

MASTER MATHS

STEVE PESCOTT



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Trevor Pescott - p. 500

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Numbers

Numbers

EXERCISE 1A

1. Write the following numbers in numeral form.

Examples

(a) **seventeen** in numeral form is **17**

(b) **three thousand, two hundred and six**
in numeral form is **3206**

- (a) thirteen (b) sixty-one (c) three hundred and twenty
- (d) five hundred and seventy-two (e) eight hundred and nine
- (f) seven thousand, eight hundred and eleven
- (g) fourteen thousand, six hundred and ninety
- (h) two hundred and seventy-one thousand, three hundred and five
- (i) nine hundred and three thousand and forty
- (j) three hundred and eighty thousand, seven hundred and one
- (k) two million, fifteen thousand, nine hundred and fifty-eight
- (l) four hundred and one million, fifty thousand and six

2. Write the following numbers in words.

- | | | | |
|----------------|---------------|-------------|-----------------|
| (a) 18 | (b) 95 | (c) 407 | (d) 3246 |
| (e) 8030 | (f) 17 179 | (g) 503 027 | (h) 618 908 |
| (i) 23 810 645 | (j) 9 000 009 | (k) 111 111 | (l) 100 010 001 |

3. Unscramble these words to find numbers.

- | | | | |
|------------|-------------|--------------|-------------|
| (a) WETLEV | (b) FURO | (c) NET | (d) TIXYS |
| (e) TOW | (f) HETER | (g) NOE | (h) VIEF |
| (i) THIGE | (j) INNE | (k) HANDOUTS | (l) TRYHIT |
| (m) TIFFY | (n) YESVENT | (o) LIMOLIN | (p) NERDHUD |

4. Find the numbers represented by these words.

- | | | | |
|---------------|------------------|-------------|--------------|
| (a) one dozen | (b) a century | (c) a score | (d) a gross |
| (e) a decade | (f) a millennium | (g) a pair | (h) a googol |

5. Use the digits below to write the **largest** number possible.

Example 2 0 9 The largest number that could be written using these digits is **920**

- | | | | |
|---------------|---------------|-----------------|-----------------|
| (a) 1 2 3 | (b) 5 8 9 1 | (c) 2 4 6 0 | (d) 3 5 1 2 |
| (e) 9 7 0 1 6 | (f) 2 6 1 1 5 | (g) 9 3 0 2 8 4 | (h) 1 3 8 2 9 9 |

6. Use the digits below to write the **smallest** number possible.

- | | | | |
|---------------|---------------|-----------------|-----------------|
| (a) 1 2 3 | (b) 5 8 9 1 | (c) 2 4 6 8 | (d) 3 5 1 2 |
| (e) 9 7 1 1 6 | (f) 2 6 1 1 5 | (g) 9 3 1 2 8 4 | (h) 1 3 8 2 9 9 |

7. The crowd sizes at a number of A.F.L. football games are given below.

Demons v Port Power	33 720	Swans v Tigers	32 099
Kangaroos v Crows	36 789	Cats v Blues	43 915
Bulldogs v Magpies	43 907	Eagles v Dockers	35 621
Lions v Bombers	32 798	Saints v Hawks	36 821

Lists the crowd sizes from the smallest to the largest.

8. List the numbers below in ascending order (from smallest to largest).

6295, 6903, 6309, 6913, 6093, 6390, 6925, 6259, 6193, 6139

9. Choose the correct symbol ($<$ or $>$) to place between the pairs of numbers below.

Examples Remember the crocodile wants to eat the **larger** number.

1. 18 28



Answer: 18 $<$ 28 18 $<$ 28

2. 397 379



Answer: 397 $>$ 379 397 $>$ 379

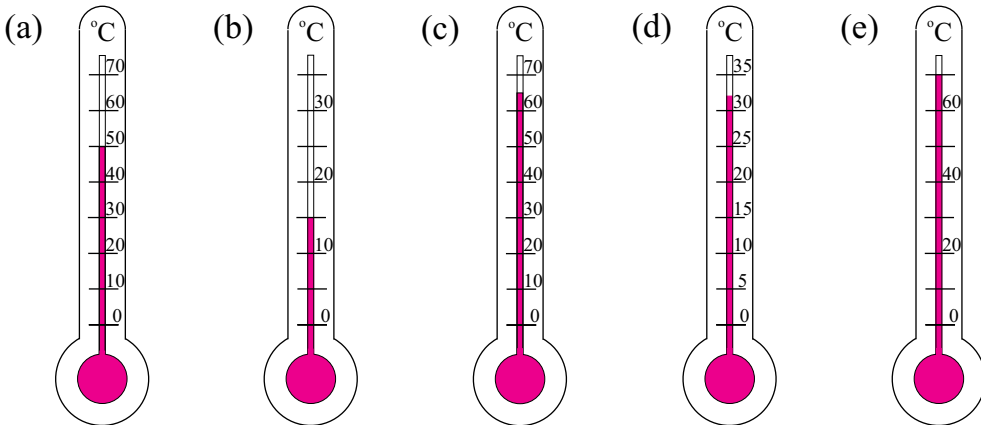
- | | | |
|---------------|-----------------|---------------------|
| (a) 36 26 | (b) 123 132 | (c) 3078 3087 |
| (d) 1203 987 | (e) 578 575 | (f) 6237 6239 |
| (g) 6804 6799 | (h) 4095 467 | (i) 12 993 12 987 |
| (j) 983 990 | (k) 39 999 4000 | (l) 675 238 675 328 |

10. (a) List all the even numbers between 791 and 807.
 (b) List all the odd numbers between 6090 and 6106.
 (c) How many even numbers are between 99 and 131?
 (d) How many odd numbers are between 3196 and 3254?
 (e) List all the different odd numbers that can be formed using all the digits **1, 3** and **8**.
 (f) List all the different even numbers that can be formed using all the digits **6, 3, 5** and **7**.
11. (a) Find the **largest even** number that can be formed using all of the following digits: 7 4 1 9 6
 (b) Find the **smallest odd** number that can be formed using all of the following digits: 3 1 8 5 2

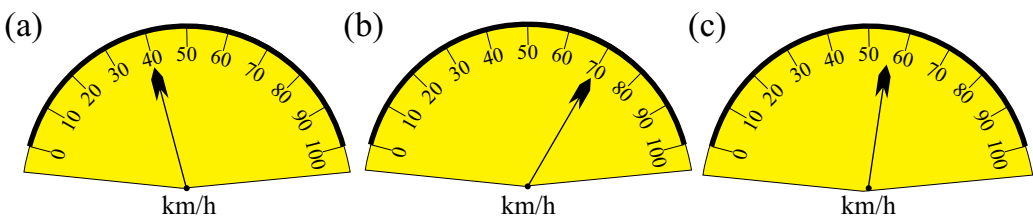
Reading Scales

EXERCISE 1B

1. Read the temperature shown on the thermometers below.

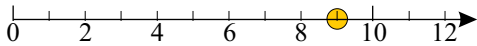


2. What speeds are shown on the speedometers below?

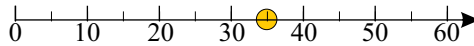


3. Write the number shown by the dot on the number lines below.

(a)



