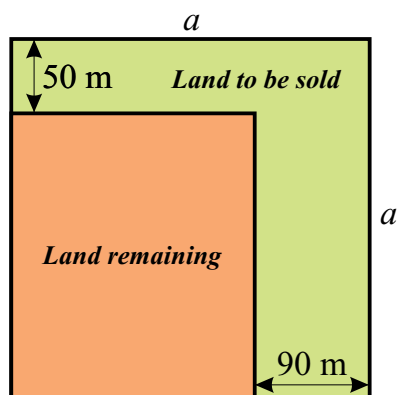
Example

$$\begin{aligned} & x^2 + 6x + 9 \\ &= (x + 3)(x + 3) \\ &= (x + 3)^2 \end{aligned}$$

PROBLEM SOLVING

1. Ruby had \$100 in her bank account. In one year she put x dollars into her account in January, $2x$ in February, $4x$ in March, $8x$ in April and so on for the year (she kept doubling the amount she saved each month).
How much will she have in her bank account at the end of the year?

2. Jeremy owned land in the shape of a square with side length a . He decided to sell part of this land by reducing it by 50 metres in one direction and 90 metres in the other (see diagram).



- (a) Find an expression in terms of a for the area of the original land.
(b) Find an expression in terms of a , in its simplest form, for the area of land Jeremy has remaining after he sells the section of land shown.
(c) Find an expression in terms of a for the area of land he sold.

PUZZLE

Each of the following expressions has $(x + 6)$ as a factor. Match the other factor and accompanying letters in the boxes below to find the answer to this riddle:

Why is the following algebraic expression like a year at school?

$$x^2 + a^2b - ab^2 - y^2$$

S	$x^2 + 8x + 12$	U	$x^2 + 2x - 24$	H	$x^2 + 7x + 6$	I	$x^2 + 6x$
T	$x^2 + 5x - 6$	A	$x^2 + 4x - 12$	E	$x^2 + x - 30$	O	$x^2 + 9x + 18$
F	$x^2 + 3x - 18$	R	$x^2 + 10x + 24$	M	$x^2 + 11x + 30$		

x	$x - 1$	$x + 1$	$x - 2$	$x + 2$	$x - 3$	$x + 3$	$x - 4$	$x + 4$	$x - 1$	$x - 5$	$x + 4$	$x + 5$	$x + 2$

CHAPTER REVIEW

1. Expand the following algebraic expressions.

(a) $5(x - 3)$

(b) $-2(a - 3b)$

(c) $5x(2y - 3z)$

2. Expand and simplify the following algebraic expressions.

(a) $2(m + 3) + 5(m - 2)$

(b) $6(x - 3) - 2(x - 5)$

(c) $(x - 3)(x + 5)$

(d) $(2y + 3)(4y - 7)$

(e) $(3n + 2m)(2n + 5m)$

(f) $(3a + 2b)(3a - 2b)$

(g) $(2x + 3)^2$

(h) $(3p - 4q)^2$

3. Factorise the following expressions.

(a) $10xy - 12y$

(b) $-6a - 9b$

(c) $3(a - 4) + b(a - 4)$

4. Factorise the following expressions.

(a) $15x + 10 + 3xy + 2y$

(b) $6ab - 8a + 9b - 12$

(c) $ab + 10 + 5b + 2a$

(d) $a^2 + 2a - ab - 2b$

5. Factorise the following expressions.

(a) $a^2 - b^2$

(b) $x^2 - 16$

(c) $25m^2 - 49n^2$

6. Factorise the following expressions.

(a) $a^2 + 11a + 18$

(b) $n^2 + 14n + 24$

(c) $p^2 - 11p + 24$

(d) $x^2 - 11x + 28$

(e) $a^2 + 5a - 36$

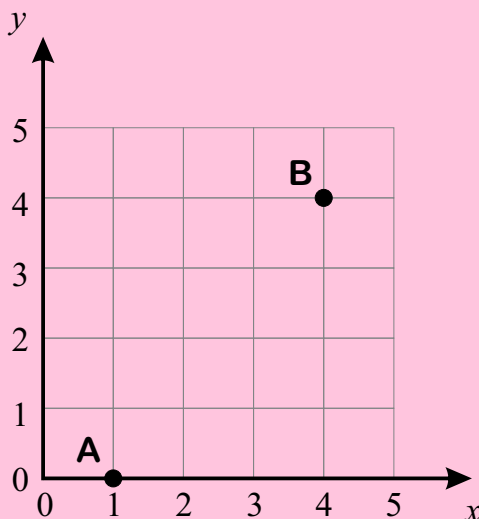
(f) $n^2 - n - 12$

Straight Lines

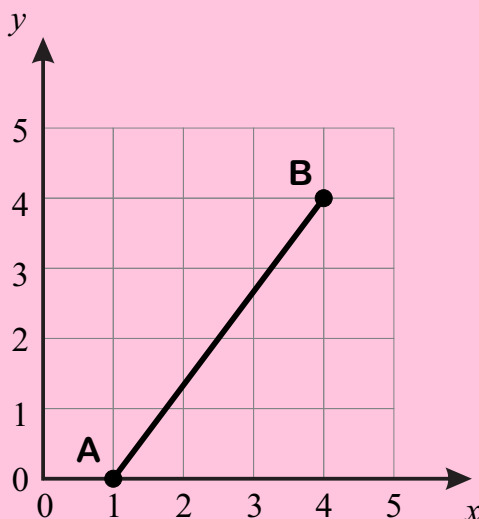
Distance Between Two Points on a Straight Line

Example 1

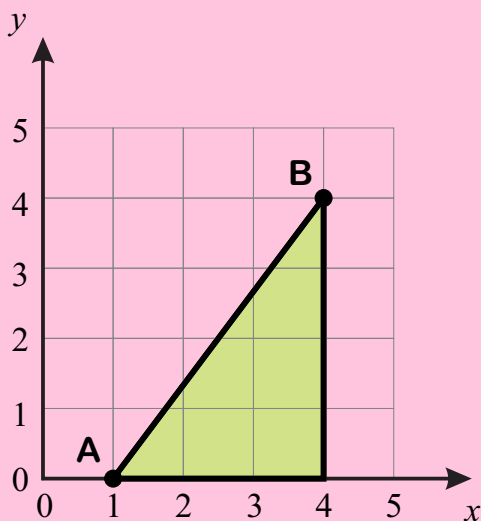
Find the distance between points **A** and **B** shown on the graph below.



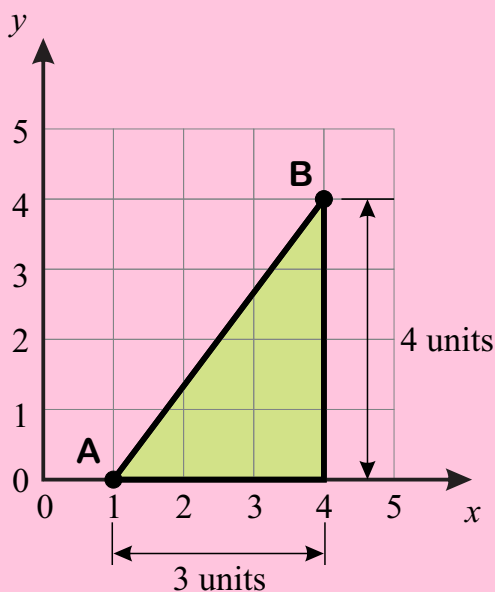
Step 1 Draw a line between **A** and **B**.



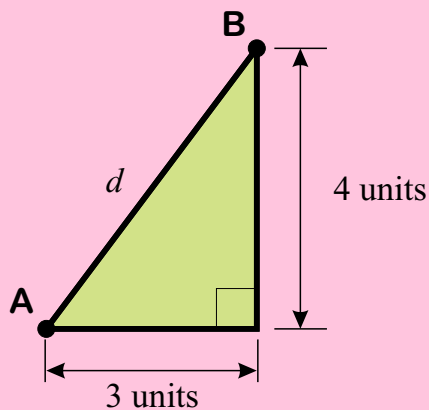
Step 2 Draw a right-angled triangle using this line as the long side of the triangle - this side is called the hypotenuse.



Step 3 Find the side lengths of the two shorter sides.



Step 4 Label the longest side d .

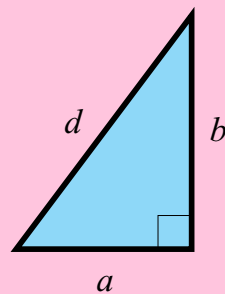


Step 5 Pythagoras' Theorem can be used to find this length.

Pythagoras' Theorem states:

$$d^2 = a^2 + b^2$$

$$d = \sqrt{a^2 + b^2}$$



Substituting in the values for this triangle.

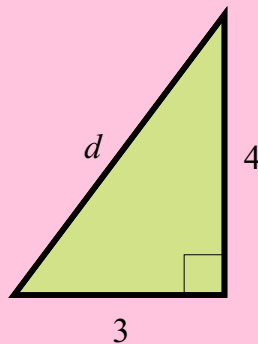
$$d = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

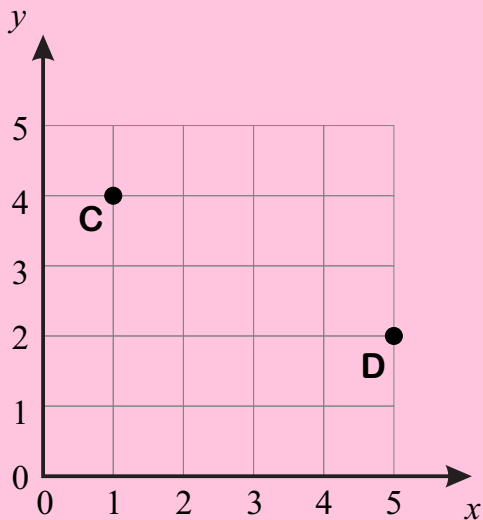
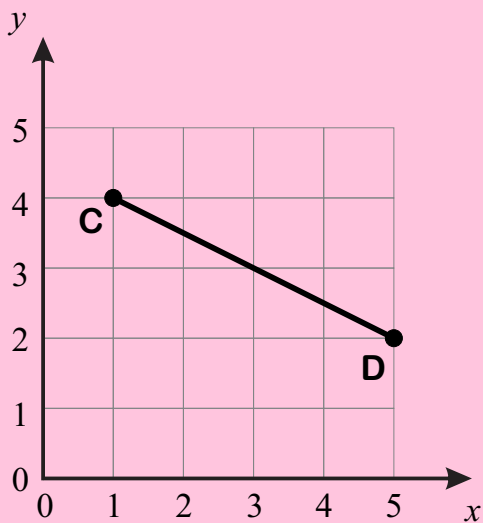
$$= \sqrt{25}$$

$$d = 5$$

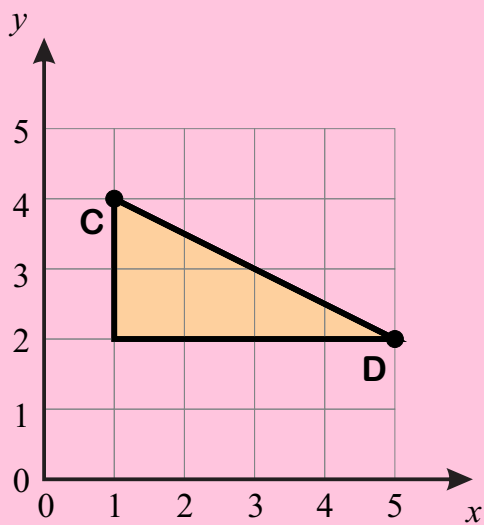


Example 2

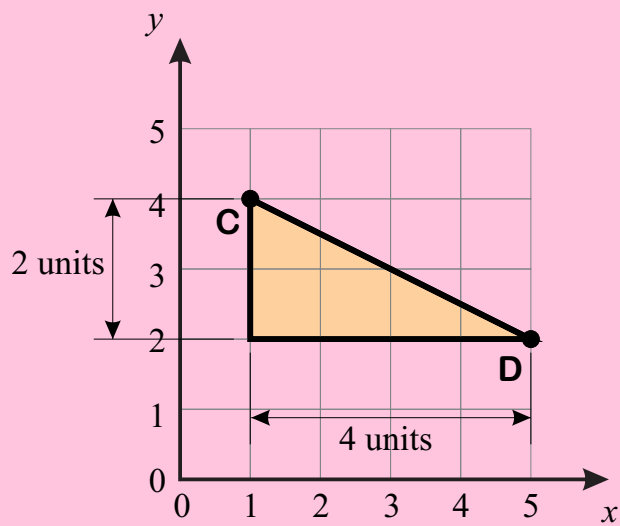
Find the distance between points **C** and **D** shown on the graph below.

**Step 1**

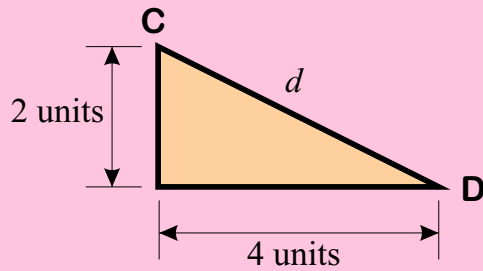
Step 2



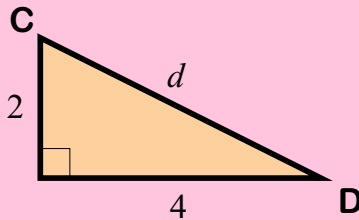
Step 3



Step 4



Step 5



$$d = \sqrt{a^2 + b^2}$$

$$= \sqrt{4^2 + 2^2}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

$$d = 4.4721$$

$$d = 4.5 \text{ (one decimal place)}$$

Example 3

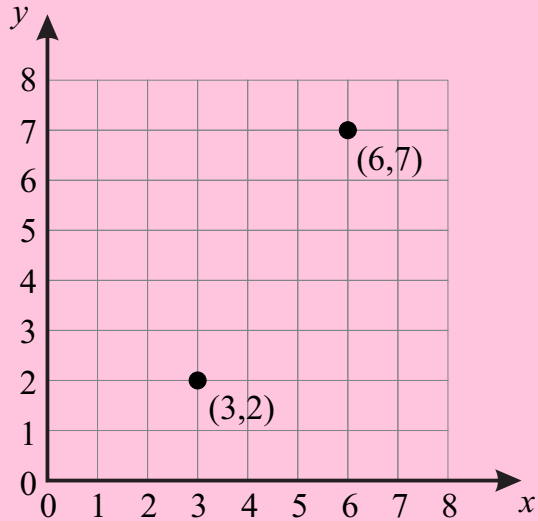
Find the distance between the points with coordinates (3,2) and (6,7).

Step 1 Locate these points on a Cartesian plane. Remember the first number in a set of coordinates is the x -ordinate and the second is the y -ordinate.

Make sure the graph is large enough to include both points.

(3,2)
x y

(6,7)
x y



Step 2 Follow the steps 1-5 from the previous examples.

$$d = \sqrt{a^2 + b^2}$$

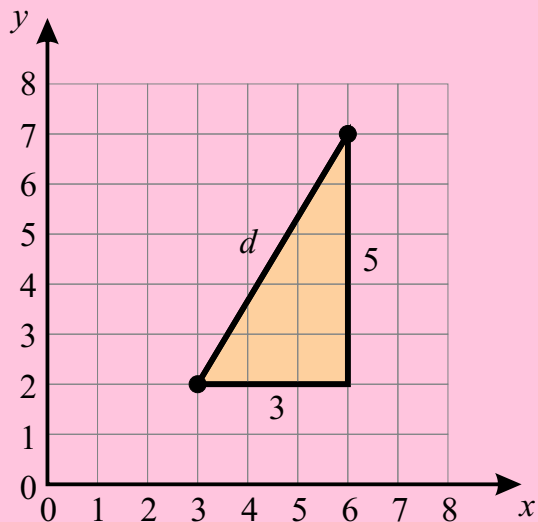
$$= \sqrt{3^2 + 5^2}$$

$$= \sqrt{9 + 25}$$

$$= \sqrt{34}$$

$$d = 5.8309$$

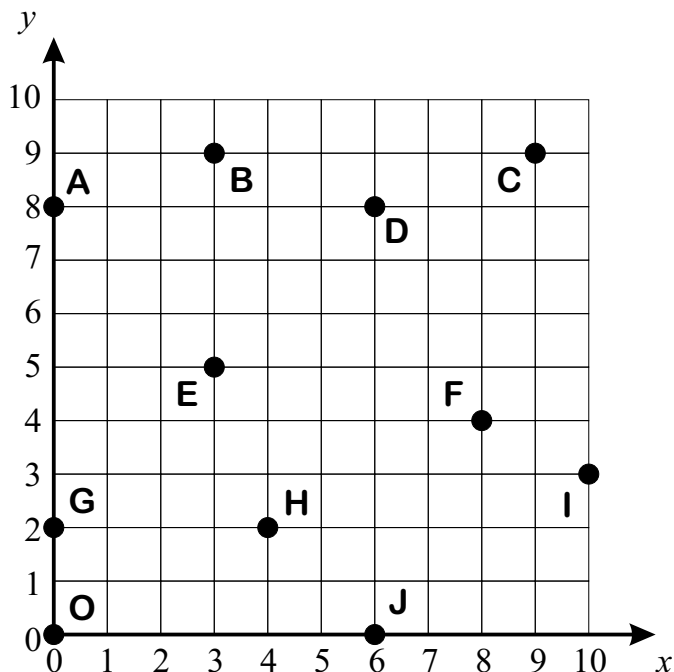
$$d = 5.8 \text{ (one dec. pl.)}$$



EXERCISE 8A

1. Find the distance between the following pairs of points shown on this Cartesian plane. Give answers correct to one decimal place.

- (a) **G** and **E**
- (b) **H** and **I**
- (c) **A** and **J**
- (d) **H** and **J**
- (e) **B** and **F**
- (f) **O** and **B**
- (g) **C** and **E**
- (h) **C** and **H**



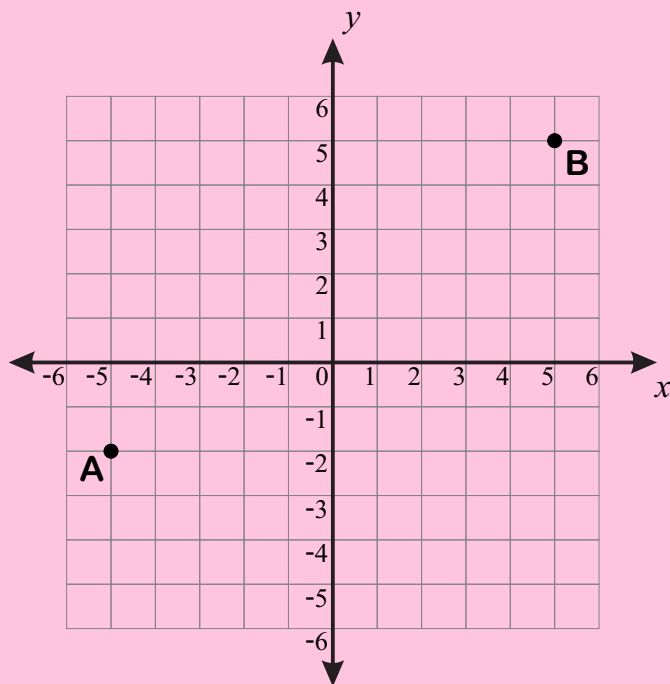
2. (a) Draw a Cartesian plane with x and y axes of 0 to 10.
 (b) Locate the following points on the Cartesian plane.

$$\mathbf{K} = (1, 9), \mathbf{L} = (0, 3), \mathbf{M} = (9, 2), \mathbf{N} = (8, 7)$$

$$\mathbf{P} = (3, 2), \mathbf{Q} = (5, 0), \mathbf{R} = (5, 6), \mathbf{S} = (1, 8)$$

- (c) Find the distance between the following pairs of points.
 Give answers correct to one decimal place.
- (i) **K** and **P**
 - (ii) **L** and **M**
 - (iii) **Q** and **N**
 - (iv) **R** and **M**
 - (v) **M** and **S**

Example Find the distance between the points **A** and **B** shown on the Cartesian plane below.



Follow the steps from the previous examples.

$$d = \sqrt{a^2 + b^2}$$

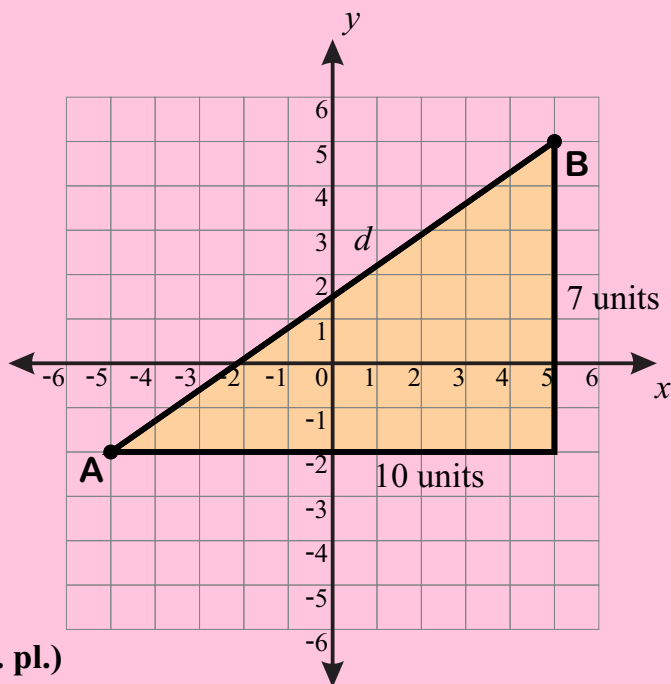
$$= \sqrt{10^2 + 7^2}$$

$$= \sqrt{100 + 49}$$

$$= \sqrt{149}$$

$$d = 12.2065$$

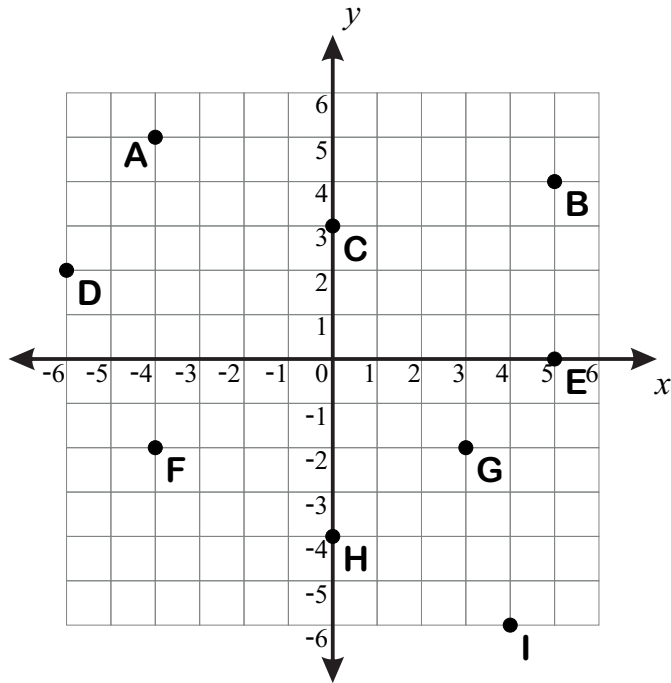
$$d = 12.2 \text{ (one dec. pl.)}$$



EXERCISE 8B

1. Find the distance between the following pairs of points shown on this Cartesian plane.

- (a) **D** and **B**
- (b) **F** and **C**
- (c) **A** and **E**
- (d) **D** and **G**
- (e) **F** and **B**
- (f) **A** and **I**
- (g) **H** and **B**



2. (a) Draw a Cartesian plane with x and y axes of -8 to 8 .
 (b) Locate the following points on the Cartesian plane.

$$\mathbf{K} = (-1, 7), \mathbf{L} = (0, -3), \mathbf{M} = (-5, -2), \mathbf{N} = (7, 6)$$

$$\mathbf{P} = (5, -4), \mathbf{Q} = (4, 0), \mathbf{R} = (-7, 6), \mathbf{S} = (-1, -8)$$

- (c) Find the distance between the following pairs of points.
 Give answers correct to one decimal place.

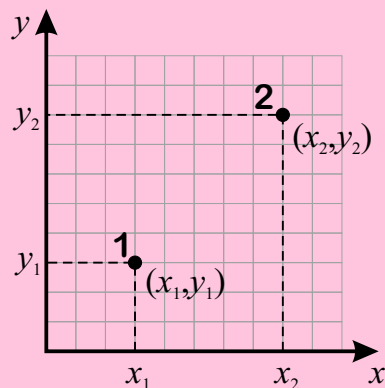
- (i) **K** and **Q**
- (ii) **N** and **M**
- (iii) **Q** and **S**
- (iv) **L** and **P**
- (v) **R** and **P**

Formula to Find the Distance Between Two Points

Consider two points **1** and **2**.

Let the coordinates of point **1** = (x_1, y_1) .

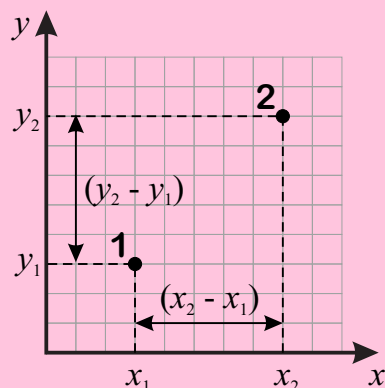
Let the coordinates of point **2** = (x_2, y_2) .



It can be seen that:

the horizontal distance between the two points = $(x_2 - x_1)$.

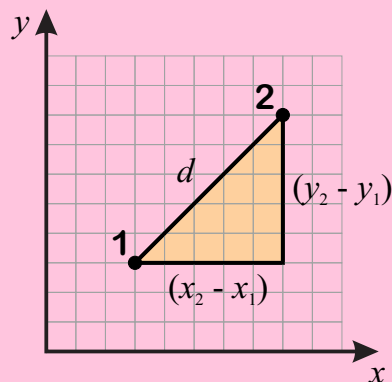
the vertical distance between the two points = $(y_2 - y_1)$.



The triangle can then be drawn and Pythagoras' Theorem used to find the formula to find d .

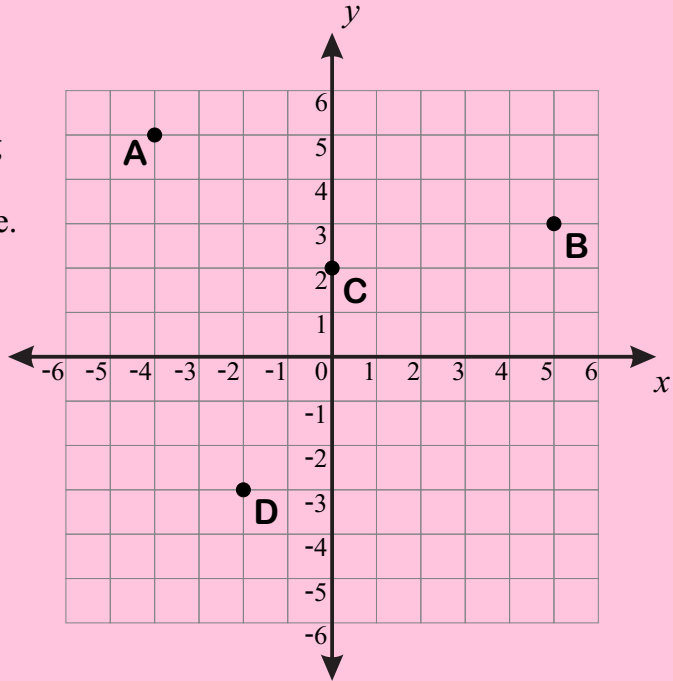
$$d = \sqrt{a^2 + b^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Example 1

Find the distance between the following pairs of points shown on this Cartesian plane. Give answers correct to one decimal place.



- (a) **A** and **B**
- (b) **B** and **C**
- (c) **A** and **D**

Answers

(a) Let point **A** = point **1**. Coordinates = $(x_1, y_1) = (-4, 5)$

Let point **B** = point **2**. Coordinates = $(x_2, y_2) = (5, 3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(5 - -4)^2 + (3 - 5)^2}$$

$$= \sqrt{(5 + 4)^2 + (-2)^2}$$

$$= \sqrt{9^2 + (-2)^2}$$

$$= \sqrt{81 + 4}$$

$$= \sqrt{85}$$

$$= 9.219$$

$$= \mathbf{9.2} \text{ (one decimal place)}$$

Either point can be used as point **1**. The answer will be the same.

(b) Let point **B** = point **1**. Coordinates = $(x_1, y_1) = (5, 3)$

Let point **C** = point **2**. Coordinates = $(x_2, y_2) = (0, 2)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(0 - 5)^2 + (2 - 3)^2} \\
 &= \sqrt{(-5)^2 + (-1)^2} \\
 &= \sqrt{25 + 1} \\
 &= \sqrt{26} \\
 &= 5.099 \\
 &= \mathbf{5.1} \text{ (one decimal place)}
 \end{aligned}$$

(c) Let point **A** = point **1**. Coordinates = $(x_1, y_1) = (-4, 5)$

Let point **D** = point **2**. Coordinates = $(x_2, y_2) = (-2, -3)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2 - -4)^2 + (-3 - 5)^2} \\
 &= \sqrt{(-2 + 4)^2 + (-8)^2} \\
 &= \sqrt{2^2 + (-8)^2} \\
 &= \sqrt{4 + 64} \\
 &= \sqrt{68} \\
 &= 8.246 \\
 &= \mathbf{8.2} \text{ (one decimal place)}
 \end{aligned}$$

Example 2

Find the distance between $(-8, 16)$ and $(-11, 7)$ on a Cartesian plane.

Let point **1** $= (-8, 16) = (x_1, y_1)$

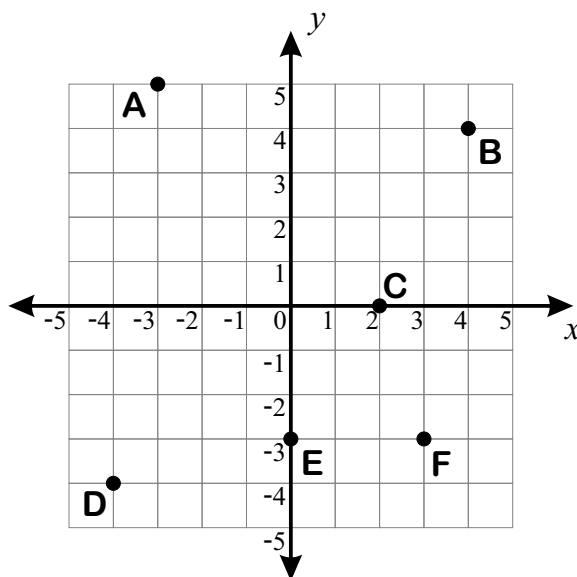
Let point **2** $= (-11, 7) = (x_2, y_2)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-11 - -8)^2 + (7 - 16)^2} \\
 &= \sqrt{(-11 + 8)^2 + (-9)^2} \\
 &= \sqrt{(-3)^2 + (-9)^2} \\
 &= \sqrt{9 + 81} \\
 &= \sqrt{90} \\
 &= 9.486 \\
 &= \mathbf{9.5} \text{ (one decimal place)}
 \end{aligned}$$

EXERCISE 8C

1. Find the distance between the following pairs of points shown on this Cartesian plane. Give answers correct to one decimal place.

- (a) **A** and **C**
- (b) **B** and **D**
- (c) **A** and **F**
- (d) **E** and **C**
- (e) **D** and **C**
- (f) **B** and **F**



2. Find the distance between the following pairs of points.

Give answers correct to one decimal place.

(a) $(6,13)$ and $(3,10)$

(b) $(-4,15)$ and $(7,-14)$

(c) $(-9,0)$ and $(9,-18)$

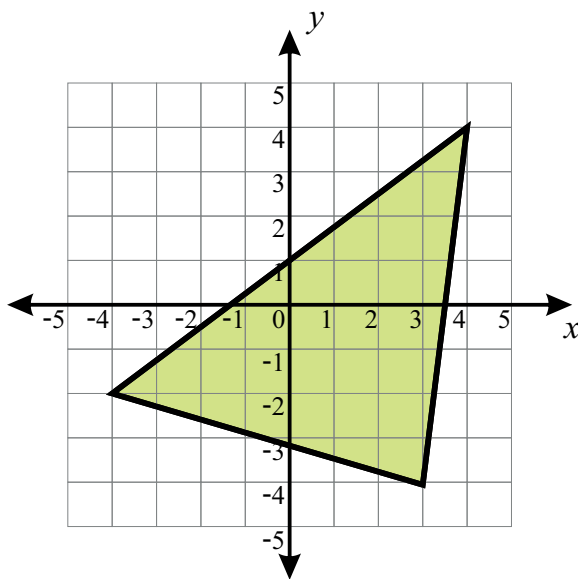
(d) $(-7,-19)$ and $(16,13)$

(e) $(17,-11)$ and $(-23,14)$

(f) $(-29,12)$ and $(-17,-15)$

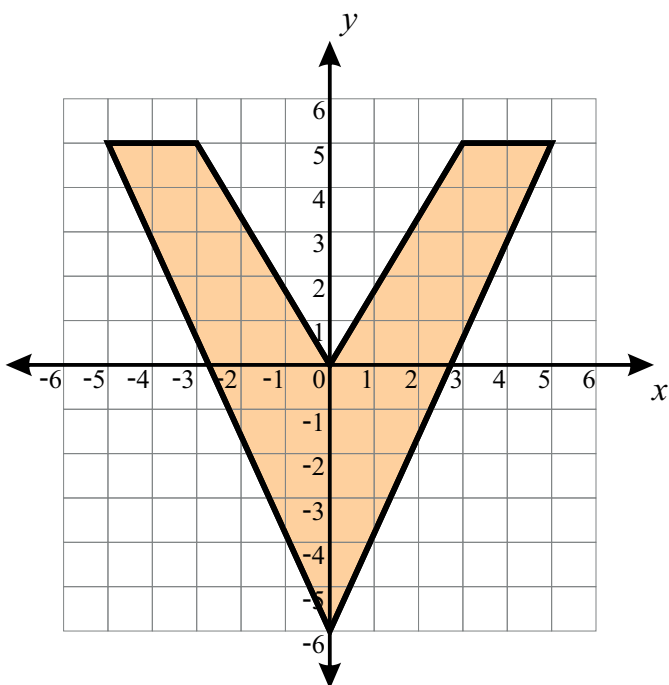
3. Find the perimeter of the triangle shown on this Cartesian plane.

Give answer correct to one decimal place.



4. Find the perimeter of the shape shown on this Cartesian plane.

Give answer correct to one decimal place.

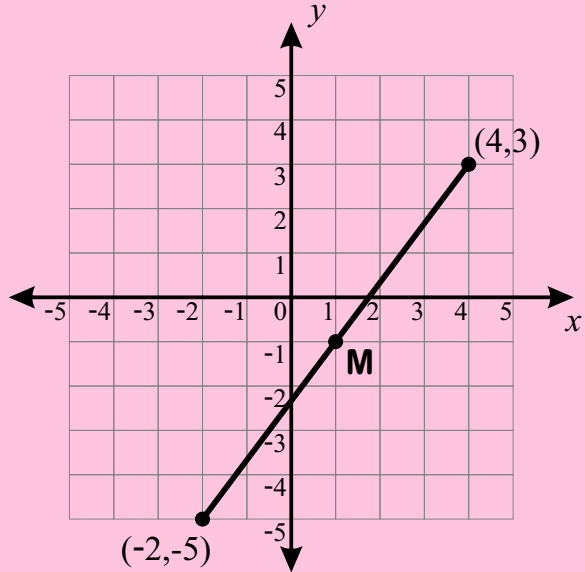


Midpoint of the Line Connecting Two Points (Line Segment)

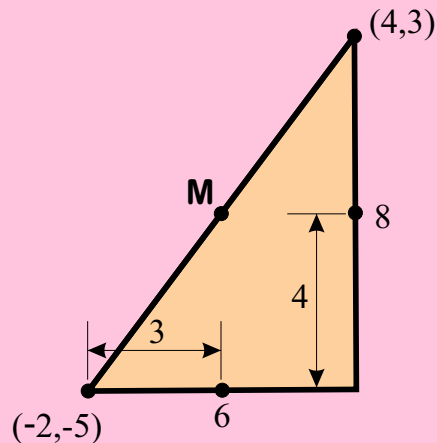
Example 1

Find the coordinates of the midpoint of the line connecting points $(-2, -5)$ and $(4, 3)$.

The coordinates of the midpoint (**M**) of the line connecting two points on a Cartesian plane (line segment) can be found by following the steps below.



Step 1 Draw a line connecting the two points and form a right-angled triangle. Find the side lengths of the triangle. Calculate half of each side length.

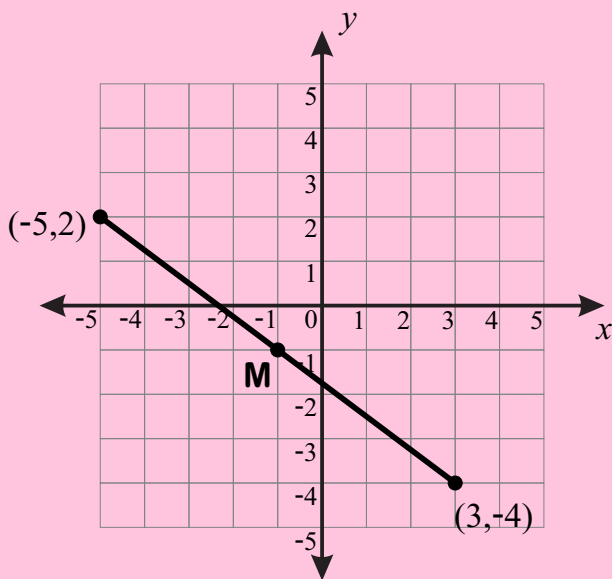


Step 2 Use these half lengths to find the coordinates the midpoint **M**.

$$\begin{aligned}\text{Coordinates of } \mathbf{M} &= (-2 + 3, -5 + 4) \\ &= \mathbf{(1, -1)}\end{aligned}$$

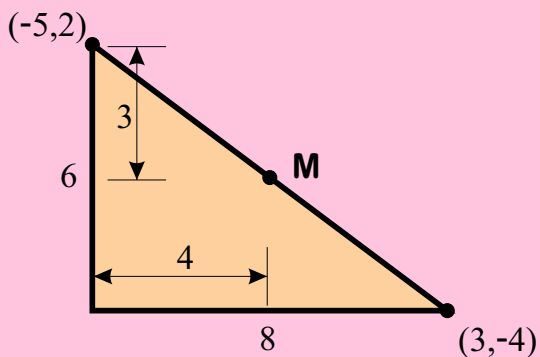
Example 2

Find the coordinates of the midpoint of the line connecting points $(-5, 2)$ and $(3, -4)$.



In this example it can be seen that the coordinates of the midpoint will be:

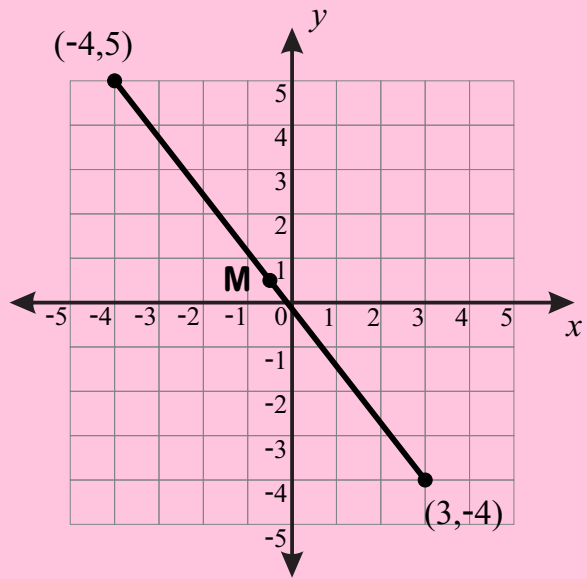
$$(-5 + 4, 2 - 3)$$



Coordinates of **M** = $(-1, -1)$

Example 3

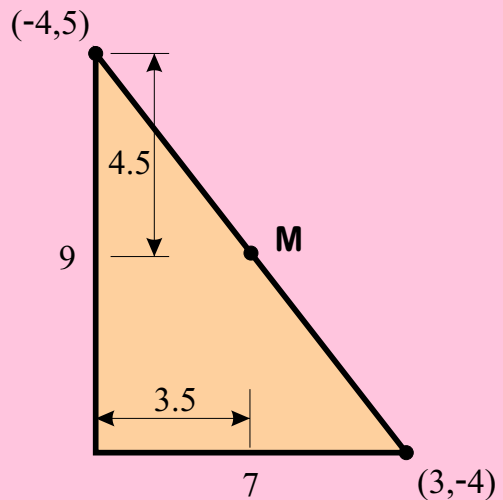
Find the coordinates of the midpoint of the line connecting points $(-4,5)$ and $(3,-4)$.



In this example it can be seen that the coordinates of the midpoint will be:

$$(-4 + 3.5, 5 - 4.5)$$

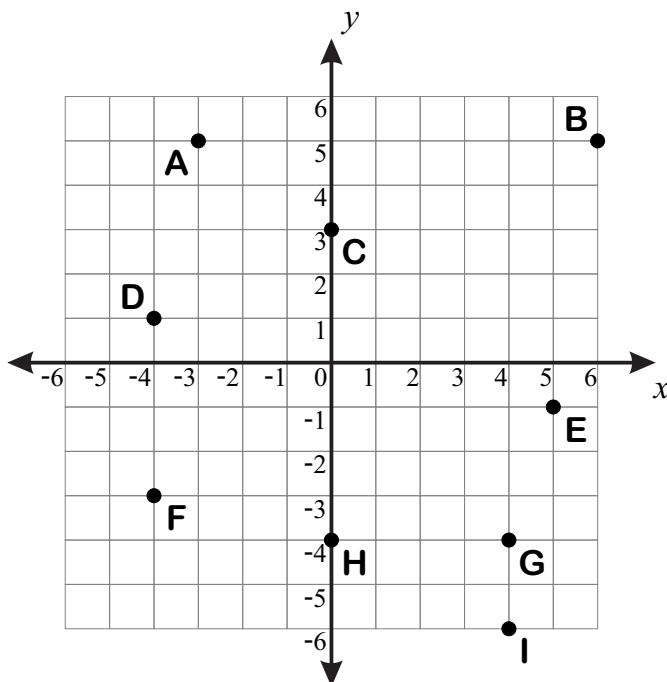
Coordinates of **M** = $(-0.5, 0.5)$



EXERCISE 8D

1. (a) Draw a Cartesian plane with x and y axes of -8 to 8.
 (b) Find the coordinates of the midpoint of the line segment connecting each of the following pairs of points.
- (i) (2,4) and (4,8)
 - (ii) (0,1) and (6,7)
 - (iii) (-3,2) and (7,4)
 - (iv) (0,7) and (6,1)
 - (v) (-5,5) and (3,-3)
 - (vi) (-1,-7) and (7,7)
 - (vii) (2,0) and (7,6)
 - (viii) (0,8) and (7,3)
 - (ix) (-4,1) and (3,7)
 - (x) (-6,-1) and (-1,-7)
 - (xi) (-5,6) and (6,-7)
2. Find the midpoint of the line segment connecting each of the following pairs of points shown on this Cartesian plane.

- (a) **D** and **B**
- (b) **F** and **C**
- (c) **A** and **E**
- (d) **D** and **G**
- (e) **F** and **B**
- (f) **A** and **I**
- (g) **H** and **B**



Formula to Find the Coordinates of the Midpoint of a Line Segment

The coordinates of the midpoint of the line segment connecting the points with coordinates (x_1, y_1) and (x_2, y_2) are:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example

Find the coordinates of the midpoint of the line segment connecting the point $(-5, -9)$ and $(13, 8)$.

$$(x_1, y_1) = (-5, -9) \quad (x_2, y_2) = (13, 8)$$

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{-5 + 13}{2}, \frac{-9 + 8}{2} \right) \\ &= \left(\frac{8}{2}, -\frac{1}{2} \right) \\ &= (4, -0.5) \end{aligned}$$

EXERCISE 8E

Find the coordinates of the midpoint of the line segment that connects the following pairs of points.

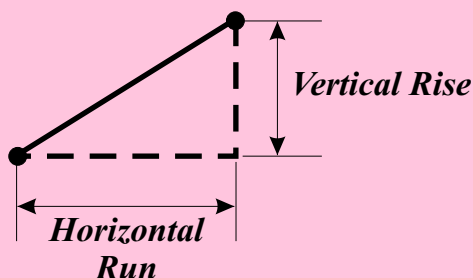
1. $(2, 8)$ and $(10, 12)$
2. $(0, 5)$ and $(14, 6)$
3. $(-3, 7)$ and $(-5, 6)$
4. $(-9, 7)$ and $(9, 15)$
5. $(11, -7)$ and $(-4, 4)$
6. $(8, 19)$ and $(24, 25)$
7. $(-13, -7)$ and $(25, -16)$
8. $(-9, 7)$ and $(-25, 36)$
9. $(18, -9)$ and $(-18, 9)$
10. $(12, -17)$ and $(-24, 44)$
11. $(13.6, 7.9)$ and $(-8.2, 15.8)$
12. $(-15.7, -21.1)$ and $(8.4, 32.8)$

Gradient

Definition

The **gradient** of the line connecting two points is defined as:

$$\frac{\text{the vertical **rise** between the two points}}{\text{the horizontal **run** between the two points}}$$

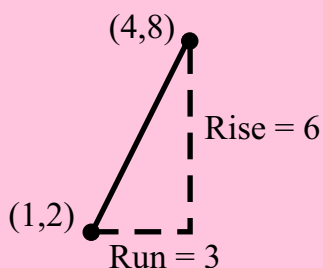
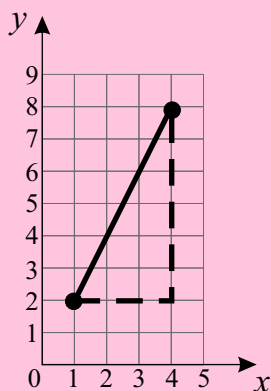


The symbol for gradient is m

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

Example

Find the gradient of the line connecting the points (1,2) and (4,8).



$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

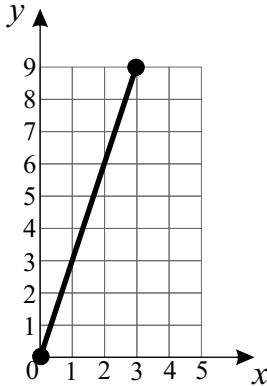
$$m = \frac{6}{3}$$

$$m = 2$$

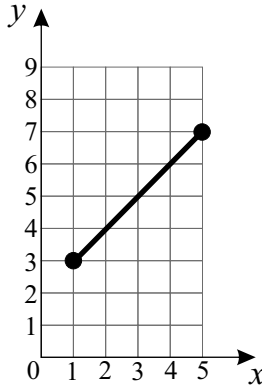
EXERCISE 8F

1. Find the gradient of the lines shown on the graphs below.

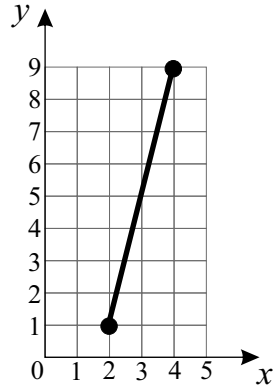
(a)



(b)



(c)



2. Find the gradient of the lines connecting the following pairs of points.

(a) (0,0) and (3,6)

(b) (1,0) and (6,5)

(c) (2,3) and (5,6)

(d) (3,2) and (6,8)

(e) (0,4) and (2,12)

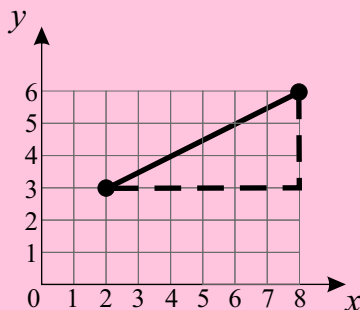
(f) (3,1) and (9,7)

(g) (4,2) and (6,8)

(h) (3,0) and (11,16)

3. Find the gradient of the lines connecting the following pairs of points.

Example (2,3) and (8,6)



Rise = 3

Run = 6

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{3}{6}$$

$$m = \frac{1}{2}$$

(a) (0,0) and (4,2)

(b) (1,2) and (7,5)

(c) (2,3) and (8,6)

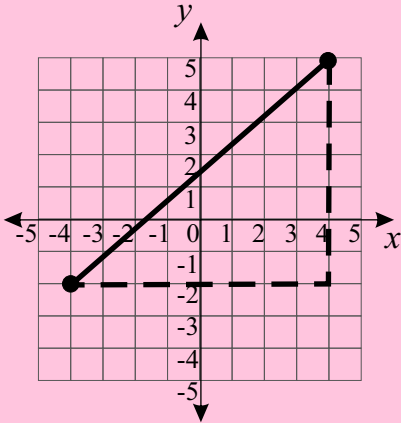
(d) (1,4) and (13,8)

(e) (0,4) and (10,7)

(f) (2,5) and (11,11)

4. Find the gradient of the lines connecting the following pairs of points.

Example $(-4, -2)$ and $(4, 5)$



$$\text{Rise} = 7$$

$$\text{Run} = 8$$

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{7}{8}$$

(a) $(-1, 2)$ and $(4, 7)$

(c) $(3, -2)$ and $(10, 5)$

(e) $(-2, -3)$ and $(5, 1)$

(g) $(-4, -4)$ and $(6, 8)$

(i) $(-3, 0)$ and $(0, 5)$

(b) $(-4, -2)$ and $(1, 8)$

(d) $(-1, 2)$ and $(7, 6)$

(f) $(-5, 3)$ and $(7, 7)$

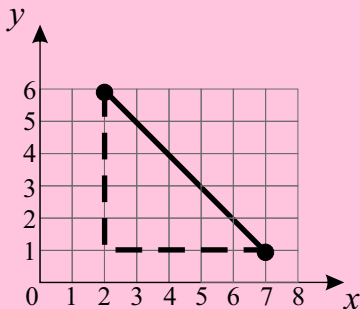
(h) $(-1, -6)$ and $(4, -1)$

(j) $(-8, 5)$ and $(-1, 8)$

Negative Gradient

If a line slopes down when going from left to right it has a **negative** gradient. It has a **negative rise**.

Example Find the gradient of the line joining $(2, 6)$ and $(7, 1)$



$$\text{Rise} = -5$$

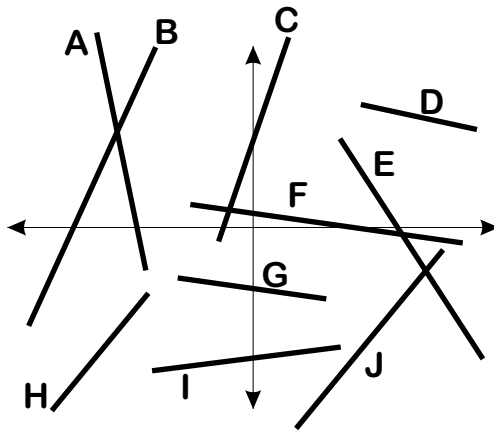
$$\text{Run} = 5$$

$$\text{Gradient} = m = \frac{\text{Rise}}{\text{Run}}$$

$$m = \frac{-5}{5}$$

$$m = -1$$

5. Which of the lines on this graph have a **negative** gradient.



6. Plot the following pairs of points on a graph and connect each pair with a straight line. State which of these lines will have a negative gradient and calculate these negative gradients.
- | | |
|------------------------|-----------------------|
| (a) (1,4) and (5,0) | (b) (2,8) and (4,4) |
| (c) (-2,1) and (3,-4) | (d) (-1,-1) and (3,1) |
| (e) (-2,-3) and (2,-5) | (f) (0,3) and (5,0) |
| (g) (2,0) and (6,-2) | (h) (-3,-4) and (4,0) |

Calculating Gradient

The gradient of the line connecting points (x_1, y_1) and (x_2, y_2) can be found by using this formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples

Find the gradient of the line connecting the following pairs of points.

1. (2,-3) and (5,7)

$$(x_1, y_1) = (2, -3)$$

$$(x_2, y_2) = (5, 7)$$

$$\begin{aligned} m &= \frac{7 - (-3)}{5 - 2} \\ &= \frac{7 + 3}{3} \end{aligned}$$

$$m = \frac{10}{3}$$

2. (-5,4) and (-2,-3)

$$(x_1, y_1) = (-5, 4)$$

$$(x_2, y_2) = (-2, -3)$$

$$\begin{aligned} m &= \frac{(-3) - 4}{(-2) - (-5)} \\ &= \frac{-3 - 4}{-2 + 5} \end{aligned}$$

$$m = \frac{-7}{3}$$

EXERCISE 8G

1. Calculate the gradient of the lines connecting the following pairs of points.

- | | |
|-----------------------|------------------------|
| (a) (2,4) and (8,10) | (b) (0,3) and (4,11) |
| (c) (1,3) and (8,17) | (d) (0,0) and (5,8) |
| (e) (2,3) and (7,9) | (f) (3,4) and (10,8) |
| (g) (-1,2) and (4,5) | (h) (-2,-3) and (5,0) |
| (i) (0,4) and (6,0) | (j) (1,8) and (7,-4) |
| (k) (-4,5) and (3,6) | (l) (-2,-5) and (6,4) |
| (m) (0,8) and (7,-4) | (n) (2,2) and (12,-7) |
| (o) (-2,8) and (5,3) | (p) (-3,-1) and (7,-5) |
| (q) (-4,0) and (0,-8) | (r) (-1,6) and (0,0) |
| (s) (2,5) and (7,5) | (t) (-3,-4) and (5,-4) |

2. (a) Which axis (x or y) is 1(s) and 1(t) parallel to?
 (b) What is the gradient of a line that is parallel to the x -axis?

3. On the graph shown here, line **A** has a gradient of 1.

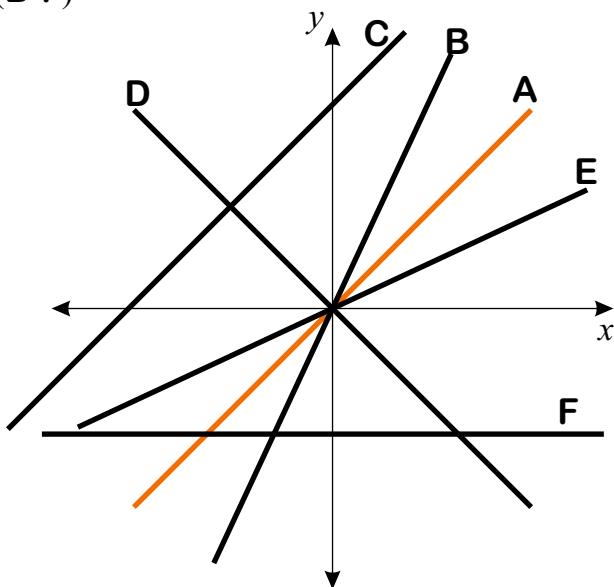
- (a) Which of the other lines (**B-F**) has a gradient of $\frac{1}{2}$?

- (b) Which of the other lines has a gradient of -1?

- (c) Which of the other lines has a gradient of 2?

- (d) Which of the other lines is parallel to line **A**?

- (e) Which of the other lines has a gradient of 0?



x- and y- Intercepts

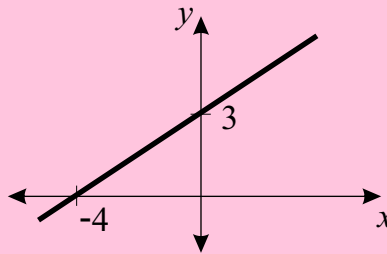
Definitions

The x -intercept is the point where a line crosses the x -axis.

The y -intercept is the point where a line crosses the y -axis.

Examples

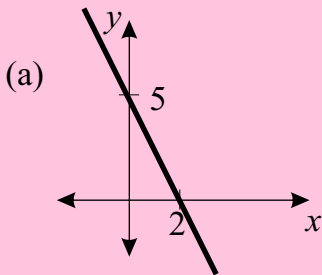
1. For the straight line shown on this graph:
the x -intercept = -4
the y -intercept = 3



The coordinates of the x -intercept are $(-4, 0)$

The coordinates of the y -intercept are $(0, 3)$

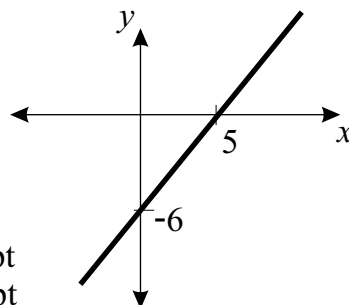
2. (a) Sketch the straight line that has the following intercepts:
 x -intercept = 2
 y -intercept = 5
(b) State the coordinates of the intercept points.



- (b) x -intercept = $(2, 0)$
 y -intercept = $(0, 5)$

EXERCISE 8H

1. For the straight line shown on this graph:
(a) state the x -intercept
(b) state the y -intercept
(c) state the coordinates of the x -intercept
(d) state the coordinates of the y -intercept



2. Sketch the straight lines that have the following x - and y -intercepts.
- (a) x -intercept = 3, y -intercept = 2
 - (b) x -intercept = -4, y -intercept = 6
 - (c) x -intercept = 5, y -intercept = -1
 - (d) x -intercept = -3, y -intercept = -5
 - (e) x -intercept = 1, y -intercept = -6
 - (f) x -intercept = -7, y -intercept = -2
3. State the coordinates of the intercept points in question 2.
4. Sketch the straight lines that have the following coordinates for their intercept points.
- (a) (4,0) and (0,-3)
 - (b) (-5,0) and (0,5)
 - (c) (-1,0) and (0,7)
 - (d) (6,0) and (0,2)

When the equation for a straight line is known, the intercept points can be found by using the following technique.

The straight line can then be sketched.

Note At the x -intercept $y = 0$
At the y -intercept $x = 0$

Step 1 Find the x -intercept by substituting $y = 0$ into the equation

Step 2 Find the y -intercept by substituting $x = 0$ into the equation

Example 1

Find the x -intercept and y -intercept of the straight line with the following equation and sketch the straight line.

$$3x + 2y = 6$$

Step 1 Find x -intercept by substituting $y = 0$ into the equation

$$3x + 2 \times 0 = 6$$

$$3x = 6$$

$$x = 2$$

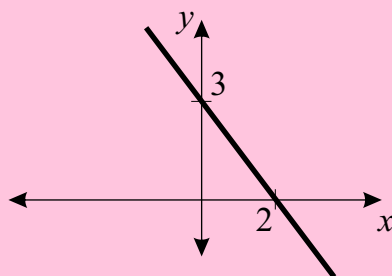
Step 2 Find y -intercept by substituting $x = 0$ into the equation

$$3 \times 0 + 2y = 6$$

$$2y = 6$$

$$y = 3$$

The straight line can now be sketched.



Example 2

Find the x -intercept and y -intercept of the straight line with the following equation and sketch the straight line.

$$y = 2x + 8$$

Step 1 Find x -intercept by substituting $y = 0$ into the equation

$$0 = 2x + 8$$

$$2x + 8 = 0$$

$$2x = -8$$

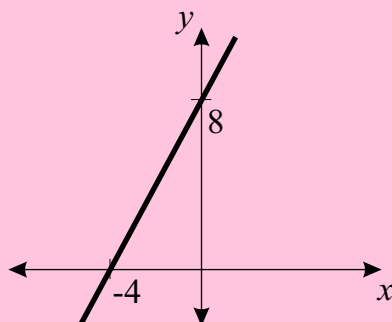
$$x = -4$$

Step 2 Find y -intercept by substituting $x = 0$ into the equation

$$y = 2 \times 0 + 8$$

$$y = 8$$

The straight line can now be sketched.



5. Find the x -intercept and y -intercept of the straight lines with the following equations and sketch the lines.

- | | |
|--------------------|--------------------|
| (a) $x + y = 3$ | (b) $2x + y = 4$ |
| (c) $3x + 3y = 6$ | (d) $2x + 5y = 10$ |
| (e) $3x + 8y = 24$ | (f) $x + 5y = 10$ |
| (g) $4x + 2y = 12$ | (h) $3x + y = 9$ |
| (i) $x - y = 2$ | (j) $x - 2y = 4$ |
| (k) $3x - 4y = 12$ | (l) $4x - 3y = 24$ |
| (m) $x + 2y = 1$ | (n) $2x - y = 1$ |

6. Find the x -intercept and y -intercept of the straight lines with the following equations and sketch the lines.

- | | |
|-------------------|-------------------|
| (a) $y = 3x - 6$ | (b) $y = 2x - 4$ |
| (c) $y = x - 3$ | (d) $y = x + 4$ |
| (e) $y = x - 1$ | (f) $y = 2x + 6$ |
| (g) $y = 3x - 12$ | (h) $y = 4x + 8$ |
| (i) $y = 2x - 10$ | (j) $y = 10 - 5x$ |
| (k) $y = 12 - 6x$ | (l) $y = 8 - 8x$ |
| (m) $y = 4x + 2$ | (n) $y = 6x - 3$ |

$$y = mx + c$$

Equations of straight lines are often written in the form:

$$y = mx + c$$

where $m = \text{the gradient}$ and $c = \text{the } y\text{-intercept}$

Examples

Equation	Gradient	y -intercept
$y = 2x + 5$	2	5
$y = -3x + 1$	-3	1
$y = \frac{2}{3}x - 4$	$\frac{2}{3}$	-4
$y = x$	1	0
$y = 7 - 4x$	-4	7

EXERCISE 8I

1. State the gradient and y -intercept of the straight lines with the following equations.

(a) $y = 3x + 4$

(c) $y = x + 5$

(e) $y = 2x - 5$

(g) $y = -3x + 2$

(i) $y = -x - 2$

(k) $y = 2x + \frac{1}{2}$

(m) $y = \frac{1}{3}x + 2$

(o) $y = 5 + 2x$

(q) $y = 6 + 3x$

(s) $y = \frac{x}{3} + 1$

(b) $y = 2x + 6$

(d) $y = x - 1$

(f) $y = 6x - 7$

(h) $y = -4x - 1$

(j) $y = -5x$

(l) $y = -7x - \frac{3}{5}$

(n) $y = -\frac{3}{4}x - \frac{1}{4}$

(p) $y = 3 - x$

(r) $y = 2 + 8x$

(t) $y = \frac{3x}{5} - 4$

2. Write the equations of the straight lines with the gradients and y -intercepts shown in this table.

	<i>Gradient</i>	<i>y-intercept</i>
(a)	5	3
(b)	2	-1
(c)	-4	5
(d)	-2	-7
(e)	1	-2
(f)	3	0
(g)	$\frac{1}{3}$	-2
(h)	2	$-\frac{2}{9}$

3. Rearrange the letters of the phrases in capital letters to make this sentence make sense!

Straight lines that are ***ALL PEARL*** have the same ***RED GIANT***.

4. State which of the following straight lines are parallel to the line with equation: $y = 3x - 4$.

A $y = 4x - 3$

C $y = x - 4$

E $y = 3x - 1$

G $y = 4x$

I $y = 2 + 3x$

B $y = 3x + 4$

D $y = x + 3$

F $y = 3x$

H $y = 2 - 3x$

J $y = 3x + 7$

5. Transpose the following equations to make y the subject and hence find the gradient and y -intercept.

- (a) $y - 3x - 5 = 0$
- (b) $y + 2x + 6 = 0$
- (c) $y - x + 3 = 0$
- (d) $y + x - 2 = 0$
- (e) $2y - 6x - 8 = 0$
- (f) $2y - 2x + 4 = 0$
- (g) $3x + 3y - 9 = 0$
- (h) $-8x + 4y - 12 = 0$
- (i) $-5 + 2y + 8x = 0$
- (j) $12 - 5x + 3y = 0$
- (k) $5y - 1 - x = 0$
- (l) $3y + 4x = 0$
- (m) $6y + 15 - 12x = 0$
- (n) $9y + 12 - 6x = 0$

Example

$$3y - 2x + 6 = 0$$

Subtract 6 from both sides

$$3y - 2x + 6 = 0$$

$$\boxed{-6} \quad \boxed{-6}$$

$$3y - 2x = -6$$

Add $2x$ to both sides

$$3y - 2x = -6$$

$$\boxed{+2x} \quad \boxed{+2x}$$

$$3y = 2x - 6$$

Divide all terms by 3

$$3y = 2x - 6$$

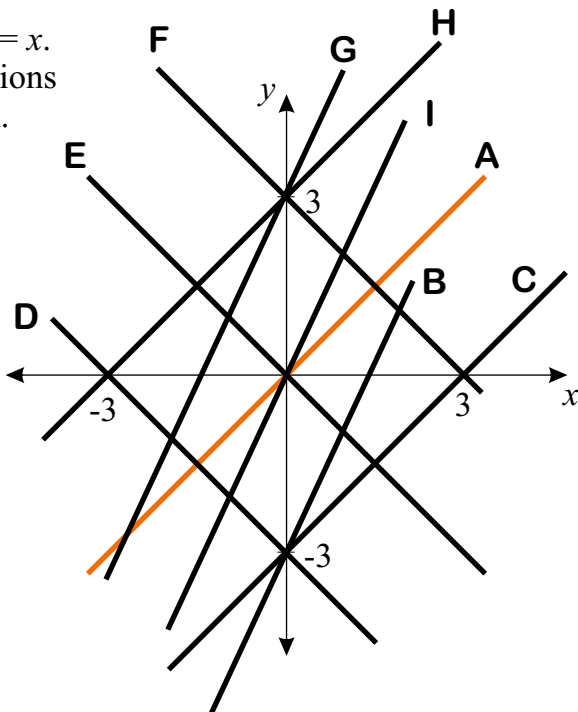
$$\boxed{\div 3} \quad \boxed{\div 3} \quad \boxed{\div 3}$$

$$y = \frac{2}{3}x - 2$$

Gradient = $\frac{2}{3}$, **y -intercept** = -2

6. On the graph shown here, the equation of line **A** is $y = x$. Match the following equations with the lines on the graph.

- (a) $y = 2x$
- (b) $y = 2x + 3$
- (c) $y = x + 3$
- (d) $y = -x$
- (e) $y = 2x - 3$
- (f) $y = -x - 3$
- (g) $y = x - 3$
- (h) $y = -x + 3$



Finding the Equation to a Straight Line Given Two Points

Example

Find the equation of the straight line that passes through the points $(-2, -11)$ and $(4, 7)$ and draw the graph showing these points and the y -intercept.

Step 1 Find the gradient of the line connecting these two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{7 - -11}{4 - -2}$$

$$= \frac{18}{6}$$

$$m = 3$$

Step 2 The value for m can be substituted into the equation for a straight line.

$$y = mx + c$$

$$y = 3x + c$$

Step 3 Substitute the x and y values from either of the points into this equation and solve for c .

$$y = 3x + c$$

Use the values from the point $(-2, -11)$.

$$-11 = 3 \times -2 + c$$

$$-11 = -6 + c$$

$$\boxed{+6} \quad \boxed{+6}$$

$$-5 = c$$

$$c = -5$$

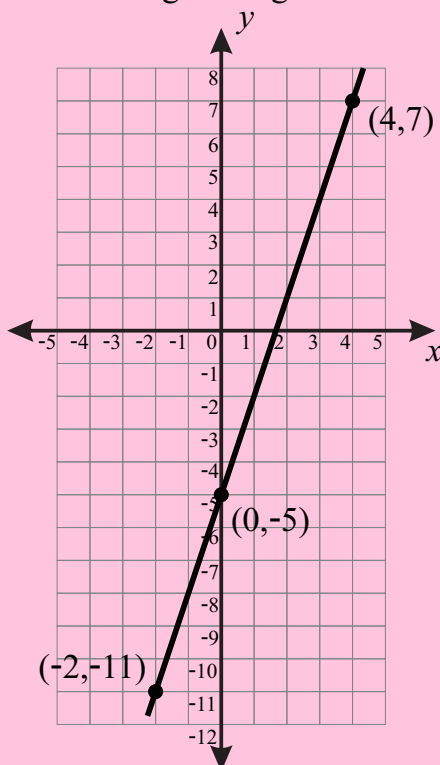
Step 4 Complete the equation.

$$y = 3x - 5$$

Step 5 Use these three points to draw the graph.

$(-2, -11)$, $(4, 7)$ and the y -intercept $(0, -5)$

Ensure the axes are large enough to include all these points.



EXERCISE 8J

Find the equation of the line that passes through each of the following pairs of points.

Construct axes large enough to draw each line and showing the two points given and the y -intercept.

1. $(-3, -9)$ and $(2, 11)$
2. $(-4, -12)$ and $(3, 2)$
3. $(-1, 7)$ and $(5, -5)$
4. $(-3, 7)$ and $(4, -14)$
5. $(-3, -1)$ and $(5, -9)$
6. $(-2, -16)$ and $(4, 14)$

Linear Relationships

EXERCISE 8K

1. (a) If $y = 2x$, find y if $x = 3$
(b) If $a = 4b + 1$, find a if $b = 4$
(c) If $M = 3N - 5$, find M if $N = 6$
(d) If $C = 2D - 8$, find C if $D = -1$

2. Copy and complete the following tables for the given rules.

Examples

1. $A = 3B$

<i>B</i>	-3	-2	-1	0	1	2	3
<i>A</i>							

Answer

<i>B</i>	-3	-2	-1	0	1	2	3
<i>A</i>	-9	-6	-3	0	3	6	9

2. $y = 2x - 3$

<i>x</i>	-3	-2	-1	0	1	2	3
<i>y</i>							

Answer

<i>x</i>	-3	-2	-1	0	1	2	3
<i>y</i>	-9	-7	-5	-3	-1	1	3

(a) $P = 2Q$

<i>Q</i>	-3	-2	-1	0	1	2	3
<i>P</i>							

(b) $n = 3m + 2$

<i>m</i>	-3	-2	-1	0	1	2	3
<i>n</i>							

(c) $b = a - 3$

<i>a</i>	-3	-2	-1	0	1	2	3
<i>b</i>							

(d) $y = 2x + 5$

<i>x</i>	-3	-2	-1	0	1	2	3
<i>y</i>							

(e) $d = 3c - 4$

<i>c</i>	-3	-2	-1	0	1	2	3
<i>d</i>							

(f) $M = 4N + 3$

<i>N</i>	-3	-2	-1	0	1	2	3
<i>M</i>							

3. Copy and complete this table for each of the rules below.

x	-3	-2	-1	0	1	2	3
y							

- (a) $y = x$ (b) $y = 2x + 3$ (c) $y = x - 5$
 (d) $y = 3x - 1$ (e) $y = 4x + 5$ (f) $y = -x$
 (g) $y = 2 - x$ (h) $y = 3 - 2x$ (i) $y = 1 - 3x$

4. If a graph was drawn for each of the equations in question 3 state the gradient (m) and y -intercept (c) of each graph.

5. For each of the tables of values below, find the rule connecting the variables.

Examples

1.

B	-3	-2	-1	0	1	2	3
A	-2	-1	0	1	2	3	4

Answer: $A = B + 1$

2.

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

Answer: $y = 2x + 1$

Hint: The y values increase by the coefficient of x

(a)

x	-3	-2	-1	0	1	2	3
y	-12	-8	-4	0	4	8	12

(b)

B	-3	-2	-1	0	1	2	3
A	0	1	2	3	4	5	6

(c)

c	-3	-2	-1	0	1	2	3
d	-5	-4	-3	-2	-1	0	1

(d)

M	-3	-2	-1	0	1	2	3
N	-7	-5	-3	-1	1	3	5

(e)

a	-3	-2	-1	0	1	2	3
b	-8	-5	-2	1	4	7	10

(f)

P	-3	-2	-1	0	1	2	3
Q	-4	-2	0	2	4	6	8

(g)

x	-3	-2	-1	0	1	2	3
y	-13	-9	-5	-1	3	7	11

(h)

m	-3	-2	-1	0	1	2	3
n	-10	-8	-6	-4	-2	0	2

(i)

C	-3	-2	-1	0	1	2	3
D	-11	-8	-5	-2	1	4	7

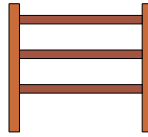
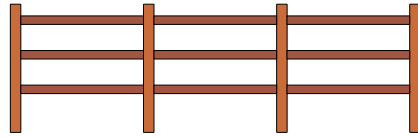
(j)

x	-3	-2	-1	0	1	2	3
y	-13	-8	-3	2	7	12	17

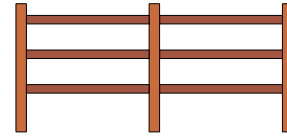
6. A farmer builds a post and rail fence as shown here.

- (a) Copy and complete the table below showing the number of rails (R) needed for a given number of posts (P).

P	2	3	4	5	6	7
R	3	6				



$P = 2$
 $R = 3$



$P = 3$
 $R = 6$

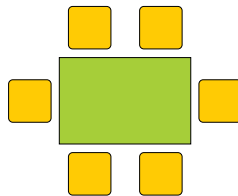
- (b) Find the rule connecting P and R : $R = \underline{\hspace{2cm}}$.

- (c) Use this rule to find the number of rails needed for the following number of posts:

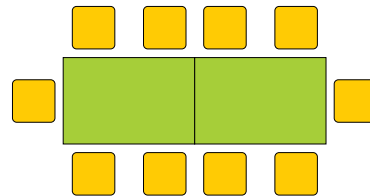
(i) 10 (ii) 25 (iii) 80

7. Organisers at a reception centre are wanting to calculate how many chairs are needed to be placed around various numbers of tables. Any number of tables can be joined end to end as shown below.

Examples



1 table
6 chairs



2 tables
10 chairs

- (a) Copy and complete this table showing the number of chairs (C) needed for a given number of tables (T).

T	1	2	3	4
C	6	10		

- (b) Find a rule connecting T and C :
 $C = \underline{\hspace{2cm}}$.

- (c) Use this rule to find the number of chairs needed for the following number of tables:

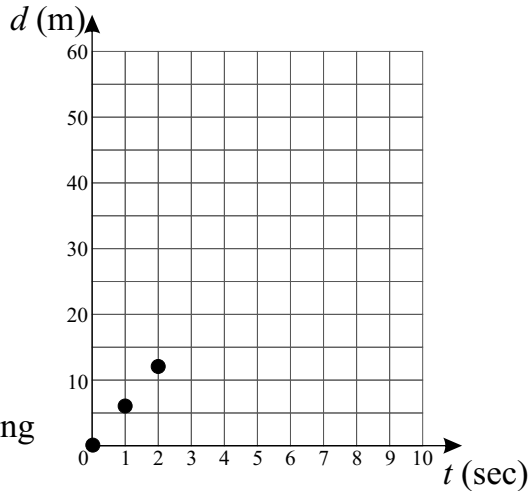
(i) 6 (ii) 10 (iii) 12

8. A cyclist rides at a speed of 6 metres per second.

- (a) Copy and complete this table showing the distance travelled (d) after t seconds.

t	0	1	2	3	4	5	6
d	0	6	12				

- (b) Copy and complete this graph showing distance travelled versus time for the first 10 seconds.
 (c) Connect the points on the graph with a line.
 (d) What shape is the line?



- (e) Complete the rule connecting distance (d) and time (t):

$$d = \underline{\hspace{2cm}}.$$

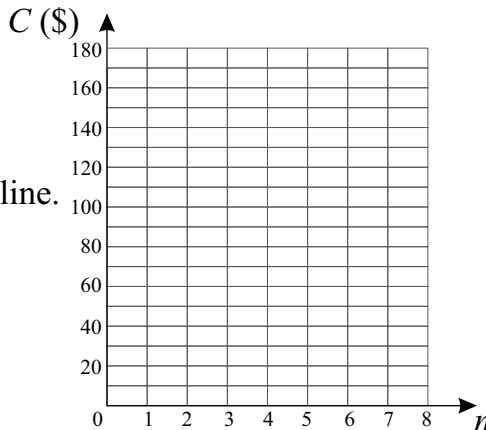
- (f) Use this rule to find the distance travelled in the following times:
 (i) 20 seconds (ii) 1 minute (iii) 1 hour

9. The cost of purchasing tickets for a concert was \$20 per ticket plus a \$10 booking fee.

- (a) Copy and complete this table showing the cost (C) of purchasing a number of tickets (n).

n	1	2	3	4	5	6	7	8
C	30	50	70					

- (b) Copy and complete this graph showing this information.
 (c) Connect the points with a line.
 (d) What shape is this line?



- (e) Complete this rule connecting cost (C) and number of tickets (n):
 $C = \underline{\hspace{2cm}}.$

- (f) Use this rule to find the cost of purchasing 25 tickets.

10. Jim and Tim own rival home gardening services.

Jim charges are \$30 per hour plus \$40 travelling fee.

Tim charges \$40 per hour and no travelling fee.

- (a) Copy and complete the following table showing the charge (C) for employing each of the gardeners for n hours.

n hours	1	2	3	4	5	6
C - Jim	70	100				
C - Tim	40	80				

- (b) Display this information on a graph with charge (C) on the vertical axis and number of hours (n) on the horizontal axis.

Show the charges for both gardeners on the one set of axes.

- (c) For what number of hours is the charge for each gardener the same?
- (d) Find a rule for each of the gardeners that could be used to calculate the charge for a given number of hours.

11. A car rental company is offering three different plans for paying for renting a car:

Plan A - \$180 + \$20 per day

Plan B - \$30 + \$40 per day

Plan C - \$50 per day

- (a) Copy and complete the following table showing the cost (C) of renting a car for a number of days (n).

n days	1	2	3	4	5	6	7	8
C - Plan A	200							
C - Plan B	70							
C - Plan C	50							

- (b) On the same set of axes draw a graph representing this information.

- (c) Which plan would be cheapest for the following number of days?

(i) 2 days (ii) 5 days (iii) 8 days

- (d) Find a rule for each of the plans that could be used to calculate the cost for a given number of rental days.

Non-linear Relationships

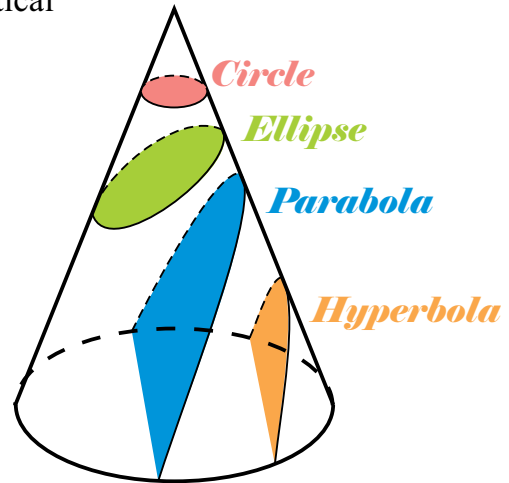
Linear relationships are those that, when graphed, produce straight line graphs. There are many other types of mathematical relationships that produce different shaped graphs.

One family of relationships are **conics** or **conic sections**.

These include ***circles***, ***ellipses***, ***parabolas*** and ***hyperbolas***.

The shapes of these curves can all be obtained by intersecting a cone with a plane as seen on this diagram.

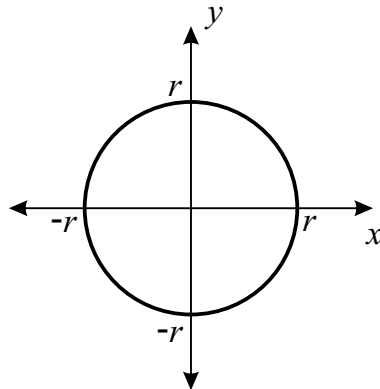
All these graphs are non-linear: not straight lines.



Circle

The equation for the basic circle is:

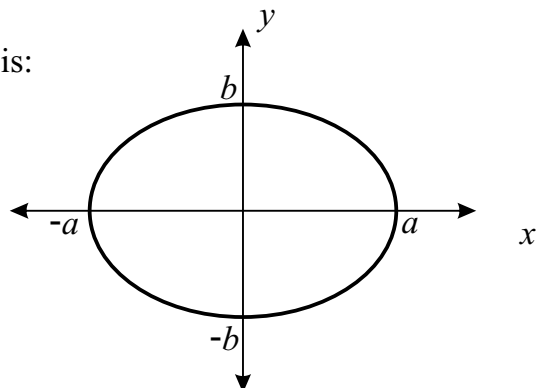
$$x^2 + y^2 = r^2$$



Ellipse

The equation for the basic ellipse is:

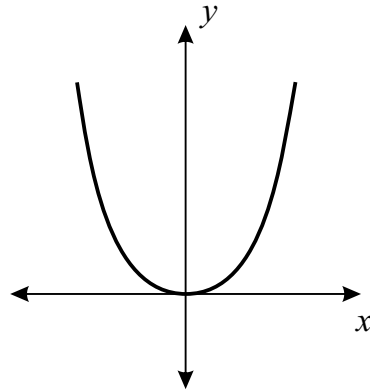
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Parabola

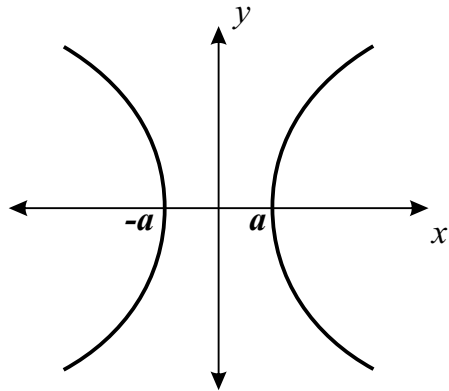
The equation for the basic parabola is:

$$y = ax^2$$

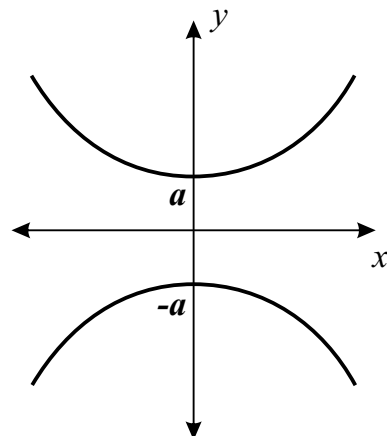
***Hyperbola***

The equation for the two basic hyperbolas are:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$



Example

Complete the table of values for the given equation and plot the graph.

$x^2 + y^2 = 9$

x	-3	-2	-1	0	1	2	3
y							

The equation needs to be transposed to make y the subject.

$x^2 + y^2 = 9$

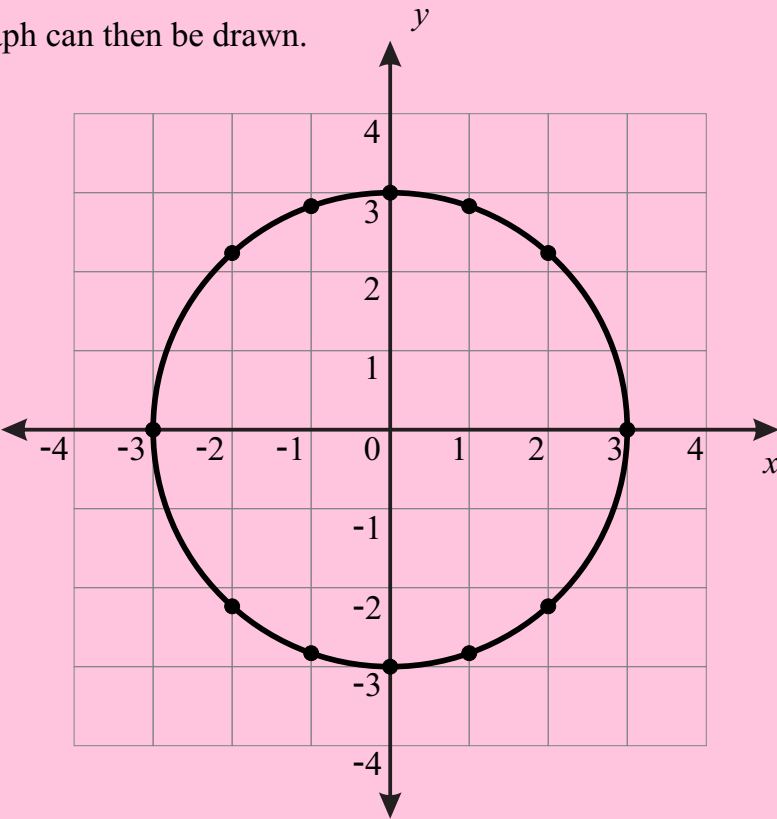
$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$

x	-3	-2	-1	0	1	2	3
y	0	$\pm\sqrt{5}$	$\pm\sqrt{8}$	± 3	$\pm\sqrt{8}$	$\pm\sqrt{5}$	0

x	-3	-2	-1	0	1	2	3
y	0	± 2.2	± 2.8	± 3	± 2.8	± 2.2	0

The graph can then be drawn.



EXERCISE 8L

1. (a) Copy and complete the table of values for the equation below.

$$x^2 + y^2 = 16$$

x	-4	-3	-2	-1	0	1	2	3	4
y									

- (b) Draw a Cartesian plane with x -axis and y -axis from -5 to 5.
Plot the points from the table of values and connect with a smooth curve.

2. (a) Copy and complete the table of values for the equation below.

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

x	-3	-2	-1	0	1	2	3
y							

- (b) Draw a Cartesian plane with x -axis and y -axis from -4 to 4.
Plot the points from the table of values and connect with a smooth curve.

3. (a) Copy and complete the table of values for the equation below.

$$y = 2x^2$$

x	-3	-2	-1	0	1	2	3
y							

- (b) Draw a Cartesian plane with x -axis from -4 to 4 and y -axis from 0 to 20.
Plot the points from the table of values and connect with a smooth curve.

4. (a) Copy and complete the table of values for the equation below.

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

x	-3	-2	-1	0	1	2	3
y							

- (b) Draw a Cartesian plane with x -axis from -4 to 4 and y -axis from -3 to 3.
Plot the points from the table of values and connect with a smooth curve.

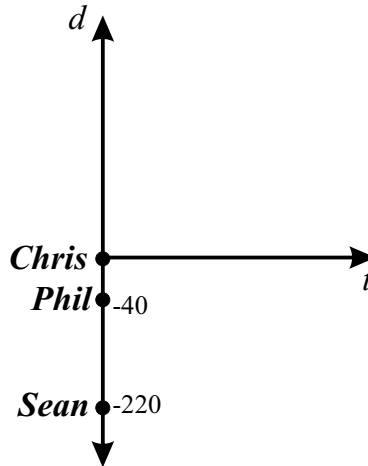
PROBLEM SOLVING

Chris, Phil and Sean were the only three riders who could win a cycling race. When Chris was 500 metres from the finish line he was 40 metres in front of Phil and 220 metres in front of Sean. Chris was riding at a speed of 10 m/s, Phil was riding at 12 m/s and Sean was riding at 15 m/s. They maintained these speeds until the finish line.

- On graph paper draw a graph of distance (d) versus time (t) from the time when Chris is 500 m from the finish line. Draw the distance-time graph for all the three riders on the same axes.

The starting points for the graphs are shown here.

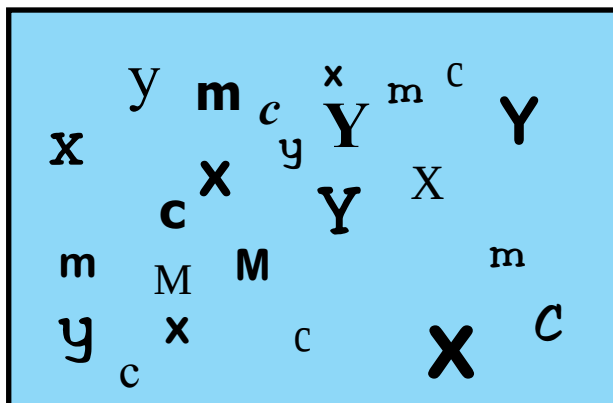
Continue each line to the point where $d = 500$ (finish line).



- Who came first, second and third in the race?
- What was the time between the three riders crossing the finish line?
- Write an equation for each of these three lines.
- Find all the points where a rider passed another rider.

PUZZLE

Use three straight lines to divide this diagram into six sections that each contain a y , m , x and c .



CHAPTER REVIEW

1. Find the distance between the points $(-6,8)$ and $(4,-7)$.
Give answer correct to one decimal place.
2. Find the coordinates of the midpoint of the line that connects the points $(-8,-7)$ and $(12,-5)$.
3. Find the gradient of the line connecting the following pairs of points.
(a) $(0,0)$ and $(2,10)$ (b) $(2,3)$ and $(5,12)$ (c) $(-1,-3)$ and $(1,5)$
(d) $(-5,4)$ and $(-1,6)$ (e) $(2,1)$ and $(10,-7)$ (f) $(-2,0)$ and $(5,-4)$
4. Sketch the straight lines with the following x - and y - intercepts.
(a) x -intercept = 2 and y -intercept = -4
(b) x -intercept = 3 and y -intercept = 2
5. State the ***coordinates*** of the x -intercept and y -intercept for the straight lines in question 5.
6. State the coordinates of the x -intercept and y -intercept for the straight lines with the following equations.
(a) $x + y = 5$ (b) $3y - 2x = 6$ (c) $y = x + 1$ (d) $y = 3x - 12$
7. State the gradient and y -intercept of the straight lines with the following equations.
(a) $y = 3x - 5$ (b) $y = -x + 2$ (c) $y = \frac{2}{3}x - 2$ (d) $y = -4x - \frac{1}{2}$
8. Write the equations for the straight lines with the following gradients and y -intercepts.
(a) gradient = 2 and y -intercept = 6
(b) gradient = -2 and y -intercept = -3
(c) gradient = $\frac{3}{4}$ and y -intercept = 0
9. Transpose the following equations to make y the subject and hence find the y -intercept and gradient.
(a) $y + 5x - 3 = 0$ (b) $2y - 3x + 6 = 0$ (c) $4x - 3y + 12 = 0$
10. Find the equation of the line that passes through the points $(-4,28)$ and $(6,-2)$.

11. Copy and complete the following tables for the given rules.

(a) $y = 3x - 4$

x	-3	-2	-1	0	1	2	3
y							

(b) $n = 3 - 2m$

m	-3	-2	-1	0	1	2	3
n							

12. For each of the tables of values below, find the rule connecting the variables.

(a)

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

(b)

b	-3	-2	-1	0	1	2	3
a	-13	-10	-7	-4	-1	2	5

13. The cost of hiring a jukebox was \$50 plus \$20 per hour.

- (a) Write a rule that can be used to calculate the cost (C) for a given number of hours (t).
 (b) How much would it cost to hire the jukebox for 8 hours?

Length and Perimeter

Length

EXERCISE 9A

From the list below find the length of the following objects.

1. Distance from the Sun to Earth.
2. The distance from the Sun to Mars.
3. The distance from the Sun to Venus.
4. The length of \$5 note.
5. The length of a \$10 note.
6. The length of a \$20 note.
7. The length of the Nile River.
8. The length of the Amazon River
9. The length of the Murray River.
10. The length of a basketball court.
11. The length of a netball court.
12. The length of a tennis court.
13. The distance from Brisbane to Fiji.
14. The distance from Melbourne to Wellington (New Zealand).
15. The distance from Perth to Singapore.
16. The height of the Sydney Centre Point tower.
17. The height of the Empire State Building.
18. The length of the human intestine.
19. The height of Mount Kosciusko above sea level.
20. The height of the Mount Everest above sea level.

227 900 000 km
138 mm
6671 km
149 500 000 km
8850 m
305 m
25 m
130 mm
3718 km
2500 km
2230 m
2800 km
24 m
448 m
30 m
6275 km
8 m
108 200 000 km
3900 km
144 mm

Conversions

The standard unit of length is the *metre*.

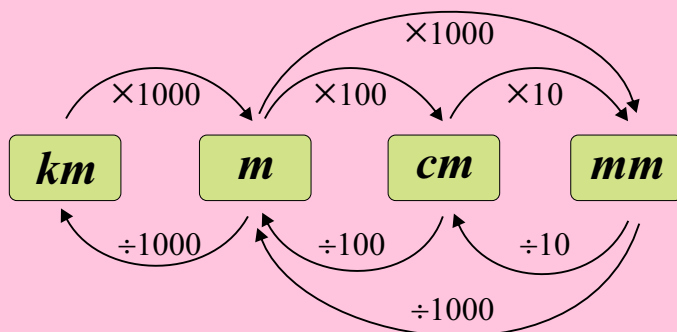
The abbreviation for metre is *m*.

Other common units used are either multiples or fractions of a metre.

The most common are *kilometres*, *centimetres* and *millimetres*.

1 kilometre (km) = 1000 metres
 1 metre = 100 centimetres (cm)
 1 metre = 1000 millimetres (mm)

This diagram can be used to find the conversion factor when converting from one unit to another.

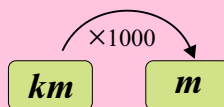


Examples

Convert the following lengths to the units shown in the brackets.
 Use the diagram above to find the conversion factors.

1. 3 km (m)

To convert from km to m: $\times 1000$
 $= 3 \times 1000$
 $= \mathbf{3000\ m}$



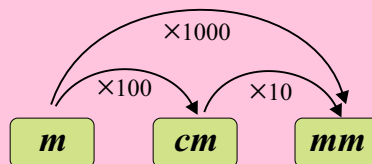
Examples continued

2. 26 m (mm)

To convert from m to mm: $\times 1000$

$$= 26 \times 1000$$

$$= \mathbf{26\,000\,mm}$$

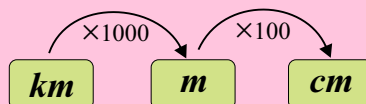


3. 4 km (cm)

To convert from km to cm: $\times 1000$ then $\times 100 = \times 100\,000$

$$= 4 \times 100\,000$$

$$= \mathbf{400\,000\,cm}$$



Remember - When *multiplying* by a power of 10 move the decimal point to the *right* the same number of decimal places as the number of zeros.

4. 9.705 m (cm)

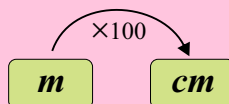
To convert from m to cm: $\times 100$

$$= 9.705 \times 100$$

Move the decimal point two places to the right (100 has two zeros)

$$= 9.705$$

$$= \mathbf{970.5\,cm}$$



5. 37.8 km (m)

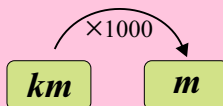
To convert from km to m: $\times 1000$

$$= 37.8 \times 1000$$

Move the decimal point three places to the right (1000 has three zeros)
Add zeros where necessary.

$$= 37.8$$

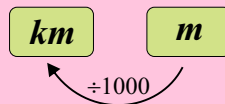
$$= \mathbf{37\,800\,m}$$



Examples continued

6. 9000 m (km)

To convert from m to km: $\div 1000$
 $= 9000 \div 1000$
 $= \mathbf{9 \text{ km}}$



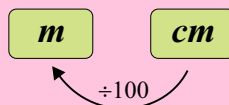
Remember - When *dividing* by a power of 10 move the decimal point to the **left** the same number of decimal places as the number of zeros.

7. 138.5 cm (m)

To convert from cm to m: $\div 100$
 $= 138.5 \div 100$

Move the decimal point two places to the left (100 has two zeros)

$= \overset{\curvearrowright}{\overset{\curvearrowright}{138.5}}$
 $= \mathbf{1.385 \text{ m}}$

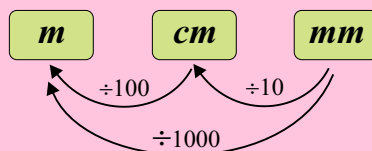


8. 37.8 mm (m)

To convert from mm to m: $\div 1000$
 $= 37.8 \div 1000$

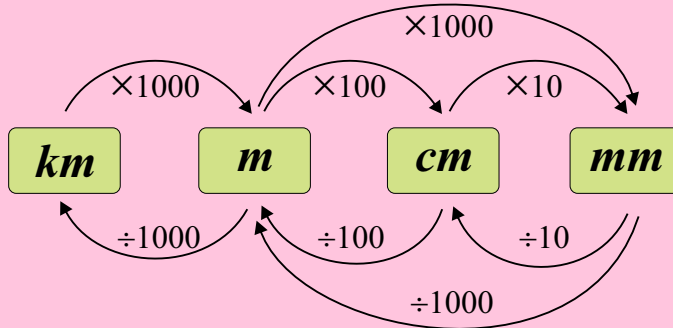
Move the decimal point three places to the left (1000 has three zeros)
 Add zeros where necessary.

$= \overset{\curvearrowright}{\overset{\curvearrowright}{\overset{\curvearrowright}{37.8}}} \div 1000$
 $= \mathbf{0.0378 \text{ m}}$



EXERCISE 9B

This diagram can be used to find the conversion factor when converting from one unit to another.