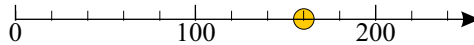
(b)



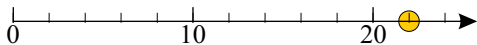
(c)



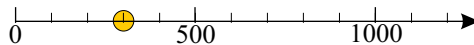
(d)



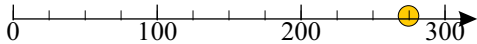
(e)



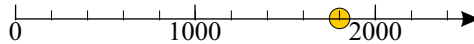
(f)



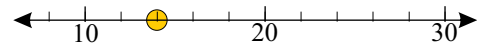
(g)



(h)



(i)



(j)

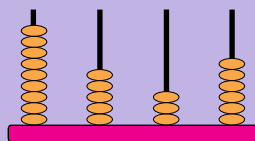


Place Value

EXERCISE 1C

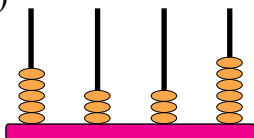
1. What number is represented on each spike abacus below?

Example

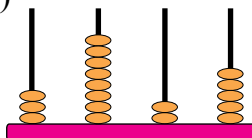


The number shown on this spike abacus is **9536**

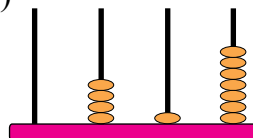
(a)



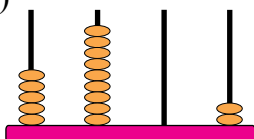
(b)



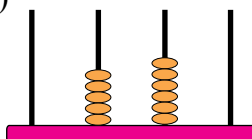
(c)



(d)

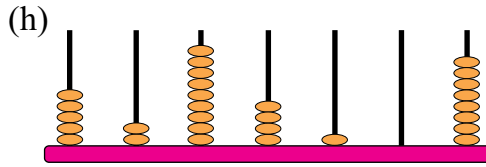
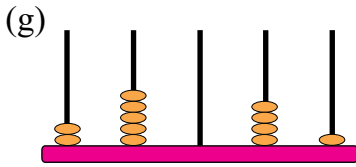


(e)



(f)





2. Write as one number.

Example

$$2000 + 600 + 50 + 1 = \mathbf{2651}$$

(a) $3000 + 200 + 10 + 5$

(b) $600 + 2$

(c) $1000 + 20$

(d) $9000 + 400 + 3$

(e) $90 + 7000 + 8$

(f) $60 + 20\ 000 + 1 + 5000$

(g) $900 + 90\ 000 + 9$

(h) $4000 + 50\ 000 + 20 + 300 + 1$

3. Add 10 to the following numbers.

(a) 73

(b) 186

(c) 752

(d) 1620

(e) 6

(f) 692

(g) 10 099

(h) 39 999

4. Subtract 10 from the following numbers.

(a) 45

(b) 278

(c) 790

(d) 105

(e) 610

(f) 906

(g) 15 096

(h) 78 001

5. Add 100 to the following numbers.

(a) 621

(b) 50

(c) 895

(d) 6073

(e) 4

(f) 12 917

(g) 999

(h) 729 945

6. Subtract 100 from the following numbers.

(a) 591

(b) 117

(c) 1086

(d) 19 051

(e) 1000

(f) 21 111

(g) 253 721

(h) 100 000

7. Find the missing numbers from the equations below.

Example

$$18 + \mathbf{10} = 28$$

(a) $26 + \square = 36$

(b) $652 + \square = 662$

(c) $27\ 350 + \square = 28\ 350$

(d) $45\ 127 + \square = 45\ 227$

(e) $21 - \square = 11$

(f) $897 - \square = 797$

(g) $6059 - \square = 5959$

(h) $42\ 898 - \square = 32\ 898$

8. Complete these patterns.

(a) 5, 15, 25, 35, , , .

(b) 305, 405, 505, , , .

(c) 1, 10, 100, 1000, , .

(d) 30 000, 3000, 300, , .

9. Round the following numbers to the nearest 10.

Examples 1. 37

- Find the multiples of 10 that the number is between
- 37 is between 30 and 40
- determine which of these numbers the number is closest to
- 37 is closest to 40
- 37 rounded to the nearest 10 would be **40**

2. 162

- 162 is between 160 and 170
- 162 rounded to the nearest 10 would be **160**

3. 275

- 275 is between 270 and 280
- if the number is in the middle, round up to the larger number
- 275 rounded to the nearest 10 would be **280**

(a) 59 (b) 111 (c) 136 (d) 9 (e) 1896 (f) 999
(g) 85 (h) 1109 (i) 13 763 (j) 405 (k) 795 (l) 8071

10. Round these numbers to the nearest 100.

Examples Use a similar method but use the multiples of 100 that the number is between.

1. 358

- 358 is between 300 and 400
- 358 rounded to the nearest 100 would be **400**

2. 5709

- 5709 is between 5700 and 5800
- 5709 rounded to the nearest 100 would be **5700**

3. 9250

- 9250 is in the middle of 9200 and 9300
- 9250 rounded to the nearest 100 would be **9300**

(a) 137 (b) 593 (c) 1724 (d) 850 (e) 2990 (f) 35 876
(g) 709 (h) 1050 (i) 8020 (j) 9971 (k) 38 (l) 21 050

PROBLEM SOLVING

1. Three friends went fishing and caught one fish each.
They weighed their fish and found the total weight of the three fish was 15 kg.
Hayley's fish was 2 kg heavier than Lee's fish and 4 kg heavier than Don's fish.
What was the weight of each of their fish?

2. Jacqui, Stephanie and Nicholas helped raise money for their tennis club.
Together they raised \$55.
Jacqui raised twice the amount that Nicholas raised and \$5 more than Stephanie.
How much money did they each raise?



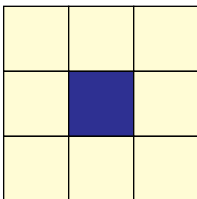
3. A=1, B=2, C=3, D=4, E=5,.....Y=25, Z=26.
Using this code the score for different words can be found by adding the value of each letter in the word.

Examples

CAT = 24 (C=3, A=1, T=20 $3+1+20=24$)

DOG = 26 (D=4, O=15, G=7 $4+15+7=26$)

- (a) Find the score for your name.
(b) Think of 4 three letter words that will fit into this grid.



Example

L	E	G
O		U
W	E	T

Try to find 4 words that will give you the biggest score.

The score for the example is 108 (L+E+G+U+T+E+W+O).

- (c) Try to find a number that has the same score as the number.
(Hint - there is one between 250 and 260)

PUZZLES

1. Change **TWO** into **TEN** in three steps by changing one letter at a time to form a new word.

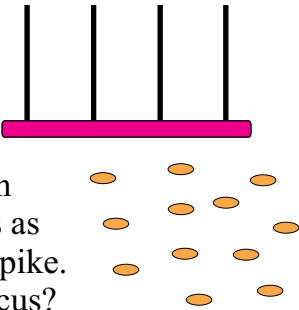
Example
CAT can be changed to **DOG** in three steps

CAT
COT
DOT
DOG

2. Find this three digit even number.
It has twice as many hundreds as tens.
Its three digits add to 18.

3. Find this two digit even number.
It has three more tens than units.
Its digits add to seven.

4. A dozen beads are placed on the spike abacus shown.
There is one more bead on the ten spike than on the thousand spike. There are three times as many beads on the ten spike as on the unit spike.
What is the number shown on the spike abacus?

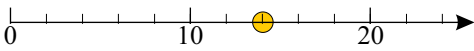
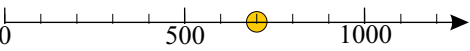


5. Find the numbers below hidden as words in the grid shown. The words could be forwards, backwards, up down or diagonally.

1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 15, 20, 30, 40, 50

M	V	C	W	G	N	Q	N	T	T	G
Q	S	O	N	E	V	E	S	X	Y	M
Y	W	E	V	L	E	W	T	Q	B	V
T	N	E	E	T	F	I	F	X	F	K
N	L	J	R	R	U	K	G	O	I	M
E	N	I	N	D	H	L	R	H	F	S
W	H	B	K	N	E	T	C	X	T	K
T	H	I	R	T	Y	W	G	U	Y	S

CHAPTER REVIEW

1. Write the following numbers in numeral form.
 (a) seventeen (b) fifty-seven (c) two thousand and seventy-one
 (d) fifty thousand, six hundred and nine
 (e) eighty-four million, twenty-three thousand, four hundred and six
2. Write the following numbers in words.
 (a) 14 (b) 586 (c) 80 963 (d) 3 042 794 (e) 701 020 879
3. Use the following digits to write (i) the largest number possible
 (ii) the smallest number possible
 (a) 6 2 5 (b) 9 1 1 4 6 (c) 7 2 8 3 5 3 (d) 9 2 9 1 9 2
4. The populations of eight cities are given below.
 List the cities from the **largest** to the **smallest**.
 Alphaton 53 791 Betaton 54 260 Gammaton 53 172 Deltaton 52 793
 Epsiton 52 780 Zeton 53 916 Eton 53 205 Theton 51 777
5. List the following numbers from **smallest** to **largest**.
 857 821 830 826 811 890 809 759 980 791 899 709 988 1000
6. What number is shown by the dot on the following number lines?
 (a)  (b) 
7. Write as one number.
 (a) $600 + 2000 + 3$ (b) $5 + 4000$ (c) $5000 + 2 + 80 + 60\,000$
8. Add 10 to the following numbers.
 (a) 84 (b) 91 (c) 897 (d) 602 (e) 3 (f) 4005 (g) 6394
9. Subtract 100 from the following numbers.
 (a) 732 (b) 2302 (c) 89 257 (d) 9085 (e) 110 054
10. Round the following numbers to the nearest 10.
 (a) 52 (b) 6 (c) 261 (d) 14 (e) 397 (f) 1203 (g) 1996
11. Round the following numbers to the nearest 100.
 (a) 621 (b) 3257 (c) 80 (d) 7060 (e) 39 (f) 289 957

Number Operations

Addition and Subtraction

EXERCISE 2A

1. Find the answers to the following calculations using *mental arithmetic*.

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| (a) $4 + 3$ | (b) $7 - 5$ | (c) $8 + 9$ | (d) $15 - 6$ |
| (e) $13 + 24$ | (f) $37 - 11$ | (g) $20 + 39$ | (h) $70 - 30$ |
| (i) $19 + 27$ | (j) $31 - 8$ | (k) $38 + 25$ | (l) $74 - 46$ |
| (m) $105 + 27$ | (n) $156 - 81$ | (o) $211 + 309$ | (p) $259 - 149$ |
| (q) $326 + 227$ | (r) $500 - 293$ | (s) $474 + 789$ | (t) $853 - 568$ |

2. Find the answers to the following calculations by:

(i) carefully setting out your workings.

(ii) checking your answer with a calculator.

- | | |
|--------------------------------|-----------------------------|
| (a) $8241 + 798 + 3085 + 33$ | (b) $5687 + 8 + 905 + 7345$ |
| (c) $987\ 620 + 85\ 234 + 789$ | (d) $980\ 284 + 659 + 8726$ |
| (e) $65\ 893 - 8351$ | (f) $6851 - 987$ |
| (g) $986 + 258 - 387$ | (h) $6054 - 589 - 1057$ |

3. (a) There were 857 students at Wenton College at the start of the year.

During the year 46 students arrived at the college and 63 students left.

How many students attended Wenton College at the end of the year?

(b) Find the sum of 45, 63, 839, 205, 500, 68, 22 and 395.

(c) Judy is 45 years old. Her daughter is 17. How old was Judy when her daughter was born?

(d) Noela has six grandchildren. Their ages are 19, 17, 15, 6, 3 and 2. Noela is five years older than the sum of her grandchildren's ages. How old is Noela?

(e) Tim is 183 cm tall. Julie is 18 cm shorter than Tim. How tall is Julie?

(f) Jay weighs 6 kg more than Emily. The sum of their weights is 98 kg. How much do they each weigh?

Multiplication

EXERCISE 2B

1. Copy and complete the following multiplication tables.

×	2	9	6	4	12	8
1						
10						
3						
11						
7						
5						

×	7	2			10	
	28		48			
				66		
			12			3
8						
						27
		10		30		

2. Write the answers to the following multiplications as quickly as you can.

- (a) 4×3 (b) 3×12 (c) 5×6 (d) 8×10 (e) 9×9
 (f) 12×2 (g) 9×4 (h) 11×5 (i) 10×6 (j) 6×6
 (k) 7×6 (l) 8×4 (m) 3×7 (n) 2×8 (o) 11×3
 (p) 8×12 (q) 7×9 (r) 5×5 (s) 7×12 (t) 6×4

3. Find the missing numbers.

- (a) $4 \times \underline{\quad} = 28$ (b) $\underline{\quad} \times 8 = 24$ (c) $5 \times \underline{\quad} = 15$ (d) $12 \times \underline{\quad} = 60$
 (e) $2 \times \underline{\quad} = 20$ (f) $\underline{\quad} \times 10 = 30$ (g) $\underline{\quad} \times 7 = 35$ (h) $9 \times \underline{\quad} = 27$
 (i) $9 \times \underline{\quad} = 18$ (j) $3 \times \underline{\quad} = 33$ (k) $9 \times \underline{\quad} = 108$ (l) $\underline{\quad} \times 11 = 99$
 (m) $8 \times \underline{\quad} = 64$ (n) $6 \times \underline{\quad} = 18$ (o) $\underline{\quad} \times 5 = 40$ (p) $9 \times \underline{\quad} = 72$

4. (a) Marcus swam 8 laps of a swimming pool every day. How many laps would he swim in a week?
 (b) Tina bought 12 kg of bird seed for \$36. What was the price of one kilogram of birdseed?
 (c) Beng played a round of golf (18 holes). She scored 3 on two holes, 4 on six holes, 5 on five holes and 6 on the other holes. What was her total score for the 18 holes?
 (d) Find this two digit number. The sum of its digits is 10. The product of its digits is 24. The number is a square number.