1. Convert the following lengths to the units shown in the brackets.

- | | | |
|--------------------|-------------------|--------------------|
| (a) 6 cm (mm) | (b) 8 m (cm) | (c) 20 km (m) |
| (d) 2.36 m (mm) | (e) 42.85 cm (mm) | (f) 0.68 km (m) |
| (g) 750 mm (cm) | (h) 75 cm (m) | (i) 76 850 m (km) |
| (j) 0.1 cm (m) | (k) 7.98 m (cm) | (l) 27.6 mm (m) |
| (m) 0.003 m (cm) | (n) 2.357 cm (mm) | (o) 0.5781 km (m) |
| (p) 0.3 km (cm) | (q) 0.43 km (mm) | (r) 0.0017 km (m) |
| (s) 0.0065 cm (mm) | (t) 4.76 cm (m) | (u) 0.006 mm (cm) |
| (v) 6.05 mm (m) | (w) 60 m (km) | (x) 56 450 mm (km) |

2. Convert the following lengths to the units shown in the brackets.

Examples

1. 3 m 2 cm (m)
= **3.02 m**

2. 2 m 18 mm (m)
= **2.018 m**

- | | |
|------------------------|------------------------|
| (a) 2 m 11 cm (m) | (b) 3 cm 2 mm (cm) |
| (c) 6 m 6 cm (m) | (d) 20 m 20 cm (m) |
| (e) 5 km 20 m (km) | (f) 18 m 5 cm (m) |
| (g) 5 m 8 cm (m) | (h) 6 m 23 cm 8 mm (m) |
| (i) 4 m 7 mm (m) | (j) 20 cm 4 mm (cm) |
| (k) 3 m 88 cm 2 mm (m) | (l) 10 m 57 mm (m) |

3. Round the following lengths to the nearest metre.

If the digit after the decimal point is less than 5 round to the existing whole number.

If the digit after the decimal point is 5 or larger then round up to the next whole number.

Examples

1. 67.4 m

The digit after the decimal point is less than 5 so round to existing whole number

= 67 m

2. 142.5 m

The digit after the decimal point is equal to 5 so round up to the next whole number

= 143 m

3. 299.803 m

The digit after the decimal point is larger than 5 so round up to the next whole number

= 300 m

(a) 2.8 m

(b) 66.2 m

(c) 14.09 m

(d) 29.7 m

(e) 15.56 m

(f) 20.199 m

(g) 3 m 15 cm

(h) 2 m 80 cm

(i) 15 m 56 cm

(j) 4 m 4 cm

(k) 8 m 9 cm

(l) 10 m 89 mm

(m) 11 m 256 mm

(n) 5 m 780 mm

(o) 48 m 458 mm

4. Round the following lengths to the nearest centimetre.

(a) 3.4 cm

(b) 2.8 cm

(c) 15.9 cm

(d) 1 cm 1 mm

(e) 13 cm 8 mm

(f) 25 cm 6 mm

(g) 9 mm

(h) 7 cm 2 mm

(i) 19 cm 9 mm

5. Toni was knitting a scarf.
She could knit 20 cm every day.
How long would the scarf be
(in *metres*) if she knitted
every day for four weeks?



6. The C-Saw Timber Yard sells timber in lengths that start at 1500 mm and increase by multiples of 300 mm (1500 mm, 1800 mm, 2100 mm, etc). The longest timber length is 6 metres.
Which of the following lengths of timber would the C-Saw Timber Company sell?

2.7 m 540 cm 3000 cm 5100 mm 3.9 m 560 cm 520 cm

7. Deanne shaved her head on her 15th birthday!
She decided that she would not get another haircut until her 21st birthday?
Deanne's hair grows 15 mm every month.
How long (in *cm*) will her hair be when she gets it cut on her 21st birthday?



8. Eric wants to use boards to cover the floor of a large room.
The room is 3.9 m $\times$ 10.4 m and the floor boards are 130 mm wide.
What is the total length of floor boards (in *m*) that Eric will need?

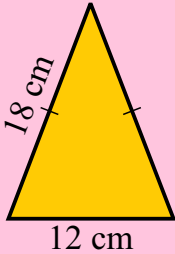
Perimeter

EXERCISE 9C

1. Find the perimeter of each of the following shapes.

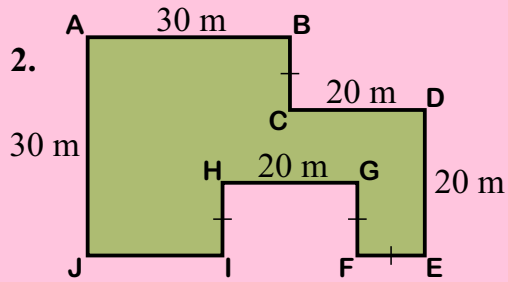
Examples

1.



$$\begin{aligned}\text{Perimeter} &= 12 + 18 + 18 \\ &= \mathbf{48\text{ cm}}\end{aligned}$$

2.



To find the perimeter, all the unknown lengths need to be calculated first.

$$\mathbf{BC + DE = AJ}$$

$$\mathbf{BC + 20 = 30}$$

$$\mathbf{BC = 10\text{ (} = EF, FG \text{ and HI)}}$$

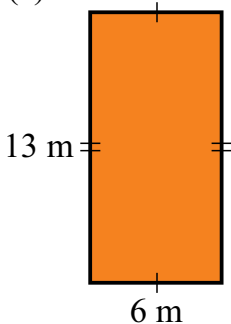
$$\mathbf{IJ + 20 + EF = AB + CD}$$

$$\mathbf{IJ + 20 + 10 = 30 + 20}$$

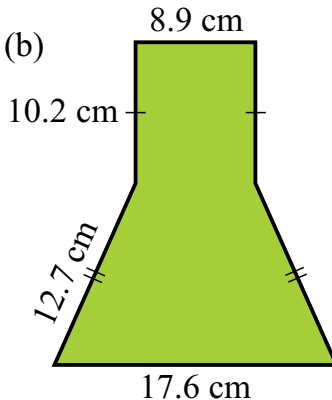
$$\mathbf{IJ = 20}$$

$$\begin{aligned}\text{Perimeter} &= 30 + 10 + 20 + 20 \\ &\quad + 10 + 10 + 20 + 10 \\ &\quad + 20 + 30 \\ &= \mathbf{180\text{ m}}\end{aligned}$$

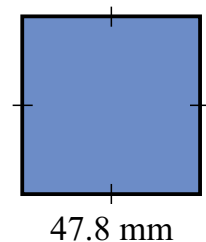
(a)

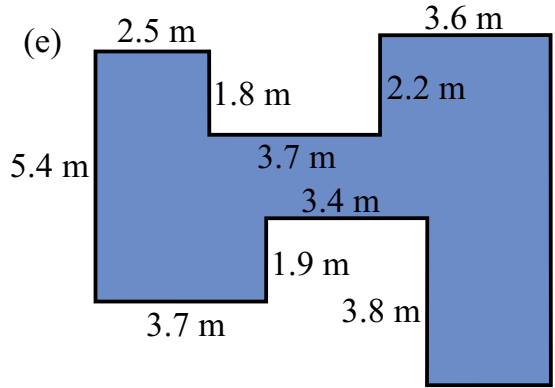
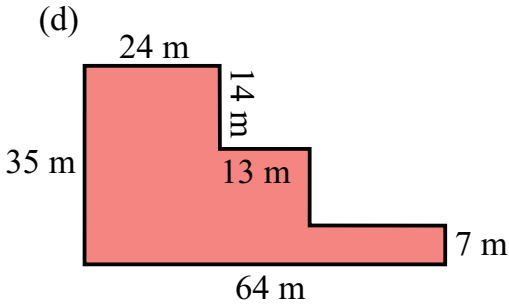


(b)

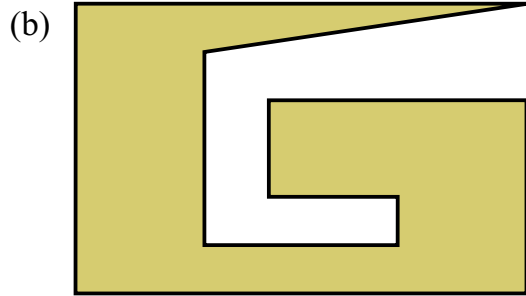
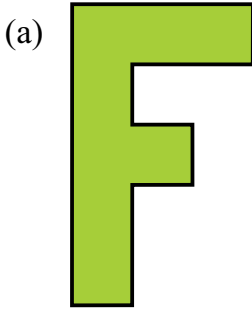


(c)



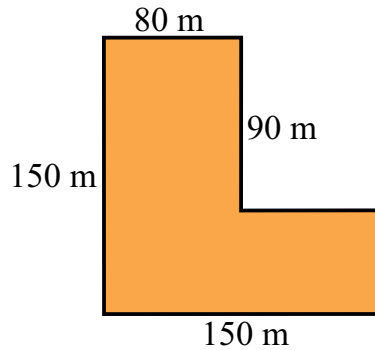


2. Use a ruler to measure all the side lengths (in *mm*) of the following shapes and hence find the perimeter of each shape.



3. A farmer needed to fence the paddock shown here.

- (a) What length of fencing will be needed?
 (b) The fence needs a post every 10 m. How many posts will be needed?



4. Olivia has 26 metres of wire mesh that she is going to use to make a pen for her ferrets.
 If the pen is **square**, what will be its side length?
5. What is the width of a rectangle that is 16.8 cm long and has a perimeter of 48.8 cm?
6. Find the width of a rectangle that has a perimeter of 36 metres and a length that is three times the width?

Circles

EXERCISE 9D

The perimeter of a circle is called its *circumference*.

The circumference (C) of a circle is found by multiplying the circle's diameter (D) by π (pi).

$$C = \pi \times D$$

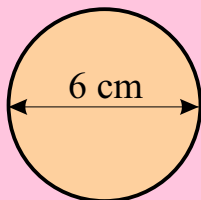
π is equal to 3.141592654...

This can be found by using the π button on your calculator.

- Find the circumference of the following circles.
Give answers correct to one decimal place.

Examples

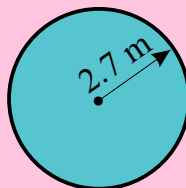
1.



The diameter = 6 cm

$$\begin{aligned} C &= \pi \times D \\ &= \pi \times 6 \\ &= 18.84955 \\ &= \mathbf{18.8 \text{ cm}} \text{ (1 dec. place)} \end{aligned}$$

2.

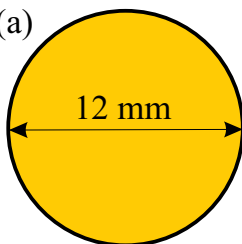


The radius = 2.7 m

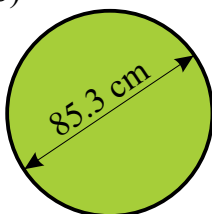
The diameter = 5.4 m

$$\begin{aligned} C &= \pi \times D \\ &= \pi \times 5.4 \\ &= 16.96460 \\ &= \mathbf{17.0 \text{ m}} \text{ (1 dec. place)} \end{aligned}$$

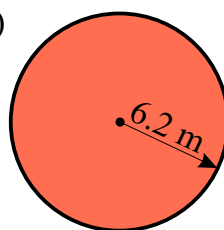
(a)



(b)



(c)



2. Find the circumference of the circles with the following **diameters**.

Give answers correct to one decimal place.

- (a) 8 mm (b) 11.8 m (c) 5.6 km (d) 200 cm
 (e) 0.58 m (f) 1.589 km (g) 11.97 mm (h) 65.89 m

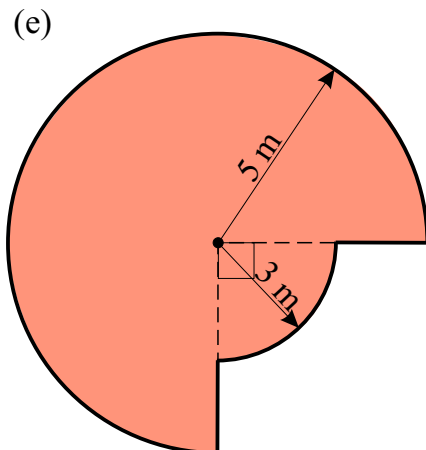
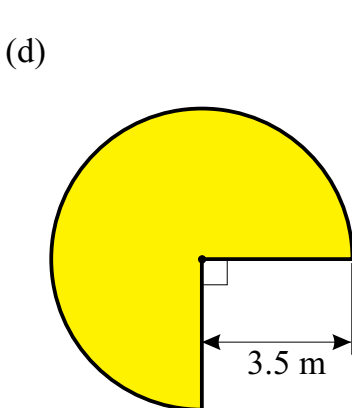
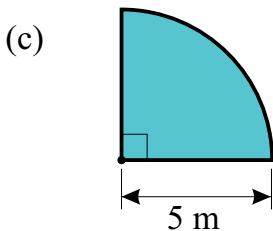
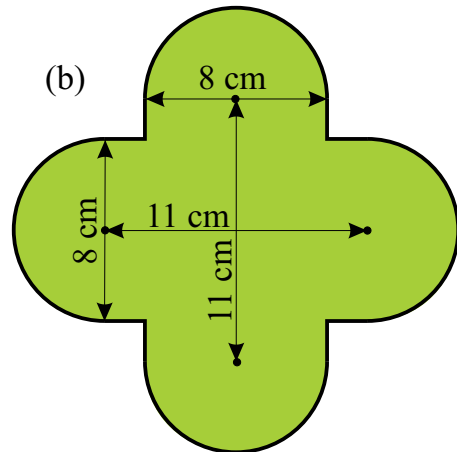
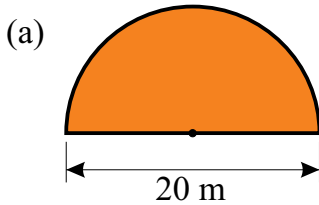
3. Find the circumference of the circles with the following **radii**.

Give answers correct to one decimal place.

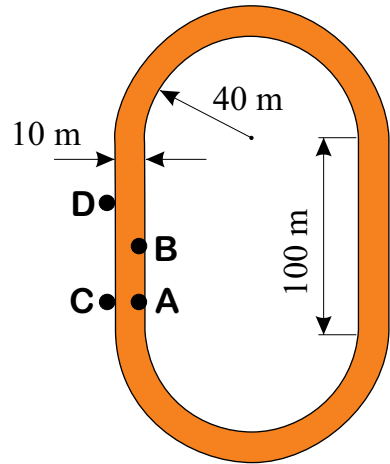
- (a) 15 mm (b) 25.8 m (c) 7.6 km (d) 500 cm
 (e) 2.59 m (f) 0.589 km (g) 10.97 mm (h) 165.89 m

4. Find the perimeter of each of the following shapes.

Give answers correct to one decimal place.



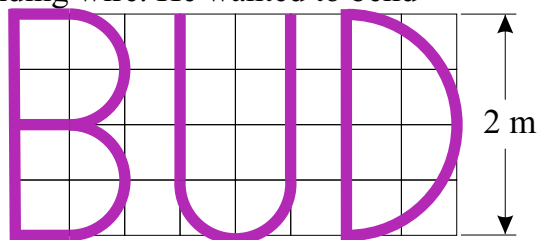
5. The diagram of an athletics track is shown here. The inside of the track consists of two semicircles of radius 40 m connected by straights that are 100 m long. The track is 10 m wide.



- (a) Candy started running at point **A** and ran anticlockwise around the inside of the track.
How far would she run in one lap?
(Give answer to one decimal place)
- (b) How far would she run (in **km**) if she ran 20 laps?
(Give answer to three decimal places)
- (c) If Candy wanted to run 5 km, how many laps would she need to run?
- (d) Candy was training for a 400 m race. She wanted to mark point **B** on the track exactly 400 m from point **A** in an anticlockwise direction. How far back from point **A** should she mark point **B**?
- (e) One day when Candy wanted to train, the track was being repaired. She had to run around the **outside** of the track. She started her 400 m run at point **C** and wanted to mark point **D** exactly 400 m from point **C** - running anticlockwise. How far back from point **C** should she mark point **D**?
6. Amelia rode her bicycle to school. She wanted to find out how far her school was from her house so she fixed a counter to her front wheel. The counter showed how many times her wheel turned. The diameter of her bicycle wheel is 600 mm. Her wheel turned 935 times riding from her house to school. What is the distance (to the nearest **metre**) from Amelia's house to her school?

7. Bud created artistic pieces by bending wire. He wanted to bend wire into the shape of his name using the grid shown here.

If he wants his name to be 2 metres tall, what is the total length (to the nearest **cm**) of wire that he will need?

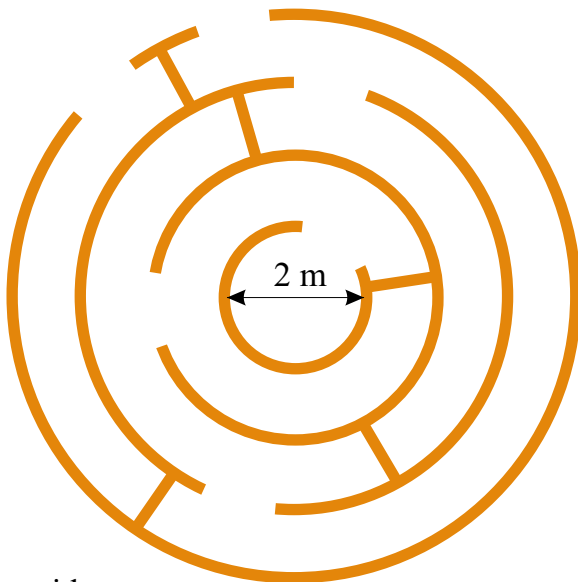


8. Tyrone plans to construct a maze in his garden. A diagram of his maze is shown here.

The centre circle has a diameter of 2 metres.

The paths are all 1 metre wide.

The openings are all 1 metre wide.



- (a) Find the total length of the fences in Tyrone's maze. Give answer in metres to one decimal place.

- (b) The fencing is to be constructed using boards that are 200 mm wide.

How many of these boards will Tyrone need?

- (c) The fencing is to be 2 metres high.

What is the total length of fencing boards that Tyrone will need?

9. Find the **diameter** of the circles with the following circumferences. Give answers correct to one decimal place.

(a) 11 m (b) 40 cm (c) 8.5 km (d) 200 mm (e) 0.94 m

10. Find the **radius** of the circles with the following circumferences. Give answers correct to one decimal place.

(a) 15 m (b) 75 cm (c) 2.7 km (d) 500 mm (e) 1.76 m

11. Alex measured the circumference of a tree to be 3450 mm. What is the diameter of the tree (to the nearest **mm**)?

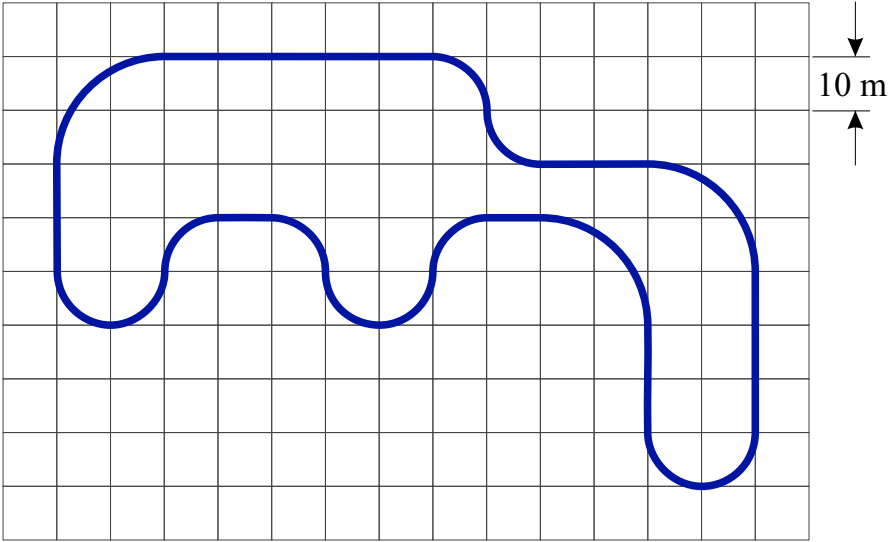
12. Petro wanted to make a 'trundle wheel'. The wheel is to have a circumference of one metre and can be used to measure distances. A counter records every revolution when it is rolled along the ground and hence how many metres. What should be the diameter (to the nearest **mm**) of the 'trundle wheel'?



PROBLEM SOLVING

The diagram below shows the drawing of a go-cart track.
The drawing is to scale and the grid lines are 10 metres apart.
All bends in the track are parts of circles.

- 1. Find the length of the track (to the nearest *m*).
- 2. Leon completed one circuit of the track in 52 seconds.
Find Leon's average speed in km/hour (to one decimal place).



PUZZLE

There are 10 words from this chapter hidden in this grid.
The words may be written forwards, backwards or diagonally.

A	E	C	N	E	R	E	F	M	U	C	R	I	C	C
L	R	T	L	H	U	K	L	E	N	G	T	H	I	U
L	F	A	N	E	C	N	A	T	S	I	D	R	T	L
E	C	Z	D	O	J	F	Y	R	G	N	C	W	U	R
E	P	E	R	I	M	E	T	E	R	L	H	A	D	M
H	D	I	L	K	U	O	R	H	E	I	G	H	T	S
W	A	M	O	J	F	S	R	E	T	E	M	A	I	D

CHAPTER REVIEW

1. Convert the following lengths to the units shown in the brackets.

(a) 14 cm (mm)	(b) 8.3 m (cm)	(c) 4 km (m)
(d) 0.56 m (mm)	(e) 9.75 cm (mm)	(f) 1.26 km (m)
(g) 83 mm (cm)	(h) 450 cm (m)	(i) 3450 m (km)
(j) 2.9 cm (m)	(k) 0.055 m (mm)	(l) 35 600 mm (m)

2. Convert the following lengths to the units shown in the brackets.

(a) 2 m 11 cm (m)	(b) 3 cm 2 mm (cm)
(c) 6 m 6 cm (m)	(d) 8 m 75 mm (m)

3. Round the following lengths to the nearest metre.

(a) 4.6 m	(b) 7 m 6 cm	(c) 3 m 350 mm
(d) 2 m 78 cm	(e) 0.7852 km	(f) 398 658 mm

4. (a) Find the circumference of the circles with the following *diameters*. (Give answers correct to one decimal place)

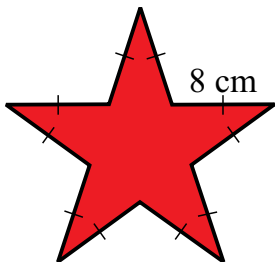
(i) 4 cm	(ii) 6.8 m	(iii) 0.6 km	(iv) 35.8 mm
----------	------------	--------------	--------------

 (b) Find the circumference of the circles with the following *radii*. (Give answers correct to one decimal place)

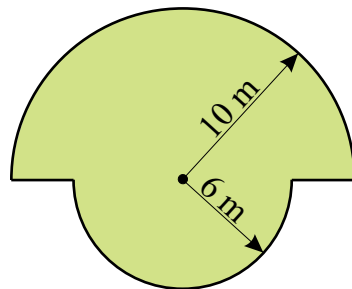
(i) 65 cm	(ii) 3 m	(iii) 6.54 m	(iv) 89.5 km
-----------	----------	--------------	--------------

5. Find the perimeter of the following shapes.
(Give answers correct to one decimal place)

(a)



(b)



Area

Common units of area are mm^2 , cm^2 , m^2 , km^2 and *hectares*.

1 mm^2 is an area equal to the area of a square with side length 1 mm.

1 cm^2 is an area equal to the area of a square with side length 1 cm.

1 m^2 is an area equal to the area of a square with side length 1 m.

1 km^2 is an area equal to the area of a square with side length 1 km.

1 hectare is an area equal to the area of a square with side length 100 m.

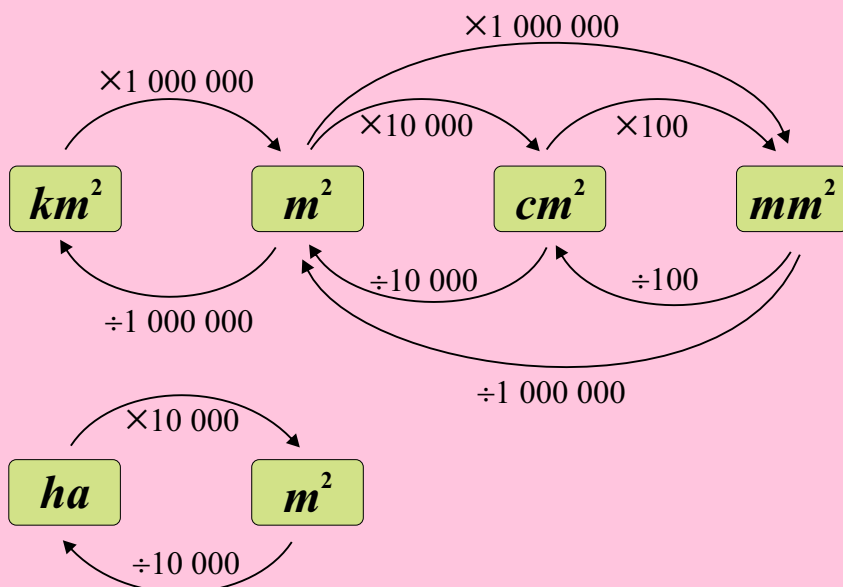
$$1 \text{ cm}^2 = 100 \text{ mm}^2$$

$$1 \text{ m}^2 = 10\,000 \text{ cm}^2$$

$$1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$$

$$1 \text{ hectare (ha)} = 10\,000 \text{ m}^2$$

These diagrams can be used to find the conversion factor when converting from one unit to another.



Conversions

EXERCISE 10A

1. Convert the following areas to the units shown in the brackets.

Examples

$$\begin{aligned} 1. \quad 6 \text{ cm}^2 \text{ (mm}^2\text{)} \\ &= 6 \times 100 \\ &= \mathbf{600 \text{ mm}^2} \end{aligned}$$

$$\begin{aligned} 2. \quad 750\,000 \text{ m}^2 \text{ (km}^2\text{)} \\ &= 750\,000 \div 1\,000\,000 \\ &= \mathbf{0.75 \text{ km}^2} \end{aligned}$$

$$\begin{aligned} 3. \quad 80\,000 \text{ m}^2 \text{ (ha)} \\ &= 80\,000 \div 10\,000 \\ &= \mathbf{8 \text{ ha}} \end{aligned}$$

(a) $700\,000 \text{ cm}^2 \text{ (m}^2\text{)}$

(b) $90 \text{ cm}^2 \text{ (mm}^2\text{)}$

(c) $640\,000 \text{ mm}^2 \text{ (cm}^2\text{)}$

(d) $0.03 \text{ km}^2 \text{ (m}^2\text{)}$

(e) $0.25 \text{ ha (m}^2\text{)}$

(f) $500\,000 \text{ m}^2 \text{ (km}^2\text{)}$

(g) $0.75 \text{ cm}^2 \text{ (mm}^2\text{)}$

(h) $420\,000 \text{ m}^2 \text{ (ha)}$

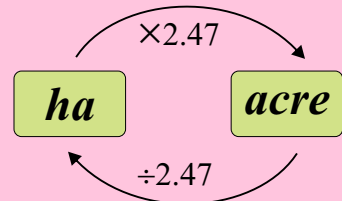
(i) $8000 \text{ cm}^2 \text{ (m}^2\text{)}$

(j) $200\,000\,000 \text{ mm}^2 \text{ (m}^2\text{)}$

2. An old unit used to measure the area of land is an **acre**. (1 hectare = 2.47 acres)

(a) How many hectares are in 148.2 acres?

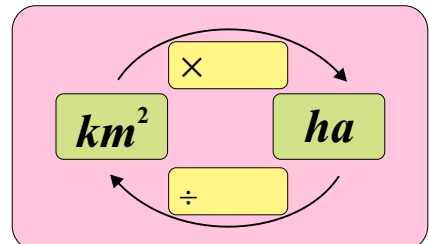
(b) How many acres are in 250 hectares?



3. (a) Evan owned a farm that was 3000 acres. Convert this to hectares. Round to the nearest hectare.

(b) Josie owned a farm that was 2500 ha. Convert this to acres.

4. Copy and complete this diagram showing the conversions between km^2 and hectares.

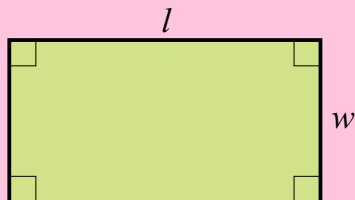


5. (a) Kakadu National Park is Australia's largest national park. It covers an area of $19\,804 \text{ km}^2$. Convert this to hectares.

(b) Australia's largest cattle station is Anna Creek in South Australia. It covers an area of $2\,367\,700 \text{ ha}$. Convert this to km^2 .

Shapes

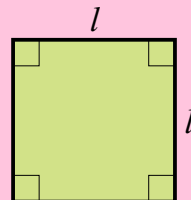
Rectangle



$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= l \times w\end{aligned}$$

$$A = lw$$

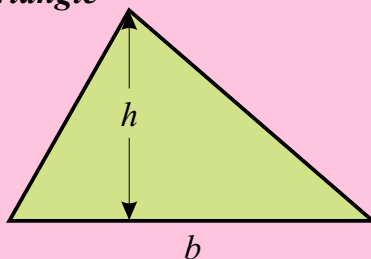
Square



$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= l \times l\end{aligned}$$

$$A = l^2$$

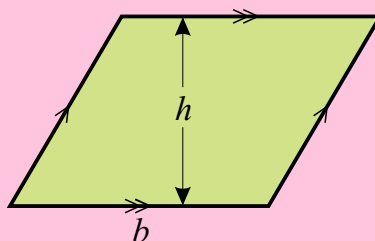
Triangle



$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times b \times h\end{aligned}$$

$$A = \frac{1}{2}bh$$

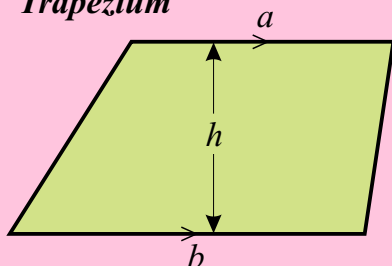
Parallelogram



$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= b \times h\end{aligned}$$

$$A = bh$$

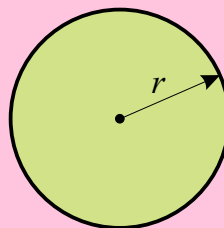
Trapezium



$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (a + b) \times \text{height} \\ &= \frac{1}{2} \times (a + b) \times h\end{aligned}$$

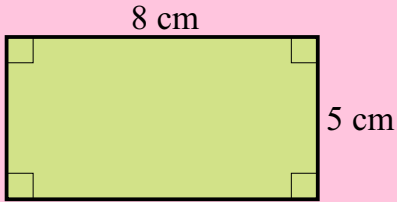
$$A = \frac{1}{2}(a + b)h$$

Circle

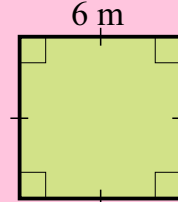


$$\text{Area} = \pi \times r^2$$

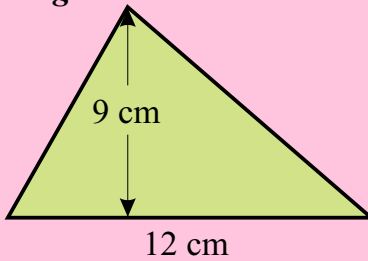
$$A = \pi r^2$$

Examples**Rectangle**

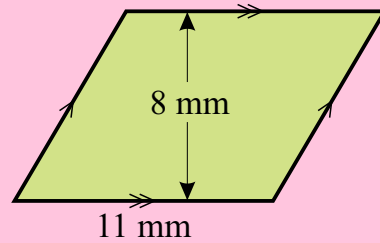
$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 8 \times 5 \\ \text{Area} &= 40 \text{ cm}^2\end{aligned}$$

Square

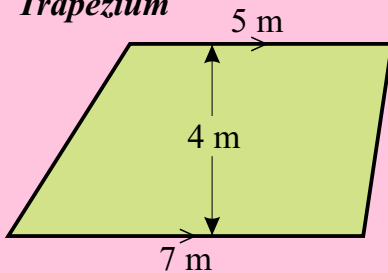
$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 6 \times 6 \\ \text{Area} &= 36 \text{ m}^2\end{aligned}$$

Triangle

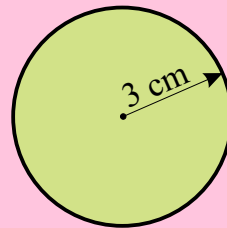
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 12 \times 9 \\ \text{Area} &= 54 \text{ cm}^2\end{aligned}$$

Parallelogram

$$\begin{aligned}\text{Area} &= \text{base} \times \text{height} \\ &= 11 \times 8 \\ \text{Area} &= 88 \text{ mm}^2\end{aligned}$$

Trapezium

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (a + b) \times \text{height} \\ &= \frac{1}{2} \times (5 + 7) \times 4 \\ \text{Area} &= 24 \text{ m}^2\end{aligned}$$

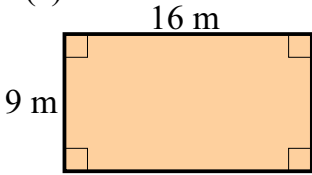
Circle

$$\begin{aligned}\text{Area} &= \pi \times r^2 \\ &= \pi \times 3^2 \\ \text{Area} &= 28.3 \text{ cm}^2 \text{ (1 dec. pl.)}\end{aligned}$$

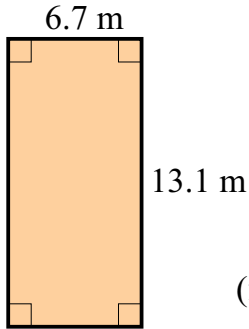
EXERCISE 10B

1. Find the area of each of the following rectangles and squares.

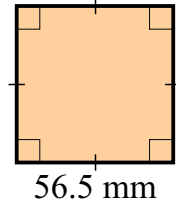
(a)



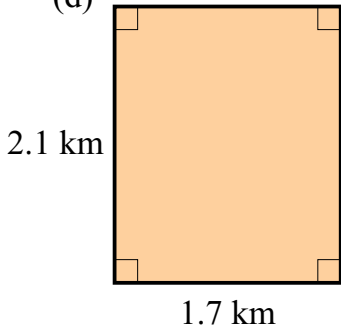
(b)



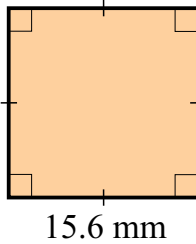
(c)



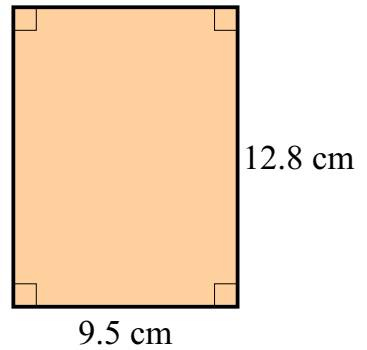
(d)



(e)

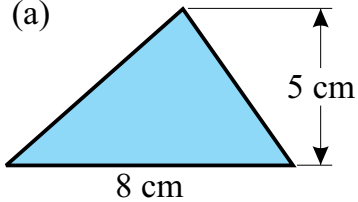


(f)

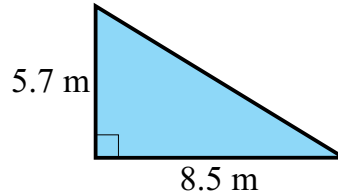


2. Find the area of each of the following triangles.

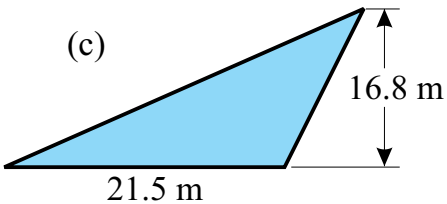
(a)



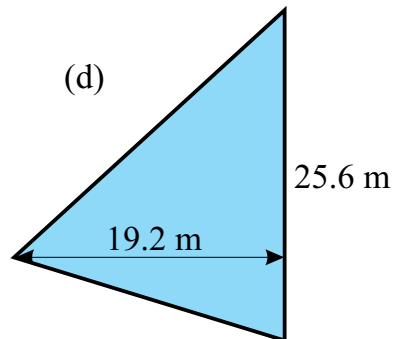
(b)



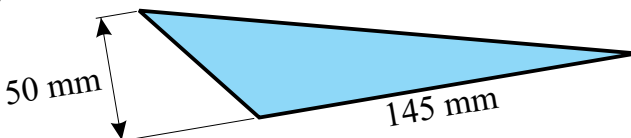
(c)



(d)

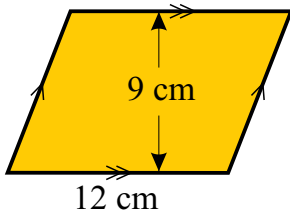


(e)

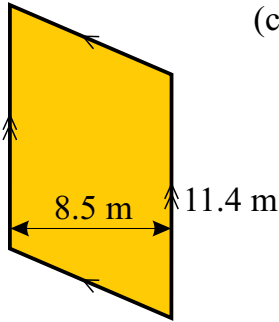


3. Find the area of each of the following parallelograms.

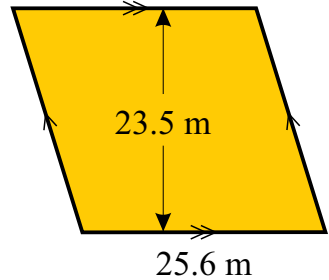
(a)



(b)

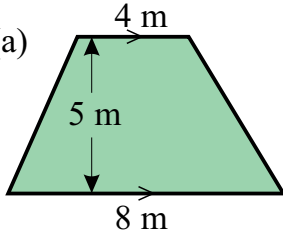


(c)

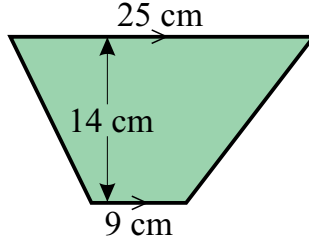


4. Find the area of each of the following trapeziums.

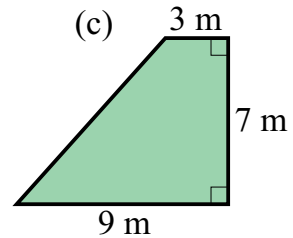
(a)



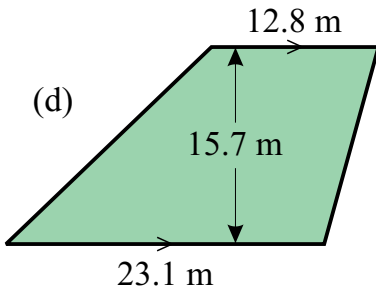
(b)



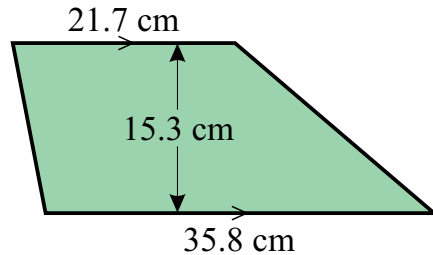
(c)



(d)



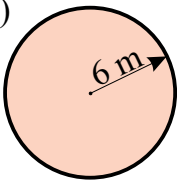
(e)



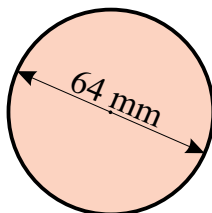
5. Find the area of each of the following circles.

Give answers correct to one decimal place.

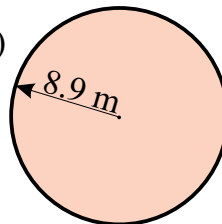
(a)



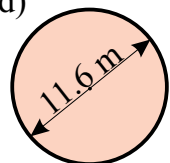
(b)



(c)



(d)

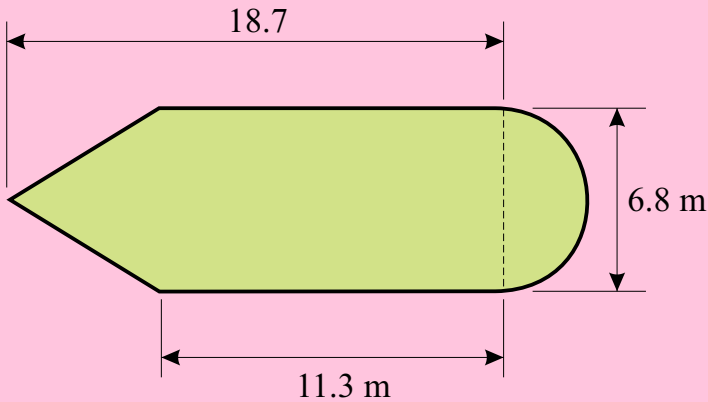


Composite Shapes

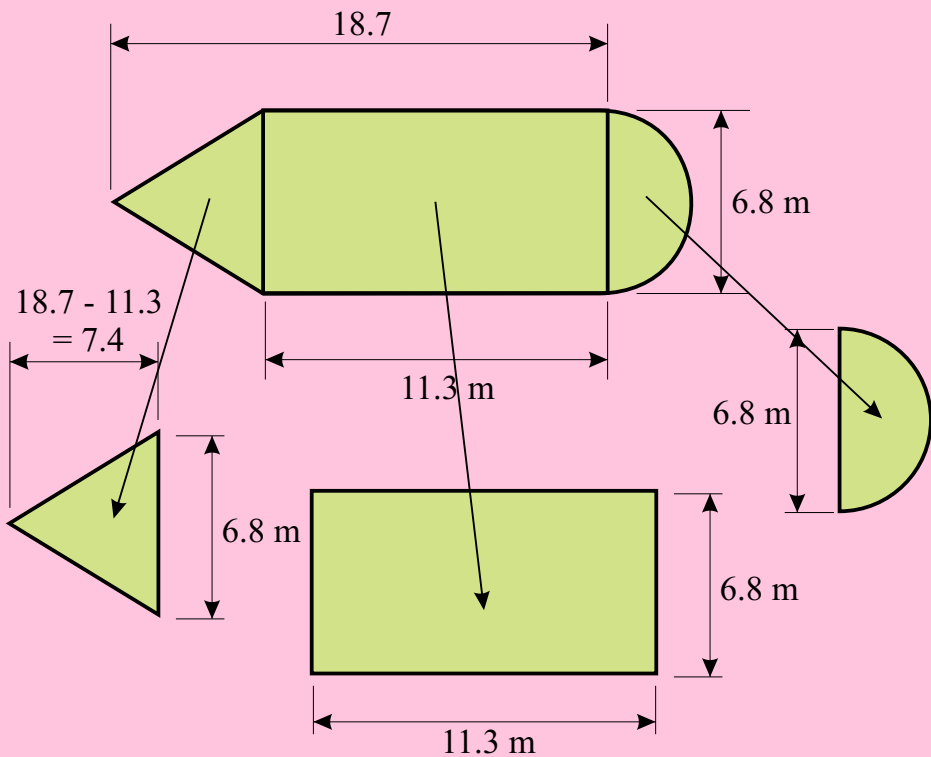
To find the area of a shape that is a composite of standard shapes calculate the area of each of the standard shapes and add or subtract them.

Examples

- Find the area of this shape. Give answer correct to two decimal places.



Step 1 Divide shape into standard shapes.

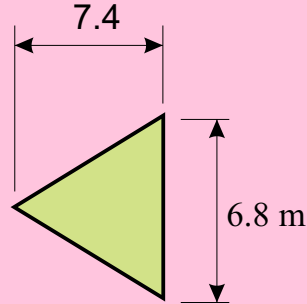


Step 2 Calculate the area of each standard shape.

Triangle

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 6.8 \times 7.4\end{aligned}$$

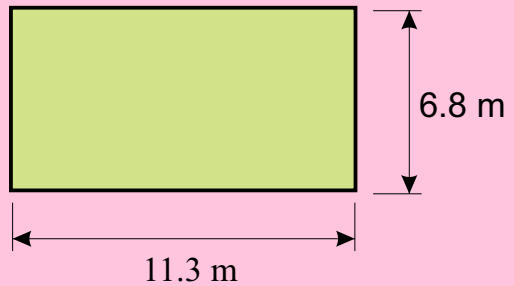
Area = 25.16 m²



Rectangle

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 11.3 \times 6.8\end{aligned}$$

Area = 76.84 m²

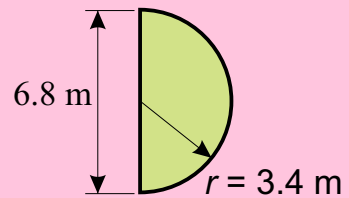


Half Circle

$$\begin{aligned}\text{Area of full circle} &= \pi \times r^2 \\ &= \pi \times 3.4^2\end{aligned}$$

Area of full circle = 36.32 m² (2 dec. pl.)

Area of half circle = 18.16 m² (2 dec. pl.)

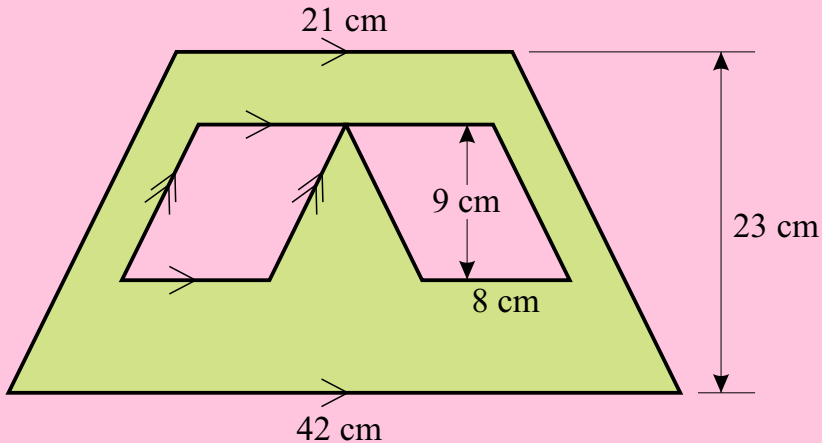


Step 3 Add all standard shape areas to find area of composite shape.

$$\text{Area} = 25.16 + 76.84 + 18.16$$

Area = 120.16 m²

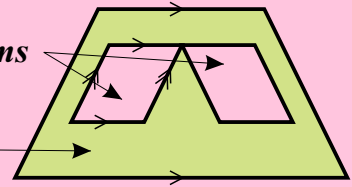
Example 2 Find the shaded area of this symmetrical shape.



Step 1 Recognise the standard shapes.

two parallelograms

trapezium



Step 2 The shaded area will be equal to the area of the trapezium minus the area of the two parallelograms.

Step 3 Calculate the areas.

$$\begin{aligned}
 \text{Area of trapezium} &= \frac{1}{2} (a + b)h \\
 &= \frac{1}{2} (21 + 42) \times 23 \\
 &= 724.5 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of each parallelogram} &= \text{base} \times \text{height} \\
 &= 9 \times 8 \\
 &= 72 \text{ cm}^2
 \end{aligned}$$

Step 4 Calculate the shaded area.

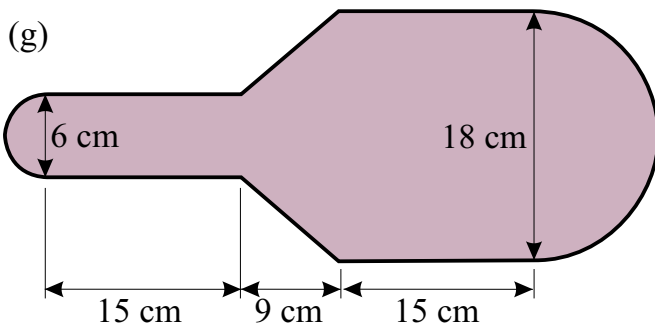
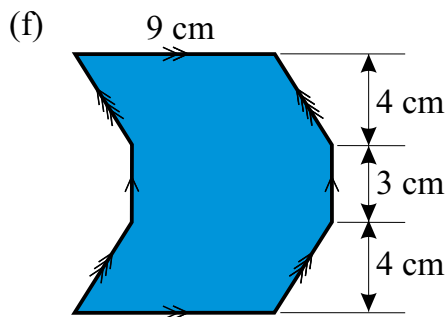
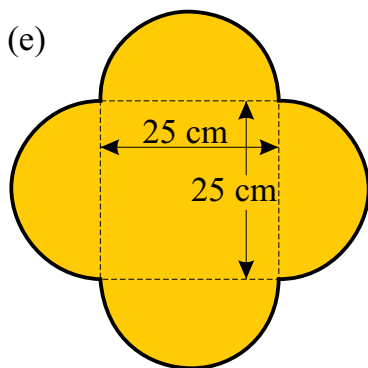
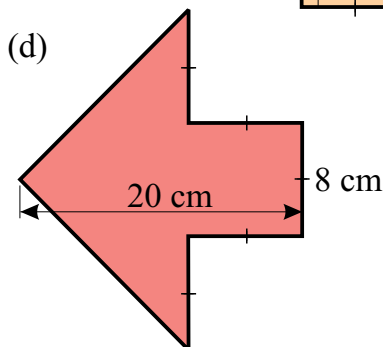
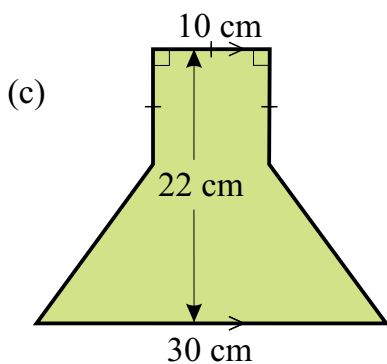
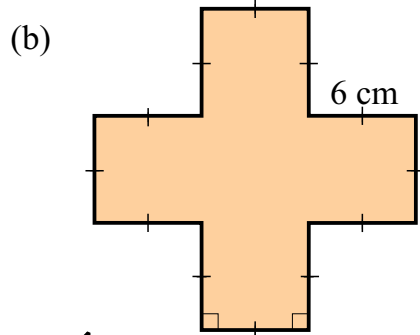
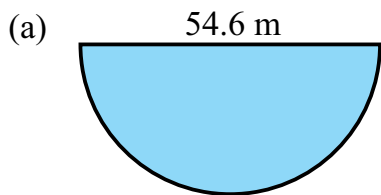
$$\text{Shaded area} = 724.5 - (2 \times 72)$$

$$\text{Shaded area} = 580.5 \text{ cm}^2$$

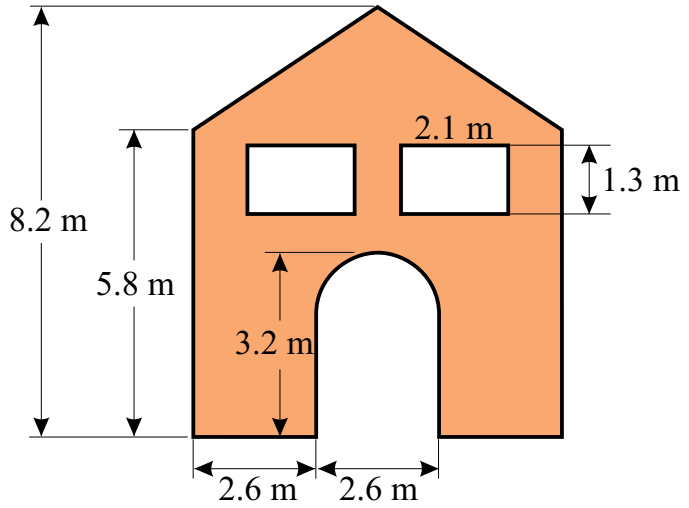
EXERCISE 10C

1. Find the area of each of the following shapes.

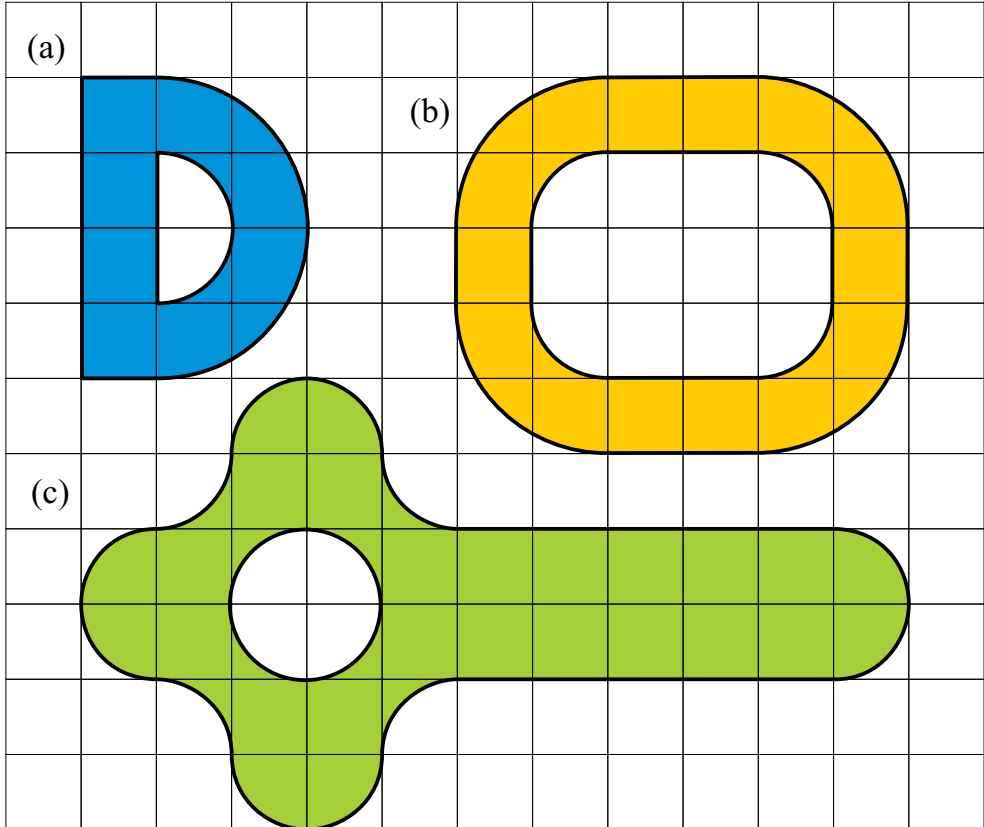
Give answers correct to one decimal place where appropriate.



2. Find the area of the *shaded* region in this symmetrical shape. Give answer correct to one decimal place.



3. The grid below consists of 1 cm squares. Find the area of each of the shaded regions on the grid. Give answers correct to one decimal place.



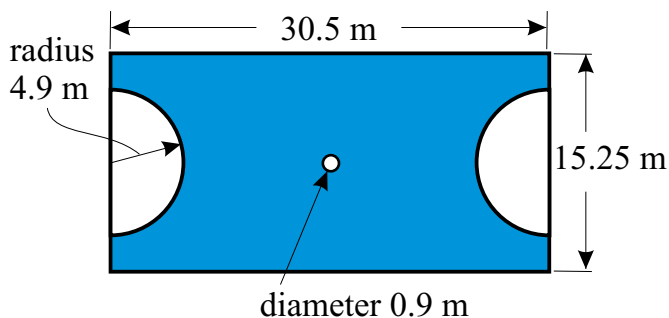
Practical Problems

EXERCISE 10D

1. The sports department at a school wanted to paint their netball court.

The dimensions of a netball court are shown here.

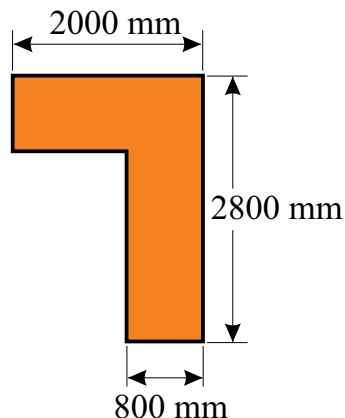
The goal circles and centre circle are to be painted white.
The remainder of the court is to be painted blue.



- (a) Find the area (to the nearest m^2) of: (i) white paint (ii) blue paint
(b) The court needed two coats of paint and each litre of paint covered 15 m^2 .
Find the number of litres (to one decimal place) of:
(i) white paint required (ii) blue paint required.

2. The kitchen bench shown here is to be tiled.

- (a) Find the area (in m^2) of the bench.
(b) The tiles are square with side length of 50 mm.
How many tiles will be needed to cover the bench?



3. Near Uluru there is a cattle station called Curtin Springs that is 1 028 960 acres.

- (a) Use the conversions at the start of this chapter to convert this area to: (i) hectares (to the nearest hectare)
(ii) km^2 (to the nearest km^2)
(b) If a property of this area was square, what would be the length of each side of the property? (Answer in km to one decimal place)
(c) Find the area (in hectares) of a property that is rectangular with dimensions $85 \text{ km} \times 54 \text{ km}$.

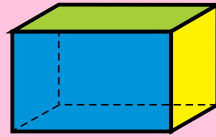
Nets

A **net**, in geometry, is defined as a collection of two dimensional shapes that when folded would form a three dimensional object.

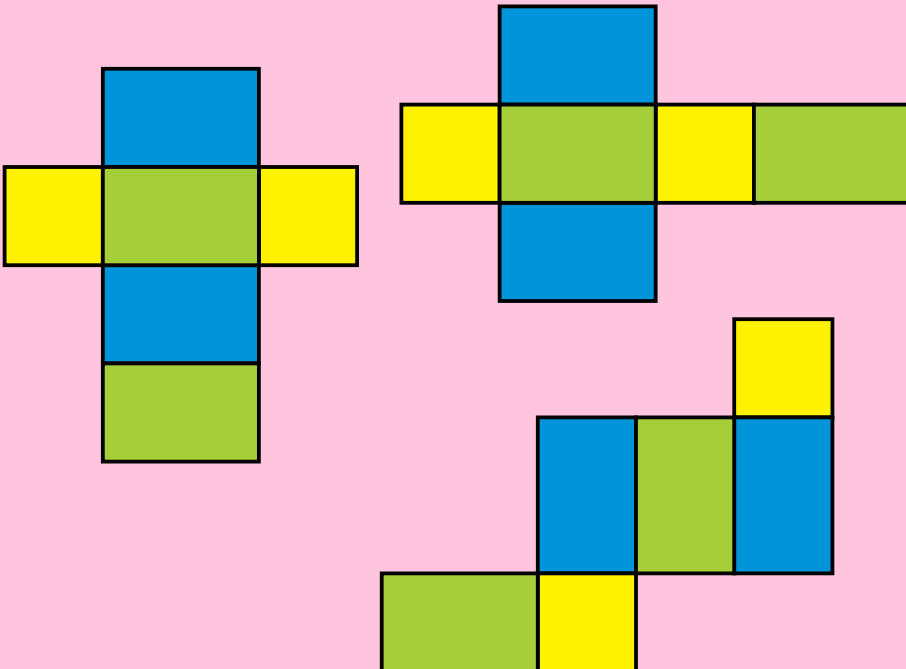
Example

The rectangular prism shown below is painted the following colours:

Top and bottom are green
Front and back are blue
 The *ends* are yellow



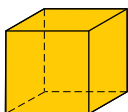
If this prism was 'unfolded' several nets could be formed. Three are shown below.



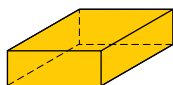
EXERCISE 10E

1. Match each object with its net.

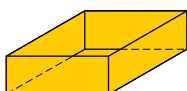
(a)



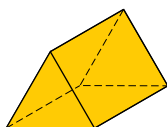
(b)



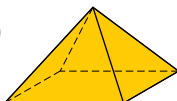
(c)



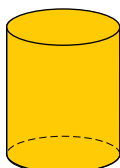
(d)



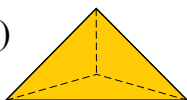
(e)



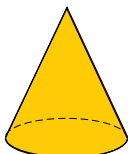
(f)



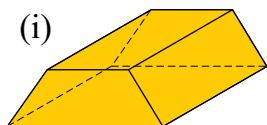
(g)



(h)



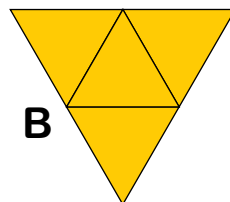
(i)



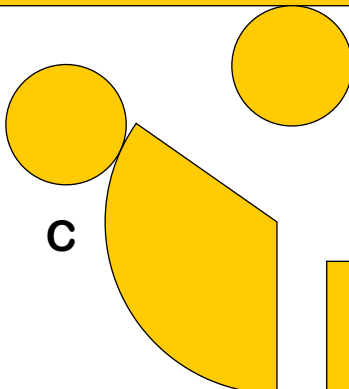
A



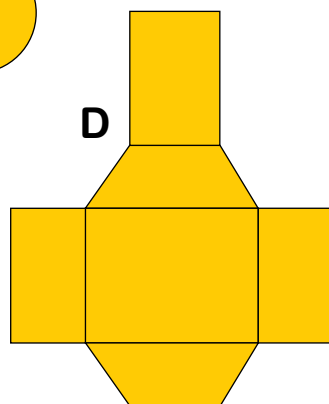
B



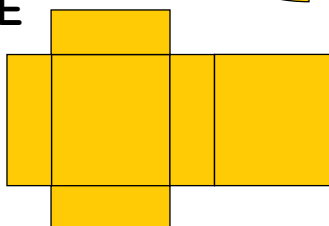
C



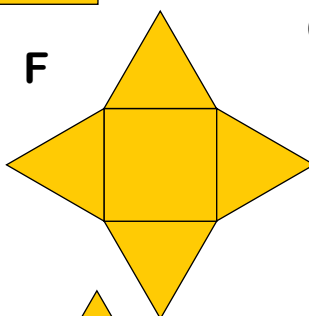
D



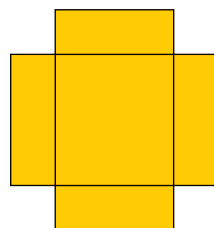
E



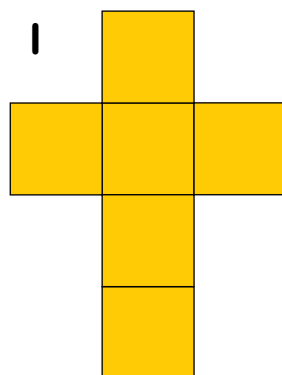
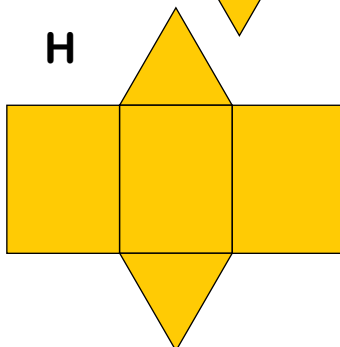
F



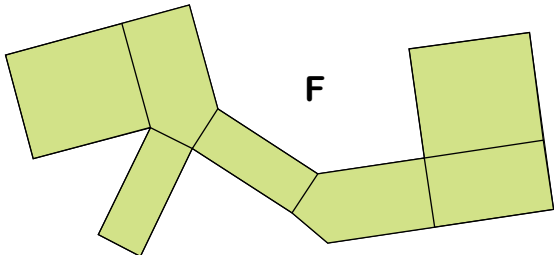
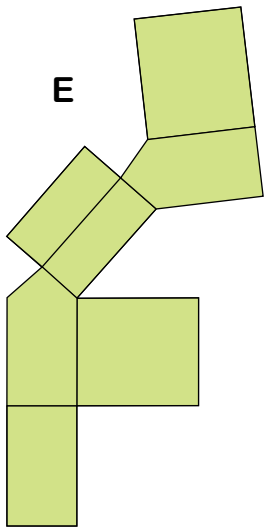
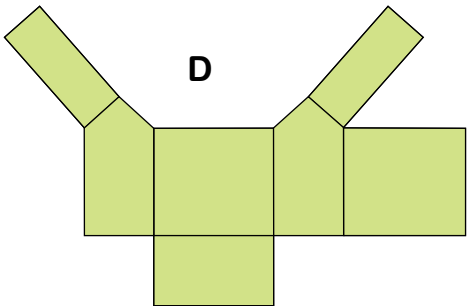
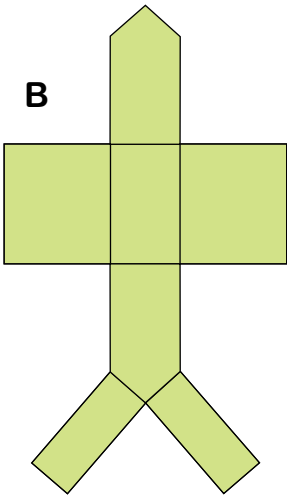
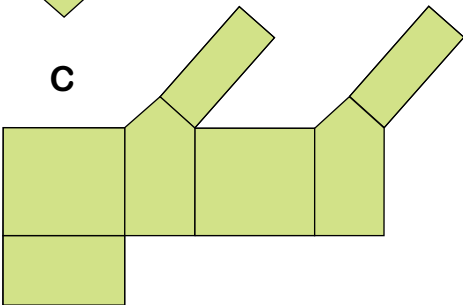
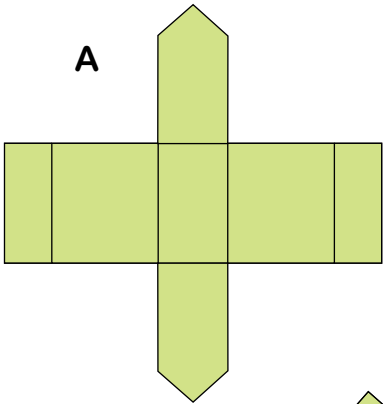
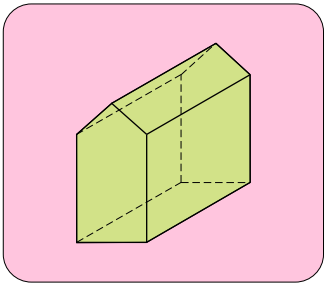
G



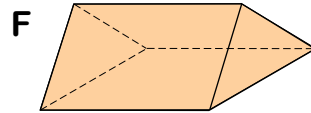
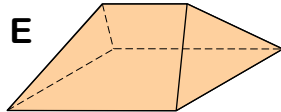
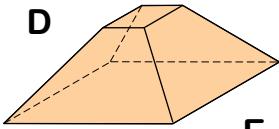
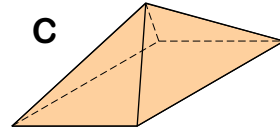
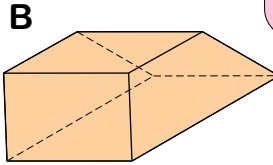
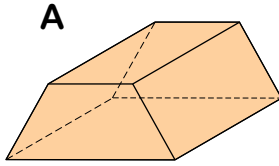
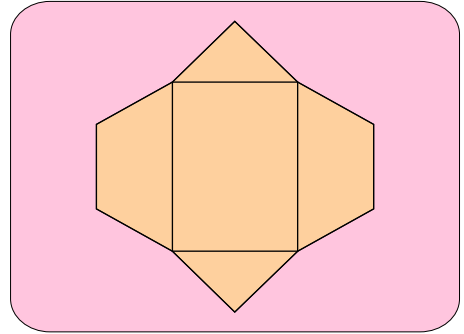
I



2. Which of the following nets could **NOT** form this object?



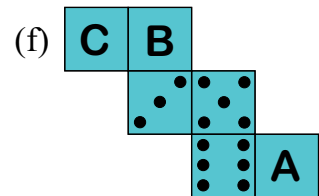
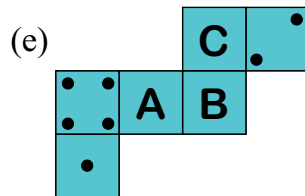
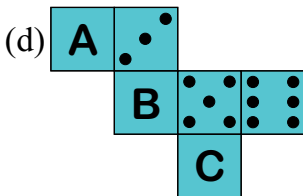
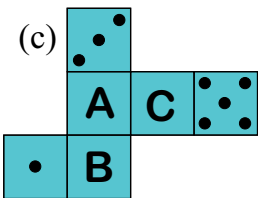
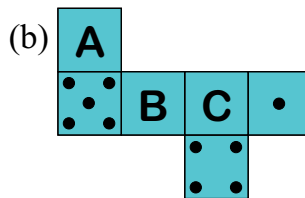
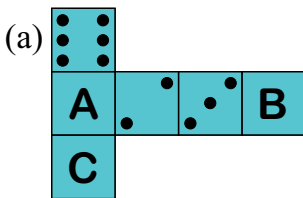
3. Which of the following objects **COULD** be formed from this net?



4. A normal die has all its opposite faces adding to 7. Find the number of dots on each of the unknown faces of the nets below so that when they are formed into dice the opposite faces add to 7.

Example

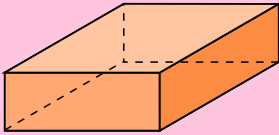
A = 3
B = 5
C = 6



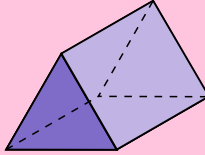
Surface Area of Prisms

A prism is a three dimensional object that has two ends that are the same shape joined by parallel lines.

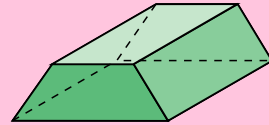
Examples



**Rectangular Prism
(Cuboid)**



Triangular Prism



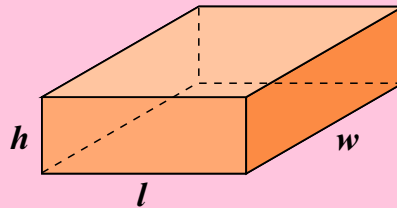
Trapezoidal Prism

The **surface area** of any object is the sum of the areas of all the faces of the object.

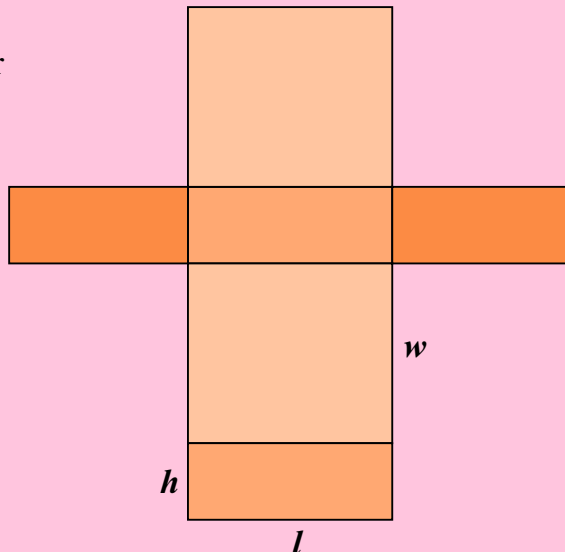
One technique for calculating the surface area is to use the net of the object to develop a formula.

Example *The Rectangular Prism*

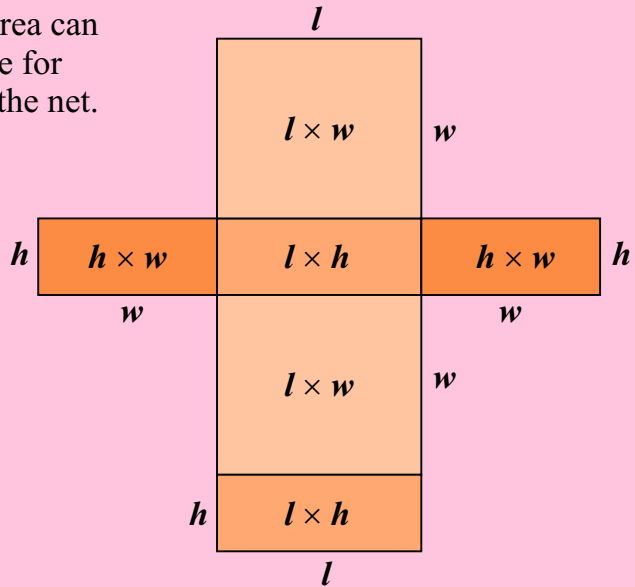
length = l
width = w
height = h



One net for this rectangular prism would be as shown here.



A formula for the surface area can be found by finding the rule for the area of each section of the net. The area of each section is shown on this diagram.



Surface Area = all these areas added

$$= (l \times w) + (l \times h) + (l \times w) + (l \times h) + (h \times w) + (h \times w)$$

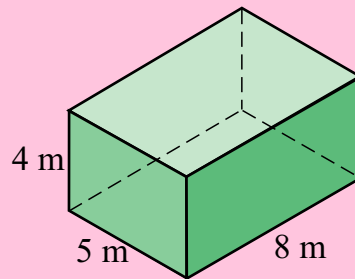
$$\text{Surface Area} = 2(l \times w) + 2(l \times h) + 2(h \times w)$$

$$\text{Surface Area} = 2lw + 2lh + 2hw$$

Example

Find the surface area of this rectangular prism.

$$\begin{aligned} l &= 5 \text{ m} \\ w &= 8 \text{ m} \\ h &= 4 \text{ m} \end{aligned}$$



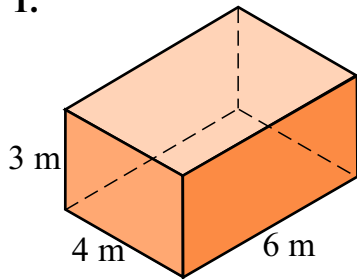
$$\begin{aligned} \text{Surface Area} &= 2(l \times w) + 2(l \times h) + 2(h \times w) \\ &= 2(5 \times 8) + 2(5 \times 4) + 2(4 \times 8) \\ &= 2 \times 40 + 2 \times 20 + 2 \times 32 \\ &= 80 + 40 + 64 \end{aligned}$$

$$\text{Surface Area} = 184 \text{ m}^2$$

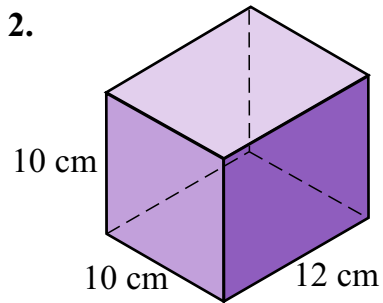
EXERCISE 10F

Find the surface area of each of the following rectangular prisms.

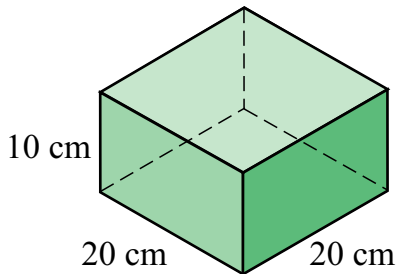
1.



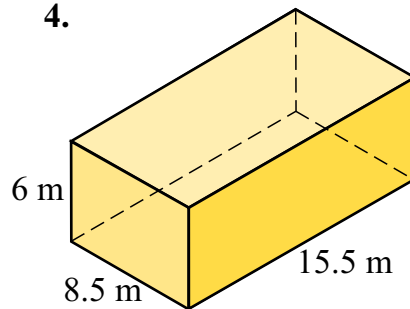
2.



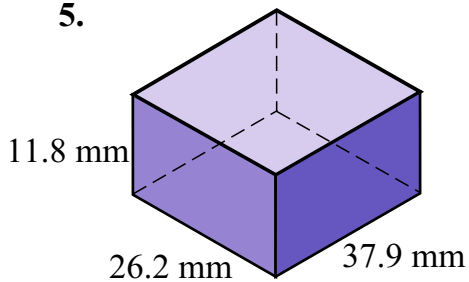
3.



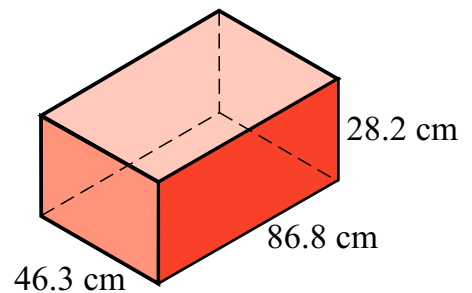
4.



5.

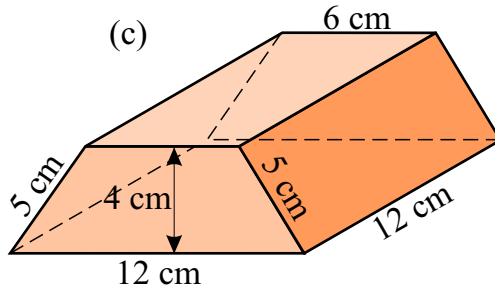
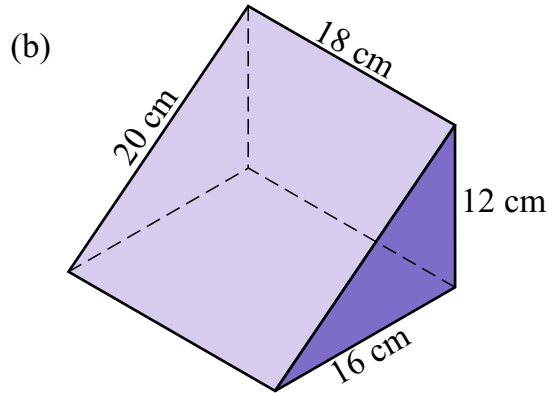
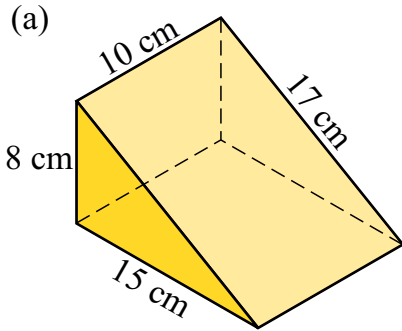


6.



EXERCISE 10G

1. Find the surface area of each of the following prisms.



2. Find the surface area of a cube with side length 9 cm.

3. Zeta's favourite colour was purple. She decided to paint her entire bedroom (walls, ceiling, floor and door) purple.

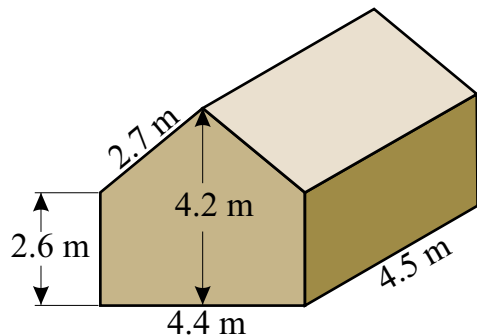
The room is $4.4 \text{ m} \times 3.7 \text{ m}$ and the walls are 2.8 m high.

There is a window ($2.1 \text{ m} \times 1.5 \text{ m}$) that Zeta did not paint!

What is the total area that Zeta painted?

4. Claudio is going to make the shed shown. He wants to calculate the amount of sheet metal he needs for the four walls and roof. He will cut the door out of one wall.

Calculate the total area of the four walls and two roof sections.



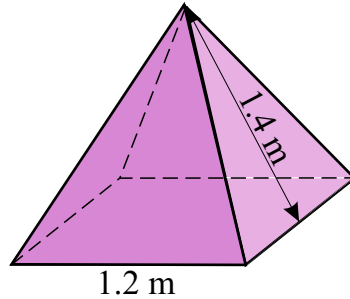
Other Objects

EXERCISE 10H

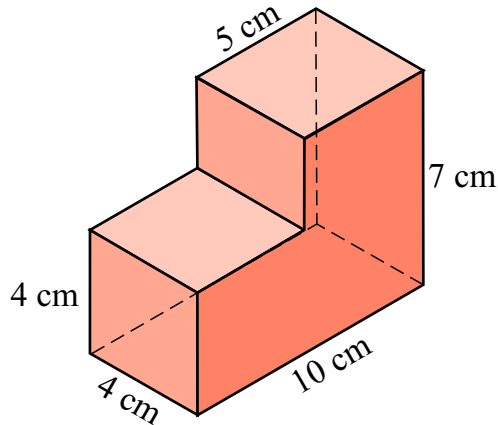
1. Paolo wants to make a model of a pyramid.

The dimensions of the pyramid are shown here.

The pyramid has a square base. Paolo is making the pyramid out of board (sides and base). What is the total area of board he will need?



2. Find the surface area of this object.

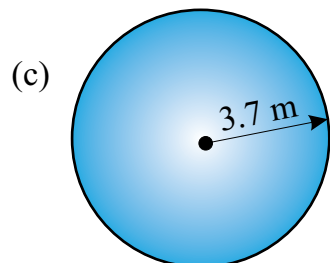
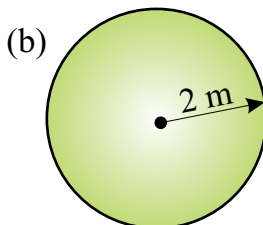
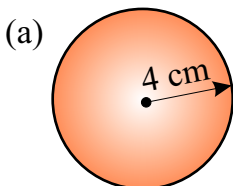


3. The surface area of a sphere (ball) is found by using the formula:

$$\text{Surface Area} = 4\pi r^2$$

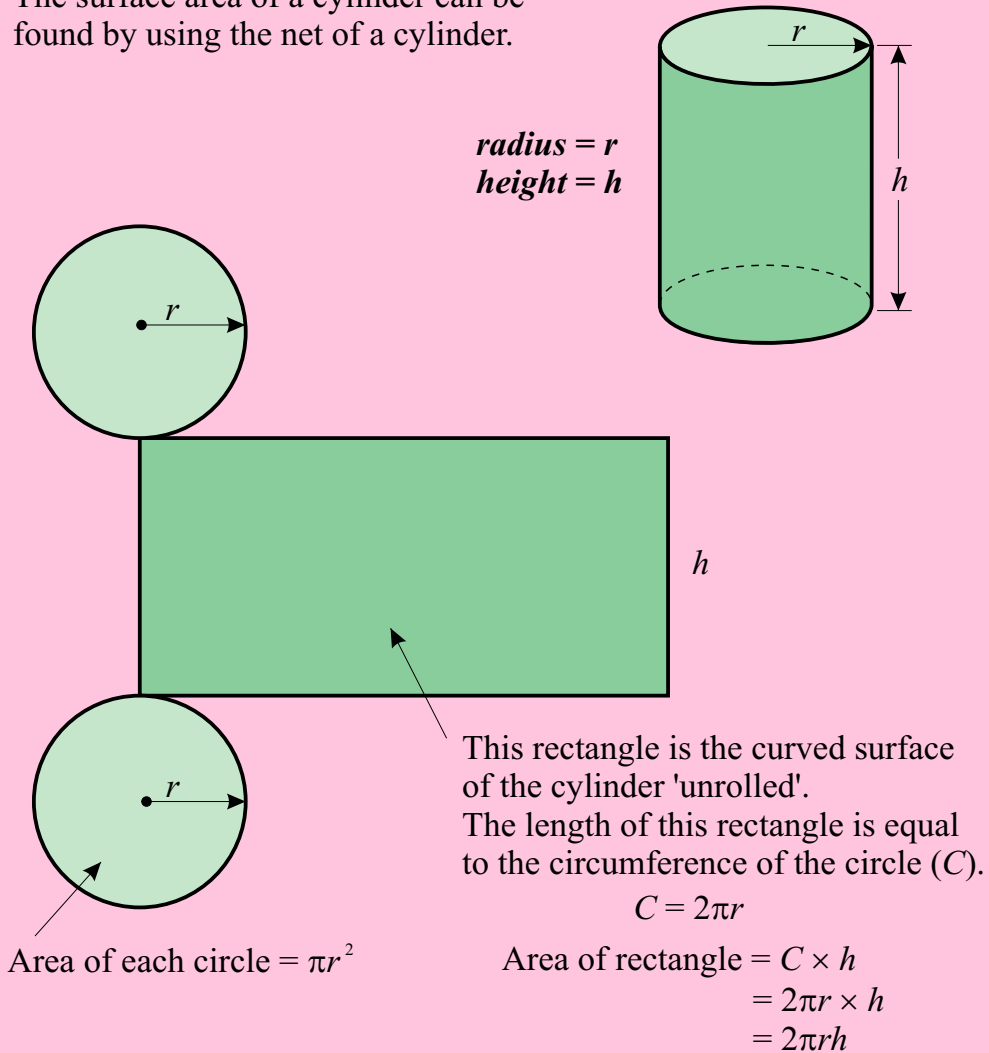
where r = the radius of the sphere

Find the surface area of the following spheres.
Give answers correct to one decimal place.



Cylinders

The surface area of a cylinder can be found by using the net of a cylinder.



The surface area of the cylinder is equal to the sum of the area of the two circles and the rectangle.

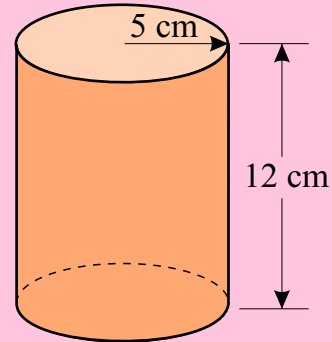
$$\begin{aligned}\text{Surface area} &= 2 \times \pi r^2 + 2\pi rh \\ &= 2\pi r^2 + 2\pi rh\end{aligned}$$

$2\pi r$ is a common factor to both these terms.

$$\text{Surface area} = 2\pi r(r + h)$$

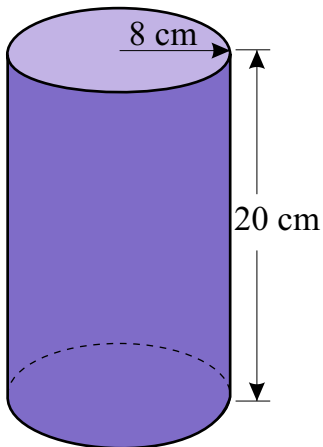
Example

$$\begin{aligned}
 \text{Surface area} &= 2\pi r(r + h) \\
 &= 2 \times \pi \times r \times (r + h) \\
 &= 2 \times \pi \times 5 \times (5 + 12) \\
 &= 2 \times \pi \times 5 \times 17 \\
 &= 534.07075 \text{ cm}^2 \\
 &= \mathbf{534.1 \text{ cm}^2} \text{ (1 dec. place)}
 \end{aligned}$$

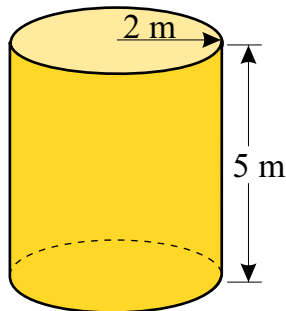
**EXERCISE 10I**

Find the surface area of the following cylinders.
Give answers correct to one decimal place.

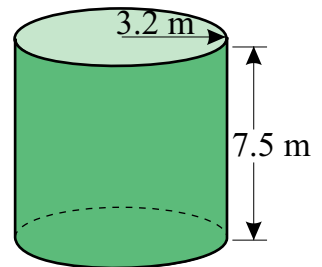
1.



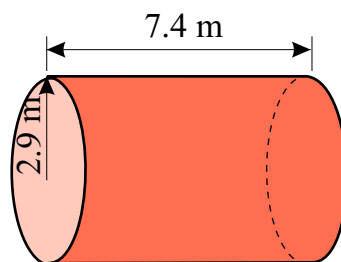
2.



3.



4.



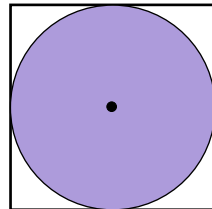
PROBLEM SOLVING

1. Clem is a farmer. He has a **square** paddock that is 4 hectares.

(a) What is the side length of this paddock?

(b) To water the paddock Clem places a large sprinkler in the centre of the paddock.

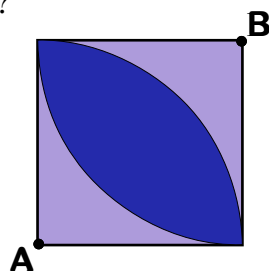
The sprinkler waters a circular area with a diameter that is the same as the length of the paddock.



(i) What area of the paddock is **not** watered?

(ii) What **percentage** of the paddock **is** watered?

(c) Clem tries another way of watering his paddock. He places a sprinkler at opposite corners of the paddock (**A** and **B**) and sets them to each water a 90° arc that has a radius equal to the length of the paddock.

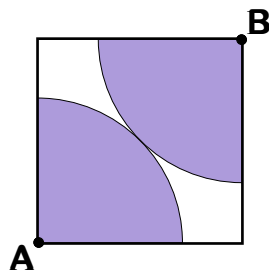


(i) What area of the paddock will be watered twice?

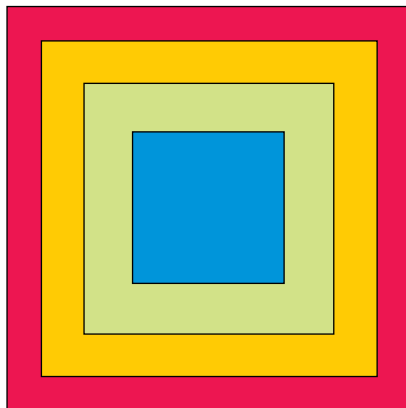
(ii) What **percentage** of the paddock will be watered twice?

(d) Clem tries another way of watering his paddock. With the sprinklers at the corners he adjusts the radius of each so they meet in the centre of the paddock.

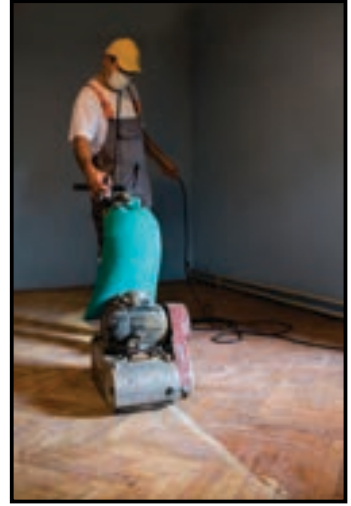
What area of the paddock is **not** watered? (You may need to use Pythagoras' Theorem)



2. Find the side length of each square in this shape so that each shaded region has an area of 100 cm^2 .

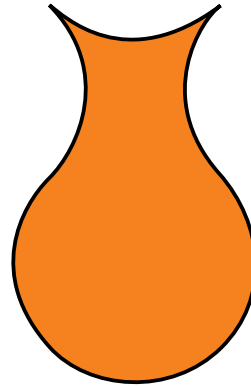


3. Sandy is a floor polisher. Josh is his apprentice. One job they had consisted of three rooms:
 Bedroom ($4 \text{ m} \times 3 \text{ m}$)
 Hallway ($10 \text{ m} \times 2 \text{ m}$)
 Living area ($9 \text{ m} \times 4 \text{ m}$)
 Sandy polished the floor in the hallway and Josh did the bedroom. They both took 4 hours to complete these rooms. They then work together in the living area. How long would it take them both to complete polishing the floor in the living area?

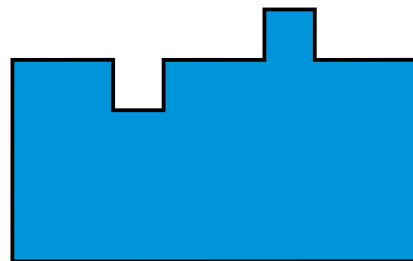
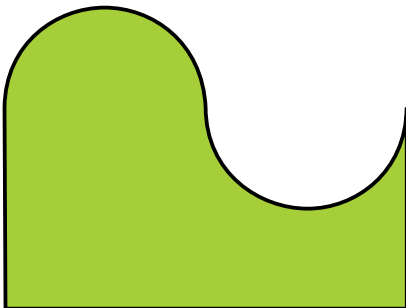


PUZZLES

1. How can this shape be cut into 3 parts using 2 straight lines so that the parts can be rearranged to form a square? All radii are the same.



2. Divide each of the shapes below into two pieces that are exactly the same.



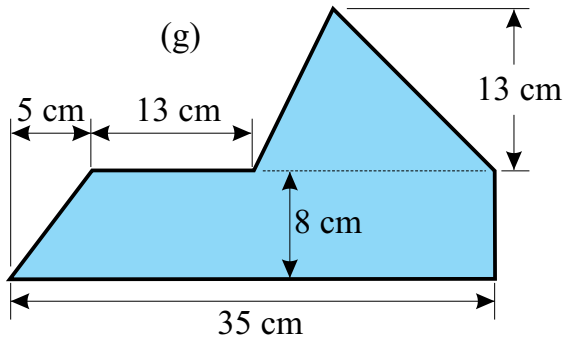
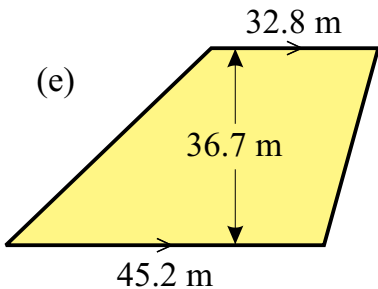
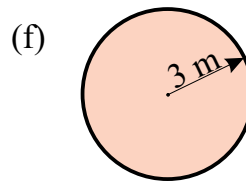
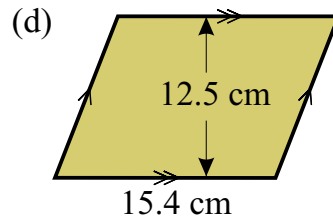
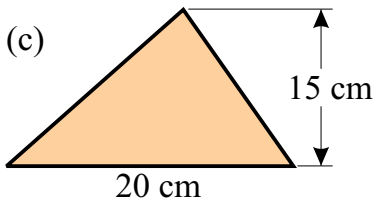
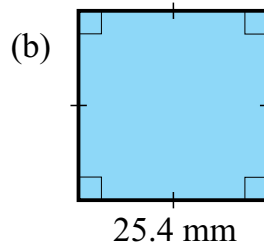
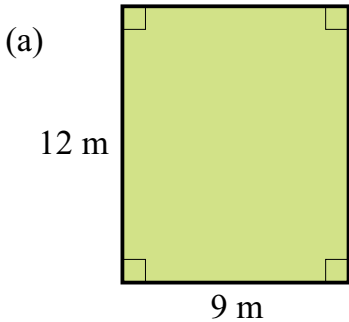
CHAPTER REVIEW

1. Convert the following areas to the units shown in the brackets.

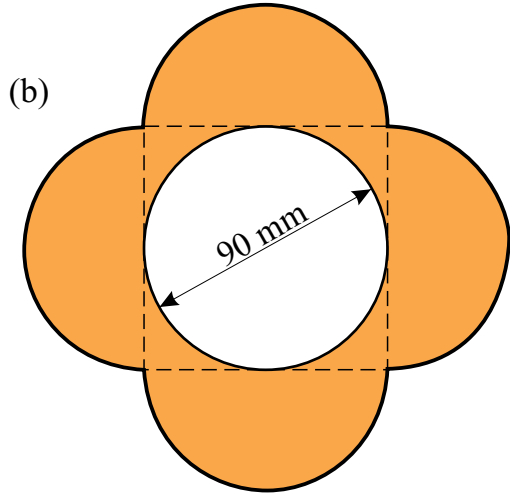
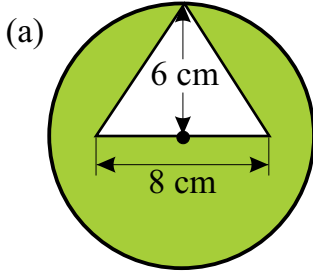
- | | |
|--|--|
| (a) $60\,000\text{ cm}^2$ (m^2) | (b) 5.8 cm^2 (mm^2) |
| (c) 9000 mm^2 (cm^2) | (d) 0.6 km^2 (m^2) |
| (e) 0.7 ha (m^2) | (f) $3\,100\,000\text{ m}^2$ (km^2) |
| (g) 0.0125 m^2 (cm^2) | (h) $40\,000\text{ m}^2$ (ha) |

2. Find the area of the following shapes.

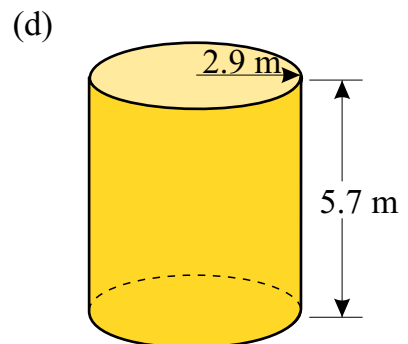
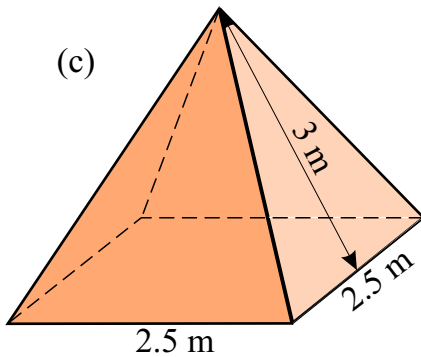
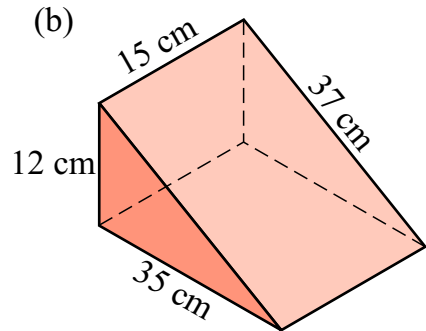
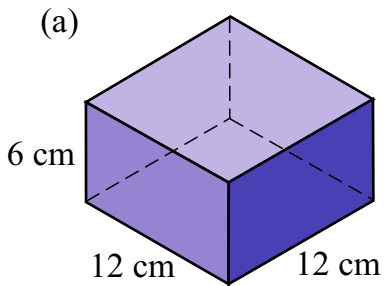
Give answers correct to one decimal place where appropriate.