Factors

EXERCISE 2C

1. Write **all** the factors of the following numbers.

Example 18

All the factors of 18 are **1, 2, 3, 6, 9, 18**

- (a) 12 (b) 36 (c) 27 (d) 80 (e) 112
(f) 56 (g) 300 (h) 23 (i) 144 (j) 171

2. Find the factors that are common to the following pairs of numbers.

Example 20 and 30

The factors of 20 are 1, 2, 4, 5, 10

The factors of 30 are 1, 2, 3, 5, 6, 10, 15

The **common factors** are **1, 2, 5, 10**

- (a) 12, 16 (b) 15, 27 (c) 16, 20 (d) 24, 36 (e) 56, 60

3. Write the highest common factor (H.C.F.) for each of the pairs of numbers in question 2.

Example The highest common factor of 20 and 30 is **10**

4. Adam wants to make a large blue and white striped flag. He has 15 metres of blue material and 12 metres of white material. The stripes are to be horizontal.

15 m

- (a) What is the length of the longest flag he could make using **all** of the material?

12 m

- (b) How many stripes of each colour would be in the flag?

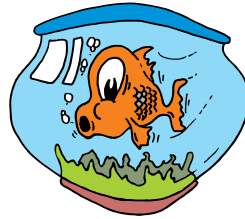
5. Mikal had two lengths of wood. One was 64 cm long and the other 48 cm. He wanted to cut them into shorter pieces all the same size.

- (a) What is the longest length of these short pieces so there was no wastage?

- (b) How many of these short pieces would there be?

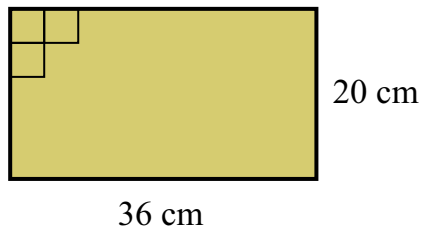
6. Susan and Craig each bought some goldfish. Susan spent exactly \$72 and Craig spent exactly \$56. The fish were all the same price and Susan bought two more than Craig.

- (a) What did each fish cost?
(b) How many fish did Susan and Craig buy?

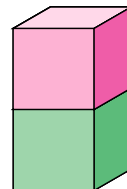


7. Evan had a piece of cardboard 36 cm long and 20 cm wide. He wanted to cut it into **squares**.

- (a) What is the side length of the largest squares he could cut out of this piece of cardboard so there was no wastage?
(b) How many squares would there be?

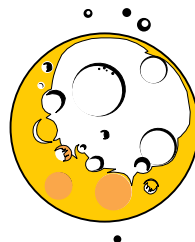


8. Three children at a play group made buildings using large cubes (larger than 1 cm). The buildings were 28 cm, 63 cm and 42 cm tall. What is the side length of the cubes?



9. Clancy has a paddock that he wants to fence. The paddock is 84 m long and 72 m wide. He wants all the fence posts to be the same distance apart and the gap to be as large as possible.
- (a) What is the largest gap he could make between the posts?
(b) How many posts will he need? (Hint - draw a diagram)

10. An astronomer was observing a moon revolving around a planet. He noticed the moon was in the same position 42 days, 56 days and 91 days after he started observing it. He knew it took longer than one day to travel around the planet. How many days does it take the moon to travel around the planet?



Divisibility

EXERCISE 2D

The following table gives tests that can be used to check if a number can be divided by certain numbers without a remainder.

The factors of numbers can be found using these tests.

The following tests can be used to check if a number is divisible by the number in the first column	
2	If the last digit is even or zero
3	If the sum of all the digits is divisible by 3
4	If the number formed by the last two digits is divisible by 4
5	If the last digit is 0 or 5
6	If the number is divisible by 2 and 3
8	If the number formed by the last three digits is divisible by 8
9	If the sum of all the digits is divisible by 9
10	If the last digit is 0

Copy and complete the table below by ticking the boxes to indicate which of the numbers at the top of the box are factors of the numbers shown.

One row is completed as an example.

	2	3	4	5	6	8	9	10
1380	✓	✓	✓	✓	✓			✓
7344								
11 496								
46 800								
410 400								

Prime and Composite Numbers

PRIME NUMBERS are numbers that have only two factors - themselves and 1.

COMPOSITE NUMBERS are any other numbers.

EXERCISE 2E

1. Copy this table into your book.

(a) Cross out 1 - this is not a prime number. Why?

(b) Circle 2 - this is the first prime number.

(c) Cross out all multiples of 2 - these are not prime numbers.

(d) Circle 3.

(e) Cross out all multiples of 3.

(f) Repeat this process for 5 and 7.

(g) Circle all the remaining numbers that have not been crossed out.

All the circled numbers are prime numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Twin primes are pairs of prime numbers that differ by 2.

Examples of twin primes are **3 and 5**, **5 and 7**.

Find all the twin primes between 1 and 100.

3. (a) Many even numbers can be written as the sum of two prime numbers. Find which two prime numbers could be added together to give the following numbers.

(i) 36 (ii) 74 (iii) 98 (iv) 128 (v) 158 (vi) 200

(b) Can you find any even numbers between 1 and 50 that cannot be written as the sum of two prime numbers?

4. Use the tests for divisibility to find which **one** of the following numbers is a prime number.

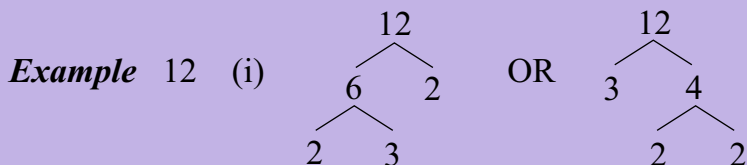
1525 1527 1528 1530 1531 1533 1539

5. Which **one** of the following numbers is **not** a prime number.

151 157 161 163 167 173

6. (i) Complete the factor trees for the following numbers.