3. Find the area of the shaded region in each of the following shapes.
Give answers to one decimal place.



4. Find the surface area of each of the following objects.



Volume

Common units of volume are mm^3 , cm^3 , litres and m^3 .

1 mm^3 is a volume equal to the volume of a cube with side length 1 mm.

1 cm^3 is a volume equal to the volume of a cube with side length 1 cm.

1 litre is a volume equal to the volume of a cube with side length 10 cm.

1 m^3 is a volume equal to the volume of a cube with side length 1 m.

$$1 \text{ cm}^3 = 1000 \text{ mm}^3$$

$$1 \text{ litre (L)} = 1000 \text{ cm}^3$$

$$1 \text{ m}^3 = 1000\,000 \text{ cm}^3$$

$$1 \text{ m}^3 = 1000 \text{ litres}$$

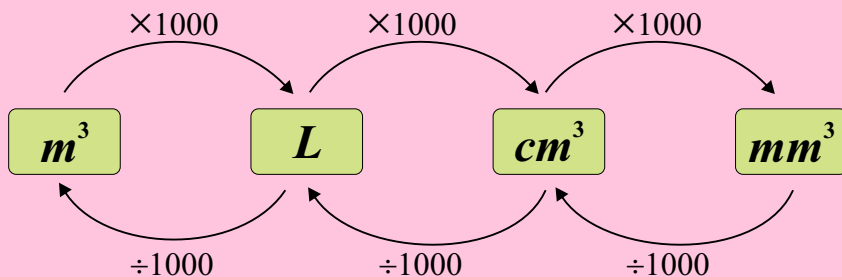
$$1 \text{ cm}^3 = 1 \text{ mL}$$

The millilitre (mL) is often used as a unit of volume.

1 millilitre (mL) is equal to 1 cubic centimetre (cm^3).

Cubic centimetre is often abbreviated to c.c.

These diagrams can be used to find the conversion factor when converting from one unit to another.



Volume

EXERCISE 11A

1. Convert the following volumes to the units shown in the brackets.

Examples

$$\begin{aligned} 1. \quad & 4 \text{ cm}^3 \text{ (mm}^3\text{)} \\ &= 4 \times 1000 \\ &= \mathbf{4000 \text{ mm}^3} \end{aligned}$$

$$\begin{aligned} 2. \quad & 7800 \text{ cm}^3 \text{ (L)} \\ &= 7800 \div 1000 \\ &= \mathbf{7.8 \text{ L}} \end{aligned}$$

$$\begin{aligned} 3. \quad & 0.52 \text{ m}^3 \text{ (L)} \\ &= 0.52 \times 1000 \\ &= \mathbf{520 \text{ L}} \end{aligned}$$

(a) $50\,000 \text{ cm}^3 \text{ (L)}$

(b) $0.02 \text{ cm}^3 \text{ (mm}^3\text{)}$

(c) 600 mL (L)

(d) $8.1 \text{ m}^3 \text{ (L)}$

(e) 0.25 L (mL)

(f) $890\,000 \text{ mm}^3 \text{ (cm}^3\text{)}$

(g) $300 \text{ L (m}^3\text{)}$

(h) $7 \text{ mL (mm}^3\text{)}$

(i) $8 \text{ cm}^3 \text{ (mL)}$

(j) $100\,000 \text{ mm}^3 \text{ (L)}$

(k) $2.4 \text{ L (cm}^3\text{)}$

(l) $0.75 \text{ mL (cm}^3\text{)}$

2. Rani bought 8 bottles of drink for a party she was organising.

Each bottle contained 750 mL.

How many *litres* of drink did she have?

3. Complete the sentences below by choosing the most appropriate unit.

m^3 L cm^3 mm^3

(a) It is recommended that people drink $1\frac{1}{2}$ ____ of water each day.

(b) Arwen had to take 15 ____ of cough medicine each day.

(c) Oliver calculated the volume of a drop of water to be 50 ____.

(d) Ed needed 2 ____ of concrete for the floor of his garage.

(e) Layla's backyard swimming pool had a capacity of 30 000 ____.

(f) There is about 5 ____ of blood in the human body.

(g) Sarah had a scoop of ice cream that was 120 ____.

(h) The average cow produces about 50 ____ of milk per day.

(i) Nicki ordered 3 ____ of firewood for winter.

(j) Xena's car had a 1.8 ____ engine.

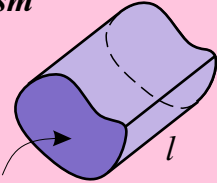
4. Terri bought a 10 litre container of water.
How many 200 mL glasses could be filled from this water container?
5. Mustafa estimated that he could lift 4 litres of sand with each scoop of his shovel.
How many shovel loads would he need to move 3 m^3 of sand?
6. Frankie had to put drops in her eyes. The bottle of eye drops contained 20 mL. Each drop was 50 mm^3 . Frankie had to put two drops in each eye twice a day.
How many days will the 20 mL bottle last?
7. Rahul donates half a litre of blood every three months.
His blood was collected at the rate of 50 cm^3 every minute.
How many minutes would it take to collect half a litre of blood at this rate?



8. Sasha has 50 rose bushes. She waters them using a drip watering system. Water is delivered to each bush at the rate of 1 cm^3 every second. Sasha waters her rose bushes for two hours twice a week.
(a) How many litres of water would she use each week on her rose bushes?
(b) Sasha is told to reduce this to 12 litres per plant per week.
For how many minutes should she now run her watering system per week?
9. Giaan bought 3 m^3 of soil for her garden. Her wheelbarrow holds 100 litres and she could fill the wheelbarrow five times an hour.
How many hours will it take her to move all the soil?

Volume of Regular Objects

Prism



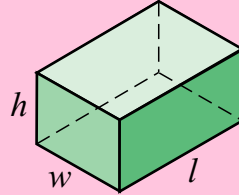
Area = A

Volume = area of end $\times$ length

$$V = A \times l$$

$$V = Al$$

Rectangular Prism (Cuboid)

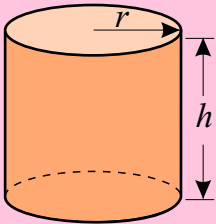


Volume = length $\times$ width $\times$ height

$$V = l \times w \times h$$

$$V = lwh$$

Cylinder

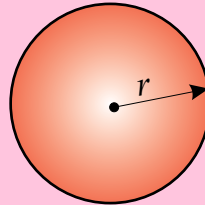


Volume = $\pi r^2 \times$ height

$$V = \pi r^2 \times h$$

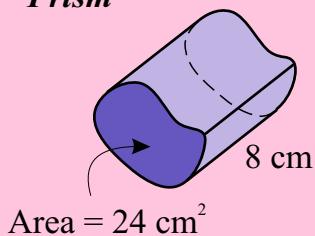
$$V = \pi r^2 h$$

Sphere



Volume = $\frac{4}{3} \pi r^3$

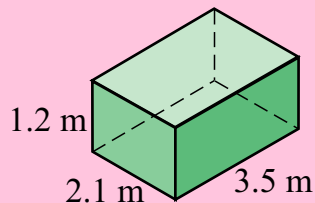
$$V = \frac{4}{3} \pi r^3$$

*Examples**Prism*

Volume = area of end $\times$ length

$$V = 24 \times 8$$

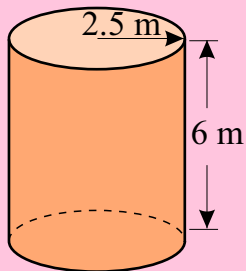
$$V = 192 \text{ cm}^3$$

Rectangular Prism (Cuboid)

Volume = length $\times$ width $\times$ height

$$V = 3.5 \times 2.1 \times 1.2$$

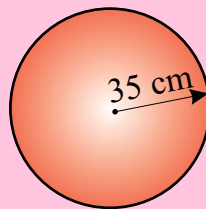
$$V = 8.82 \text{ m}^3$$

Cylinder

Volume = $\pi r^2 \times$ height

$$V = \pi \times 2.5^2 \times 6$$

$$V = 117.8 \text{ m}^3 \text{ (1 dec. place)}$$

Sphere

Volume = $\frac{4}{3} \pi r^3$

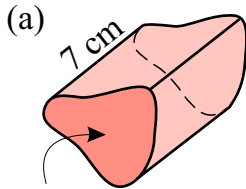
$$V = \frac{4}{3} \pi \times 35^3$$

$$V = 179\,594.4 \text{ cm}^3$$

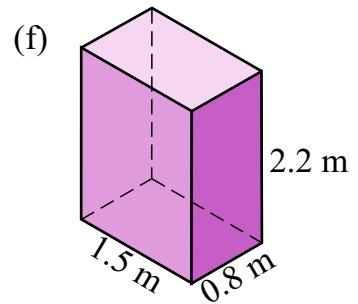
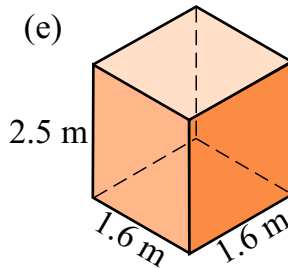
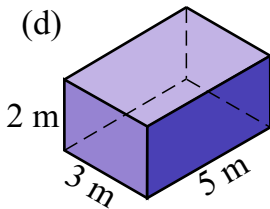
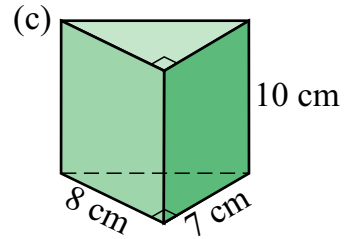
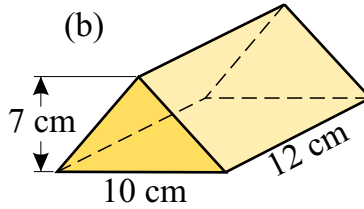
$$V = 179.6 \text{ L (1 dec. place)}$$

EXERCISE 11B

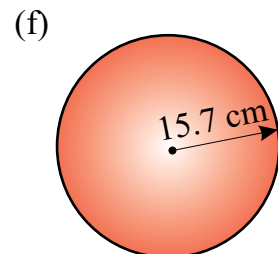
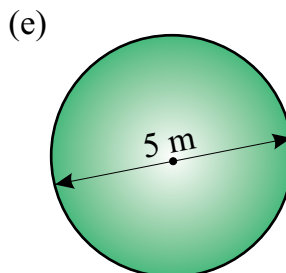
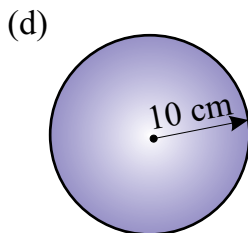
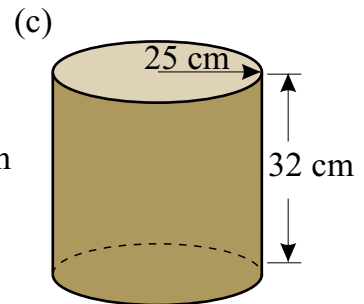
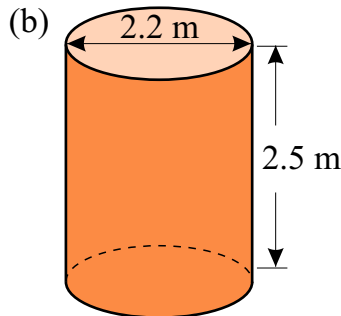
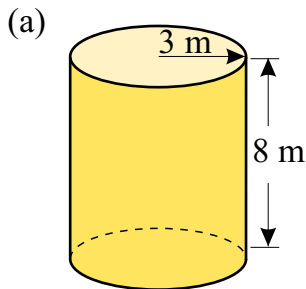
1. Find the volume of each of the following objects.



Area = 30 cm^2

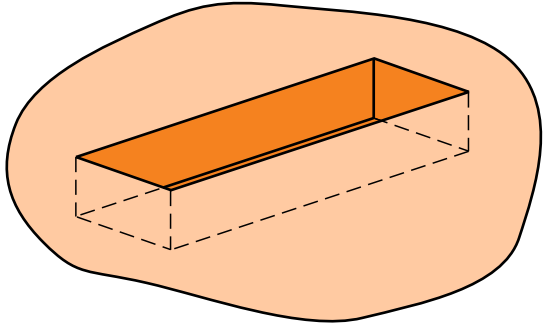


2. Find the volume of each of the following objects.
Give answers to one decimal place.



3. The trench shown here is to be filled with concrete as the base for a wall.

The trench is 5 m long, 90 cm wide and 50 cm deep. What volume of concrete (m^3) is needed to fill this trench?



4. A grain container on a train is 10 m long, 4 m wide and 3 m deep. A farmer has a cylindrical silo of wheat that is full to the top of the cylinder (not in the roof cover). The silo has a diameter of 15 m and is 10 m high.

- What is the capacity of a grain container on the train?
- What volume of wheat is stored in the silo? (Answer to one decimal place)
- How many grain containers will be needed to transport all of the grain from the silo?



5. Phaedra has a sphere of solid glass that she wanted to melt and form into a cube. The sphere had a diameter of 20 cm. What would be the side length (in cm) of the cube? Give answer to one decimal place.
6. The roof area of Sachin's house is 200 m^2 . He has a water tank that collects all the rain that falls on the roof.
- If 10 mm of rain falls, calculate the volume of water (in litres) that will be collected in the tank.
 - If the water tank holds 20 000 litres, how many millimetres of rain would need to fall to fill the tank if it was empty?
 - If the average annual rainfall is 75 cm, find the total volume of water (in litres) collected in a year.
 - Sachin calculated that his household uses an average of 500 litres of water per day. In an average rainfall year, is enough rain collected to supply the household needs?

7. A container company has two sizes of containers.

Container **A** is a cube (length, width and height are the same).

Container **B** is the same height as container **A** but is one metre wider and two metres longer than container **A**.

Container **B** has three times the volume of container **A**.

What are the dimensions of each container?

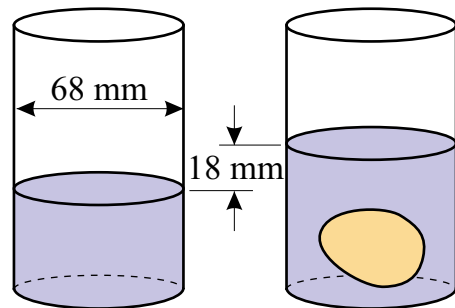
8. (a) *Estimate* the volume (in cm^3) of an egg.

- (b) *Calculate* the volume of an egg using the following information.

An egg was placed into a cylindrical drinking glass containing some water.

The diameter of the glass was 68 mm.

The water level rose 18 mm when the egg was placed in the water. Give answer to one decimal place.



9. Julia has a fish tank that is 120 cm long, 50 cm wide and 50 cm high.

- (a) Julia decides to have the water level 5 cm from the top of the tank. How many litres of water will she need to pour into the tank to achieve this?

- (b) If she wants to have one fish for every 15 litres of water, how many fish should she purchase?

- (c) Julia added four rocks to the fish tank and the water level rose by 6 mm. What was the average volume (in cm^3) of each of these rocks?

- (d) Julia buys a ceramic model of an old diver's helmet and puts this into the tank. The model is spherical in shape with a diameter of 30 cm. How far will the water level rise when she puts the model into the water? (Give answer in cm to one decimal place).

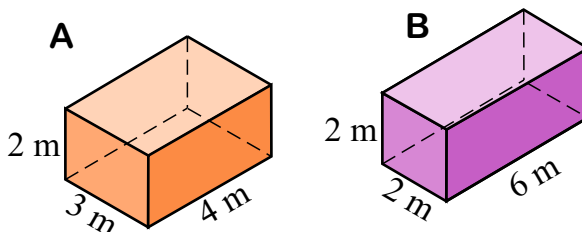


Volume and Surface Area

EXERCISE 11C

Refer to the chapter on Surface Area for information needed on surface area in the following questions.

1. (a) Calculate the volume of each of these two rectangular prisms.
- (b) Calculate the surface area of each of these rectangular prisms.
- (c) Find the height of a rectangular prism that has a length of 8 m, width of 1 m and the same volume as the two rectangular prisms shown here.
- (d) Copy and complete the table below showing length, width, height and surface area of rectangular prisms that all have the same volume.



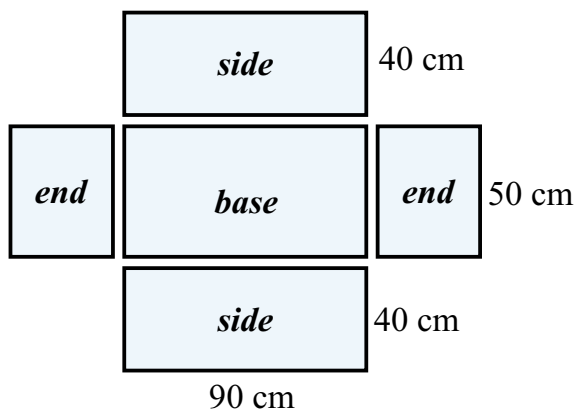
<i>Length (m)</i>	<i>Width (m)</i>	<i>Height (m)</i>	<i>Surface Area (m²)</i>
4	3	2	
6	2	2	
8	1		
	1	1	
4		1	
	2	1	

2. (a) Find the volume of a rectangular prism with a length of 4 cm, width of 2 cm and height of 1 cm.
- (b) Find the surface area of this rectangular prism.
- (c) Find the surface area of a **cube** that has the same volume as this rectangular prism.

3. (a) Find the surface area of a rectangular prism with a length of 11 cm, width of 2 cm and height of 2 cm.
 (b) Find the volume of this rectangular prism.
 (c) Find the volume of a **cube** that has the same surface area as this rectangular prism.

4. Kory cuts five pieces of glass that he is going to use to construct a fish tank. The pieces of glass and their dimensions are shown here.

- (a) How many litres of water will the fish tank hold?
 (b) The glass cost \$60 per square metre.
 What was the cost of the glass for Kory's fish tank?

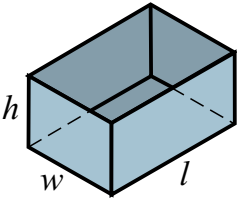
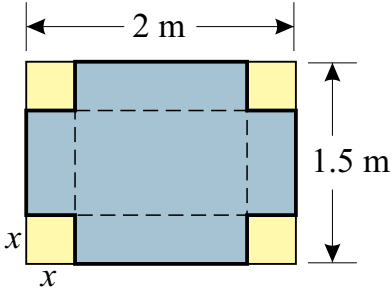


5. Many drinks are packaged in containers that are rectangular prisms. (Some have a triangular top as shown in this picture)

- (a) What is the volume of a drink container that has a square base with side length 73 mm and a height of 188 mm. Give answer in litres.
 (b) What would be the height of a two litre drink container that has a square base with side length of 97 mm? Give answer to the nearest mm.
 (c) Find the height of a one litre drink container that has base dimensions of 92 mm and 58 mm. Give answer to the nearest mm.
 (d) What is the volume of a drink container that has dimensions of 61 mm, 38 mm and 108 mm? Give answer to the nearest mL.
 (e) What would be the side length of a square based one litre drink container that has a height of 244 mm? Give answer to the nearest mm.



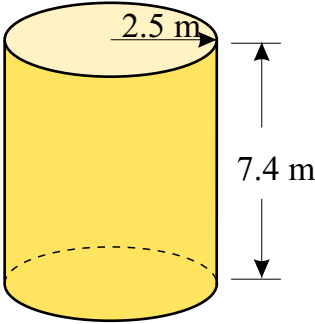
6. Gerry wants to make a water trough for his horse. He has a sheet of metal that is 2 metres long and 1.5 metres wide. He plans to cut squares pieces from the corners, fold the metal along the dotted lines and weld the joins to form a rectangular prism that he can fill with water. He wants the trough to hold as much water as possible so he investigates the effect the size of the square cutout side length (x) has on the volume of water the resultant trough could hold.



- (a) Copy and complete the table below showing the length (l), width (w) and height (h), all in cm, of the water trough and the volume of water (V), in litres, the trough will hold for different sized square cutouts.
- (b) Based on these figures, what is side length of the cutout that would result in a trough that would hold the most water?

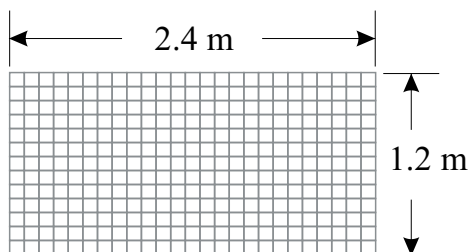
x (cm)	l (cm)	w (cm)	h (cm)	V (L)
10				
20				
30				
40				
50				
60				

7. (a) Find the volume of the cylinder shown here. Give answer correct to four decimal places.
- (b) Convert this to litres. Give answer to the nearest litre.
- (c) Find the surface area of this cylinder. Give answer correct to one decimal place.

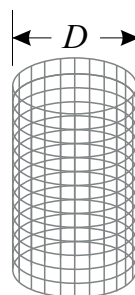


8. Georgina wanted to make a compost bin by rolling a sheet of wire mesh into the shape of a cylinder.

The dimensions of the wire mesh are shown here.

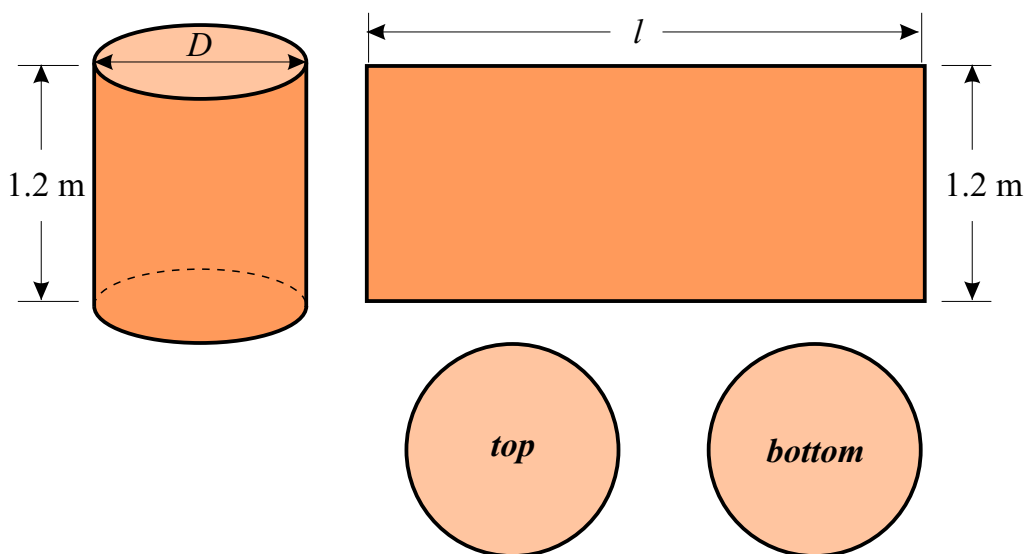


- What is the circumference of the mesh cylinder?
- What is the diameter (D) of the mesh cylinder? Give answer correct to three decimal places.
- What volume of compost could this mesh cylinder hold? Give answer to the nearest litre.
- Another sheet of wire mesh has the dimensions of 3.0 m long and 1.5 m high. If this sheet of wire mesh was rolled to form a cylinder, what would be the volume of compost it would hold? Give answer to the nearest litre.



9. Larry wants to make a 200 litre cylindrical water tank from sheet metal. He will use three pieces of sheet metal:- a rectangle of length l and width 1.2 m to form the cylinder and two circles for the top and bottom. The sheet metal he uses has a width of 1.2 metres.

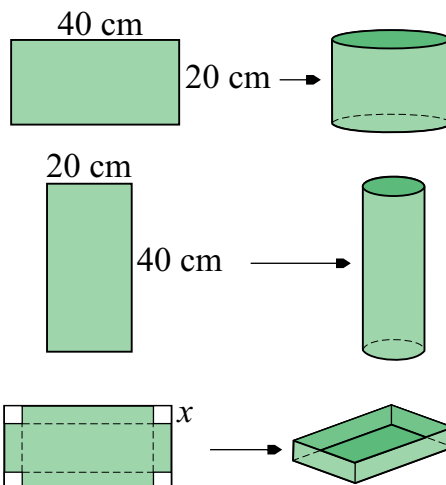
- Find the diameter of the cylindrical water tank. Give answer to the nearest mm.
- Find the length of the rectangular sheet of metal. Give answer to the nearest mm.



PROBLEM SOLVING

1. Jorja has a sheet of board ($40\text{ cm} \times 20\text{ cm}$) that she wants to form into a container. She can think of three different options.

- (i) Roll the board to form a cylinder 20 cm high and add a circular base.
- (ii) Roll the board to form a cylinder 40 cm high and add a circular base.
- (iii) Cut out a square in each corner and fold to form a box.



- (a) Find the radius of each of the two cylinders. (To one dec. pl.)
 - (b) Find the volume of each of the two cylinders. (To one dec. pl.)
 - (c) Find the volume of the box if the square cut out has a side length (x) of 2 cm.
 - (d) Use a different value for x and find the volume of the box.
 - (e) Find the value for x (to one decimal place) that would give the box with the maximum volume. Use of a spreadsheet will make this process quicker.
2. Gabriel likes to make his own salad dressing using olive oil and vinegar. He has 600 mL of each liquid. One day he thought he would try a new combination of the olive oil and vinegar. He poured 200 mL of the olive oil into the vinegar container. After mixing, he then poured 200 mL from the vinegar and oil mix back into the olive oil.
- (a) Find the amount of olive oil in the vinegar.
 - (b) Find the amount of vinegar in the olive oil.

PUZZLE

Explain how a 9 litre and a 4 litre container can be used to measure exactly 6 litres.

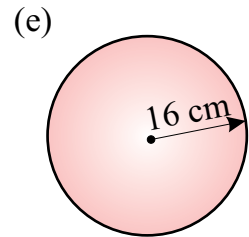
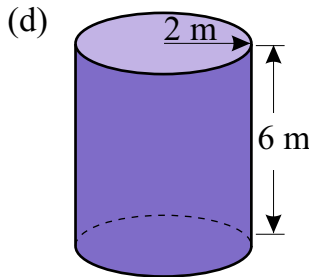
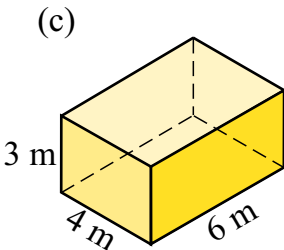
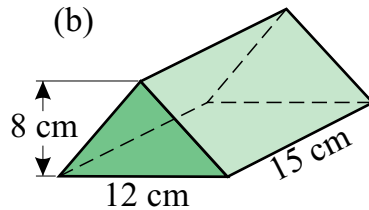
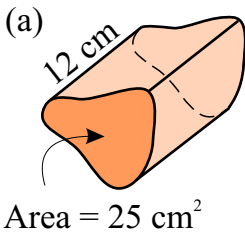
CHAPTER REVIEW

1. Convert the following volumes to the units shown in the brackets.

- | | |
|--|--|
| (a) 8000 cm^3 (L) | (b) 7 cm^3 (mm^3) |
| (c) 900 mL (L) | (d) 0.55 m^3 (L) |
| (e) 0.01 L (mL) | (f) $30\,000 \text{ mm}^3$ (cm^3) |
| (g) 750 L (m^3) | (h) 9.35 mL (mm^3) |
| (i) 15.6 cm^3 (mL) | (j) $50\,000 \text{ mm}^3$ (L) |
| (k) 3.95 L (cm^3) | (l) 0.15 mL (cm^3) |

2. Find the volume of each of the following objects.

Give answers to one decimal place where appropriate.



3. (a) Find the volume of a cube with side length 25 cm.

(b) An ice-cream company wants to produce a 2 litre tub of ice-cream in the shape of a cube.

What should be the side length, in cm, (one dec. pl.) of the 2 litre cube?

4. A truck has a fuel tank in the shape of a cylinder that is 1.2 m long and has a diameter of 80 cm.

How many litres would this fuel tank hold?

Give answer to the nearest litre.

5. Find the volume of a cube that has a surface area of 600 cm^2 .

Similarity

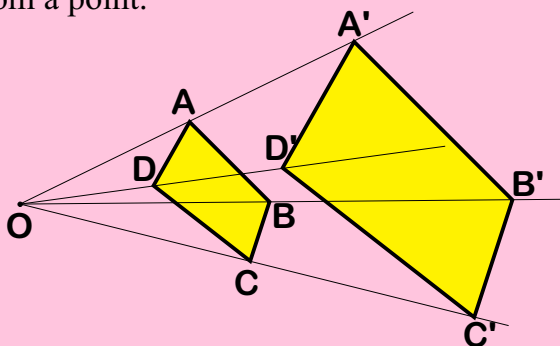
Similarity

The concept of *similarity* is used in many applications: maps, scale drawings, building plans, sign writing, clothing design, models of cars, components, etc.

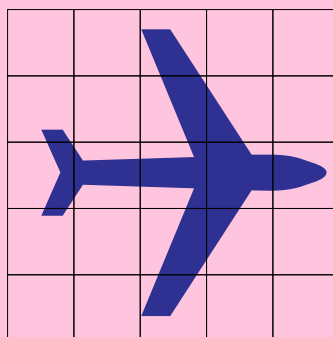
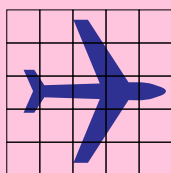
When one object or shape is an enlargement of another, they are said to be *similar*.

There are several ways that similar shapes can be created. Two are shown below.

1. Construction lines from a point.



2. Using a grid.



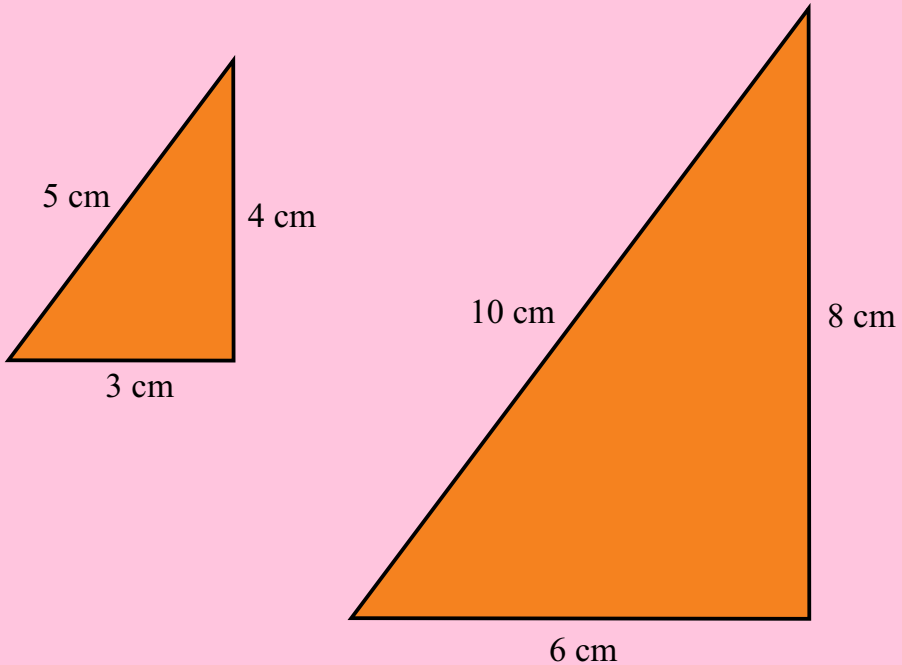
Similar Triangles

When two similar shapes are compared several relationships are observed. These relationships can be used to test for similarity and used to create similar shapes.

Similar triangles are often used in real life situations and these can be used to study the rules for similarity.

Example 1

The two triangles below are similar triangles.



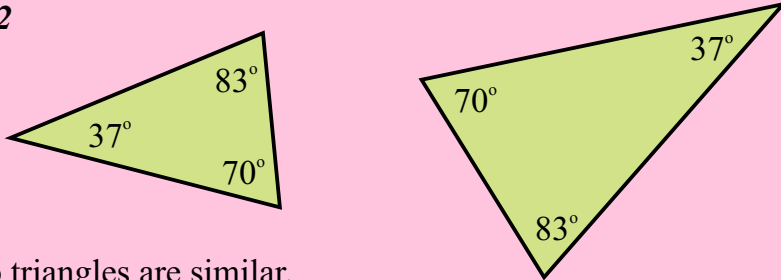
It can be seen that each side length in the enlarged triangle is double the corresponding side length in the smaller triangle.

This is true of all similar shapes.

The ratio of corresponding side lengths is the same for all the sides. This ratio is called the enlargement factor.

Triangles can be similar even if they have been rotated or reflected.

Example 2



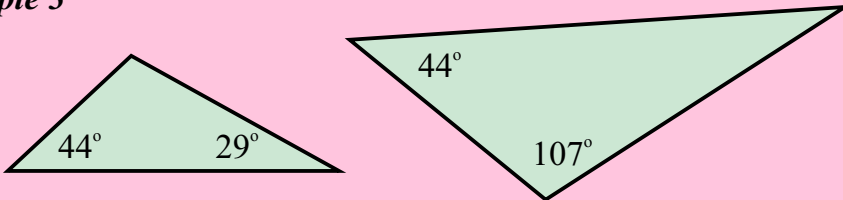
These two triangles are similar.

It can be seen that the angles in the smaller triangle are the same as the angles in the enlarged triangle.

This is true for all similar triangles.

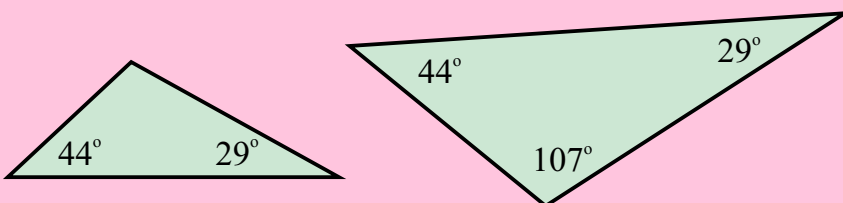
The corresponding angles in similar triangles are equal.

Example 3



These two triangles are similar because all the corresponding angles are the same. If the size of the third angle in each triangle is calculated it can be seen that all the corresponding angles are equal.

Remember: the sum of the angles in a triangle is 180° .



Tests for Similar Triangles

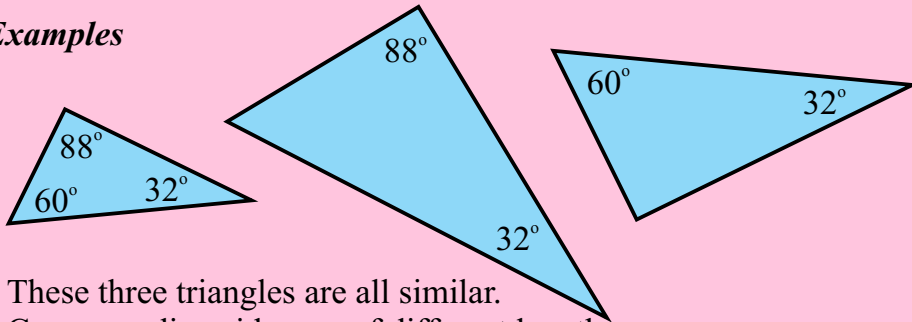
There are four similarity tests for triangles.

1. Angle Angle Angle (AAA)

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

It is sufficient to prove that only two pairs of angles are respectively equal to each other. Because the angle sum of a triangle is always 180° , the third pair of angles will automatically be equal.

Examples

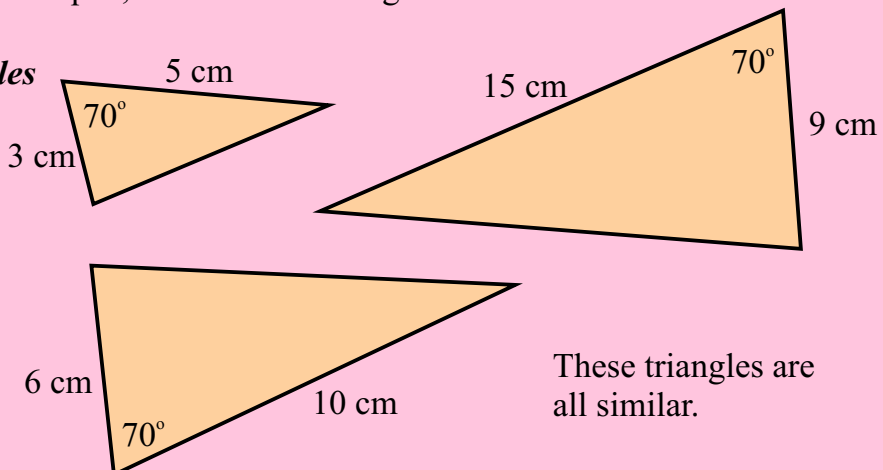


These three triangles are all similar.
Corresponding sides are of different length.

2. Side Angle Side (SAS)

If the ratio of the lengths of two sides of one triangle is equal to the ratio of the lengths of two sides of another triangle, and the included angles are equal, then the two triangles are similar.

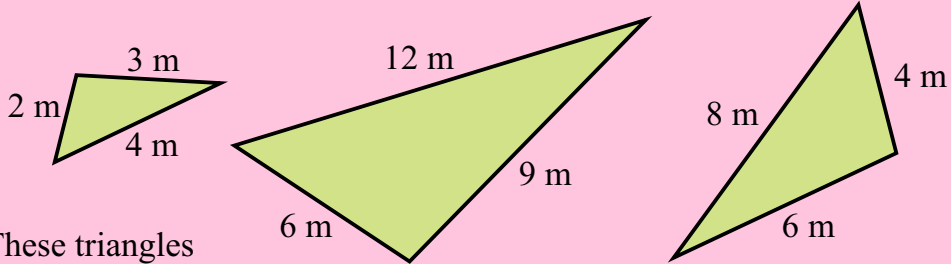
Examples



These triangles are all similar.

3. Side Side Side (SSS)

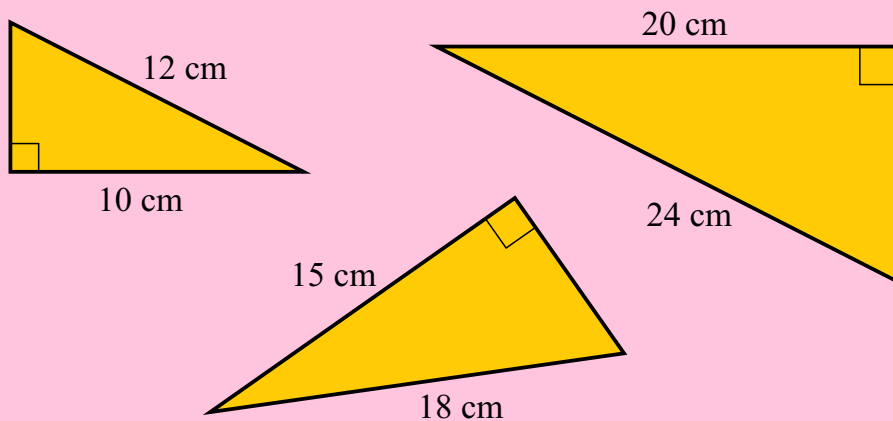
If each of the sides of one triangle can be matched up with each of the sides of another so that the ratios of matching sides are equal, then the two triangles are similar.



These triangles are all similar.

4. Right-angle Hypotenuse Side (RHS)

If the ratio of the hypotenuse and one side of a right-angled triangle is equal to the ratio of the hypotenuse and one side of another right-angled triangle, then the two triangles are similar.



These triangles are all similar.

Summary

The ratio of corresponding side lengths is the same for all the sides of similar triangles.

This ratio is called the enlargement factor.

The corresponding angles in similar triangles are equal.

Remember: the sum of the angles in a triangle is 180° .

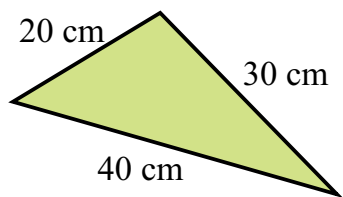
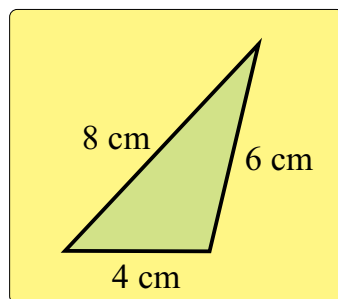
Triangles can be similar even if they have been rotated or reflected.

There are four tests that can be used to verify if triangles are similar.

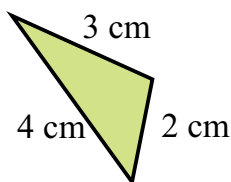
- 1. Angle Angle Angle (AAA)*
- 2. Side Angle Side (SAS)*
- 3. Side Side Side (SSS)*
- 4. Right-angle Hypotenuse Side (RHS)*

EXERCISE 12A

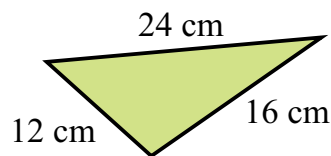
1. Which of the triangles **A-C** below are similar to the triangle shown here.
Triangles not drawn to scale.
One or more answers.



A

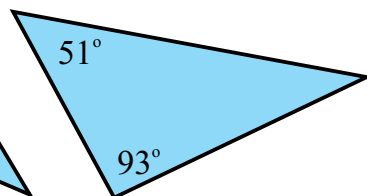
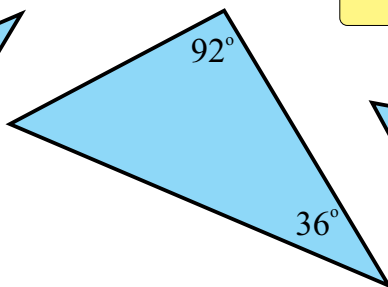
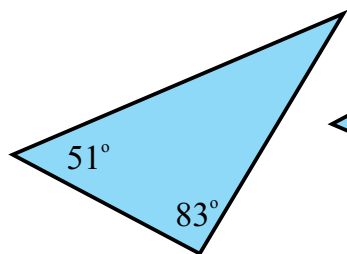
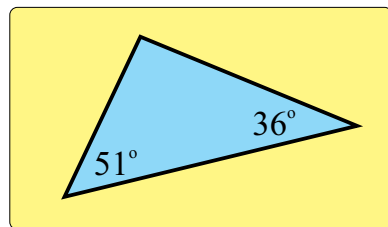


B

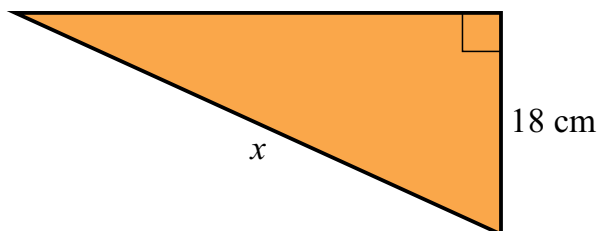
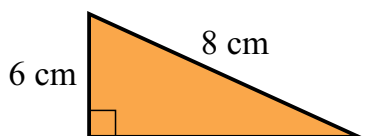


C

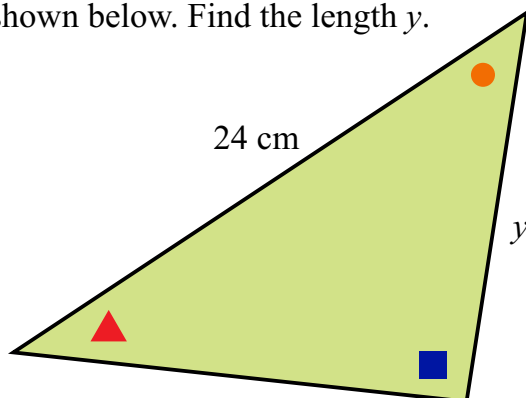
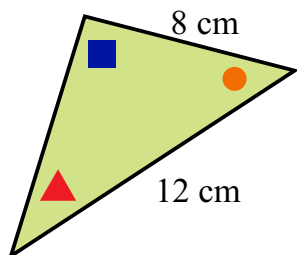
2. Which of the triangles **A-C** below are similar to the triangle shown here.
One or more answers.



3. Two similar triangles are shown below. Find the length x .

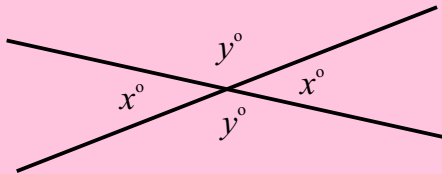


4. Two similar triangles are shown below. Find the length y .



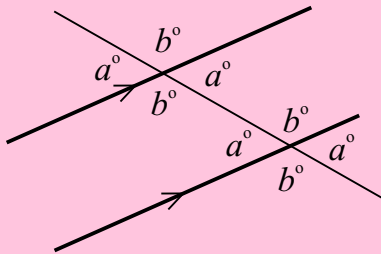
Other geometric facts can be used to find similar triangles.

1. When two lines intersect, two pairs of equal angles are formed.

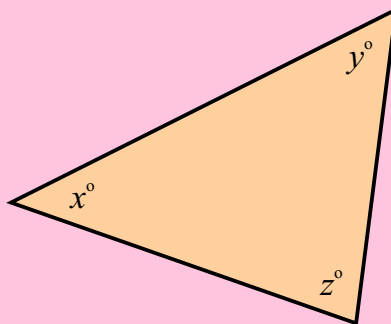


$$x^\circ + y^\circ = 180^\circ$$

2. A line crossing parallel lines with produce equal angles.

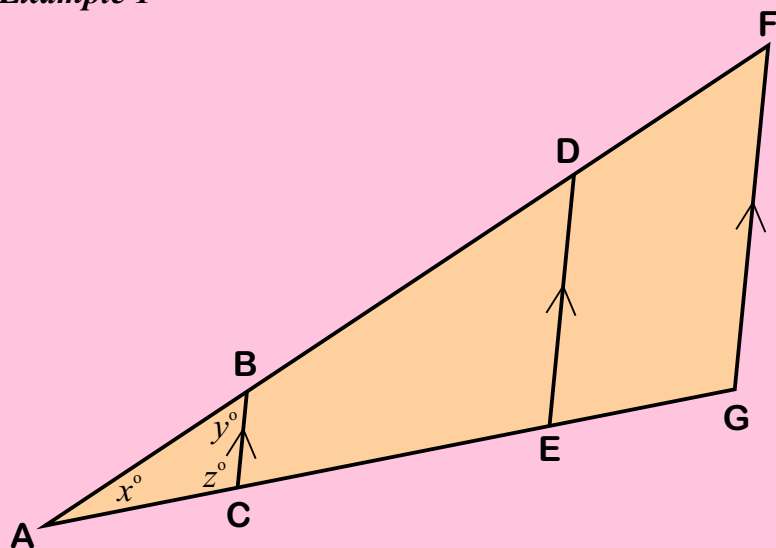


$$a^\circ + b^\circ = 180^\circ$$

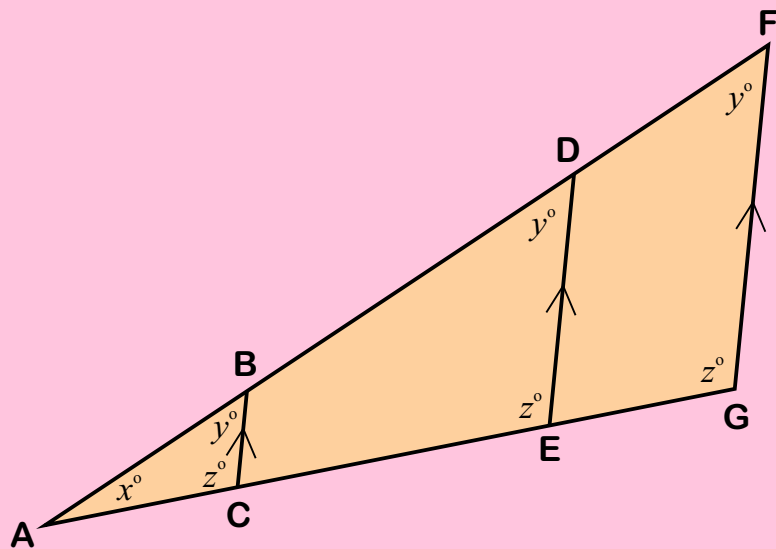


Remember - the sum of the angles in a triangle is 180° .

$$x^\circ + y^\circ + z^\circ = 180^\circ$$

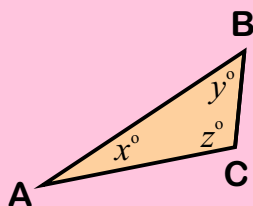
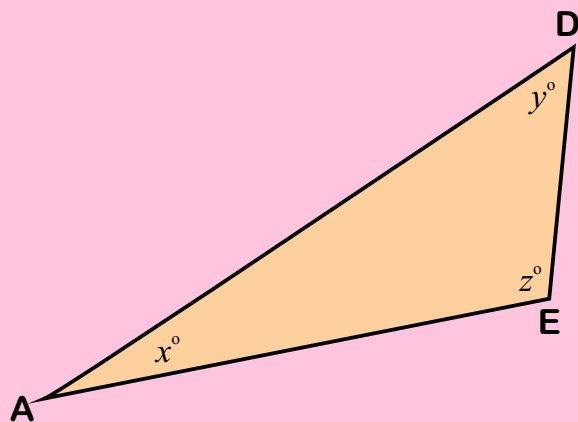
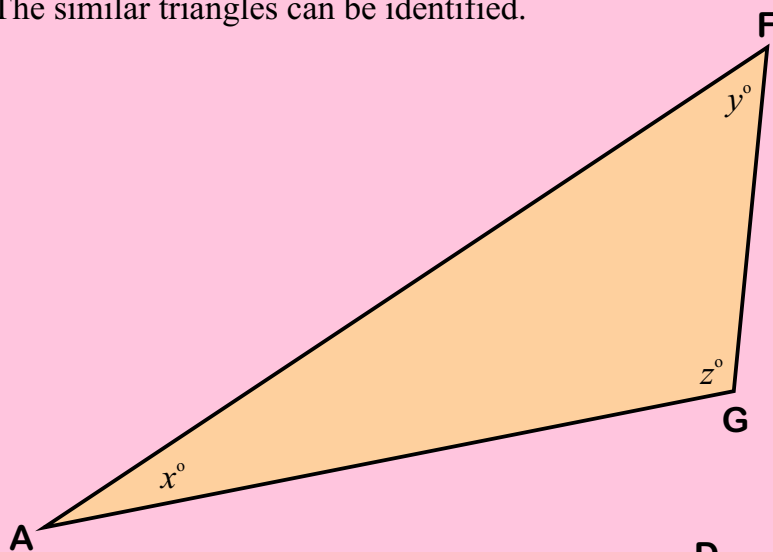
Example 1

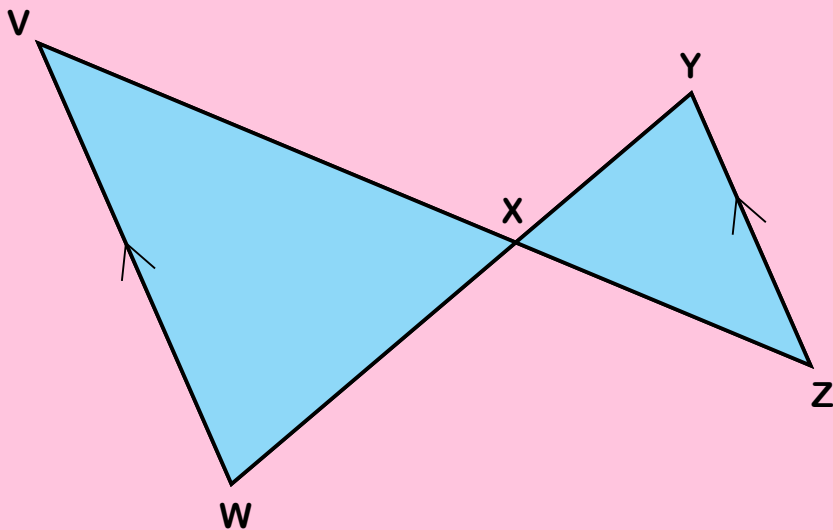
Labelling the equal angles will help identify the similar triangles.



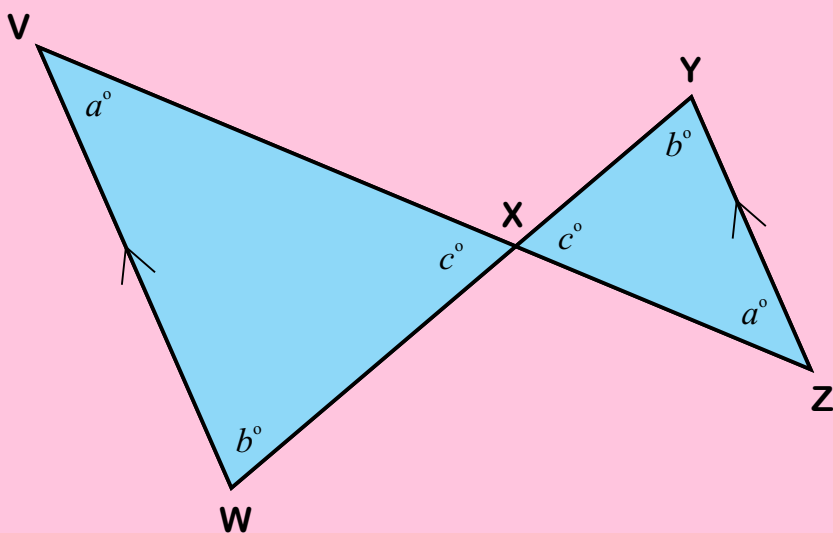
Example 1 continued

The similar triangles can be identified.



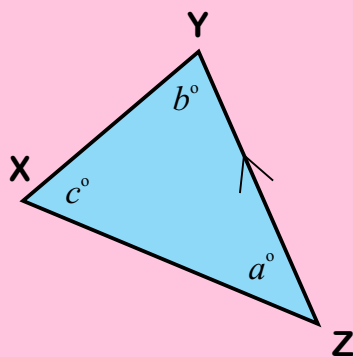
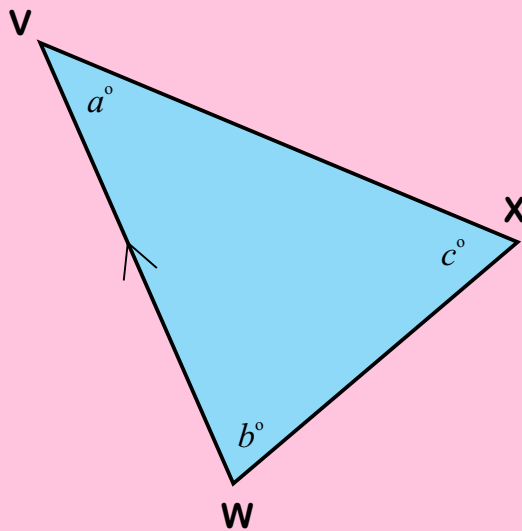
Example 2

Labelling the equal angles will help identify the similar triangles.



Example 2 continued

The similar triangles can be identified.

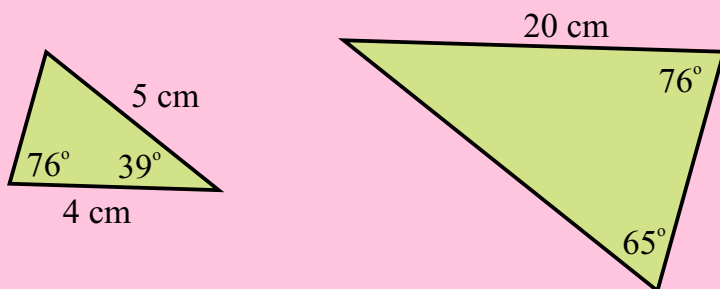


Enlargement Factors

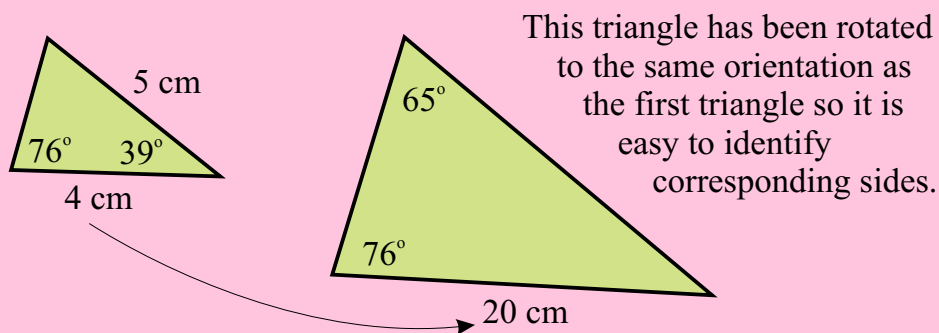
Example 1

1. Find the enlargement factor for the following similar triangles.
2. Find the ratio between the side lengths of following similar triangles.
Give the ratio in its simplest form.

Note: Triangles not drawn to scale.



Step 1 Rotate and/or reflect one of the triangles so the corresponding sides can be easily identified.



Step 2 Find the enlargement factor by using the side lengths of *corresponding sides*.

$$\text{enlargement factor} = \frac{\text{side length of second triangle}}{\text{side length of first triangle}}$$

Comparing corresponding sides that the lengths are known:

side length of first triangle = 4 cm

side length of second triangle = 20 cm

$$\text{enlargement factor} = \frac{\text{side length of second triangle}}{\text{side length of first triangle}}$$

$$= \frac{20}{4}$$

$$\text{enlargement factor} = 5$$

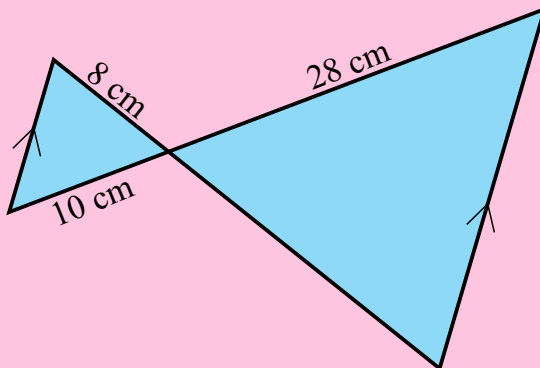
ratio of side lengths = 4:20

ratio of side lengths = 1:5

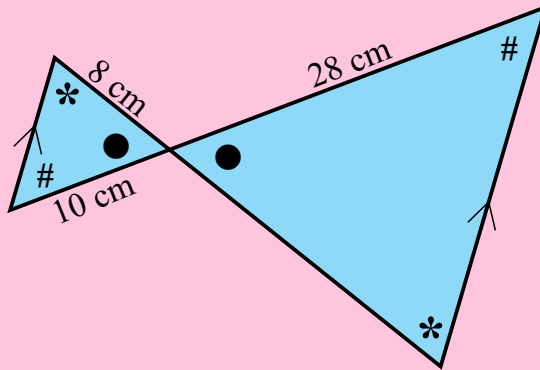
Example 2

- Find the enlargement factor for the following similar triangles.
Give answer as a decimal.
- Find the ratio between the side lengths of following similar triangles.
Give the ratio in its simplest form.

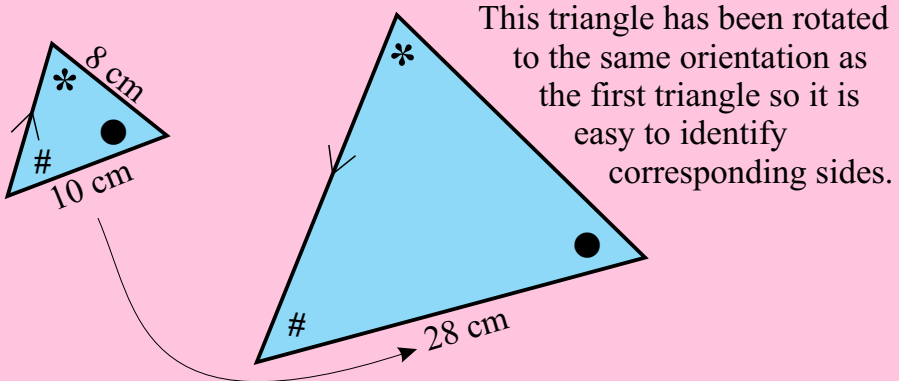
Note: Triangles not drawn to scale.



Step 1 Identify the equivalent angles in the two triangles.



Step 2 Rotate and/or reflect one of the triangles so that the corresponding sides can be easily identified.



Step 3 Calculate the enlargement factor and ratio between the triangles.

$$\begin{aligned} \text{enlargement factor} &= \frac{\text{side length of second triangle}}{\text{side length of first triangle}} \\ &= \frac{28}{10} \end{aligned}$$

$$\text{enlargement factor} = 2.8$$

$$\text{ratio of side lengths} = 10:28$$

$$\text{ratio of side lengths} = 5:14$$

EXERCISE 12B

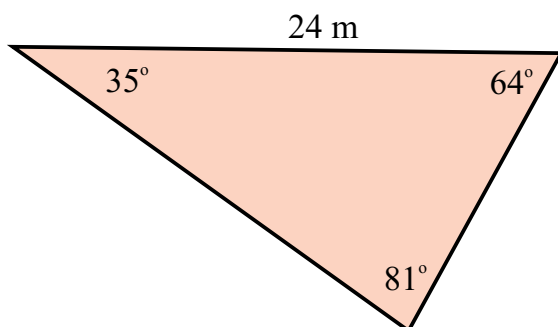
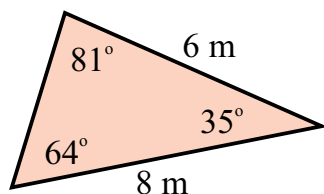
For the following pairs of similar triangles:

(a) find the enlargement factor (as a decimal)

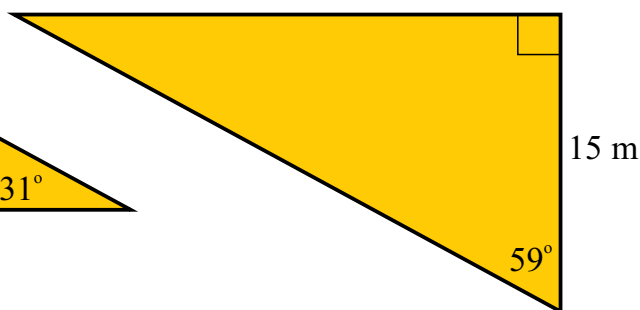
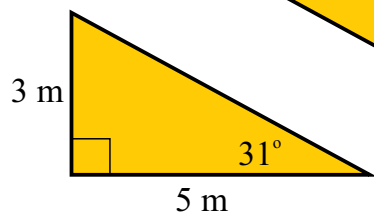
(b) find the ratio of side lengths

Triangles are not drawn to scale.

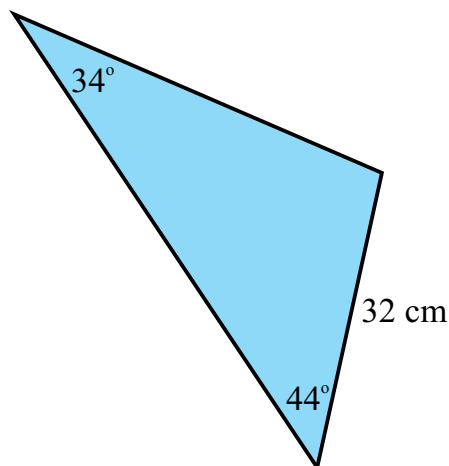
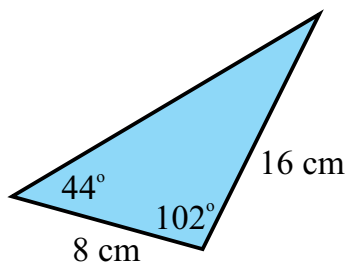
1.



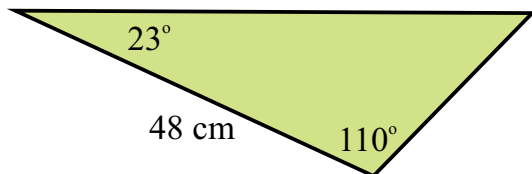
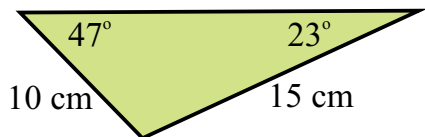
2.



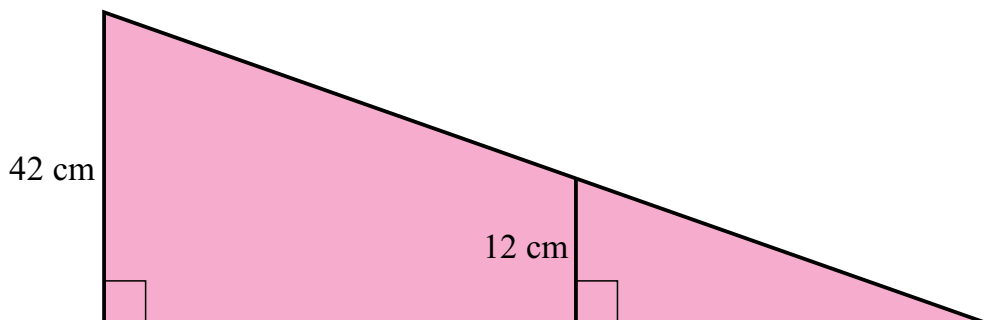
3.



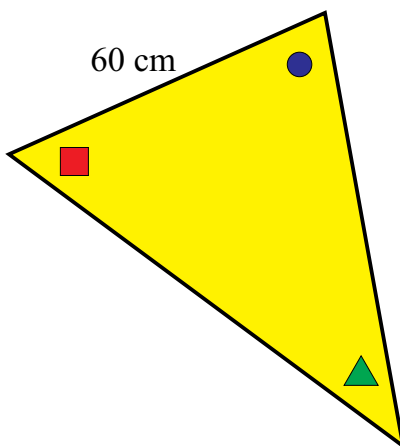
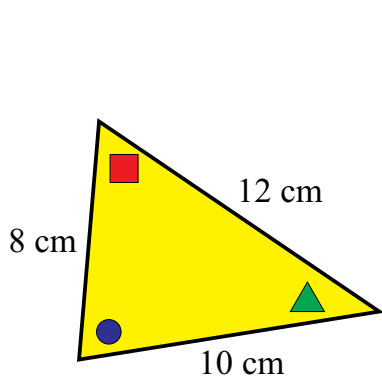
4.



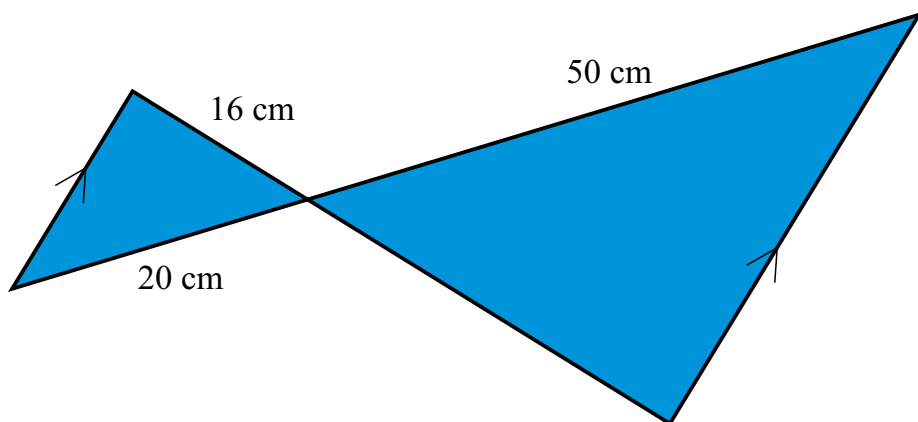
5.



6.



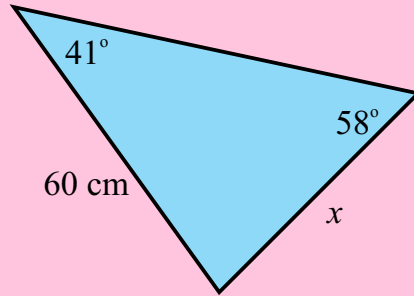
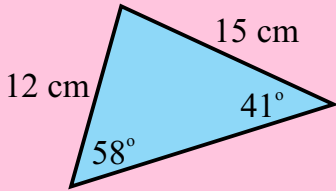
7.



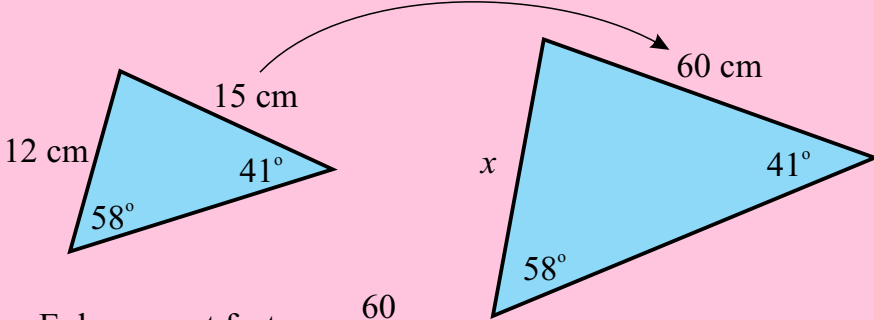
Finding Unknown Lengths

Example 1

Find the unknown length in the following similar triangles.



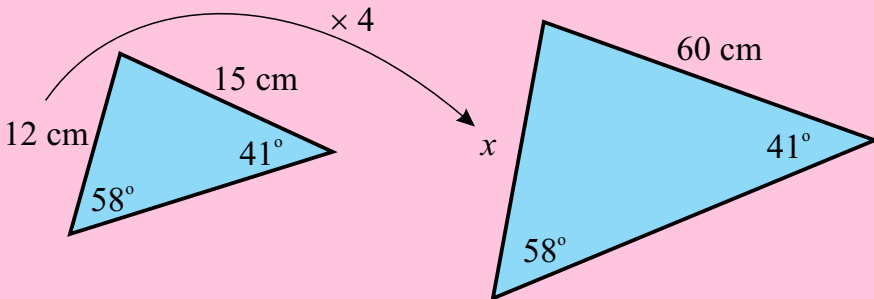
Step 1 Find the enlargement factor.



$$\text{Enlargement factor} = \frac{60}{15}$$

$$\text{Enlargement factor} = 4$$

Step 2 Use the enlargement factor to find the unknown length.

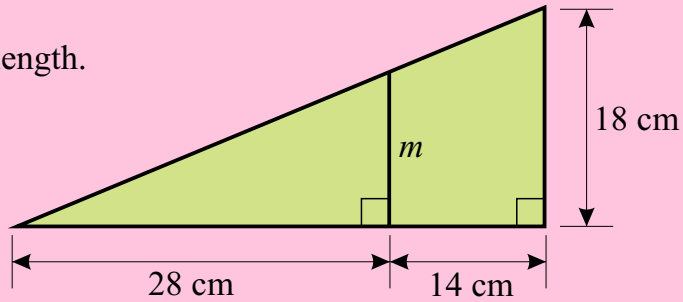


$$x = 12 \times 4$$

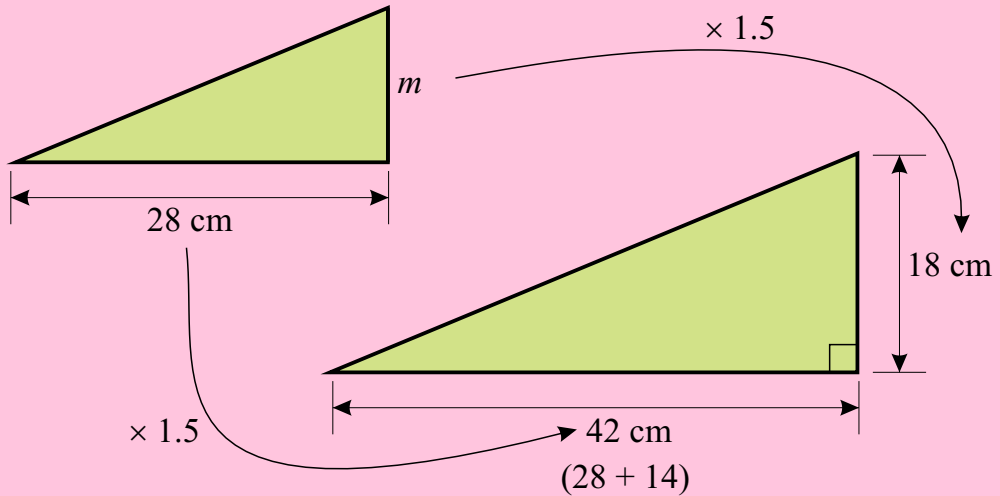
$$x = 48 \text{ cm}$$

Example 2

Find the unknown length.



Step 1 Find the enlargement factor.



$$\text{Enlargement factor} = \frac{42}{28}$$

$$\text{Enlargement factor} = \mathbf{1.5}$$

Step 2 Use the enlargement factor to find the unknown length.

$$m \times 1.5 = 18$$

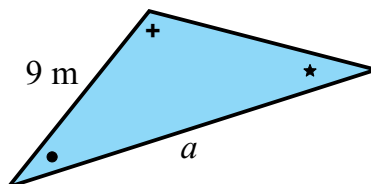
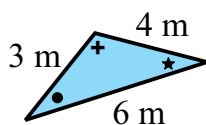
$$m = \frac{18}{1.5}$$

$$\mathbf{m = 12 \text{ cm}}$$

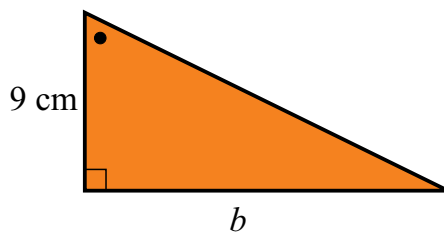
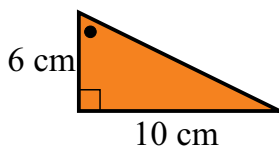
EXERCISE 12C

Find the unknown lengths.

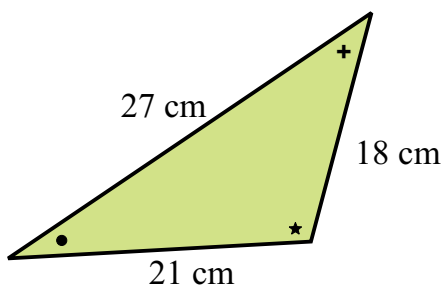
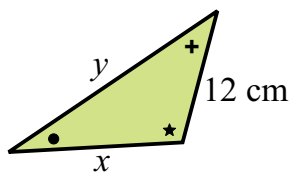
1.



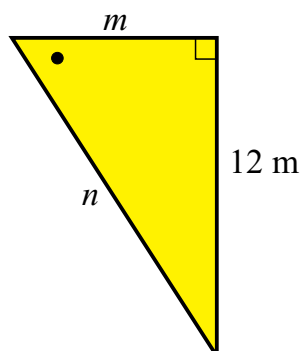
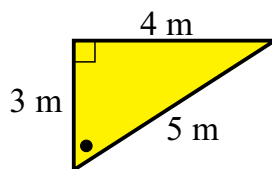
2.



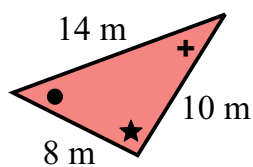
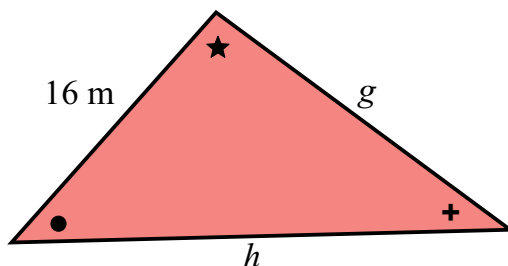
3.



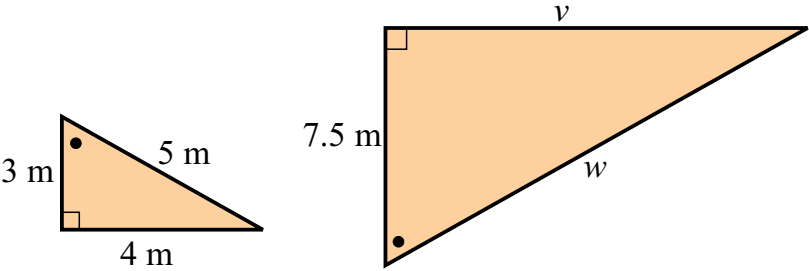
4.



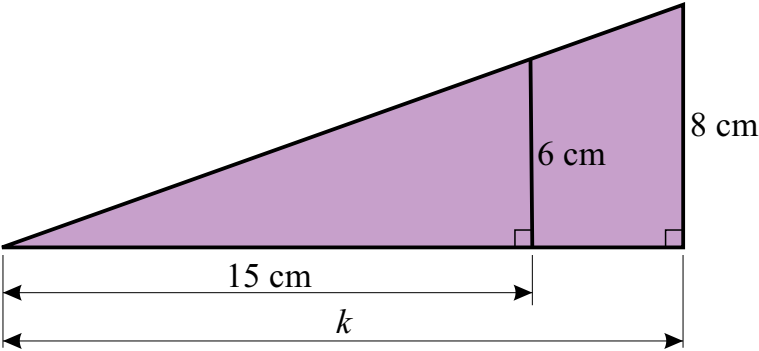
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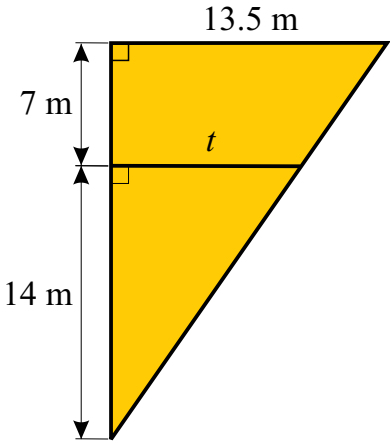
6.



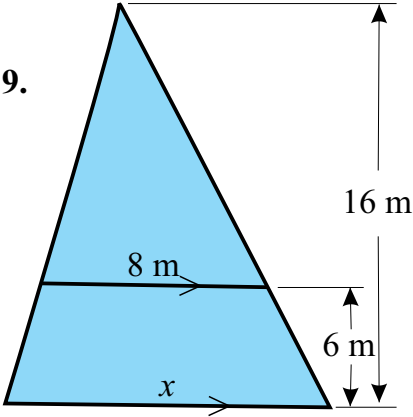
7.



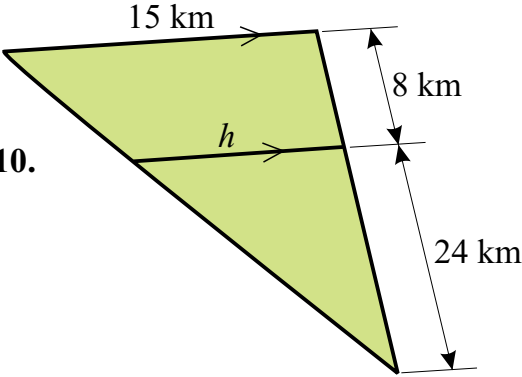
8.



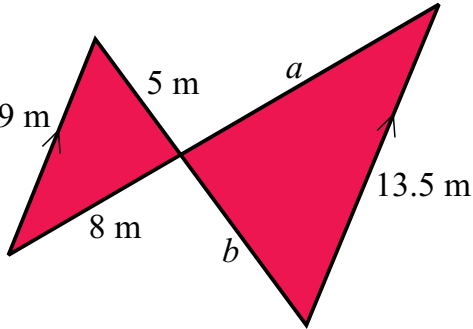
9.



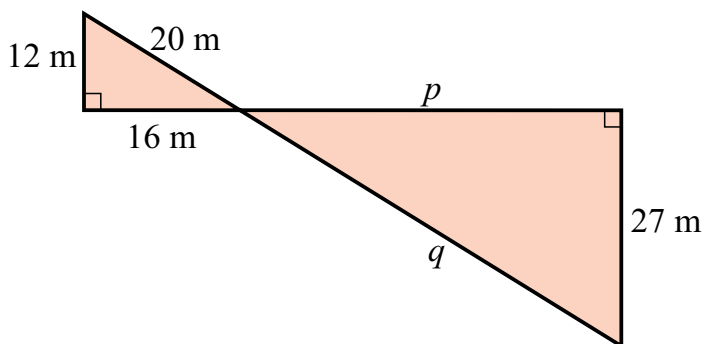
10.



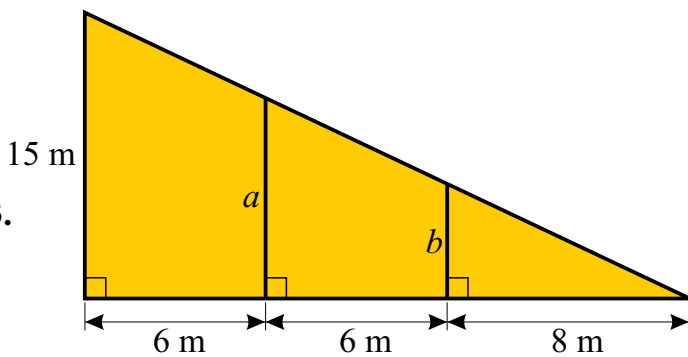
11.



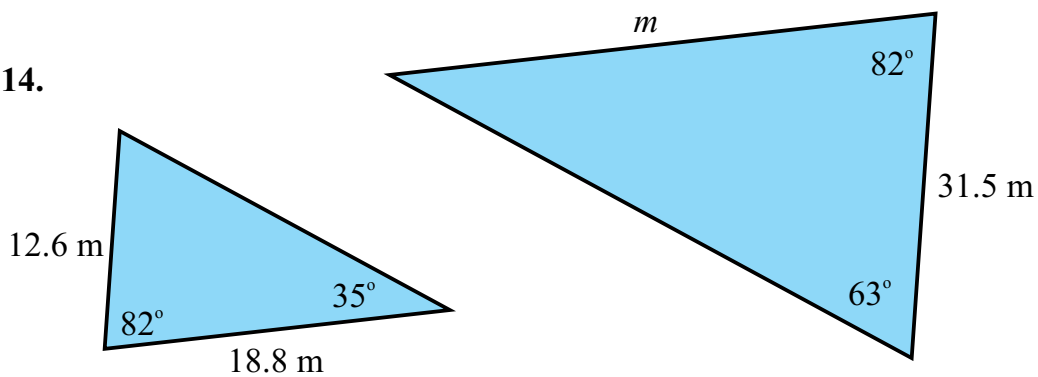
12.



13.



14.



Practical Problems

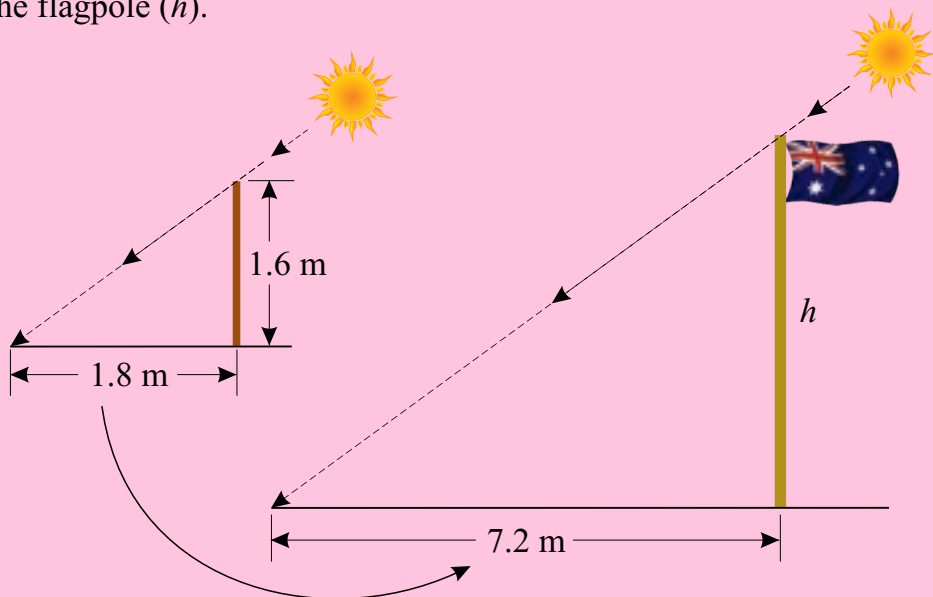
Example

Students in a mathematics class wanted to find the height of a flagpole. On a sunny day they measured the shadow of the flagpole to be 7.2 m long.

They also measured the shadow of a 1.6 m post to be 1.8 m long.

They realised these two triangles would be similar because the Sun's rays would make the same angle at the ground and both the flagpole and post were vertical.

They used their knowledge of similar triangles to calculate the height of the flagpole (h).



Step 1 Calculate the enlargement factor.

$$\text{Enlargement factor} = \frac{7.2}{1.8}$$

$$\text{Enlargement factor} = 4$$

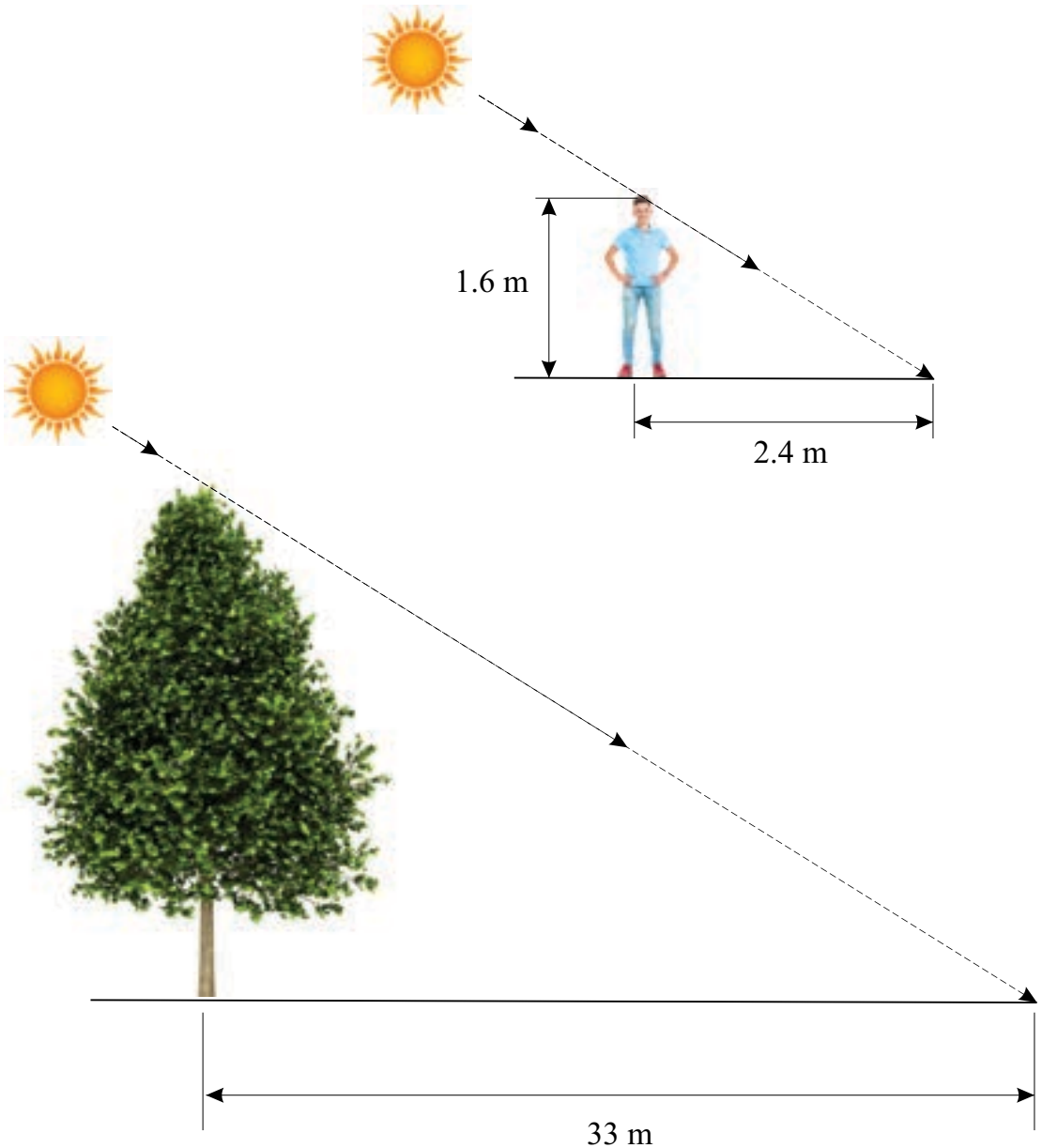
Step 2 Calculate the height of the flagpole.

$$h = 4 \times 1.6$$

$$h = 6.4 \text{ m}$$

EXERCISE 12D

1. Jayden is 1.6 m tall. He measured his shadow one day to be 2.4 m long. The shadow of a nearby tree was 33 m long. How tall is the tree?

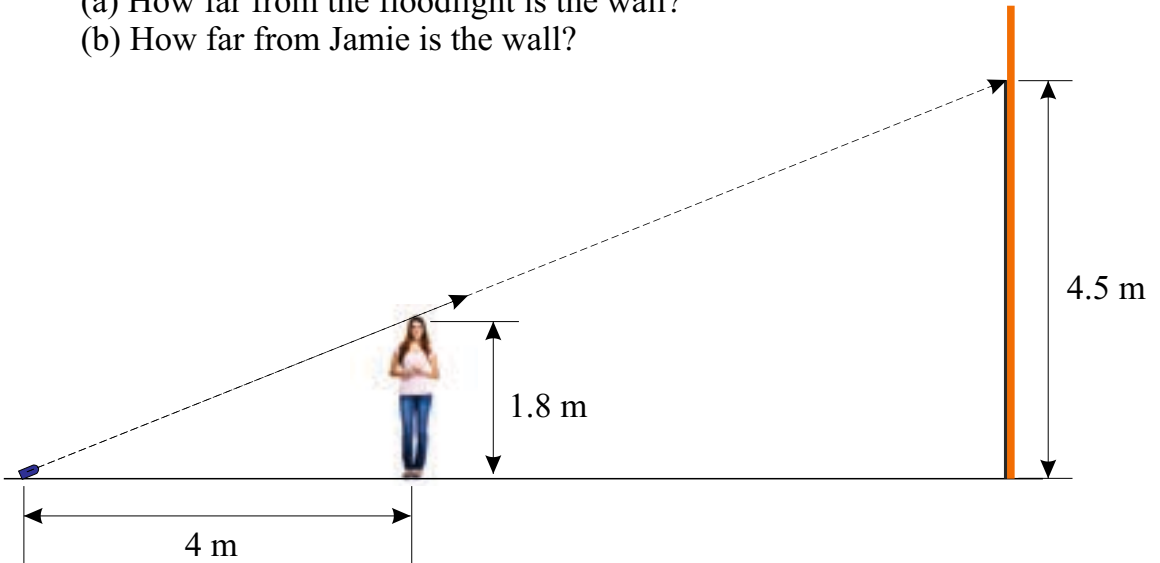


2. Jamie is standing 4 m from a floodlight.

Jamie is 1.8 m tall and her shadow on a wall is 4.5 m high.

(a) How far from the floodlight is the wall?

(b) How far from Jamie is the wall?



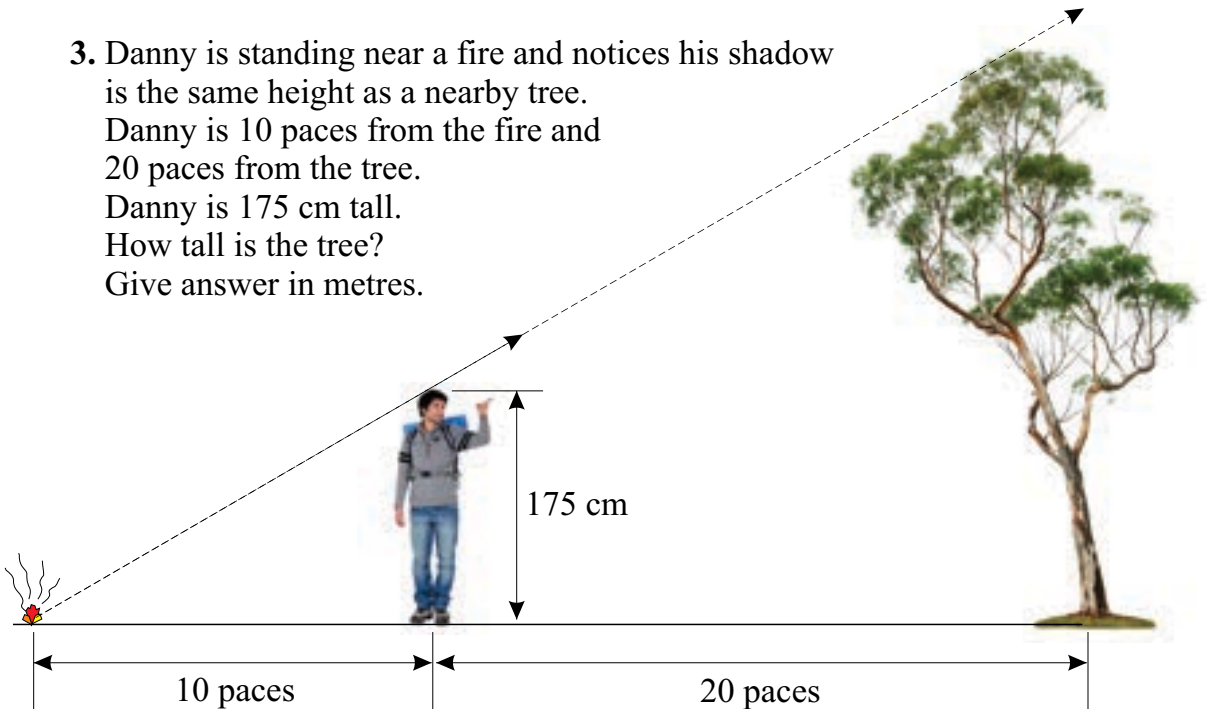
3. Danny is standing near a fire and notices his shadow is the same height as a nearby tree.

Danny is 10 paces from the fire and 20 paces from the tree.

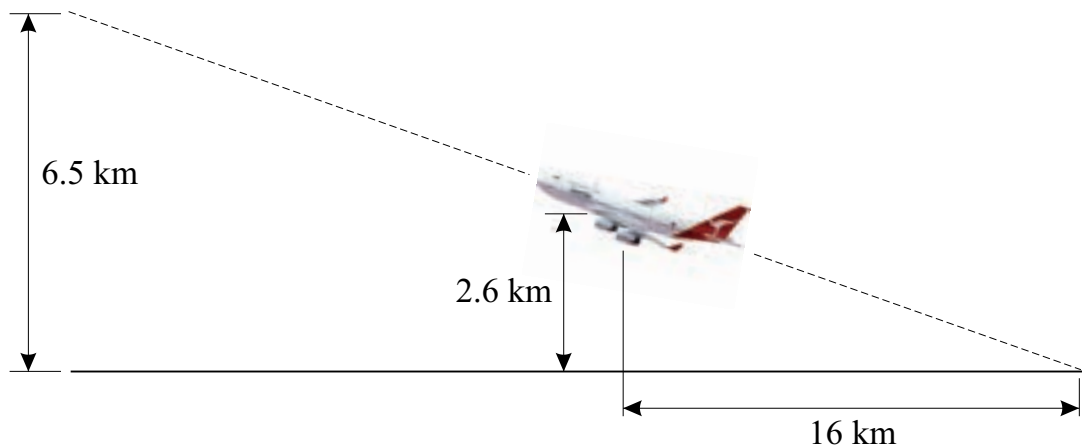
Danny is 175 cm tall.

How tall is the tree?

Give answer in metres.



4. An aeroplane takes off and increases its altitude at a constant angle.
At a distance of 16 km from the airport it is 2.6 km above the ground.
(a) How far from the airport is the aeroplane when it reaches its maximum altitude of 6.5 km?
(b) What altitude was the aeroplane when it was 24 km from the airport?



5. Julie wants to calculate the width of the river shown. There are trees at the points **A - E**.

Tree **A** and tree **B** are on one bank of the river.

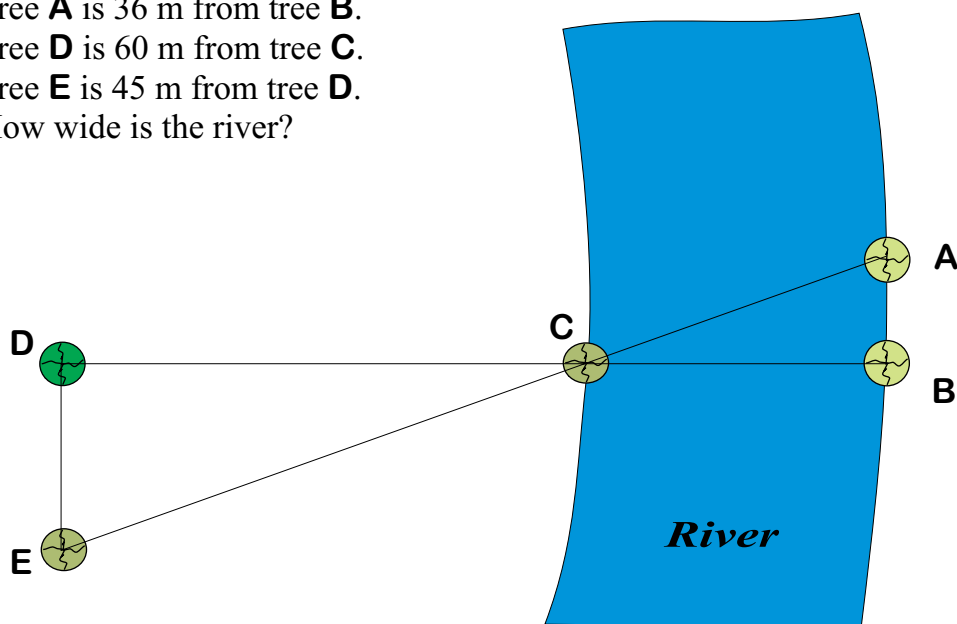
Tree **C** is on the other bank.