(ii) Write each number as a product of its prime factors.



Both these factor trees are correct.

(ii) The prime factors are $2 \times 3 \times 2$ or $3 \times 2 \times 2$

(a) 24 (b) 18 (c) 56 (d) 64 (e) 108

7. Write the products of prime factors from question 6 in index notation.

Examples $12 = 2 \times 3 \times 2 = 2 \times 2 \times 3 = 2^2 \times 3$
 $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5$

8. Find the numbers represented by the products of prime factors shown below.

Example $2^2 \times 3^4 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 324$

(a) $2^3 \times 3^2$ (b) $2^4 \times 3$ (c) $3^2 \times 5$ (d) $5^2 \times 7$
 (e) 2×3^3 (f) 3×5^2 (g) $5^3 \times 7^2$ (h) $2^2 \times 3^3 \times 5$

Square Numbers

EXERCISE 2F

1. Find the following.

(a) 1^2 (b) 2^2 (c) 3^2 (d) 4^2 (e) 5^2

2. (a) Complete the following pattern.

1, 4, 9, 16, 25, __, __, __, __, __, __.

(b) These numbers are called **square numbers**.

The first square number is 1 (1^2)

The second square number is 4 (2^2)

The third square number is 9 (3^2), etc

Find (i) the tenth square number

(ii) the twentieth square number

(iii) the thirty-fifth square number

3. Without using a calculator, find the following square numbers.

- (a) 20^2 (b) 30^2 (c) 40^2 (d) 50^2 (e) 60^2
 (f) 70^2 (g) 80^2 (h) 90^2 (i) 100^2 (j) 110^2

4. Without using a calculator find the following.

Example Choose which of the following alternatives is equal to 32^2

A 624 **B** 824 **C** 1024 **D** 1624

Method: Find the multiples of 10 the number is between.

In this example 32 is between 30 and 40.

So 32^2 will be between 30^2 and 40^2 .

The answer will be between 900 and 1600.

The only alternative between 900 and 1600 is 1024.

Answer: C

(a) Choose which of the following alternatives is equal to 18^2 .

A 96 **B** 324 **C** 424 **D** 624

(b) Choose which of the following alternatives is equal to 27^2 .

A 269 **B** 329 **C** 729 **D** 1029

(c) Choose which of the following alternatives is equal to 45^2 .

A 1525 **B** 2025 **C** 2525 **D** 2825

(d) Choose which of the following alternatives is equal to 71^2 .

A 3441 **B** 4671 **C** 4861 **D** 5041

(e) Choose which of the following alternatives is equal to 83^2 .

A 3489 **B** 4789 **C** 5689 **D** 6889

5.

Example Choose which of the following alternatives is equal to 21^2

A 381 **B** 441 **C** 781 **D** 941

Method: Find the multiples of 10 the number is between.
In this example 21 is between 20 and 30.
So 21^2 will be between 20^2 and 30^2 .
The answer will be between 400 and 900.
There are two alternatives between 400 and 900.
So the answer could be 441 or 781.
But 21 is closer to 20 so 21^2 will be closer to 20^2 .
So the answer must be closer to 400 than 900.
The answer must be 441.

Answer: B

(a) Choose which of the following alternatives is equal to 29^2 .

A 341 **B** 421 **C** 841 **D** 981

(b) Choose which of the following alternatives is equal to 31^2 .

A 961 **B** 1481 **C** 1661 **D** 1891

(c) Choose which of the following alternatives is equal to 58^2 .

A 1524 **B** 2024 **C** 2684 **D** 3364

(d) Choose which of the following alternatives is equal to 62^2 .

A 3444 **B** 3844 **C** 4844 **D** 5044

(e) Choose which of the following alternatives is equal to 49^2 .

A 1321 **B** 1641 **C** 1821 **D** 2401

Square Roots

EXERCISE 2G

The opposite of the square of a number is the **square root** of a number. The symbol for the square root is $\sqrt{\quad}$.

Example $5^2 = 25$ $\sqrt{25} = 5$

1. Find the following.

(a) $\sqrt{36}$ (b) $\sqrt{81}$ (c) $\sqrt{16}$ (d) $\sqrt{1}$ (e) $\sqrt{100}$

(f) $\sqrt{49}$ (g) $\sqrt{64}$ (h) $\sqrt{144}$ (i) $\sqrt{4}$ (j) $\sqrt{400}$

Example Between which two consecutive whole numbers is $\sqrt{72}$?

Method: Find the nearest square numbers the number is between.

In this example 72 is between 64 and 81.

So $\sqrt{72}$ will be between $\sqrt{64}$ and $\sqrt{81}$.

So $\sqrt{72}$ will be between 8 and 9.

Answer: 8 and 9

2. Between which two consecutive whole numbers are the following square roots?

- (a) $\sqrt{61}$ (b) $\sqrt{7}$ (c) $\sqrt{13}$ (d) $\sqrt{95}$ (e) $\sqrt{30}$
 (f) $\sqrt{42}$ (g) $\sqrt{3}$ (h) $\sqrt{19}$ (i) $\sqrt{79}$ (j) $\sqrt{110}$

3. Use a calculator to find the values of the square roots in question 2 correct to one decimal place.

Example $\sqrt{72} = 8.48528\dots$

= **8.5** (correct to one decimal place)

4. Without using a calculator, match the following square roots with their decimal equivalents from the list below. (All decimal numbers have been rounded to one decimal place)

- (a) $\sqrt{26}$ (b) $\sqrt{62}$ (c) $\sqrt{80}$ (d) $\sqrt{52}$ (e) $\sqrt{102}$ (f) $\sqrt{35}$

A 7.2 **B** 7.9 **C** 5.1 **D** 10.1 **E** 8.9 **F** 5.9

5. Without using a calculator, match the following squares with their decimal equivalents from the list below. (All decimal numbers have been rounded to one decimal place)

- (a) 4.1^2 (b) 8.2^2 (c) 9.8^2 (d) 3.6^2 (e) 7.3^2 (f) 2.9^2

A 13.0 **B** 8.4 **C** 16.8 **D** 67.2 **E** 96.0 **F** 53.3

Multiples

EXERCISE 2H

1. For the pairs of numbers below:

- list the first eight multiples of each number
- underline the multiples common to both numbers
- circle the lowest common multiple (L.C.M.)

Example 6, 8

6, 12, 18, 24, 30, 36, 42, 48
8, 16, 24, 32, 40, 48, 56, 64

24 is the lowest common multiple of 6 and 8

- | | | | |
|----------|-----------|----------|-----------|
| (a) 3, 8 | (b) 6, 10 | (c) 4, 6 | (d) 8, 12 |
| (e) 6, 3 | (f) 8, 12 | (g) 5, 7 | (h) 7, 3 |

2. Find the lowest common multiple of the following groups of numbers.

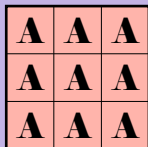
- | | | | |
|----------|------------|-------------|-------------|
| (a) 7, 8 | (b) 12, 20 | (c) 3, 4, 8 | (d) 4, 5, 6 |
|----------|------------|-------------|-------------|

3. A dice company makes three large dice A, B and C with sizes shown here. They wanted to make different sized square trays to hold the dice.

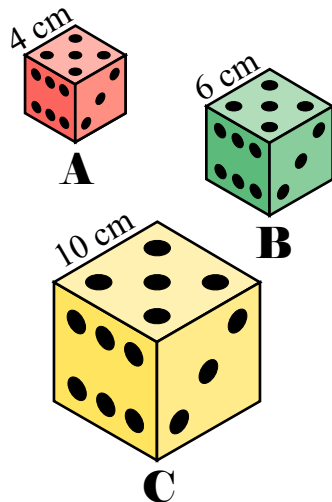
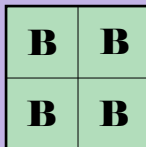
Example

A 12 cm square tray would be the smallest square tray that would hold a number of dice A or B.

It would hold 9 A dice or 4 B dice.



12 cm



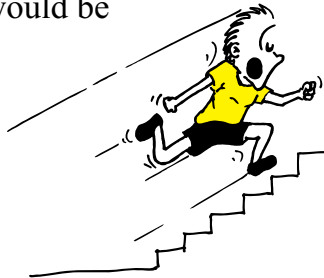
Find the side length of the smallest square tray that would hold the following dice and state how many of each dice the tray would hold.

- | | | |
|------------|------------|-----------------|
| (a) A or C | (b) B or C | (c) A or B or C |
|------------|------------|-----------------|

4. Neil can jump 5 steps at a time. Meryl can jump 4 steps at a time.

(a) If they both ran up stairs, what would be the number of the first step they **both** stepped on?

(b) If they both start with their right foot on the ground, what is the first step both their right feet touch?



5. Daniel and Peter are in a soccer team. They go for regular evening runs to keep fit. They both go for a run on July 1. Daniel then runs every third night and Peter every fifth night.

(a) On what date will they next run on the same night?

Their friend Josh runs with them on this night. He runs on every fourth night after that.

(b) On what date will all three friends next run on the same night?

Multiplication

EXERCISE 2I

1. Without using a calculator evaluate the following products.

- | | | | |
|----------------------------|-----------------------------|----------------------------|------------------------|
| (a) 15×2 | (b) 37×5 | (c) 123×4 | (d) 97×6 |
| (e) 389×5 | (f) 2804×7 | (g) 78×21 | (h) 318×66 |
| (i) $52 \times 2 \times 3$ | (j) $57 \times 89 \times 0$ | (k) $55 \times 2 \times 4$ | (l) $54\,806 \times 1$ |

2. Without using a calculator evaluate the following products.

- | | | | |
|-----------------------|------------------------|-----------------------------|----------------------------|
| (a) 6×10 | (b) 23×10 | (c) 51×100 | (d) 201×100 |
| (e) 2000×100 | (f) 1032×1000 | (g) $3 \times 10 \times 10$ | (h) $62 \times 2 \times 5$ |

3. In cricket, a bowler bowls 6 balls every over.
How many balls would be bowled in 16 overs?

4. Mandy bought 15 dozen bottles of drink for a party.

(a) How many bottles is this?

(b) If each dozen cost \$6, what was the total cost of the drinks?

(c) Mandy allowed for each person at the party to have two bottles of drink. How many people were at the party?

5. Alvira can ride her bike at an average speed of 250 metres a minute. How many metres can she ride in two and a half hours?
6. In A.F.L. football a goal is worth six points and a behind is worth one point.
- (a) How many points did a team get if they scored 19 goals and 21 behinds.
- (b) A team scored 108 points and kicked four more goals than they kicked behinds. How many goals did they kick?
7. A grain store supplied different grains to farmers. A bag of oats from the store weighed 25 kg. A bag of wheat weighed 30 kg.
- (a) A truck loaded on 80 bags of oats and 100 bags of wheat. What was the total weight of its load?
- (b) Another truck had a load of 200 bags that weighed a total of 5400 kg. How many bags of oats and how many bags of wheat did this truck have in its load?



EXERCISE 2J

1. Without using a calculator evaluate the following.
- (a) 50×60 (b) 20×40 (c) 800×20 (d) 700×300
 (e) 5×400 (f) 400×5000 (g) 20×3000 (h) $2 \times 30 \times 400$
2. Round these numbers to the nearest 10 before multiplying.
- Example** 37×53
 Round to the nearest 10 then evaluate the answer:
 $40 \times 50 = 2000$
- (a) 21×59 (b) 68×33 (c) 9×93 (d) 47×98
 (e) 102×76 (f) $27 \times 9 \times 19$ (g) 123×98 (h) $62 \times 47 \times 98$
3. Round these numbers to the nearest 100 before multiplying.
- (a) 115×196 (b) 220×492 (c) 178×327 (d) 712×308
 (e) 837×313 (f) 562×380 (g) 2416×95 (h) 987×3258

4. Copy and complete the following table.
One line is completed as an example.

Problem	Answer Using Calculator	Problem with Rounded Numbers	Answer Using Rounded Numbers	Difference Between the Answers
23×57	1311	20×60	1200	111
68×9				
97×31				
86×44				
79×73				

For the following questions, round the numbers, then choose the *best* answer without using a calculator.

5. A builder bought 18 kg of nails. There were 215 nails in a kg.
Approximately how many nails did he buy?
A 2000 **B** 20 000 **C** 4000 **D** 40 000
6. A trip was being organised for 57 people to visit a park. The cost was \$ 33 per person.
What was the approximate total cost for the group of people?
A \$1500 **B** \$1800 **C** \$15 000 **D** \$18 000
7. If a book weighed 112 grams what would be the approximate weight of a box of 95 books?
A 900 grams **B** 1000 grams **C** 10 000 grams **D** 100 000 grams
8. The manager of a trout farm wanted to find the number of trout in one of his ponds. He counted 27 trout in 1 square metre.
If the pond was 216 square metres what would be the approximate number of trout in the pond?
A 600 **B** 6000 **C** 60 000 **D** 600 000

Division

EXERCISE 2K

1. Without using a calculator find the following divisions.

- (a) $24 \div 8$ (b) $36 \div 12$ (c) $88 \div 11$ (d) $72 \div 9$
 (e) $40 \div 2$ (f) $150 \div 5$ (g) $66 \div 3$ (h) $246 \div 2$
 (i) $76 \div 2$ (j) $232 \div 4$ (k) $392 \div 7$ (l) $810 \div 3$
 (m) $1170 \div 5$ (n) $783 \div 9$ (o) $1048 \div 8$ (p) $1470 \div 6$

2. Use a calculator to find the following divisions.

- (a) $10\,488 \div 23$ (b) $11\,908 \div 26$ (c) $5572 \div 398$
 (d) $\frac{4158}{27}$ (e) $\frac{13\,122}{243}$ (f) $\frac{(1300 + 772)}{(25 + 12)}$

3. Find the following divisions by cancelling first.

Cancel the same number of zeros in each number.