Tree **A** is 36 m from tree **B**.

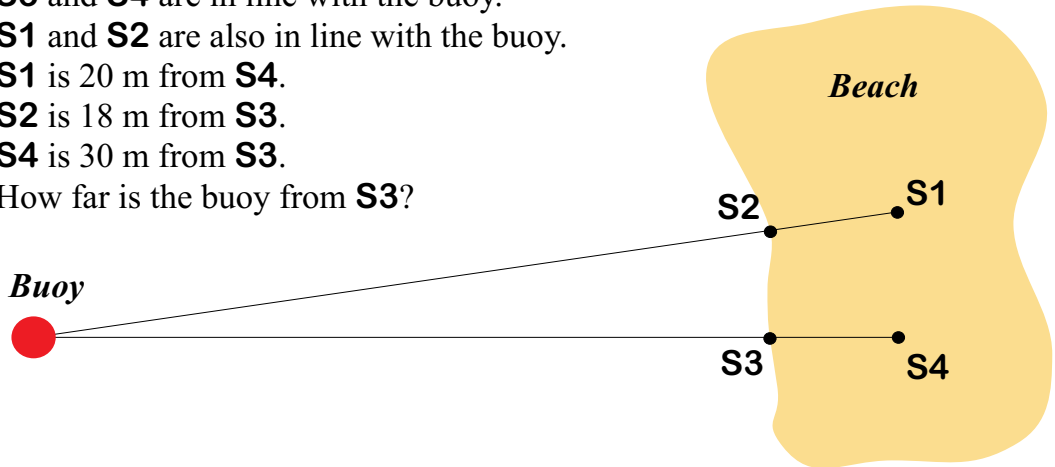
Tree **D** is 60 m from tree **C**.

Tree **E** is 45 m from tree **D**.

How wide is the river?



6. Adam is going to swim from a beach to a buoy floating in the water. He wants to calculate the distance from the beach to buoy. He hammers four stakes (**S1** - **S4**) into the sand on the beach. **S3** and **S4** are in line with the buoy. **S1** and **S2** are also in line with the buoy. **S1** is 20 m from **S4**. **S2** is 18 m from **S3**. **S4** is 30 m from **S3**. How far is the buoy from **S3**?



7. Stephanie wants to run from **A** to **C**. There are three different routes she could take:

A-B-C

A-F-B-D-C

A-F-E-D-C

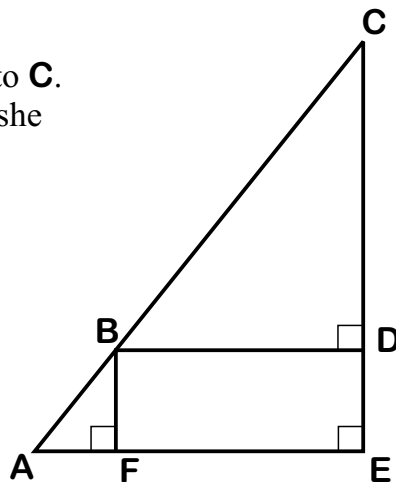
Some of the distances are as follows:

A-F = 300 m

F-B = 400 m

B-D = 900 m

B-C = 1500 m



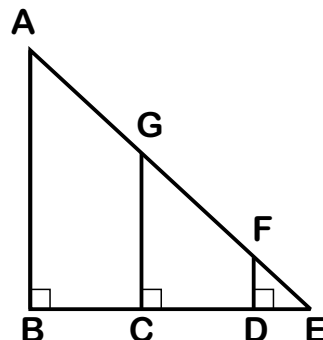
Find the distance Stephanie would run for each of the routes.

8. Use the information below to find **BE** (the distance from **B** to **E**).

$\overline{CD} = 4 \text{ m}$

$\overline{CG} = 3\overline{FD}$

$\overline{AB} = 5\overline{FD}$

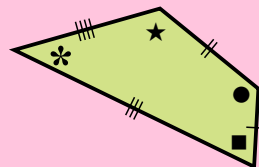
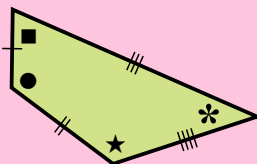
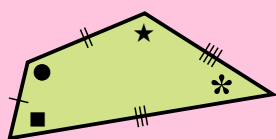


Congruence

Definition: Two shapes are called *congruent* if they are *identical*. They can be rotated or reflected but if they have identical corresponding side lengths and angles they are *congruent*.

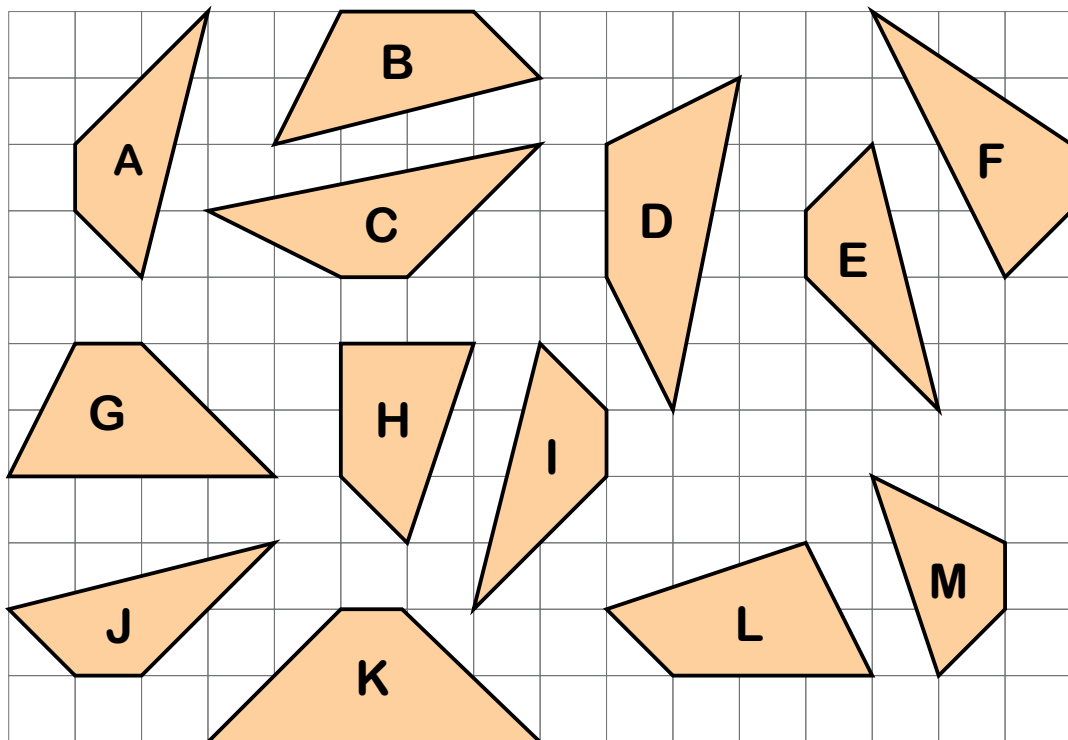
Examples

These three shapes are congruent.

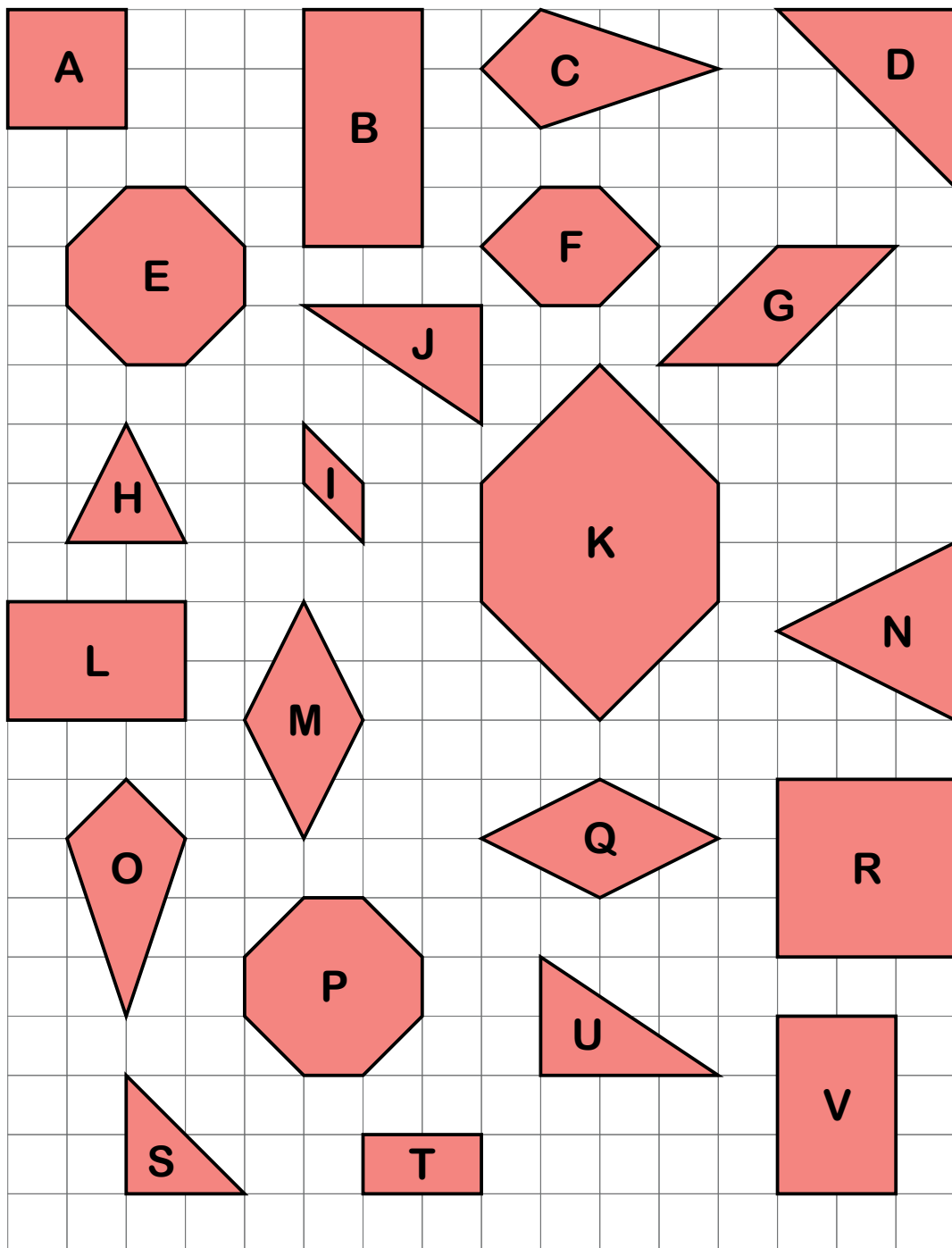


EXERCISE 12E

1. Find the four congruent shapes below.

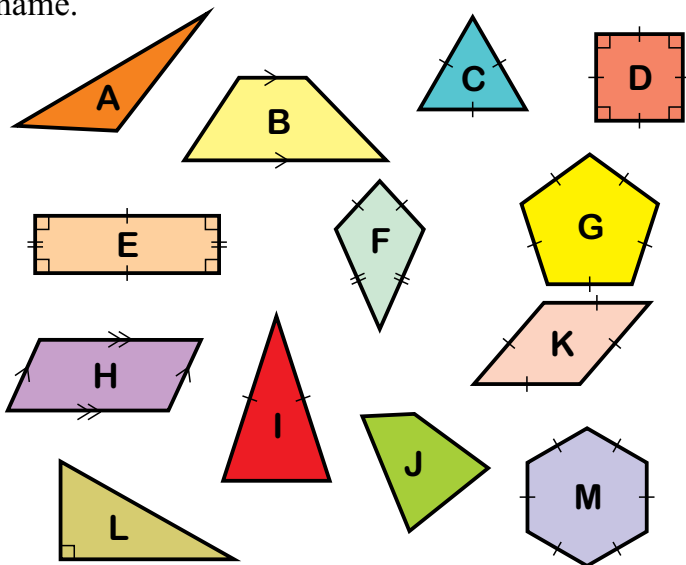


2. (a) Find the pairs of *similar* shapes below.
(b) Find the pairs of *congruent* shapes below.



3. Match the shape with its name.

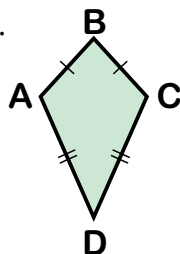
- (a) square
- (b) rectangle
- (c) parallelogram
- (d) rhombus
- (e) trapezium
- (f) irregular quadrilateral
- (g) kite
- (h) equilateral triangle
- (i) isosceles triangle
- (j) scalene triangle
- (k) right-angled triangle
- (l) regular pentagon
- (m) regular hexagon

4. Using the mathematical definition for **similarity**, state if the following statements are true (T) or false (F).

- (a) Squares of different sizes are all similar.
- (b) Rectangles of different sizes are all similar.
- (c) Regular hexagons of different sizes are all similar.
- (d) Equilateral triangles of different sizes are all similar.
- (e) Isosceles triangles of different sizes are all similar.
- (f) Rhombuses of different sizes are all similar.

5. State if the following statements are true (T) or false (F).

- (a) Drawing a diagonal on a square would divide it into two congruent triangles.
- (b) Drawing a diagonal on a rectangle would divide it into two similar triangles.
- (c) Drawing a diagonal on a rhombus would divide it into two similar triangles.
- (d) Drawing two diagonals on a rhombus would divide it into four congruent triangles.
- (e) A regular hexagon is composed of six congruent triangles.
- (f) Drawing a diagonal from **A** to **C** on the kite shown here would result in two similar triangles.
- (g) Drawing a diagonal from **B** to **D** on the kite shown here would result in two similar triangles.

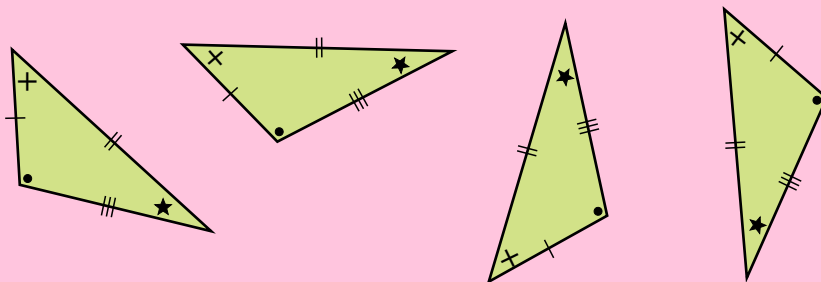


Congruent Triangles

Definition: Triangles are called *congruent triangles* if they are exactly the same:

- all their corresponding angle sizes are the same
- all their corresponding side lengths are the same

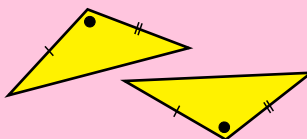
Example: All the triangles below are congruent triangles



One of four tests can be used to identify congruent triangles.

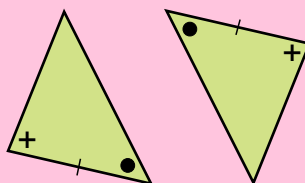
1. Side Angle Side (SAS)

If two side lengths and the contained angle are the same.



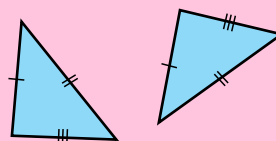
2. Angle Side Angle (ASA)

If two angles and the contained side length are the same.



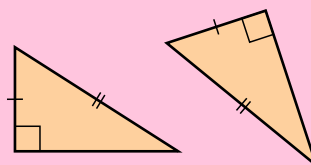
3. Side Side Side (SSS)

If two triangles have the same side lengths they are congruent.



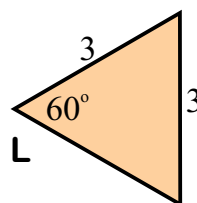
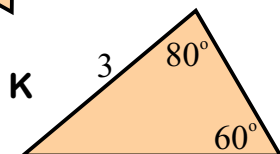
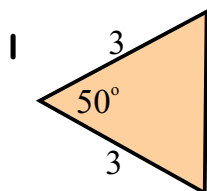
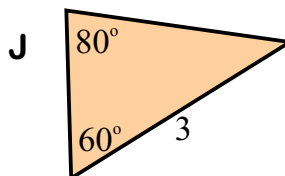
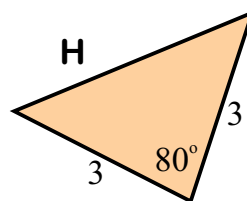
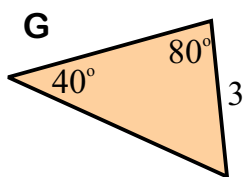
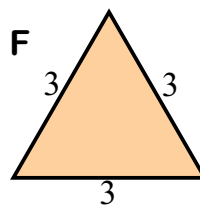
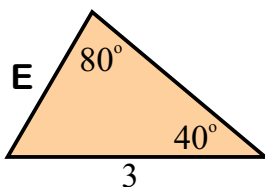
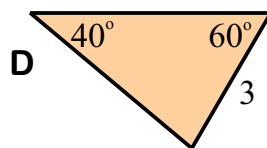
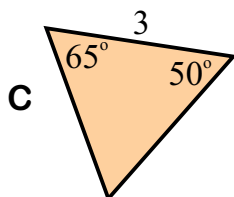
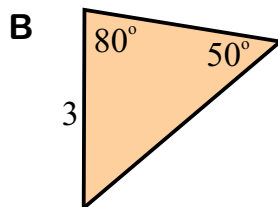
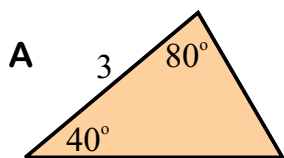
4. Right-angle Hypotenuse Side (RHS)

If the length of the hypotenuse and another side are the same and they are right-angle triangles.



EXERCISE 12F

Match all the pairs of congruent triangles from the following triangles.



Scales

Scales are used on maps, in architecture, science and many other applications as a method of representing an object that is large or small. Scales allow measurements to be taken from the scale drawing and converted into the actual measurements.

Example 1

The scale on a map may be 1:20 000.

This means that a measurement taken from the map needs to be multiplied by 20 000 to find the actual measurement.

It also means that to construct the map the actual distances need to be divided by 20 000 to find the distance on the map.

It is important to remember that the unit of the actual measurement will be the same as the unit used to measure the distance on the map.

Use the scale of 1:20 000 to answer the following questions.

- Find the actual distances (in *metres*) of the following lengths measured off the map.

(a) 3 cm (b) 18 mm

- (a) If 3 cm is measured off the map the actual distance is:

$$\begin{aligned} & 3 \text{ cm} \times 20\,000 \\ &= 60\,000 \text{ cm} \\ &= \mathbf{600 \text{ m}} \end{aligned}$$

- (b) If 18 mm is measured off the map the actual distance is:

$$\begin{aligned} & 18 \text{ mm} \times 20\,000 \\ &= 360\,000 \text{ mm} \\ &= \mathbf{360 \text{ m}} \end{aligned}$$

2. Find the lengths on the map (in **mm**) of the following distances.

(a) 3 km (b) 850 m

(a) The length on the map representing 3 km would be:

$$\begin{aligned} 3 \text{ km} &\div 20\,000 \\ &= 0.00015 \text{ km} \\ &= \mathbf{150 \text{ mm}} \end{aligned}$$

(b) The length on the map representing 850 m would be:

$$\begin{aligned} 850 \text{ m} &\div 20\,000 \\ &= 0.0425 \text{ m} \\ &= \mathbf{42.5 \text{ mm}} \end{aligned}$$

Example 2

An engineer made a model of bridge he was designing. The actual bridge was 250 metres long and the model was 125 cm long. What scale was the model?

Step 1 Convert both measurements to the same unit.

$$\begin{aligned} \text{Model} &= 125 \text{ cm} \\ \text{Actual bridge} &= 250 \text{ m} = 25\,000 \text{ cm} \end{aligned}$$

Step 2 Write as a ratio in its simplest form.

$$\begin{aligned} 125:25\,000 \\ &= \mathbf{1:200} \end{aligned}$$

Example 3

Sometimes the scales on maps or scale drawings are shown as a comparative measurement.

$$\mathbf{\text{Example}} \quad 1 \text{ cm} = 500 \text{ m}$$

This can be easily changed to a scale ratio by converting to the same units and writing the ratio.

$$\begin{aligned} 1 \text{ cm} &= 500 \text{ m} \\ 1 \text{ cm} &= 50\,000 \text{ cm} \\ &\mathbf{1:50\,000} \end{aligned}$$

EXERCISE 12G

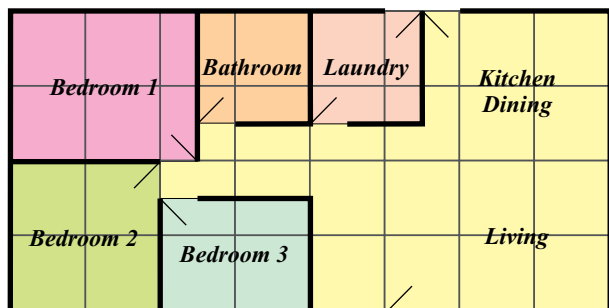
1. The scale on a map was 1:10 000.
 - (a) Find the actual distances (in **metres**) of the following lengths that were measured off the map.
 - (i) 6 cm (ii) 38 mm (iii) 9.5 cm (iv) 52 mm
 - (b) Find the lengths on the map (in **mm**) of the following distances.
 - (i) 450 m (ii) 80 m (iii) 2 km (iv) 1540 m

2. The scale on a map was 1:2 000 000.
 - (a) Find the actual distances (in **km**) of the following lengths on the map.
 - (i) 8 cm (ii) 15.5 cm (iii) 75 mm (iv) 122 mm
 - (b) Find the lengths on the map (in **mm**) of the following distances.
 - (i) 500 km (ii) 178 km (iii) 336 km (iv) 90 km

3. An architect built a scale model of a building that she was designing. The building was to be 60 metres tall and the scale of the model was 1:50.
 - (a) How tall (in **cm**) was the model of the building?
 - (b) The model was 50 cm wide. How wide was the building?

4. Nerada had the model of a house. The model was 30 cm long. The house was 30 metres long. What is the scale of the model?

5. The plan of a house is shown here.
The scale of the plan is 1:200.
The grid shown over the plan is 1 cm squares.
Find the following lengths in **metres**.



- (a) The length of the house.
- (b) The width of the house.
- (c) The dimensions of:
 - (i) bedroom 1 (ii) bedroom 3 (iii) the laundry

6. Sally has a doll's house which was made to a scale of 1:15.
She wants to make some furniture for her doll's house.
Find the dimensions of the following items of furniture for her doll's house (in **cm**) given the full size dimensions below.
- (a) Chair - 90 cm high.
 - (b) Table - 120 cm long, 75 cm wide.
 - (c) Wardrobe - 210 cm high, 150 cm wide.
 - (d) Bed - 2.25 m long, 1.35 m wide.

7. (a) Caleb is making a model of a helicopter. The rotor blades of the actual helicopter are 8 metres long.
The scale factor for Caleb's model is 1:50.
Find the length of the rotor blades (in **cm**) on Caleb's model.
- (b) Caleb has a model of an aeroplane that is 18 cm long.
The scale factor of the aeroplane model is also 1:50.
How long, in **metres**, is the actual aeroplane?



8. Peter and Izza were hiking. They were using a map that had a scale of 1:50 000.
- (a) They decided to walk in a straight line from their campsite to a lake.
The measurement on the map was 17 cm.
How far, in **metres**, was the lake from their campsite?
 - (b) The next day they wanted to walk 10 km from their campsite.
What will be the measurement, in **cm**, on their map?

9. Harry and Lola were sailing around some tropical islands.
On one day they were sailing at a speed of 10 km/h.
They decided to sail from one island to another and the map they were using had a scale of 1:200 000.
The distance between the islands on the map was 15 cm.
How long would it take them to sail between the two islands?

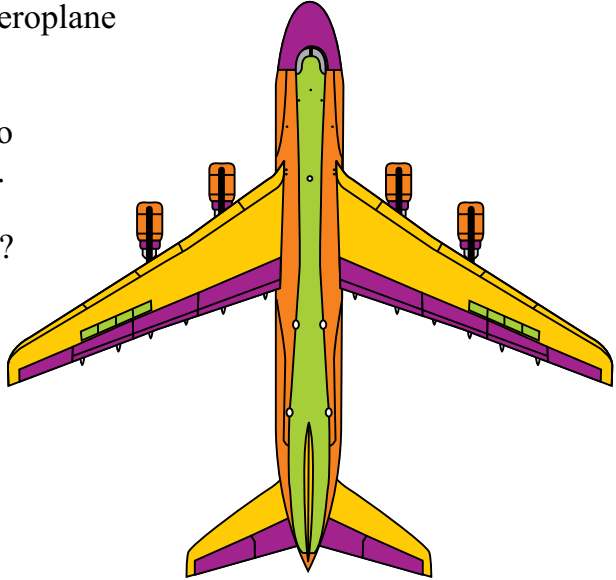


10. A scaled drawing of an aeroplane is shown here.

The scale is 1:500.

Use a ruler and the scale to find the following lengths.

- (a) What is the wingspan (in *m*) of the aeroplane?
- (b) What is the length (in *m*) of the aeroplane?



11. A scaled map of the United Kingdom is shown below.

Several of the major cities are shown.

The scale of the map is 1:10 000 000.

Use a ruler and the scale to find the distance between the following cities (in *km*).

- (a) London and Dublin
- (b) London and Liverpool
- (c) Dublin and Belfast
- (d) Edinburgh and Glasgow



Very Large and Very Small

When creating models or diagrams of very large or very small objects it is easier to use a comparative measurement as the scale.

Example 1

The diameter of the Earth is 12 742 km.

The diameter of the Moon is 3474 km.

Jasper wanted to make a model of the Earth and the Moon with the model of the Earth having the same diameter as a basketball - about 25 cm.

If this was converted to a scale it would be:

$$\begin{aligned} & 25 \text{ cm} : 12\,742 \text{ km} \\ &= 25 \text{ cm} : 1\,274\,200\,000 \text{ cm} \\ &= 1 : 50\,968\,000 \end{aligned}$$

Even if this was rounded off to 1:51 000 000 it is very clumsy to work with.

It is easier to compare the dimensions directly.

$$\text{Moon model diameter} : \text{Earth model diameter} = 3474 \text{ km} : 12\,742 \text{ km}$$

$$\frac{\text{Moon model diameter}}{\text{Earth model diameter}} = \frac{3474 \text{ km}}{12\,742 \text{ km}}$$

$$\frac{\text{Moon model diameter}}{25} = \frac{3474}{12\,742}$$

$$\text{Moon model diameter} = \frac{3474}{12\,742} \times 25$$

$$\text{Moon model diameter} = 6.8 \text{ cm (one decimal place)}$$

Sometimes when wanting to compare quantities that are very large or very small it is convenient to let one quantity be equal to one unit as a reference point. This reference point could be the smallest quantity or a particular chosen quantity.

Example 2

The population of three countries at the end of 2016 rounded to the nearest million is shown below.

Let the smallest of these populations equal one unit and find the relative sizes of the other two countries.

Japan - 126 000 000
India - 1 327 000 000
Germany - 81 000 000

Step 1 Arrange in order from the smallest to the largest.

Germany - 81 000 000
Japan - 126 000 000
India - 1 327 000 000

Step 2 Let the smallest (Germany) equal one unit and calculate the relative sizes the other two.

$$\frac{\text{population of Japan}}{\text{population of Germany}} = \frac{126\,000\,000}{81\,000\,000}$$

$$\frac{\text{population of Japan}}{1} = \frac{126\,000\,000}{81\,000\,000}$$

$$\text{population of Japan} = 1.6 \text{ (one decimal place)}$$

$$\frac{\text{population of India}}{\text{population of Germany}} = \frac{1\,327\,000\,000}{81\,000\,000}$$

$$\frac{\text{population of India}}{1} = \frac{1\,327\,000\,000}{81\,000\,000}$$

$$\text{population of India} = 16.4 \text{ (one decimal place)}$$

Example 3

The land area of three countries is shown below. The areas are given in million km² rounded to one decimal place.

Let the land area of Australia equal one unit and find the relative sizes of the other two countries.

China - 9.6 million km²

Australia - 7.7 million km²

Spain - 0.5 million km²

Let Australia's area equal one unit and calculate the relative area of the other countries.

$$\frac{\text{Area of China}}{\text{Area of Australia}} = \frac{9.6 \text{ million km}^2}{7.7 \text{ million km}^2}$$

$$\frac{\text{Area of China}}{1} = \frac{9.6}{7.7}$$

$$\text{Area of China} = 1.25 \text{ (Two decimal places)}$$

$$\frac{\text{Area of Spain}}{\text{Area of Australia}} = \frac{0.5 \text{ million km}^2}{7.7 \text{ million km}^2}$$

$$\frac{\text{Area of Spain}}{1} = \frac{0.5}{7.7}$$

$$\text{Area of Spain} = 0.06 \text{ (Two decimal places)}$$

EXERCISE 12H

1. Jasper (from example 1) was so happy with his models of the Earth and Moon he decided to make models of all the other planets in our Solar System and the Sun. The diameters of all the planets and the Sun are shown in the table below.

Copy and complete this table showing the diameters of the models of the planets and the Sun given that the diameter of the model of the Earth is 25 cm.

Give diameters to the nearest cm.

<i>Planet/Sun</i>	<i>Diameter (km)</i>	<i>Diameter of Models (cm)</i>
Sun	1 391 400	
Mercury	4879	
Venus	12 104	
Earth	12 742	25
Mars	6779	
Jupiter	139 822	
Saturn	116 464	
Uranus	50 724	
Neptune	49 244	



2. After making his models of the planets (except the Sun!) Jasper decided to use the school oval to arrange the planets as they are in the Solar System. The school oval is 100 metres long. Jasper placed a picture of the Sun at one end of the oval and his model of Neptune at the other end of the oval. The table below shows the distances of the planets from the Sun. Copy and complete the table showing the distance the model of each planet should be from the picture of the Sun on the oval. Give distances in metres to one decimal place.

<i>Planet/Sun</i>	<i>Distance from the Sun (millions of km)</i>	<i>Distance from 'the Sun' on the oval (metres)</i>
Mercury	58	
Venus	108	
Earth	150	
Mars	228	
Jupiter	779	
Saturn	1429	
Uranus	2871	
Neptune	4498	100



3. Copy and complete the table below showing the masses of the planets in our Solar System and the Sun. Let the Earth equal one unit in the relative masses column and calculate the relative mass of each of the other planets and the Sun.

Give relative masses correct to two decimal places.

<i>Planet/Sun</i>	<i>Mass ($\times 10^{23}$ kg)</i>	<i>Relative Mass</i>
Sun	20 000 000	
Mercury	3	
Venus	49	
Earth	60	1
Mars	6	
Jupiter	19 000	
Saturn	5700	
Uranus	870	
Neptune	1000	

4. The areas of the seven continents of the world are shown in the table below. The continents are listed in alphabetical order.

- (a) Copy the table **BUT** list the continents in order from the smallest to the largest.
- (b) In the relative area column give the smallest continent a relative area of one unit and calculate all the other relative areas. (Two decimal places)

<i>Continent</i>	<i>Area (km²)</i>	<i>Relative Area</i>
Africa	30 065 000	
Antarctica	13 209 000	
Asia	44 579 000	
Australia	7 687 000	
Europe	9 938 000	
North America	24 256 000	
South America	17 819 000	

5. The sizes of several parts of the human body are shown in the table below.

- (a) Copy this table but list the parts from smallest to largest.
 (b) In the relative size column give the smallest part a relative value of one and calculate the relative size of each of the other parts.

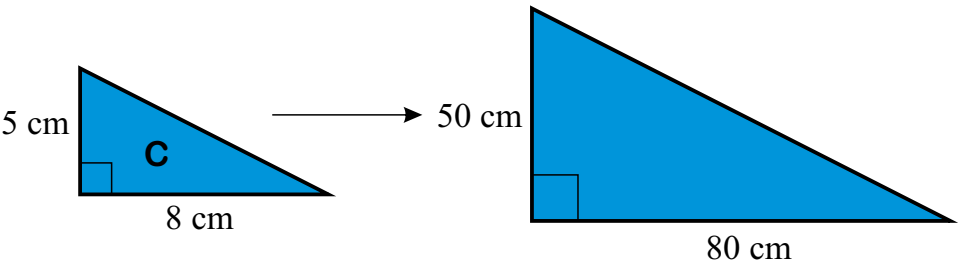
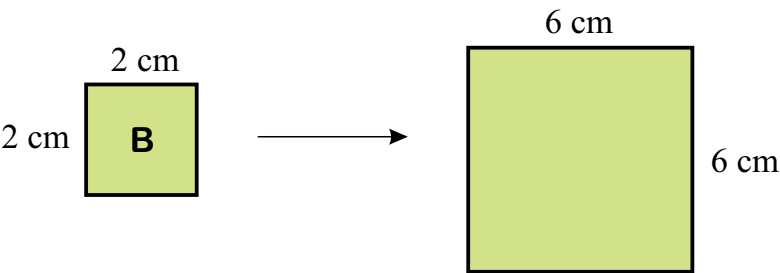
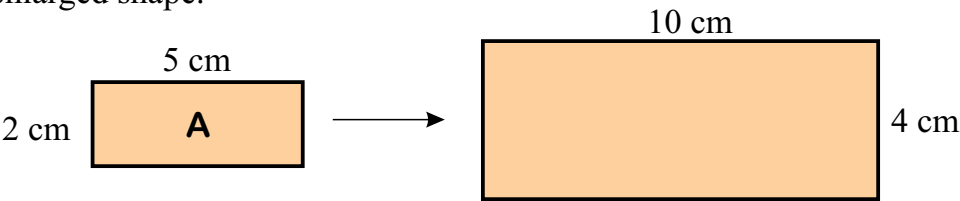
<i>Part</i>	<i>Size (mm)</i>	<i>Relative Size</i>
Diameter of Human Egg	0.2	
Length of Human Sperm	0.05	
Width of Hair	0.08	
Thickness of Fingernail	0.4	
Diameter of Blood Cell	0.002	
Diameter of Skin Cell	0.03	
Length of Gut Bacteria	0.01	
Diameter of Brain Cell	0.1	



Area

EXERCISE 12I

1. For the pairs of similar shapes below copy and complete the table showing the area of the first shape, the enlargement factor and the area of the enlarged shape.

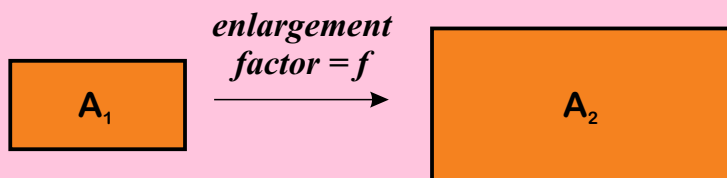


Shape	Area of Shape	Enlargement Factor	Area of Enlarged Shape
A			
B			
C			

2. Can you see a relationship between the areas of the shapes, the enlargement factors and the areas of the enlarged shapes?

You may have found that the area of the enlarged shape is equal to the area of the original shape multiplied by the square of the enlargement factor. This applies to all enlargements.

$$\text{area of enlarged shape} = \text{area of original shape} \times \text{square of the enlargement factor}$$



$$A_2 = A_1 \times f^2$$

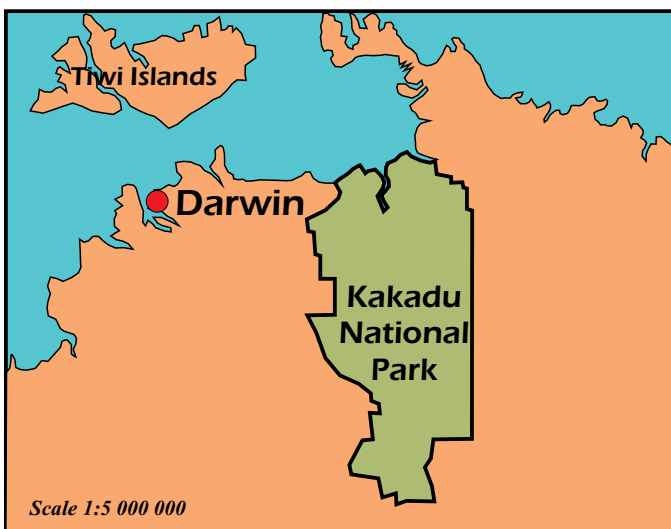
Example

A shape with an area of 20 cm^2 is enlarged by a factor of 4. Find the area of the enlarged shape.

$$\begin{aligned} A_2 &= A_1 \times f^2 \\ &= 20 \times 4^2 \\ &= 20 \times 16 \\ A_2 &= 320 \text{ cm}^2 \end{aligned}$$

3. Find the area of the enlarged shapes if the following shapes are enlarged by the given enlargement factors.
- A shape with an area of 8 cm^2 is enlarged by a factor of 2.
 - A shape with an area of 30 cm^2 is enlarged by a factor of 3.
 - A shape with an area of 15 cm^2 is enlarged by a factor of 5.
 - A shape with an area of 8.5 cm^2 is enlarged by a factor of 10.
 - A square with side length of 2.4 cm is enlarged by a factor of 6.
 - A rectangle with side lengths of 8.5 cm and 15.6 cm is enlarged by a factor of 20.

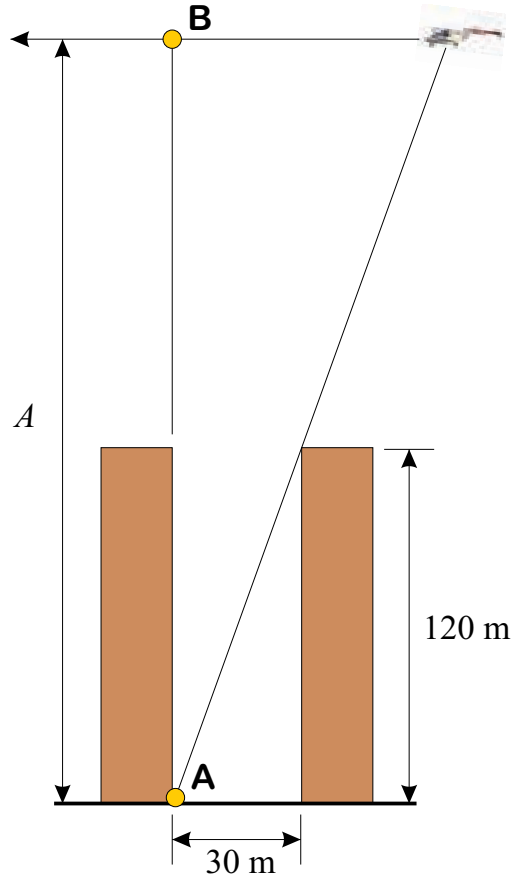
4. (a) A shape with an area of 30 cm^2 is enlarged by a factor of 100.
Find the area of the enlarged shape in: (i) cm^2 (ii) m^2
- (b) A shape with an area of 65 cm^2 is enlarged by a factor of 2000.
Find the area of the enlarged shape in: (i) cm^2 (ii) m^2
- (c) A shape with an area of 120 mm^2 is enlarged by a factor of 5000.
Find the area of the enlarged shape in: (i) mm^2 (ii) cm^2 (iii) m^2
5. The plan of a building has a scale of 1:500. A room on the plan measures $6 \text{ cm} \times 8 \text{ cm}$. Find the area of the room in the building.
Give answer in m^2 .
6. A block of land is drawn using a scale of 1:2000. The scale drawing measures $10 \text{ cm} \times 25 \text{ cm}$.
Find the area of the block of land.
Give answer in: (a) m^2 (b) hectares
7. The area of a rectangular camping ground measured $4 \text{ cm} \times 3 \text{ cm}$ on a map that had a scale of 1:40 000.
Find the area of the camping ground.
Give answer in: (a) m^2 (b) hectares
8. A map of Kakadu National Park is shown below. The scale is 1:5 000 000. The area of Kakadu National Park on this map could be approximated to a rectangle that is $2 \text{ cm} \times 4 \text{ cm}$.
What is the approximate area of Kakadu National Park.
Give answer in km^2 .



PROBLEM SOLVING

Nerali was standing at the base of a building, point **A** on this diagram, between two buildings that were both 120 m tall.

She noticed an aeroplane in line with the top corner of the building opposite to where she was standing. The aeroplane took 5 seconds to fly to point **B**, directly above Nerali. Nerali had a keen interest in aircraft and knew this aeroplane would be flying at a speed of 360 km/hour. Use this information and similar triangles to calculate the altitude, A , at which the aeroplane was flying.



PUZZLE

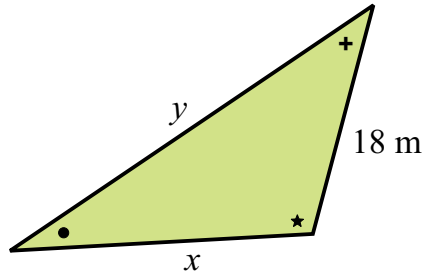
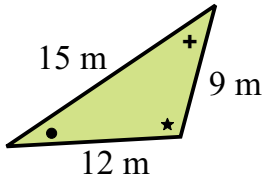
How many words of four letters or more can you make using the letters shown here. Each word must include the central letter and there is at least one nine letter word (from this chapter).

A	G	R
S	T	N
I	E	L

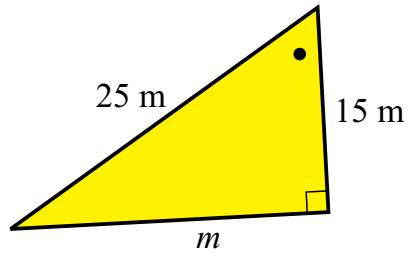
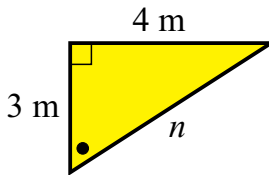
CHAPTER REVIEW

1. Find the unknown lengths.

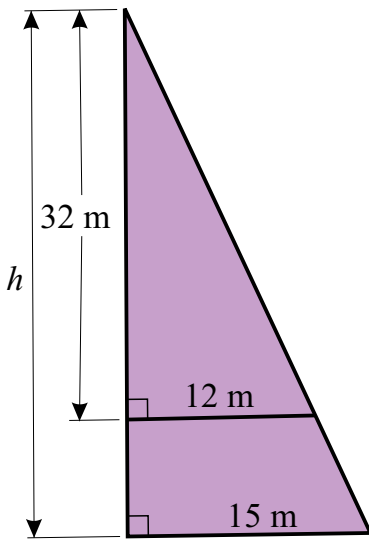
(a)



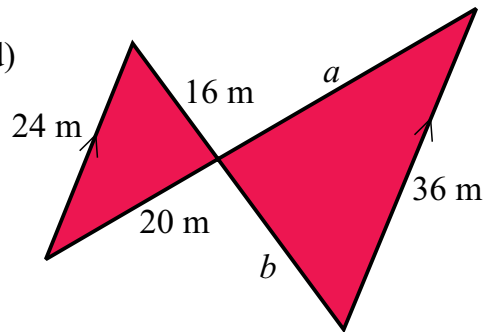
(b)



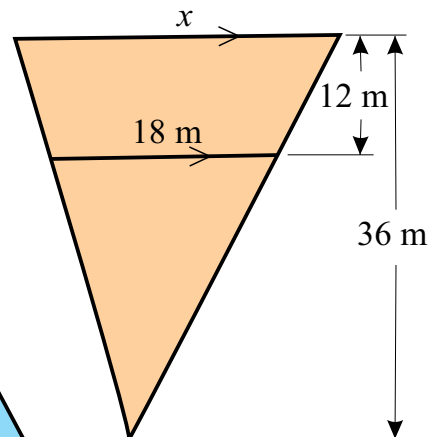
(c)



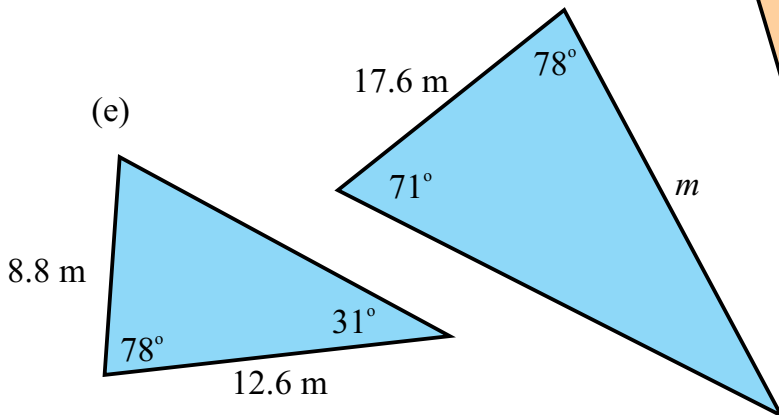
(d)



(f)



(e)



2. State if the following statements are true (**T**) or false (**F**).
- Two congruent shapes must be the same size.
 - The corresponding angles in two similar shapes must be the same size.
 - All squares of different sizes are similar.
 - All isosceles triangles of different sizes are similar.
 - All equilateral triangles of different sizes are similar.
3. The scale on a map is 1:5000.
- Find the actual distances (in *metres*) of the following lengths that were measured off the map.

(i) 2 cm	(ii) 63 mm	(iii) 11.5 cm
----------	------------	---------------
 - Find the lengths on the map (in *mm*) of the following distances.

(i) 800 m	(ii) 2 km	(iii) 620 m
-----------	-----------	-------------
4. A model of a 200 m long ship was 40 cm long.
What is the scale of the model?
5. Harriet made models of three buildings. The buildings and their heights are:
- | |
|---------------------------------|
| <i>Tango Towers</i> - 570 m |
| <i>Alpha Apartments</i> - 480 m |
| <i>Bravo Building</i> - 630 m |
- She made the model of the Tango Towers 95 cm tall.
If the other two models were to be the same scale, what would be the height of these models?
6. An astronomer found three asteroids that had not been discovered before. He named them after his three pet dogs - Basil, Polly and Sybil. Their approximate masses are shown below:
- | |
|---------------------------------------|
| <i>Basil</i> - 400 000 000 000 kg |
| <i>Polly</i> - 56 000 000 000 000 kg |
| <i>Sybil</i> - 280 000 000 000 000 kg |

If he let the relative mass of the smallest asteroid be one unit, find the relative masses of the other two asteroids.

Pythagoras

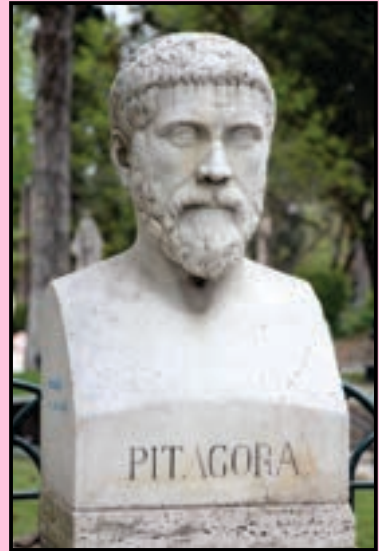
***Pythagoras was a Greek philosopher, mathematician and scientist.
(570-495 B.C.)***

Pythagoras contributed to many fields including philosophy, astronomy, mathematics, music, politics and medicine.

His philosophies influenced many philosophers including Plato.

He founded the Pythagorean School of Mathematics and it was from this school that the well known Pythagoras' Theorem was developed.

***Along with his contributions to the fields of science and mathematics his writings and poetry are well known.
Two of his quotes are shown below.***



***Rome, Italy. Bust statue of Pythagoras, famous philosopher, mathematician and scientist.
Sculpture in Villa Borghese park.***

**ABOVE ALL THINGS,
RESPECT YOURSELF**

$$A^2 + B^2 = C^2$$

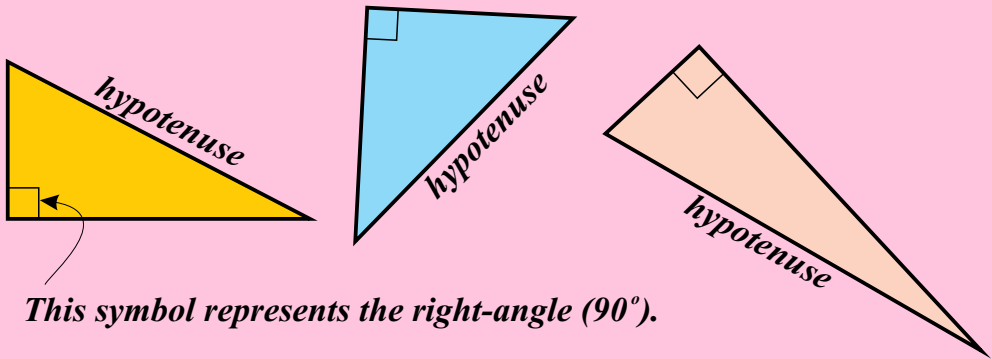
**IN NO WAY NEGLECT THE
HEALTH OF YOUR BODY**

Pythagoras' Theorem

Remember:

- A right-angled triangle is a triangle that has one angle that is 90° .
- The longest side of a right-angled triangle is opposite the right-angle and is called the ***hypotenuse***.

Examples of right-angled triangles showing their hypotenuse.



This symbol represents the right-angle (90°).

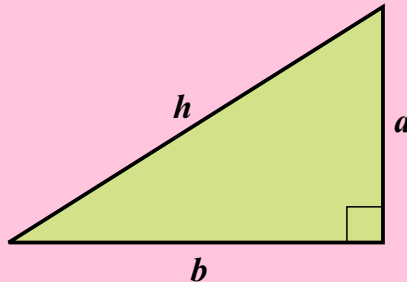
Pythagoras' Theorem states that:

For any ***right-angled triangle*** the square of the longest side (the ***hypotenuse***) is equal to the sum of the squares of the two shorter sides.

$$h^2 = a^2 + b^2$$

or

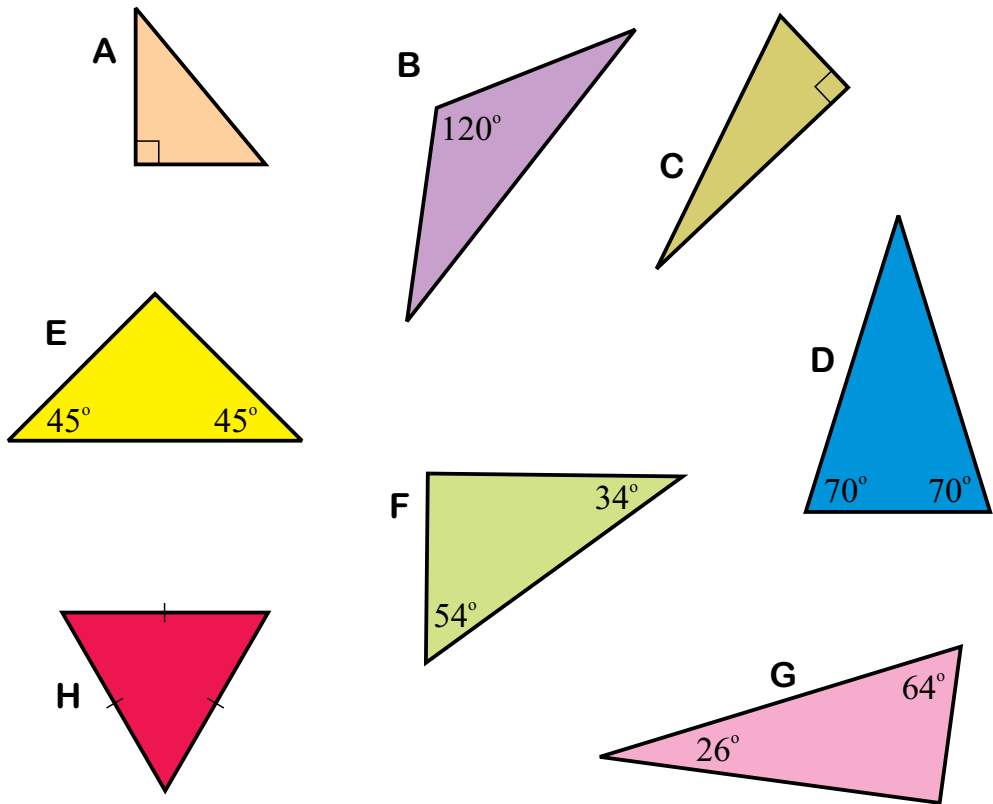
$$a^2 + b^2 = h^2$$



EXERCISE 13A

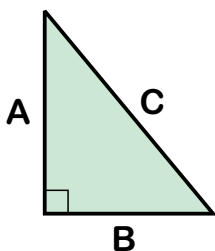
1. Which of the following triangles are *right-angled triangles*?

Remember: The sum of the angles in a triangle is 180°

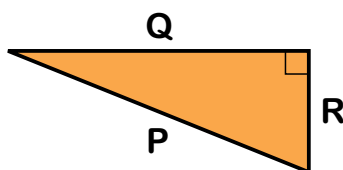


2. Which of the sides in each of the following triangles is the hypotenuse?

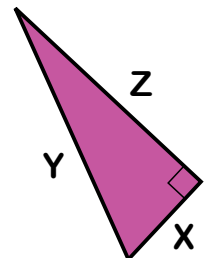
(a)



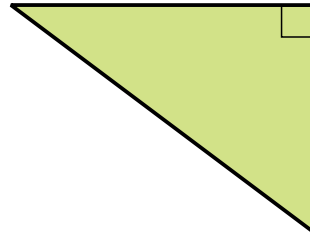
(b)



(c)



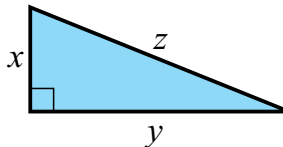
3. (a) Use a ruler to measure the length of the hypotenuse in this triangle.
 (b) Use a ruler to measure the other two sides of this triangle.
 (c) Show that this triangle obeys Pythagoras' Theorem.



4. State whether the following statements are **TRUE** or **FALSE**.

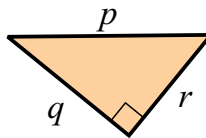
- (a) In this triangle:

$$z^2 = x^2 + y^2$$



- (b) In this triangle:

$$r^2 = p^2 + q^2$$



- (c) The hypotenuse is always opposite the right-angle.
 (d) The hypotenuse is always the longest side of a triangle.
 (e) In this triangle:

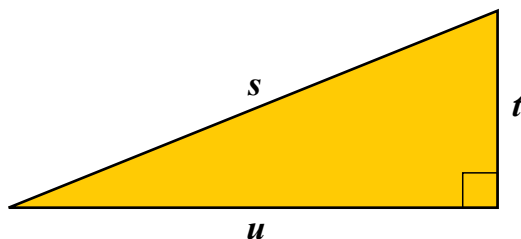
(i) $s^2 + t^2 + u^2 = 0$

(ii) $s^2 - t^2 - u^2 = 0$

(iii) $t^2 = s^2 - u^2$

(iv) $u^2 = s^2 - t^2$

(v) $s = \sqrt{t^2 + u^2}$



- (f) It is impossible to have a right-angled equilateral triangle.
 (g) It is impossible to have a right-angled isosceles triangle.
 (h) It is impossible to have a right-angled scalene triangle.

A **Pythagorean Triad** is a set of three numbers that follow Pythagoras' theorem.

Examples

1. 3, 4, 5

If three numbers follow Pythagoras' Theorem then the sum of the squares of the two smaller numbers should equal the square of the larger number.

$$\begin{array}{rcl}
 3^2 + 4^2 & \text{should equal} & 5^2 \\
 3^2 + 4^2 & & 5^2 \\
 = 9 + 16 & & = 25 \\
 = 25 & &
 \end{array}$$

These are equal so 3, 4, 5 is a Pythagorean Triad

2. 36, 39, 15

The two smaller numbers are 15 and 36.

For these three numbers to be a Pythagorean Triad then:

$$\begin{array}{rcl}
 15^2 + 36^2 & \text{should equal} & 39^2 \\
 15^2 + 36^2 & & 39^2 \\
 = 225 + 1296 & & = 1521 \\
 = 1521 & &
 \end{array}$$

These are equal so 15, 36, 39 is a Pythagorean Triad

3. 9, 43, 40

The two smaller numbers are 9 and 40.

For these three numbers to be a Pythagorean Triad then:

$$\begin{array}{rcl}
 9^2 + 40^2 & \text{should equal} & 43^2 \\
 9^2 + 40^2 & & 43^2 \\
 = 81 + 1600 & & = 1849 \\
 = 1681 & &
 \end{array}$$

These are not equal so 9, 43, 40 is not a Pythagorean Triad

5. Use a calculator to find the following.

- (a) 18^2 (b) 6.5^2 (c) 13.4^2 (d) 8.72^2

6. Which of the following sets of numbers are Pythagorean Triads?

- (a) 9, 40, 41 (b) 16, 63, 65 (c) 11, 60, 63
 (d) 15, 112, 113 (e) 13, 83, 85 (f) 264, 23, 265
 (g) 25, 7, 24 (h) 82, 18, 79 (i) 164, 160, 36
 (j) 131, 78, 105 (k) 50, 48, 14 (l) 224, 305, 207

7. Find the three numbers from the following list that form a Pythagorean Triad.

78 301 317 321
 308 75

8. There are many ways to prove Pythagoras' Theorem.

Here is one of them.

Make this construction on paper that you can cut.

Step 1 - Draw a right-angled triangle **ABC**.

Step 2 - Construct three **squares** on the sides of the triangle (**ABED**, **CBHI** and **ACFG**).

Step 3 - Find the **centre** of the middle sized square (**O**).

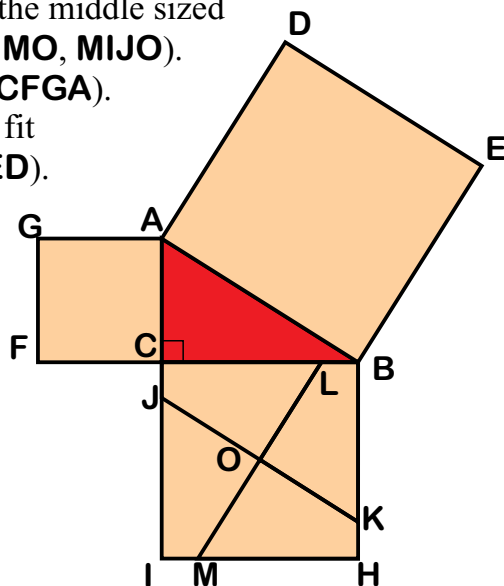
Step 4 - Draw a line (**JK**) through **O** that is parallel to **AB**.

Step 5 - Draw a line (**LM**) through **O** that is parallel to **EB**.

Step 6 - Cut out the four sections of the middle sized square (**JCLO**, **LBKO**, **KHMO**, **MIJO**).

Step 7 - Cut out the smaller square (**CFGA**).

Step 8 - Arrange these five pieces to fit into the largest square (**ABED**).



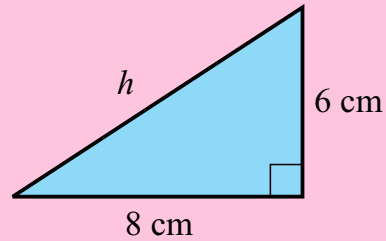
Explain how this proves Pythagoras' Theorem.

Finding the Hypotenuse

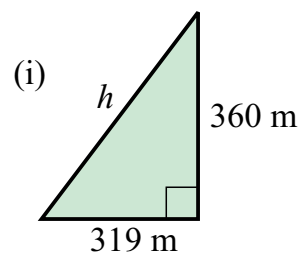
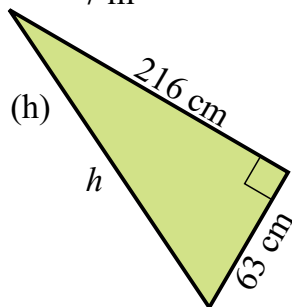
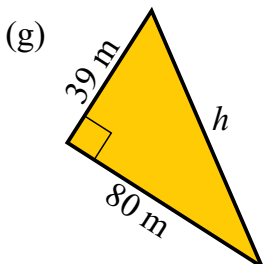
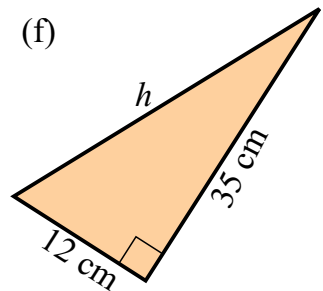
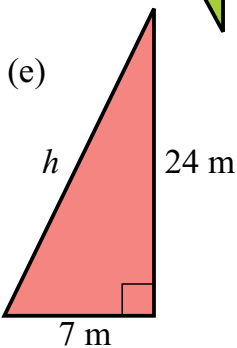
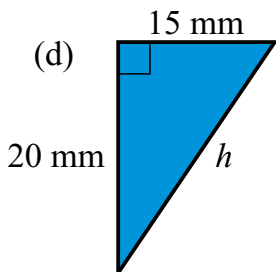
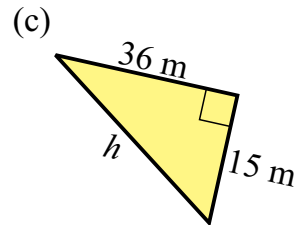
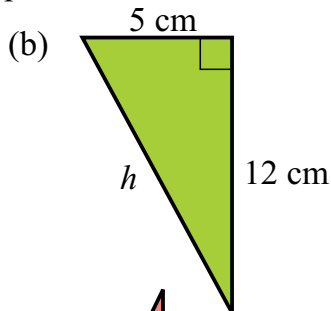
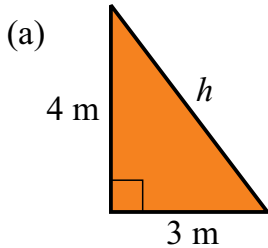
EXERCISE 13B

Example 1 Find the length of the hypotenuse in the following right-angled triangle.

$$\begin{aligned} h^2 &= a^2 + b^2 \\ h^2 &= 6^2 + 8^2 \\ h^2 &= 36 + 64 \\ h^2 &= 100 \\ h &= \sqrt{100} \\ h &= 10 \text{ cm} \end{aligned}$$



1. Find the length of the hypotenuse in each of the following triangles.



Example 2 Find the length of the hypotenuse in the following right-angled triangle.

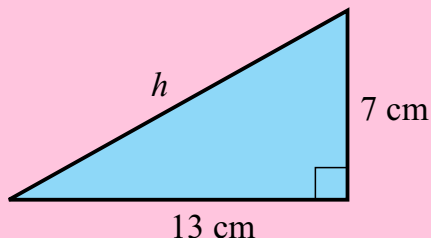
$$h^2 = a^2 + b^2$$

$$h^2 = 13^2 + 7^2$$

$$h^2 = 169 + 49$$

$$h^2 = 218$$

$$h = \sqrt{218}$$



Note this answer is a surd (irrational number) and an exact answer cannot be calculated. The answer can be given two ways:

(a) as a surd

(b) as a decimal number correct to a given number of decimal places.

In this example the answer would be:

(a) $h = \sqrt{218} \text{ cm}$

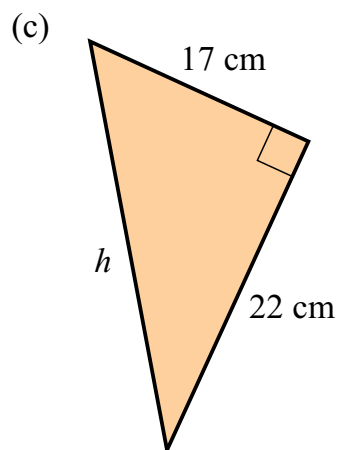
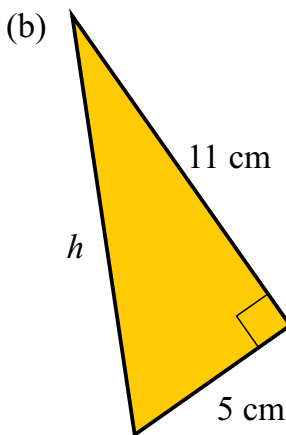
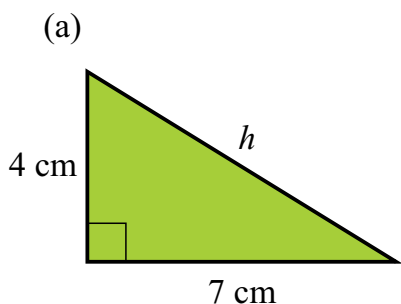
(b) $h = \sqrt{218}$
 $= 14.764823.....$

$h = 14.8 \text{ cm}$ (one decimal place)

2. Find the length of the hypotenuse in the following right-angled triangles.

Give answers as: (i) surds

(ii) decimal numbers correct to one decimal place



Example 3 Find the length of the hypotenuse in the following right-angled triangle.

$$h^2 = a^2 + b^2$$

$$h^2 = \left(\frac{8}{9}\right)^2 + \left(\frac{2}{3}\right)^2$$

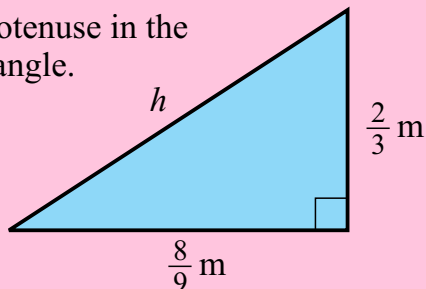
$$h^2 = \frac{64}{81} + \frac{4}{9}$$

$$h^2 = \frac{64}{81} + \frac{36}{81} \quad \leftarrow \text{Common denominator so the fractions can be added}$$

$$h^2 = \frac{100}{81}$$

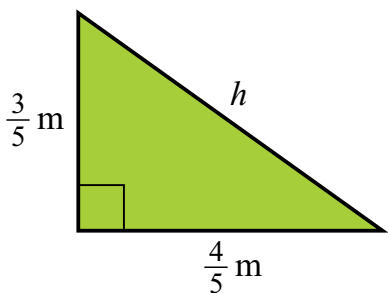
$$h = \sqrt{\frac{100}{81}}$$

$$h = \frac{10}{9} \text{ m}$$

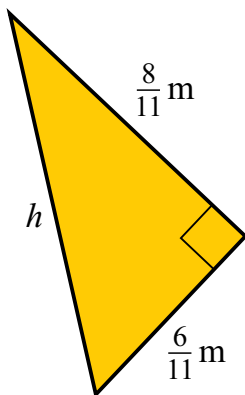


3. Find the length of the hypotenuse in the following right-angled triangles.

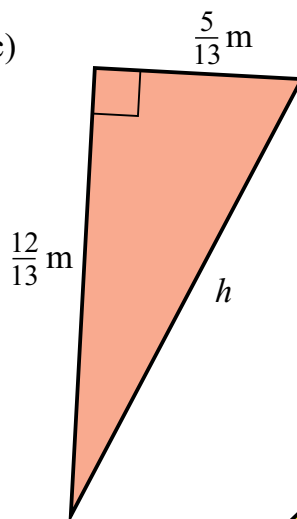
(a)



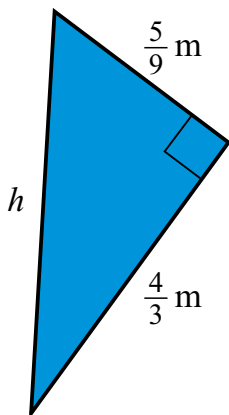
(b)



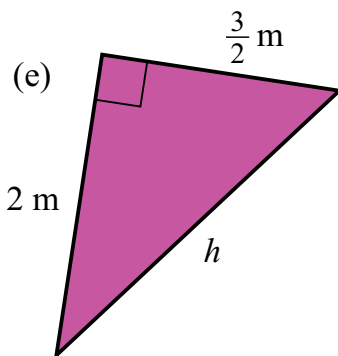
(c)



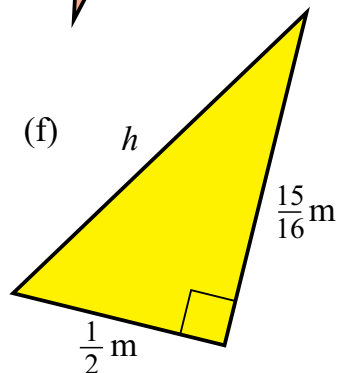
(d)



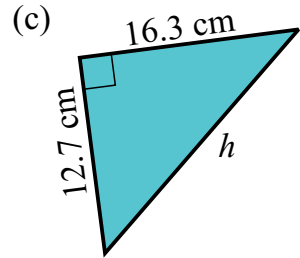
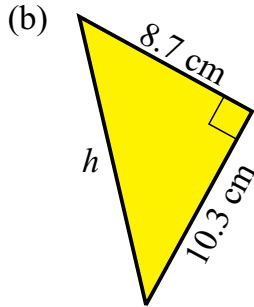
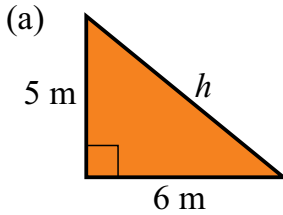
(e)



(f)

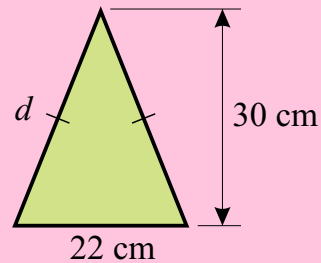


4. Find the length of the hypotenuse in each of the following triangles.
Give answers correct to one decimal place.



Example Find the unknown length in the following triangle.
Give answer correct to one decimal place.

To find the unknown length
a right-angled triangle needs
to be formed to use Pythagoras'
Theorem.



$$d^2 = a^2 + b^2$$

$$d^2 = 11^2 + 30^2$$

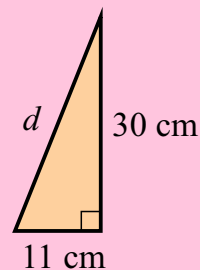
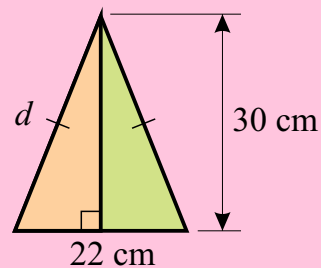
$$d^2 = 121 + 900$$

$$d^2 = 1021$$

$$d = \sqrt{1021}$$

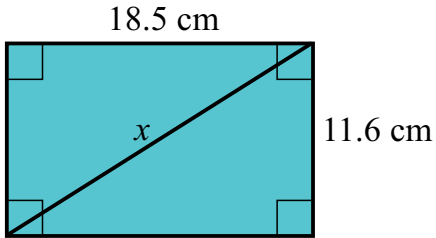
$$d = 31.95309...$$

$$d = 32.0 \text{ cm}$$

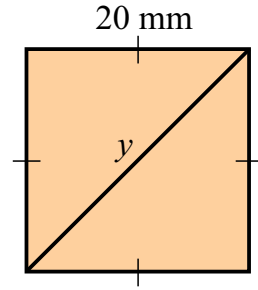


5. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

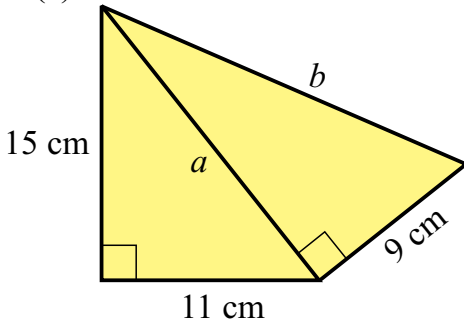
(a)



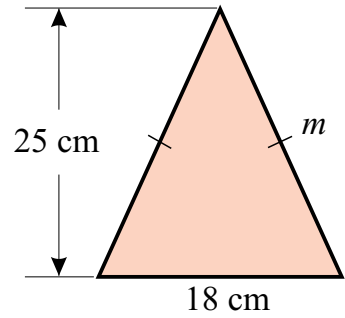
(b)



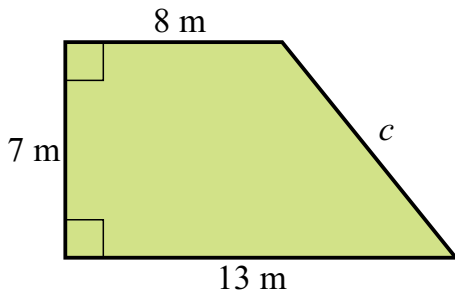
(c)



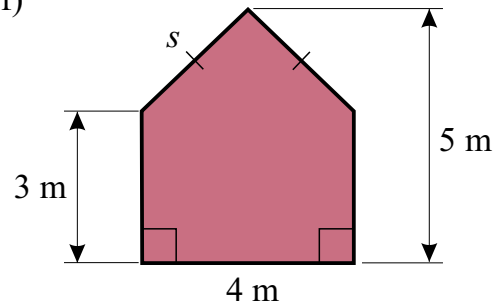
(d)



(e)



(f)



Finding One of the Shorter Sides

EXERCISE 13C

Example Find the length of the unknown side in the following right-angled triangle.

Note - use $a^2 + b^2 = h^2$ when finding a shorter side

$$a^2 + b^2 = h^2$$

$$a^2 + 20^2 = 25^2$$

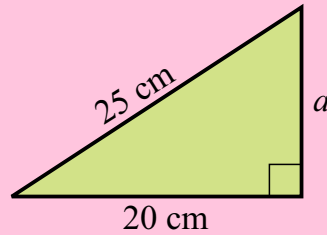
$$a^2 + 400 = 625$$

$$a^2 = 625 - 400$$

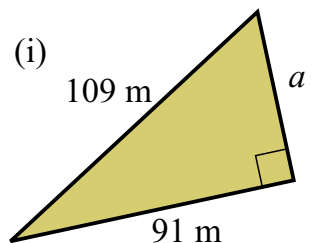
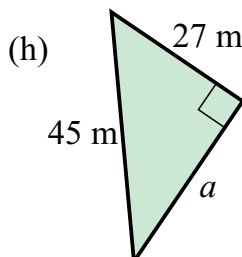
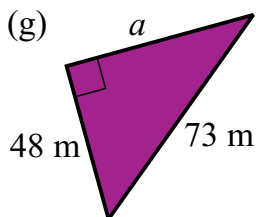
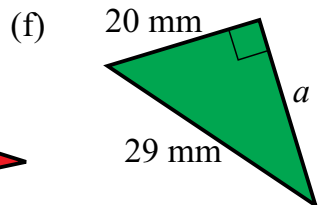
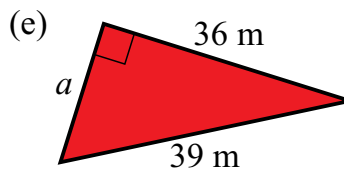
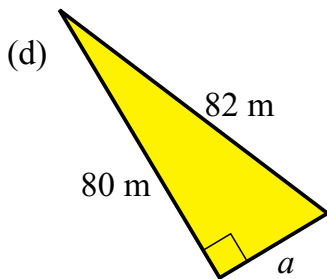
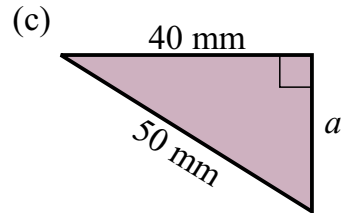
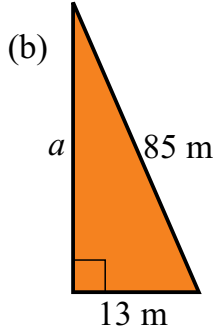
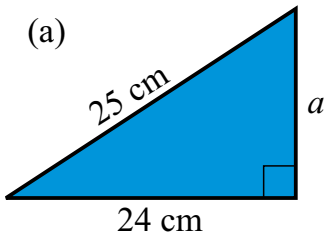
$$a^2 = 225$$

$$a = \sqrt{225}$$

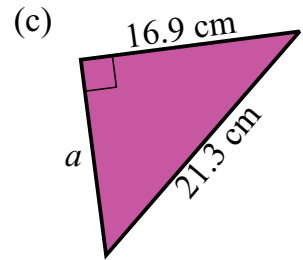
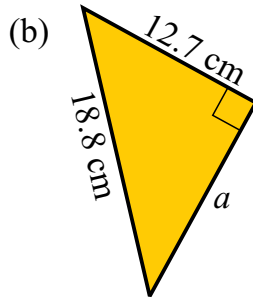
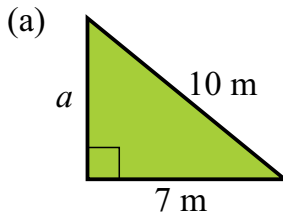
$$a = 15 \text{ cm}$$



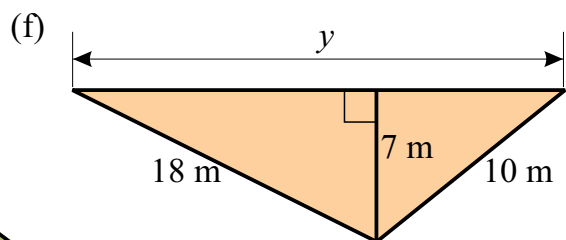
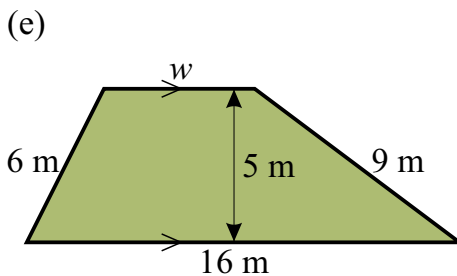
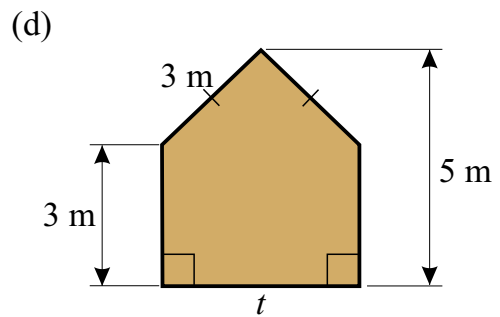
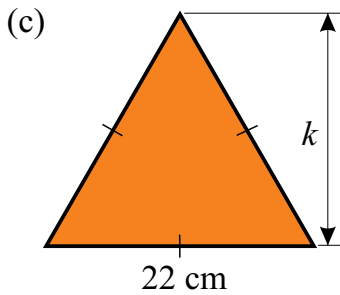
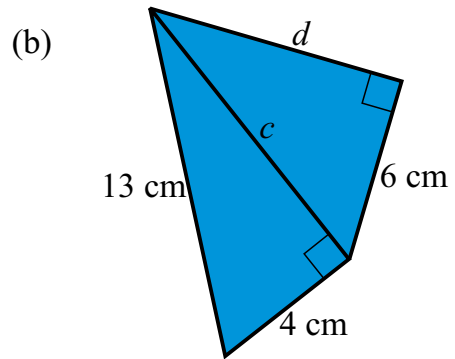
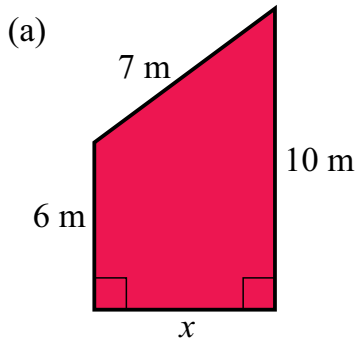
1. Find the length of the unknown side in each of the following triangles.



2. Find the length of the unknown side in each of the following triangles.
Give answers correct to one decimal place.



3. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

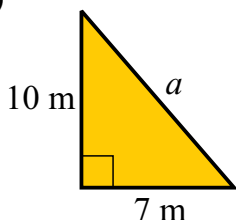


Mixed Problems

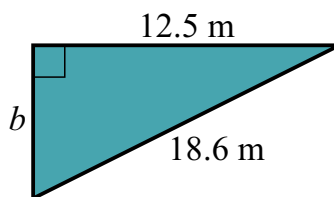
EXERCISE 13D

1. Calculate the length of a diagonal of a square with side length 25 cm. Give answer correct to one decimal place.
2. Calculate the width of a rectangle that is 208 cm long and has a diagonal that is 233 cm long.
3. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

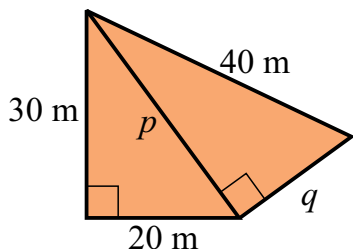
(a)



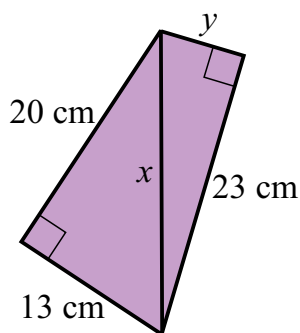
(b)



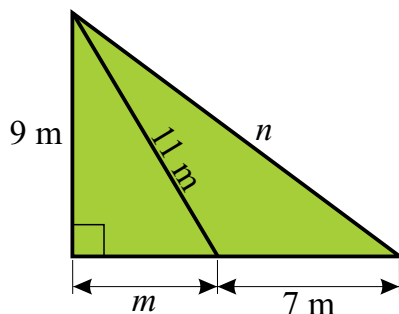
(c)



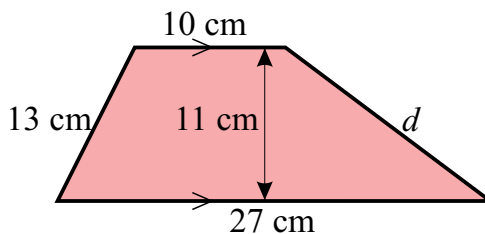
(d)



(e)



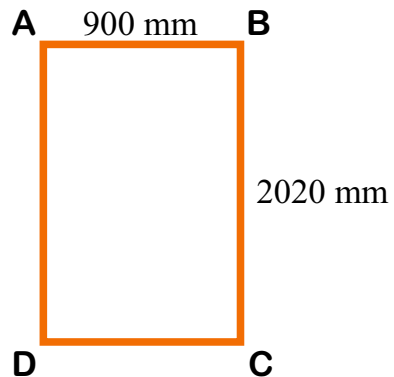
(f)



Practical Problems

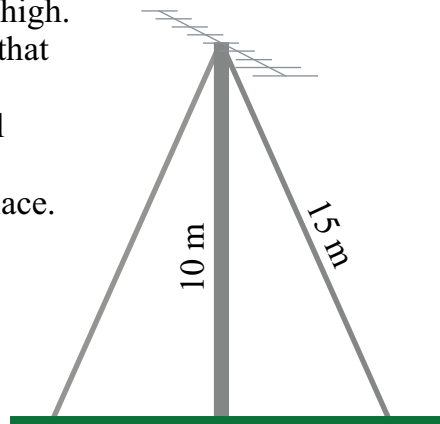
EXERCISE 13E

1. A carpenter wants to check if a door frame (**ABCD**) is 'square' (all angles are 90°) by measuring the diagonals. These will be equal only if all the angles of the frame are 90° . The dimensions of the door frame are $2020 \text{ mm} \times 900 \text{ mm}$ as shown.



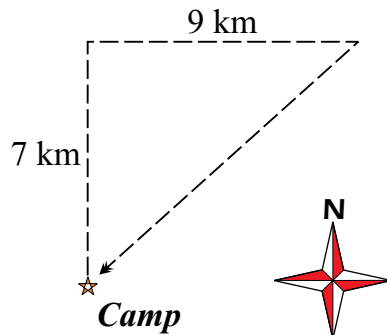
- (a) What should be the length of each diagonal (**AC** & **BD**)?
Give answer to the nearest mm.
- (b) The carpenter found that **AC** measured 2230 mm.
Which of the angles (**A**, **B**, **C**, **D**) are *greater* than 90° ?

2. (a) An antenna support pole is 10 metres high. The pole is to be supported by cables that are 15 metres long. How far from the base of the pole will the cables be fixed to the ground?
Give answer correct to one decimal place.

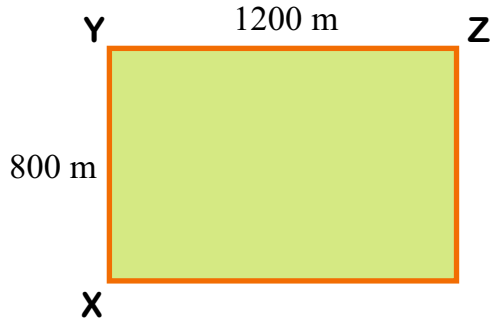


- (b) It was found that the pole was still unstable. More cables were fixed from the top of the pole to points 25 metres from the base of the pole. What will be the length of these cables?
Give answer correct to one decimal place.

3. Two bush walkers hiked 7 km north from their camp, then 9 km east and then walked in a straight line back to their camp. How far did they hike?
Give answer correct to one decimal place.

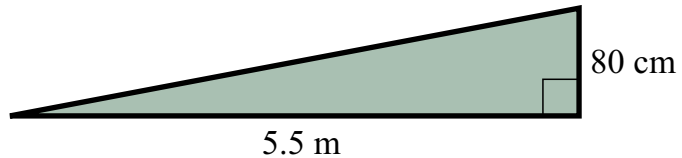


4. Noel and Paddy were at one corner (**X**) of a rectangular paddock. They wanted to run to the opposite corner (**Z**). Noel ran around the paddock (**X-Y-Z**) at a speed of 8 metres per second.



- Paddy ran straight from **X** to **Z**, through the paddock, at a speed of 5 metres per second.
 (a) How far did Paddy have to run (to the nearest metre)?
 (b) How many seconds did it take each runner to get to point **Z**?

5. A ramp for wheelchair access to a building is to be built. The ramp is to start 5.5 metres from the building and finish at the doorway which is 80 cm above the ground. How long is the ramp?
 Give answer in metres correct to three decimal places.



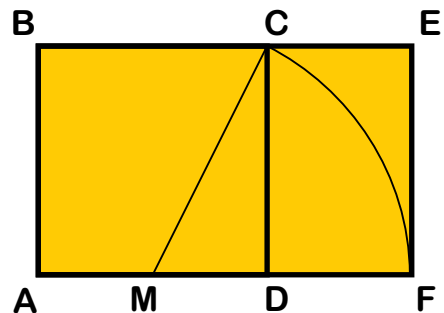
6. An artist wants to make a rectangular frame for a painting using the **Golden Ratio**. A rectangle with this shape is said to be the most pleasing to look at and is called a **Golden Rectangle**. To achieve this she must follow these steps:

Step 1 - Draw a square **ABCD**.

Step 2 - Mark a point (**M**) midway along **AD**.

Step 3 - Connect **M** to **C** and draw an arc using **M** as the centre to find point **F**.

Step 4 - **ABEF** is called a **Golden Rectangle**.



Find the length (**AF**) of the artist's frame if it is to be 70 cm wide (**AB**).
 Give answer correct to one decimal place.

7. A Royal Flying Doctor has to visit three cattle stations.

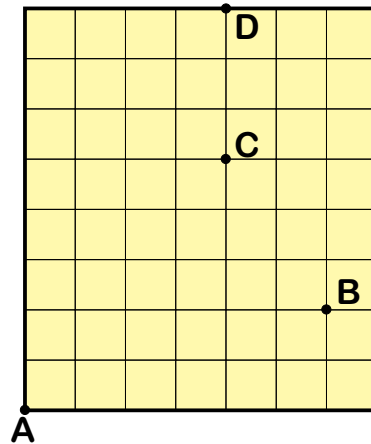
The map shown here indicates the position of the airport (**A**) and the three stations (**B**, **C** and **D**).

The lines on the grid are 50 km apart.

The doctor needs to visit station **B** first but needs to work out which will be the shortest route to visit all the stations and return to the airport:

- (a) **A-B-C-D-A** or
(b) **A-B-D-C-A**.

Calculate the total length of each route to find out which is the shortest.
Give answers correct to one decimal place.

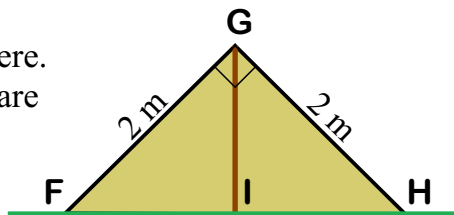


8. The end view of a tent is shown here.

The sides of the tent (**FG** & **GH**) are 2 metres and are at right angles.

Find the height of the tent post (**GI**).

Give answer correct to three decimal places.



9. (a) Complete this saying:

The shortest distance between two points is:

A _ _ _ A _ _ _ T _ I _ _

- (b) Rearrange the letters of each of the words below to find a saying that means to travel in a straight line between two places.

SA HET WORC SLIFE

Pythagoras in 3D

EXERCISE 13F

Example

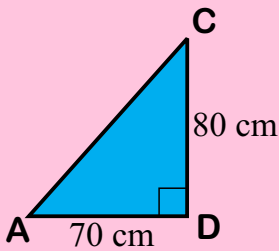
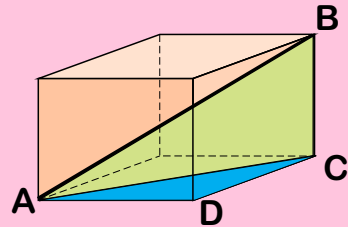
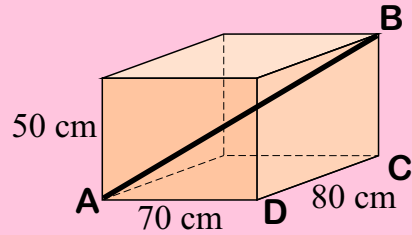
Find the length of the diagonal **AB** in the box shown.

Step 1 - Identify **AB** as the hypotenuse of triangle **ABC**.

Step 2 - Identify **AC** as the hypotenuse of triangle **ACD**.

Step 3 - Calculate **AC**.

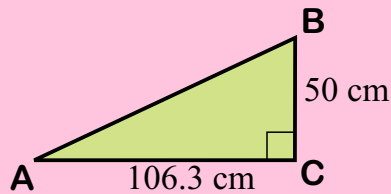
Step 4 - Calculate **AB**.



$$AC^2 = 70^2 + 80^2$$

$$AC^2 = 11300$$

$$AC = 106.3 \text{ cm}$$



$$AB^2 = 106.3^2 + 50^2$$

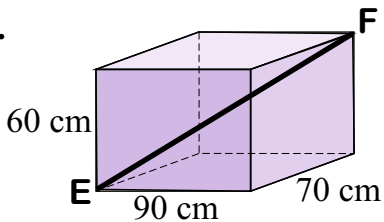
$$AB^2 = 13800$$

$$AB = 117.5 \text{ cm}$$

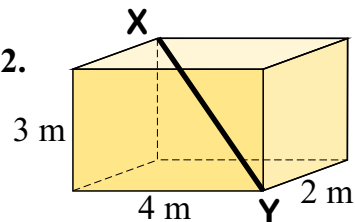
Find the length of the diagonals shown in the cuboids below.

Give answers correct to one decimal place.

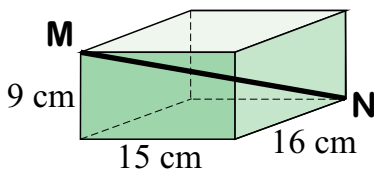
1.



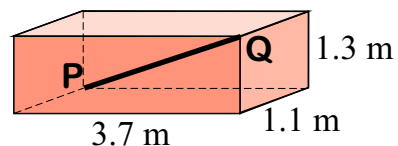
2.



3.



4.



PROBLEM SOLVING

A contour map is shown here.

The grid on the map is 100 m squares.

Heidi is walking from **A** to **B**.

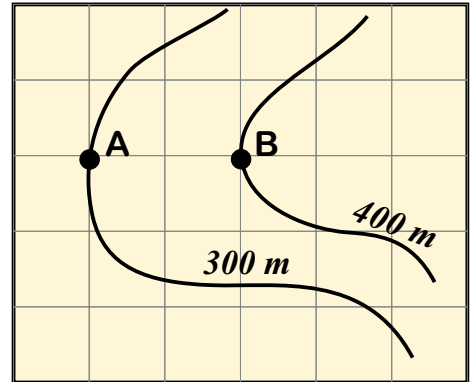
Point **A** is 300 m above sea level.

Point **B** is 400 m above sea level.

There is a constant slope between points **A** and **B**.

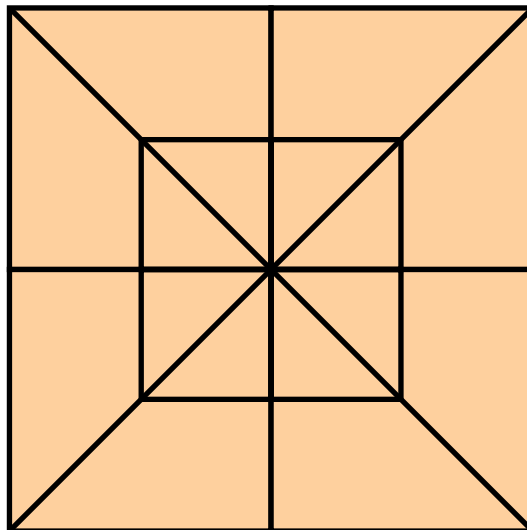
Use the grid and Pythagoras' Theorem to calculate the distance Heidi has to walk between points **A** and **B**.

Give correct to one decimal place.



PUZZLE

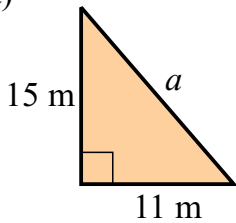
How many right-angled triangles are in this shape?



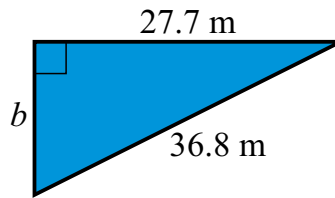
CHAPTER REVIEW

1. Find the unknown lengths in the following shapes.
Give answers correct to one decimal place.

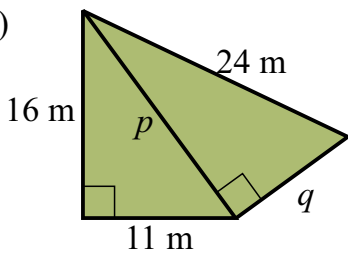
(a)



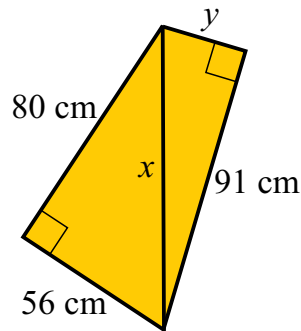
(b)



(c)

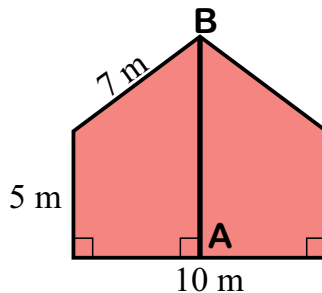


(d)

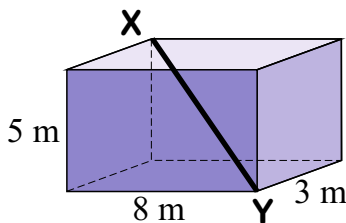


2. A yacht sails 15 km north from the yacht club then 12.5 km west.
How far is the yacht from the yacht club?
Give answer correct to one decimal place.

3. Find the length of **AB**.
Give answer correct to one decimal place.



4. Find the length of the **XY**
in the cuboid below.
Give answer correct to one decimal place.

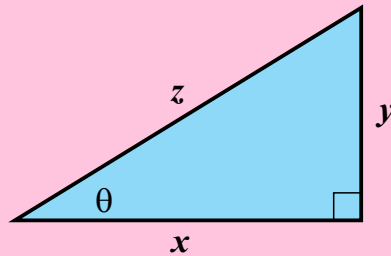
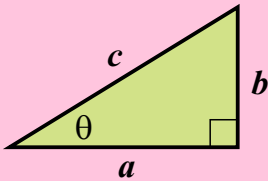


Trigonometry

Trigonometry is the branch of mathematics that studies triangles and in particular right-angled triangles.

From the chapter on **similarity** it can be seen that if right-angled triangles of different sizes have corresponding angles of the same size, they are similar and the ratio of side lengths is the same.

Example



These two triangles are **similar** because the corresponding angles are the same so the ratio of side lengths will be the same.

In particular the following ratios are used in trigonometry and given special names.

$$\frac{b}{c} = \frac{y}{z} = \textit{sine of the angle } \theta - \text{abbreviated to } \sin \theta$$

$$\frac{a}{c} = \frac{x}{z} = \textit{cosine of the angle } \theta - \text{abbreviated to } \cos \theta$$

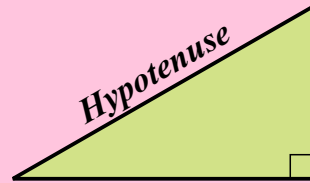
$$\frac{b}{a} = \frac{y}{x} = \textit{tangent of the angle } \theta - \text{abbreviated to } \tan \theta$$

These are called the **trigonometric functions** and will be studied in detail in this chapter.

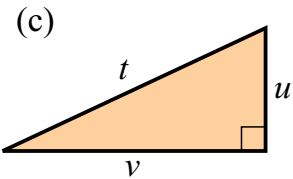
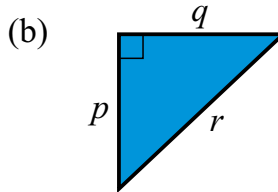
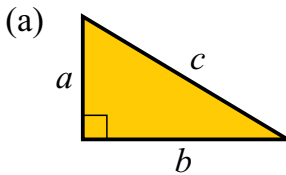
Right-angled Triangles

EXERCISE 14A

The ***hypotenuse*** is the longest side of a right-angled triangle. It is always opposite the right-angle.



1. Which side is the hypotenuse in the following triangles?



The two other sides of a right-angled triangle are labelled ***opposite*** and ***adjacent*** depending on the angle.

Example

For angle θ :

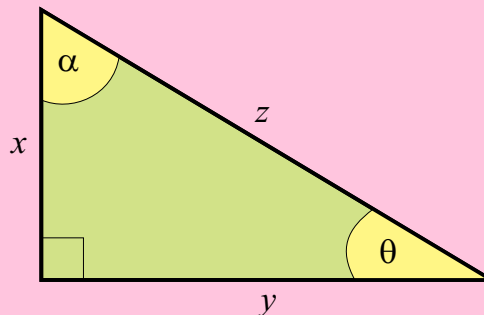
x is the ***opposite*** side

y is the ***adjacent*** side

For angle α :

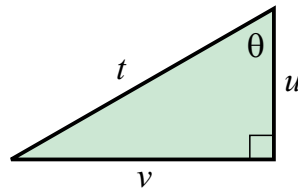
y is the ***opposite*** side

x is the ***adjacent*** side



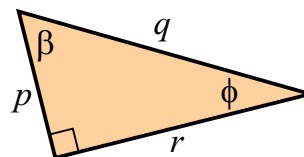
2. For the triangle shown:

- which side is the hypotenuse?
- which side is opposite angle θ ?
- which side is adjacent to angle θ ?



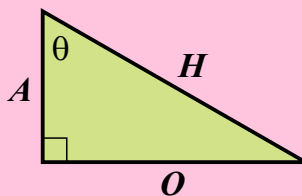
3. For this triangle:

- which side is opposite angle β ?
- which side is adjacent to angle β ?
- which side is opposite angle ϕ ?
- which side is adjacent to angle ϕ ?
- which side is the hypotenuse?

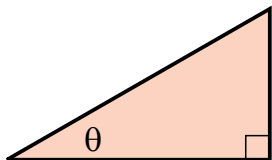


4. Copy the following triangles and label the sides **O** (opposite), **A** (adjacent) and **H** (hypotenuse) for the angle shown.

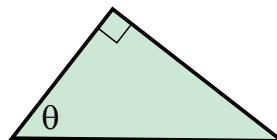
Example



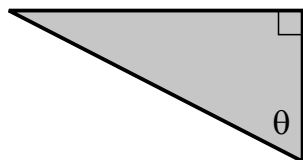
(a)



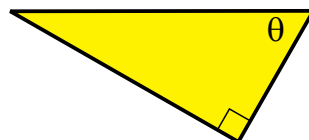
(b)



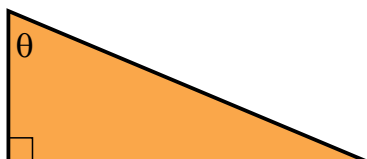
(c)



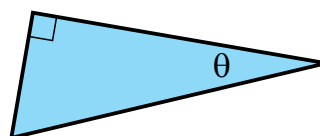
(d)



(e)



(f)



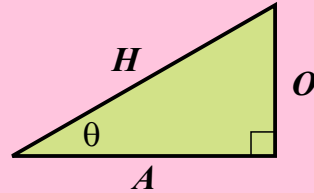
Sine

EXERCISE 14B

The **sine** of an angle is defined as the ratio of the side opposite the angle to the hypotenuse.

The sine of θ is abbreviated to ***sin*** θ .

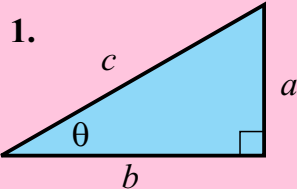
$$\text{Sine of } \theta = \sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$



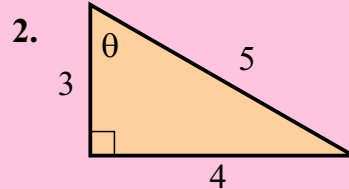
$$\sin \theta = \frac{O}{H}$$

1. Find $\sin \theta$ in the following triangles and write in fraction form.

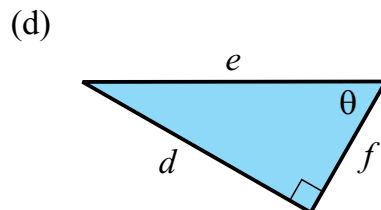
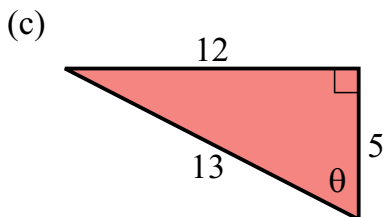
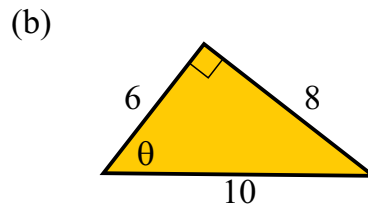
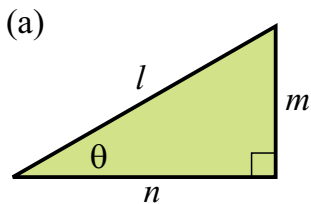
Examples



$$\sin \theta = \frac{a}{c}$$

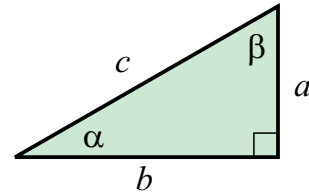


$$\sin \theta = \frac{4}{5}$$



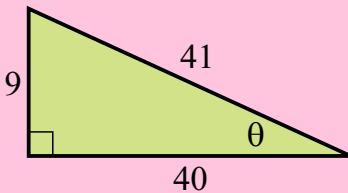
2. For the triangle shown here find the correct angles to complete the following:

(a) $\sin _ = \frac{a}{c}$ (b) $\sin _ = \frac{b}{c}$



3. Find $\sin \theta$ in the following triangles and write as decimal numbers correct to four decimal places.

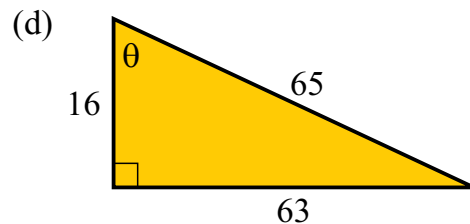
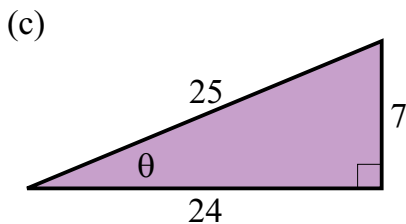
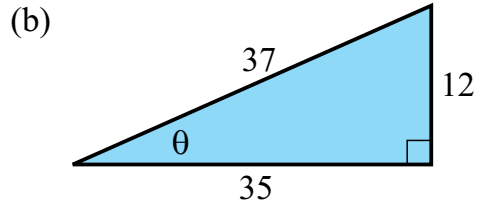
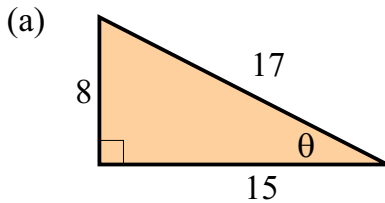
Example



$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{9}{41}$$

$$\sin \theta = 0.2195 \text{ (4 dec. pl.)}$$



4. Use a calculator to find the following values correct to four decimal places.

Example

$$\sin 43^\circ = 0.6820$$

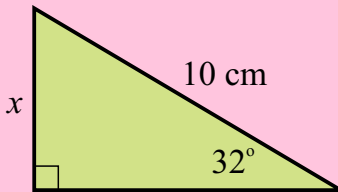
Check the calculator is in DEGREE mode when the angles are given in degrees.

- | | | |
|-----------------------|---------------------|---------------------|
| (a) $\sin 56^\circ$ | (b) $\sin 45^\circ$ | (c) $\sin 60^\circ$ |
| (d) $\sin 38.2^\circ$ | (e) $\sin 35^\circ$ | (f) $\sin 30^\circ$ |
| (g) $\sin 5^\circ$ | (h) $\sin 89^\circ$ | (i) $\sin 90^\circ$ |

Finding the Opposite Side Using Sine

EXERCISE 14C

Example



$$\sin \theta = \frac{O}{H}$$

$$\sin 32^\circ = \frac{x}{10}$$

$$10 \times \sin 32^\circ = x$$

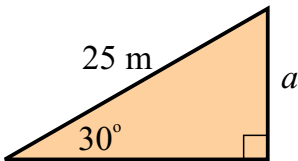
$$x = 10 \times \sin 32^\circ$$

$$x = 10 \times 0.5299$$

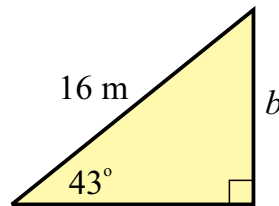
$$x = \mathbf{5.3 \text{ cm}} \text{ (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

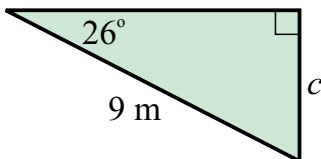
1.



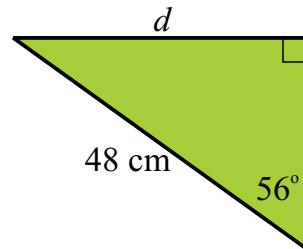
2.



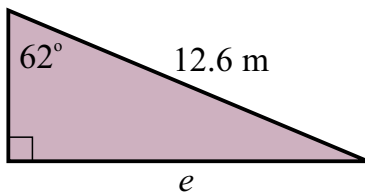
3.



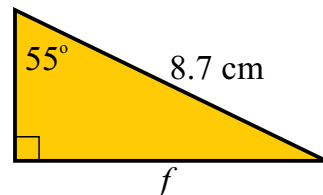
4.



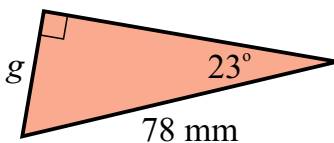
5.



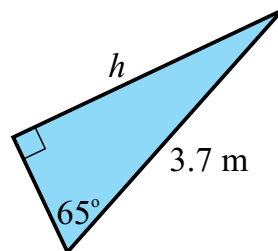
6.



7.



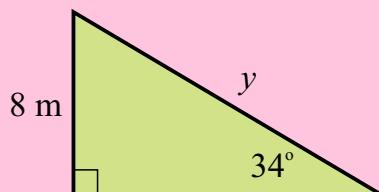
8.



Finding the Hypotenuse Using Sine

EXERCISE 14D

Example



$$\sin \theta = \frac{O}{H}$$

$$\sin 34^\circ = \frac{8}{y}$$

$$y \times \sin 34^\circ = 8$$

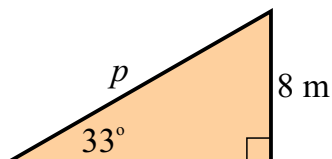
$$y = \frac{8}{\sin 34^\circ}$$

$$y = \frac{8}{0.5592}$$

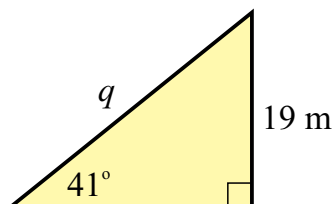
$$y = 14.3 \text{ m (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

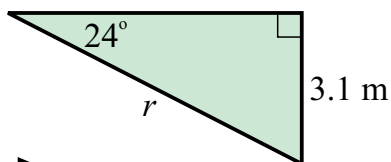
1.



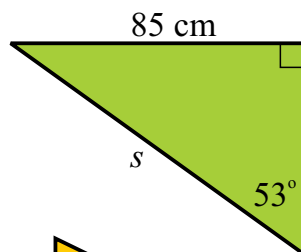
2.



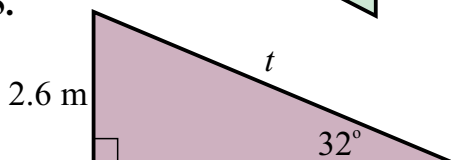
3.



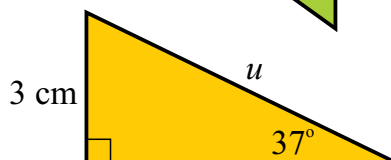
4.



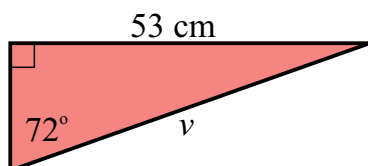
5.



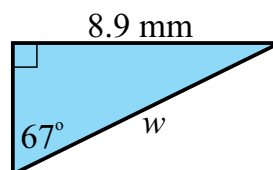
6.



7.



8.

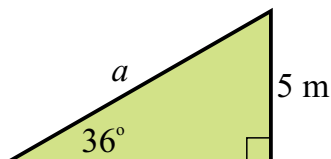


Mixed Problems Using Sine

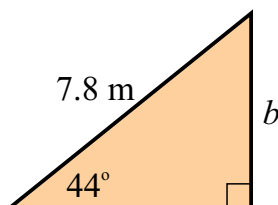
EXERCISE 14E

1. Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

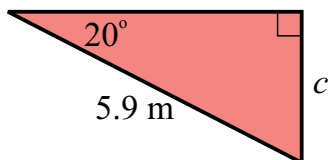
(a)



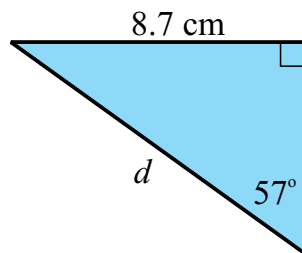
(b)



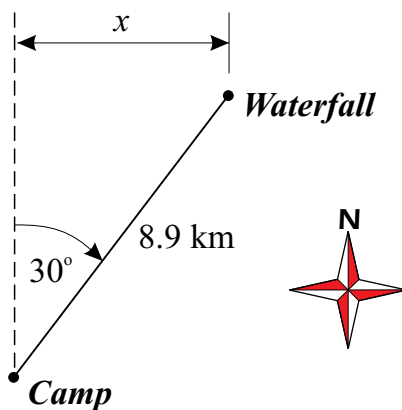
(c)



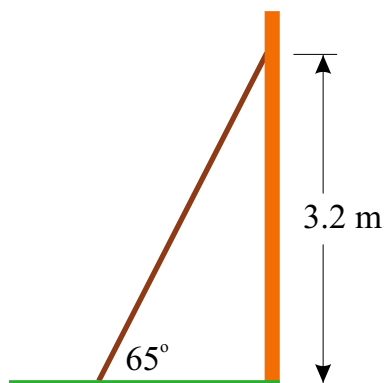
(d)



2. Ros and Mareyke were camping.
One day they walked 8.9 km in a direction $N 30^\circ E$ from their camp to see a waterfall.
How far east from their camp (x) is the waterfall?



3. A ladder is placed against a wall.
The top of the ladder is 3.2 m from the ground. The ladder makes an angle of 65° with the ground.
How long is the ladder?
Give answer correct to two decimal places.



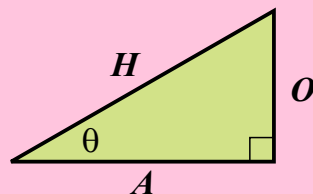
Cosine

EXERCISE 14F

The **cosine** of an angle is defined as the ratio of the side adjacent to the angle to the hypotenuse.

The cosine of θ is abbreviated to **cos** θ .

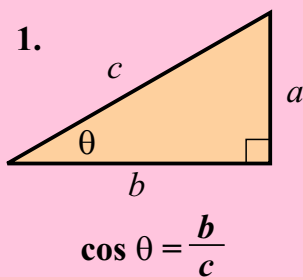
$$\text{Cosine of } \theta = \cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$



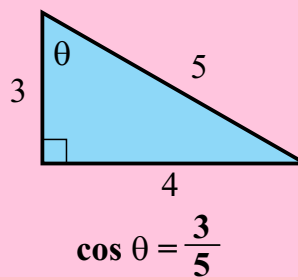
$$\cos \theta = \frac{A}{H}$$

1. Find $\cos \theta$ in the following triangles and write in fraction form.

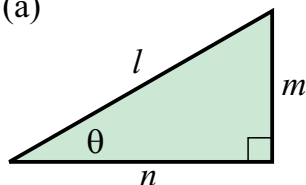
Examples 1.



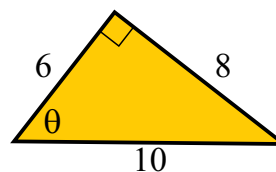
2.



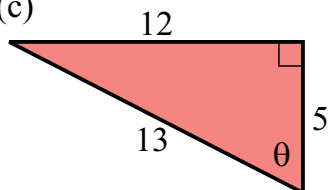
(a)



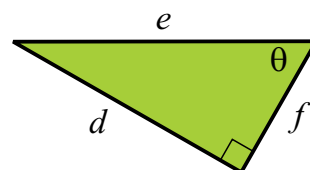
(b)



(c)

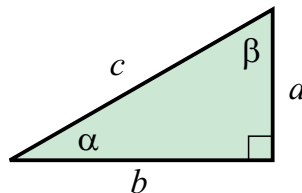


(d)



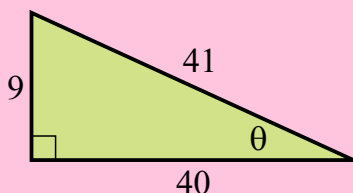
2. For the triangle shown here find the correct angles to complete the following:

(a) $\cos __ = \frac{a}{c}$ (b) $\cos __ = \frac{b}{c}$



3. Find $\cos \theta$ in the following triangles and write as decimal numbers correct to four decimal places.

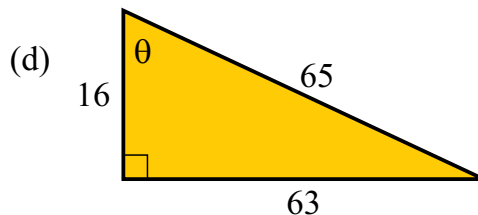
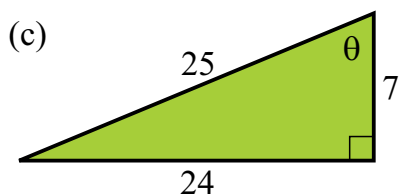
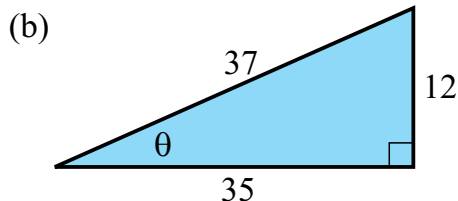
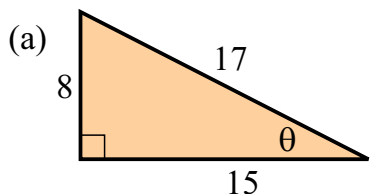
Example



$$\cos \theta = \frac{A}{H}$$

$$\cos \theta = \frac{40}{41}$$

$$\cos \theta = \mathbf{0.9756} \text{ (4 dec. pl.)}$$



4. Use a calculator to find the following values correct to four decimal places.

Example
 $\cos 43^\circ = 0.7314$

**Check the calculator is in DEGREE mode
when the angles are given in degrees.**

(a) $\cos 56^\circ$

(b) $\cos 45^\circ$

(c) $\cos 60^\circ$

(d) $\cos 38.2^\circ$

(e) $\cos 35^\circ$

(f) $\cos 30^\circ$

(g) $\cos 16.5^\circ$

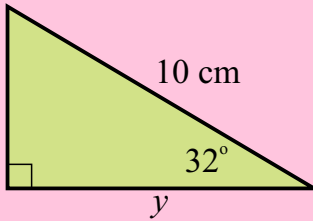
(h) $\cos 0^\circ$

(i) $\cos 90^\circ$

Finding the Adjacent Side Using Cosine

EXERCISE 14G

Example



$$\cos \theta = \frac{A}{H}$$

$$\cos 32^\circ = \frac{y}{10}$$

$$10 \times \cos 32^\circ = y$$

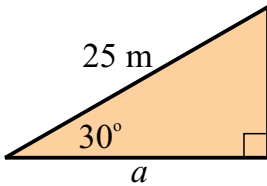
$$y = 10 \times \cos 32^\circ$$

$$y = 10 \times 0.8480$$

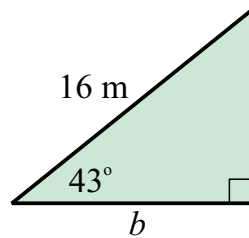
$$y = \mathbf{8.5 \text{ cm}} \text{ (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

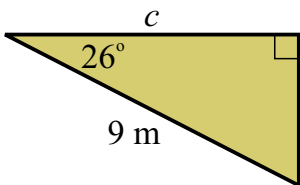
1.



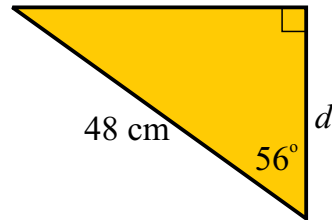
2.



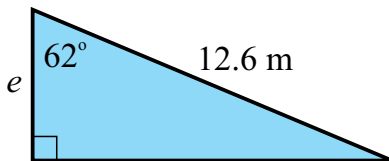
3.



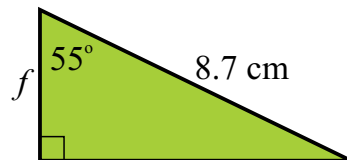
4.



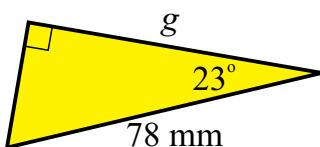
5.



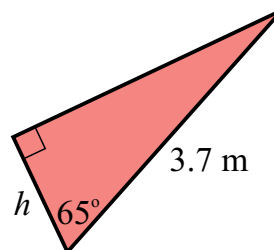
6.



7.



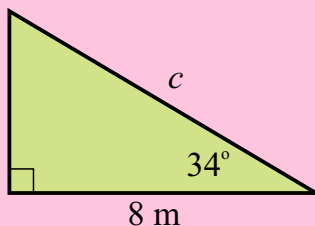
8.



Finding the Hypotenuse Using Cosine

EXERCISE 14H

Example



$$\cos \theta = \frac{A}{H}$$

$$\cos 34^\circ = \frac{8}{c}$$

$$c \times \cos 34^\circ = 8$$

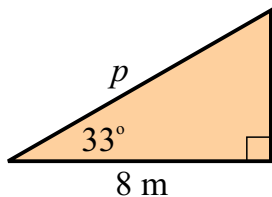
$$c = \frac{8}{\cos 34^\circ}$$

$$c = \frac{8}{0.8290}$$

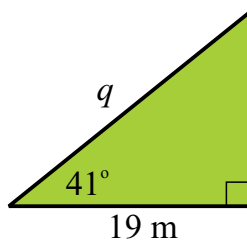
$$c = 9.6 \text{ m (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

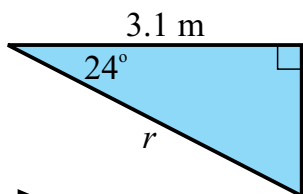
1.



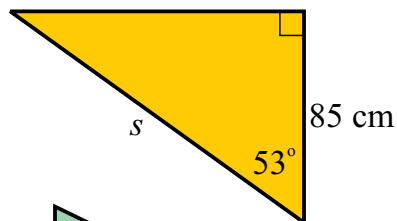
2.



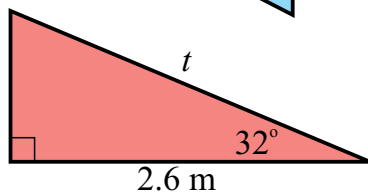
3.



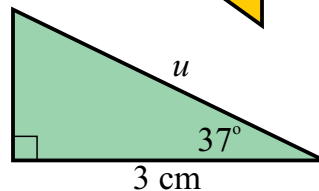
4.



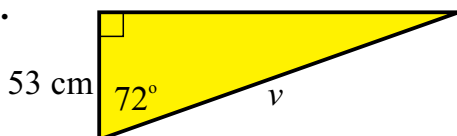
5.



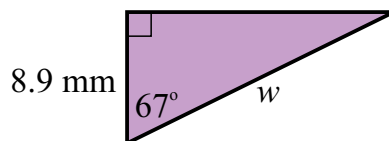
6.



7.



8.



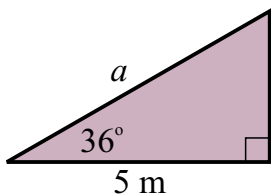
Mixed Problems Using Cosine

EXERCISE 14I

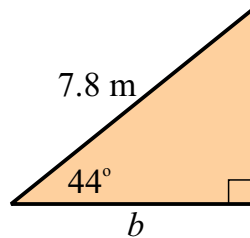
1. Find the unknown length in each of the following triangles.

Give answers correct to one decimal place.

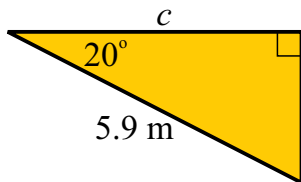
(a)



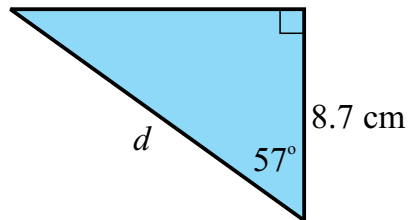
(b)



(c)



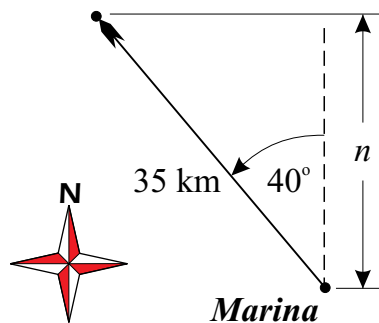
(d)



2. Con and Jaan sailed their yacht for 35 km in a direction of N 40° W from the marina.

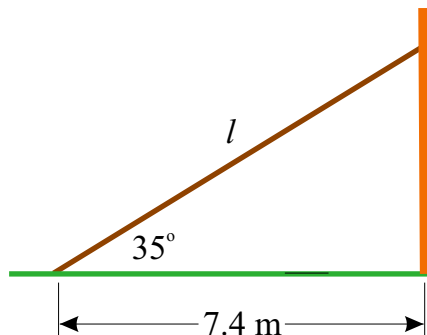
How far north from the marina (n) are they?

Give answer to one decimal place.



3. Achim wants to use a long pole to support a wall. He is going to fix the pole to the ground 7.4 m from the wall and he wants the pole to make an angle of 35° to the ground.

How long should the pole be (l)? Give answer to one decimal place.



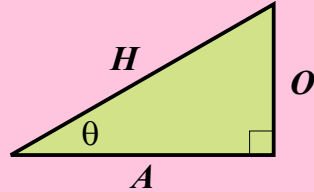
Tangent

EXERCISE 14J

The **tangent** of an angle is defined as the ratio of the side opposite the angle to the side adjacent to the angle.

The tangent of θ is abbreviated to **tan** θ .

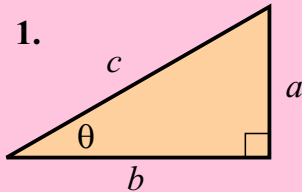
$$\text{Tangent of } \theta = \tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$



$$\tan \theta = \frac{O}{A}$$

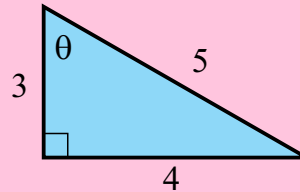
1. Find $\tan \theta$ in the following triangles and write in fraction form.

Examples 1.



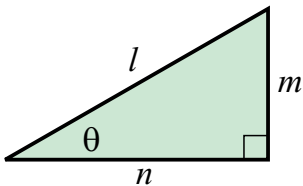
$$\tan \theta = \frac{a}{b}$$

2.

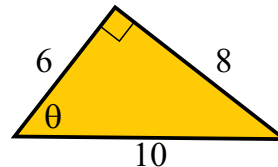


$$\tan \theta = \frac{4}{3}$$

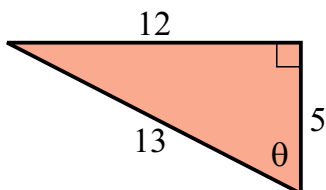
(a)



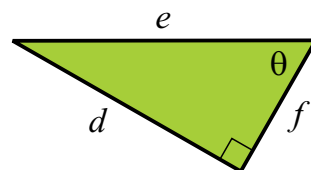
(b)



(c)

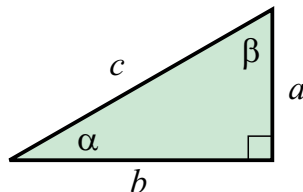


(d)



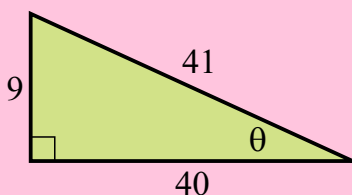
2. For the triangle shown here find the correct angles to complete the following:

(a) $\tan __ = \frac{a}{b}$ (b) $\tan __ = \frac{b}{a}$



3. Find $\tan \theta$ in the following triangles and write as decimal numbers correct to four decimal places.

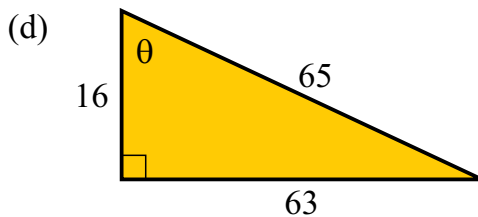
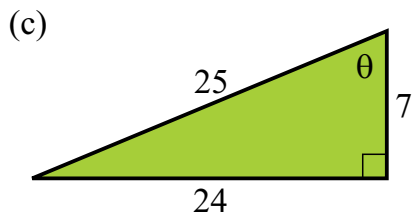
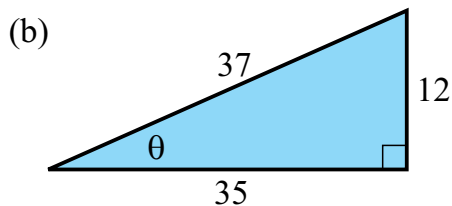
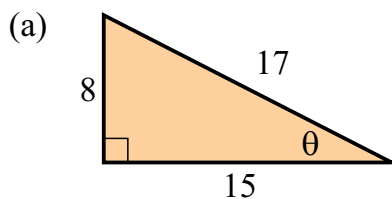
Example



$$\tan \theta = \frac{O}{A}$$

$$\tan \theta = \frac{9}{40}$$

$$\tan \theta = 0.2250 \text{ (4 dec. pl.)}$$



4. Use a calculator to find the following values correct to four decimal places.

Example

$$\tan 43^\circ = 0.9325$$

Check the calculator is in DEGREE mode when the angles are given in degrees.

(a) $\tan 56^\circ$

(b) $\tan 45^\circ$

(c) $\tan 60^\circ$

(d) $\tan 38.2^\circ$

(e) $\tan 35^\circ$

(f) $\tan 30^\circ$

(g) $\tan 5^\circ$

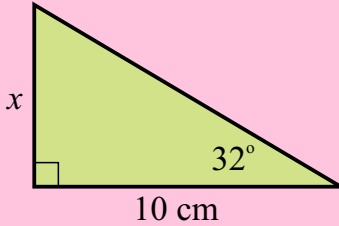
(h) $\tan 80.7^\circ$

(i) $\tan 88^\circ$

Finding the Opposite Side Using Tangent

EXERCISE 14K

Example



$$\tan \theta = \frac{O}{A}$$

$$\tan 32^\circ = \frac{x}{10}$$

$$10 \times \tan 32^\circ = x$$

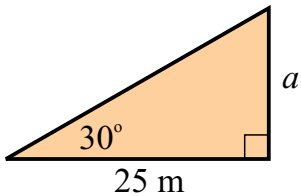
$$x = 10 \times \tan 32^\circ$$

$$x = 10 \times 0.6249$$

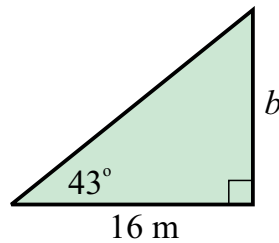
$$x = \mathbf{6.2 \text{ cm}} \text{ (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

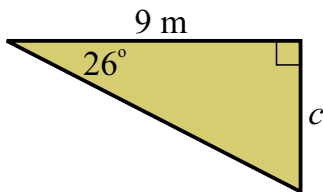
1.



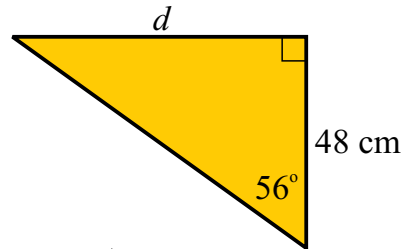
2.



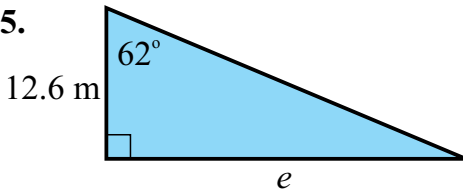
3.



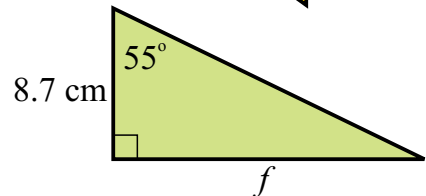
4.



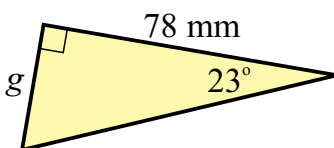
5.



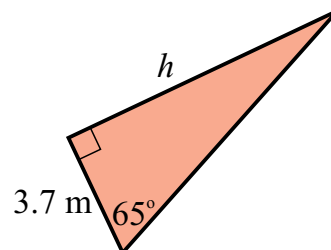
6.



7.



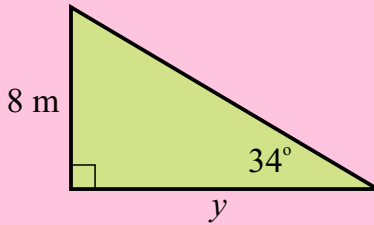
8.



Finding the Adjacent Side Using Tangent

EXERCISE 14L

Example



$$\tan \theta = \frac{O}{A}$$

$$\tan 34^\circ = \frac{8}{y}$$

$$y \times \tan 34^\circ = 8$$

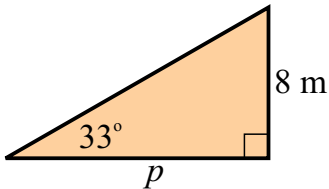
$$y = \frac{8}{\tan 34^\circ}$$

$$y = \frac{8}{0.6745}$$

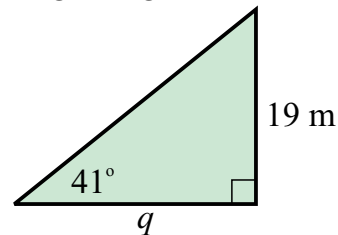
$$y = 11.9 \text{ m (one dec. pl.)}$$

Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

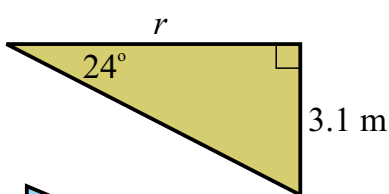
1.



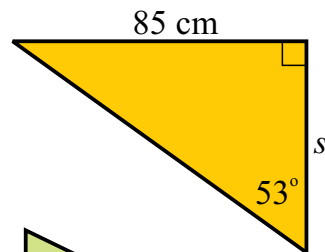
2.



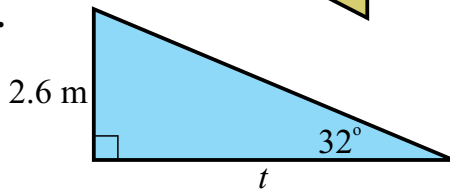
3.



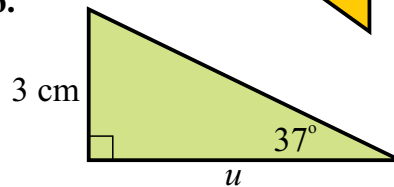
4.



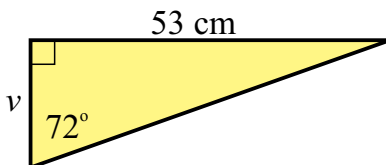
5.



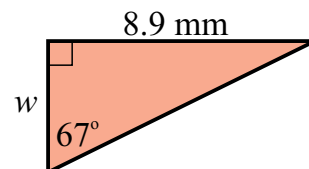
6.



7.



8.

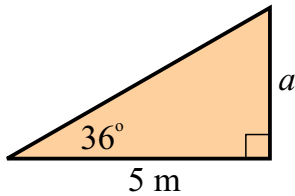


Mixed Problems Using Tangent

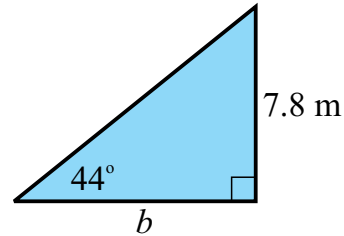
EXERCISE 14M

1. Find the unknown length in each of the following triangles.
Give answers correct to one decimal place.

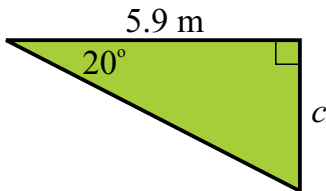
(a)



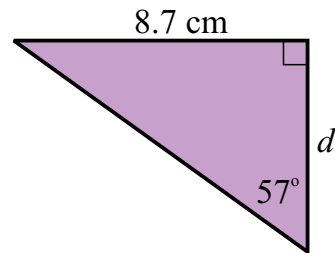
(b)



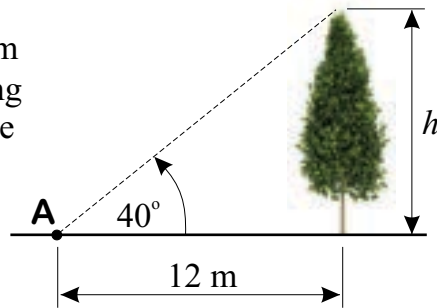
(c)



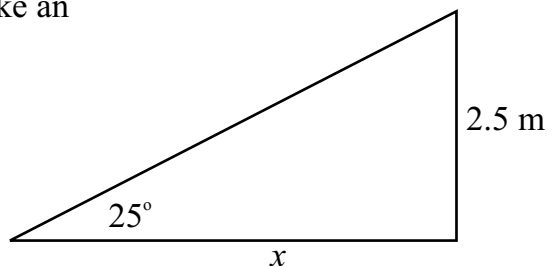
(d)



2. Tanja wanted to find the height of a tree. She measured 12 metres from the base of the tree to point **A**. Using a clinometer she measured the angle from point **A** to the top of the tree to be 40° . Calculate the height of the tree (h).
Give answer to one decimal place.



3. The ramp for a ski jump is to make an angle of 25° to the ground. If the height of the jump is 2.5 metres, calculate the horizontal distance (x) from the start to the end of the jump.



Mixed Problems

When finding a side length using sine, cosine or tangent follow these steps:

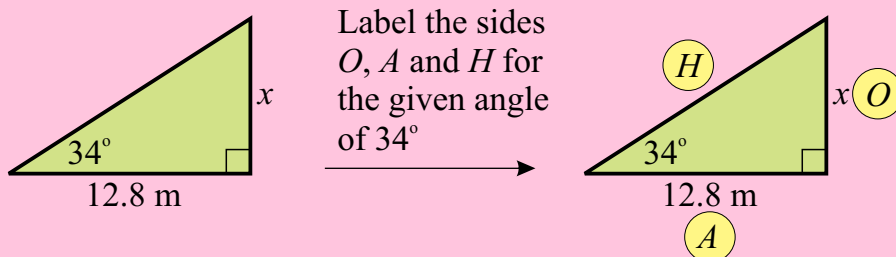
- Step 1** Label the sides (O , A and H) based on the given angle.
Step 2 Determine which function (sine, cosine or tangent) to use.
Step 3 Write the rule for the function.
Step 4 Solve the unknown length.

SOHCAHTOA can be used to remember the functions

$$\sin \theta = \frac{O}{H} \quad \cos \theta = \frac{A}{H} \quad \tan \theta = \frac{O}{A}$$

Example 1

Find the unknown length in the following triangle.



The unknown length (x) is the opposite side (O) and the known length (12.8 m) is the adjacent side (A).

We therefore need to use **tangent** to solve the unknown length in this triangle.

$$\tan \theta = \frac{O}{A}$$

$$\tan 34^\circ = \frac{x}{12.8}$$

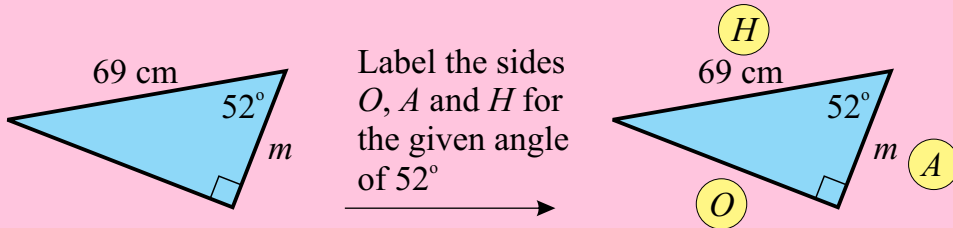
$$12.8 \times \tan 34^\circ = x$$

$$x = 12.8 \times \tan 34^\circ$$

$$x = \mathbf{8.6 \text{ m}} \text{ (one dec. place)}$$

Example 2

Find the unknown length in the following triangle.



The unknown length (m) is the adjacent side (A) and the known length (69 cm) is the hypotenuse (H).

We therefore need to use **cosine** to solve the unknown length in this triangle.

$$\cos \theta = \frac{A}{H}$$

$$\cos 52^\circ = \frac{m}{69}$$

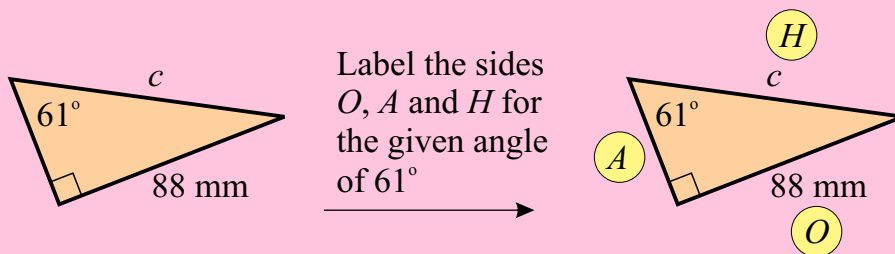
$$69 \times \cos 52^\circ = m$$

$$m = 69 \times \cos 52^\circ$$

$$m = 42.5 \text{ cm (one dec. place)}$$

Example 3

Find the unknown length in the following triangle.



The unknown length (c) is the hypotenuse (H) and the known length (88 mm) is the opposite side (O).

We therefore need to use **sine** to solve the unknown length in this triangle.

$$\sin \theta = \frac{O}{H}$$

$$\sin 61^\circ = \frac{88}{c}$$

$$c \times \sin 61^\circ = 88$$

$$c = \frac{88}{\sin 61^\circ}$$

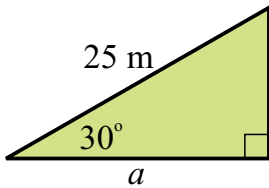
$$c = \frac{88}{0.8746}$$

$$c = 100.6 \text{ mm (one dec. place)}$$

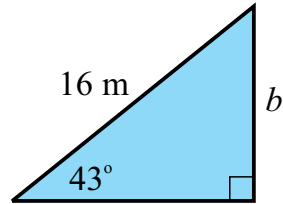
EXERCISE 14N

Find the unknown lengths in the following triangles.
Give answers correct to one decimal place.

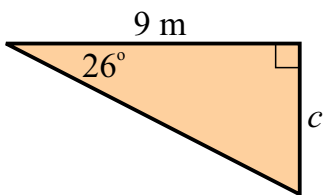
1.



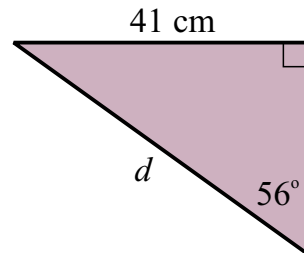
2.



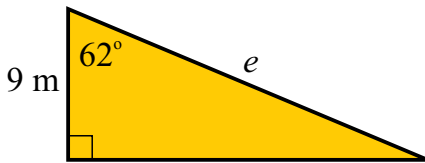
3.



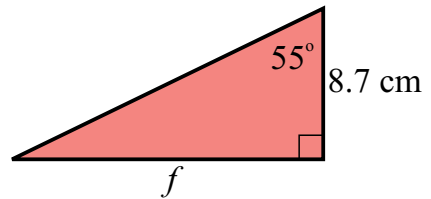
4.



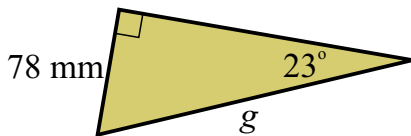
5.



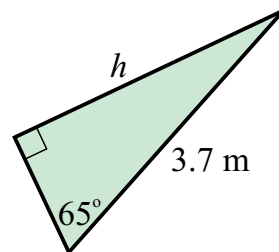
6.



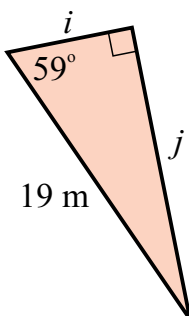
7.



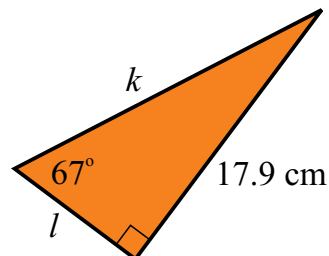
8.



9.



10.



Practical Problems

EXERCISE 140

Give all answers correct to one decimal place.

1. A hang-glider is flying at an altitude of 2000 metres.

The pilot wants to descend

to the ground at an angle

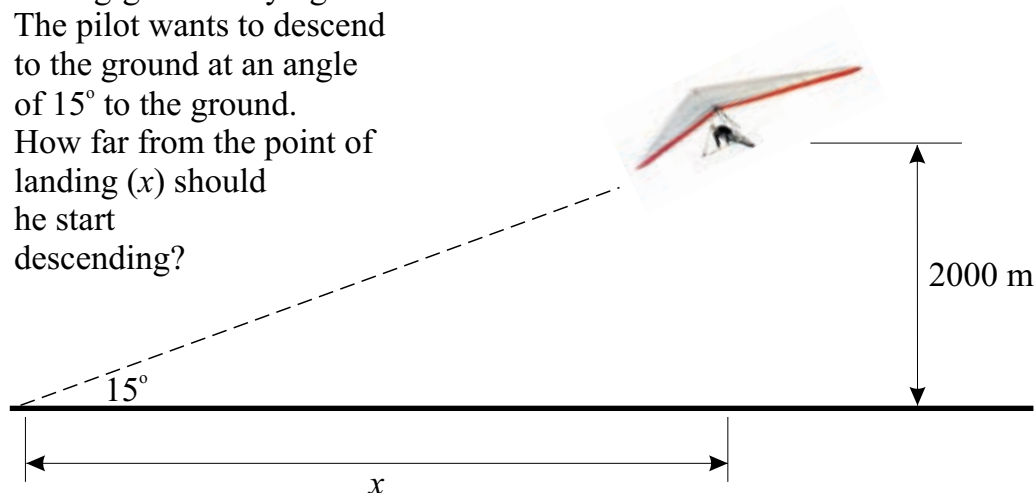
of 15° to the ground.

How far from the point of

landing (x) should

he start

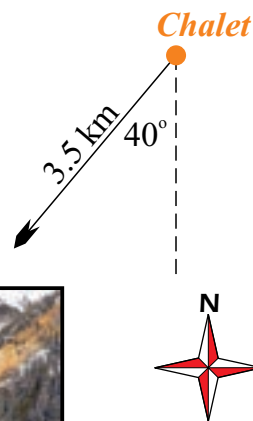
descending?



2. Two cross-country snow skiers travel 3.5 kilometres in a direction S 40° W from their chalet.

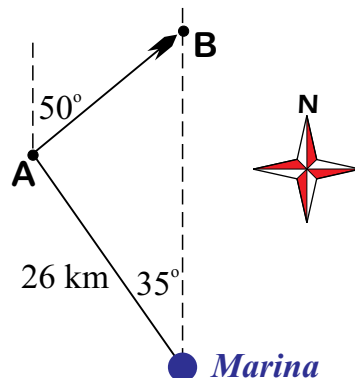
(a) How far south from the chalet are the skiers?

(b) How far west from the chalet are the skiers?



3. A yacht sails 26 km in a direction N 35° W from the marina to point **A**. It then sails in a direction N 50° E to point **B** which is due north from the marina.

- (a) How far west from the marina is point **A**?
 (b) How far north from the marina is point **B**?



4. Crazy Craig is a stunt motorcycle rider. His new stunt is to ride fast up a ramp and jump over four cars. He has calculated that the ramp needs to be at an angle of 20° to the ground and the end of the ramp is to be 3 metres above the ground.
- (a) How long is the ramp?
 (b) How far from the end of the ramp is the start of the ramp?



5. The sun's rays on a particular part of a day were making an angle of 62° to the ground.

(a) Hollie is 158 cm tall. Calculate the length of her shadow.

(b) Hollie measured the shadow of a tree to be 5.8 m.

How tall is the tree?

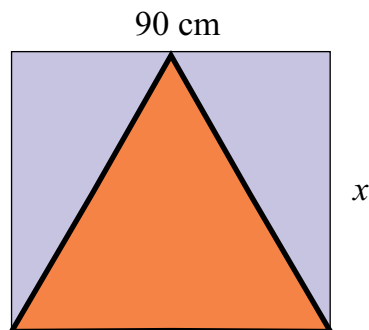
6. Imogen wanted to make a poster in the shape of an equilateral triangle with side length 90 cm.

She was going to cut the equilateral triangle from a rectangular board.

The length of the board is 90 cm.

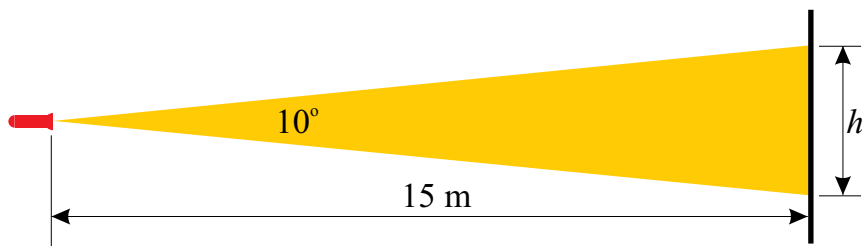
Calculate the width of the board (x).

(Hint: divide the triangle in half to form two right-angled triangles)



7. The beam of light from a torch makes an angle of 10° .

What is the height (h) of the area of light shining on a wall 15 metres from the torch?

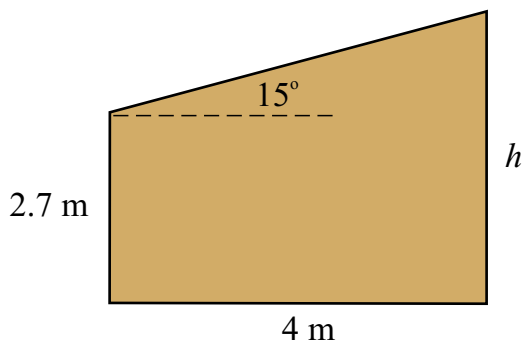


8. Find the height (h) of the back wall of the shed shown.

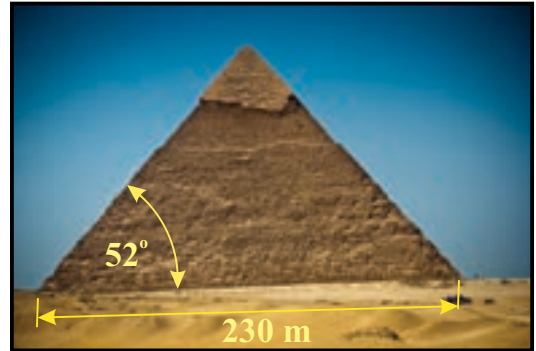
The front wall is 2.7 m high.

The shed is 4 m wide.

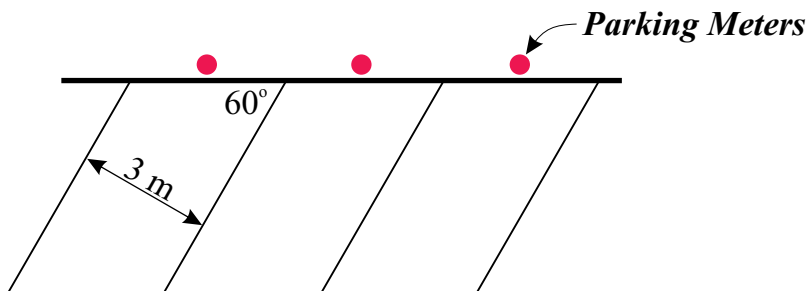
The angle of the roof (pitch) is 15° .



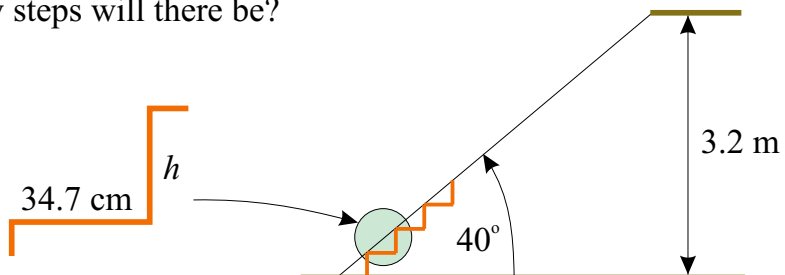
9. An archeologist studying one of the pyramids in Egypt calculated the angle of each face to be 52° to the ground.
The base length is 230 metres.
How high is the pyramid?



10. A local council wanted to make angle car parking along a street.
The car parks were to be at an angle of 60° to the side of the street
and were to be 3 metres wide.
The parking meters are to be in the centre of the car park spaces
on the footpath.
What is the distance between the parking meters?



11. A builder is designing stairs for a two storey house.
The second floor is 3.2 metres above the ground floor.
The stairs are to be at an angle of 40° .
The width of each step is to be 34.7 cm.
(a) Find the height (h) of each step.
(b) How many steps will there be?



Calculating Angles

To find an angle in a right-angled triangle knowing two sides follow these steps:

Step 1 Label the sides (*O*, *A* or *H*) based on the angle to be found.

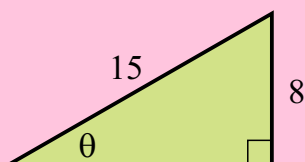
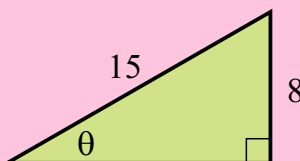
Step 2 Determine which function (sine, cosine or tangent) to use.

Step 3 Write the rule for the function.

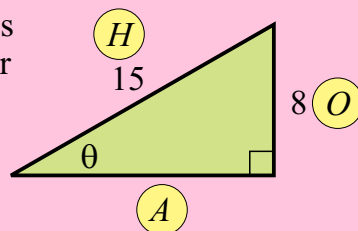
Step 4 Find the angle by calculating the inverse of the angle using a calculator: $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.

Example

Find the angle (θ) shown in this triangle.



Label the sides
O, *A* and *H* for
angle θ .



The two known sides are the opposite side (*O*) and the hypotenuse (*H*). We therefore need to use **sine** to solve the angle in this triangle.

$$\sin \theta = \frac{O}{H}$$

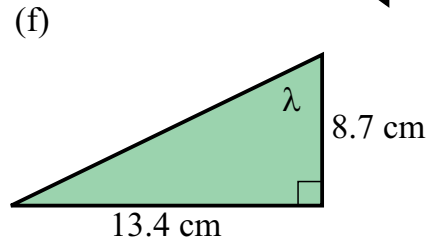
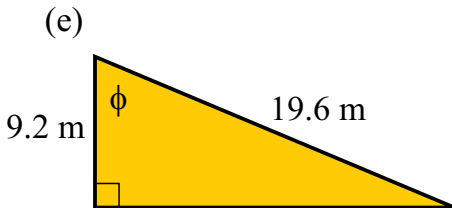
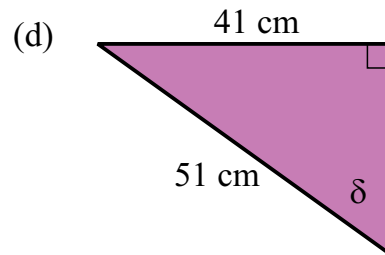
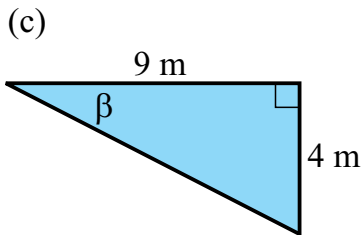
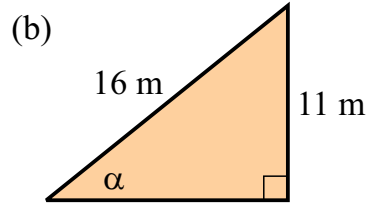
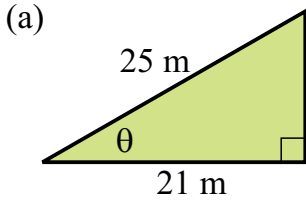
$$\sin \theta = \frac{8}{15}$$

$$\theta = \sin^{-1}\left(\frac{8}{15}\right)$$

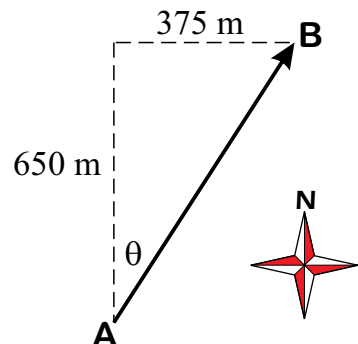
$$\theta = 32.2^\circ \text{ (1 decimal place)}$$

EXERCISE 14P

1. Find the unknown angles in the following triangles.
Give answers correct to one decimal place.



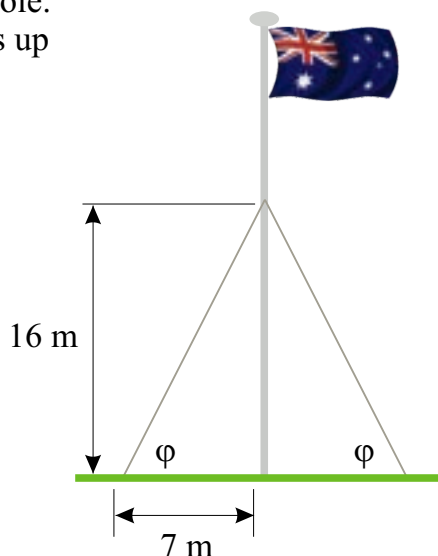
2. An orienteering course required competitors to run in a straight line between points **A** and **B** through bush.
Point **B** is 650 metres north and 375 metres east of point **A**.
At what angle (θ) should the runners set their compasses?



3. Sydney is approximately 450 km north and 500 km east from Melbourne.
- In what direction should an aeroplane fly to travel in a straight line from Melbourne to Sydney?
 - In what direction should an aeroplane fly to travel in a straight line from Sydney to Melbourne?

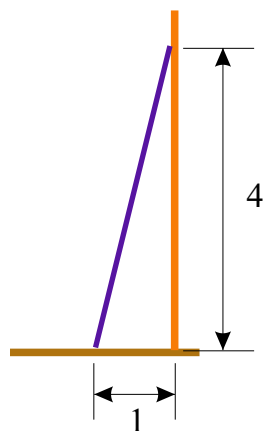
4. Cables are used to support a large flagpole. The cables are fixed to points 16 metres up the flagpole and 7 metres from the base of the flagpole.

Calculate the angle (ϕ) that the cables make with the ground.



5. Ladder manufacturers recommend that for safe use of a ladder the slope it makes should be one in four. This means the top of the ladder should be a distance up the wall which is four times the distance the bottom is from the wall.

- What angle should a ladder make with the ground in this safe position?

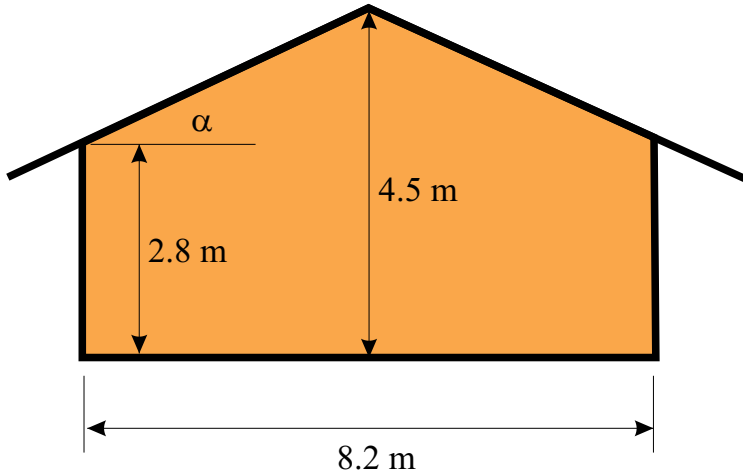


Chloe is using a ladder 4.2 metres long to clean windows.

The bottom of the ladder is 1.7 metres from the wall.

- What angle does her ladder make with the ground?
- Does she need to move it closer to, or further from, the wall so it is in the safe position?

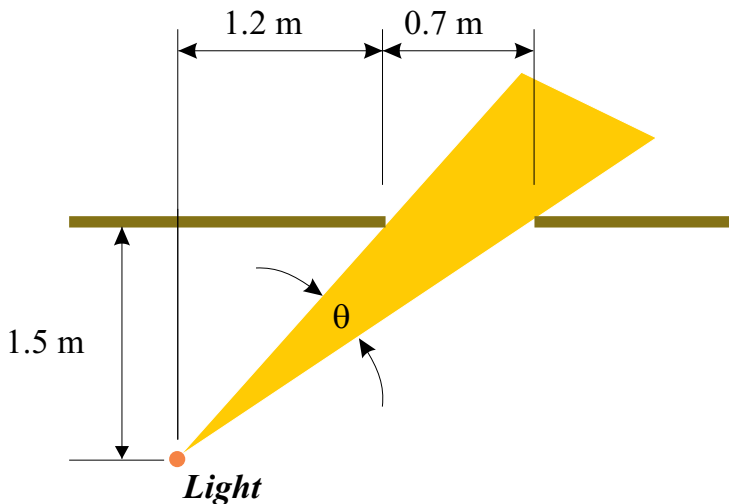
6. Find the angle of the roof (α) in this building.



7. A light is shining near a wall. Part of the light's rays are passing through a window in the wall.

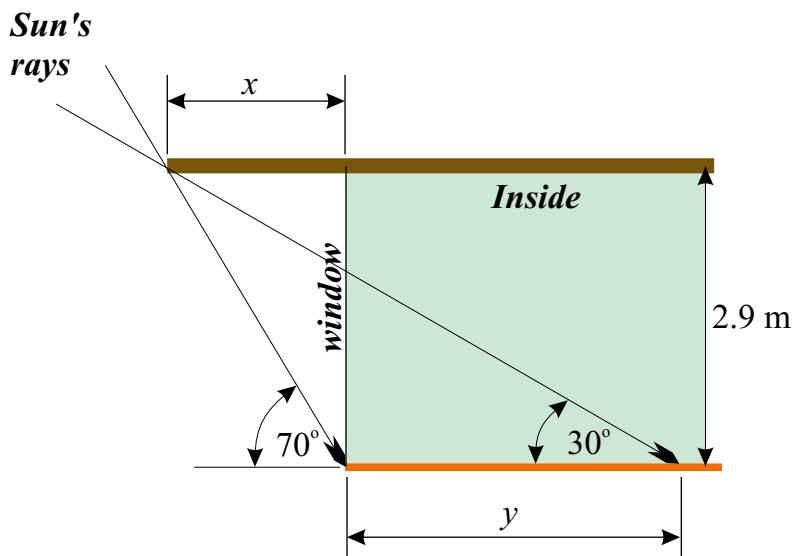
The position of the light and size of the window are shown in the diagram below.

What is the angle of the light ray (θ) that passes through the window?



PROBLEM SOLVING

1. Desha is designing a house. There is a large floor to ceiling window on the north side of the house. She wants to design the verandah so that in summer very little of the sun comes through the window but in winter a large amount of sun comes through to warm the house. She decides to have the verandah come out far enough for its shadow to touch the bottom of the window when the angle of the sun's rays is 70° to the ground. The window is 2.9 metres high.
- Calculate the width of the verandah (x) to two decimal places.
 - Calculate the distance the sun's rays will come into the house (y) when they make an angle of 30° to the ground. (Two dec. pl.)



2. A yacht sails from a marina for 2 hours in a direction $N 40^\circ W$ then turns and travels for 3 hours in a direction $S 50^\circ W$. The yacht sailed at an average speed of 15 km/hour.
- In which direction should the yacht sail to travel in a straight line back to the marina?
 - How far is the yacht from the marina?
 - How long will it take to sail back to the marina?

PUZZLES

The following three riddles can be solved by using the code and triangles below.

1. What do you call two male trigonometry teachers lying in the sun?

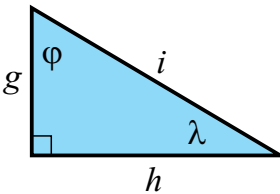
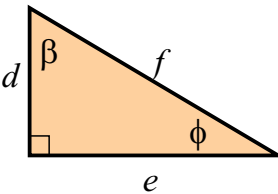
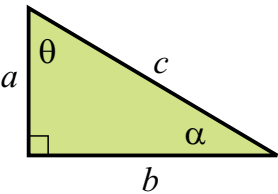
Answer: $\cos \lambda \sin \alpha \tan \phi \tan \phi \tan \alpha \sin \theta$
 $\tan \theta \tan \alpha \tan \phi \cos \lambda \cos \phi$

2. What do you call someone who solves trigonometry using a paint brush?

Answer: $\cos \theta \sin \lambda \cos \beta \tan \phi \tan \alpha$
 $\tan \phi \tan \beta \sin \phi \cos \lambda \tan \alpha \tan \beta$

3. How does a deaf person teach trigonometry?

Answer: $\cos \phi \sin \phi \tan \phi \tan \alpha$
 $\cos \phi \sin \alpha \tan \phi \tan \theta \tan \lambda \cos \theta \tan \theta \tan \alpha$

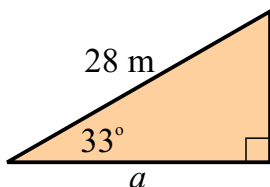


$\frac{a}{c}$	$\frac{b}{c}$	$\frac{a}{b}$	$\frac{b}{a}$	$\frac{d}{f}$	$\frac{e}{f}$	$\frac{d}{e}$	$\frac{e}{d}$	$\frac{g}{i}$	$\frac{h}{i}$	$\frac{g}{h}$	$\frac{h}{g}$
A	D	E	G	I	L	N	R	S	T	U	W

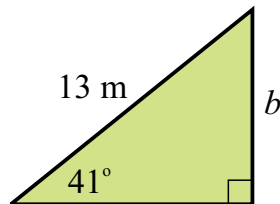
CHAPTER REVIEW

1. Find the unknown lengths in the following triangles.
Give answers correct to one decimal place.

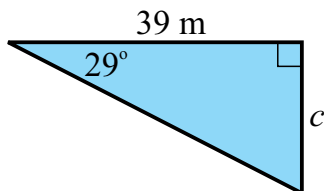
(a)



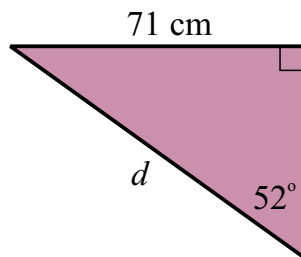
(b)



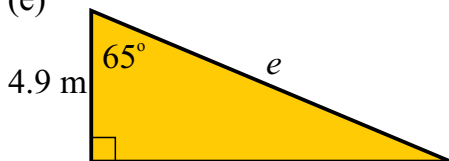
(c)



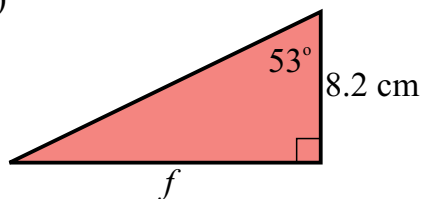
(d)



(e)

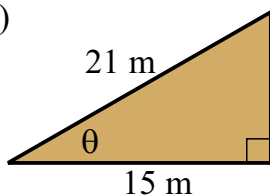


(f)

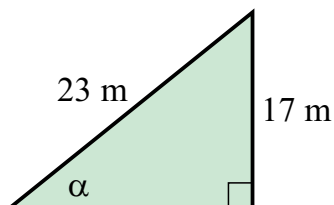


2. Find the unknown angles in the following triangles.
Give answers correct to one decimal place.

(a)



(b)



3. Murali is flying a kite. He notices the kite is flying at the same height as an 8.5 metre building nearby. It is also directly above his friend Isha who is 12 metres away. Murali is holding the end of the string 1 metre above the ground.
- (a) What angle does the kite string make to the ground?
- (b) How long is the kite string?

Probability

Probability

Probability is the branch of mathematics that studies the chance or likelihood of events occurring.

There are many situations in real life where the probability that a particular event will occur is of interest.

What is the probability of rain tomorrow?

What is the probability of a train being late?

What is the probability that a netball player will miss a shot at goal?

What is the probability of a team winning the grand final?

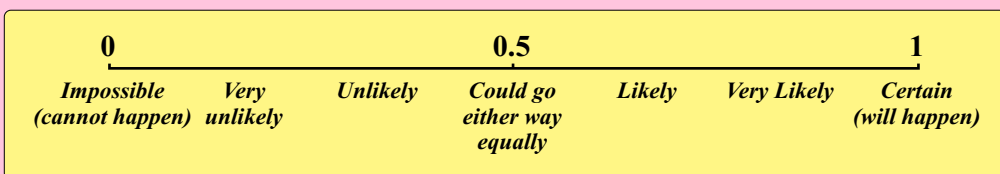
What is the probability of being hit by lightning?

etc., etc.

In the mathematics of probability, a value is placed on the likelihood of an event occurring.

This value can be a decimal or fraction between 0 (impossible) and 1 (definitely will occur) or a percentage between 0 and 100%.

The scale below shows this range of values.



In some situations this value can be calculated exactly.

Example Tossing a coin.

There are two possible outcomes - **heads** and **tails**.

There is an equal probability of each occurring so the probability that either occurs is 0.5, $\frac{1}{2}$ or 50%.

In some situations the probability can only be evaluated by conducting experiments or collecting data.

Examples Weather, populations, sporting statistics, etc.

Calculating Probabilities

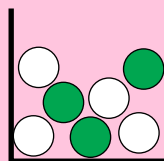
The probability of an event occurring (successful outcome) can be calculated by finding the ratio of the number of successful outcomes to the total number of outcomes.

The probability of a successful outcome abbreviates to $P(\text{success})$.

$$P(\text{success}) = \frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$$

Examples

- Find the probability of randomly choosing a white ball from this box.



$$P(\text{choosing a white ball}) = \frac{\text{Number of white balls}}{\text{Total number of balls}}$$

$$P(\text{choosing a white ball}) = \frac{4}{7}$$

- Find the probability of randomly choosing a male from the following group of people:

Jack, Bob, Sandra, Bill, Helen and Matthew

$$P(\text{choosing a male}) = \frac{\text{Number of males}}{\text{Total number of people}}$$

Jack, Bob, Sandra, Bill, Helen and Matthew

$$P(\text{choosing a male}) = \frac{4}{6} = \frac{2}{3} \text{ (Simplest form)}$$

EXERCISE 15A

Write all answers to the following questions as fractions in their simplest form.

1. In a class of 25 students there are 12 boys and 13 girls.
 - (a) What is the probability of randomly choosing a boy?
 - (b) What is the probability of randomly choosing a girl?
2. In another class there are 10 boys and 16 girls. From this class:
 - (a) what is the probability of randomly choosing a boy?
 - (b) what is the probability of randomly choosing a girl?
3. The probability of randomly choosing a girl from another class is $\frac{3}{8}$.
 - (a) What is the probability of randomly choosing a boy from this class?
 - (b) There are 6 girls in the class. How many students are in this class?
4. The heights of six students are given below:

152 cm, 147 cm, 163 cm, 158 cm, 159 cm, 173 cm

If a student is chosen at random, find the probability that:

- (a) they will be taller than 160 cm.
 - (b) they will be shorter than 150 cm.
5. A box of sweets contains eight chocolates and six pieces of licorice.
 - (a) What is the probability of randomly choosing a chocolate?
 - (b) Simone eats two chocolates from the box. What is the probability that she randomly chooses another chocolate?
6. A pencil box contains six blue pens, four red pens, three green pens and two black pens.
 - (a) What is the probability of Peter randomly choosing a blue pen from the box?
 - (b) Peter keeps the blue pen and randomly chooses another pen. What is the probability that it will be red?

7. Alvira has eight black socks, four red socks, four white socks and four purple socks in her sock draw. One dark morning she randomly chooses a sock then a second sock.
- (a) What is the probability that the first sock is purple?
 - (b) What is the probability that the second sock is also purple?
8. (a) What is the probability that your birthday will fall on a weekend next year?
- (b) What is the probability that someone is born on Christmas Day?
 - (c) What is the probability that someone's star sign is Aries?



9. A standard six sided die is rolled.
- (a) What is the probability of rolling a six?
 - (b) What is the probability of rolling an even number?
 - (c) What is the probability of rolling a number larger than four?
 - (d) What is the probability of rolling a seven?
 - (e) What is the probability of rolling a number less than seven?



10. Dean randomly chose a letter (**A-Z**) and a number (0-50) as a code.
- (a) What is the probability that the letter is a vowel?
 - (b) What is the probability of the letter being '**D**'?
 - (c) What is the probability that the number has one digit?

Probabilities and Relative Frequencies

The probability of an event occurring in many situations needs to be found by conducting experiments or using data.

Example

Lizzie was organising a party for her birthday in May and wanted to find the probability that there will be no rain on the day.

She looked at rainfall records over the last 10 years and found that on her birthday it rained on four occasions.

Based on these records she calculated the probability of rain:

$$P(\text{rain}) = \frac{4}{10} = \frac{2}{5} \text{ (Simplest form)}$$

Note the probability can also be written as a decimal or percentage:

$$P(\text{rain}) = \frac{2}{5} = 0.4 = 40\%$$

Note - the sum of all the probabilities of possible outcomes equals 1.

In this example:

$$P(\text{rain}) + P(\text{no rain}) = 1$$

$$P(\text{no rain}) = \frac{3}{5} = 0.6 = 60\%$$

It should also be noted that when calculating a probability using experiments or data the accuracy of the probability depends on the amount of information obtained. The larger the sample size the more accurate will be the probability.

In this example if Lizzie had looked at the records for more than 10 years the value of the probability would be more accurate.

When collecting data or recording results from an experiment or trial, the number of each item recorded is called the **frequency**. To obtain a comparison between the number of different items, the frequencies are divided by the total number. The resultant numbers are called the **relative frequencies**. The relative frequencies can be used as probabilities.

Example

20 people who worked in a city were asked what mode of transport they used to get to work.

The results are shown in this table.

The relative frequencies are found by dividing each of the frequencies by the total (20).

Mode of Transport	Number (Frequency)	Relative Frequency
Bicycle	4	0.2
Bus	6	0.3
Train	7	0.35
Car	2	0.1
Walk	1	0.05

Note - The total of the relative frequencies will always be 1.

The relative frequencies can be used as probabilities.

Example

Based on the above figures, what is the probability that someone in this city rides their bicycle to work?

Answer 0.2

Note - The accuracy of the probabilities used from a survey will depend on the size of the survey. In the above example only 20 people were asked. For a more accurate picture of this type of information more people need to be surveyed.

EXERCISE 15B

Give answers to the following questions as decimals

1. 100 people were randomly surveyed and asked if they preferred to live in a major city, a small city or a rural town. The results are shown in this table.

<i>Preferred place to live</i>	<i>Number (Frequency)</i>
Major city	46
Small city	33
Rural town	21

Based on these figures:

- (a) what is the probability that someone wants to live in a:
- (i) major city?
 - (ii) small city?
 - (iii) rural town?
- (b) what is the probability that someone does **not** want to live in a major city?

2. 20 students completed a course in graphic design.

One year after completing the course, 12 of the students had obtained employment in this field. In the following year another four students had started work in the field of graphic design.

Based on these figures:

- (a) what is the probability of getting a job in graphic design within one year of completing the course?
- (b) what is the probability of getting a job in graphic design within two years of completing the course?
- (c) what is the probability of **not** getting a job in graphic design within two years of completing the course?

3. Andy is a regular train traveller. He wanted to calculate the probability that the train he catches each morning left at the correct time. The train is scheduled to leave at 8:30 am. Andy recorded the time it left each day for 20 days. The times are listed below.

8:30 8:34 8:31 8:37 8:30 8:33 8:39 8:41 8:30 8:36
8:46 8:32 8:30 8:40 8:33 8:30 8:30 8:39 8:32 8:37

Based on these figures:

- what is the probability that the train leaves *after* 8:30?
 - what is the probability that the train is more than five minutes late leaving?
 - what is the probability that the train is more than 10 minutes late leaving?
 - what is the probability that the train is more than 20 minutes late leaving?
4. Veronica makes jackets for dogs and sells them through a website *keepyourpoochwarm.com*. The number of site visits and sales each day for a week are shown below.

<i>Day</i>	<i>Number of Visits</i>	<i>Number of Sales</i>
Monday	20	14
Tuesday	24	15
Wednesday	28	20
Thursday	25	17
Friday	27	16
Saturday	48	37
Sunday	54	42

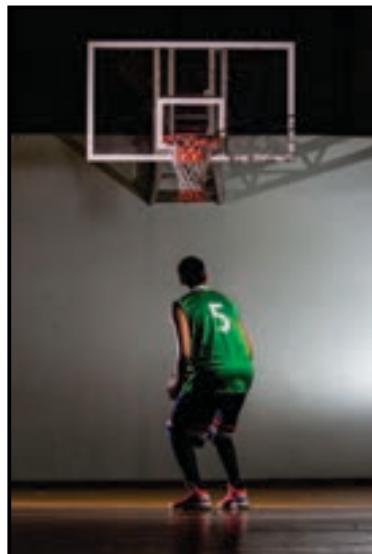
Based on these figures, calculate the probability that someone who looks at the website purchases a dog jacket.

Give answer correct to two decimal places.

5. Matty, Patty and Dante play basketball. The number of free throw shots they each had in a season and the number of times they scored are shown in the table below.

<i>Player</i>	<i>Shots Taken</i>	<i>Shots Made</i>
Matty	16	13
Patty	29	19
Dante	65	49

- (a) Based on these figures, which player has the highest probability of scoring from a free throw?
 (b) The probability for which of the players is the most accurate? Why?



6. Tycho has a keen interest in astronomy. He wanted to find the probability of seeing a meteor on any particular night of star gazing. He went out every night in September and noted that on the following dates he observed at least one meteor.

3rd, 9th, 15th, 22nd, 23rd, 27th

Based on these figures, what is the probability of seeing at least one meteor on a given night of star gazing?



Predicting Outcomes

If the probability of success is known for a particular event then the number of successful outcomes can be predicted for a given number of total outcomes. The following formula can be used.

$$\text{Number of successful outcomes} = \text{Probability of success} \times \text{Total number of outcomes}$$

Example 1

If a coin is tossed 30 times, how many times would **heads** be expected to result?

In this example:

Number of successful outcomes = the number of times heads is tossed

Probability of success = the probability of tossing heads = **0.5**

Total number of outcomes = **30**

$$\text{Number of successful outcomes} = \text{Probability of success} \times \text{Total number of outcomes}$$

$$\text{Number of times heads is tossed} = 0.5 \times 30$$

$$\text{Number of times heads is tossed} = 15$$

So if a coin is tossed 30 times it would be expected that heads is tossed 15 times.

Example 2

Kirby enjoys playing a card game of patience. He wanted to find the probability of winning the game. He played the game 500 times and won on 100 occasions.

Based on these figures:

1. what is the probability of winning the game?
2. if Kirby played the game 25 times, how many could he expect to win?

Answers

1.

$$P(\text{success}) = \frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{success}) = \frac{100}{500} = \frac{1}{5} = 0.2 = 20\%$$

2.

$$\text{Number of successful outcomes} = \text{Probability of success} \times \text{Total number of outcomes}$$

$$\text{Number of successful outcomes} = 0.2 \times 25$$

$$\text{Number of successful outcomes} = 5$$

If Kirby played the game 25 times he could expect to win five of them.

EXERCISE 15C

1. Natasha is the goal-shooter in a netball team. In one season she had 350 attempts at goal. She scored 210 goals.
- Justine is the goal-attack in the same netball team. In the season Justine had 280 attempts at goal and scored 182 goals.
- (a) Based on these figures, find the probability (as a decimal) of:
- (i) Natasha scoring a goal.
 - (ii) Justine scoring a goal.
- (b) Who has a better probability of scoring a goal?
- (c) If they both had 40 attempts at goal in one game, what would be the expected team score?
(Assume Natasha and Justine were the only players who scored goals in the team)



2. Lleyton is a tennis player. He wanted to work out the probability of hitting his first serve into play. He recorded that from 250 first serves, 190 went into play.
- (a) Based on these figures, what is the probability (as a percentage) of Lleyton's first serve going into play?
- (b) If Lleyton had 65 first serves in a match, how many could he expect to hit into play?
(Round to the nearest serve)



3. A balloon manufacturer tested 200 balloons and found that 8 exploded before they reached a diameter of 40 cm.
- Based on these figures, what is the probability (as a percentage) of a balloon exploding before it reaches a diameter of 40 cm?
 - Based on these figures, what is the probability (as a percentage) that a balloon reaches a diameter of 40 cm without exploding?
 - Ivan bought 500 balloons for a party. How many would be expected to explode before they reach a diameter of 40 cm?



4. The number of days that it rains in each month at the Great Barrier Reef is shown in the table below.

<i>Month</i>	J	F	M	A	M	J	J	A	S	O	N	D
<i>Days of Rain</i>	16	17	17	15	13	7	7	6	5	7	9	12

Use these figures to answer the following questions.

- In which month is it *least* likely to rain?
 - What is the probability (as a percentage to one decimal place) of it raining on a day in the following months?
 - January
 - February (non-leap year)
 - March
 - June
 - July
 - September
 - October
 - What is the probability (as a percentage to one decimal place) of it raining on Christmas Day on the Great Barrier Reef?
5. The four main blood types and the probability that someone is in each are as follows:
Type O (45%), **Type A** (41%), **Type B** (10%) and **Type AB** (4%)
 If there were 3 million people in a city, how many would be expected to be in each blood type?

Listing Possible Outcomes for Multiple Events - Tree Diagrams

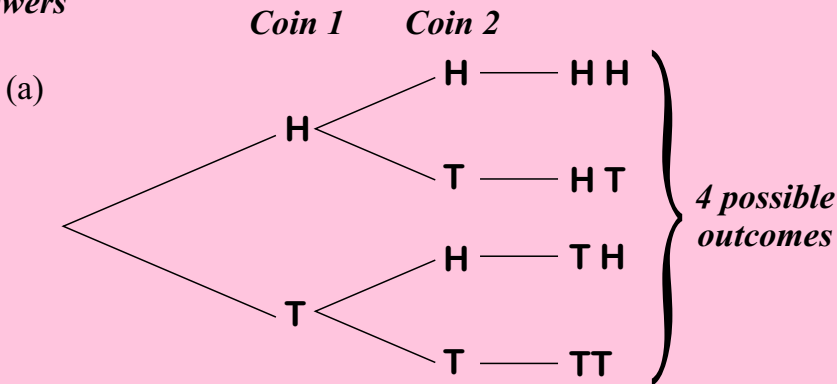
Tree diagrams can be used to list all the possible outcomes from situations where there are *multiple events*.

Example 1

Two coins are tossed.

- Draw a tree diagram to show all the possible outcomes.
- List all the possible outcomes.
- Find the probability of tossing two heads, $P(\mathbf{HH})$.
- Find the probability of tossing a head and tail, $P(\mathbf{HT}$ or $\mathbf{TH})$.

Answers



- (b) The possible outcomes are **HH, HT, TH, TT**

$$P(\text{success}) = \frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$$

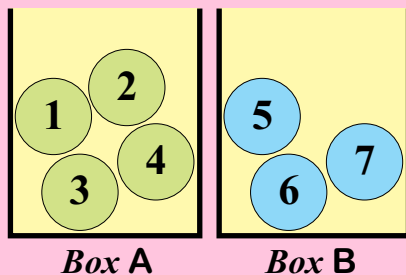
(c) $P(\mathbf{HH}) = \frac{1}{4}$

(d) $P(\mathbf{HT} \text{ or } \mathbf{TH}) = \frac{2}{4} = \frac{1}{2}$

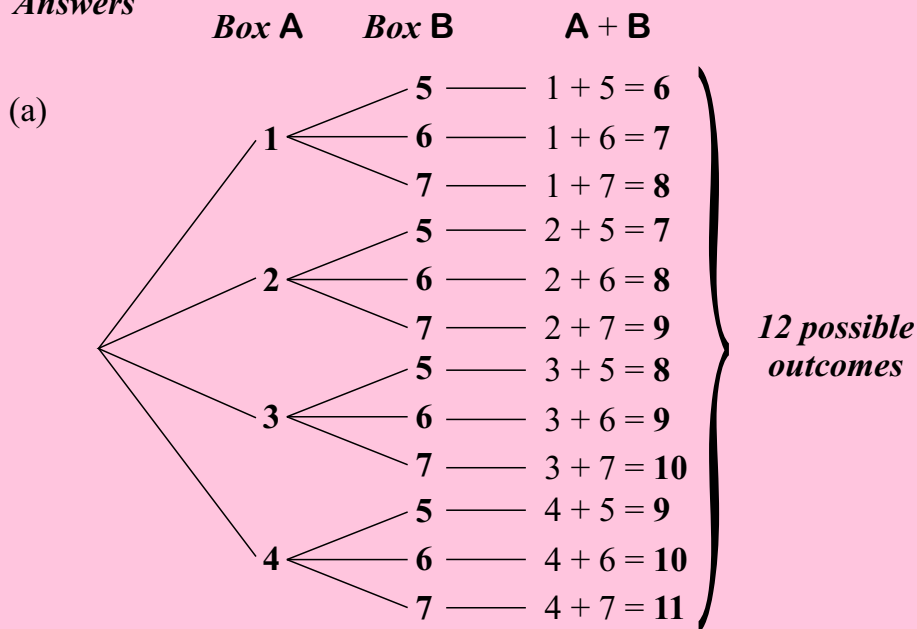
Example 2

A ball is taken randomly from each of the two boxes shown - **Box A** and **Box B**.

The numbers on the two balls are *added*.



- Construct a tree diagram to show all the possible outcomes.
- Find the probability that the sum of the two numbers is equal to 8, $P(A + B = 8)$.
- Find the probability that the sum of the two numbers is less than 10, $P(A + B < 10)$.

Answers

- (b) There are three of the possible outcomes that add to 8.

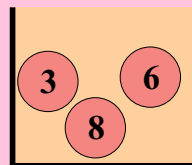
$$P(A + B = 8) = \frac{3}{12} = \frac{1}{4}$$

- (c) There are nine of the possible outcomes that have a sum less than 10.

$$P(A + B < 10) = \frac{9}{12} = \frac{3}{4}$$

Example 3

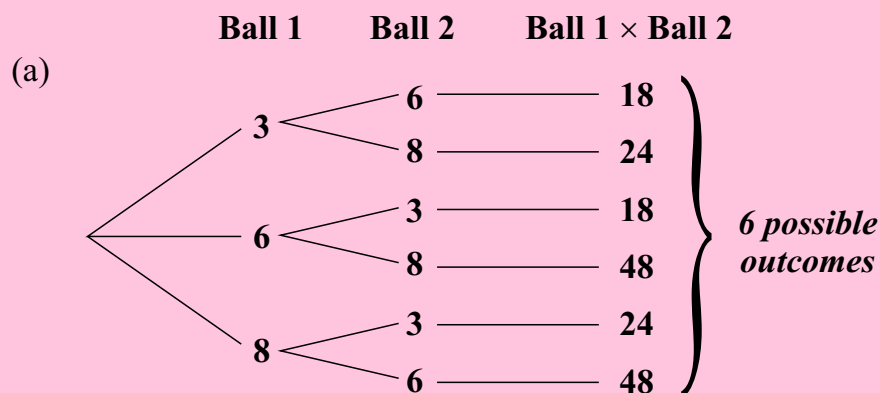
A box contains three balls with the numbers 3, 6 and 8 as shown here.



One ball is taken from the box then *without replacing* the first ball a second ball is taken from the box.

The numbers on the two balls are *multiplied*.

- Construct a tree diagram to show all the possible outcomes.
- Find the probability that the product of the numbers is less than 20, $P(\text{Ball 1} \times \text{Ball 2} < 20)$.
- Find the probability that the product of the numbers is greater than 40, $P(\text{Ball 1} \times \text{Ball 2} > 40)$.
- Find the probability that the product of the numbers is between 10 and 30, $P(10 < \text{Ball 1} \times \text{Ball 2} < 30)$.

Answers

- (b) There are two of the possible outcomes that are less than 20.

$$P(\text{Ball 1} \times \text{Ball 2} < 20) = \frac{2}{6} = \frac{1}{3}$$

- (c) There are two of the possible outcomes that are greater than 40.

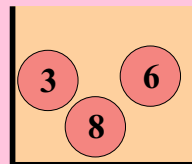
$$P(\text{Ball 1} \times \text{Ball 2} > 40) = \frac{2}{6} = \frac{1}{3}$$

- (d) There are four of the possible outcomes that are between 10 and 30.

$$P(10 < \text{Ball 1} \times \text{Ball 2} < 30) = \frac{4}{6} = \frac{2}{3}$$

Example 4

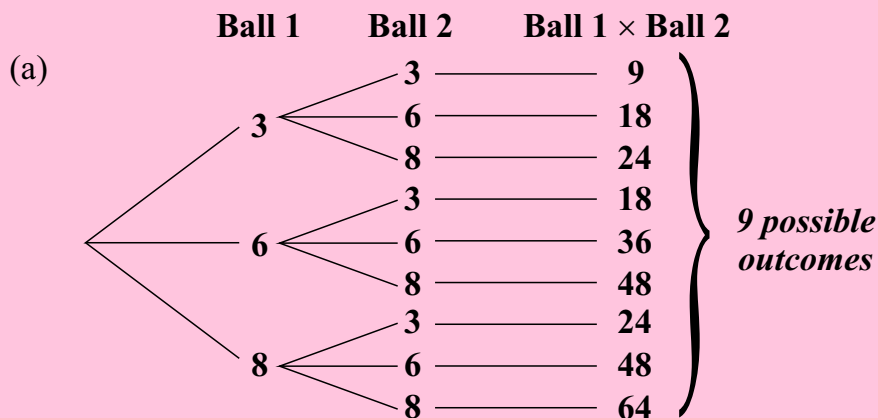
A box contains three balls with the numbers 3, 6 and 8 as shown here.



One ball is taken from the box then *replaced* into the box and then a second ball is taken from the box.

The numbers on the two balls are *multiplied*.

- Construct a tree diagram to show all the possible outcomes.
- Find the probability that the product of the numbers is less than 10, $P(\text{Ball 1} \times \text{Ball 2} < 10)$.
- Find the probability that the product of the numbers is greater than 40, $P(\text{Ball 1} \times \text{Ball 2} > 40)$.
- Find the probability that the product of the numbers is between 10 and 30, $P(10 < \text{Ball 1} \times \text{Ball 2} < 30)$.

Answers

- (b) There is one of the possible outcomes that is less than 10.

$$P(\text{Ball 1} \times \text{Ball 2} < 10) = \frac{1}{9}$$

- (c) There are three of the possible outcomes that are greater than 40.

$$P(\text{Ball 1} \times \text{Ball 2} > 40) = \frac{3}{9} = \frac{1}{3}$$

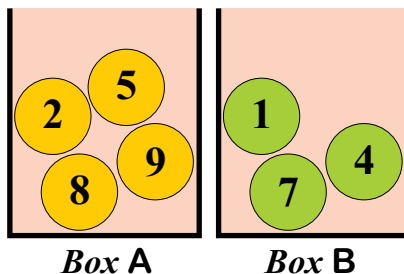
- (d) There are four of the possible outcomes that are between 10 and 30.

$$P(10 < \text{Ball 1} \times \text{Ball 2} < 30) = \frac{4}{9}$$

EXERCISE 15D

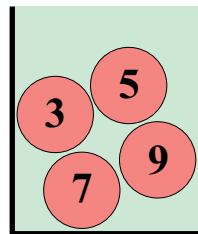
Write answers to the following questions as fractions in their simplest form

1. A ball is taken randomly from each of these boxes - **Box A** and **Box B**. The two numbers are **added**.



- (a) Construct a tree diagram to show all the possible outcomes.
- (b) Find the probability that the sum of the numbers is equal to 9,
 $P(A + B = 9)$.
- (c) Find the probability that the sum of the two numbers is less than 10,
 $P(A + B < 10)$.
- (d) Find the probability that the sum of the two numbers is greater than 10,
 $P(A + B > 10)$.
- (e) Find the probability that the sum of the two numbers is less than 20,
 $P(A + B < 20)$.
- (f) Find the probability that the sum of the two numbers is greater than 20,
 $P(A + B > 20)$.

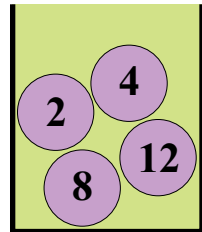
2. One ball is taken from the box shown here and then **without replacing** this ball a second ball is taken from the box. The two numbers are **multiplied**.



- (a) Construct a tree diagram to show all the possible outcomes.
- (b) Find the probability that the product of the two numbers is greater than 20,
 $P(\text{Ball 1} \times \text{Ball 2} > 20)$.
- (c) Find the probability that the product of the two numbers is an odd number,
 $P(\text{Ball 1} \times \text{Ball 2 is an odd number})$.
- (d) Find the probability that the product of the two numbers is an even number,
 $P(\text{Ball 1} \times \text{Ball 2 is an even number})$.
- (e) Find the probability that the product of the two numbers is a multiple of 3.
 $P(\text{Ball 1} \times \text{Ball 2 is a multiple of 3})$.

3. A ball is randomly taken from the box shown here then *replaced* and a second ball is then taken.

The two numbers are *multiplied*.



- (a) Construct a tree diagram to show all the possible outcomes.
- (b) Find the probability that the product of the two numbers is greater than 50,
 $P(\text{Ball 1} \times \text{Ball 2} > 50)$.
- (c) Find the probability that the product of the two numbers is an odd number,
 $P(\text{Ball 1} \times \text{Ball 2 is an odd number})$.
- (d) Find the probability that the product of the two numbers is less than 50,
 $P(\text{Ball 1} \times \text{Ball 2} < 50)$.
- (e) Find the probability that the product of the two numbers is a multiple of 16.
 $P(\text{Ball 1} \times \text{Ball 2 is a multiple of 16})$.

4. Two mixed tennis teams are playing each other. The teams' members are shown below.

Racketeers: Mary, Jack, Elle, Bob
Netters: Barry, Jenny, Natalie, Eve

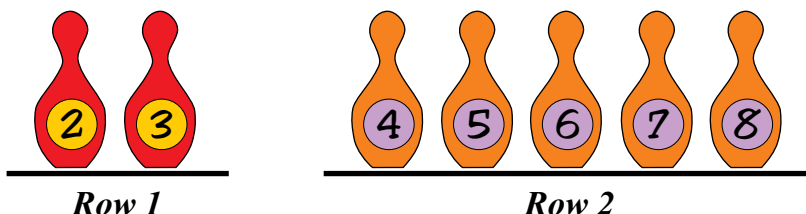
One player is randomly chosen from each team to clean the courts and pack up the nets after they play.