Examples $3000 \div 100 = 30\cancel{00} \div 1\cancel{00} = 30 \div 1 = \mathbf{30}$
 $6000 \div 20 = 600\cancel{0} \div 2\cancel{0} = 600 \div 2 = \mathbf{300}$

- (a) $200 \div 10$ (b) $8000 \div 40$ (c) $20\,000 \div 50$ (d) $60\,000 \div 3000$
 (e) $6000 \div 500$ (f) $1300 \div 20$ (g) $90\,000 \div 50$ (h) $105\,200 \div 400$

4. Copy and complete the following table.

One line is completed as an example.

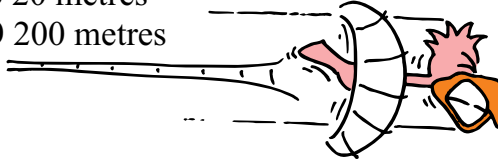
Problem	Answer Using Calculator	Problem with Rounded Numbers	Answer Using Rounded Numbers	Difference Between the Answers
$1957 \div 19$	103	$2000 \div 20$	100	3
$611 \div 13$				
$899 \div 31$				
$2009 \div 49$				
$1368 \div 72$				

For the following questions, round the numbers, then choose the **best** answer without using a calculator.

5. A bird was observed to fly 790 metres in 38 seconds.

Approximately how far would the bird fly in one second?

- A** 2 metres **B** 20 metres
C 50 metres **D** 200 metres



6. Graham walked 5420 metres in an hour. Approximately how far did he walk every minute?

- A** 9 metres **B** 90 metres **C** 900 metres **D** 9000 metres

7. A water tank contains 55 000 litres of water. It is used to fill a 525 litre drinking trough for a horse. Approximately how many times could the trough be filled from the tank?

- A** 11 **B** 110 **C** 1100 **D** 11 000

8. An apple orchard had 396 trees. In one season there were 41 bins of apples picked. Each bin held an average of 8960 apples.

(a) Approximately how many apples were picked?

- A** 360 **B** 3600 **C** 36 000 **D** 360 000

(b) Approximately how many apples were on each tree?

- A** 90 **B** 900 **C** 9000 **D** 90 000

9. The distance from the Sun to Earth is about 150 million kilometres. Light travels at a speed of 300 thousand kilometres per second. Using these figures, how long does it take light to travel from the Sun to Earth?

- A** 5 seconds **B** 50 seconds **C** 500 seconds **D** 5000 seconds

10. The Hubble satellite travels about 42 000 km in one orbit of the Earth. It travels at a speed of about 7 km per sec. Using these figures, how many **minutes** does it take the satellite to travel around the Earth?

- A** 1 minute **B** 10 minutes **C** 100 minutes **D** 1000 minutes

BODMAS

When solving problems that involve **brackets**, **of** (eg. $\frac{1}{2}$ of 8) and/or the **four operations** ($\div$, $\times$, $+$, $-$) the following rules must be observed.

1. The order that the operations are performed follow BODMAS:
Brackets must be calculated first, followed by **Of**'s, followed by **Divisions** and **Multiplications** and finally **Additions** and **Subtractions**.
2. If the problem does not involve all the operations then the order remains the same ignoring the operations missing.
3. If there is a succession of division and multiplication operations these are performed in order from the left to the right.
4. If there is a succession of addition and subtraction operations these are performed in order from the left to the right.

BRACKETS
OF
 { **DIVISION**
 { **MULTIPLICATION**
 { **ADDITION**
 { **SUBTRACTION**

Examples

1. $6 + 5 \times 4$ There are no **brackets** or **of's**
 $= 6 + 20$ The next operations are **division** or **multiplication**
 $= 26$

2. $2 \times (8 - 5)$
 $= 2 \times 3$ **Brackets** first
 $= 6$

3. $8 + (13 - 1) \div \frac{1}{2} \text{ of } 6$
 $= 8 + 12 \div \frac{1}{2} \text{ of } 6$ **Brackets** first
 $= 8 + 12 \div 3$ **Of** next
 $= 8 + 4$ **Division** next
 $= 12$

4. $10 - 8 + 4 + 3$ When there is a succession
 $= 2 + 4 + 3$ of **additions** and **subtractions**
 $= 6 + 3$ perform these in order from
 $= 9$ left to right

Examples continued

$$\begin{aligned}
 5. \quad & 4 \times 6 \div 8 \times 12 \div 6 \div 3 \\
 &= 24 \div 8 \times 12 \div 6 \div 3 \\
 &= 3 \times 12 \div 6 \div 3 \\
 &= 36 \div 6 \div 3 \\
 &= 6 \div 3 \\
 &= 2
 \end{aligned}$$

When there is a succession of *divisions* and *multiplications* perform these in order from left to right

$$6. \quad 7 + 8 \div 4 - \frac{1}{2} \text{ of } 6 \times (5 - 3)$$

$$\begin{aligned}
 &= 7 + 8 \div 4 - \frac{1}{2} \text{ of } 6 \times 2 && \text{Brackets first} \\
 &= 7 + 8 \div 4 - 3 \times 2 && \text{Of next} \\
 &= 7 + 2 - 6 && \text{Division and Multiplication next} \\
 &= 9 - 6 && \text{Addition and Subtraction from} \\
 &= 3 && \text{left to right}
 \end{aligned}$$

$$7. \quad 6 \times (8 - 3) + (9 + 3) \times 2 - 10 \div 5$$

$$\begin{aligned}
 &= 6 \times 5 + 12 \times 2 - 10 \div 5 \\
 &= 30 + 24 - 2 \\
 &= 54 - 2 \\
 &= 52
 \end{aligned}$$

$$8. \quad (6 \times 7 - 2) - \left(\frac{1}{4} \text{ of } 20 + 7 \right) \div 3 + 6 \times 2 - 7$$

$$\begin{aligned}
 &= (6 \times 7 - 2) - (5 + 7) \div 3 + 6 \times 2 - 7 \\
 &= (42 - 2) - 12 \div 3 + 6 \times 2 - 7 \\
 &= 40 - 12 \div 3 + 6 \times 2 - 7 \\
 &= 40 - 4 + 12 - 7 \\
 &= 36 + 12 - 7 \\
 &= 48 - 7 \\
 &= 41
 \end{aligned}$$