- (a) Draw a tree diagram to shown all the possible combinations.
- (b) Find the probability that the two chosen are both males.
- (c) Find the probability that the two chosen are both females.
- (d) Find the probability that the two chosen have the same initial for their first name.
- (e) Find the probability that Mary is chosen.
- (f) Find the probability that either Jack or Barry, but *not* both, are chosen.
- (g) Find the probability that Elle and/or Eve are chosen.

5. At a fun parlour one of the games involved throwing balls at two rows of skittles (shown below). To win a prize you need to knock over one skittle from each row. The numbers from the two skittles knocked over are *multiplied*.

If the product of the two numbers is greater than 20 you win a CD.
If the product of the two numbers is less than 10 you win \$10.

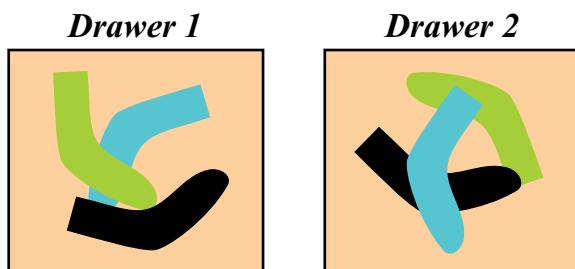
- (a) Construct a tree diagram to show all the possible outcomes.
(b) What is the probability of winning a CD?
(c) What is the probability of winning \$10?



6. Milton has three pairs of socks. One of each pair is in each of two drawers.

He randomly chooses a sock from each drawer.
Construct a tree diagram to help find the probability that:

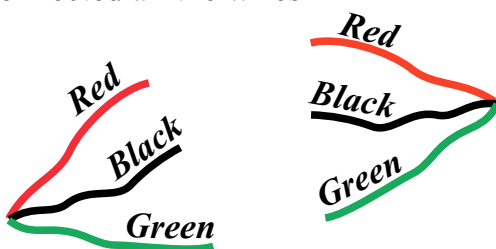
- (a) he chooses a pair.
(b) he chooses the black pair.



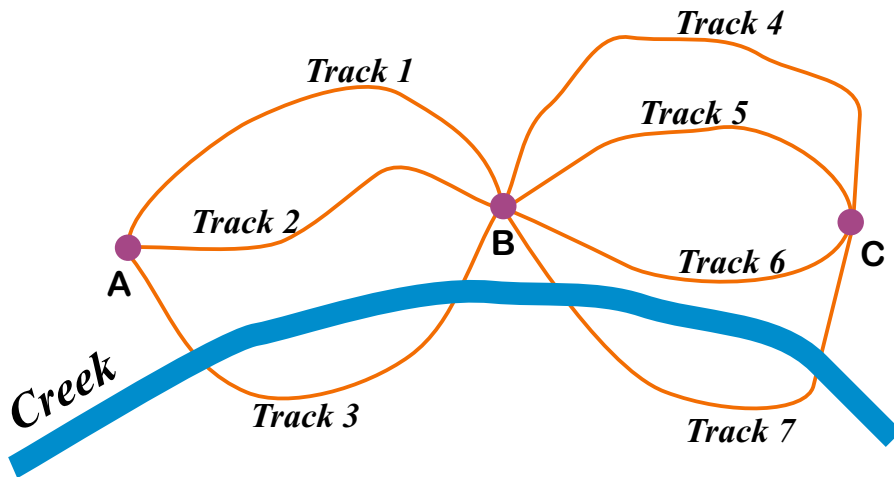
7. Marat's stereo system was not working. He took the back off and noticed three wires were broken. The wires were coloured red, black and green. The problem was Marat is colour-blind.

He decided to have a guess and randomly connect the wires.
Construct a tree diagram to help find the following probabilities.

- (a) What is the probability that he connected all the wires correctly?
(b) What is the probability that he connected *any* of the wires correctly?



8. Rodney is going for a hike from **A** to **B** to **C**. There are three tracks from **A** to **B** and four tracks from **B** to **C** as shown below. There is a creek that crosses two of the tracks. Rodney randomly chooses the tracks. Construct a tree diagram to show all possible outcomes. What is the probability that he will need to cross over the creek?



9. Jimi has a four string bass guitar. He decides to replace all the strings. His younger brother wants to help and puts the new strings on but didn't realise they had to go in a certain order. Construct a tree diagram to help find the following probabilities.
- What is the probability that the younger brother put the strings in the correct order?
 - What is the probability that at least two strings are in the correct places?

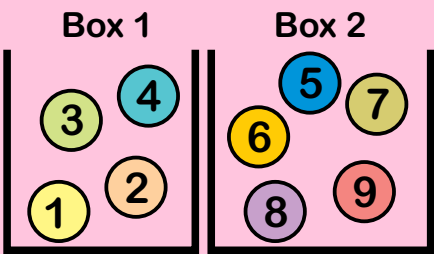


Two-way Tables (Arrays)

Two-way tables (also known as arrays) can be used when there is a large number of possible outcomes.

Example 1

A ball is chosen at random from each of these boxes and the numbers *added*.



1. What is the probability that the sum of the numbers is greater than 10?

$P(\text{Box 1} + \text{Box 2} > 10)$

2. What is the probability that the sum of the numbers is equal to 10?

$P(\text{Box 1} + \text{Box 2} = 10)$

Step 1 Construct a two-way table as shown here.

		Box 1			
		1	2	3	4
Box 2	5	6	7	8	9
	6	7	8	9	10
	7	8	9	10	11
	8	9	10	11	12
	9	10	11	12	13

Total number of outcomes = 20

Step 2 Use this to find the probabilities.

		Box 1			
		1	2	3	4
Box 2	5	6	7	8	9
	6	7	8	9	10
	7	8	9	10	11
	8	9	10	11	12
	9	10	11	12	13

6 of the 20 outcomes are greater than 10

1. $P(\text{Box 1} + \text{Box 2} > 10) = \frac{6}{20} = \frac{3}{10}$ (Simplest form)

		Box 1			
		1	2	3	4
Box 2	5	6	7	8	9
	6	7	8	9	10
	7	8	9	10	11
	8	9	10	11	12
	9	10	11	12	13

4 of the 20 outcomes are equal to 10

$$2. P(\text{Box 1} + \text{Box 2} = 10) = \frac{4}{20} = \frac{1}{5} \quad (\text{Simplest form})$$

Example 2

The names of the players in two mixed basketball teams are shown below.

The Beebees: Adam, Jenny, Ben, Damien, Kate

The Hotshots: Andrew, William, Emma, Brooke, James, Mike

After these teams play each other, one player is chosen from each team to score the next game in the competition. If one player is chosen at random from each team, what is the probability they will both be boys?

Solution

Using the letters **B** (boys) and **G** (girls) construct a two-way table.

		Beebees				
		B	G	B	B	G
Hotshots	B	BB	BG	BB	BB	BG
	B	BB	BG	BB	BB	BG
	G	GB	GG	GB	GB	GG
	G	GB	GG	GB	GB	GG
	B	BB	BG	BB	BB	BG
	B	BB	BG	BB	BB	BG

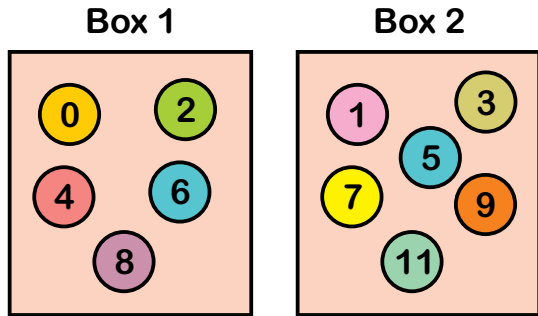
12 of the 30 possible outcomes are both boys

$$\text{The probability that they are both boys} = P(\text{BB}) = \frac{12}{30} = \frac{2}{5} \quad (\text{Simplest form})$$

EXERCISE 15E

Write the answers to the following questions as fractions in their simplest form.

1. A ball is taken at random from each of these boxes and the numbers are **added**.



- (a) Construct a two-way table to show all the possible outcomes.
- (b) What is the probability that the sum of the two numbers is less than 12?
- (c) What is the probability the sum of the two numbers is equal to 9?
- (d) What is the probability the sum of the two numbers is greater than 5?
2. A ball is taken at random from each of the boxes in question 1 and the numbers are **multiplied**.
- (a) Construct a two-way table to show all the possible outcomes.
- (b) What is the probability that the product of the two numbers is less than 50?
- (c) What is the probability the product of the two numbers is equal to 50?
- (d) What is the probability that the product of the two numbers is greater than 70?
3. Tammy has five skirts in one drawer - **blue**, **green**, **pink**, **red** and **orange**. She has five shirts in another drawer that are the same colours as the skirts - **blue**, **green**, **pink**, **red** and **orange**. One morning, in the dark, she randomly chose a skirt and shirt.
- (a) Construct a two-way table to show all the possible outcomes for the combinations of skirt and shirt.
- (b) What is the probability that she chose the **green** skirt and the **red** shirt?
- (c) What is the probability that she chose two colours that were the same?
- (d) What is the probability that she chose a **blue** and **orange** combination?
- (e) What is the probability that she chose two colours that were different?
- (f) What is the probability that **at least** one item of clothing was **pink**?



4. Two dice are rolled and the numbers are ***added***.

- (a) Construct a two-way table to show all the possible outcomes.
- (b) What is the probability that the sum of the two numbers is 5?
- (c) What is the probability that the sum of the two numbers is less than 5?
- (d) What is the probability that the sum of the two numbers is greater than 8?
- (e) What is the probability that the sum of the two numbers is an even number?
- (f) What is the probability that the sum of the two numbers is an odd number?
- (g) What sum/s of the numbers has the highest probability of occurring?
- (h) What sum/s of the numbers has the lowest probability of occurring?



5. Two dice are rolled and the numbers are ***multiplied***.

- (a) Construct a two-way table to show all the possible outcomes.
- (b) What is the probability that the product of the two numbers is 12?
- (c) What is the probability that the product of the two numbers is less than 20?
- (d) What is the probability that the product of the two numbers is greater than 20?
- (e) What is the probability that the product of the two numbers is between 10 and 20?
- (f) What is the probability that the product of the two numbers is a single digit number?
- (g) What is the probability that the product of the two numbers is an even number?
- (h) What is the probability that the product of the two numbers is an odd number?
- (i) What product/s of the numbers has the highest probability of occurring?
- (j) What product/s of the numbers has the lowest probability of occurring?

Probability Trees

When the individual probabilities of different outcomes are different, **probability trees** can be used to find the probabilities for multiple events.

Example 1

A school group was going camping on Tuesday and Wednesday in March. The probability of rain on a day in March, based on historic data, was 0.3.

Find the following probabilities.

1. It rains on both days.
2. It rains on just one of the two days.
3. There is no rain on both days.

To construct a probability tree all the probabilities need to be known. In this example there are two possible outcomes:

Rain (R)

No rain (N)

The probability of rain is 0.3 so the probability of no rain will be 0.7.

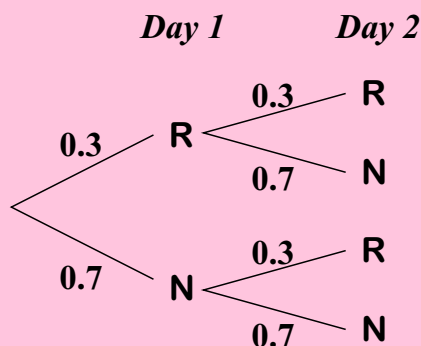
Remember the sum of all possible probabilities is 1.

$$P(R) = 0.3$$

$$P(N) = 0.7$$

A probability tree can now be constructed.

On each 'branch' of the tree include the probability of that outcome.



The probabilities can now be calculated.

The following rules need to be used to calculate probabilities of multiple events.

To find the probability of one event *and* another occurring the individual probabilities are *multiplied*.

To find the probability of one event *or* another occurring the individual probabilities are *added*.

The probability of rain on one day *and* the next day = $P(RR)$

$$P(RR) = 0.3 \times 0.3 = 0.09$$

The multiple probabilities can now be all calculated.

	Day 1	Day 2	Outcome	Probability
0.3	R	0.3 R	RR	$0.3 \times 0.3 = 0.09$
		0.7 N	RN	$0.3 \times 0.7 = 0.21$
0.7	N	0.3 R	NR	$0.7 \times 0.3 = 0.21$
		0.7 N	NN	$0.7 \times 0.7 = 0.49$

Note - the sum of all the probabilities will always equal 1.

Answers

1. The probability it rains on both days = $P(RR) = 0.09$
2. The probability it rains on just one of the two days = $P(RN \text{ or } NR)$
To find the probability of one event *or* another, the individual probabilities are *added*.

$$P(RN \text{ or } NR) = 0.21 + 0.21 = 0.42$$

3. The probability of no rain on both days = $P(NN) = 0.49$

Example 2

A box contains five caramel flavoured chocolates and three peppermint flavoured chocolates.

Willy takes two chocolates from the box.

1. Draw a probability tree to show the probability of all possible outcomes.
2. Find the following probabilities.
 - (a) Both are caramel (**C**).
 - (b) One of each flavour.
 - (c) Both are peppermint (**P**).

Write all probabilities as fractions in their simplest form.

The probability of choosing a caramel = $P(\mathbf{C}) = \frac{5}{8}$
 $\swarrow$ 5 caramels
 $\nwarrow$ 8 chocolates

The probability of choosing a peppermint = $P(\mathbf{P}) = \frac{3}{8}$
 $\swarrow$ 3 peppermints
 $\nwarrow$ 8 chocolates

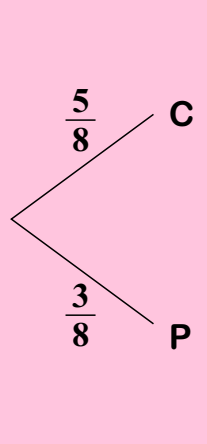
After one of the chocolates has been taken out there are seven chocolates remaining and this needs to be remembered when calculating the next probabilities:

Example $P(\mathbf{CC})$

$$\begin{array}{ccccccc}
 \text{Initially 5 caramels} & & & \text{4 caramels remaining} & & & \\
 \downarrow & & & \downarrow & & & \\
 P(\mathbf{CC}) = \frac{5}{8} & \times & \frac{4}{7} & = & \frac{20}{56} & = & \frac{5}{14} \\
 \uparrow & & \uparrow & & & & \\
 \text{Initially 8 chocolates} & & & \text{7 chocolates remaining} & & &
 \end{array}$$

Answers

1. The probability tree can now be completed.

	<i>Pick 1</i>	<i>Pick 2</i>	<i>Outcome</i>	<i>Probability</i>
	$\frac{5}{8}$	$\frac{4}{7}$ C	CC	$\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$
		$\frac{3}{7}$ P	CP	$\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$
	$\frac{3}{8}$	$\frac{5}{7}$ C	PC	$\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$
		$\frac{2}{7}$ P	PP	$\frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$

It is always a good idea to check that all the probabilities add to 1.

$$\begin{aligned}
 & \frac{5}{14} + \frac{15}{56} + \frac{15}{56} + \frac{3}{28} \\
 &= \frac{20}{56} + \frac{15}{56} + \frac{15}{56} + \frac{6}{56} \\
 &= \frac{56}{56} \\
 &= 1
 \end{aligned}$$

2. (a) $P(\mathbf{CC}) = \frac{5}{14}$

(b) $P(\mathbf{CP \text{ or } PC}) = \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$

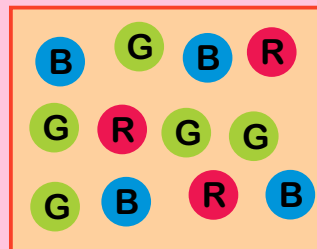
(c) $P(\mathbf{PP}) = \frac{3}{28}$

Example 3

A box contains four blue balls (**B**), five green balls (**G**) and three red balls (**R**).

Two balls are chosen at random.

- Construct a probability tree to show the probability of all possible outcomes.
- Find the following probabilities.
 - Both balls are green.
 - Neither ball is green.
 - At least one of the balls is green.
 - Both balls are the same colour.
 - The balls are different colours.

**Answers**

There are 12 balls.

The probabilities of choosing each of the colours as the first ball are:

$$P(\mathbf{B}) = \frac{4}{12} = \frac{1}{3}$$

$$P(\mathbf{G}) = \frac{5}{12}$$

$$P(\mathbf{R}) = \frac{3}{12} = \frac{1}{4}$$

- Complete the probability tree.

	Ball 1	Ball 2	Outcome	Probability
$\frac{1}{3}$ $\frac{5}{12}$ $\frac{1}{4}$	B	$\frac{3}{11}$ — B	BB	$\frac{1}{3} \times \frac{3}{11} = \frac{3}{33} = \frac{1}{11}$
		$\frac{5}{11}$ — G	BG	$\frac{1}{3} \times \frac{5}{11} = \frac{5}{33}$
		$\frac{3}{11}$ — R	BR	$\frac{1}{3} \times \frac{3}{11} = \frac{3}{33} = \frac{1}{11}$
	G	$\frac{4}{11}$ — B	GB	$\frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$
		$\frac{4}{11}$ — G	GG	$\frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$
		$\frac{3}{11}$ — R	GR	$\frac{5}{12} \times \frac{3}{11} = \frac{15}{132} = \frac{5}{44}$
	R	$\frac{4}{11}$ — B	RB	$\frac{1}{4} \times \frac{4}{11} = \frac{4}{44} = \frac{1}{11}$
		$\frac{5}{11}$ — G	RG	$\frac{1}{4} \times \frac{5}{11} = \frac{5}{44}$
		$\frac{2}{11}$ — R	RR	$\frac{1}{4} \times \frac{2}{11} = \frac{2}{44} = \frac{1}{22}$

Check that the probabilities add to 1.

$$\begin{aligned} & \frac{1}{11} + \frac{5}{33} + \frac{1}{11} + \frac{5}{33} + \frac{5}{33} + \frac{5}{44} + \frac{1}{11} + \frac{5}{44} + \frac{1}{22} \\ &= \frac{12}{132} + \frac{20}{132} + \frac{12}{132} + \frac{20}{132} + \frac{20}{132} + \frac{15}{132} + \frac{12}{132} + \frac{15}{132} + \frac{6}{132} \\ &= 1 \end{aligned}$$

2. (a) Probability that both balls are green.

$$P(\mathbf{GG}) = \frac{5}{33}$$

(b) Probability that neither ball is green.

$$P(\mathbf{neither\ ball\ is\ G}) = P(\mathbf{BB\ or\ BR\ or\ RB\ or\ RR})$$

$$P(\mathbf{BB\ or\ BR\ or\ RB\ or\ RR}) = \frac{1}{11} + \frac{1}{11} + \frac{1}{11} + \frac{1}{22}$$

$$P(\mathbf{neither\ ball\ is\ G}) = \frac{7}{22}$$

(c) Probability that at least one of the balls is green.

$$P(\mathbf{at\ least\ one\ ball\ is\ G}) = P(\mathbf{BG\ or\ GB\ or\ GG\ or\ GR\ or\ RG})$$

$$P(\mathbf{BG\ or\ GB\ or\ GG\ or\ GR\ or\ RG}) = \frac{5}{33} + \frac{5}{33} + \frac{5}{33} + \frac{5}{44} + \frac{5}{44}$$

$$P(\mathbf{at\ least\ one\ ball\ is\ G}) = \frac{90}{132} = \frac{15}{22}$$

(d) Probability that both balls are the same colour.

$$P(\mathbf{both\ balls\ the\ same\ colour}) = P(\mathbf{BB\ or\ GG\ or\ RR})$$

$$P(\mathbf{BB\ or\ GG\ or\ RR}) = \frac{1}{11} + \frac{5}{33} + \frac{1}{22}$$

$$P(\mathbf{both\ balls\ the\ same\ colour}) = \frac{38}{132} = \frac{19}{66}$$

(e) Probability that both balls are different colours.

$$P(\mathbf{both\ balls\ different\ colours}) = 1 - P(\mathbf{both\ balls\ the\ same\ colour})$$

$$P(\mathbf{both\ balls\ different\ colours}) = 1 - \frac{19}{66} = \frac{47}{66}$$

EXERCISE 15F

Construct probability trees to help solve the following problems.

1. Skye wanted to go skydiving on the upcoming weekend. The probability that the weather conditions would be suitable for skydiving on a particular day was 0.8.

Based on this figure, find the probability that she will be able to skydive on:

- (a) only one of the two days.
- (b) both days.
- (c) neither Saturday nor Sunday.



2. Kerry was the opening batter for her cricket team. After batting many times she has calculated that the probability that she scores at least 50 runs in an innings is 0.6.

Based on this figure, find the probability that in the two innings of her next match she scores:

- (a) at least 50 runs in both innings.
- (b) less than 50 runs in both innings.
- (c) at least 50 runs in only one of the two innings.

3. Milly is a border collie and had a litter of nine puppies of which six were males. Kyle wanted to buy two of the puppies and picked the two quietest without knowing whether they were males or females.

What is the probability that the two he chose were:

- (a) both males?
- (b) both females?
- (c) one male and one female?



4. In a volleyball team of seven players Chappy and Kappy are brothers. After the team played one night, two players were chosen at random from the team to umpire the next game.

What is the probability that:

- (a) both brothers were chosen as umpires?
- (b) neither brother was chosen?
- (c) at least one of the brothers was chosen?



5. Hattie did not enjoy cooking. She had several frozen meals in her freezer from which she would often choose her evening meal. At the start of one weekend the frozen meals in her freezer were:

3 butter chickens
4 lasagnes
2 vegetarian pastas

For her two evening meals over the weekend she randomly chose one of the frozen meals each night.

What is the probability that over the weekend she ate:

- (a) two lasagnes?
 - (b) two vegetarian pastas?
 - (c) one lasagne and one butter chicken?
 - (d) two butter chickens?
 - (e) at least one butter chicken?
6. In a Year 9 class there were 15 girls and 5 boys. Two of the students were chosen at random to represent the class at an ANZAC service.

What is the probability that:

- (a) both students chosen were boys?
- (b) both students chosen were girls?
- (c) there was one boy and one girl?
- (d) there was at least one girl?

7. Brydon enjoyed playing chess. He calculated from all the games he had played that the probability of him winning a game was 0.7, the probability of him losing was 0.2 and the probability of him drawing a game was 0.1. Brydon was to play two games against his arch-rival Issa.

Based on the above figures, what is the probability that Brydon will:

- (a) win both games?
- (b) lose both games?
- (c) draw both games?
- (d) win one and lose one?
- (e) win one and draw one?
- (f) win at least one game?



8. Rosie wanted to plant two rose bushes in her garden. She went to the local nursery and found, in the rose section, 12 roses without labels and without flowers. The nurseryman told her the labels had been lost but did know that there were six red roses, four yellow roses and two pink roses.

Rosie randomly chose two of the roses.

What is the probability that:

- (a) both roses were red?
- (b) both roses were pink?
- (c) one was red and one pink?
- (d) she didn't get a yellow rose?
- (e) she got at least one red rose?



Venn Diagrams

Venn diagrams are another way of representing data and providing a way to help calculate probabilities.

Example 1

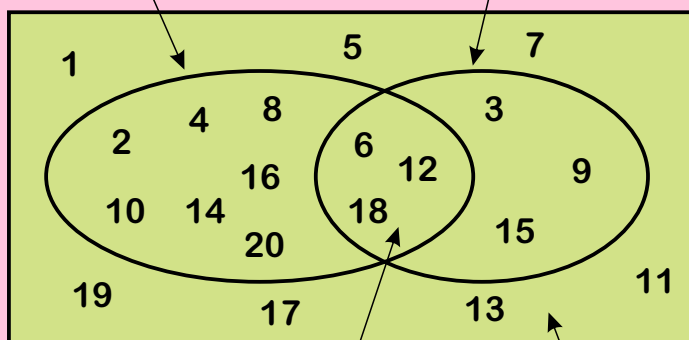
From the numbers 1-20, a number is chosen at random.

- What is the probability that it is a multiple of 2?
- What is the probability that it is a multiple of 3?
- What is the probability that it is a multiple of 2 **and** 3?
- What is the probability that it is **not** a multiple of 2 **or** 3?
- What is the probability that it is a multiple of 2 but **not** 3?
- What is the probability that it is a multiple of 3 but **not** 2?
- Given that the number is **greater** than 12, what is the probability that it is a multiple of 3?
- Given that the number is **less** than 10, what is the probability that it is a multiple of 2 and 3?

The Venn diagram for this problem is shown below.

Numbers in this ellipse are multiples of 2.

Numbers in this ellipse are multiples of 3.



Numbers in the section where the two ellipses overlap are multiples of 2 and 3.

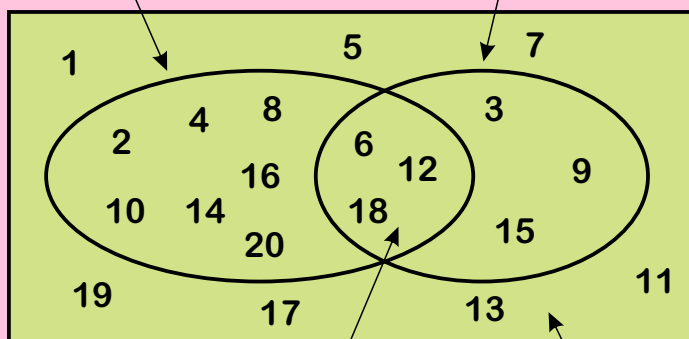
Numbers outside the ellipses are **not** multiples of 2 or 3.

Continued on next page

To calculate the probabilities, the number of numbers in each section need to be stated.

10 numbers in this ellipse
(multiples of 2)

6 numbers in this ellipse
(multiples of 3)



3 numbers in the section where
the two ellipses overlap
(multiples of 2 and 3)

7 numbers outside the ellipses
(not multiples of 2 or 3)

(a) Probability that it is a multiple of 2 $= \frac{10}{20} = \frac{1}{2}$ (Simplest form)

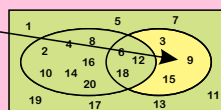
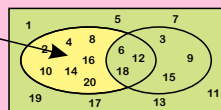
(b) Probability that it is a multiple of 3 $= \frac{6}{20} = \frac{3}{10}$ (Simplest form)

(c) Probability that it is a multiple of 2 **and** 3 $= \frac{3}{20}$

(d) Probability that it is **not** a multiple of 2 **or** 3? $= \frac{7}{20}$

(e) Probability that it is a multiple of 2 but **not** 3? $= \frac{7}{20}$

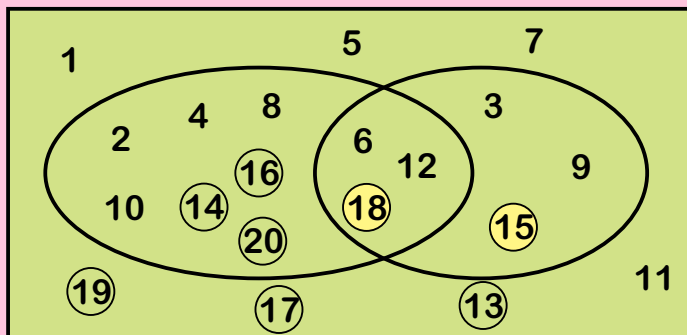
(f) Probability that it is a multiple of 3 but **not** 2? $= \frac{3}{20}$



- (g) Given that the number is **greater** than 12, what is the probability that it is a multiple of 3?

This means that the number chosen is greater than 12. There are eight numbers greater than 12 and these are circled below.

The numbers from these that are multiples of 3 are coloured. There are two of these.

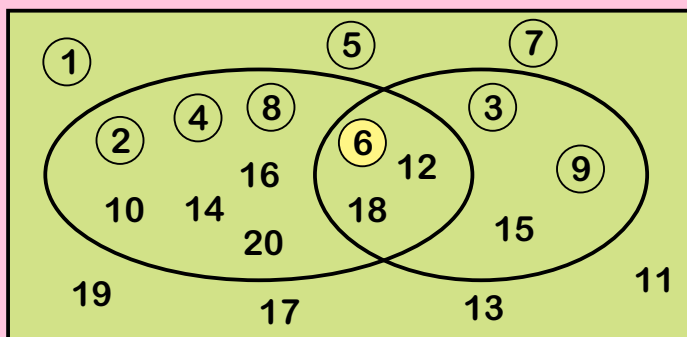


Given that the number is **greater** than 12, the probability that it is a multiple of 3 $= \frac{2}{8} = \frac{1}{4}$ (Simplest form)

- (h) Given that the number is **less** than 10, what is the probability that it is a multiple of 2 and 3?

This means that the number chosen is less than 10. There are nine numbers less than 10 and these are circled below.

The numbers from these that are multiples of 2 and 3 are coloured. There is one of these.



Given that the number is **less** than 10, the probability that it is a multiple of 2 and 3 $= \frac{1}{9}$

EXERCISE 15G

1. (a) Draw a Venn diagram with two ellipses. Write all the numbers from 1 to 20 such that:

- One ellipse shows the numbers that are multiples of 3.
- The other ellipse shows the numbers that are multiples of 4.
- Note there will be some numbers in the section where the two ellipses overlap.

From the numbers 1-20, a number is chosen at random.

- (b) What is the probability that it is a multiple of 3?
- (c) What is the probability that it is a multiple of 4?
- (d) What is the probability that it is a multiple of 3 **and** 4?
- (e) What is the probability that it is **not** a multiple of 3 **or** 4?
- (f) What is the probability that it is a multiple of 3 but **not** 4?
- (g) Given that the number is **greater** than 10, what is the probability that it is a multiple of 3?
- (h) Given that the number is **greater** than 10, what is the probability that it is a multiple of 4?
- (i) Given that the number is **greater** than 10, what is the probability that it is a multiple of 3 and 4?
- (j) Given that the number is **greater** than 10, what is the probability that it is a multiple of 3 or 4?
- (k) Given that the number is **less** than 10, what is the probability that it is a multiple of 3?
- (l) Given that the number is **less** than 10, what is the probability that it is a multiple of 3 and 4?

2. (a) Draw a Venn diagram with two ellipses that shows the following.

- All the numbers from 1 to 30.
- One ellipse shows all the numbers that are multiples of 4.
- The other ellipse shows all the numbers that are multiples of 5.

From the numbers 1-30 a number is chosen at random.

- (b) What is the probability that it is a multiple of 4?
- (c) What is the probability that it is a multiple of 4 **and** 5?
- (d) What is the probability that it is a multiple of 4 **or** 5?
- (e) What is the probability that it is **not** a multiple of 4?
- (f) What is the probability that it is a multiple of 4 but **not** 5?
- (g) Given that the number is **greater** than 10, what is the probability that it is a multiple of 4?
- (h) Given that the number is **less** than 16, what is the probability that it is a multiple of 5?

Example 2

15 people attended a music camp.

Jimi, Eric, Carlos, Suzi and Angus played only guitar.

Elton, Joni and Chris played only piano.

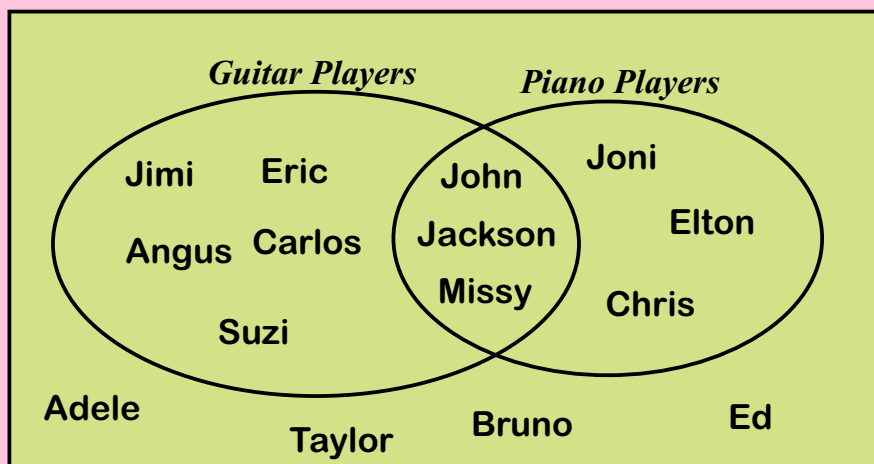
John, Jackson and Missy played guitar and piano.

Adele, Taylor, Bruno and Ed did not play guitar or piano.

1. Draw a Venn diagram to display this information.
2. A person is chosen at random from the group. Find the probability of the following.
 - (a) The person plays guitar.
 - (b) The person does not play guitar.
 - (c) The person plays guitar or piano.
 - (d) The person only plays piano.
 - (e) The person plays guitar and piano.
 - (f) Given that the person chosen plays guitar and/or piano, find the probability that they only play guitar.
 - (g) Given that the person chosen plays guitar and/or piano, find the probability that they play guitar and piano.

Answers

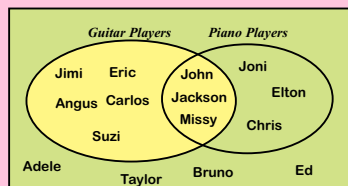
1.



2. (a) The probability that the person plays guitar.

There are 8 out of the 15 people that play guitar.

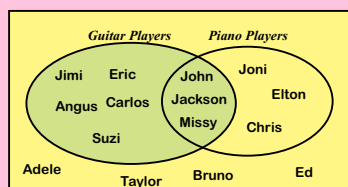
$$\text{Probability that a person chosen at random plays guitar} = \frac{8}{15}$$



- (b) The probability that the person does not play guitar.

There are 7 out of the 15 people that do not play guitar.

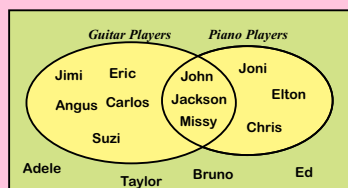
$$\text{Probability that a person chosen at random does not play guitar} = \frac{7}{15}$$



- (c) The probability that the person plays guitar or piano.

There are 11 out of the 15 people that play guitar or piano.

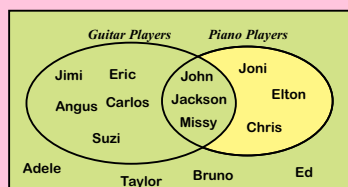
$$\text{Probability that a person chosen at random plays guitar or piano} = \frac{11}{15}$$



- (d) The probability that the person only plays piano.

There are 3 out of the 15 people that only play piano.

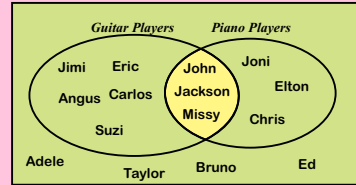
$$\begin{aligned} \text{Probability that a person chosen at random only plays piano} &= \frac{3}{15} \\ &= \frac{1}{5} \quad (\text{Simplest form}) \end{aligned}$$



- (e) The probability that the person plays guitar and piano.

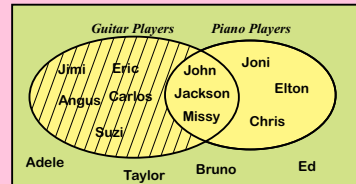
There are 3 out of the 15 people that play guitar and piano.

$$\begin{aligned} \text{Probability that a person} &= \frac{3}{15} \\ \text{chosen at random plays} & \\ \text{guitar and piano} & \\ &= \frac{1}{5} \quad (\text{Simplest form}) \end{aligned}$$



- (f) Given that the person chosen plays an instrument, find the probability that they only play guitar.

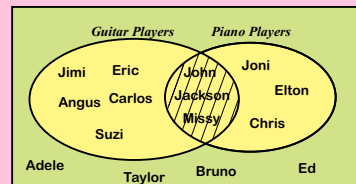
There are 11 people that play an instrument. **Given that the person chosen plays an instrument** means one of these 11 people is chosen at random. Of these, 5 play guitar only.



Given that the person chosen plays an instrument the probability that they only play guitar. $= \frac{5}{11}$

- (g) Given that the person chosen plays an instrument, find the probability that they play guitar and piano.

There are 11 people that play an instrument. **Given that the person chosen plays an instrument** means one of these 11 people is chosen at random. Of these, 3 play guitar and piano.



Given that the person chosen plays an instrument the probability that they play guitar and piano. $= \frac{3}{11}$

EXERCISE 15H

Write the answers to the following probabilities as fractions in their simplest form.

1. 20 students were asked if they played basketball and/or tennis.
Amelia, Ben, Carrie, Delta, Eve and Faran played basketball but not tennis.
Garth, Harry, Indie, Jaan and Kiera played tennis but not basketball.
Liam, Mia, Nellie and Ollie played both basketball and tennis.
Pat, Quinn, Rose, Sian and Tilly did not play either sport.
- (a) Draw a Venn diagram to display this information.
A student is chosen at random from the group. Find the probability of the following.
- (b) The student plays basketball but not tennis.
(c) The student plays tennis.
(d) The student plays tennis and basketball.
(e) The student does not play either sport.
(f) The student does not play basketball.
(g) The student does not play both sports.
(h) Given that the student chosen plays basketball and/or tennis, find the probability that they play both sports.
(i) Given that the student chosen plays basketball and/or tennis, find the probability that they play only basketball.
(j) Given that the student chosen does not play basketball, find the probability that they play tennis.
(k) Given that the student chosen does not play tennis, find the probability that they do not play one of the sports.



2. Students in a visual art class were asked if they wanted to study painting, photography or another type of visual arts.

Nell, Lonie, Keiren, Xavier, Tye, Peta, Georgia and Marvin wanted to study painting but not photography.

Kyle, Oscar, Hilda, Josh, Wanda and Fleur wanted to study photography but not painting.

Rohan, Darian, Jueban and Bessy wanted to study both painting and photography.

Zena, Will, Tasha, Crosby, Vera and Clara wanted to study other types of visual arts.

(a) Draw a Venn diagram to display this information.

A student is chosen at random from the group. Find the probability of the following.

- (b) The student wants to study only painting.
- (c) The student does not want to study painting.
- (d) The student does not want to study painting or photography.
- (e) The student wants to study painting and photography.
- (f) Given that the student chosen wants to study painting and/or photography, what is the probability that they want to study both?
- (g) Given that the student chosen wants to study painting and/or photography, what is the probability that they want to study painting?
- (h) Given that the student chosen does not want to study painting, what is the probability that they want to study photography?
- (i) Given that the student chosen does not want to study photography, what is the probability that they don't want to study painting?



Example 3

There are 25 students in a class.

14 of them play a musical instrument.

18 of them play sport.

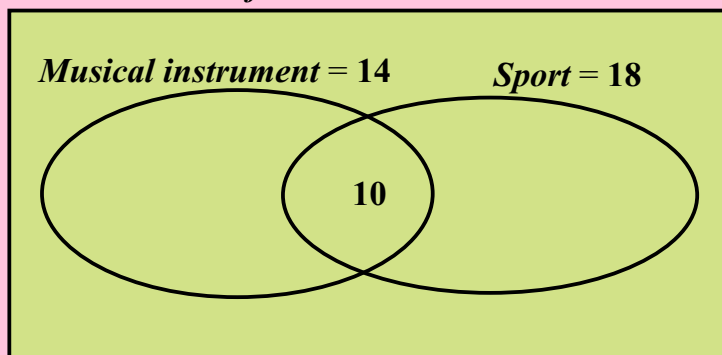
10 play a musical instrument and play sport.

1. Draw a Venn diagram to display this information.
2. Complete all sections of the Venn diagram.
3. Find the following:
 - (a) How many students play a musical instrument but don't play sport?
 - (b) How many students play sport but not a musical instrument?
 - (c) How many students don't play a musical instrument and don't play sport?
4. A student is chosen at random.
 - (a) What is the probability that they sport?
 - (b) What is the probability that they play a musical instrument but don't play sport?
 - (c) Given that the student chosen plays sport, what is the probability that they also play a musical instrument?

Answers

1. The Venn diagram with known information is shown below.

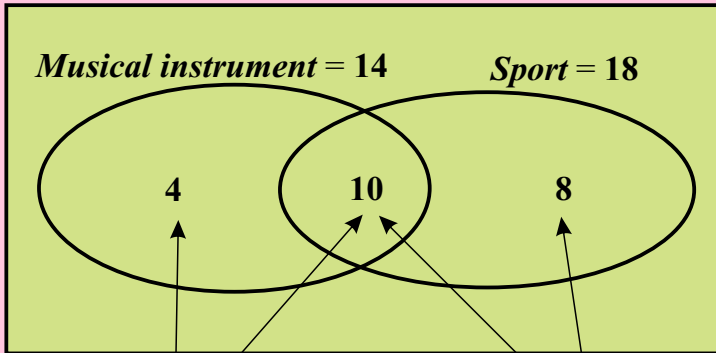
Total number of students = 25



2. Complete the other sections of the Venn diagram in two steps.

Step 1

Total number of students = 25

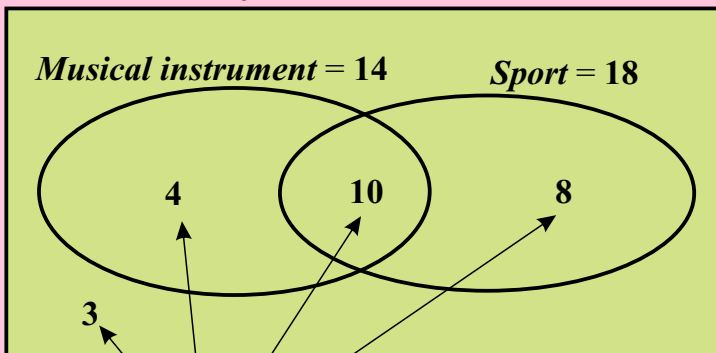


These two add to 14
(students playing a
musical instrument)

These two add to 18
(students playing sport)

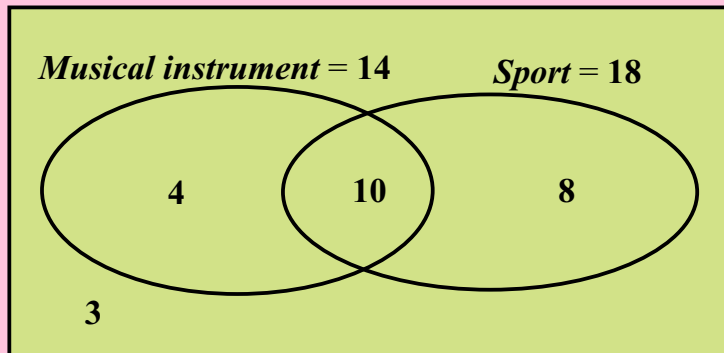
Step 2

Total number of students = 25



These four must add to 25
(the total number of students)

Total number of students = 25



Use the Venn diagram to solve the following questions.

3. (a) **4** students play a musical instrument but don't play sport.
 (b) **8** students play sport but don't play a musical instrument.
 (c) **3** students don't play a musical instrument and don't play sport.
4. (a) The probability a student chosen at random plays sport = $\frac{18}{25}$
 (b) The probability a student chosen at random plays a musical instrument but doesn't play sport = $\frac{4}{25}$
 (c) Given that the student plays sport, the probability that they also play a musical instrument = $\frac{10}{18} = \frac{5}{9}$ (Simplest form)

EXERCISE 15I

1. 50 people were asked if they had a pet dog and/or cat.
26 had a dog, 15 had a cat and 8 of these had both.
- (a) Draw a Venn diagram to show this information.
 - (b) Complete all sections of the Venn diagram.
 - (c) How many of the people only had a dog?
 - (d) How many of the people only had a cat?
 - (e) How many had neither a dog nor a cat?
- A person is chosen at random from the group.
- (f) What is the probability that the person owns a dog and cat?
 - (g) What is the probability that the person only owns a dog?
 - (h) Given that the person chosen owns a dog, what is the probability that they also own a cat?



2. At a school there were 60 students in Year 9.
32 studied graphics, 18 studied art and 6 of these studied both.
- (a) Draw a Venn diagram to show this information.
 - (b) Complete all sections of the Venn diagram.
 - (c) How many students studied graphics but not art?
 - (d) How many students studied art but not graphics?
 - (e) How many students did not study graphics or art?
- A student is chosen at random from the group.
- (f) What is the probability that they studied art?
 - (g) What is the probability that they studied art and graphics?
 - (h) Given that they studied art, what is the probability that they also studied graphics?

3. 80 people were asked if they had been to see an AFL game or visit an art exhibition in the last year.

35 had seen an AFL game, 23 had seen an art exhibition and 12 of these had seen both.

- Draw a Venn diagram to show this information.
- Complete all sections of the Venn diagram.
- How many of the people had only seen an AFL game?
- How many of the people had only seen an art exhibition?
- How many had seen neither?

One of these people was chosen at random.

- What is the probability that they had been to an AFL game and an art exhibition?
- What is the probability that they had seen an art exhibition but not been to an AFL game?
- Given that the person had been to an AFL game, what is the probability that they had not seen an art exhibition?



4. 65 students were asked if they had cereal and/or fruit juice for breakfast.

42 had cereal, 27 had both cereal and fruit juice and 9 had neither cereal nor fruit juice for breakfast.

- Draw a Venn diagram to show this information.
- Complete all sections of the Venn diagram.
- How many had cereal but not fruit juice for breakfast?
- How many had fruit juice but not cereal?
- How many had fruit juice for breakfast?
- If one of the students was chosen at random, what is the probability that they had both cereal and fruit juice for breakfast?

5. 100 people were asked if they had travelled overseas and/or to another state in Australia.
37 had not travelled overseas or to another state.
6 had travelled overseas but not interstate.
16 had travelled overseas and interstate.
(a) Draw a Venn diagram to show this information.
(b) Complete all sections of the Venn diagram.
(c) How many of the people had travelled overseas?
(d) How many of the people had travelled interstate but not overseas?
(e) How many of the people had travelled interstate?
(f) If one of these people was chosen at random, what is the probability that they had travelled to another state in Australia?

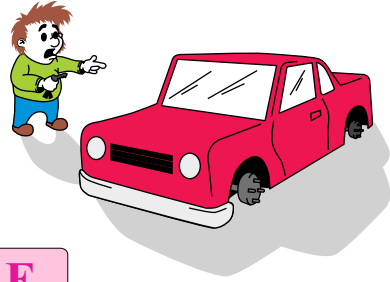


6. 120 companies were surveyed to see if they used the internet or newspapers for advertising.
48 used only the internet.
16 used only newspapers.
13 didn't use either.
(a) Draw a Venn diagram to show this information.
(b) Complete all sections of the Venn diagram.
(c) If one of the companies was chosen at random, what is the probability that they had advertised in a newspaper and the internet?
7. An ice-cream seller recorded how many people had chocolate and/or crushed nuts on their ice-creams. Out of 110 ice-creams sold, 75 had chocolate, 36 had crushed nuts and 21 had neither chocolate nor crushed nuts.
(a) Draw a Venn diagram to show this information.
(b) Complete all sections of the Venn diagram.
(c) If one of these people was chosen at random, what is the probability that they had both chocolate and crushed nuts?

PROBLEM SOLVING

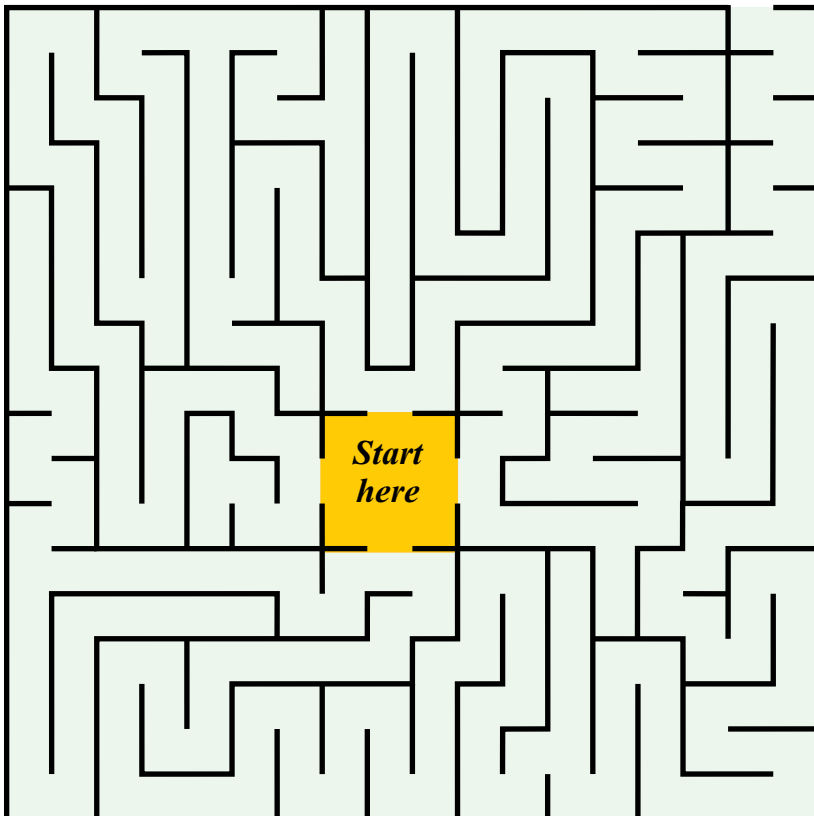
Seymour wanted to change the position of all the wheels on his car to balance the wear on each tyre. He took all the wheels off but then forgot where each wheel came from.

What is the probability that when he puts the wheels back on the car each one is in a different position to where it was originally?

**PUZZLE**

There are 16 'dead ends' in this maze.

The probability of getting out of the maze first attempt is $\frac{1}{17}$.
See how many attempts you need to get from the starting point out of the maze.



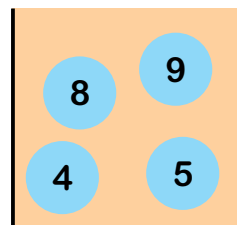
CHAPTER REVIEW

1. A box of sweets contained 8 strawberry, 6 caramel and 4 peppermint flavoured chocolates.
 - (a) What is the probability of randomly choosing a strawberry flavoured chocolate?
 - (b) If 3 strawberry chocolates are eaten what then is the probability of choosing another strawberry flavoured chocolate?

2. Ally scored 24 goals in a netball game. She missed 16 shots at goal.
 - (a) What is the probability (as a percentage) that she would score a goal?
 - (b) If she had 320 shots at goal over the season, how many goals would she be expected to score?

3. Spud is a potato farmer. He wanted to get an idea of the size of his potatoes. He dug up 50 kg and found that 35 kg of them were larger than 250 g each.
 - (a) What is the probability (as a percentage) that a potato is larger than 250 g?
 - (b) If he expects to have a total crop of 1200 kg, how many kilograms of the potatoes would be larger than 250 g?

4. One ball is taken from this box and *without replacing* the first ball a second ball is taken from the box. The two numbers are *multiplied*.
 - (a) Construct a tree diagram to show all possible outcomes.
 - (b) Find the following probabilities:
 - (i) $P(\text{Ball 1} \times \text{Ball 2 is an odd number})$
 - (ii) $P(\text{Ball 1} \times \text{Ball 2} < 30)$
 - (iii) $P(\text{Ball 1} \times \text{Ball 2} > 80)$
 - (iv) $P(30 < \text{Ball 1} \times \text{Ball 2} < 50)$

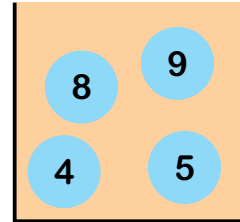


5. One ball is taken from this box then *replaced* and another ball is randomly taken from the box. The two numbers are *multiplied*.

(a) Construct an array to show all possible outcomes.

(b) Find the following probabilities:

- (i) $P(\text{Ball 1} \times \text{Ball 2 is an odd number})$
- (ii) $P(\text{Ball 1} \times \text{Ball 2} < 30)$
- (iii) $P(\text{Ball 1} \times \text{Ball 2} > 80)$
- (iv) $P(30 < \text{Ball 1} \times \text{Ball 2} < 50)$



6. There are two mixed basketball teams.

The names of the players in each team are:

Jumpers - James, Michael, William, Chrissie, Emma
Screeners - Nathan, Hayley, Jacqui, Xavier, Meredith

One player from each team is chosen at random to umpire the next game.

- (a) Construct a two-way table to show all the possible combinations of boy (**B**) and girl (**G**) from each team.
- (b) What is the probability that two boys are chosen?
- (c) What is the probability that two girls are chosen?
- (d) What is the probability that a boy and girl are chosen?

7. The probability of snow falling on a given day at a ski resort was 0.8.

A school outdoor education class was going to the resort for two days. Construct a probability tree to help find the probability that:

- (a) snow is falling on both days the school group is at the resort.
- (b) snow does not fall on either day that the group is there.
- (c) snow falls on at least one of the two days.

8. Randolph and Remmie buy 10 movies at random from an online movie store. They know seven of them are comedies and the other three are mysteries. One night they randomly chose two of the movies to watch.

- (a) Construct a probability tree to show all possible outcomes.
- (b) What is the probability that:
 - (i) both movies are comedies?
 - (ii) neither of the movies are comedies?
 - (iii) at least one of the movies is a comedy?

9. 60 people were asked if they had been on an aeroplane or a cruise ship. 35 had been on an aeroplane, 18 had been on a cruise ship and of these 11 had been on both.
- (a) Draw a Venn diagram to show this information.
 - (b) Complete all sections of the Venn diagram.
- A person is chosen at random from the group.
- (c) What is the probability they had been on an aeroplane and cruise ship?
 - (d) What is the probability they had not been on either an aeroplane or cruise ship?
 - (e) Given that the person chosen had been on an aeroplane, what is the probability they had also been on a cruise ship?

Data

Types of Data

The collection of data is very important in many areas of society. Federal, state and local governments need information on which to base their policies in areas such as education, transport, health, environment, etc. Businesses need information to help them decide which products to produce or stock.

Medical researchers need data to see which research is needed and how research is progressing.

Farmers need information on weather trends, which crops to plant or which livestock to farm.

There are companies that collect data using surveys and sell them to organisations, governments and individuals.

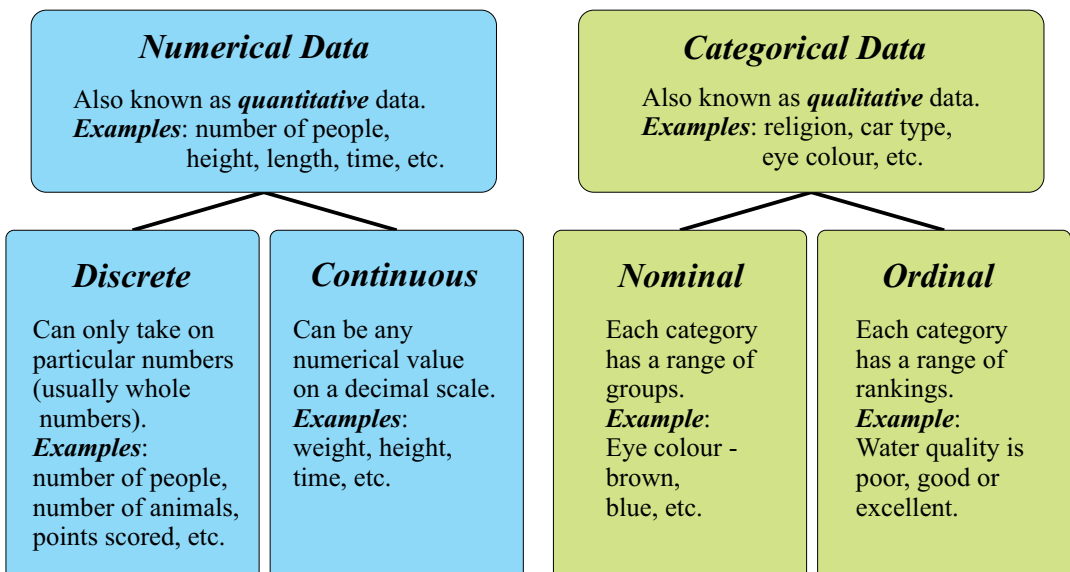
Data can be grouped in several ways.

1. Numerical and Categorical Data

There are two types of numerical data - *discrete* and *continuous*.

There are two types of categorical data - *nominal* and *ordinal*.

The diagram below details these different types of data.



2. Primary and Secondary Data

Primary data

Data that is collected by the people, organisation, etc. who are going to use the data.

Sources

Surveys, questionnaires, observations, experiments, interviews, telephone, focus groups

Examples

- ♦ A medical research group may interview a group of people to investigate the effects of a medical treatment.
- ♦ A local government may conduct a survey to find out the opinions of residents on a proposed development.

Advantages

- ♦ Direct personal investigation can be more accurate.
- ♦ The information can be more reliable.
- ♦ Can target the particular people, areas, etc. that are being studied.

Disadvantages

- ♦ Direct contact with people may lead to bias or reluctance of people to answer truthfully depending on the topic.
- ♦ May be expensive and time consuming.
- ♦ Due to restrictions of money and time may lead to an investigation that is smaller than what is required to obtain accurate results.

Secondary data

Data that has been collected by a group or organisation other than the people who are going to use the information.

Sources

Internet, newspapers, magazines, Australian Bureau of Statistics (ABS)

Examples

- ♦ The government may want information on which form of transport people use to decide whether to build more roads or expand public transport.
- ♦ Medical researchers may need statistics on health areas that need more research.

Advantages

- ♦ In the case of a country wide census the information will be more accurate due to the number of people surveyed.
- ♦ It already exists so there is no cost or time involved with conducting surveys or studies. The only cost may be in the use of the data.
- ♦ Offers access to information that may otherwise be difficult to obtain.
- ♦ May provide information that was obtained using equipment that would otherwise have been difficult to access.
- ♦ Offers access to data obtained over a period of time.
- ♦ Offers access to data obtained in areas that would have been difficult to access - geographically, politically, etc.

Disadvantages

- ♦ It may be difficult to confirm the accuracy, quality or independence of the data.
- ♦ The data may be obsolete.
- ♦ Some data may be expensive to get approval to use.

EXERCISE 16A

Classify the following data as *continuous numerical*, *discrete numerical* or *categorical*.

1. Crowd number at a cricket match.
2. Student's favourite type of music.
3. The number of pages in a book.
4. Amount of water in a reservoir.
5. Price of a pair of jeans.
6. Time taken to get to school.
7. Distance an athlete throws a discus.
8. Level of satisfaction with the performance of a computer program - poor, good, excellent.
9. Number of students on a bus.
10. Number of points scored by a basketball player in a season.
11. Types of crops grown by a farmer.
12. Amount of crops produced by a farmer.
13. Number of people employed by a farmer.
14. Amount of money spent by the government on education.
15. The different portfolios on which government divide budget spending.
16. The number of people in a meeting.



Samples and Population

When collecting data, it is often not possible to include every person or object that is affected. In these situations a smaller representative sample is used.

In statistics, the total number of people or objects affected is called the **population**. A smaller representative group is called a **sample**.

For example, a local council wants to find out if the residents want a library or a swimming pool. 500 residents are surveyed to get an idea of what they prefer.

In this example all the residents in the area would be the **population**. The 500 that were surveyed is the **sample**.

To make the results of a survey as accurate as possible:

1. The sample must be as large as possible.
2. The people or items surveyed must be a true representation of the population.

EXERCISE 16B

1. List the population that would be affected by the following surveys.
 - (a) A State Government wants to investigate raising the driving age to 19 to help lower the road toll.
 - (b) The Federal Government wants to find out how many people are going to vote for them in the next election.
 - (c) A park ranger wants to find the number of sugar gliders that are in a state park.
 - (d) A clothing company wants to find out which of their latest jean designs will be the most popular.
 - (e) A local radio station wants to find out how many people listen to them.
 - (f) A factory that produces blocks of chocolate that have nuts in them, wants to find out how many nuts are in each block.
 - (g) A local football team wants to find out which player is considered to be the most valuable.

2. For the surveys listed below, a list of four possible sample groups is given.

State the one that you think would give the most accurate results and give your reasons for not choosing the other three.

- (a) A school wants to investigate whether the students should wear school uniform. There are 800 students at the school.

Sample Group **A** - 50 students from each year level

Sample Group **B** - 10 students from each year level and 20 teachers

Sample Group **C** - 100 randomly chosen students, 10 teachers, 100 parents and 20 people from the local community

Sample Group **D** - all the teachers and 100 parents

- (b) The state government wants to find out secondary students' attitude towards a television advertisement aimed at stopping people drinking and driving.

Sample Group **A** - 50 year 7 students from every secondary school in the state

Sample Group **B** - 50 students randomly chosen from every secondary school in the state

Sample Group **C** - 100 students randomly chosen from every secondary school in the capital city

Sample Group **D** - every year 9 student in the state

- (c) A local council wants to investigate whether to build a skate park in the town (population 5000).

Sample Group **A** - 10 people randomly chosen

Sample Group **B** - 1000 people between the ages of 40 and 60

Sample Group **C** - all students at the local secondary school

Sample Group **D** - 1 in 10 people chosen randomly

3. State an appropriate sample group for each of the surveys in Question 1.

Example

A sample group of 50 students from a school were asked if they wanted a music band or a sports clinic to visit the school.

There are 800 students at the school.

35 students from the sample group chose the music band.

15 students from the sample group chose the sports clinic.

1. What percentage of the sample group of 50 chose the music band and what percentage chose the sports clinic?
2. Based on this sample group, how many students in the school would have chosen the music band and how many would have chosen the sports clinic?

1. Percentage that chose music band = $\frac{35}{50} \times 100 = \mathbf{70\%}$

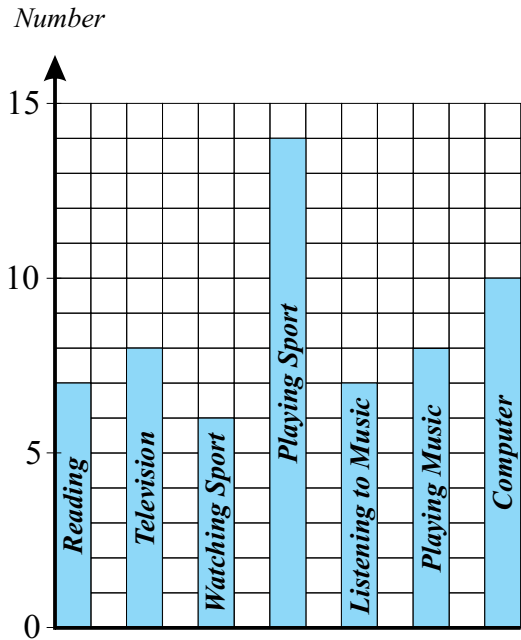
Percentage that chose sports clinic = $\frac{15}{50} \times 100 = \mathbf{30\%}$

2. 70% of 800 = **560 students** would want the music band
 30% of 800 = **240 students** would want the sports clinic

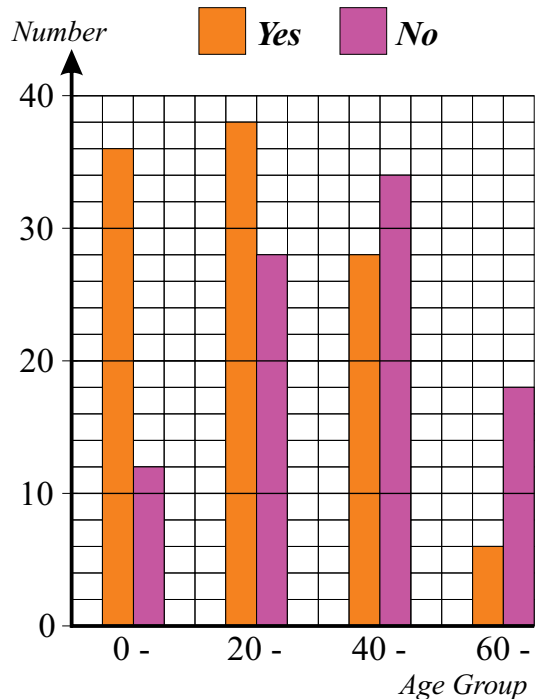
4. A town of 12 000 people has three television stations to watch: Channels 1, 2 and 3.
 200 people were asked what channel they preferred.
 90 chose Channel 1, 75 chose Channel 2 and 35 chose Channel 3.
- (a) What percentage of the sample group chose each channel?
 - (b) Based on this survey, how many people in the town watch each television channel?
5. 15 000 people attended the final of a tennis tournament.
 400 of them were asked their ages and the results are shown in the table below.
- (a) What percentage of the sample group were in each age group?
 - (b) Based on this sample, how many people at the final were in each age group?

<i>Age Group</i>	<i>Number</i>
- 20	50
- 30	86
- 40	107
- 50	89
over 50	68

6. 60 students from a college were asked to state their favourite leisure activity. The results are shown in this column graph. There are 720 students at the college. Based on the results from this survey, how many students from the college would choose each of the leisure activities?



7. A local government surveyed 200 people to find out if they wanted a new sports stadium to be built. The age group of the people was noted (0-, 20-, 40-, 60-). The results of this survey are presented on this column graph.



- What is the total number of people surveyed who **do** want the new stadium?
- What percentage of the people surveyed **do not** want the new stadium?
- What percentage of the people under 20 who were surveyed **do** want the new stadium?
- If there are 8000 people in the town, how many, based on this survey, would want the new sports stadium?

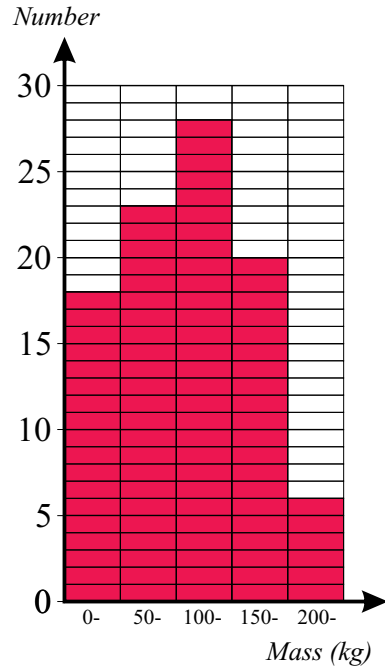
8. Marine biologists wanted to find out how many turtles were in a marine park and the mass of the turtles.

They captured, weighed and released every turtle in an area of 10 km^2 .

The results of the study are shown in this graph.

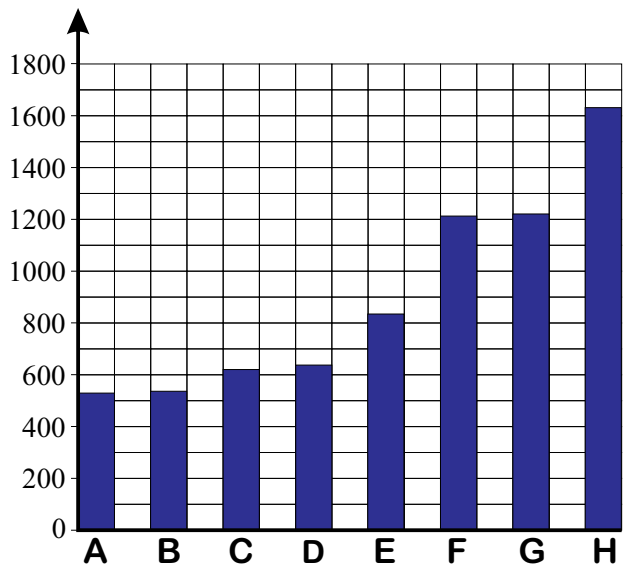
The marine park has a total area of 200 km^2 .

- (a) How many turtles were captured and weighed in the 10 km^2 area?
 (b) What percentage of these turtles was less than 100 kg ? (one dec. pl.)
 (c) What percentage of the turtles was 200 kg or heavier? (one dec. pl.)
 (d) Based on these figures, how many turtles would be expected to be in the marine park?
 (e) How many turtles in the marine park would be 100 kg or heavier?



9. This graph shows the average annual rainfall (in mm) for all of the capital cities of Australia (arranged from lowest to highest). List all the capital cities in Australia. Using your knowledge of geography and weather try to match the capital cities and the annual rainfalls (**A - H**).

*Average
Annual
Rainfall
(mm)*



10. Lloyd is the forestry manager at a state park. He has been given the task of recording the diameter of the trunks of all the eucalyptus trees in the park.

The park has a total area of 20 000 hectares.

Lloyd decides to measure all the trees in an area of 5 hectares and use these figures to calculate the trunk diameters for the park.

The trunk diameters (in *centimetres*) of all the trees in the 5 hectares are given below.

23	45	13	48	12	60	51	19	10	54	32	40	5	31	11	25	38	8
13	20	31	46	6	17	22	30	6	15	19	23	28	39	7	15	20	34
5	29	26	21	41	50	10	18	14	24	8	17	36	45	33	18	21	12
19	14	6	33	7	21	42	14	27	24	27	25	51	38	10	17	2	19
29	37	17	8	11	26	45	7	14	16	6	34	15	4	28	18	29	16

- (a) Copy this tally sheet and use it to find out how many trees were in each diameter range
(0- , 10- , 20- , 30- , 40- , 50-)

Diameter Range (cm)	Tally	Number (Frequency)
0 -		
10 -		
20 -		
30 -		
40 -		
50 -		
Total		

- (b) Construct a column graph to display this data.
(c) Find the percentage of trees in each diameter range. (one dec. pl.)
(d) Calculate how many trees would be expected to be in each diameter range for the entire park.

11. The Krunchee Biscuit Company makes chocolate chip biscuits. They guarantee that at least 90% of the biscuits have 6 chocolate chips or more. Willie is the quality control officer. Every day his job is to count the number of chocolate chips in 50 biscuits. His findings for one day are listed below.

- (a) Record this information on a column graph.
(b) What percentage of the 50 biscuits had 6 chocolate chips or more?

The number of chocolate chips in each of 50 biscuits:

8	9	7	7	8	9	6	5	7	9	10	9	6	7	9	6	7	8	7	8	9	6	6	7	8
8	8	8	7	9	6	7	8	8	6	6	7	8	8	9	5	6	7	8	4	7	8	6	8	7

Sources of Secondary Data

Project

The internet is an extremely powerful and convenient method of finding data.

1. (a) Use the internet to find the median age of people in Australia, Cambodia, Tonga and two other countries of your own choice.
 (b) Where was this data displayed?
 (c) From which organisation was the data collected?
2. (a) Use the internet to find the subjects studied by 14 year old students in Australia, Japan and Timor-Leste.
 (b) State the source of this data.
3. (a) Use the internet to find and compare the rainfall in several parts of Australia, Pakistan, New Guinea and Malaysia.
 (b) State the source of this data.
4. What does **ABS** stand for in Australian data collection?
5. How often is the Australian census taken?
6. Name three different items of information collected in the Australian census.
7. How is the information in the Australian census collected?



Mean, Median, Mode and Range

Mean, median, mode and **range** are values calculated in numerical data that are used to give a measure of the location (centre) and spread of the data. They are also used to compare different groups of data.

$$\textbf{Mean (average)} = \frac{\text{the sum of all numerical data}}{\text{the number of items of data}}$$

Median = the middle number when the data is arranged in numerical order from smallest to largest

Mode = the item of data that occurs the most

Range = difference between largest and smallest value
(largest value - smallest value)

Examples

For the following sets of data find the:
 (a) mean (b) median (c) mode (d) range

1.

4, 9, 9, 10, 11, 16, 18

$$\begin{aligned}
 \text{(a) mean} &= \frac{4+9+9+10+11+16+18}{7} \\
 &= \frac{77}{7} \\
 \text{mean} &= 11
 \end{aligned}$$

(b) the numbers are in numerical order from smallest to largest

$$\begin{array}{ccccccc}
 4 & 9 & 9 & 10 & 11 & 16 & 18
 \end{array}$$

median = 10

The median is the middle number

(c) the number that occurs the most is 9

$$\text{mode} = 9$$

(d) range = largest - smallest

$$= 18 - 4$$

$$\text{range} = 14$$

Examples continued

2.

13, 16, 19, 12, 13, 22, 20, 12, 16, 20, 13

$$(a) \text{ mean} = \frac{13+16+19+12+13+22+20+12+16+20+13}{11}$$

$$= \frac{176}{11}$$

$$\text{mean} = 16$$

(b) the numbers need to be arranged in numerical order from the smallest to largest to find the median

12 12 13 13 13 16 16 19 20 20 22

$$\text{median} = 16$$

The median is the middle number

(c) the number that occurs the most is 13

$$\text{mode} = 13$$

(d) range = largest - smallest

$$= 22 - 12$$

$$\text{range} = 10$$

Examples continued

3.

13, 16, 10, 11, 8, 12, 11, 13, 14, 13

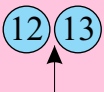
$$(a) \text{ mean} = \frac{13+16+10+11+8+12+11+13+14+13}{10}$$

$$= \frac{121}{10}$$

$$\text{mean} = 12.1$$

(b) the numbers need to be arranged in numerical order from the smallest to largest to find the median

8 10 11 11 12 13 13 14 16



When there is an even number of data items, the median is midway between the centre two numbers

$$\text{median} = 12.5$$

(c) the number that occurs the most is 13

$$\text{mode} = 13$$

(d) range = largest - smallest

$$= 16 - 8$$

$$\text{range} = 8$$

EXERCISE 16C

1. For each of the following sets of numbers find the:

(i) mean (ii) median (iii) mode (iv) range

(a) 4, 4, 5, 7, 10

(b) 13, 13, 14, 14, 14, 15, 15

(c) 9, 11, 15, 12, 18, 15, 7, 8, 12, 15, 21

(d) 12, 5, 13, 13, 17, 15, 14, 7, 12, 8, 12, 14, 14

(e) 4, 6, 10, 12, 14, 14

(f) 15, 16, 21, 21, 23, 23, 23, 26

(g) 10, 11, 13, 15, 13, 9, 11, 12, 17, 20

(h) 5, 5, 7, 8, 9, 2, 7, 8, 9, 7

(i) 7, 8, 9, 10, 9, 6, 7, 6, 7, 9, 10, 11, 10, 9, 8, 8, 7, 6, 9, 10

2. Ten people went on a trek to Machu Picchu in Peru.

Their ages were:

28, 36, 40, 19, 21, 23, 38, 57, 18, 41

(a) What is the mean age of the trekkers?

(b) What is the median age of the trekkers?

(c) What is the range of the trekkers' ages?



3. A 400 metre runner had the following times (in seconds) in eight races.

48.7 49.6 47.3 50.2 53.1 52.7 49.8 50.2

- What is the mean time for the races?
- What is the median time for the races?
- What is the range of times run by the runner?



4. The scores of two batsmen in a cricket team in 12 innings are shown below.

Hervey Hooker 0, 3, 7, 8, 329, 8, 4, 1, 13, 0, 111, 8
Cary Cutter 42, 43, 53, 38, 44, 46, 41, 45, 30, 48, 21, 41

- Find the mean of both batsman's scores.
 - Find the median of both batsman's scores.
 - Find the range of runs scored by each batter?
 - Comment on which batsman you think played better in the 12 innings. Give reasons for your answer.
5. The manager of a factory earns \$2900 per week. There are seven employees at the factory that each earn \$500 per week.
- Find the mean weekly wage.
 - Find the median weekly wage.
 - Which is the better indicator of the amount earned by the people at the factory?

6. The prices of 20 houses that sold in a town are shown here.

Find the following:

- (a) the mean house price
- (b) the median house price
- (c) the range of house prices

\$235 000	\$425 000
\$576 500	\$337 200
\$420 000	\$584 000
\$395 000	\$400 000
\$720 000	\$328 000
\$386 000	\$299 000
\$292 000	\$389 000
\$478 000	\$725 000
\$542 000	\$472 000
\$412 000	\$248 000

7. The amount of money spent by 50 people in a clothes shop is shown below.

Find the following:

- (a) the mean amount spent
- (b) the median amount spent
- (c) the range of amounts spent

\$250	\$310	\$110	\$110	\$150
\$180	\$290	\$140	\$425	\$140
\$325	\$115	\$285	\$95	\$225
\$440	\$180	\$120	\$180	\$240
\$200	\$40	\$480	\$190	\$55
\$590	\$170	\$190	\$145	\$150
\$180	\$80	\$180	\$120	\$110
\$70	\$190	\$90	\$100	\$85
\$180	\$280	\$65	\$120	\$190
\$280	\$330	\$380	\$210	\$180



8. The prices of seven houses sold in two towns are shown in the tables below.

Wattleton

\$350 000
\$375 000
\$325 000
\$295 000
\$400 000
\$380 000
\$360 000

Bankston

\$375 000
\$360 000
\$385 000
\$315 000
\$300 000
\$1 205 000
\$350 000

- (a) For each town find:
- (i) the mean
 - (ii) the median
 - (iii) the range
- (b) Remove the most expensive house price from each list and using the remaining prices find:
- (i) the mean
 - (ii) the median
 - (iii) the range
- (c) Comment on how one large price affected the mean, median and range of the two lists.
- (d) Explain why the median house price is often used, rather than the mean, to compare house prices in different areas.



Dot Plots

When there is a large number of data items, with many repeated, several techniques can be used to make it easier to analyse the data. One of those techniques is a *dot plot*.

Example 1

The goals scored by a soccer team in 25 games are shown below.

0, 2, 3, 1, 2, 3, 4, 0, 3, 1, 3, 6, 3, 2, 1, 4, 2, 3, 2, 2, 1, 2, 3, 1, 0

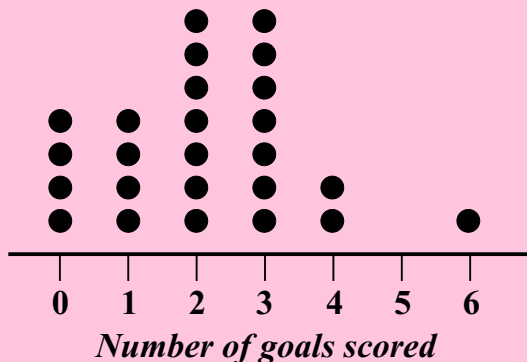
1. Display this data on a dot plot.
2. Find the following:
 - (a) the mean
 - (b) the median
 - (c) the mode
 - (d) the range of scores

Answers

1. To construct a dot plot following these steps:

Step 1 Identify the smallest and largest item of data so the horizontal axis can be drawn.

Step 2 Represent each item of data by a dot. Work through the data and place a dot on the graph for each item.



Note - it is always a good idea to check that the number of dots equals the number of items of data.

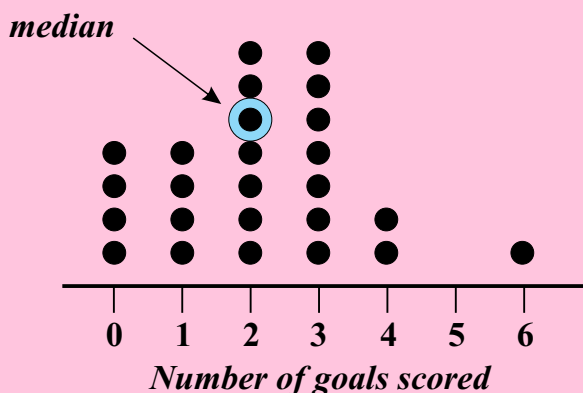
2. (a) The dot plot can make calculating the mean easier. The number of each item can be identified and multiplied by each numerical data item.

$$\begin{aligned}
 \text{mean} &= \frac{(4 \times 0) + (4 \times 1) + (7 \times 2) + (7 \times 3) + (2 \times 4) + (1 \times 6)}{25} \\
 &= \frac{0 + 4 + 14 + 21 + 8 + 6}{25} \\
 &= \frac{53}{25}
 \end{aligned}$$

$$\text{mean} = 2.12$$

- (b) The median is also easier to find because the dots representing the data are in numerical order.

There are 25 items of data so the median will be the 13th item.



$$\text{median} = 2$$

- (c) It can be seen that there are two modes in this data.

$$\text{modes} = 2 \text{ and } 3$$

- (d) $\text{range} = 6 - 0$

$$\text{range} = 6$$

Example 2

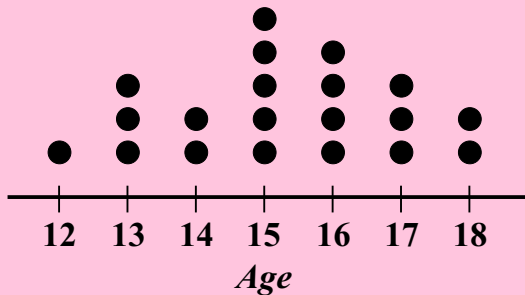
The ages of 20 members of a youth theatrical group are shown below.

**12, 14, 15, 17, 15, 17, 16, 18, 16, 17,
13, 15, 18, 16, 15, 14, 13, 15, 13, 16**

1. Display this data on a dot plot.
2. Find the following:
 - (a) the mean age
 - (b) the median age
 - (c) the mode
 - (d) the range of ages

Answers

1.

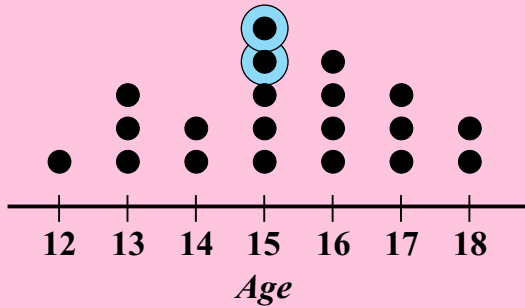


2. (a) **mean** =

$$\begin{aligned}
 & \frac{(1 \times 12) + (3 \times 13) + (2 \times 14) + (5 \times 15) + (4 \times 16) + (3 \times 17) + (2 \times 18)}{20} \\
 &= \frac{12 + 39 + 28 + 75 + 64 + 51 + 36}{20} \\
 &= \frac{305}{20}
 \end{aligned}$$

$$\text{mean} = 15.25$$

(b) The median will be midway between the 10th and 11th items.



median = midway between 15 and 15

median = 15

(c) **mode = 15**

(d) **range = 18 - 12**

range = 6

EXERCISE 16D

1. The number of people who boarded a bus was recorded for 40 bus stops and this data is shown below.

1, 2, 1, 4, 5, 0, 3, 4, 2, 3, 4, 7, 4, 2, 1, 0, 3, 4, 0, 1,
2, 1, 3, 2, 6, 4, 2, 1, 1, 2, 0, 1, 3, 3, 0, 1, 2, 4, 2, 3

(a) Display this data on a dot plot.

(b) Find the following:

- (i) the mean
- (ii) the median
- (iii) the mode/s
- (iv) the range

2. A breeder of Siberian Huskies recorded the number of puppies in 30 litters and this data is shown below.

4, 5, 3, 3, 5, 4, 6, 2, 6, 7,
2, 3, 4, 5, 6, 5, 4, 5, 3, 5,
4, 5, 4, 6, 5, 3, 4, 5, 5, 4



- (a) Display this data on a dot plot.
(b) Find the following:
(i) the mean
(ii) the median
(iii) the mode
(iv) the range

3. Lloyd enjoyed swimming. He would swim for half an hour every second day and recorded the number of laps he swam for 50 sessions. These are shown here.

- (a) Display this data on a dot plot.

- (b) Find the following:

- (i) the mean
(ii) the median
(iii) the mode
(iv) the range

40, 42, 36, 38, 43, 41, 42, 41, 42, 37,
38, 41, 39, 40, 41, 36, 38, 39, 40, 36,
38, 43, 42, 38, 42, 43, 36, 38, 39, 40,
37, 37, 38, 41, 42, 43, 37, 36, 40, 40,
41, 43, 40, 39, 38, 37, 38, 39, 41, 40

4. There were 24 students in a Year 9 mathematics class. They were all given a challenging problem to solve and the number of minutes taken by each student to solve the problem was recorded.

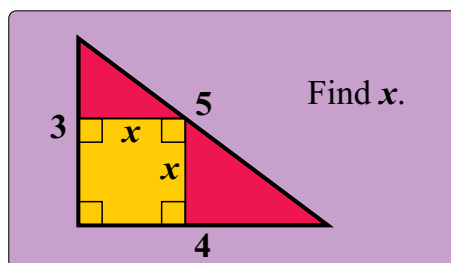
These times are shown below.

8, 10, 9, 15, 12, 13, 9, 12, 11, 10, 10, 11,
12, 9, 13, 15, 14, 11, 13, 14, 13, 12, 11, 9

- (a) Display this data on a dot plot.

- (b) Find the following:

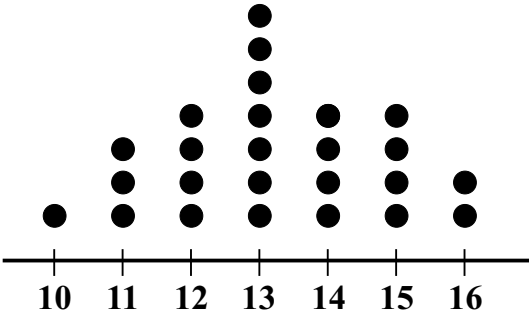
- (i) the mean
(ii) the median
(iii) the mode/s
(iv) the range



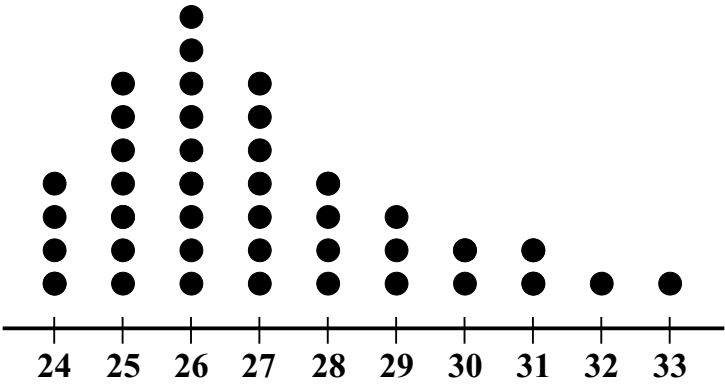
5. For the following dot plots find:

- (i) the mean
- (ii) the median
- (iii) the mode/s
- (iv) the range

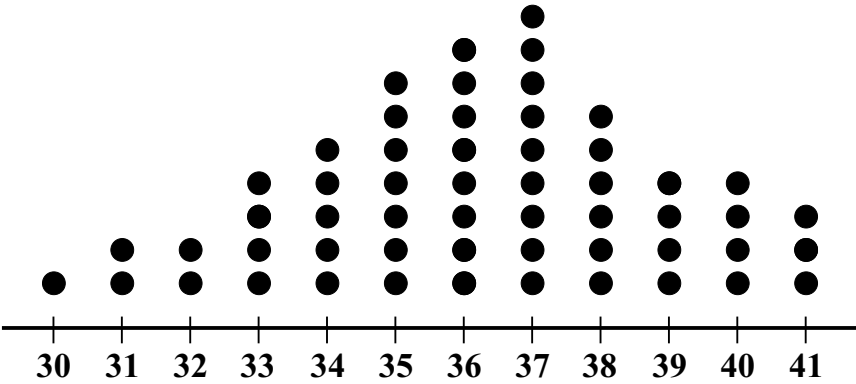
(a)



(b)



(c)



Histograms

It is often easier to represent large amounts of data on frequency tables or frequency graphs called *histograms*.

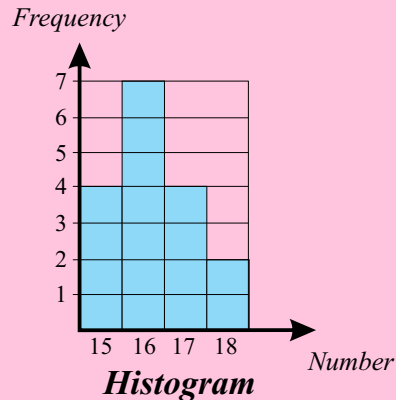
Example 1

The following data can be displayed on the frequency table and frequency graph shown below.

15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 16, 17, 17, 17, 17, 18, 18

Number	Frequency
15	4
16	7
17	4
18	2
Total	17

Frequency Table



Histogram

$$(a) \text{ Mean} = \frac{(4 \times 15) + (7 \times 16) + (4 \times 17) + (2 \times 18)}{17}$$

$$\text{Mean} = 16.2 \text{ (one decimal place)}$$

$$(b) \text{ Median} = 16$$

$$(c) \text{ Mode} = 16$$

$$(d) \text{ Range} = 18 - 15 = 3$$

Example 2

For the data in this frequency table find:

- (a) the mean (one decimal place)
- (b) the median
- (c) the mode
- (d) the range

Frequency Table

Number	Frequency
40	7
41	11
42	15
43	12
44	8

- (a) To find the mean the total number of data items needs to be calculated:

$$7 + 11 + 15 + 12 + 8 = \mathbf{53}$$

$$\text{Mean} = \frac{(7 \times 40) + (11 \times 41) + (15 \times 42) + (12 \times 43) + (8 \times 44)}{53}$$

$$\text{Mean} = \mathbf{42.1} \text{ (one decimal place)}$$

- (b) The median is the middle item of data. There are 53 items of data so the median will be the 27th item. This can be found by looking at the 'running total' in the frequency column - this is called the **cumulative frequency**. Note the last value in the cumulative frequency column is the total number of items.

Frequency Table

Number	Frequency	Cumulative Frequency
40	7	7
41	11	18
42	15	33
43	12	45
44	8	53

*The 27th item will be in this group.
Median = 42*

(c) **Mode** = 42

(d) **Range** = 44 - 40 = 4

Example 3

For the data in this frequency table find:

- (a) the mean (one decimal place)
- (b) the median
- (c) the mode
- (d) the range

Frequency Table

Number	Frequency
18	12
19	13
20	11
21	19
22	17

- (a) To find the mean the total number of data items needs to be calculated:

$$12 + 13 + 11 + 19 + 17 = 72$$

$$\text{Mean} = \frac{(12 \times 18) + (13 \times 19) + (11 \times 20) + (19 \times 21) + (17 \times 22)}{72}$$

$$\text{Mean} = 20.2 \text{ (one decimal place)}$$

- (b) The median is the middle item of data. There are 72 items of data so the median will be midway between the 36th item and 37th item.

Frequency Table

Number	Frequency	Cumulative Frequency
18	12	12
19	13	25
20	11	36
21	19	55
22	17	72

The 36th item is the last item in this group (20).

The 37th item is the first item in this group (21).

Median is midway between 20 and 21.

$$\text{Median} = 20.5$$

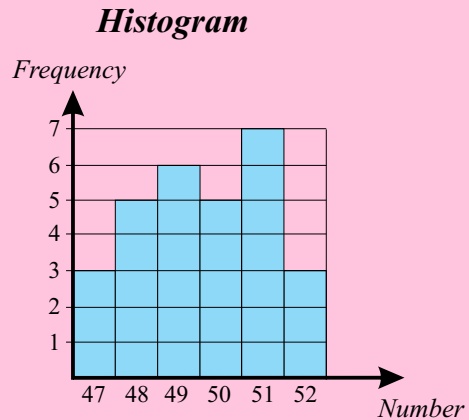
(c) **Mode** = 21

(d) **Range** = $22 - 18 = 4$

Example 4

For the data in this histogram find:

- (a) the mean (one decimal place)
- (b) the median
- (c) the mode
- (d) the range

**Frequency Table**

<i>Number</i>	<i>Frequency</i>	<i>Cumulative Frequency</i>
47	3	3
48	5	8
49	6	14
50	5	19
51	7	26
52	3	29

A frequency table with cumulative frequency column can assist with calculating the mean and median.

$$(a) \text{ Mean} = \frac{(3 \times 47) + (5 \times 48) + (6 \times 49) + (5 \times 50) + (7 \times 51) + (3 \times 52)}{29}$$

$$\text{Mean} = 49.6 \text{ (one decimal place)}$$

(b) There are 29 items of data so the median will be the 15th item.

$$\text{Median} = 50$$

$$(c) \text{ Mode} = 51$$

$$(d) \text{ Range} = 52 - 47 = 5$$

Example 5

For the following data items find:

- (a) the mean (one decimal place)
- (b) the median
- (c) the mode
- (d) the range

92, 87, 88, 99, 87, 88, 91, 92, 88, 92, 88, 90, 92, 90, 90, 91, 92, 91, 91, 91, 88, 91, 92, 91, 92, 87, 94, 88, 94, 87, 95, 88, 96, 88, 98

Frequency Table

Displaying the data on a frequency table makes it easy to analyse.

For large numbers of data items using a tally column can help when the data is not in numerical order.

Cross off each data item as it is recorded in the tally column.

The smallest and largest items need to be identified first.

<i>Number</i>	<i>Tally</i>	<i>Frequency</i>	<i>Cumulative Frequency</i>
87		4	4
88		8	12
89		0	12
90		3	15
91		8	23
92		7	30
93		0	30
94		2	32
95		1	33
96		1	34
97		0	34
98		1	35
99		1	36

(a) **Mean** = $\frac{(4 \times 87) + (8 \times 88) + \dots}{36} = \mathbf{90.8}$ (one decimal place)

(b) **Median** = **91** (midway between the 18th and 19th items)

(c) **Mode** = **88 and 91**

(d) **Range** = $99 - 87 = \mathbf{12}$

EXERCISE 16E

1. Display each of the following sets of data as a frequency table and histogram.

(a) 9, 9, 9, 10, 10, 10, 10, 10, 11, 11, 11, 12, 12

(b) 20, 22, 22, 22, 23, 23, 23, 23, 23, 23, 24, 24, 25

(c) 4, 4, 4, 5, 5, 5, 5, 6, 6, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10, 11, 12

2. For each set of data in question 1 find the:

(i) mean (to one decimal place)

(ii) median

(iii) mode

(iv) range

3. For the data in the following frequency tables find the:

(i) mean (1 dec. pl.) (ii) median (iii) mode (iv) range

(a)

Number	Frequency
8	4
9	6
10	7
12	2
Total	19

(b)

Number	Frequency
22	8
25	10
28	13
31	9
32	6
Total	46

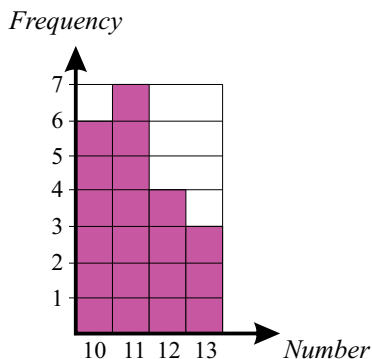
(c)

Number	Frequency
12	10
14	13
17	16
18	11
19	7

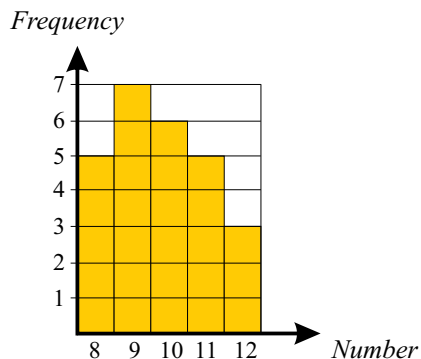
4. For the data in the following histograms find the:

(i) mean (1 dec. pl.) (ii) median (iii) mode (iv) range

(a)



(b)



5. Display each of the following sets of data as a frequency table and histogram.

(a) 21, 20, 20, 21, 19, 20, 21, 22, 20, 19, 20, 22, 20, 21

(b) 53, 50, 53, 54, 53, 51, 54, 53, 54, 51, 54, 53, 53, 54, 55, 51, 56, 52

6. For each set of data in question 5 find the:

(i) mean (to one decimal place)

(ii) median

(iii) mode

(iv) range

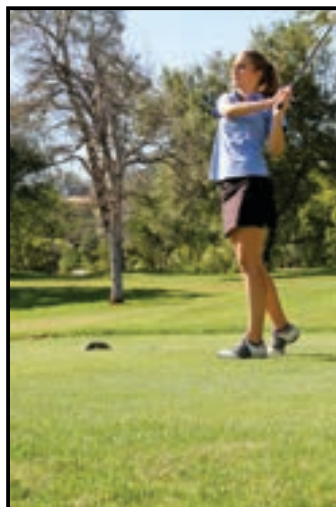
7. Penelope regularly plays golf. The number of strokes she took on each hole of an 18 hole round of golf are listed below.

4, 5, 4, 6, 7, 6, 4, 3, 5, 4, 6, 6, 5, 6, 4, 4, 5, 4

(a) Construct a frequency table to display this information.

(b) Calculate the average (mean) number of strokes she took per hole. (one decimal place)

(c) Calculate the median.



8. Jaryd worked as a fruit picker during his school holidays. He worked eight hours each day and the number of kilograms of apples he picked each hour for his first four days is shown below .

66, 64, 65, 68, 65, 70, 69, 65, 65, 66, 67, 66, 67, 66, 67, 67, 67, 68, 67, 66, 70, 67, 66, 67, 68, 68, 65, 64, 69, 69, 66, 69

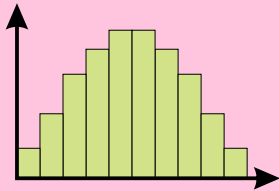
(a) Construct a frequency table and histogram to display this information.

(b) Find the mean number of kilograms of apples he picked each hour. (one decimal place)

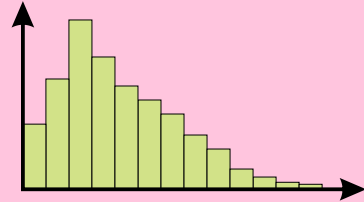
(c) Use this figure to find the mean number of kilograms he picked each day.

Types of Histograms

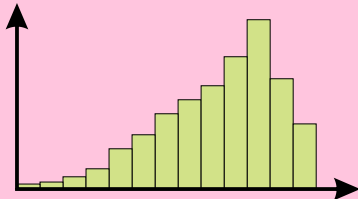
There are four general types of histograms.



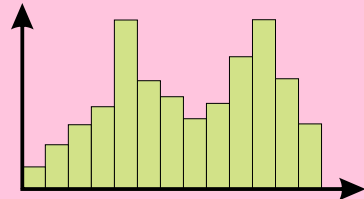
Symmetric



*Right-Skewed
(Positively Skewed)*



*Left-Skewed
(Negatively Skewed)*



Bimodal

When a set of data is summarised, the mean, median and/or mode are often used to give an indication of the centre of the data. They are measures of the values about which the data is spread and are called the **central tendency** of the data.

The mean, median and mode/s can give a different indication of the central tendency of data depending on the shape of the histogram.

For **symmetric** histograms the **mean, median and mode are very similar**.

For **right-skewed** histograms the **mean is generally higher than the median which is generally higher than the mode**.

For **left-skewed** histograms the **mean is generally lower than the median which is generally higher than the mode**.

For **bimodal** (or multimodal) histograms the **location of the mean and median will depend on the position of the modes**.

EXERCISE 16F

1. For the following sets of data:

- (i) construct a frequency table
- (ii) draw the histogram
- (iii) find the mean (one decimal place)
- (iv) find the median
- (v) find the mode/s
- (vi) state if the histogram is symmetric, right-skewed, left-skewed or bimodal.

- (a) 22, 23, 24, 25, 25, 26, 26, 27, 27, 27, 28, 28, 28, 28, 28, 29, 29, 29, 29, 29, 29, 29, 30, 30, 30, 30, 30, 30, 30, 30, 30, 31, 31, 31, 31, 31, 31, 31, 32, 32, 32

- (b) 8, 8, 9, 9, 9, 10, 10, 10, 10, 10, 11, 11, 11, 12, 12, 13, 13, 13, 14, 14, 14, 14, 14, 15, 15, 15, 16, 17

- (c) 20, 19, 22, 19, 20, 21, 23, 21, 20, 22, 21, 21, 22, 24, 22, 21, 22, 22, 23, 22, 23, 23, 23, 24, 22, 23, 24, 20, 24, 25, 19, 25, 25

- (d) 63, 54, 64, 54, 65, 54, 61, 59, 57, 57, 56, 65, 65, 63, 63, 57, 55, 57, 57, 55, 58, 55, 65, 65, 55, 64, 64, 61, 66, 67, 57, 65, 56, 58, 58, 56, 56, 67, 56, 64, 66, 66, 61, 61, 62, 64, 65, 63, 57, 57, 57, 58, 59, 59, 60, 60, 62, 63, 64, 65, 65, 66, 66, 67

2. A group of students were asked how many hours they slept each night. The results are shown below.

8, 3, 9, 7, 7, 6, 5, 9, 9, 8, 8, 7, 5, 8, 8,
4, 7, 8, 4, 9, 7, 6, 8, 8, 10, 6, 6, 7, 6, 10,
7, 3, 7, 7, 8, 5, 8, 8, 8, 5, 8, 9, 9, 4, 9, 9

- (a) Construct a frequency table and histogram to display this data.
 (b) Find:
 (i) the mean
 (ii) the median
 (iii) the mode
 (c) state if the histogram is symmetric, right-skewed, left-skewed or bimodal.
3. The wave height (to the nearest metre) at a popular surfing beach was measured each day for a number of days. The results are shown below.

3, 4, 1, 0, 2, 0, 3, 3, 4, 4, 0, 1, 2, 2, 1, 0, 3, 5,
1, 3, 3, 1, 1, 2, 2, 1, 6, 1, 5, 7, 8, 3, 3, 1, 1, 2, 2,
2, 2, 3, 2, 4, 2, 3, 5, 5, 2, 4, 4, 2, 6, 6, 7, 3, 8

- (a) Construct a frequency table and histogram to display this data.
 (b) Find:
 (i) the mean
 (ii) the median
 (iii) the mode
 (c) state if the histogram is symmetric, right-skewed, left-skewed or bimodal.



Stem-and-Leaf Plots

Stem-and-leaf plots are another way of displaying numerical data so it can be easily analysed.

The example below shows the steps involved with constructing stem-and-leaf plots.

Example

1. Construct a non-ordered stem-and-leaf plot for the data below.
2. Construct an ordered stem-and-leaf plot.
3. Find the mean and median of the data.

24, 35, 26, 48, 51, 17, 34, 26, 28, 40, 20, 19, 22, 37, 41, 49, 51, 53

1. To construct a non-ordered stem-and-leaf plot follow these steps:

Step 1 Identify the smallest and largest numbers.

In the above data these are **17** and **53**.

Step 2 The **stems** in the stem-and-leaf plot are the first digits.

In this example the stems range from **1** (the smallest) to **5** (the largest).

Step 3 The stem-and-leaf plot can now be started and the stems included - all numbers from 1 to 5.

Stem	Leaf
1	
2	
3	
4	
5	

Step 4 The leaves are the second digits.

These are now added to the stem-and-leaf plot in the order the data is given.

The first number is 24 so the stem is 2 and the leaf is 4.

Stem	Leaf
1	
2	4
3	
4	
5	

Step 5 Complete the stem-and-leaf plot by adding all the other numbers. The next number is 35 so the stem is 3 and leaf is 5, etc..

Stem	Leaf
1	7 9
2	4 6 6 8 0 2
3	5 4 7
4	8 0 1 9
5	1 1 3

$$\overline{1|7} = 17$$

This key is added as an example to show the number represented by the stem and leaf

Ordered Stem-and-leaf Plot

Step 6 The above stem-and-leaf plot is **non-ordered** because the numbers are not in numerical order. The **ordered** stem-and-leaf plot shows all the numbers in numerical order.

Stem	Leaf
1	7 9
2	0 2 4 6 6 8
3	4 5 7
4	0 1 8 9
5	1 1 3

$$\overline{1|7} = 17$$

Step 7 The mean can be calculated. There are 18 items of data.

$$\text{Mean} = \frac{17+19+20+22+24+26+26+28+34+35+37+40+41+48+49+51+51+53}{18}$$

$$\text{Mean} = 34.5$$

Step 8 The median can be found using the ordered stem-and-leaf plot. There are 18 items of data so the median will be midway between the 9th and 10th items (34 and 35).

$$\text{Median} = 34.5$$

Stem	Leaf
1	7 9
2	0 2 4 6 6 8
3	4 5 7
4	0 1 8 9
5	1 1 3

EXERCISE 16G

1. For the following ordered stem-and-leaf plots find:

- (i) the largest number
- (ii) the smallest number
- (iii) the range of numbers (largest - smallest)
- (iv) the mean (one decimal place)
- (v) the median

(a)

Stem	Leaf
1	6 8 9
2	0 1 3 5 7 8
3	1 3 4 6 8
4	0 2 5 7 9
5	0 1 1 3

$$1\overline{6} = 16$$

(b)

Stem	Leaf
3	0 1 2 3 8
4	1 2 2 4 6 9
5	2 3 6 8
6	0 4 4 7 9 9
7	1 2 3 3 7
8	0 2 6

$$3\overline{0} = 30$$

(c)

Stem	Leaf
0	2 3 6 8
1	0 4 5 5 8 9
2	1 2 4 6 7 7 8
3	0 4 6 7 8
4	1 1 2 4

$$0\overline{2} = 0.2$$

2. Construct ordered stem-and-leaf plots for the following non-ordered stem-and-leaf plots.

(a)

Stem	Leaf
2	3 0 9 8
3	6 2 5 7 0 1
4	9 0 3 5 2 8
5	1 3 0 7 4 2
6	9 0 2 1 0

$$2\overline{6} = 26$$

(b)

Stem	Leaf
7	9 0 2
8	3 5 2 0 8
9	0 7 2 4 1
10	5 1 5 2 1
11	8 0 1 5 6 5
12	7 2 1 7 3

$$7\overline{0} = 70$$

(c)

Stem	Leaf
4	7 6 5 3 9
5	2 7 1 6 3 2 8
6	9 0 2 5 2 1 1
7	6 5 9 2 3 1
8	5 3 7 0 1 5 5

$$4\overline{5} = 4.5$$

3. For the following sets of numbers:

- (i) construct a non-ordered stem-and-leaf plot
- (ii) construct an ordered stem-and-leaf plot
- (iii) calculate the mean (one decimal place)
- (iv) find the median

(a)

24, 15, 36, 40, 12, 25, 19, 36, 48, 41, 30, 22, 20, 37, 49, 29, 30

(b)

35, 48, 49, 56, 78, 81, 64, 80, 70, 40, 58, 59, 64, 39, 31, 42, 56

4. A number of spectators at a tennis match were asked their age.
The results are given below.

25, 45, 31, 37, 36, 29, 28, 43, 19, 18, 24, 48, 36, 27, 26, 37, 17, 42,
51, 28, 36, 45, 40, 20, 28, 37, 25, 38, 41, 20, 23, 33, 45, 29, 27, 21

- (a) Construct a non-ordered stem-and-leaf plot for these ages.
- (b) Use this to construct an ordered stem-and-leaf plot.
- (c) What is the age of the youngest person asked?
- (d) What is the age of the oldest person asked?
- (e) How many people were asked their age?
- (f) Calculate the mean age (one decimal place).
- (g) Find the median age.

5. The ages when a number of people married are shown below:

25, 31, 34, 23, 40, 25, 27, 29, 30, 19, 27, 39, 35, 33, 24, 27, 20, 21

- (a) Construct a non-ordered and ordered stem-and-leaf plot for this data.
- (b) Find the range for the ages.
- (c) Find the mean age when these people married (one decimal place).
- (d) Find the median age when these people married.

Two sets of data can be displayed next to each other on a **back-to-back stem-and-leaf plot**.

Example

The minimum and maximum temperatures for 10 days in a town are listed below.

Minimum temp. (°C) 10, 12, 15, 8, 9, 16, 18, 11, 21, 20

Maximum temp. (°C) 21, 22, 27, 18, 31, 35, 22, 41, 33, 30

These can be displayed on a **non-ordered** back-to-back stem-and-leaf as shown here.

Minimum Temp (°C)		Maximum Temp (°C)
Leaf	Stem	Leaf
9 8	0	
1 8 6 5 2 0	1	8
0 1	2	1 2 7 2
	3	1 5 3 0
	4	1

These can be then be arranged to form an **ordered** back-to-back stem-and-leaf as shown here.

Note the data on the left side start on the right end next to the stem.

Minimum Temp (°C)		Maximum Temp (°C)
Leaf	Stem	Leaf
9 8	0	
8 6 5 2 1 0	1	8
1 0	2	1 2 2 7
	3	0 1 3 5
	4	1

6. The ages of the players in two cricket teams are listed below.

The Slow Overs: 38, 42, 51, 34, 47, 42, 52, 50, 39, 48, 51
The Rushed Singles: 19, 20, 19, 22, 25, 30, 31, 19, 24, 18, 26

- Construct a non-ordered and ordered back-to-back stem-and-leaf plot for this data.
- Find the range of ages for each team.
- Find the mean age for each team. (one decimal place)
- Find the median age for each team.

7. The heights (in cm) of the players in two basketball teams are listed below.

Slammers: 198, 186, 178, 201, 185, 190, 200, 205, 189, 192
Swish: 200, 185, 184, 177, 179, 190, 186, 191, 186, 180

- Construct a non-ordered and ordered back-to-back stem-and-leaf plot for this data.

Use a stem and leaf of: $\overline{18} | \overline{3} = 183$

- Find the mean height for each team. (one decimal place)
- Find the median height for each team.

8. Charles recorded all his test results (as percentages) in mathematics for Year 7 and Year 8. The results are shown below.

Year 7: 45, 53, 39, 61, 54, 60, 52, 62, 48, 57
Year 8: 67, 83, 89, 79, 90, 75, 68, 73, 82, 70

- Construct a non-ordered and ordered back-to-back stem-and-leaf plot for this data.
- Find the range of test results in each year.
- Find the mean test result for each year. (one decimal place)
- Find the median test result for each year.
- Did getting a Mathematics tutor in Year 8 help him?

PROBLEM SOLVING

1. Lucien averaged 20 points a game over the season for his basketball team. His average for his best three games was 27 points and for the other games he averaged 17 points.
 - (a) How many games were in the season?
 - (b) If he scored 28 points in each of his two best games, how many points did he score in his third best game?

2. Tarzan has four pet snakes - Smooth, Scaley, Slick and Skinny.

Smooth is 0.75 metres longer than Slick.
 Smooth and Slick together measure 4.45 metres.
 Smooth is shorter than Skinny.
 The mean length of the four snakes is 2.35 metres.
 The median length of the four snakes is 2.45 metres.
 Find the length of each snake.

PUZZLES

1. Change **DATA** into **FACT** in four steps by changing one letter at a time to form a new word.

Example

CAT can be changed to **DOG** in three steps

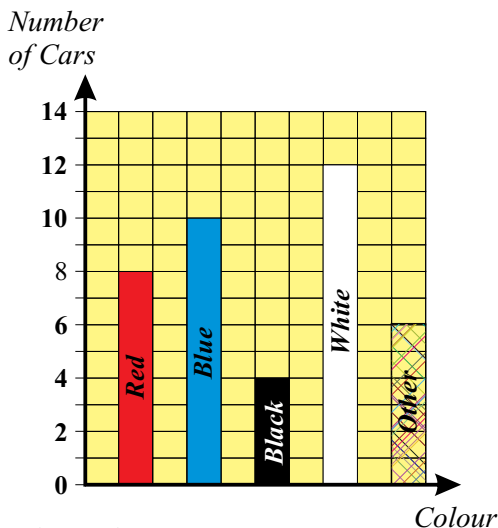
CAT
COT
DOT
DOG

2. Rearrange the letters from the following phrases to make words or phrases from this chapter. (Words may be repeated)

(a) SLAP ME (c) SCOUT UNION (e) A FIRM NOTION (g) A GOAT CIRCLE (i) LUNAR MICE (k) NICE MURAL	(b) LONG CAR HUMP (d) VERY US (f) PUT IN A POOL (h) TREE DISC (j) A COSY NERD (l) A GRIM SHOT
--	--

CHAPTER REVIEW

1. The colour of a number of cars was recorded and displayed on this column graph.



- (a) How many cars had their colour recorded?
- (b) What was the most popular colour?
- (c) What was the least popular colour?
- (d) What fraction of the cars were blue?
- (e) If 80 cars were observed, how many could be expected to be red?

2. For the following sets of data find:

- (i) the mean (one decimal place)
- (ii) the median
- (iii) the range

(a) 21, 20, 15, 32, 33, 27, 28, 19, 18, 22, 23, 24, 29

(b) 37, 33, 45, 50, 29, 36, 41, 50, 42, 40, 39, 27, 39, 48, 46, 30, 55, 37

3. For the following set of data:

- (a) construct a dot plot
- (b) construct a frequency table
- (c) construct a histogram
- (d) find the mean (one decimal place)
- (e) find the median
- (f) find the mode
- (g) is the data symmetric, right-skewed, left-skewed or bimodal?

19, 15, 14, 22, 17, 14, 13, 15, 15
 13, 14, 16, 14, 14, 24, 14, 15, 15
 15, 15, 16, 16, 15, 16, 14, 17, 16
 17, 18, 15, 18, 19, 13, 20, 15, 15

4. The heights of 15 eucalyptus trees (in metres) in two sections of a large national park are shown below.

Ferny Gully: 32, 25, 33, 27, 41, 29, 30, 40, 42, 36, 33, 32, 28, 31, 26
Bald Hill: 12, 22, 24, 20, 19, 15, 25, 19, 15, 22, 25, 28, 18, 19, 17

- (a) Construct a non-ordered and ordered back-to-back stem-and-leaf plot for this data.
- (b) For each set of trees find:
- (i) the mean height (one decimal place)
 - (ii) the median height
 - (iii) the range of heights



Answers

Chapter 1

Exercise 1A

- six hundred and fifty-one
 - nine hundred and eighteen
 - six thousand and five
 - four thousand, two hundred and eighty-three
 - nineteen thousand and forty-seven
 - fifty thousand and forty
 - seventy-five thousand, eight hundred and six
 - one hundred and three thousand, nine hundred and sixty-seven
 - seven hundred thousand, seven hundred
 - eight hundred and twenty-three thousand and nine
 - three million, fifty-seven thousand and forty-one
 - eleven million, eight hundred and twenty-one thousand, six hundred and fifteen
- 657 (b) 3095 (c) 24 801 (d) 700 467 (e) 2 084 079 (f) 394 650 820
- 106 (b) 891 (c) 1056 (d) 8171 (e) 63 567 (f) 5052 (g) 60 001 (h) 36 000 099
- 7567 (b) 63 200 (c) 9003 (d) 699 508 (e) 49 971 (f) 2 299 157 (g) 8 039 222 (h) 4 999 000
- 12 (b) 13 (c) 100 (d) 144 (e) 1000
 - 1 with one hundred zeros after it (g) 20
 - one million million (one thousand million in US and commonly when referring to large amounts of money)

- 1 with 18 zeros (1 with 12 zeros in US)
- 20 (b) 80 (c) 10 (d) 100 (e) 230 (f) 400 (g) 41 000 (h) 130 000
- 100 (b) 100 (c) 0 (d) 800 (e) 1000 (f) 3300 (g) 19 000 (h) 190 000
- 7000 (b) 2000 (c) 1000 (d) 11 000 (e) 33 000 (f) 107 000 (g) 100 000 (h) 1 930 000
- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71 (b) 97 (c) 997 (d) 71, 73

Exercise 1B

- 72 (b) 42 (c) 60 (d) 121 (e) 7 (f) 7 (g) 3 (h) 30 (i) 69 (j) 155 (k) 400 (l) 432 (m) 51 (n) 160 (o) 188 (p) 573 (q) 23 (r) 67 (s) 64 (t) 178

2.

×	3	5	7	11	9	12	6
2	6	10	14	22	18	24	12
10	30	50	70	110	90	120	60
8	24	40	56	88	72	96	48
4	12	20	28	44	36	48	24
9	27	45	63	99	81	108	54
7	21	35	49	77	63	84	42
6	18	30	42	66	54	72	36

- 14 (b) 10 (c) 0 (d) 0 (e) 42 (f) 6 (g) 3 (h) 11 (i) 24 (j) 15 (k) 15
- 2^4 (b) $2^2 \times 3^2$ (c) $2^3 \times 5^2$ (d) $2^4 \times 3^2$ (e) $2^2 \times 3 \times 5^2$ (f) $2 \times 3 \times 13$ (g) $2^3 \times 5^3$ (h) 19×23 (i) $2^3 \times 3^2 \times 13$ (j) 2^{12} (k) 23^2 (l) 3×37

Exercise 1C

1. (a) 24 (b) 57 (c) 99 (d) 1666
 (e) 79 (f) 295 (g) 954 (h) 3222
 (i) 154 (j) 901 (k) 2010 (l) 444
2. (a) XXXVI (b) LXXXIV
 (c) CCL (d) DCCXCIII
 (e) MMMCDVIII (f) MCXI
 (g) MMCDXLIX
 (h) CMXXXVIII (i) CCCXXXIII
 (j) MMMDCCCLXXVII
 (k) MMXX (l) DCLXXII
- 3.

**Exercise 1D**

1. (a) 4 (b) 10 (c) 6 (d) 12 (e) 5
 (f) 8 (g) 7 (h) 1 (i) 16 (j) 49
 (k) 23 (l) 625
2. (a) 9 (b) 25 (c) 16 (d) 100
 (e) 8 (f) 1000 (g) 49 (h) 14
 (i) 32 (j) 100 000 (k) 121 (l) 7

Exercise 1E

1. (a) 200 (b) 3000 (c) 20 000
 (d) 600 (e) 3000 (f) 140 000
 (g) 90 000 (h) 480 000 (i) 20
 (j) 5 (k) 600 (l) 70 (m) 50
 (n) 40 (o) 20 (p) 30
2. (a) $70 \times 20 = 1400$
 (b) $60 \times 100 = 6000$
 (c) $220 \times 10 = 2200$
 (d) $70 \times 110 = 7700$
 (e) $90 \div 30 = 3$
 (f) $720 \div 90 = 8$
 (g) $640 \div 80 = 8$
 (h) $540 \div 60 = 9$
3. (a)(i) $40 \times 120 = 4800$ emus
 (ii) 4636 emus
 (b)(i) $30 \times 50 + 20 \times 10 = 1700$
 (ii) 1798

Exercise 1F

1. 7 2. 7 3. 10 4. 1
 5. 7 6. 0 7. 1

Exercise 1G

1. 1354
2. Sally is 18, Fiona is 12
3. (a) **A** = - , **B** = $\times$, **C** = + , **D** = $\div$
 (b)(i) 6 (ii) $^{-}13$
4. $1 = 3 + 2 - 4 \times 1$
 $2 = 3 \times 2 - 4 \times 1$
 $3 = (4 - 1) \times (3 - 2)$
 $4 = 4 \times 1 \times (3 - 2)$
 $5 = (4 + 1) \times (3 - 2)$
 $6 = (4 \times 2) - 3 + 1$
 $7 = (3 + 4) \times (2 - 1)$
 $8 = 4 + 3 + 2 - 1$
 $9 = 4 + 3 + (2 \times 1)$
 $10 = 1 + 2 + 3 + 4$
 $11 = 4 \times 2 + 3 \times 1$
 $12 = 4 \times 3 \times (2 - 1)$
 $13 = 4 \times 3 + 2 - 1$
 $14 = 4 \times 3 + 2 \times 1$
 $15 = (4 + 2 - 1) \times 3$
 $16 = 4 \times 2 \times (3 - 1)$
 $17 = (4 + 2) \times 3 - 1$
 $18 = (4 + 2) \times 3 \times 1$
 $19 = (4 + 2) \times 3 + 1$
 $20 = (3 + 2) \times 4 \times 1$

5.

$$\begin{array}{r}
 \boxed{3} \ 7 \ 9 \ 8 \\
 + \quad 6 \ \boxed{4} \ 1 \ \boxed{5} \\
 \hline
 \boxed{1} \ \boxed{0} \ \boxed{2} \ 1 \ 3
 \end{array}$$

6. O = E, 1 = R, 2 = T, 3 = S, 4 = A,
 5 = H, 6 = M

7.

$$\begin{array}{r}
 1 \ \boxed{3} \ 0 \ \boxed{6} \ 1 \ \boxed{7} \\
 - \quad 9 \ \boxed{7} \ 2 \ \boxed{8} \ 4 \\
 \hline
 3 \ 3 \ 3 \ 3 \ 3
 \end{array}$$

8. Arthur is 52, Martha is 39

9. There may be more than one solution.

- (a) $2 \times 55c, 1 \times 35c$
 (b) $4 \times 45c$
 (c) $2 \times 35c, 3 \times 55c$
 (d) $3 \times 35c, 5 \times 55c$
 (e) $2 \times 35c, 5 \times 45c$
 (f) $3 \times 35c, 6 \times 55c$

Chapter Review

1. (a) Six hundred and four thousand, two hundred and ninety-five
 (b) 156 708
 2. (a) 700 (b) 1900 (c) 24 000 (d) 0
 3. (a) 54 (b) 84 (c) 9 (d) 7 (e) 108 (f) 211 (g) 122 (h) 178
 4. (a) 10 (b) 26 (c) 12
 5. $2^2 \times 3^3 \times 5$
 6. (a) LXXVIII (b) CCCXCIV (c) CDLXXXIX (d) MMDCXXIV
 7. (a) 12 (b) 10 (c) 14 (d) 9 (e) 9 (f) 10 000 (g) 49 (h) 11
 8. (a) 900 (b) 8000 (c) 20 (d) 200
 9. (a) $30 \times 30 = 900$
 (b) $80 \times 50 = 4000$
 (c) $400 \div 20 = 20$
 (d) $640 \div 80 = 8$
 10. 1
 11. 12, 12, 24, 72, 288, **1440, 8640, 60 480**

Chapter 2

Exercise 2A

1. (a) $\frac{3}{7}$ (b) $\frac{9}{20}$ (c) $\frac{4}{9}$ (d) $\frac{12}{25}$
 2. (a) $\frac{4}{7}$ (b) $\frac{11}{20}$ (c) $\frac{5}{9}$ (d) $\frac{13}{25}$
 3. (a) $\frac{5}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{7}{8}$ (e) $\frac{4}{5}$
 (f) $\frac{41}{48}$
 4. (a) $\frac{12}{48} = \frac{1}{4}$ (b) $\frac{8}{12} = \frac{2}{3}$
 5. Examples are:



6. (a) $\frac{16}{20}$ (b) $\frac{8}{28}$ (c) $\frac{56}{64}$ (d) $\frac{45}{81}$
 (e) $\frac{3}{4} = \frac{12}{16} = \frac{15}{20} = \frac{21}{28} = \frac{33}{44} = \frac{27}{36} = \frac{60}{80}$
 7. (a) $\frac{3}{4} = \frac{21}{28}$, $\frac{5}{7} = \frac{20}{28}$, $\frac{5}{7} = \frac{3}{4}$
 (b) $\frac{5}{12} = \frac{10}{24}$, $\frac{3}{8} = \frac{9}{24}$, $\frac{1}{3} = \frac{8}{24}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{12}$
 (c) $\frac{5}{9} = \frac{20}{36}$, $\frac{3}{4} = \frac{27}{36}$, $\frac{7}{12} = \frac{21}{36}$, $\frac{5}{9}$, $\frac{7}{12}$, $\frac{3}{4}$
 (d) $\frac{2}{3} = \frac{48}{72}$, $\frac{5}{8} = \frac{45}{72}$, $\frac{7}{9} = \frac{56}{72}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{7}{9}$
 8. (a) $\frac{10}{200} = \frac{1}{20}$ (b) $\frac{8}{40} = \frac{1}{5}$ (c) $\frac{200}{3000} = \frac{1}{15}$
 (d) $\frac{60}{500} = \frac{3}{25}$ (e) $\frac{400}{6000} = \frac{1}{15}$ (f) $\frac{50}{2000} = \frac{1}{40}$
 (g) $\frac{25}{50} = \frac{1}{2}$ (h) $\frac{520}{8000} = \frac{13}{200}$ (i) $\frac{90}{360} = \frac{1}{4}$
 9. (a) $\frac{7}{2}$ (b) $\frac{13}{5}$ (c) $\frac{17}{3}$ (d) $\frac{25}{4}$ (e) $\frac{29}{6}$
 (f) $\frac{71}{8}$ (g) $\frac{66}{7}$ (h) $\frac{91}{12}$
 10. (a) $4\frac{1}{2}$ (b) $4\frac{1}{3}$ (c) $5\frac{2}{5}$ (d) $6\frac{3}{10}$
 (e) $8\frac{1}{4}$ (f) $6\frac{5}{6}$ (g) $8\frac{4}{9}$ (h) $6\frac{5}{8}$
 (i) $13\frac{6}{7}$ (j) $24\frac{1}{6}$

Exercise 2B

1. (a) 4 (b) 8
 2. (a) 4 (b) 8 (c) 12 (d) 16
 3. (a) 20 kg (b) \$30 (c) 30 m (d) 150 L (e) \$210 (f) 30 cm (g) 240 kg (h) \$90 (i) 100 g
 4. (a) $\frac{7}{9}$ (b) $\frac{3}{11}$ (c) $\frac{13}{15}$ (d) $\frac{5}{8}$ (e) $\frac{17}{24}$
 (f) $\frac{1}{4}$ (g) $1\frac{4}{5}$ (h) $1\frac{1}{2}$ (i) $6\frac{1}{10}$ (j) $1\frac{11}{12}$
 (k) $7\frac{5}{6}$ (l) $2\frac{19}{20}$ (m) $14\frac{31}{40}$ (n) $\frac{37}{56}$
 (o) $1\frac{3}{8}$
 5. (a) $\frac{24}{35}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $6\frac{3}{5}$ (e) $5\frac{5}{7}$
 (f) $11\frac{1}{4}$ (g) $7\frac{1}{2}$ (h) $11\frac{1}{4}$ (i) $2\frac{1}{2}$
 (j) $38\frac{1}{2}$ (k) $58\frac{1}{2}$ (l) 12
 6. (a) 10 (b) $\frac{1}{24}$ (c) $\frac{2}{49}$ (d) $\frac{5}{6}$ (e) $1\frac{1}{2}$
 (f) $\frac{2}{3}$ (g) $\frac{1}{12}$ (h) $1\frac{1}{2}$ (i) $2\frac{1}{3}$
 7. (a) 22 (b) 23 (c) 29 (d) 100
 (e) 15 (f) $2\frac{2}{5}$ (g) $\frac{5}{8}$

Exercise 2C

1. $71\frac{1}{2}$
 2. $2\frac{101}{176}$
 3. \$2414
 4. \$23 076
 5. 5
 6. $1\frac{201}{280}$
 7. $63\frac{89}{162}$
 8. $11\frac{7}{10}$
 9. $1\frac{31}{170}$

Exercise 2D

- \$9
- 10 litres
- (a) 90 seconds (b) 285 seconds
- Length 1 = 64 cm
Length 2 = 48 cm
Length 4 = 27 cm
- $\frac{2}{3}$
- 120 mice
- $\frac{2}{3}$
- $\frac{1}{6}$
- (a) 6 hours (360 minutes)
(b) 30 rose bushes
- (a) $\frac{81}{100}$ (b) 2430 kg
- $\frac{1}{11}$
- 480 litres
- 3456 koalas
- (a) $\frac{5}{11}$ (b) $\frac{2}{11}$
- 24 students

Chapter Review

- $\frac{6}{8} = \frac{3}{4}$
- $\frac{3}{5} = \frac{9}{15} = \frac{6}{10} = \frac{18}{30} = \frac{36}{60} = \frac{48}{80}$
- $\frac{2}{3} = \frac{20}{30}$, $\frac{3}{5} = \frac{18}{30}$, $\frac{1}{2} = \frac{15}{30}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{2}{3}$
- (a) $\frac{8}{200} = \frac{1}{25}$ (b) $\frac{50}{300} = \frac{1}{6}$ (c) $\frac{600}{3000} = \frac{1}{5}$
- $\frac{59}{8}$
- $11\frac{2}{7}$
- (a) \$40 (b) 120 kg (c) $1\frac{4}{15}$ (d) $\frac{13}{72}$
(e) $4\frac{5}{8}$ (f) $\frac{30}{77}$ (g) $\frac{5}{44}$ (h) 12 (i) 10
- 38
- $24\frac{47}{56}$
- (a) $\frac{9}{20}$ (b) 1600 people

Chapter 3**Exercise 3A**

- (a) 1.54 (b) 11.842
- 1.7
- 123.456789
- 3.87
- 17.835
- (a) 6.23 (b) 1.242 (c) 0.0365
- 8.8918
- False
- 34.7779

10. 200, 20, 4, 1.2, 0.48, **0.24, 0.144**

11. 0.089

12. Two hundred and two ten-thousandths

Exercise 3B

1.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{7}{10}$	$\frac{1}{100}$	$\frac{9}{100}$	$\frac{23}{100}$	$\frac{53}{100}$	$\frac{1}{1000}$
Decimal	0.5	0.25	0.75	0.1	0.3	0.7	0.01	0.09	0.23	0.53	0.001

- (a) 0.07 (b) 0.009 (c) 0.17 (d) 0.011
(e) 6.9 (f) 12.029 (g) 3.157
(h) 0.0021 (i) 0.0009 (j) 23.0579
- (a) $\frac{3}{1000}$ (b) $\frac{39}{1000}$ (c) $\frac{2357}{10\ 000}$ (d) $\frac{591}{10\ 000}$
(e) $4\frac{31}{100}$ (f) $\frac{7}{100\ 000}$ (g) $8\frac{92}{10\ 000}$ (h) $5\frac{2001}{10\ 000}$
(i) $\frac{59}{100\ 000}$ (j) $9\frac{43431}{1\ 000\ 000}$
- (a) 0.375 (b) 7.125 (c) 0.6
(d) 11.8 (e) 0.55 (f) 0.5625
(g) 6.1875 (h) 11.72 (i) 9.195
(j) 2.826
- (a) $0.\dot{3}$ (b) $2.\dot{8}\dot{3}$ (c) $0.\dot{2}$ (d) $13.\overline{27}$
(e) 0.86 (f) 0.846153
(g) 5.857142 (h) 9.904761
(i) $3.\overline{528}$ (j) 2.8816

Exercise 3C

- (a) 0.4 (b) 6.7 (c) 0.1 (d) 12.0
(e) 13.9 (f) 0.6 (g) 1.0 (h) 16.0
- (a) 0.79 (b) 10.51 (c) 9.57
(d) 90.91 (e) 124.57 (f) 6.60
(g) 130.00 (h) 67.91
- (a) 0.446 (b) 9.605 (c) 0.068
(d) 100.803 (e) 8.106 (f) 45.020
(g) 7.990 (h) 0.100

Exercise 3D

- (a) 5.43×10^6 (b) 6.0901×10^7
(c) 2.3×10^8 (d) 4.1×10^3
(e) 7×10^9 (f) 6×10^{-4}
(g) 6.754×10^{-5} (h) 7.845×10^{-4}
(i) 7.804×10^{-4} (j) 5.6×10^{10}
(k) 7.7×10^{-8} (l) 8.971×10^{-4}

2. (a) 1.496×10^8 km
 (b) 2.279×10^8 km
 (c) 7.83×10^7 km
 (d) 2×10^6 m
 (e) 1.5×10^{-4} m
 (f) 1.9×10^5 kg
 (g) 1.5×10^7 °C

Exercise 3E

1. 8794.2 2. 566 000 3. 0.6
 4. 7.85 5. 8900 6. 45 000
 7. 0.0452 8. 45.046 9. 0.3
 10. 1.04026 11. 0.0005678
 12. 0.0005

Exercise 3F

1. Fleet - 11.9 seconds
 Dan - 12.4 seconds
 2. Jodie - 27.85 seconds
 Jessie - 27.97 seconds
 Jacqui - 28.17 seconds
 3. 30.04 seconds
 4. 1.75 m and 2.15 m
 5. (a) (i) 27.75 km/hour
 (ii) 15.725 km/hour
 (iii) 51.8 km/hour
 (b) (i) 10.8 knots
 (ii) 18.9 knots
 (iii) 24.9 knots
 (c) 27 hours
 6. (a) 40 (b) 41.7 grams
 7. 0.004 seconds
 8. (a) 43.3 (b) 0.0231 seconds
 9. 600 km
 10. Kate - 8.2353 m/s
 Mike - 8.2372 m/s
 Skye - 8.2389 m/s
 Skye, Mike, Kate
 11. (a) 0.9 litres
 (b) Container A - 0.6 litres
 Container B - 0.45 litres
 (c) Container A - 0.3 litres
 Container B - 0.45 litres

12. 3.9 litres

13. (a) (i) 2.004546×10^{-26}
 (ii) 2.672728×10^{-26}
 (iii) 1.67×10^{-27}
 (b) 4.98866×10^{25}

14. (a) 10.05c (b) 150.75c (c) 5.36c
 (d) 13.40c (e) 2.23c

Chapter Review

1. 6.173
 2. (a) 0.25 (b) 0.875 (c) 5.6
 (d) 0.009 (e) 17.19
 3. (a) $\frac{4}{10} (\frac{2}{5})$ (b) $6\frac{9}{100}$ (c) $3\frac{197}{10\ 000}$
 (d) $15\frac{217}{1000}$ (e) $8\frac{7}{100\ 000}$
 4. (a) $0.\dot{6}$ (b) $3.1\dot{6}$ (c) $7.\dot{7}$
 (d) $3.\overline{428571}$ (e) $0.0\dot{6}$
 5. (a) 6.24 (b) 0.10 (c) 18.40
 (d) 4.79 (e) 29.99
 6. (a) 4.317 (b) 1.094 (c) 11.900
 (d) 83.218
 7. (a) 7.82×10^4 (b) 3×10^5
 (c) 1.72×10^{-3} (d) 4.2×10^{-6}
 8. (a) 3490 (b) 1.24 (c) 56.245
 (d) 6.2 (e) 0.008341 (f) 0.000973
 9. 250
 10. 6 cm
 11. (a) 2.469 acres (b) 12.15 hectares

Chapter 4

Exercise 4A

1. (a) 20% (b) 28% (c) 36%
 2. (a) 80% (b) 72% (c) 64%
 3. (a) C (b) B (c) A
 4. (a) 45.5% (b) Kirk
 5. 13%
 6. 1.41%
 7. Aaron - 10%, Clara - 20%,
 Barton - 30%, Felicity - 40%
 8. Munchers - 65%, Bashers - 35%
 9. Local - 49%, interstate - 33%,
 overseas - 18%

Exercise 4B

1. (a) 10% (b) 30% (c) 25%
 (d) 50% (e) 20% (f) 75%
 (g) 35% (h) 26% (i) 48%
 (j) 12.5% (k) 43.75%
 (l) 118.75% (m) 285%
 (n) 137.5% (o) 325.6%
2. (a) $\frac{11}{50}$ (b) $\frac{19}{20}$ (c) $\frac{3}{25}$ (d) $1\frac{1}{5}$ (e) $\frac{1}{50}$
 (f) $\frac{31}{200}$ (g) $\frac{5}{8}$ (h) $\frac{83}{400}$ (i) $\frac{1}{500}$ (j) $\frac{53}{1000}$
3. (a) 50% (b) 62.5% (c) 60%

Exercise 4C

1. (a) 40% (b) 25% (c) 9% (d) 5%
 (e) 20% (f) 5% (g) 60% (h) 10%
 (i) 25% (j) 8% (k) 19% (l) 27%
2. Percentages - 86.7%
 Algebra - 77.6%
 Graphs - 80%
3. Right-handed - 83.7%
 Left-handed - 14.3%
 Ambidextrous - 2%
4. Private land - 62.7%
 Aboriginal & Torres Strait
 Islander land - 14.3%
 Public land - 23%

Exercise 4D

1. (a) 33% (b) 19% (c) 87%
 (d) 6% (e) 40% (f) 29.5%
 (g) 5.6% (h) 0.8% (i) 200%
 (j) 10.57%
2. (a) 0.59 (b) 0.24 (c) 0.76
 (d) 0.3 (e) 0.08 (f) 0.1 (g) 1
 (h) 0.654 (i) 0.058 (j) 0.0025

3.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
10%	$\frac{1}{10}$	0.1
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
90%	$\frac{9}{10}$	0.9
66%	$\frac{33}{50}$	0.66
18%	$\frac{9}{50}$	0.18
32%	$\frac{8}{25}$	0.32
85%	$\frac{17}{20}$	0.85
9%	$\frac{9}{100}$	0.09
3%	$\frac{3}{100}$	0.03
4%	$\frac{1}{25}$	0.04
$33\frac{1}{3}\%$	$\frac{1}{3}$	0. $\dot{3}$
$66\frac{2}{3}\%$	$\frac{2}{3}$	0. $\dot{6}$

Exercise 4E

1. (a) 4 m (b) \$10 (c) \$5 (d) 3 kg
 (e) 6 kg (f) 9 m (g) \$60
 (h) \$100 (i) 10 km (j) 25 kg
 (k) \$4000 (l) 100 kg (m) \$2.50
 (n) 30c (o) \$2.60
2. (a) 279 kg (b) 161.5 m
 (c) \$593.40 (d) \$1128
 (e) 3013.5 m (f) 0.385 m
 (g) \$48.75 (h) \$3570
 (i) \$10 799.25
3. \$140
4. (a) 1242 (b) 35 742
5. 33
6. (a) Scott - 69.6%, Lucien - 72.8%
 (b) Lucien
 (c) Scott - 42, Lucien - 44
7. (a) 2665 (b) 2732
8. 1 119 300 litres
9. 187 200
10. (a) Further training - 40%
 Tertiary - 45%
 Job - 15%
 (b) Further training - 392
 Tertiary - 441
 Job - 147

Exercise 4F

- (a) \$500 (b) 225 kg (c) \$1250
(d) 625 g (e) \$7000 (f) \$800
- 125 000
- (a) 400 mL (b) 380 mL
- 50 seconds

Exercise 4G

- (a) \$60 (b) \$340 (c) \$13 440
(d) \$1875 (e) \$32.30 (f) \$23.92
(g) \$71.50
- (a) \$150 (b) \$50 (c) \$500
(d) \$4.50 (e) \$250 (f) \$280
(g) \$480

Exercise 4H

- (a) \$30 (b) 25% (c) 20%
- (a) \$100 (b) 100% (c) 50%
- (a) \$200 (b) 57.1% (c) 36.4%
- (a) \$60 (b) 42.9% (c) 30%

Exercise 4I

- \$120
- \$509.60
- (a) \$2475 (b) \$8175 (c) \$855
- (a) Option A - \$367.50
Option B - \$495
(b) Option A - \$735
Option B - \$740
(c) Option A - \$1102.50
Option B - \$985
- \$494.55

Exercise 4J

- (a) \$420 (b) \$81 (c) \$2200
(d) \$2868.75 (e) \$270 (f) \$400
- (a)(i) \$600 (ii) \$4600
(b)(i) \$720 (ii) \$12 720
(c)(i) \$1562.50 (ii) \$4062.50
(d)(i) \$2220 (ii) \$4070
(e)(i) \$5250 (ii) \$25 250
(f)(i) \$268.125 (ii) \$6768.125

Chapter Review

- 29.2%

- | Fraction | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{5}{8}$ |
|------------|---------------|---------------|---------------|---------------|---------------|
| Decimal | 0.5 | 0.2 | 0.25 | 0.125 | 0.625 |
| Percentage | 50% | 20% | 25% | 12.5 | 62.5 |

- 62.9%
- (a) \$1575 (b) 11.84 m
- 1060
- \$72
- \$450
- 30%
- \$180
- \$900

Chapter 5**Exercise 5A**

- (a)(i) \$36 (ii) \$54 (iii) \$28.80
(iv) \$7.20
(b)(i) 2 kg (ii) 1.5 kg (iii) 1.2 kg
(iv) 0.4 kg
- (a)(i) \$3.60 (ii) \$0.60 (iii) \$12
(iv) \$19.20
(b)(i) 200 g (ii) 167 g
(iii) 417 g (iv) 875 g
- (a)(i) \$1.30 (ii) \$3.90 (iii) \$0.65
(iv) \$0.26
(b)(i) 0.77 kg (ii) 1.23 kg
(iii) 1.92 kg (iv) 9.23 kg
- (a) 20 tablets (b) 14 tablets
(c) 5 tablets
- 27.5 mL
- (a)(i) \$1.20 (ii) \$16 (iii) \$3.40
(b)(i) 5 minutes (ii) 3.2 minutes
(iii) 12.6 minutes
- 240 seconds
- (a)(i) 1400 kJ (ii) 2100 kJ
(iii) 500 kJ (iv) 2000 kJ
(b)(i) 300 g (ii) 400 g
(iii) 350 g (iv) 240 g
(c)(i) 3000 kJ (ii) 1050 kJ
(iii) 500 kJ (iv) 1500 kJ
(v) 420 kJ
(d) 45 minutes (e) 400 g (f) 50 g

9. (a)(i) 16 litres (ii) 4 litres
(iii) 6 litres (iv) 25.6 litres
(b)(i) 25 km (ii) 300 km
(iii) 250 km (iv) 625 km
10. (a) 3 hours (b) 4 hours
(c) 50 minutes (d) 66.7 minutes
(e) 40 bricks (f) 15 hours

Exercise 5B

1. 4.5 m/s
2. (a) 3150 m (b) 210 seconds
(c) 3.5 minutes
3. (a) 16.8 km/h (b) 4.7 m/s
(c) 280 m/min
4. 3 m/s
5. (a) 30 m/s (b) 10 seconds
(c) 360 m
6. (a) 31 557 600 seconds
(b) 29.8 km/s
7. (a) Pedro - 8.10 am
Iain - 8.25 am
(b) 9.05 am
8. 0.405 seconds

Exercise 5C

1. (a) DP (b) DP (c) DP
2. $M = 5N$
3. $P = 0.25Q$ or $P = \frac{1}{4}Q$
4. (a)(i) DP (ii) $b = 6a$
(b)(i) DP (ii) $W = \frac{1}{3}T$
(c)(i) NDP
(d)(i) DP (ii) $B = 9g$
(e)(i) DP (ii) $R = 0.6Q$ or $R = \frac{3}{5}Q$
(f)(i) NDP
(g)(i) DP (ii) $H = 2.5I$ or $H = \frac{5}{2}I$
(h)(i) DP (ii) $Y = 0.3j$ or $Y = \frac{3}{10}j$
5. (a) $A = 6H$
(b)(i) 30 (ii) 120 (iii) 48 (iv) 270
(v) 15 (vi) 91.2
(c)(i) 10 (ii) 15 (iii) 4 (iv) 30
(v) 12.6 (vi) 35.9

6. (a) $R = 8T$
(b)(i) 40 (ii) 24 (iii) 72 (iv) 120
(v) 196 (vi) 420.8
(c)(i) 10 (ii) 7.5 (iii) 1 (iv) 12
(v) 8.9 (vi) 15.3
7. (a) DP
(b) $P = 2.4Y$ or $P = \frac{12}{5}Y$
(c)(i) 7.2 (ii) 24 (iii) 84 (iv) 13.2
(v) 78.48

8. (a)

<i>N</i>	0	5	7	8.5	12.6	20.5
<i>M</i>	0	15	21	25.5	37.8	61.5

- (b)

<i>K</i>	5	15	24	32	36.5	41.5
<i>Q</i>	4	12	19.2	25.6	29.2	33.2

- (c)

<i>a</i>	0.4	6	7.4	8.8	12.5	15.3
<i>c</i>	1.8	27	33.3	39.6	56.25	68.85

- (d)

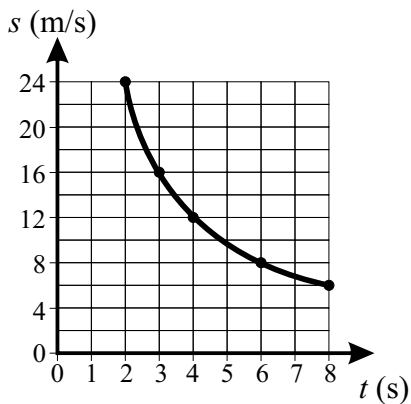
<i>b</i>	2.5	5.3	8.9	12	15.1	20.6
<i>T</i>	1.75	3.71	6.23	8.4	10.57	14.42

9. (a) Yes
(b) $V = 12I$
(c)(i) 96 volts (ii) 114 volts
(iii) 206.4 volts
(d)(i) 3 amps (ii) 14.5 amps
(iii) 24.7 amps
10. (a) $D = 2.2N$
(b) 3509 m
(c) 4545.5 revolutions
11. (a) $D = 7.5t$
(b)(i) 75 m (ii) 187.5 m (iii) 270 m
(c)(i) 6 min (ii) 15 min (iii) 26 min
(d) 40 min
(e) 37.5 min

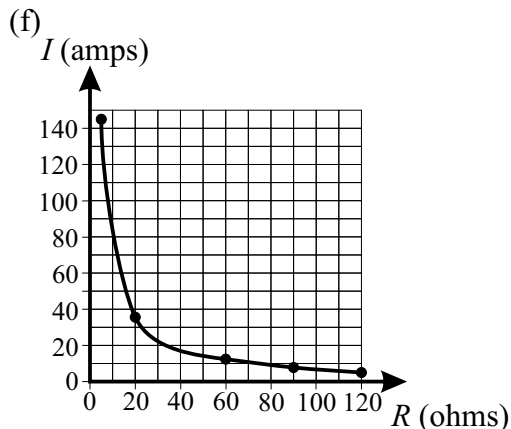
12. (a) $P = 2.5A$
 (b) $Y = 1.5A$
 (c)(i) 7500 kg (ii) 4500 tonnes
 (d)(i) \$17 500 (ii) \$262 500
 (e)(i) \$120 000 (ii) \$1 800 000

Exercise 5D

1. (a) Yes
 (b) $k = 48$
 (c) $s = \frac{48}{t}$
 (d) 3 m/s
 (e) 48 s
 (f)



2. (a) Yes
 (b) $k = 720$
 (c) $I = \frac{720}{R}$
 (d)(i) 360 amps (ii) 18 amps
 (iii) 2 amps
 (e)(i) 720 ohms (ii) 24 ohms
 (iii) 4 ohms



3. (a) Yes
 (b) $k = 480\,000$
 (c) $T = \frac{480\,000}{N}$
 (d)(i) 80 min (ii) 96 min
 (iii) 24 min
 (e) 3200 lights
 4. (a) Yes
 (b) $k = 7200$
 (c) $P = \frac{7200}{V}$
 (d)(i) 240 kPa (ii) 144 kPa
 (iii) 60 kPa
 (e)(i) 72 cm^3 (ii) 48 cm^3
 (iii) 40 cm^3

Exercise 5E

1. D
 2. I
 3. D
 4. I
 5. D
 6. I
 7. D

Chapter Review

1. (a)(i) \$1.60 (ii) \$5.60 (iii) \$6.80
 (b)(i) 10 kg (ii) 12.5 kg (iii) 3.75 kg
 2. (a) 13 min 20 s (b) 26 min 40 s
 (c) 1125 m

3. 30 km/h
 4. 25 min
 5. (a) NDP (b) DP (c) DP
 6. (b) $h = 4g$ (c) $z = \frac{1}{3}y$
 7. (a) $P = 5Q$
 (b)(i) 30 (ii) 95 (iii) 260
 (c)(i) 14 (ii) 26 (iii) 33.6
 (d) $k = 5$
 8. (a) $L = 1.5K$
 (b)(i) 27 (ii) 37.5 (iii) 14.4
 (iv) 56.7
 (c)(i) 16 (ii) 34 (iii) 19 (iv) 65.3
 9. (a)

<i>N</i>	0	5	6	9.5	12.5	20.5
<i>M</i>	0	20	24	38	50	82

(b)

<i>K</i>	2	4	21	25.5	30.5	40.2
<i>Q</i>	4.8	9.6	50.4	61.2	73.2	96.48

10. (a) Yes
 (b) $E = 3M$
 (c)(i) 1.2 cm (ii) 3.9 cm
 (iii) 8.1 cm
 (d)(i) 0.1 kg (ii) 1.4 kg
 (iii) 1.8 kg
 11. (a) Inversely proportional
 (b) $k = 6$
 (c) $a = \frac{6}{M}$
 (d)(i) 12 m/s^2 (ii) 5 m/s^2
 (iii) 2.5 m/s^2
 (e) 7.5 kg

Chapter 6

Exercise 6A

1. (a) $2 \times 2 \times 2 \times 2$
 (b) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
 (c) 10×10
 (d) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
 (e) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
 $8 \times 8 \times 8$

2. (a) 6^4 (b) 9^6 (c) 2^7 (d) 1^{10}
 3. (a) $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
 (b) $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
 (c) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^8$
 (d) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^8$
 (e) $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$
 (f) $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^8$
 (g) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^7$
 (h) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^{11}$

4. Add the indices to find the index of the answer.

5. (a) 6^{10} (b) 5^{12} (c) 4^{11} (d) 8^7 (e) 2^{15}
 (f) 13^{14} (g) 3^9 (h) 15^{11} (i) 9^{27}
 (j) 17^{22} (k) 11^{13} (l) 26^{16}

Exercise 6B

1. (a) $\frac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4} = 4^3$
 (b) $\frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} = 2^2$
 (c) $\frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7} = 7^4$
 (d) $\frac{9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9} = 9^4$
 (e) $\frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5} = 5^3$
 (f) $\frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10} = 10^6$
 (g) $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3} = 3^3$
 (h) $\frac{6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6}{6 \times 6} = 6^7$
 (i) $\frac{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8 \times 8} = 8^2$
 (j) $\frac{11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11}{11 \times 11 \times 11 \times 11 \times 11} = 11^4$
 (k) $\frac{2 \times 2 \times 2 \times 2 \times 2}{2} = 2^4$
 (l) $\frac{13 \times 13 \times 13 \times 13}{13 \times 13 \times 13} = 13^1 = 13$
 (m) $\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = 3^1 = 3$

$$\mathbf{1. (n) \quad \frac{15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15}{15 \times 15 \times 15 \times 15 \times 15} = 15^2}$$

$$(o) \frac{20 \times 20 \times 20 \times 20}{20} = 20^3$$

(p) $\frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7} = 7^2$

2. Subtract the index on the denominator (bottom line) from the index on the numerator (top line) to find the index of the answer.

3. (a) 5^5 (b) 3^4 (c) 4^5 (d) 7^2 (e) 6^8
(f) 12^6 (g) 2^7 (h) 8 (i) 9 (j) 15^8
(k) 9^4 (l) 17^6

4. (a) 9^3 (b) 12^6 (c) 7^3 (d) 8^2 (e) 10^4
(f) 6^7 (g) 5^6 (h) 2^8 (i) 3^8 (j) 20^3
(k) 4^{10} (l) 17^{11}

Exercise 6C

1. (a) $\frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = \frac{1}{1} = 1$

$$(b) \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{1} = 1$$

$$(c) \frac{4 \times 4 \times 4}{4 \times 4 \times 4} = \frac{1}{1} = 1$$

$$(d) \frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7 \times 7} = \frac{1}{1} = 1$$

2. (a) 5^0 (b) 3^0 (c) 4^0 (d) 7^0

3. (a) $5^0 = 1$ (b) $3^0 = 1$ (c) $4^0 = 1$
(d) $7^0 = 1$

4. (a) 6 (b) 13 (c) 13 (d) 1 (e) 25
(f) 5 (g) 1 (h) 1 (i) 27 (j) 5
(k) 3 (l) 3

Exercise 6D

1. (a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^8$

(b) $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^6$

[illegible]

(d) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^{10}$

(e) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times$
 $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{16}$

(f) $9 \times 9 \times 9 \times 9 = 9^4$

1. (g) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times$
 $5 \times 5 \times 5 \times 5 \times 5 = 5^{12}$

[illegible]

(i) $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times$
 $12 \times 12 \times 12 \times 12 \times 12 = 12^{12}$

(j) $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times$
 $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^{14}$

(k) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times$
 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^{14}$

$$(l) \quad 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times$$

$$2 \times 2 \times 2 \times 2 = 2^{18}$$

2. Multiply the indices.

3. (a) 5^{24} (b) 6^{25} (c) 2^{28} (d) 9^{56} (e) 7^{40}
 (f) 8^{36} (g) 10^{50} (h) 11^{54} (i) 13^{42} (j) 9^{72}
 (k) 3^{63} (l) 8^{160}

Exercise 6E

1. (a) $3^{30} \times 4^{12}$ (b) $7^{12} \times 2^{24}$ (c) $6^{49} \times 9^{28}$
(d) $5^{48} \times 8^{54}$

2. (a) $\frac{3^{42}}{8^{21}}$ (b) $\frac{5^{36}}{6^{20}}$ (c) $\frac{4^{15}}{9^{55}}$ (d) $\frac{2^{72}}{7^{96}}$

Exercise 6F

1. (a) $\frac{1}{6^6}$ (b) $\frac{1}{8^9}$ (c) $\frac{1}{4^{13}}$ (d) $\frac{1}{5^7}$

(e) $\frac{1}{12^8}$

2. (a) $\frac{3}{7^9}$ (b) $\frac{8}{2^5}$ (c) $\frac{3}{9^4}$ (d) $\frac{2}{4^6}$

(e) $\frac{6^2}{4^8}$ (f) $\frac{12^3}{3^8}$ (g) $\frac{4^6}{7^2}$ (h) $\frac{7^{12}}{8^{10}}$

3. (a) 8^3 (b) 5^7 (c) 4×2^9 (d) 3×6^2

(e) $\frac{4 \times 8^4}{9}$ (f) $\frac{7 \times 2^9}{3}$ (g) $\frac{5 \times 7^3}{6}$

(h) $\frac{8 \times 4^5}{11}$ (i) $\frac{5^8}{6 \times 7^2}$ (j) $\frac{5 \times 6^3}{4 \times 7^2}$

(k) $\frac{3^5 \times 7^2}{9^8 \times 5^4}$ (l) $\frac{2^6 \times 9^8}{3^2 \times 4^7}$

Exercise 6G

1. 4^{13} 2. 7 3. 8^8
 4. 1 5. 5^{56} 6. 8^5
 7. 9^{17} 8. 6^4 9. 9^{34}
 10. 8^{11} 11. $\frac{1}{5^{26}}$ 12. 7^{119}
 13. 1 14. $\frac{1}{3^{37}}$ 15. $\frac{1}{9^{39}}$
 16. 2^{36} 17. 4^{72} 18. $\frac{1}{4^7}$
 19. 6^8 20. 2^3

Exercise 6H

1. (a) 4 (b) 8 (c) $\frac{1}{16}$ (d) 32 (e) 64
 (f) $\frac{1}{128}$ (g) 256 (h) $\frac{1}{512}$ (i) 1024
 (j) 2048 (k) 9 (l) 27 (m) $\frac{1}{81}$ (n) 25
 (o) 125 (p) 100 (q) $\frac{1}{1000}$ (r) 10 000
 (s) $\frac{1}{100\,000}$ (t) 1 000 000 (u) 512
 (v) $\frac{1}{81}$ (w) $\frac{1}{216}$ (x) $\frac{1}{64}$ (y) 2401
 2. (a) $\frac{1}{1000}$ (b) 512 (c) 1 (d) 81
 (e) $\frac{1}{49}$ (f) 36 (g) 12 (h) 50 (i) 1024

Exercise 6I

1. (a) 50 625 (b) 176 812 (c) 2613
 (d) 1 108 809 (e) 262 144 (f) 324
 (g) 3010 (h) 4096 (i) 5 (j) 370 942
 (k) 117 649 (l) 2 131 936
 2. (a) 14.51 (b) 34.36 (c) -455.25
 (d) 29.45 (e) 2.76 (f) 3.03
 (g) 11.10 (h) 3.47 (i) -2.37

Exercise 6J

1. (a) a^8 (b) n^8 (c) y^{11} (d) b^7 (e) m^9
 (f) x^{10} (g) a^{14} (h) x^{16} (i) y^9
 2. (a) $18a^8$ (b) $8x^{11}$ (c) $6m^{10}$
 (d) $12n^{18}$ (e) $2a^5b^7$ (f) $6x^4y^7$
 (g) $12a^7y^7$ (h) $14a^7b^9c^2$
 (i) $12m^8n^8$ (j) $12x^9y^6z^8$
 (k) $24a^{10}b^4c^8$ (l) $120m^{17}n^8$

Exercise 6K

1. (a) x^3 (b) a^3 (c) n^6 (d) m^7 (e) b^3
 (f) y^5 (g) p^6 (h) w (i) m^7 (j) x^9
 (k) n^{14} (l) m
 2. (a) $2x^3y^5$ (b) $4m^3n^6$ (c) $3a^3b^3$
 (d) $\frac{4}{3}m^2n^3$ (e) $\frac{2}{3}x^3y^3$ (f) $\frac{4}{3}a^3b^3$
 (g) $\frac{3}{2}x^2y^5$ (h) $\frac{5}{6}m^7n$ (i) $\frac{5}{4}a^3b^{11}$
 (j) $2x^3y$ (k) $\frac{6}{5}m^7n^{11}$ (l) $\frac{6}{7}ab^9$
 (m) $3m^2n^6$ (n) $\frac{2}{3}x^6y$ (o) $3mn$
 (p) $\frac{2}{3}hi^5$ (q) $\frac{1}{2}a^6$ (r) $\frac{4}{3}ac$

Exercise 6L

1. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 1
 (g) 1 (h) 1
 2. (a) 5 (b) 3 (c) 4 (d) $2a$ (e) 2
 (f) 3 (g) 9 (h) 5 (i) 3 (j) $7x^2$
 (k) 4 (l) 7
 3. (a) 1 (b) z^2 (c) $2m^3$ (d) $\frac{4b^6}{5}$
 (e) 2 (f) $\frac{2m}{3}$ (g) $\frac{3y^2z}{4}$ (h) $2ab$

Exercise 6M

1. (a) m^{12} (b) x^{15} (c) a^{24} (d) c^{18}
 (e) p^{30} (f) b^{21} (g) n^{36} (h) y^{20}
 2. (a) $9x^4$ (b) $16m^{12}$ (c) $27m^{15}$
 (d) $16n^{12}$ (e) $36c^{14}$ (f) $-32n^{20}$
 (g) $-27a^{12}$ (h) $-81u^{24}$ (i) $216a^9$
 (j) $-125y^{15}$ (k) $-64c^8$ (l) $1024y^{15}$

Exercise 6N

1. (a) a^8b^{12} (b) x^8y^{14} (c) p^8q^{36}
 (d) $u^{30}v^{18}$ (e) $m^{25}n^{40}o^{30}$
 (f) $a^{18}b^9c^{27}$ (g) $9x^6y^{10}$ (h) $16a^6b^2$
 (i) $-125m^3n^{18}$ (j) $64a^{10}b^6$
 (k) $64a^{24}b^6c^{18}$ (l) $81p^8q^{14}r^6$
 (m) $-8m^9n^{18}$ (n) $-81x^{16}y^{20}$
 (o) $-125u^{18}v^{21}$ (p) a^9b^{12}
 (q) $-256m^{24}n^{12}$ (r) $128a^{21}b^{14}$
 (s) $216x^{12}y^3z^{15}$ (t) $2401a^{16}b^{12}c^4$
 (u) $3125p^{15}q^{35}r^{40}$

2. (a) $\frac{a^{15}}{b^6}$ (b) $\frac{m^{14}}{n^6}$ (c) $\frac{x^{21}}{y^{15}}$
 (d) $\frac{m^{10}}{n^8}$ (e) $\frac{a^{18}}{b^{21}}$ (f) $\frac{e^{20}}{r^{10}}$
 (g) $\frac{m^{12}n^{16}}{p^{16}q^{12}}$ (h) $\frac{a^{12}b^6}{x^4y^{16}}$ (i) $\frac{m^{30}p^{18}}{n^{24}q^{12}}$
 (j) $\frac{8a^9b^{12}}{m^6n^{15}}$ (k) $\frac{x^{12}}{27y^9z^{15}}$ (l) $\frac{16m^4n^{10}}{p^{10}q^8}$
 (m) $\frac{64m^9n^{18}}{27a^6b^3}$ (n) $\frac{25x^6y^8}{36z^{14}}$
 (o) $\frac{9a^8b^{18}}{49x^{10}y^{14}}$ (p) $\frac{8a^3b^3c^9}{27x^6y^9z^{12}}$
 (q) $\frac{625}{m^8n^{12}p^8}$ (r) $\frac{343m^3n^{21}}{1000p^{12}}$

Exercise 60

1. (a) $\frac{1}{x^8}$ (b) $\frac{1}{3^4}$ (c) $\frac{1}{a^2}$
 (d) $\frac{1}{e^{10}}$ (e) $\frac{1}{c}$ (f) $\frac{3}{m^4}$
 (g) $\frac{x}{y^2}$ (h) $\frac{4m}{n^7}$ (i) $\frac{6x^2}{y^3}$
 (j) $\frac{8b^9}{a^2}$

2. (a) $\frac{3m^2}{n^6}$ (b) $\frac{7a^3}{b^5}$ (c) $\frac{a^4b^3}{c^6}$
 (d) $\frac{6y^3}{x^5}$ (e) $\frac{3m^2}{4n^3}$ (f) $\frac{5a^3}{7b^4}$
 (g) $\frac{y^3}{2x^4}$ (h) $\frac{x^3y}{3z^2}$ (i) $\frac{4a^5c}{9b^3}$
 (j) $\frac{2p^4}{7q^3r^2}$ (k) $\frac{3}{8a^2b^3c^2}$ (l) $\frac{w^2}{5x^7v^3}$

Exercise 6P

1. (a) x^8 (b) m^7 (c) c^3 (d) a^5b^5 (e) m^5n^8
 (f) $3x^4$ (g) $2m^2$ (h) $9x^9$ (i) $\frac{3c^3}{2}$
 (j) $\frac{b^2}{4}$ (k) $\frac{5x^2}{3}$ (l) 6
 (m) $8x^3$ (n) $\frac{3m^3}{2}$ (o) $\frac{8x^{15}}{5}$

2. (a) x^5y^{10} (b) $\frac{1}{m^{13}n^{12}}$ (c) $a^{37}b^{15}$ (d) $4x^{10}y^9$
 (e) $\frac{4mn^2}{3}$ (f) $6x^5y^9$ (g) $\frac{1}{6x^7y^8}$
 (h) $\frac{3n^4}{2}$ (i) $\frac{2ab^3}{3}$ (j) $4x^4y^4$
 (k) $\frac{3m^7n^2}{8}$ (l) $2a^{30}b^{60}$

3. (a) $\frac{5x^3}{7}$ (b) $6c^4$ (c) $\frac{8a^3}{3}$
 (d) $6a^5$ (e) $\frac{x^6y^2}{6}$ (f) 1
 (g) $2xy^2$ (h) $2x^9y^{13}$ (i) $8a^3$
 (j) $\frac{3m^6n^5}{4}$

Exercise 6Q

1. (a) 9×10^3 (b) 5×10^4 (c) 7×10^8
 (d) 2.75×10^5 (e) 3.2×10^5
 (f) 8.21×10^7 (g) 3.6547×10^8
 (h) 2.3×10^{11} (i) 5.0001×10^4
 (j) 6.587×10^3 (k) 2.5434×10^2
 (l) 1.5378×10^3

2. (a) 5×10^{-4} (b) 9×10^{-6}
 (c) 4×10^{-9} (d) 1.3×10^{-4}
 (e) 6.92×10^{-3} (f) 5.79×10^{-5}
 (g) 2.045×10^{-3} (h) 6×10^{-6}
 (i) 6.81×10^{-8} (j) 2.0204×10^{-5}
 (k) 2×10^{-10} (l) 5.7×10^{-9}

3. (a) 36 000 (b) 8 000 000
 (c) 671 000 (d) 245 500 000
 (e) 72 010 000 (f) 382 000 000 000
 (g) 0.0027 (h) 0.000 07
 (i) 0.000 93 (j) 0.000 000 483
 (k) 0.000 000 005 917
 (l) 0.000 000 000 100 8
4. (a) 9.109×10^{-31} kg
 (b) 1.6726×10^{-27} kg
 (c) 1.6749×10^{-27} kg
 (d) 2.009×10^{-26} kg
5. (a) 1.99×10^{30} kg (b) 1.5×10^7 °C
6. (a) Mercury - 3.3×10^{23} kg
 Venus - 4.87×10^{24} kg
 Earth - 5.98×10^{24} kg
 Mars - 6.42×10^{23} kg
 Jupiter - 1.9×10^{27} kg
 Saturn - 5.69×10^{26} kg
 Uranus - 8.69×10^{25} kg
 Neptune - 1.02×10^{26} kg
 Pluto - 1.2×10^{22} kg
 (b) Pluto, Mercury, Mars, Venus,
 Earth, Uranus, Neptune,
 Saturn, Jupiter
7. 4×10^{11}
8. (a) 1×10^{-10} m (b) 1.2×10^{-4} m
 (c) 2×10^{-6} m (d) 7×10^{-6} m
 (e) 1×10^{-4} m
9. (a) 3×10^5 km/s (b) 1.5×10^8 km
 (c)(i) 500 seconds
 (ii) 8 minutes 20 seconds
 (d) 1.28 seconds
10. 0.24 seconds
11. (a) 1.1×10^{-2} s (b) 7×10^{-4} s
 (c) 3×10^{-6} s (d) 2×10^{-9} s
 (e) 1×10^{-10} s
12. (a) 1.1×10^7 ns (b) 7×10^5 ns
 (c) 3×10^3 ns (d) 2 ns
 (e) 1×10^{-1} ns

Chapter Review

1. 6^7
2. (a) 3^9 (b) 6^5 (c) 4^8 (d) 10 (e) 7^{20}
 (f) 5^{20} (g) $\frac{9^{24}}{5^{12}}$ (h) $\frac{4^{20}}{7^{32}}$
3. (a) $\frac{1}{6^5}$ (b) $\frac{5}{3^8}$ (c) $\frac{4 \times 7^3}{8}$
 (d) $\frac{2^9 \times 5^8}{4^3 \times 7^6}$
4. (a) x^9 (b) $12m^{11}n^6$ (c) a^{12}
 (d) $32m^{15}n^{10}$ (e) $-a^6b^9$ (f) $-81p^8q^{12}r^8$
 (g) 1 (h) 7 (i) $3a^{10}$ (j) x^2y^{16}
 (k) $3mn$ (l) $\frac{4x^5y^4}{9}$
5. (a) $\frac{1}{x^6}$ (b) $\frac{8m^2}{n^3}$ (c) $\frac{2x^5}{5y^2z^4}$
6. (a) 8.62×10^9 (b) 5.72×10^{-8}
7. (a) 67 100 (b) 0.000 003 01
8. (a) 2800 (b) 64 (c) 5

Chapter 7

Exercise 7A

1. (a) xz (b) $G + H$ (c) $p - 2$
 (d) $d + 7$ (e) $w + y$ (f) $4t + 9w$
 (g) $m - 7$ (h) $H/3$ (i) $t(5 - q)$ (j) $15b$
2. (a) term (b) expression
 (c) expression (d) term
 (e) term (f) expression
 (g) term (h) expression
3. (a) coefficient = 7, pronumeral = a
 (b) coefficient = 34, pronumeral = abc^2
 (c) coefficient = -2, pronumeral = p
 (d) coefficient = 1, pronumeral = F
 (e) coefficient = $\frac{3}{5}$, pronumeral = m^2n^3
 (f) coefficient = $-\frac{2}{7}$, pronumeral = w^2z^5
4. $(2m, -4m)$, $(4n^2, \frac{1}{4}n^2)$, $(6mn, \frac{1}{4}mn)$,
 $(n, 4n)$, $(6m^2, 3m^2)$, $(4m^2n, -3m^2n)$
5. (a) $7y$ (b) $3b$ (c) $8k$ (d) $9z$ (e) $11ab$
 (f) $3xy$ (g) $5mn$ (h) $8xyz$ (i) $-2d$
 (j) $5pq^2$ (k) $3x^2$ (l) $-3y^2$ (m) $-2a$
 (n) $-3mn$ (o) $-8t$ (p) $3y$ (q) $-6xy$ (r) 0

6. (a) $9x + 8y$ (b) $7a + 9b$ (c) $7r + 9t$
 (d) $13u + 2w$ (e) $7y + 2z$ (f) $3k + 3m$
 (g) $7w + 6h$ (h) 0 (i) $-5x - 6y$
 (j) 0 (k) $k^2l - 3kl^2$ (l) $5ab^2 + 2a^2b$
 (m) xyz (n) $2x^2y - 5xy^2$
 (o) $a^2b + 4a^2 + 4b$ (p) 0
7. (a) $22a$ (b) $2p + 6q$ (c) $4x + 10y$
8. (a) $5x$ (b) $35x$
9. (a) $10ab$ (b) $8abc$ (c) $6xyz$ (d) $8d^2$
 (e) $30pq$ (f) $18tuv$ (g) $18qrs$
 (h) $24a^2b^2$ (i) $30x^2y^2z$ (j) $60a^2b^2c^2$
 (k) $252defg$ (l) $24lm^2n^2$
10. (a) $24n^2$ (b) $6xy$ (c) $21ab$
11. (a) $4a$ (b) ay (c) $5az$

Exercise 7B

1. (a) $3y + 6$ (b) $2x - 8$ (c) $4m - 24$
 (d) $7a - 21$ (e) $12 + 15m$ (f) $16 + 6d$
 (g) $3m - 3n$ (h) $18p + 6q$
 (i) $14t + 21u$ (j) $-3x - 9$ (k) $-5a - 35$
 (l) $-32 - 8p$ (m) $-5a - 5b$ (n) $-12 - 6z$
 (o) $-28y - 35z$ (p) $-3x + 15$
 (q) $-4a + 24$ (r) $-6y + 9$ (s) $-6a + 10b$
 (t) $24m - 18n$ (u) $10a - 15b + 20c$
 (v) $-28x + 8y$ (w) $-15g - 30h$
 (x) $-12x + 8y + 20z$
2. (a) $2a^2 + 3a$ (b) $2mn + 5mp$
 (c) $3xy - 2xz$ (d) $6pq - 14p$
 (e) $6x^2 - 10xy$ (f) $18ab + 21ac$
 (g) $15m^2 - 10mn$ (h) $2b^2 - 14bc$
 (i) $6t^2 - 20tu$ (j) $-2x^2 - xy$
 (k) $-6a^2 - 9ac$ (l) $-6ax - 4bx$
 (m) $-8ab + 10a$ (n) $-6mp + 9mq$
 (o) $-10vw + 25vz$
 (p) $-24mn + 16mp$ (q) $-8ad - 12dx$
 (r) $-6a^2 + 15ab + 6ac$
 (s) $-20a^2 + 10a^2b$ (t) $-12nt + 16t^2$
 (u) $-10x^2 + 4xy - 6xz$
 (v) $6x^2y - 9xyz$ (w) $-6x^2y - 9xy^2$
 (x) $6m^2n - 4m^2 - 10mn$
3. (a) $5a + 17$ (b) $5x + 12$ (c) $9y + 2$
 (d) $7m + 5n$ (e) $5x - 13$ (f) $y - 6$
 (g) $2n + 2$ (h) $2p + 11$ (i) $11y^2 - 9y$
3. (j) $8a^2 + 5ab$ (k) $8x^2 - x$
 (l) $14m - 27mn$ (m) $-2a^2 + 43a$
 (n) $24x^2 - 16xy$ (o) $9m - 16m^2$
 (p) $-2c^2 + 15cd$ (q) $33xy + xz$
 (r) $2mp$ (s) $-6a^2 + 4b^2$
 (t) $-34xy + 15x^2 + 18y^2$
4. (a) $4x + 8$ (b) $4a + 6$ (c) $6n + 18$
5. $5x^2 + 8x$
6. (a) $x^2 + 5x + 6$ (b) $y^2 + 6y + 5$
 (c) $m^2 + 8m + 12$ (d) $c^2 + 11c + 24$
 (e) $a^2 + 3a + 2$ (f) $x^2 + 15x + 50$
 (g) $p^2 + 16p + 63$ (h) $p^2 + 3p - 18$
 (i) $x^2 + 3x - 10$ (j) $y^2 + 2y - 3$
 (k) $m^2 + m - 12$ (l) $n^2 + 4n - 21$
 (m) $w^2 - 2w - 8$ (n) $y^2 - 5y - 6$
 (o) $x^2 + 2x - 8$ (p) $y^2 - 11y + 24$
 (q) $z^2 - 9z + 14$ (r) $b^2 - 10b + 24$
 (s) $x^2 - 5x + 6$ (t) $y^2 - 7y + 12$
 (u) $n^2 - 17n + 70$ (v) $n^2 - 13n + 42$
 (w) $y^2 - 9$ (x) $x^2 - 16$ (y) $m^2 - 1$
 (z) $p^2 - 36$
7. (a) $6x^2 + 5x + 1$ (b) $6m^2 + 13m + 6$
 (c) $6p^2 + 19p + 10$ (d) $8m^2 + 26m + 15$
 (e) $5c^2 + 17c + 6$ (f) $4y^2 + 15y + 14$
 (g) $6m^2 + 7m - 5$ (h) $4n^2 - 29n - 24$
 (i) $6c^2 + 11c - 10$ (j) $10d^2 + 3d - 18$
 (k) $6m^2 + 11m - 10$ (l) $6x^2 + x - 15$
 (m) $6y^2 - 19y + 10$ (n) $10p^2 - 17p + 3$
 (o) $8y^2 - 22y + 5$ (p) $18y^2 - 45y + 28$
 (q) $6c^2 - 37c + 45$ (r) $8a^2 - 42a + 49$
8. (a) $6x^2 + 11xy + 3y^2$ (b) $6a^2 + 7ab + 2b^2$
 (c) $2x^2 + xy - y^2$ (d) $a^2 - b^2$
 (e) $9m^2 - 9mn - 10n^2$
 (f) $8p^2 - 22pq + 15q^2$ (g) $9w^2 - 4z^2$
 (h) $15n^2 - 19np - 10p^2$
 (i) $12x^2 - 23xy + 10y^2$
 (j) $6x^2 - 4xy - 42y^2$ (k) $4v^2 - 25w^2$
 (l) $9m^2 - 12mn + 4n^2$
 (m) $25p^2 - 20pq + 4q^2$
 (n) $100a^2 - 49b^2$ (o) $9x^2y^2 - 4a^2b^2$
 (p) $25x^2 + 60xy + 36y^2$

9. (a) $x^2 + 10x + 25$ (b) $y^2 + 14y + 49$
 (c) $a^2 - 4a + 4$ (d) $m^2 - 8m + 16$
 (e) $4n^2 + 12n + 9$ (f) $9a^2 + 30a + 25$
 (g) $25t^2 - 20t + 4$ (h) $16d^2 - 56d + 49$
 (i) $64n^2 - 48n + 9$ (j) $a^2 + 2ab + b^2$
 (k) $n^2 - 2mn + m^2$ (l) $x^2 + 2xy + y^2$
 (m) $4x^2 + 12xy + 9y^2$
 (n) $16m^2 - 24mn + 9n^2$
 (o) $4p^2 + 20pq + 25q^2$
 (p) $49a^2 - 42ab + 9b^2$
 (q) $64x^2 + 48xy + 9y^2$
 (r) $100p^2 - 60pq + 9q^2$

10. (a) xy
 (b) $(x + 3)(y + 2) = xy + 2x + 3y + 6$
 (c) $2x + 3y + 6$

11. $(a - 7)(b - 4) = ab - 4a - 7b + 28$

Exercise 7C

1. (a) 1, 2, 5, 10 (b) 1, 2, 3, 6, x
 (c) 1, 2, 3, 6, 9, 18, a
 (d) 1, 2, 3, 4, 6, 9, 12, 18, 36, x , x^2
 (e) 1, 2, 3, 4, 6, 8, 12, 24, p , q , q^2
 (f) -1, 1, 2, 3, 6, 9, 18, m
 (g) 1, 2, 3, 5, 6, 10, 15, 30, x , x^2 , y , z
 (h) -1, 1, 2, 4, 7, 14, 28, c , c^2
 (i) -1, 1, 3, 9, 27, p , q
 (j) -1, 1, 2, 3, 4, 6, 8, 12, 16, 24, 48, m , m^2 , n , n^2
2. (a) 4 (b) 9 (c) 10 (d) x (e) 2
 (f) 4 (g) -4 (h) $2x$ (i) x (j) b
 (k) $4m$ (l) $-5x$ (m) $2x$ (n) $4a$
 (o) xy (p) $4a$ (q) $-2x$ (r) $6m$
 (s) $-q$ (t) $-3m$ (u) $16g$ (v) $18ab^2$
 (w) $-8a^2b$
3. (a) $2(4p + 5)$ (b) $2(3c - 5)$
 (c) $4(3x + 4)$ (d) $2(2a - 7)$
 (e) $6(3 + 2m)$ (f) $6(p + 2)$
 (g) $5(m - 3)$ (h) $9(q + 3)$
 (i) $5(3t - 5)$ (j) $12(2 + 3n)$
 (k) $7(a - 3)$ (l) $9(v + 5)$
 (m) $6(1 - 4y)$ (n) $5(5y - 1)$
 (o) $6(3a + 1)$ (p) $5(7x - 5)$

4. (a) $4(p + 3q)$ (b) $4(3a - 7b)$
 (c) $3(2m + 3n)$ (d) $2b(5a + 7)$
 (e) $4p(2q - 3)$ (f) $4y(3x + 4)$
 (g) $3c(d + 2)$ (h) $4m(4n - 5)$
 (i) $2a(b + 12)$ (j) $4a(1 + 7b)$
 (k) $6x(1 - 3y)$ (l) $3b(a - 5)$
 (m) $2ab(3c + 4)$ (n) $5yz(x + 2)$
 (o) $8mp(2n + 3)$

5. (a) $-3(3p + 4q)$ (b) $-2(2m + 3n)$
 (c) $-3(a + 4b)$ (d) $-4(a + 2)$
 (e) $-2(3x + 5y)$ (f) $-2(5b + 3c)$
 (g) $-4(5p + 6q)$ (h) $-2(4x + 9)$
 (i) $-12(2 + m)$ (j) $-3(2x + 1)$
 (k) $-5(4p + 1)$ (l) $-4(3x + 1)$
 (m) $-12(3p + 2q)$ (n) $-8(5s + t)$
 (o) $-18(2a + b)$ (p) $-8(4x + 3y)$

6. (a) $2n(3n + 4)$ (b) $2a(a - 2)$
 (c) $2p(3p - 10)$ (d) $2p(5p + 7q)$
 (e) $2r(3r - 1)$ (f) $4m(2m + 1)$
 (g) $4m(3n + 4m)$ (h) $-5x(2x + 3)$
 (i) $-6b(1 + 3b)$ (j) $-2x(x + 8)$
 (k) $-2m(3m + 5n)$ (l) $-7xy(2x + 1)$

Exercise 7D

1. (a) $(a + 4)(2 + b)$ (b) $(b - 2)(4 + c)$
 (c) $(d + 3)(5 - c)$ (d) $(x - y)(3 + z)$
 (e) $(m + 4)(7 - n)$ (f) $(p - 1)(6 + q)$
 (g) $(a - 5)(9 + b)$ (h) $(n + 2)(m + 3)$
2. (a) $(a + 3)(2 + b)$ (b) $(m - 2)(3 + n)$
 (c) $(p + 2)(5 + q)$ (d) $(x + 3)(4 + y)$
 (e) $(n - 5)(3 + m)$ (f) $(p + 1)(6 + q)$
 (g) $(2a + 3)(3 + b)$ (h) $(2x - 1)(4 + y)$
 (i) $(3x + 2)(5 + y)$ (j) $(2q + 1)(p + 4)$
 (k) $(3b - 5)(a + 6)$ (l) $(3v - 4)(2u + 3)$
3. (a) $(3 + z)(y + 5)$ (b) $(b + 7)(2 + c)$
 (c) $(m - 3)(5 + n)$ (d) $(c + 5)(4 + d)$
 (e) $(n - 1)(3 + m)$ (f) $(y - 3)(4 + x)$
 (g) $(a + 3)(b - 2)$ (h) $(m + 5)(n + 4)$
 (i) $(p + 5)(q + 2)$ (j) $(b + 3)(a + 7)$
 (k) $(x + 3)(x + y)$ (l) $(a - c)(b + 2)$
 (m) $(m + n)(m + 1)$ (n) $(n + p)(m - 3)$
 (o) $(q - 5)(p + r)$ (p) $(c + d)(b + 7)$

4. (a) $(m+2)(4-n)$ (b) $(b+5)(3-c)$
 (c) $(x+4)(2-y)$ (d) $(n+3)(m-3)$
 (e) $(b+5)(a-4)$ (f) $(h+3)(g-7)$
 (g) $(d+4)(c-6)$ (h) $(p+5)(4-q)$
 (i) $(x+2)(x-y)$ (j) $(a+b)(a-3)$
 (k) $(n+5)(n-p)$ (l) $(6+w)(w-y)$
5. (a) $(a-3)(3-b)$ (b) $(m-5)(2-n)$
 (c) $(p-1)(4-q)$ (d) $(x-4)(3-y)$
 (e) $(y-2)(2-z)$ (f) $(b-3)(5-c)$
 (g) $(m-5)(6-n)$ (h) $(a-3)(4-b)$
 (i) $(a-5)(3-b)$ (j) $(n-3)(6-p)$
 (k) $(x-7)(4-y)$ (l) $(q-5)(p-r)$
 (m) $(x-3)(x-y)$ (n) $(a-4)(a-b)$
6. (a) $(n+3)(m+3)$ (b) $(b-2)(a-7)$
 (c) $(a+6)(3-c)$ (d) $(q+7)(p-8)$
 (e) $(b+c)(a+d)$ (f) $(n-10)(m-5)$
 (g) $(3-a)(5-b)$ (h) $(b+2c)(3a-7)$
 (i) $(x+5)(x-y)$ (j) $(a-b)(a-3)$
 (k) $(p-5)(q-2)$ (l) $(w-y)(z-6)$
 (m) $(b-c)(b+d)$ (n) $(d-5)(c-6)$
 (o) $(2y+3x)(x-z)$
 (p) $(2a-5)(3a+2b)$
 (q) $(q-2p)(4p+5)$
 (r) $(mp+ab)(a+b)$

Exercise 7E

1. (a) $(m-n)(m+n)$ (b) $(A-B)(A+B)$
 (c) $(v-w)(v+w)$ (d) $(y-3)(y+3)$
 (e) $(5-c)(5+c)$ (f) $(x-7)(x+7)$
 (g) $(p-10)(p+10)$ (h) $(g-4)(g+4)$
 (i) $(b-1)(b+1)$
2. (a) $(2a-3b)(2a+3b)$
 (b) $(4p-7q)(4p+7q)$
 (c) $(10x-9y)(10x+9y)$
 (d) $(3n-m)(3n+m)$
 (e) $(8g-11h)(8g+11h)$
 (f) $(7a-8b)(7a+8b)$
 (g) $(5x-y)(5x+y)$
 (h) $(6a-b)(6a+b)$
 (i) $(13n-15m)(13n+15m)$
3. (a) $3(3n-m)(3n+m)$
 (b) $2(4a-b)(4a+b)$
 (c) $6(2x-3y)(2x+3y)$
 (d) $2(2a-3b)(2a+3b)$

3. (e) $2(5g-h)(5g+h)$
 (f) $2(4p-7q)(4p+7q)$
 (g) $5(2x-y)(2x+y)$
 (h) $8(5a-b)(5a+b)$
 (i) $5(3n-2m)(3n+2m)$

Exercise 7F

1. (a) $(x+2)(x+3)$ (b) $(x+3)(x+4)$
 (c) $(n+2)(n+7)$ (d) $(a+2)(a+8)$
 (e) $(p+2)(p+9)$ (f) $(a+2)(a+11)$
 (g) $(y+4)(y+4)$ (h) $(x+3)(x+8)$
 (i) $(n+4)(n+6)$ (j) $(x+2)(x+12)$
 (k) $(a+1)(a+10)$ (l) $(y+1)(y+8)$
 (m) $(x+5)(x+8)$ (n) $(a+6)(a+8)$
 (o) $(b+2)(b+25)$ (p) $(x+7)(x+9)$
 (q) $(v+2)(v+21)$ (r) $(x+5)(x+9)$
 (s) $(a+1)(a+17)$
 (t) $(m+3)(m+30)$
2. (a) $(x-3)(x-4)$ (b) $(m-4)(m-4)$
 (c) $(a-3)(a-8)$ (d) $(c-5)(c-6)$
 (e) $(z-4)(z-8)$ (f) $(x-2)(x-15)$
 (g) $(n-3)(n-12)$ (h) $(y-2)(y-14)$
3. (a) $(x-3)(x+8)$ (b) $(a-3)(a+10)$
 (c) $(n+3)(n-10)$ (d) $(c+2)(c-10)$
 (e) $(m-2)(m+8)$ (f) $(y-2)(y+6)$
 (g) $(x-2)(x+9)$ (h) $(a+3)(a-8)$
 (i) $(y-4)(y+9)$ (j) $(x+5)(x-8)$
 (k) $(n-6)(n+7)$ (l) $(a-3)(a+4)$
 (m) $(x+4)(x-5)$ (n) $(g+5)(g-6)$
4. (a) $(x+6)(x+9)$ (b) $(c-5)(c-8)$
 (c) $(z+2)(z-9)$ (d) $(a-5)(a-7)$
 (e) $(x+6)(x+10)$ (f) $(n-7)(n-8)$
 (g) $(y+6)(y-11)$ (h) $(b+8)(b-9)$
 (i) $(x-5)(x-20)$ (j) $(m-9)(m-11)$
 (k) $(a-7)(a+9)$ (l) $(x-11)(x-12)$
 (m) $(y-2)(y+20)$ (n) $(r+4)(r+8)$
 (o) $(a+3)(a+4)$ (p) $(m+3)(m-6)$
 (q) $(p-4)(p-6)$ (r) $(r+3)(r-9)$
 (s) $(y-4)(y-11)$ (t) $(b-7)(b+8)$
 (u) $(n-4)(n-9)$ (v) $(q+2)(q+16)$
5. (a) $(a+4)^2$ (b) $(m+10)^2$ (c) $(n+9)^2$
 (d) $(c-5)^2$ (e) $(y+7)^2$ (f) $(x-6)^2$
 (g) $(y+8)^2$ (h) $(a-10)^2$ (i) $(u+11)^2$
 (j) $(p-12)^2$

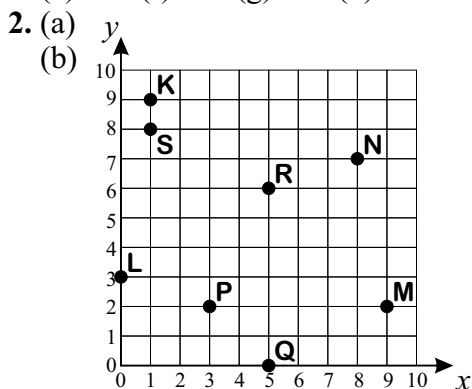
Chapter Review

- (a) $5x - 15$ (b) $-2a + 6b$
(c) $10xy - 15xz$
- (a) $7m - 4$ (b) $4x - 8$
(c) $x^2 + 2x - 15$ (d) $8y^2 - 2y - 21$
(e) $6n^2 + 19mn + 10m^2$
(f) $9a^2 - 4b^2$ (g) $4x^2 + 12x + 9$
(h) $9p^2 - 24pq + 16q^2$
- (a) $2y(5x - 6)$ (b) $-3(2a + 3b)$
(c) $(a - 4)(3 + b)$
- (a) $(5 + y)(3x + 2)$ (b) $(3b - 4)(2a + 3)$
(c) $(a + 5)(b + 2)$ (d) $(a + 2)(a - b)$
- (a) $(a - b)(a + b)$ (b) $(x - 4)(x + 4)$
(c) $(5m - 7n)(5m + 7n)$
- (a) $(a + 2)(a + 9)$ (b) $(n + 2)(n + 12)$
(c) $(p - 3)(p - 8)$ (d) $(x - 4)(x - 7)$
(e) $(a - 4)(a + 9)$ (f) $(n + 3)(n - 4)$

Chapter 8

Exercise 8A

- (a) 4.2 (b) 6.1 (c) 10.0 (d) 2.8
(e) 7.1 (f) 9.5 (g) 7.2 (h) 8.6

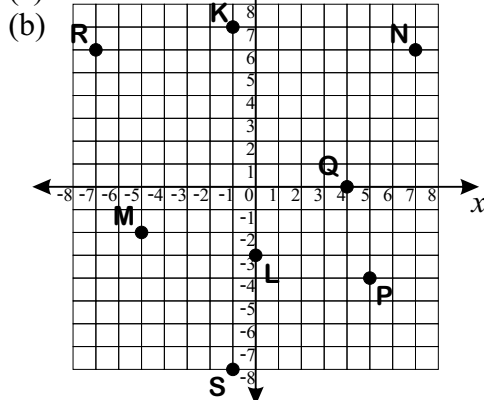


- (c)(i) 7.3 (ii) 9.1 (iii) 7.6 (iv) 5.7
(v) 10.0

Exercise 8B

- (a) 11.2 (b) 6.4 (c) 10.3 (d) 9.8
(e) 10.8 (f) 13.6 (g) 9.4

2. (a)



- (c)(i) 8.6 (ii) 14.4 (iii) 9.4 (iv) 5.1
(v) 15.6

Exercise 8C

- (a) 7.1 (b) 11.3 (c) 10.0 (d) 3.6
(e) 7.2 (f) 7.1
- (a) 4.2 (b) 31.0 (c) 25.5 (d) 39.4
(e) 47.2 (f) 29.5
- 25.3
- 39.8

Exercise 8D

- (b)(i) (3, 6) (ii) (3, 4) (iii) (2, 3)
(iv) (3, 4) (v) (-1, 1) (vi) (3, 0)
(vii) (4.5, 3) (viii) (3.5, 5.5)
(ix) (-0.5, 4) (x) (-3.5, -4)
(xi) (0.5, -0.5)
- (a) (1, 3) (b) (-2, 0) (c) (1, 2) (d) (0, -1.5)
(e) (1, 1) (f) (0.5, -0.5) (g) (3, 0.5)

Exercise 8E

- | | |
|------------------|-------------------|
| 1. (6, 10) | 2. (7, 5.5) |
| 3. (-4, 6.5) | 4. (0, 11) |
| 5. (3.5, -1.5) | 6. (16, 22) |
| 7. (6, -11.5) | 8. (-17, 21.5) |
| 9. (0, 0) | 10. (-6, 13.5) |
| 11. (2.7, 11.85) | 12. (-3.65, 5.85) |

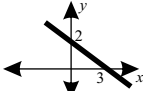
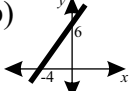
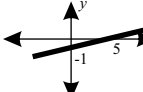
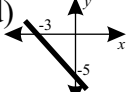
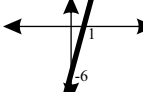
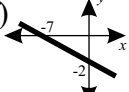
Exercise 8F

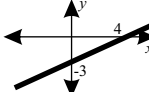
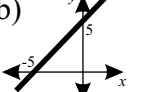
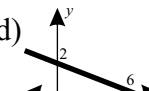
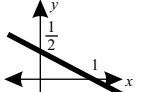
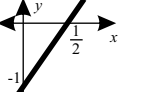
1. (a) 3 (b) 1 (c) 4
 2. (a) 2 (b) 1 (c) 1 (d) 2 (e) 4
 (f) 1 (g) 3 (h) 2
 3. (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
 (e) $\frac{3}{10}$ (f) $\frac{2}{3}$
 4. (a) 1 (b) 2 (c) 1 (d) $\frac{1}{2}$ (e) $\frac{4}{7}$
 (f) $\frac{1}{3}$ (g) $\frac{6}{5}$ (h) 1 (i) $\frac{5}{3}$
 (j) $\frac{3}{7}$
 5. A, D, E, F, G
 6. (a) -1 (b) -2 (c) -1 (d) + (e) $-\frac{1}{2}$
 (f) $-\frac{3}{5}$ (g) $-\frac{1}{2}$ (h) +

Exercise 8G

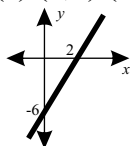
1. (a) 1 (b) 2 (c) 2 (d) $\frac{8}{5}$ (e) $\frac{6}{5}$
 (f) $\frac{4}{7}$ (g) $\frac{3}{5}$ (h) $\frac{3}{7}$ (i) $-\frac{2}{3}$
 (j) -2 (k) $\frac{1}{7}$ (l) $\frac{9}{8}$ (m) $-\frac{12}{7}$
 (n) $-\frac{9}{10}$ (o) $-\frac{5}{7}$ (p) $-\frac{2}{5}$ (q) -2
 (r) -6 (s) 0 (t) 0
 2. (a) x-axis (b) 0
 3. (a) E (b) D (c) B (d) C (e) F

Exercise 8H

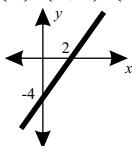
1. (a) 5 (b) -6 (c) (5,0) (d) (0,-6)
 2. (a)  (b) 
 (c)  (d) 
 (e)  (f) 
 3. (a) (3,0) and (0,2)
 (b) (-4,0) and (0,6)
 (c) (5,0) and (0,-1)
 (d) (-3,0) and (0,-5)
 (e) (1,0) and (0,-6)
 (f) (-7,0) and (0,-2)

4. (a)  (b) 
 (c)  (d) 
 5. (a) (3,0) (0,3) (b) (2,0) (0,4)
 (c) (2,0) (0,2) (d) (5,0) (0,2)
 (e) (8,0) (0,3) (f) (10,0) (0,2)
 (g) (3,0) (0,6) (h) (3,0) (0,9)
 (i) (2,0) (0,-2) (j) (4,0) (0,-2)
 (k) (4,0) (0,-3) (l) (6,0) (0,-8)
 (m) (1,0) (0, $\frac{1}{2}$) (n) ($\frac{1}{2}$, 0) (0, -1)
 

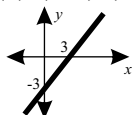
6. (a) (2,0) (0,-6)



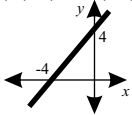
(b) (2,0) (0,-4)



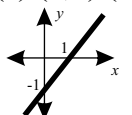
(c) (3,0) (0,-3)



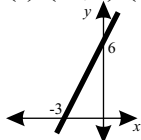
(d) (-4,0) (0,4)



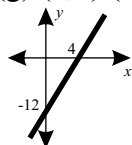
(e) (1,0) (0,-1)



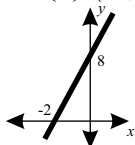
(f) (-3,0) (0,6)



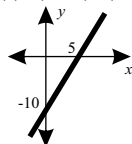
(g) (4,0) (0,-12)



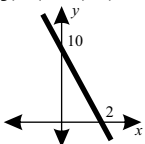
(h) (-2,0) (0,8)



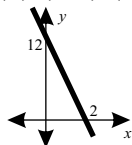
(i) (5,0) (0,-10)



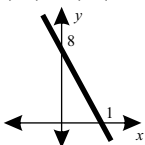
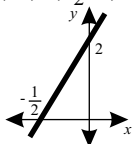
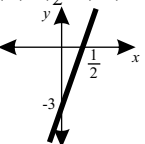
(j) (2,0) (0,10)



(k) (2,0) (0,12)



(l) (1,0) (0,8)

(m) $(-\frac{1}{2}, 0)$ (0,2)(n) $(\frac{1}{2}, 0)$ (0,-3)**Exercise 8I**

1. (a) gradient = 3, y-intercept = 4
 (b) gradient = 2, y-intercept = 6
 (c) gradient = 1, y-intercept = 5
 (d) gradient = 1, y-intercept = -1
 (e) gradient = 2, y-intercept = -5
 (f) gradient = 6, y-intercept = -7
 (g) gradient = -3, y-intercept = 2
 (h) gradient = -4, y-intercept = -1
 (i) gradient = -1, y-intercept = -2
 (j) gradient = -5, y-intercept = 0
 (k) gradient = 2, y-intercept = $\frac{1}{2}$
 (l) gradient = -7, y-intercept = $-\frac{3}{5}$
 (m) gradient = $\frac{1}{3}$, y-intercept = 2
 (n) gradient = $-\frac{3}{4}$, y-intercept = $-\frac{1}{4}$
 (o) gradient = 2, y-intercept = 5
 (p) gradient = -1, y-intercept = 3
 (q) gradient = 3, y-intercept = 6
 (r) gradient = 8, y-intercept = 2
 (s) gradient = $\frac{1}{3}$, y-intercept = 1
 (t) gradient = $\frac{3}{5}$, y-intercept = -4

2. (a) $y = 5x + 3$ (b) $y = 2x - 1$
 (c) $y = -4x + 5$ (d) $y = -2x - 7$
 (e) $y = x - 2$ (f) $y = 3x$
 (g) $y = \frac{1}{3}x - 2$ (h) $y = 2x - \frac{2}{9}$

3. Straight lines that are **parallel** have the same **gradient**.

4. B, E, F, I, J

5. (a) $y = 3x + 5$, $m = 3$, $c = 5$
 (b) $y = -2x - 6$, $m = -2$, $c = -6$
 (c) $y = x - 3$, $m = 1$, $c = -3$
 (d) $y = -x + 2$, $m = -1$, $c = 2$
 (e) $y = 3x + 4$, $m = 3$, $c = 4$
 (f) $y = x - 2$, $m = 1$, $c = -2$
 (g) $y = -x + 3$, $m = -1$, $c = 3$
 (h) $y = 2x + 3$, $m = 2$, $c = 3$
 (i) $y = -4x + \frac{5}{2}$, $m = -4$, $c = \frac{5}{2}$
 (j) $y = \frac{5}{3}x - 4$, $m = \frac{5}{3}$, $c = -4$
 (k) $y = \frac{1}{3}x + \frac{1}{5}$, $m = \frac{1}{3}$, $c = \frac{1}{5}$
 (l) $y = -\frac{4}{3}x$, $m = -\frac{4}{3}$, $c = 0$
 (m) $y = 2x - \frac{5}{2}$, $m = 2$, $c = -\frac{5}{2}$
 (n) $y = \frac{2}{3}x - \frac{4}{3}$, $m = \frac{2}{3}$, $c = -\frac{4}{3}$
6. (a) I (b) G (c) H (d) E (e) B
 (f) D (g) C (h) F

Exercise 8J

1. $y = 4x + 3$
2. $y = 2x - 4$
3. $y = -2x + 5$
4. $y = -3x - 2$
5. $y = -x - 4$
6. $y = 5x - 6$

Exercise 8K

1. (a) $y = 6$ (b) $a = 17$ (c) $M = 13$
(d) $C = -10$

2. (a)

Q	-3	-2	-1	0	1	2	3
P	-6	-4	-2	0	2	4	6

- (b)

m	-3	-2	-1	0	1	2	3
n	-7	-4	-1	2	5	8	11

- (c)

a	-3	-2	-1	0	1	2	3
b	-6	-5	-4	-3	-2	-1	0

- (d)

x	-3	-2	-1	0	1	2	3
y	-1	1	3	5	7	9	11

- (e)

c	-3	-2	-1	0	1	2	3
d	-13	-10	-7	-4	-1	2	5

- (f)

N	-3	-2	-1	0	1	2	3
M	-9	-5	-1	3	7	11	15

3. (a)

x	-3	-2	-1	0	1	2	3
y	-3	-2	-1	0	1	2	3

- (b)

x	-3	-2	-1	0	1	2	3
y	-3	-1	1	3	5	7	9

- (c)

x	-3	-2	-1	0	1	2	3
y	-8	-7	-6	-5	-4	-3	-2

- (d)

x	-3	-2	-1	0	1	2	3
y	-10	-7	-4	-1	2	5	8

- (e)

x	-3	-2	-1	0	1	2	3
y	-7	-3	1	5	9	13	17

- (f)

x	-3	-2	-1	0	1	2	3
y	3	2	1	0	-1	-2	-3

- (g)

x	-3	-2	-1	0	1	2	3
y	5	4	3	2	1	0	-1

- (h)

x	-3	-2	-1	0	1	2	3
y	9	7	5	3	1	-1	-3

- (i)

x	-3	-2	-1	0	1	2	3
y	10	7	4	1	-2	-5	-8

4. (a) $m = 1, c = 0$
(b) $m = 2, c = 3$
(c) $m = 1, c = -5$
(d) $m = 3, c = -1$
(e) $m = 4, c = 5$
(f) $m = -1, c = 0$
(g) $m = -1, c = 2$
(h) $m = -2, c = 3$
(i) $m = -3, c = 1$

5. (a) $y = 4x$ (b) $A = B + 3$
(c) $d = c - 2$ (d) $N = 2M - 1$
(e) $b = 3a + 1$ (f) $Q = 2P + 2$
(g) $y = 4x - 1$ (h) $n = 2m - 4$
(i) $D = 3C - 2$ (j) $y = 5x + 2$

6. (a)

P	2	3	4	5	6	7
R	3	6	9	12	15	18

- (b) $R = 3P - 3$

- (c)(i) 27 (ii) 72 (iii) 237

7. (a)

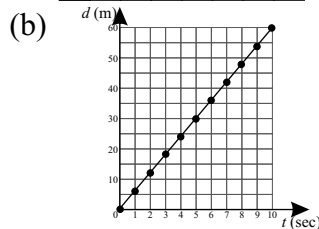
T	1	2	3	4
C	6	10	14	18

- (b) $C = 4T + 2$

- (c)(i) 26 (ii) 42 (iii) 50

8. (a)

t	0	1	2	3	4	5	6
d	0	6	12	18	24	30	36



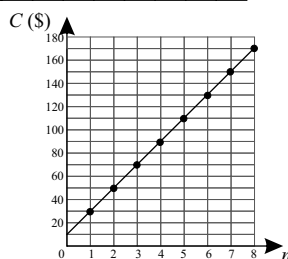
- (d) Straight line (e) $d = 6t$

- (f)(i) 120 m (ii) 360 m (iii) 21 600 m

9. (a)

- (b)

n	1	2	3	4	5	6	7	8
C	30	50	70	90	110	130	150	170

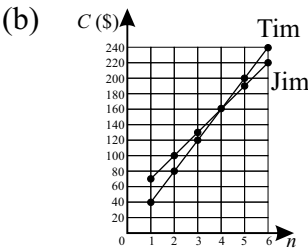


- (d) Straight line (e) $C = 20n + 10$

- (f) \$510

10. (a)

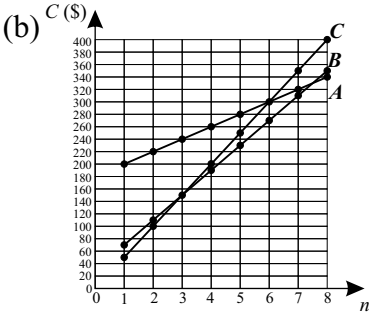
n hours	1	2	3	4	5	6
C - Jim	70	100	130	160	190	220
C - Tim	40	80	120	160	200	240



- (c) 4 hours
(d) Jim: $C = 30n + 40$
Tim: $C = 40n$

11. (a)

n days	1	2	3	4	5	6	7	8
C - Plan A	200	220	240	260	280	300	320	340
C - Plan B	70	110	150	190	230	270	310	350
C - Plan C	50	100	150	200	250	300	350	400

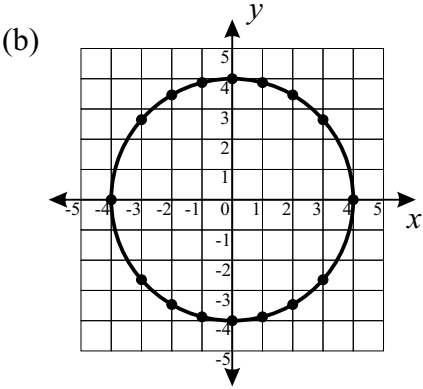


- (c)(i) C (ii) B (iii) A
(d) Plan A: $C = 20n + 180$
Plan B: $C = 40n + 30$
Plan C: $C = 50n$

Exercise 8L

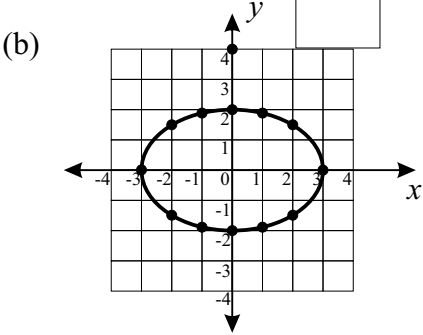
1. (a)

x	-4	-3	-2	-1	0	1	2	3	4
y	0	± 2.6	± 3.5	± 3.9	± 4	± 3.9	± 3.5	± 2.6	0



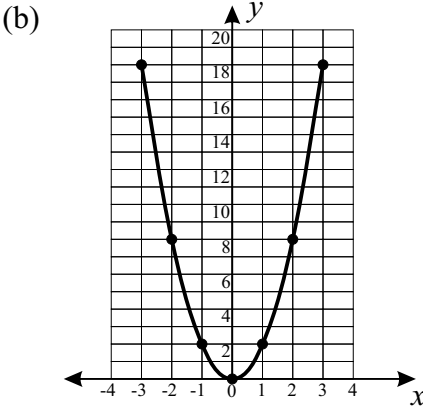
2. (a)

x	-3	-2	-1	0	1	2	3
y	0	± 1.5	± 1.9	± 2	± 1.9	± 1.5	0



3. (a)

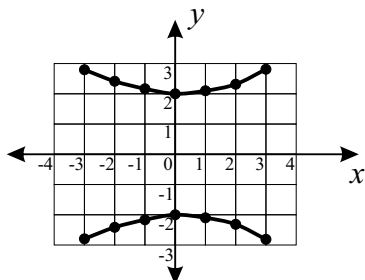
x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18



4. (a)

x	-3	-2	-1	0	1	2	3
y	± 2.8	± 2.4	± 2.1	± 2	± 2.1	± 2.4	± 2.8

(b)



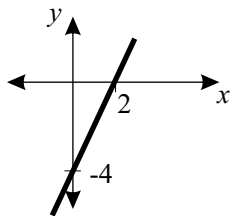
Chapter Review

1. 18.0

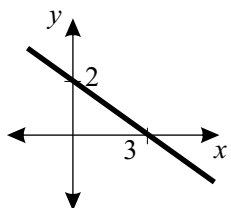
2. (2, -6)

3. (a) 5 (b) 3 (c) 4 (d) $\frac{1}{2}$ (e) -1 (f) $-\frac{4}{7}$

4. (a)



(b)



5. (a) (2, 0) and (0, -4)

(b) (3, 0) and (0, 2)

6. (a) (5, 0), (0, 5) (b) (-3, 0), (0, 2)
(c) (-1, 0) (0, 1) (d) (4, 0) (0, -12)7. (a) $m = 3$, $c = -5$ (b) $m = -1$, $c = 2$ (c) $m = \frac{2}{3}$, $c = -2$ (d) $m = -4$, $c = -\frac{1}{2}$ 8. (a) $y = 2x + 6$ (b) $y = -2x - 3$ (c) $y = \frac{3}{4}x$ 9. (a) $y = -5x + 3$, $m = -5$, $c = 3$ (b) $y = \frac{3}{2}x - 3$, $m = \frac{3}{2}$, $c = -3$ (c) $y = \frac{4}{3}x + 4$, $m = \frac{4}{3}$, $c = 4$ 10. $y = -3x + 16$

11. (a)

y	-3	-2	-1	0	1	2	3
x	-13	-10	-7	-4	-1	2	5

(b)

m	-3	-2	-1	0	1	2	3
n	9	7	5	3	1	-1	-3

12. (a) $y = 2x + 3$ (b) $a = 3b - 4$ 13. (a) $C = 20t + 50$ (b) \$210

Chapter 9

Exercise 9A

1. 149 500 000 km

2. 227 900 000 km

3. 108 200 000 km

4. 130 mm

5. 138 mm

6. 144 mm

7. 6671 km

8. 6275 km

9. 3718 km

10. 25 m

11. 30 m

12. 24 m

13. 2800 km

14. 2500 km

15. 3900 km

16. 305 m

17. 448 m

18. 8 m

19. 2230 m

20. 8850 m

Exercise 9B

1. (a) 60 mm (b) 800 cm

(c) 20 000 m (d) 2360 mm

(e) 428.5 mm (f) 680 m (g) 75 cm

(h) 0.75 m (i) 76.85 km

(j) 0.001 m (k) 798 cm

(l) 0.0276 m (m) 0.3 cm

(n) 23.57 mm (o) 578.1 m

(p) 30 000 cm (q) 430 000 mm

(r) 1.7 m (s) 0.065 mm (t) 0.0476 m

(u) 0.0006 cm (v) 0.00605 m

(w) 0.06 km (x) 0.05645 km

2. (a) 2.11 m (b) 3.2 cm (c) 6.06 m
(d) 20.2 m (e) 5.02 km (f) 18.05 m
(g) 5.08 m (h) 6.238 m (i) 4.007 m
(j) 20.4 cm (k) 3.882 m (l) 10.057 m
3. (a) 3 m (b) 66 m (c) 14 m (d) 30 m
(e) 16 m (f) 20 m (g) 3 m (h) 3 m
(i) 16 m (j) 4 m (k) 8 m (l) 10 m
(m) 11 m (n) 6 m (o) 48 m
4. (a) 3 cm (b) 3 cm (c) 16 cm
(d) 1 cm (e) 14 cm (f) 26 cm
(g) 1 cm (h) 7 cm (i) 20 cm
5. 5.6 m
6. 2.7 m, 540 cm, 5100 mm, 3.9 m
7. 108 cm
8. 312 m

Exercise 9C

1. (a) 38 m (b) 72.3 cm (c) 191.2 mm
(d) 198 m (e) 42.4 m
2. (a) 144 mm (b) 347 mm
3. (a) 600 m (b) 60 posts
4. 6.5 m
5. 7.6 cm
6. 4.5 m

Exercise 9D

1. (a) 37.7 mm (b) 268.0 cm
(c) 39.0 m
2. (a) 25.1 mm (b) 37.1 m
(c) 17.6 km (d) 628.3 cm
(e) 1.8 m (f) 5.0 km (g) 37.6 mm
(h) 207.0 m
3. (a) 94.2 mm (b) 162.1 m
(c) 47.8 km (d) 3141.6 cm
(e) 16.3 m (f) 3.7 km
(g) 68.9 mm (h) 1042.3 m
4. (a) 51.4 m (b) 62.3 cm
(c) 17.9 m (d) 23.5 m (e) 32.3 m
5. (a) 451.3 m (b) 9.027 km
(c) 11 laps (d) 51.3 m
(e) 114.2 m
6. 1762 m
7. 1635 cm
8. (a) 61.8 m (b) 309 (c) 618 m

9. (a) 3.5 m (b) 12.7 cm (c) 2.7 km
(d) 63.7 mm (e) 0.3 m
10. (a) 2.4 m (b) 11.9 cm (c) 0.4 km
(d) 79.6 mm (e) 0.3 m
11. 1098 mm
12. 318 mm

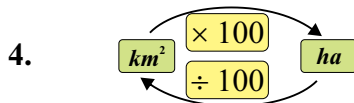
Chapter Review

1. (a) 140 mm (b) 830 cm
(c) 4000 m (d) 560 mm
(e) 97.5 mm (f) 1260 m
(g) 8.3 cm (h) 4.5 m (i) 3.45 km
(j) 0.029 m (k) 55 mm (l) 35.6 m
2. (a) 2.11 m (b) 3.2 cm (c) 6.06 m
(d) 8.075 m
3. (a) 5 m (b) 7 m (c) 3 m (d) 3 m
(e) 785 m (f) 399 m
4. (a)(i) 12.6 cm (ii) 21.4 m
(iii) 1.9 km (iv) 112.5 mm
(b)(i) 408.4 cm (ii) 18.8 m
(iii) 41.1 m (iv) 562.3 km
5. (a) 80 cm (b) 58.3 m

Chapter 10

Exercise 10A

1. (a) 70 m^2 (b) 9000 mm^2
(c) 6400 cm^2 (d) $30\,000 \text{ m}^2$
(e) 2500 m^2 (f) 0.5 km^2
(g) 75 mm^2 (h) 42 ha (i) 0.8 m^2
(j) 200 m^2
2. (a) 60 ha (b) 617.5 acres
3. (a) 1215 ha (b) 6175 acres



5. (a) 1 980 400 ha (b) $23\,677 \text{ km}^2$

Exercise 10B

1. (a) 144 m^2 (b) 87.77 m^2
(c) 3192.25 mm^2 (d) 3.57 km^2
(e) 243.36 mm^2 (f) 121.6 cm^2
2. (a) 20 cm^2 (b) 24.225 m^2 (c) 180.6 m^2
(d) 245.76 m^2 (e) 3625 mm^2

3. (a) 108 cm^2 (b) 96.9 m^2
 (c) 601.6 m^2
 4. (a) 30 m^2 (b) 238 cm^2 (c) 42 m^2
 (d) 281.815 m^2 (e) 439.875 cm^2
 5. (a) 113.1 m^2 (b) 3217.0 mm^2
 (c) 248.8 m^2 (d) 105.7 m^2

Exercise 10C

1. (a) 1170.7 m^2 (b) 180 cm^2
 (c) 340 cm^2 (d) 208 cm^2
 (e) 1606.7 cm^2 (f) 99 cm^2
 (g) 609.4 cm^2
 2. 41.5 m^2
 3. (a) 8.7 cm^2 (b) 15.4 cm^2
 (c) 26 cm^2

Exercise 10D

1. (a)(i) 76 m^2 (ii) 389 m^2
 (b)(i) 10.1 litres (ii) 51.9 litres
 2. (a) 3.2 m^2 (b) 1280 tiles
 3. (a)(i) 416 583 ha (ii) 4166 km^2
 (b) 64.5 km (c) 459 000 ha

Exercise 10E

1. (a) **I** (b) **E** (c) **G** (d) **H** (e) **F**
 (f) **A** (g) **B** (h) **C** (i) **D**
 2. **D**
 3. **E**
 4. (a) **A** = 4, **B** = 5, **C** = 1
 (b) **A** = 3, **B** = 6, **C** = 2
 (c) **A** = 2, **B** = 4, **C** = 6
 (d) **A** = 2, **B** = 1, **C** = 4
 (e) **A** = 5, **B** = 3, **C** = 6
 (f) **A** = 4, **B** = 1, **C** = 2

Exercise 10F

1. 108 m^2
 2. 680 cm^2
 3. 1600 cm^2
 4. 551.5 m^2
 5. 3498.72 mm^2
 6. $15\,544.52 \text{ m}^2$

Exercise 10G

1. (a) 520 cm^2 (b) 1056 cm^2 (c) 408 cm^2
 2. 486 cm^2
 3. 74.77 m^2
 4. 77.62 m^2

Exercise 10H

1. 4.8 m^2
 2. 246 cm^2
 3. (a) 201.1 cm^2 (b) 50.3 m^2 (c) 172.0 m^2

Exercise 10I

1. 1407.4 cm^2
 2. 88.0 m^2
 3. 215.1 m^2
 4. 187.7 m^2

Chapter Review

1. (a) 6 m^2 (b) 580 mm^2
 (c) 90 cm^2 (d) $600\,000 \text{ m}^2$
 (e) 7000 m^2 (f) 3.1 km^2
 (g) 125 cm^2 (h) 4 ha
 2. (a) 108 m^2 (b) 645.2 mm^2
 (c) 150 cm^2 (d) 192.5 cm^2
 (e) 1431.3 m^2 (f) 28.3 m^2
 (g) 370.5 cm^2
 3. (a) 89.1 cm^2 (b) $14\,461.7 \text{ mm}^2$
 4. (a) 576 cm^2 (b) 1680 cm^2
 (c) 21.25 m^2 (d) 156.7 m^2

Chapter 11

Exercise 11A

1. (a) 50 L (b) 20 mm^3 (c) 0.6 L
 (d) 8100 L (e) 250 mL
 (f) 890 cm^3 (g) 0.3 m^3
 (h) 7000 mm^3 (i) 8 mL (j) 0.1 L
 (k) 2400 cm^3 (l) 0.75 cm^3
 2. 6 L
 3. (a) L (b) cm^3 (c) mm^3 (d) m^3
 (e) L (f) L (g) cm^3 (h) L (i) m^3
 (j) L
 4. 50 glasses

5. 750 shovels
 6. 50 days
 7. 10 minutes
 8. (a) 720 L (b) 200 minutes
 9. 6 hours

Exercise 11B

1. (a) 210 cm^3 (b) 420 cm^3
 (c) 280 cm^3 (d) 30 m^3 (e) 6.4 m^3
 (f) 2.64 m^3
 2. (a) 226.2 m^3 (b) 9.5 m^3
 (c) $62\,831.9 \text{ cm}^3$ (d) 4188.8 cm^3
 (e) 65.4 m^3 (f) $16\,210.2 \text{ cm}^3$
 3. 2.25 m^3
 4. (a) 120 m^3 (b) 1767.1 m^3
 (c) 15 grain containers
 5. 16.1 cm
 6. (a) 2000 L (b) 100 mm
 (c) 150 000 L
 (d) No (182 500 L used)
 7. **A** - 2 m cube, **B** - $2 \text{ m} \times 3 \text{ m} \times 4 \text{ m}$
 8. (b) 65.4 cm^3
 9. (a) 270 L (b) 18 fish (c) 900 cm^3
 (d) 2.4 cm

Exercise 11C

1. (a) **A** - 24 cm^3 , **B** - 24 cm^3
 (b) **A** - 52 cm^2 , **B** - 56 cm^2
 (c) 3 cm
 (d)

Length	Width	Height	Surface Area
4	3	2	52
6	2	2	56
8	1	3	70
24	1	1	98
4	6	1	68
12	2	1	76

2. (a) 8 cm^3 (b) 28 cm^2 (c) 24 cm^2
 3. (a) 96 cm^2 (b) 44 cm^3 (c) 64 cm^3
 4. (a) 180 L (b) \$94.20
 5. (a) 1 L (b) 213 mm (c) 187 mm
 (d) 250 mL (e) 64 mm

6. (a)

x (cm)	l (cm)	w (cm)	h (cm)	V (L)
10	180	130	10	234
20	160	110	20	352
30	140	90	30	378
40	120	70	40	336
50	100	50	50	250
60	80	30	60	144

- (b) 30 cm
 7. (a) 145.2987 m^3 (b) 145 299 L
 (c) 155.5 m^2
 8. (a) 2.4 m (b) 0.764 m (c) 550 L
 (d) 1074 L
 9. (a) 461 mm (b) 1447 mm

Chapter Review

1. (a) 8 L (b) 7000 mm^3 (c) 0.9 L
 (d) 550 L (e) 10 mL (f) 30 cm^3
 (g) 0.75 m^3 (h) 9350 mm^3
 (i) 15.6 mL (j) 0.05 L
 (k) 3950 cm^3 (l) 0.15 cm^3
 2. (a) 300 cm^3 (b) 720 cm^3
 (c) 72 m^3 (d) 75.4 m^3
 (e) $17\,157.3 \text{ cm}^3$
 3. (a) $15\,625 \text{ cm}^3$ (b) 12.6 cm
 4. 603 L
 5. 1000 cm^3

Chapter 12

Exercise 12A

1. **A, B**
 2. **C**
 3. 24 cm
 4. 16 cm

Exercise 12B

1. (a) 3 (b) 1:3
 2. (a) 5 (b) 1:5
 3. (a) 4 (b) 1:4
 4. (a) 3.2 (b) 5:16
 5. (a) 3.5 (b) 2:7
 6. (a) 7.5 (b) 2:15
 7. (a) 2.5 (b) 2:5

Exercise 12C

- $a = 18 \text{ m}$
- $b = 15 \text{ cm}$
- $x = 14 \text{ cm}, y = 18 \text{ cm}$
- $m = 9 \text{ m}, n = 15 \text{ m}$
- $g = 20 \text{ m}, h = 28 \text{ m}$
- $v = 10 \text{ m}, w = 12.5 \text{ m}$
- $k = 20 \text{ cm}$
- $t = 9 \text{ m}$
- $x = 12.8 \text{ m}$
- $h = 11.25 \text{ km}$
- $a = 12 \text{ m}, b = 7.5 \text{ m}$
- $p = 36 \text{ m}, q = 45 \text{ m}$
- $a = 10.5 \text{ m}, b = 6 \text{ m}$
- $m = 47 \text{ m}$

Exercise 12D

- 22 m
- (a) 10 m (b) 6 m
- 5.25 m
- (a) 40 km (b) 3.9 km
- 48 m
- 270 m
- A-B-C** = 2000 m
A-F-B-D-C = 2800 m
A-F-E-D-C = 2800 m
- $\overline{BE} = 10 \text{ m}$

Exercise 12E

- A, E, I, J**
- (a) **A & R, B & T, D & S, F & K, G & I, H & N**
 (b) **C & O, E & P, J & U, L & V, M & Q**
- (a) **D** (b) **E** (c) **H** (d) **K** (e) **B**
 (f) **J** (g) **F** (h) **C** (i) **I** (j) **A**
 (k) **L** (l) **G** (m) **M**
- (a) **T** (b) **F** (c) **T** (d) **T** (e) **F** (f) **F**
- (a) **T** (b) **F** (c) **F** (d) **T** (e) **T** (f) **F**
 (g) **F**

Exercise 12F

A & K, B & H, C & I, D & G, E & J, F & L,

Exercise 12G

- (a)(i) 600 m (ii) 380 m (iii) 950 m
 (iv) 520 m
 (b)(i) 45 mm (ii) 8 mm (iii) 200 mm
 (iv) 154 mm
- (a)(i) 160 km (ii) 310 km (iii) 150 km
 (iv) 244 km
 (b)(i) 250 mm (ii) 89 mm (iii) 168 mm
 (iv) 45 mm
- (a) 120 cm (b) 25 m
- 1:100
- (a) 16 m (b) 8 m
 (c)(i) $5 \text{ m} \times 4 \text{ m}$ (ii) $4 \text{ m} \times 3 \text{ m}$
 (iii) $3 \text{ m} \times 3 \text{ m}$
- (a) 6 cm (b) 8 cm long, 5 cm wide
 (c) 14 cm high, 10 cm wide
 (d) 15 cm long, 9 cm wide
- (a) 16 cm (b) 9 m
- (a) 8500 m (b) 20 cm
- 3 hours
- (a) 40 m (b) 37.5 m
- (a) 470 km (b) 250 km (c) 120 km
 (d) 60 km

Exercise 12H

1.

<i>Planet/Sun</i>	<i>Diameter (m)</i>	<i>Diameter of Models (cm)</i>
Sun	1 391 400	2730
Mercury	4879	10
Venus	12 104	24
Earth	12 742	25
Mars	6779	13
Jupiter	139 822	274
Saturn	116 464	229
Uranus	50 724	100
Neptune	49 244	97

2.

Planet/Sun	Distance from the Sun (millions of km)	Distance from 'the Sun' on the oval (metres)
Mercury	58	1.3
Venus	108	2.4
Earth	150	3.3
Mars	228	5.1
Jupiter	779	17.3
Saturn	1429	31.8
Uranus	2871	63.8
Neptune	4498	100

3.

Planet/Sun	Mass ($\times 10^{21}$ kg)	Relative Mass
Sun	20 000 000	333 333.33
Mercury	3	0.05
Venus	49	0.82
Earth	60	1
Mars	6	0.1
Jupiter	19 000	316.67
Saturn	5700	95
Uranus	870	14.5
Neptune	1000	16.67

4.

Continent	Area (km ²)	Relative Area
Australia	7 687 000	1
Europe	9 938 000	1.29
Antarctica	13 209 000	1.72
South America	17 819 000	2.32
North America	24 256 000	3.16
Africa	30 065 000	3.91
Asia	44 579 000	5.80

5.

Part	Size (mm)	Relative Size
Diameter of Blood Cell	0.002	1
Length of Gut Bacteria	0.01	5
Diameter of Skin Cell	0.03	15
Length of Human Sperm	0.05	25
Width of Hair	0.08	40
Diameter of Brain Cell	0.1	50
Diameter of Human Egg	0.2	100
Thickness of Fingernail	0.4	200

Exercise 12I

1.

Shape	Area of Shape	Enlargement Factor	Area of Enlarged Shape
A	10 cm ²	2	40 cm ²
B	4 cm ²	3	36 cm ²
C	20 cm ²	10	2000 cm ²

2. The area of the enlarged shape is equal to the area of the original shape multiplied by the square of the enlargement factor.

3. (a) 32 cm² (b) 270 cm² (c) 375 cm²
(d) 850 cm² (e) 207.36 cm²
(f) 53 040 cm²

4. (a)(i) 300 000 cm² (ii) 30 m²
(b)(i) 260 000 000 cm² (ii) 26 000 m²
(c)(i) 3 000 000 000 mm²
(ii) 30 000 000 cm²
(iii) 3000 m²

5. 1200 m²

6. (a) 100 000 m² (b) 10 hectares

7. (a) 1 920 000 m² (b) 192 hectares

8. 20 000 km²

Chapter Review

1. (a) $x = 24$ m, $y = 30$ m

(b) $n = 5$ m, $m = 20$ m

(c) 40 m

(d) $a = 30$ m, $b = 24$ m

(e) $m = 25.2$ m

(f) 27 m

2. (a) T (b) T (c) T (d) F (e) T

3. (a)(i) 100 m (ii) 315 m (iii) 575 m

(b)(i) 160 mm (ii) 400 mm

(iii) 124 mm

4. 1:500

5. Alpha Apartments - 80 cm

Bravo Building - 105 m

6. Basil - 1

Polly - 140

Sybil - 700

Chapter 13

Exercise 13A

- (a) **A, C, E, G**
- (a) **C** (b) **P** (c) **Y**
- (a) 5 cm (b) 3 cm & 4 cm
(c) $3^2 + 4^2 = 25$, $5^2 = 25$
- (a) True (b) False (c) True
(d) True (e)(i) False (ii) True
(iii) True (iv) True (v) True
(f) True (g) False (h) False
- (a) 324 (b) 42.25 (c) 179.56
(d) 76.0384
- (a) Yes (b) Yes (c) No (d) Yes
(e) No (f) Yes (g) Yes (h) No
(i) Yes (j) No (k) Yes (l) Yes
- 308, 75, 317
- The sum of the squares of the two shorter sides (the two smaller areas) is equal to the square of the larger side (the larger area).

Exercise 13B

- (a) 5 m (b) 13 cm (c) 39 m
(d) 25 mm (e) 25 m (f) 37 cm
(g) 89 m (h) 225 cm (i) 481 m
- (a)(i) $\sqrt{65}$ cm (ii) 8.1 cm
(b)(i) $\sqrt{146}$ cm (ii) 12.1 cm
(c)(i) $\sqrt{773}$ cm (ii) 27.8 cm
- (a) 1 m (b) $\frac{10}{11}$ m (c) 1 m (d) $\frac{13}{9}$ m
(e) $\frac{5}{2}$ m (f) $\frac{17}{16}$ m
- (a) 7.8 m (b) 13.5 cm (c) 20.7 cm
- (a) $x = 21.8$ cm (b) $y = 28.3$ cm
(c) $a = 18.6$ cm, $b = 20.7$ cm
(d) $m = 26.6$ cm (e) $c = 8.6$ m
(f) $s = 2.8$ m

Exercise 13C

- (a) 7 cm (b) 84 m (c) 30 mm
(d) 18 m (e) 15 m (f) 21 mm
(g) 55 m (h) 36 m (i) 60 m
- (a) 7.1 m (b) 13.9 cm (c) 13.0 cm
- (a) $x = 5.7$ m
(b) $c = 12.4$ cm, $d = 10.8$ cm
(c) $k = 19.1$ cm (d) $t = 4.5$ m
(e) $w = 5.2$ m (f) $y = 23.7$ m

Exercise 13D

- 35.4 cm
- 105.0 cm
- (a) $a = 12.2$ m (b) $b = 13.8$ m
(c) $p = 36.1$ m, $q = 17.3$ m
(d) $x = 23.9$ cm, $y = 6.3$ cm
(e) $m = 6.3$ m, $n = 16.1$ m
(f) $d = 14.9$ cm

Exercise 13E

- (a) 2211 mm (b) **B & D**
- (a) 11.2 m (b) 26.9 m
- 27.4 km
- (a) 1442 m
(b) Noel - 250 sec
Paddy - 288.4 sec
- 5.558 m
- 113.3 cm
- (a) 1093.7 km (b) 1102.6 km
A-B-C-D-A is shorter
- 1.414 m
- (a) A STRAIGHT LINE
(b) AS THE CROW FLIES

Exercise 13F

- 128.8 cm
- 5.4 m
- 23.7 cm
- 4.1 m

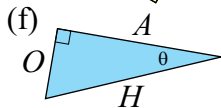
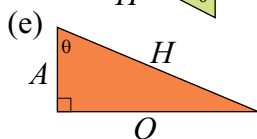
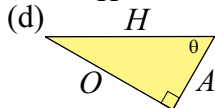
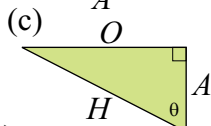
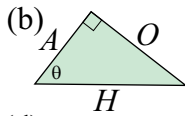
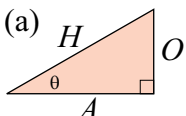
Chapter Review

- (a) $a = 18.6$ m (b) $b = 24.2$ m
(c) $p = 19.4$ m, $q = 14.1$ m
(d) $x = 97.7$ cm, $y = 35.4$ cm
- 19.5 km
- 9.9 m
- 9.9 m

Chapter 14

Exercise 14A

- (a) c (b) r (c) t
- (a) t (b) v (c) u
- (a) r (b) p (c) p (d) r (e) q
- (a)



Exercise 14B

- (a) $\frac{m}{l}$ (b) $\frac{8}{10}$ (c) $\frac{12}{13}$ (d) $\frac{d}{e}$
- (a) $\sin \alpha = \frac{a}{c}$ (b) $\sin \beta = \frac{b}{c}$
- (a) 0.4706 (b) 0.3243 (c) 0.2800 (d) 0.9692
- (a) 0.8290 (b) 0.7071 (c) 0.8660 (d) 0.6184 (e) 0.5736 (f) 0.5000 (g) 0.0872 (h) 0.9998 (i) 1.0000

Exercise 14C

- $a = 12.5$ m (2) $b = 10.9$ m
- $c = 3.9$ m (4) $d = 39.8$ cm
- $e = 11.1$ m (6) $f = 7.1$ cm
- $g = 30.5$ mm (8) $h = 3.4$ m

Exercise 14D

- $p = 14.7$ m (2) $q = 29.0$ m
- $r = 7.6$ m (4) $s = 106.4$ cm
- $t = 4.9$ m (6) $u = 5.0$ cm
- $v = 55.7$ cm (8) $w = 9.7$ mm

Exercise 14 E

- (a) $a = 8.5$ m (b) $b = 5.4$ m (c) $c = 2.0$ m (d) $d = 10.4$ cm
- 4.45 km
- 3.53 m

Exercise 14F

- (a) $\frac{n}{l}$ (b) $\frac{6}{10}$ (c) $\frac{5}{13}$ (d) $\frac{f}{e}$
- (a) $\cos \beta = \frac{a}{c}$ (b) $\cos \alpha = \frac{b}{c}$
- (a) 0.8824 (b) 0.9459 (c) 0.2800 (d) 0.2462
- (a) 0.5592 (b) 0.7071 (c) 0.5000 (d) 0.7859 (e) 0.8192 (f) 0.8660 (g) 0.9588 (h) 1.0000 (i) 0

Exercise 14G

- $a = 21.7$ m (2) $b = 11.7$ m
- $c = 8.1$ m (4) $d = 26.8$ cm
- $e = 5.9$ m (6) $f = 5.0$ cm
- $g = 71.8$ mm (8) $h = 1.6$ m

Exercise 14H

- $p = 9.5$ m (2) $q = 25.2$ m
- $r = 3.4$ m (4) $s = 141.2$ cm
- $t = 3.1$ m (6) $u = 3.8$ cm
- $v = 171.5$ cm (8) $w = 22.8$ mm

Exercise 14I

- (a) $a = 6.2$ m (b) $b = 5.6$ m (c) $c = 5.5$ m (d) $d = 16.0$ cm
- 26.8 km
- 9.0 m

Exercise 14J

- (a) $\frac{m}{n}$ (b) $\frac{8}{6}$ (c) $\frac{12}{5}$ (d) $\frac{d}{f}$
- (a) $\tan \alpha = \frac{a}{b}$ (b) $\tan \beta = \frac{b}{a}$
- (a) 0.5333 (b) 0.3429 (c) 3.4286 (d) 3.9375
- (a) 1.4826 (b) 1.0000 (c) 1.7321 (d) 0.7869 (e) 0.7002 (f) 0.5774 (g) 0.0875 (h) 6.1066 (i) 28.6363

Exercise 14K

- $a = 14.4$ m (2) $b = 14.9$ m
- $c = 4.4$ m (4) $d = 71.2$ cm
- $e = 23.7$ m (6) $f = 12.4$ cm
- $g = 33.1$ mm (8) $h = 7.9$ m

Exercise 14L

1. $p = 12.3$ m 2. $q = 21.9$ m
3. $r = 7.0$ m 4. $s = 64.1$ cm
5. $t = 4.2$ m 6. $u = 4.0$ cm
7. $v = 17.2$ cm 8. $w = 3.8$ mm

Exercise 14M

1. (a) $a = 3.6$ m (b) $b = 8.1$ m
(c) $c = 2.1$ m (d) $d = 5.6$ cm
2. 10.1 m
3. 5.4 m

Exercise 14N

1. $a = 21.7$ m 2. $b = 10.9$ m
3. $c = 4.4$ m 4. $d = 49.5$ cm
5. $e = 19.2$ m 6. $f = 12.4$ cm
7. $g = 199.6$ mm 8. $h = 3.4$ m
9. $i = 9.8$ m, $j = 16.3$ m
10. $k = 19.4$ cm, $l = 7.6$ cm

Exercise 14O

1. 7464.1 m
2. (a) 2.7 km (b) 2.2 km
3. (a) 14.9 km (b) 33.8 km
4. (a) 8.8 m (b) 8.2 m
5. (a) 84.0 cm (b) 10.9 m
6. 77.9 cm
7. 2.6 m
8. 3.8 m
9. 147.2 m
10. 3.5 m
11. (a) 29.1 cm (b) 11 steps

Exercise 14P

1. (a) $\theta = 32.9^\circ$ (b) $\alpha = 43.4^\circ$
(c) $\beta = 24.0^\circ$ (d) $\delta = 53.5^\circ$
(e) $\phi = 62.0^\circ$ (f) $\lambda = 57.0^\circ$
2. N 30° E
3. (a) N 48° E (b) S 48° W
4. 66.4°
5. (a) 76.0° (b) 66.1° (c) closer
6. 22.5°
7. 13.1°

Chapter Review

1. (a) $a = 23.5$ m (b) $b = 8.5$ m
(c) $c = 21.6$ m (d) $d = 90.1$ cm
(e) $e = 11.6$ m (f) $f = 10.9$ cm
2. (a) $\theta = 44.4^\circ$ (b) $\alpha = 47.7^\circ$
3. (a) 32.0° (b) 14.2 m

Chapter 15**Exercise 15A**

1. (a) $\frac{12}{25}$ (b) $\frac{13}{25}$
2. (a) $\frac{10}{26} = \frac{5}{13}$ (b) $\frac{16}{26} = \frac{8}{13}$
3. (a) $\frac{5}{8}$ (b) 16
4. (a) $\frac{2}{6} = \frac{1}{3}$ (b) $\frac{1}{6}$
5. (a) $\frac{8}{14} = \frac{4}{7}$ (b) $\frac{6}{12} = \frac{1}{2}$
6. (a) $\frac{6}{15} = \frac{2}{5}$ (b) $\frac{4}{14} = \frac{2}{7}$
7. (a) $\frac{4}{20} = \frac{1}{5}$ (b) $\frac{3}{19}$
8. (a) $\frac{2}{7}$ (b) $\frac{1}{365}$ (non-leap year)
(c) $\frac{1}{12}$
9. (a) $\frac{1}{6}$ (b) $\frac{3}{6} = \frac{1}{2}$ (c) $\frac{2}{6} = \frac{1}{3}$ (d) 0 (e) 1
10. (a) $\frac{5}{26}$ (b) $\frac{1}{26}$ (c) $\frac{10}{51}$

Exercise 15B

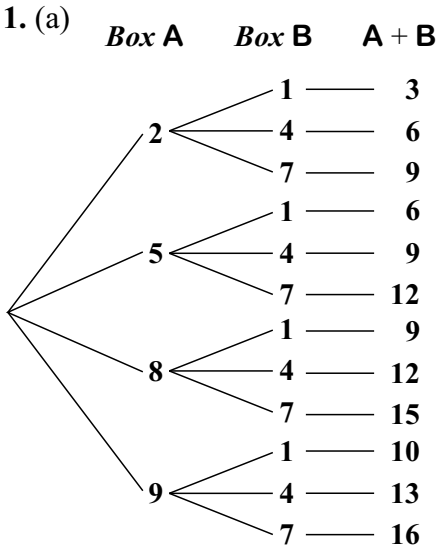
1. (a)(i) 0.46 (ii) 0.33 (iii) 0.21
(b) 0.54
2. (a) 0.6 (b) 0.8 (c) 0.2
3. (a) 0.7 (b) 0.4 (c) 0.1 (d) 0
4. 0.71
5. (a) **Matty - 0.81**, Patty - 0.66,
Dante - 0.75
(b) Dante - larger sample size
6. 0.2

Exercise 15C

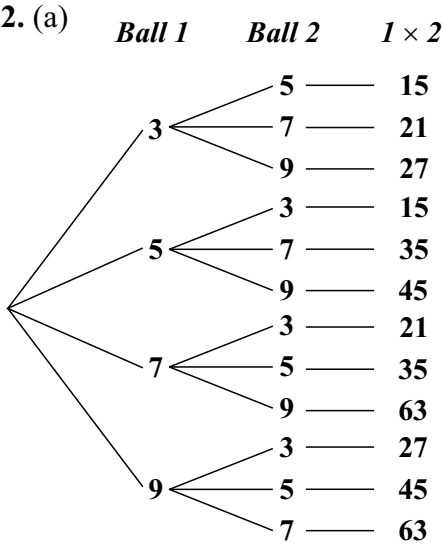
1. (a)(i) 0.6 (ii) 0.65 (b) Justine
(c) $24 + 26 = 50$
2. (a) 76% (b) 49
3. (a) 4% (b) 96% (c) 20
4. (a) September
(b)(i) 51.6% (ii) 60.7% (iii) 54.8%
(iv) 23.3% (v) 22.6%
(vi) 16.7% (vii) 22.6%
(c) 38.7%

5. Type O - 1 350 000
Type A - 1 230 000
Type B - 300 000
Type AB - 120 000

Exercise 15D

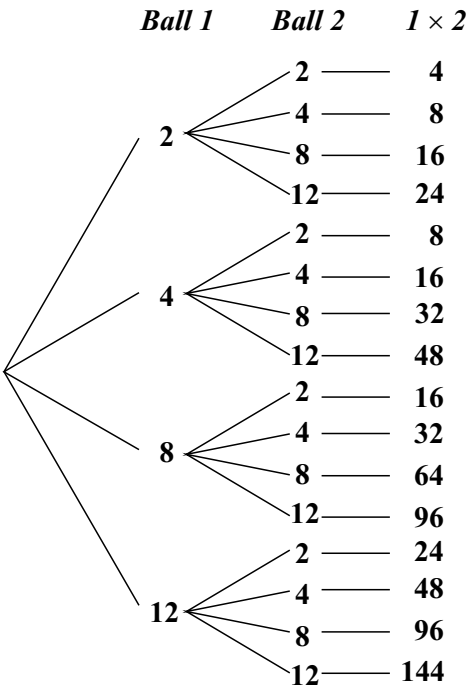


- (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{5}{12}$ (e) 1 (f) 0



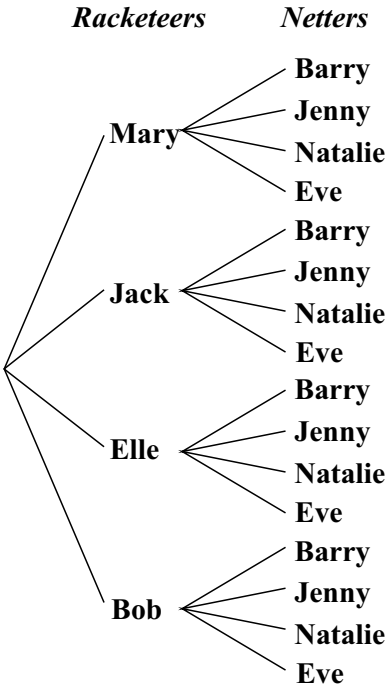
- (b) $\frac{5}{6}$ (c) 1 (d) 0 (e) $\frac{5}{6}$

3. (a)



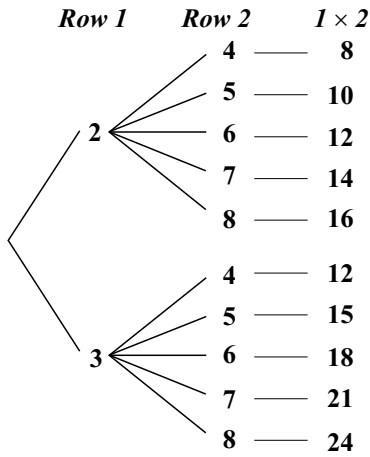
- (b) $\frac{1}{4}$ (c) 0 (d) $\frac{3}{4}$ (e) $\frac{11}{16}$

4. (a)

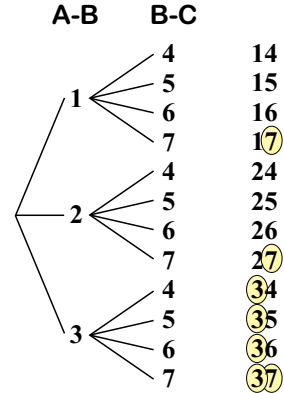


- (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) $\frac{3}{16}$ (e) $\frac{1}{4}$ (f) $\frac{3}{8}$ (g) $\frac{7}{16}$

5. (a)



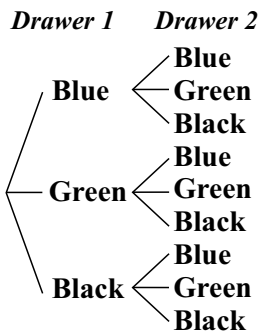
8. $\frac{6}{12} = \frac{1}{2}$



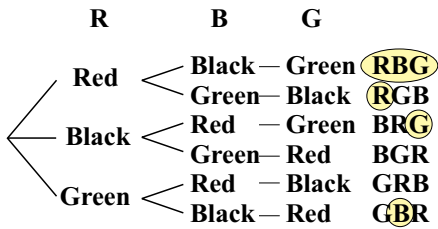
(b) $\frac{2}{10} = \frac{1}{5}$ (c) $\frac{1}{10}$

6. (a) $\frac{3}{9} = \frac{1}{3}$

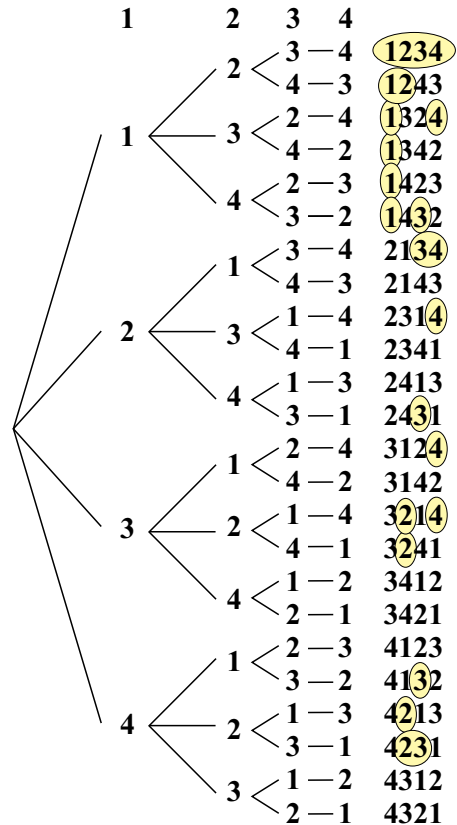
(b) $\frac{1}{9}$



7. (a) $\frac{1}{6}$ (b) $\frac{4}{6} = \frac{2}{3}$



9. (a) $\frac{1}{24}$ (b) $\frac{7}{24}$



Exercise 15E

1. (a)

		Box 1				
		0	2	4	6	8
Box 2	1	1	3	5	7	9
	3	3	5	7	9	11
	5	5	7	9	11	13
	7	7	9	11	13	15
	9	9	11	13	15	17
	11	11	13	15	17	19

(b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{4}{5}$

2. (a)

		Box 1				
		0	2	4	6	8
Box 2	1	0	2	4	6	8
	3	0	6	12	18	24
	5	0	10	20	30	40
	7	0	14	28	42	56
	9	0	18	36	54	72
	11	0	22	44	66	88

(b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{15}$

3. (a)

		Skirt				
		B	G	P	R	O
Shirt	B	BB	BG	BP	BR	BO
	G	GB	GG	GP	GR	GO
	P	PB	PG	PP	PR	PO
	R	RB	RG	RP	RR	RO
	O	OB	OG	OP	OR	OO

(b) $\frac{1}{25}$ (c) $\frac{1}{5}$ (d) $\frac{2}{25}$ (e) $\frac{4}{5}$ (f) $\frac{9}{25}$

4. (a)

		Die 1					
		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(b) $\frac{1}{9}$ (c) $\frac{1}{6}$ (d) $\frac{5}{18}$ (e) $\frac{1}{2}$ (f) $\frac{1}{2}$
(g) 7 (h) 2 and 12

5. (a)

		Die 1					
		1	2	3	4	5	6
Die 2	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

(b) $\frac{1}{9}$ (c) $\frac{7}{9}$ (d) $\frac{1}{6}$ (e) $\frac{1}{4}$ (f) $\frac{17}{36}$
(g) $\frac{3}{4}$ (h) $\frac{1}{4}$ (i) 6 and 12
(j) 1, 9, 16, 25 and 36

Exercise 15F

1. Day 1 Day 2 Outcome Probability

0.8	S	0.8	S	SS	$0.8 \times 0.8 = 0.64$
		0.2	N	SN	$0.8 \times 0.2 = 0.16$
	N	0.8	S	NS	$0.2 \times 0.8 = 0.16$
		0.2	N	NN	$0.2 \times 0.2 = 0.04$

(a) 0.32 (b) 0.64 (c) 0.04

2. Inn. 1 Inn. 2 Outcome Probability

0.6	F	0.6	F	FF	$0.6 \times 0.6 = 0.36$
		0.4	N	FN	$0.6 \times 0.4 = 0.24$
	N	0.6	F	NF	$0.4 \times 0.6 = 0.24$
		0.4	N	NN	$0.4 \times 0.4 = 0.16$

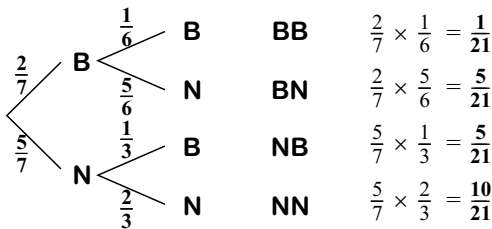
(a) 0.36 (b) 0.16 (c) 0.48

3. Puppy 1 Puppy 2 Outcome Probability

$\frac{2}{3}$	M	$\frac{5}{8}$	M	MM	$\frac{2}{3} \times \frac{5}{8} = \frac{5}{12}$
		$\frac{3}{8}$	F	MF	$\frac{2}{3} \times \frac{3}{8} = \frac{1}{4}$
	F	$\frac{3}{4}$	M	FM	$\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$
		$\frac{1}{4}$	F	FF	$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

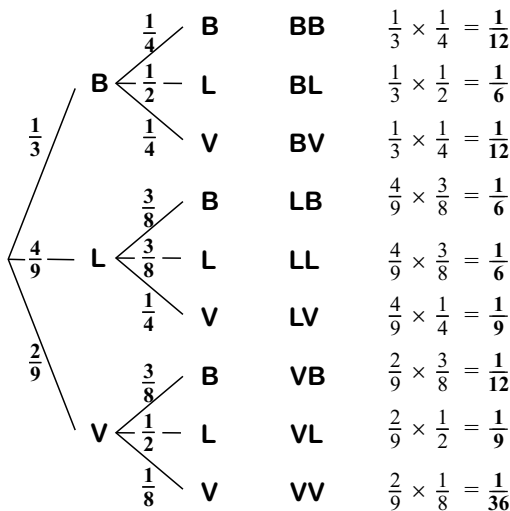
(a) $\frac{5}{12}$ (b) $\frac{1}{12}$ (c) $\frac{1}{2}$

4. Ump. 1 Ump. 2 Outcome Probability



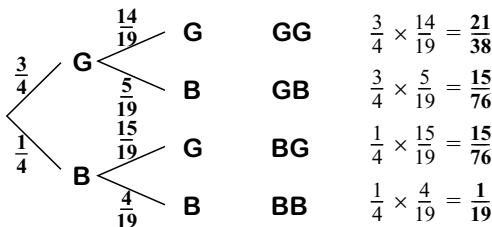
(a) $\frac{1}{21}$ (b) $\frac{10}{21}$ (c) $\frac{11}{21}$

5. Meal 1 Meal 2 Outcome Probability



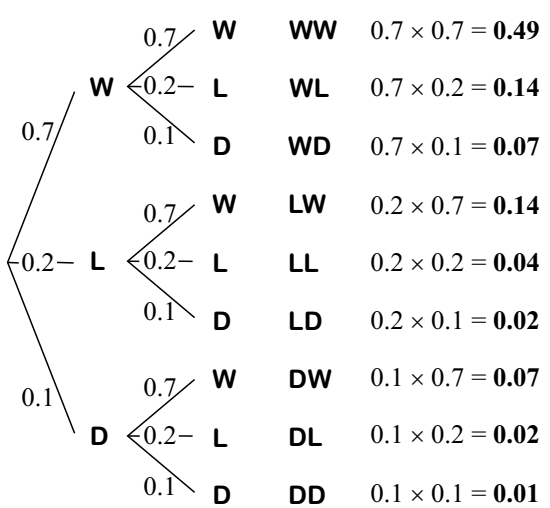
(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{3}$ (d) $\frac{1}{12}$ (e) $\frac{7}{12}$

6. Stud. 1 Stud. 2 Outcome Probability



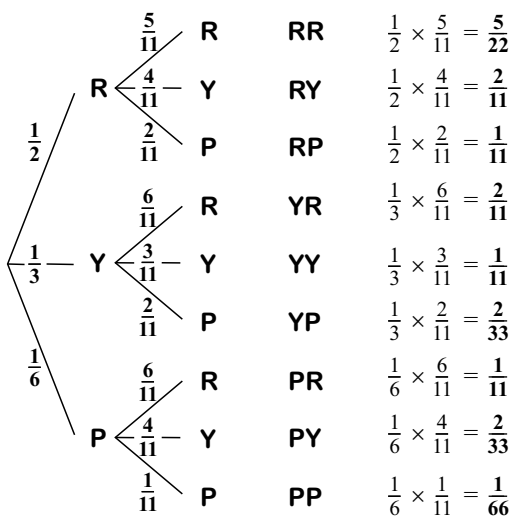
(a) $\frac{1}{19}$ (b) $\frac{21}{38}$ (c) $\frac{15}{38}$ (d) $\frac{18}{19}$

7. Game 1 Game 2 Outcome Probability



(a) 0.49 (b) 0.04 (c) 0.01 (d) 0.28
(e) 0.91

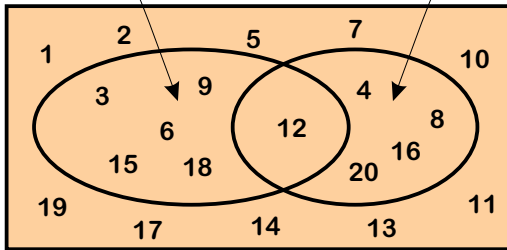
8. Rose 1 Rose 2 Outcome Probability



(a) $\frac{5}{22}$ (b) $\frac{1}{66}$ (c) $\frac{2}{11}$ (d) $\frac{14}{33}$ (e) $\frac{17}{22}$

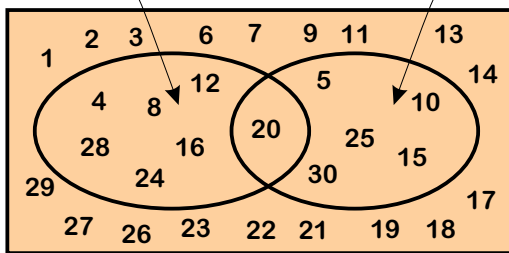
Exercise 15G

1. (a)

*Multiples of 3**Multiples of 4*

- (b) $\frac{3}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{20}$ (e) $\frac{1}{2}$ (f) $\frac{1}{4}$
 (g) $\frac{3}{10}$ (h) $\frac{3}{10}$ (i) $\frac{1}{10}$ (j) $\frac{1}{2}$ (k) $\frac{1}{3}$
 (l) 0

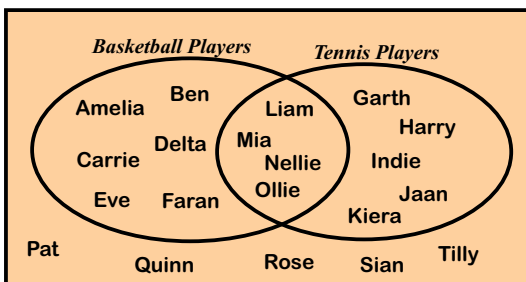
2. (a)

*Multiples of 4**Multiples of 5*

- (b) $\frac{7}{30}$ (c) $\frac{1}{30}$ (d) $\frac{2}{5}$ (e) $\frac{23}{30}$ (f) $\frac{1}{5}$
 (g) $\frac{1}{4}$ (h) $\frac{1}{5}$

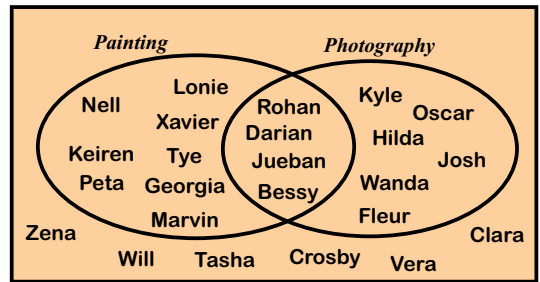
Exercise 15H

1. (a)



- (b) $\frac{3}{10}$ (c) $\frac{9}{20}$ (d) $\frac{1}{5}$ (e) $\frac{1}{4}$ (f) $\frac{1}{2}$
 (g) $\frac{4}{5}$ (h) $\frac{4}{15}$ (i) $\frac{2}{5}$ (j) $\frac{1}{2}$ (k) $\frac{5}{11}$

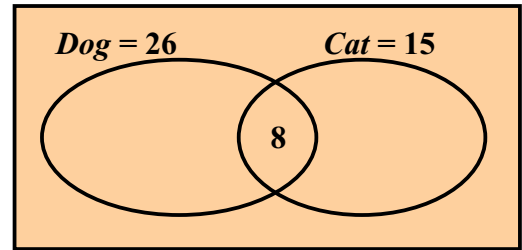
2. (a)



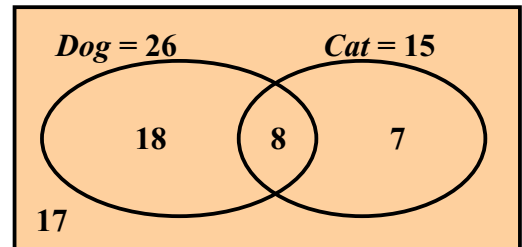
- (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$ (e) $\frac{1}{6}$ (f) $\frac{2}{9}$
 (g) $\frac{2}{3}$ (h) $\frac{1}{2}$ (i) $\frac{3}{7}$

Exercise 15I

1. (a)

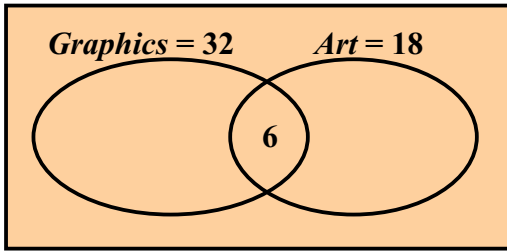
Total number of people asked = 50

(b)

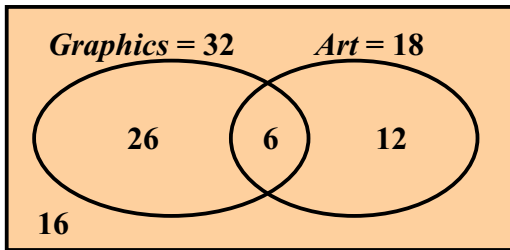
Total number of people asked = 50

- (c) 18 (d) 7 (e) 17 (f) $\frac{4}{25}$ (g) $\frac{9}{25}$ (h) $\frac{4}{13}$

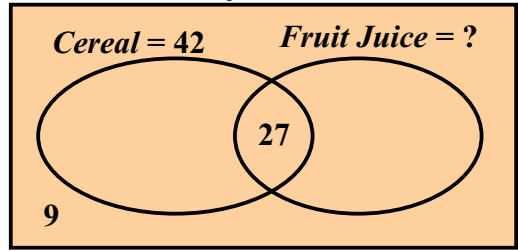
2. (a)

Total number of students = 60

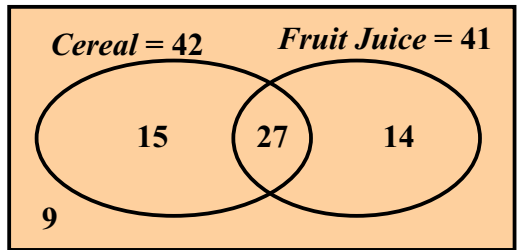
(b)

Total number of students = 60(c) 26 (d) 12 (e) 16 (f) $\frac{3}{10}$ (g) $\frac{1}{10}$ (h) $\frac{1}{3}$

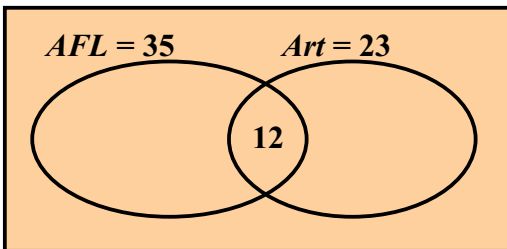
4. (a)

Total number of students = 65

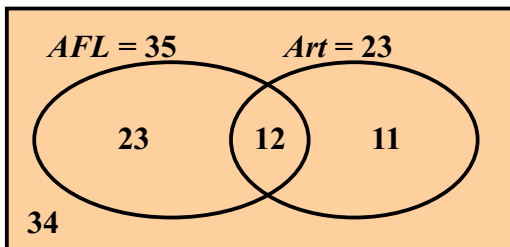
(b)

Total number of students = 65(c) 15 (d) 14 (e) 41 (f) $\frac{27}{65}$

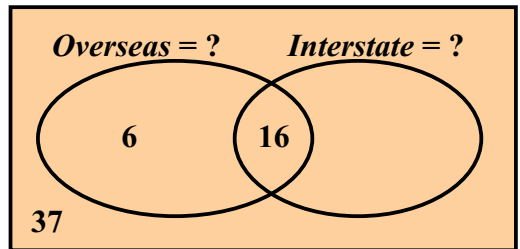
3. (a)

Total number of people asked = 80

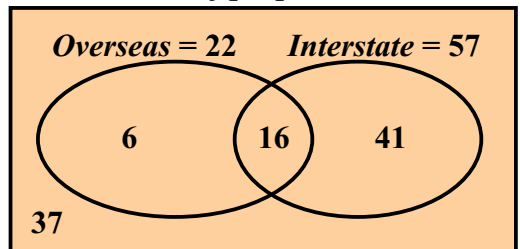
(b)

Total number of people asked = 80(c) 23 (d) 11 (e) 34 (f) $\frac{3}{20}$ (g) $\frac{11}{80}$
(h) $\frac{23}{35}$

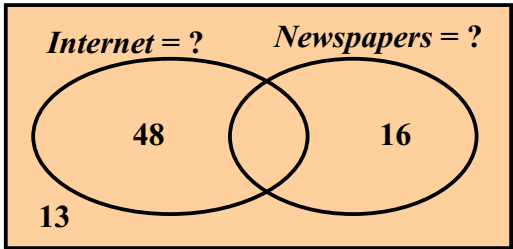
5. (a)

Total number of people asked = 100

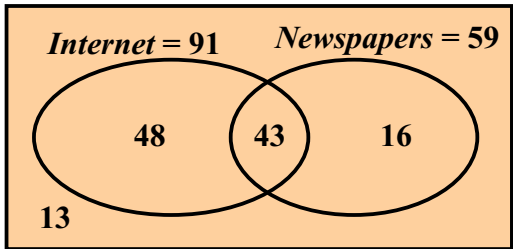
(b)

Total number of people asked = 100(c) 22 (d) 41 (e) 57 (f) $\frac{57}{100}$

6. (a)
Total number of companies = 120

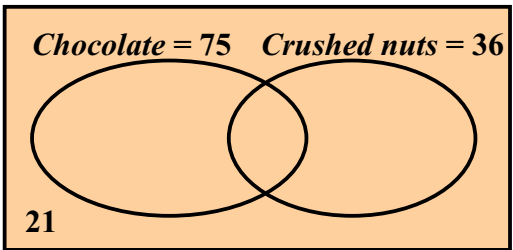


(b)
Total number of companies = 120

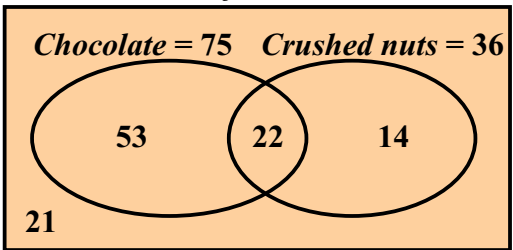


(c) $\frac{43}{120}$

7. (a)
Total number of ice-creams = 110



(b)
Total number of ice-creams = 110



(c) $\frac{1}{5}$

Chapter Review

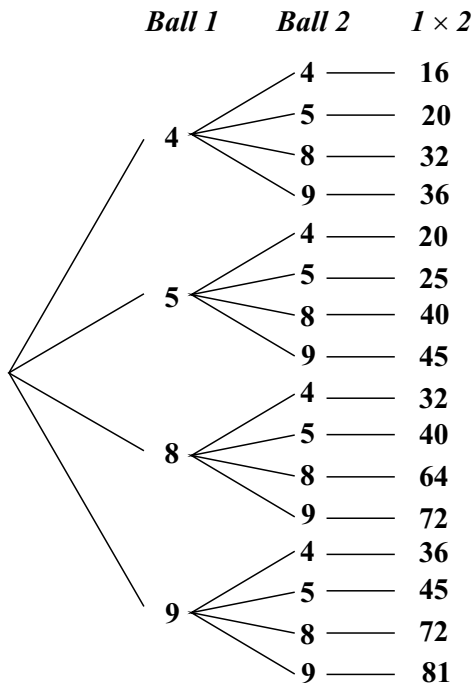
1. (a) $\frac{8}{14} = \frac{4}{7}$ (b) $\frac{5}{15} = \frac{1}{3}$
2. (a) 60% (b) 192
3. (a) 70% (b) 840 kg

4. (a)

Ball 1	Ball 2	1×2
4	5	20
	8	32
	9	36
5	4	20
	8	40
	9	45
8	4	32
	5	40
	9	72
9	4	36
	5	45
	8	72

(b)(i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) 0 (iv) $\frac{2}{3}$

5. (a)



(b)(i) $\frac{1}{4}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{16}$ (iv) $\frac{1}{2}$

6. (a)

		Jumpers				
		B	B	B	G	G
Screeners	B	BB	BB	BB	BG	BG
	G	GB	GB	GB	GG	GG
	G	GB	GB	GB	GG	GG
	B	BB	BB	BB	BG	BG
	G	GB	GB	GB	GG	GG

(b) $\frac{6}{25}$ (c) $\frac{6}{25}$ (d) $\frac{13}{25}$

7.

	Day 1	Day 2	Outcome	Probability
0.8	S	0.8 S	SS	$0.8 \times 0.8 = 0.64$
		0.2 N	SN	$0.8 \times 0.2 = 0.16$
0.2	N	0.8 S	NS	$0.2 \times 0.8 = 0.16$
		0.2 N	NN	$0.2 \times 0.2 = 0.04$

(a) 0.64 (b) 0.04 (c) 0.96

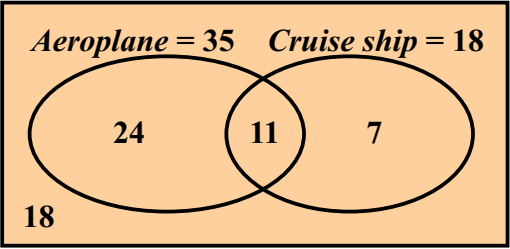
8. (a)

	Movie 1	Movie 2	Outcome	Probability
$\frac{7}{10}$	C	$\frac{2}{3}$ C	CC	$\frac{7}{10} \times \frac{2}{3} = \frac{14}{30} = \frac{7}{15}$
		$\frac{1}{3}$ M	CM	$\frac{7}{10} \times \frac{1}{3} = \frac{7}{30}$
$\frac{3}{10}$	M	$\frac{7}{9}$ C	MC	$\frac{3}{10} \times \frac{7}{9} = \frac{21}{90} = \frac{7}{30}$
		$\frac{2}{9}$ M	MM	$\frac{3}{10} \times \frac{2}{9} = \frac{6}{90} = \frac{1}{15}$

(b) $\frac{7}{15}$ (c) $\frac{1}{15}$ (d) $\frac{14}{15}$

9. (a)(b)

Total number of people asked = 60



(c) $\frac{11}{60}$ (d) $\frac{3}{10}$ (e) $\frac{11}{35}$

Chapter 16

Exercise 16A

1. Discrete numerical
2. Categorical
3. Discrete numerical
4. Continuous numerical
5. Discrete numerical
6. Continuous numerical
7. Continuous numerical
8. Categorical
9. Discrete numerical
10. Discrete numerical
11. Categorical
12. Continuous numerical
13. Discrete numerical
14. Discrete numerical
15. Categorical
16. Discrete numerical

Exercise 16B

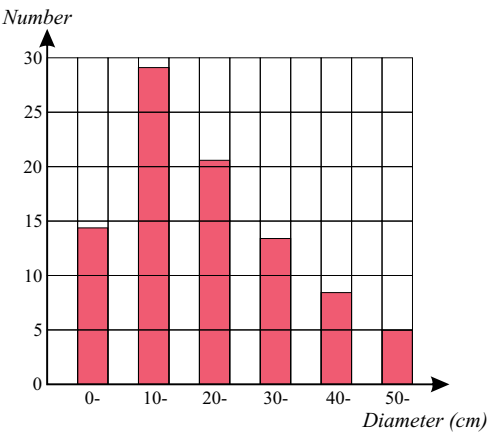
1. (a) Everyone in the state
(b) All voters in the country
(c) All the sugar gliders in the park
(d) Everyone who wears jeans
(e) Everyone in the area
(f) All the blocks of chocolate
(g) Everyone interested in the football team
2. (a) Sample Group **C** would give the best results. It covers more people and a wider range of people than the others.
(b) Sample Group **B** would give the best results. It covers a larger range of students.
(c) Sample Group **D** would give the best results. It covers more people and a wider range of people than the others.

3. (a) As many people as possible from across the state.
(b) As many voters as possible.
(c) As many sugar gliders as possible.
(d) As many people who wear jeans as possible.
(e) As many people in the area as possible.
(f) As many blocks of chocolate as possible.
(g) As many people as possible that are involved with the team.
4. (a) Channel 1 - 45%
Channel 2 - 37.5%
Channel 3 - 17.5%
(b) Channel 1 - 5400
Channel 2 - 4500
Channel 3 - 2100
5. (a) -20 = 12.5%, -30 = 21.5%,
-40 = 26.75%, -50 = 22.25%,
over 50 = 17%
(b) -20 = 1875, -30 = 3225,
-40 = 4013, -50 = 3338,
over 50 = 2550
6. Reading - 84
Television - 96
Watching Sport - 72
Playing Sport - 168
Listening to Music - 84
Playing Music - 96
Computer - 120
7. (a) 108 (b) 46% (c) 75% (d) 4320
8. (a) 95 (b) 43.2% (c) 6.3%
(d) 1900 (e) 1080
9. **A** - Hobart (529 mm)
B - Adelaide (530 mm)
C - Canberra (625 mm)
D - Melbourne (655 mm)
E - Perth (869 mm)
F - Brisbane (1215 mm)
G - Sydney (1219 mm)
H - Darwin (1661 mm)

10. (a)

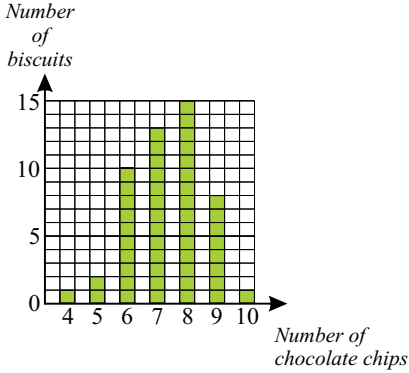
Diameter Range (cm)	Tally	Number (Frequency)
0 -		14
10 -		29
20 -		21
30 -		13
40 -		8
50 -		5
Total		90

(b)



- (c) 0- = 15.6%, 10- = 32.2%,
20- = 23.3%, 30- = 14.4%,
40- = 8.9%, 50- = 5.6%
- (d) 0- = 56 000, 10- = 116 000,
20- = 84 000, 30- = 52 000,
40- = 32 000, 50- = 20 000

11. (a)



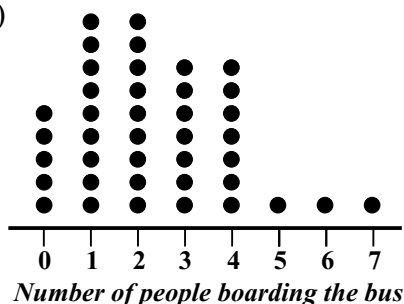
(b) 94%

Exercise 16C

1. (a)(i) 6 (ii) 5 (iii) 4 (iv) 6
 (b)(i) 14 (ii) 14 (iii) 14 (iv) 2
 (c)(i) 13 (ii) 12 (iii) 15 (iv) 14
 (d)(i) 12 (ii) 13 (iii) 12, 14 (iv) 12
 (e)(i) 10 (ii) 11 (iii) 14 (iv) 10
 (f)(i) 21 (ii) 22 (iii) 23 (iv) 11
 (g)(i) 13.1 (ii) 12.5 (iii) 11, 13 (iv) 11
 (h)(i) 6.7 (ii) 7 (iii) 7 (iv) 7
 (i)(i) 8.3 (ii) 8.5 (iii) 9 (iv) 5
2. (a) 32.1 (b) 32 (c) 39
3. (a) 50.2 sec (b) 50 sec (c) 5.8 sec
4. (a) Hervey - 41, Cary - 41
 (b) Hervey - 7.5, Cary - 42.5
 (c) Hervey - 329, Cary - 32
 (d) Hervey had two big innings but all others were very low. Cary had consistent scores.
5. (a) \$800 (b) \$500
 (c) The median best shows the earnings of most workers.
6. (a) \$433 185 (b) \$406 000
 (c) 490 000
7. (a) \$198.80 (b) \$180 (c) \$550
8. (a) Wattleton: (i) \$355 000
 (ii) \$360 000
 (iii) \$105 000
 Bankston: (i) \$470 000
 (ii) \$360 000
 (iii) \$905 000
 (b) Wattleton: (i) \$347 500
 (ii) \$355 000
 (iii) \$85 000
 Bankston: (i) \$347 500
 (ii) \$355 000
 (iii) \$85 000
 (c) The large price affected the mean and range for the Bankston prices significantly more than the median.
 (d) The median is a better indicator of the 'central tendency' of a set of prices because it is affected less than the mean by very large or small prices.

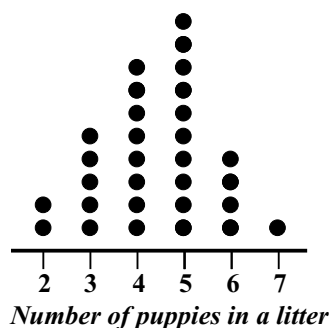
Exercise 16D

1. (a)



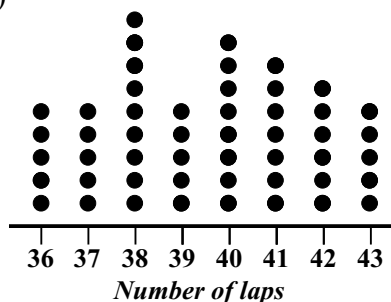
(b)(i) 2.35 (ii) 2 (iii) 1 & 2 (iv) 7

2. (a)



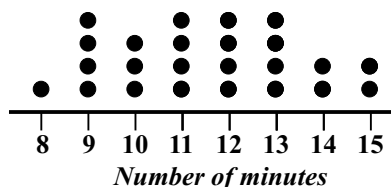
(b)(i) 4.4 (ii) 4.5 (iii) 5 (iv) 5

3. (a)



(b)(i) 39.52 (ii) 40 (iii) 38 (iv) 7

4. (a)

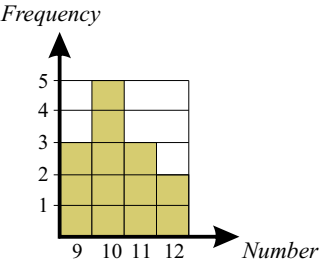
(b)(i) 11.5 (ii) 11.5 (iii) 9, 11, 12, 13
 (iv) 7

5. (a)(i) 13.2 (ii) 13 (iii) 13 (iv) 6
 (b)(i) 27 (ii) 26.5 (iii) 26 (iv) 9
 (c)(i) 36.2 (ii) 36 (iii) 37 (iv) 11

Exercise 16E

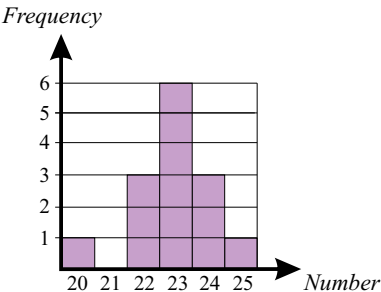
1. (a)

Number	Frequency
9	3
10	5
11	3
12	2
Total	13



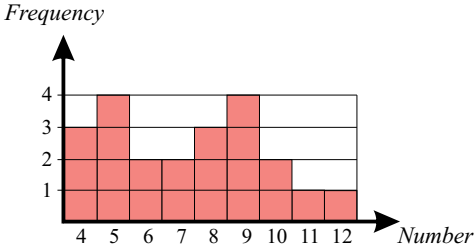
(b)

Number	Frequency
20	1
21	0
22	3
23	6
24	3
25	1
Total	14



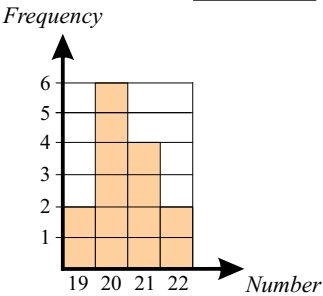
(c)

Number	Frequency
4	3
5	4
6	2
7	2
8	3
9	4
10	2
11	1
12	1
Total	22



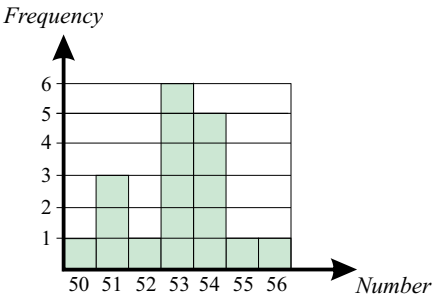
2. (a)(i) 10.3 (ii) 10 (iii) 10 (iv) 3
 (b)(i) 22.9 (ii) 23 (iii) 23 (iv) 5
 (c)(i) 7.3 (ii) 7.5 (iii) 5 & 9 (iv) 8
 3. (a)(i) 9.5 (ii) 9 (iii) 10 (iv) 4
 (b)(i) 27.4 (ii) 28 (iii) 28 (iv) 10
 (c)(i) 15.9 (ii) 17 (iii) 17 (iv) 7
 4. (a)(i) 11.2 (ii) 11 (iii) 11 (iv) 3
 (b)(i) 9.8 (ii) 10 (iii) 9 (iv) 4
 5. (a)

Number	Frequency
19	2
20	6
21	4
22	2
Total	14



(b)

Number	Frequency
50	1
51	3
52	1
53	6
54	5
55	1
56	1
Total	18



6. (a)(i) 20.4 (ii) 20 (iii) 20 (iv) 3
(b)(i) 53 (ii) 53 (iii) 53 (iv) 6

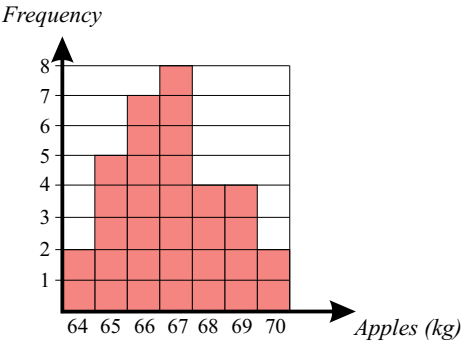
7. (a)

Strokes	Frequency
3	1
4	7
5	4
6	5
7	1
Total	18

(b) 4.9 (c) 5

8. (a)

Apples (kg)	Frequency
64	2
65	5
66	7
67	8
68	4
69	4
70	2
Total	32



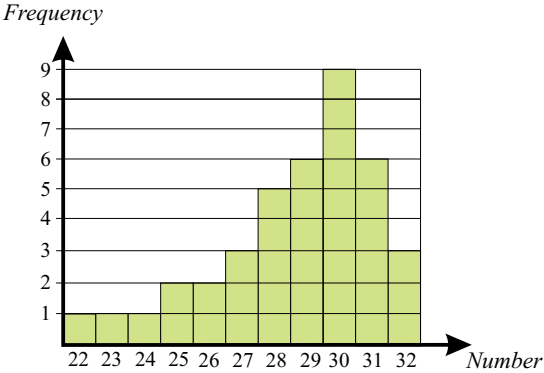
(b) 66.8 (c) 534.4

Exercise 16F

1. (a)(i)

Number	Frequency
22	1
23	1
24	1
25	2
26	2
27	3
28	5
29	6
30	9
31	6
32	3
Total	39

(ii)

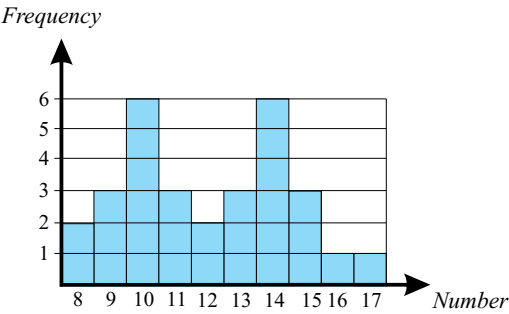


(iii) 28.7 (iv) 29 (v) 30
(vi) left-skewed

(b)(i)

Number	Frequency
8	2
9	3
10	6
11	3
12	2
13	3
14	6
15	3
16	1
17	1
Total	30

(ii)

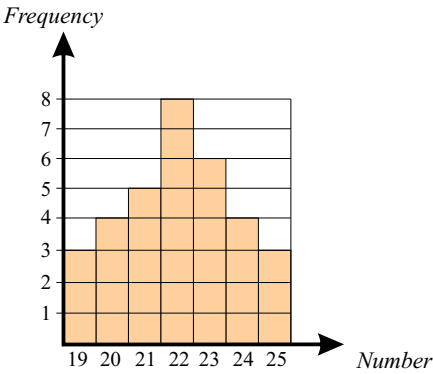


(iii) 12.0 (iv) 12 (v) 10 & 14
(vi) bimodal

(c)(i)

Number	Frequency
19	3
20	4
21	5
22	8
23	6
24	4
25	3
Total	33

(ii)

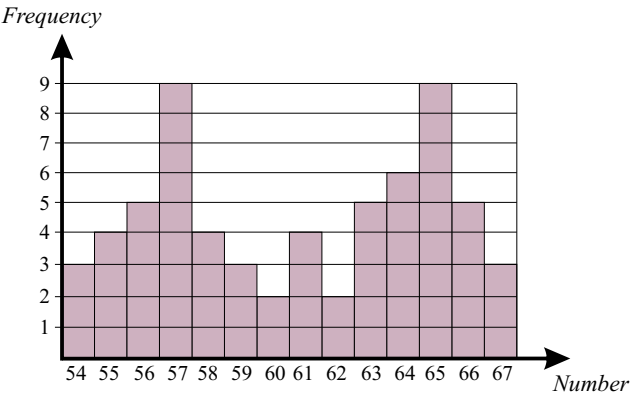


(iii) 22.0 (iv) 22 (v) 22
(vi) symmetric

(d)(i)

Number	Frequency
54	3
55	4
56	5
57	9
58	4
59	3
60	2
61	4
62	2
63	5
64	6
65	9
66	5
67	3
Total	64

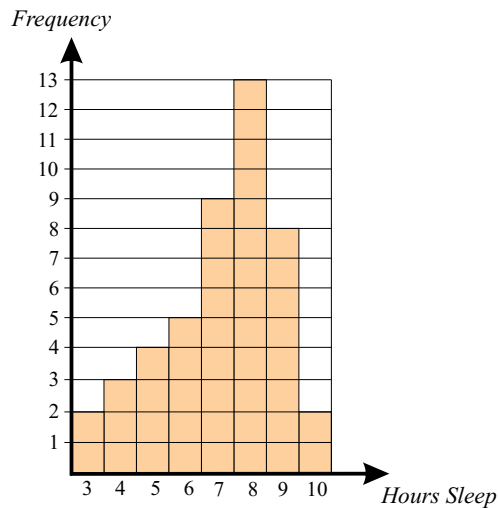
(ii)



(iii) 60.7 (iv) 61 (v) 57 & 65
(vi) bimodal

2. (a)

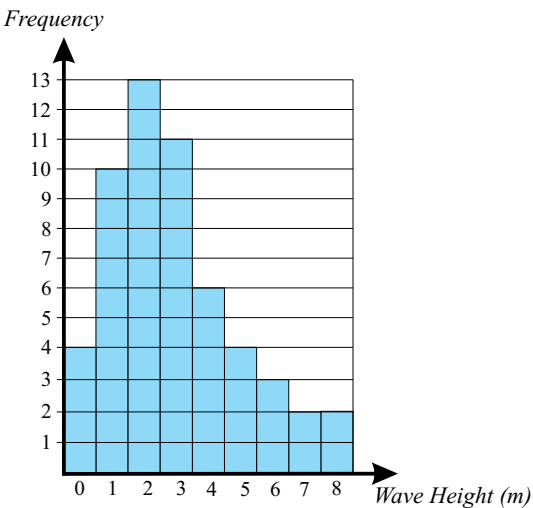
Hours sleep	Frequency
3	2
4	3
5	4
6	5
7	9
8	13
9	8
10	2
Total	46



- (b)(i) 7.1 (ii) 7.5 (iii) 8
(c) left-skewed

3. (a)

Wave Height (m)	Frequency
0	4
1	10
2	13
3	11
4	6
5	4
6	3
7	2
8	2
Total	55



- (b)(i) 2.9 m (ii) 3 m (iii) 2 m
(c) right-skewed

Exercise 16G

1. (a)(i) 53 (ii) 16 (iii) 37 (iv) 34.7
 (v) 34
 (b)(i) 86 (ii) 30 (iii) 56 (iv) 57.0
 (v) 58
 (c)(i) 4.4 (ii) 0.2 (iii) 4.2 (iv) 2.4
 (v) 2.5

2. (a)

Stem	Leaf
2	0 3 8 9
3	0 1 2 5 6 7
4	0 2 3 5 8 9
5	0 1 2 3 4 7
6	0 0 1 2 9

$$\overline{2|6} = 26$$

(b)

Stem	Leaf
7	0 2 9
8	0 2 3 5 8
9	0 1 2 4 7
10	1 1 2 5 5
11	0 1 5 5 6 8
12	1 2 3 7 7

$$\overline{7|0} = 70$$

(c)

Stem	Leaf
4	3 5 6 7 9
5	1 2 2 3 6 7 8
6	0 1 1 2 2 5 9
7	1 2 3 5 6 9
8	0 1 3 5 5 5 7

$$\overline{4|5} = 4.5$$

3. (a)(i)

Stem	Leaf
1	5 2 9
2	4 5 2 0 9
3	6 6 0 7 0
4	0 8 1 9

$$\overline{2|6} = 26$$

(ii)

Stem	Leaf
1	2 5 9
2	0 2 4 5 9
3	0 0 6 6 7
4	0 1 8 9

$$\overline{2|6} = 26$$

(iii) 30.2 (iv) 30

(b)(i)

Stem	Leaf
3	5 9 1
4	8 9 0 2
5	6 8 9 6
6	4 4
7	8 0
8	1 0

$$\overline{2|6} = 26$$

(ii)

Stem	Leaf
3	1 5 9
4	0 2 8 9
5	6 6 8 9
6	4 4
7	0 8
8	0 1

$$\overline{2|6} = 26$$

(iii) 55.9 (iv) 56

4. (a)

Stem	Leaf
1	9 8 7
2	5 9 8 4 7 6 8 0 8 5 0 3 9 7 1
3	1 7 6 6 7 6 7 8 3
4	5 3 8 2 5 0 1 5
5	1

$$\overline{2|6} = 26$$

(b)

Stem	Leaf
1	7 8 9
2	0 0 1 3 4 5 5 6 7 7 8 8 8 9 9
3	1 3 6 6 6 7 7 7 8
4	0 1 2 3 5 5 5 8
5	1

$$\overline{2|6} = 26$$

(c) 17 (d) 51 (e) 36 (f) 32.1 (g) 30

5. (a)

Non-ordered stem-and-leaf plot

Stem	Leaf
1	9
2	5 3 5 7 9 7 4 7 0 1
3	1 4 0 9 5 3
4	0

$$\overline{2|6} = 26$$

Ordered stem-and-leaf plot

Stem	Leaf
1	9
2	0 1 3 4 5 5 7 7 7 9
3	0 1 3 4 5 9
4	0

$$\overline{2|6} = 26$$

(b) 21 (c) 28.3 (d) 27

6. (a)

*Non-ordered stem-and-leaf plot**The Slow
Overs**The Rushed
Singles*

Leaf	Stem	Leaf
	1	9 9 9 8
	2	0 2 5 4 6
9 4 8	3	0 1
8 2 7 2	4	
1 0 2 1	5	

*Ordered stem-and-leaf plot**The Slow
Overs**The Rushed
Singles*

Leaf	Stem	Leaf
	1	8 9 9 9
	2	0 2 4 5 6
9 8 4	3	0 1
8 7 2 2	4	
2 1 1 0	5	

(b) The Slow Overs - 18
The Rushed Singles - 13(c) The Slow Overs - 44.9
The Rushed Singles - 23(d) The Slow Overs - 47
The Rushed Singles - 22

7. (a)

Non-ordered stem-and-leaf plot

<i>Slammers</i>		<i>Swish</i>
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>
8	17	7 9
9 5 6	18	5 4 6 6 0
2 0 8	19	0 1
5 0 1	20	0

Ordered stem-and-leaf plot

<i>Slammers</i>		<i>Swish</i>
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>
8	17	7 9
9 6 5	18	0 4 5 6 6
8 2 0	19	0 1
5 1 0	20	0

- (b) Slammers - 192.4 cm
Swish - 185.8 cm
- (c) Slammers - 191 cm
Swish - 185.5 cm

8. (a)

Non-ordered stem-and-leaf plot

<i>Year 7</i>		<i>Year 8</i>
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>
9	3	
8 5	4	
7 2 4 3	5	
2 0 1	6	7 8
	7	9 5 3 0
	8	3 9 2
	9	0

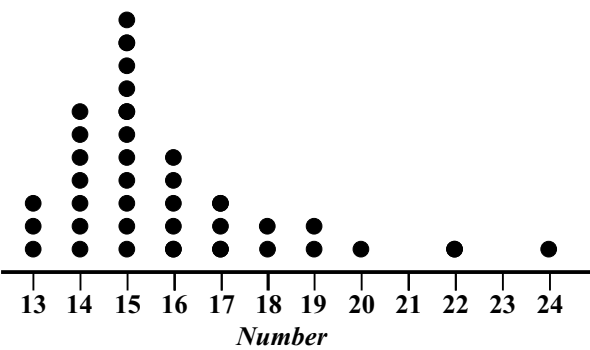
Ordered stem-and-leaf plot

<i>Year 7</i>		<i>Year 8</i>
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>
9	3	
8 5	4	
7 4 3 2	5	
2 1 0	6	7 8
	7	0 3 5 9
	8	2 3 9
	9	0

- (b) Year 7 - 23%
Year 8 - 23%
- (c) Year 7 - 53.1%
Year 8 - 77.6%
- (d) Year 7 - 53.5%
Year 8 - 77%
- (e) Yes

Chapter Review

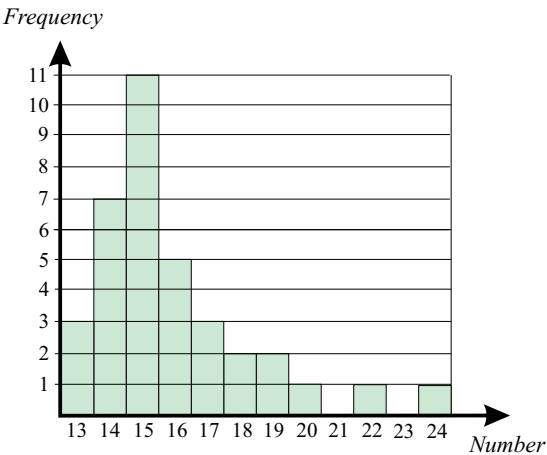
1. (a) 40 (b) white (c) black
(d) $\frac{1}{4}$ (e) 16
2. (a)(i) 23.9 (ii) 23 (iii) 18
(b)(i) 40.2 (ii) 39.5 (iii) 28
3. (a)



(b)

Number	Frequency
13	3
14	7
15	11
16	5
17	3
18	2
19	2
20	1
21	0
22	1
23	0
24	1
Total	36

(c)



- (d) 15.9 (e) 15 (f) 15 (g) right-skewed

4. (a)

Non-ordered stem-and-leaf plot

Ferny Gully

Bald Hill

Leaf	Stem	Leaf
6 8 9 7 5	1	2 9 5 9 5 8 9 7
1 2 3 6 0 3 2	2	2 4 0 5 2 5 8
2 0 1	3	
	4	

Ordered stem-and-leaf plot

Ferny Gully

Bald Hill

Leaf	Stem	Leaf
9 8 7 6 5	1	2 5 5 7 8 9 9 9
6 3 3 2 2 1 0	2	0 2 2 4 5 5 8
2 1 0	3	
	4	

- (b) (i) Ferny Gully - 32.3 m
Bald Hill - 20 m
(ii) Ferny Gully - 32 m
Bald Hill - 19 m
(iii) Ferny Gully - 17 m
Bald Hill - 16 m