EXERCISE 2L

1. Solve the following problems using BODMAS without using a calculator.

- | | | |
|--|---|-----------------------------------|
| (a) $5 + 3 \times 2$ | (b) $3 \times 2 + 5$ | (c) $3 \times (2 + 5)$ |
| (d) $10 \div 2 + 3$ | (e) $10 \div (2 + 3)$ | (f) $12 - \frac{1}{2}$ of 10 |
| (g) $\frac{1}{2}$ of $8 + 2$ | (h) $80 - 2 \times 3$ | (i) $8 + 5 \times 3 - 6$ |
| (j) $12 - 3 \times 2 + 2$ | (k) $10 \div 5 \times 2 + 6$ | (l) $3 \times 5 + 10 \div 2$ |
| (m) $3 \times (4 + 2) - 7$ | (n) $\frac{1}{2}$ of $(8 + 4) \times 3$ | (o) $4 + 2 \times 3 - 5 \times 2$ |
| (p) $3 \times 8 + 2 \times (16 - 10)$ | (q) $10 + 2 \times 2 - 8 \div 4$ | |
| (r) $(6 \times 3 + 2) \div 10 + 10$ | (s) $20 - 5 \times 2 + 2 \times (3 + 5)$ | |
| (t) $8 + (3 \times 5) - \frac{1}{2}$ of $(16 - 10)$ | (u) $8 + 2 - 5 + 6 - 2 + 5 - 6 + 1$ | |
| (v) $6 \times 4 \div 2 \times 3 \div 9 \times 4$ | (w) $[(2 + 3) \times 4] + [6 \times (3 + 1)]$ | |
| (x) $\frac{1}{2}$ of $(8 + 4) + 6 \times 2 \div (5 + 1) + 8 \div 4 \times 3 - 3 \times \frac{1}{2}$ of $(7 + 1)$ | | |

2. Rewrite these equations and include the symbols (+, -, $\times$ or $\div$) to make the equations correct.

Example

$$2 * 3 * 5 = 17$$

This equation would be $2 + 3 \times 5 = 17$

- | | | |
|---------------------------|--------------------------|----------------------|
| (a) $6 * 4 * 2 = 14$ | (b) $10 * 2 * 4 = 9$ | (c) $5 * 4 * 2 = 7$ |
| (d) $3 * 2 * 4 = 24$ | (e) $20 * 2 * 6 = 16$ | (f) $4 * 4 * 2 = 12$ |
| (g) $10 * 4 * 2 * 6 = 12$ | (h) $6 * 2 * 3 * 4 = 15$ | |

3. Rewrite these equations and include brackets to make them correct.

- | | |
|-------------------------------|---------------------------------------|
| (a) $2 \times 3 + 4 = 14$ | (b) $2 + 5 \times 2 + 3 = 27$ |
| (c) $6 \times 3 + 4 - 5 = 37$ | (d) $10 + 3 \times 5 - 2 = 19$ |
| (e) $6 + 3 \times 2 + 3 = 45$ | (f) $8 - 4 \div 2 + 2 = 7$ |
| (g) $4 + 7 \times 2 + 3 = 25$ | (h) $12 - 10 \div 2 + 3 \times 2 = 7$ |

4. See how many different equations can be formed by including brackets into the following statement. Include the answer for each equation.

$$24 + 24 \div 6 + 2 \times 3$$

Number Laws

Commutative Law

The ***commutative law*** deals with the order that numbers are added, subtracted, multiplied or divided. For the commutative law to apply, the order that the numbers are operated on does not affect the result.

Addition

Use an example to see if the commutative law applies for addition:

Example $8 + 4 = 12$

Change the order of the numbers to see if the result is the same:

$$4 + 8 = 12$$

The result is the same therefore the ***commutative law DOES apply for addition***

Subtraction

Use an example to see if the commutative law applies for subtraction:

Example $8 - 4 = 4$

Change the order of the numbers to see if the result is the same:

$$4 - 8 = -4$$

The result is ***not*** the same therefore the ***commutative law DOES NOT apply for subtraction***

Multiplication

Use an example to see if the commutative law applies for multiplication:

Example $8 \times 4 = 32$

Change the order of the numbers to see if the result is the same:

$$4 \times 8 = 32$$

The result is the same therefore the ***commutative law DOES apply for multiplication***

Division

Use an example to see if the commutative law applies for division:

Example $8 \div 4 = 2$

Change the order of the numbers to see if the result is the same:

$$4 \div 8 = \frac{1}{2}$$

The result is **not** the same therefore the **commutative law DOES NOT apply for division**

EXERCISE 2M

1. State whether the following statements are **true** (T) or **false** (F).

- | | |
|-----------------------------------|---------------------------------|
| (a) $6 + 5 = 5 + 6$ | (b) $12 - 5 = 5 - 12$ |
| (c) $8 \times 7 = 7 \times 8$ | (d) $15 \div 3 = 3 \div 15$ |
| (e) $78 + 93 = 93 + 78$ | (f) $652 - 347 = 347 - 652$ |
| (g) $56 \times 79 = 79 \times 56$ | (h) $562 \div 34 = 34 \div 562$ |
| (i) $a + b = b + a$ | (j) $m - n = n - m$ |
| (k) $p \times q = q \times p$ | (l) $A \div B = B \div A$ |
| (m) $3 \times y = y \times 3$ | (n) $2 + b = b + 2$ |
| (o) $8 \div K = K \div 8$ | (p) $7 - D = D - 7$ |

2. Rewrite the following statements including one of the following symbols: either $=$ or $\neq$.

$=$ means **is equal to**

$\neq$ means **is not equal to**

Examples	1. $P + M$ $M + P$	2. $3 - x$ $x - 3$
	$P + M$ $=$ $M + P$	$3 - x$ $\neq$ $x - 3$

- | | |
|--|--|
| (a) 3×7 <input type="text"/> 7×3 | (b) $8 + 13$ <input type="text"/> $13 + 8$ |
| (c) $21 - 9$ <input type="text"/> $9 - 21$ | (d) $8 \div 5$ <input type="text"/> $5 \div 8$ |
| (e) $78 + 34$ <input type="text"/> $34 + 78$ | (f) $128 - 77$ <input type="text"/> $77 - 128$ |
| (g) $A \times 7$ <input type="text"/> $7 \times A$ | (h) $B + C$ <input type="text"/> $C + B$ |
| (i) $D \div F$ <input type="text"/> $F \div D$ | (j) $3a - 7$ <input type="text"/> $7 - 3a$ |

Associative Law

When three terms are being added, subtracted, multiplied or divided the ***associative law*** deals with the grouping of terms two at a time. If the order that the terms are grouped does not affect the result then the associative law applies.

Addition

Use an example to see if the associative law applies for addition:

Example $8 + 4 + 2$

These can be grouped two ways: $(8 + 4) + 2$ or $8 + (4 + 2)$

$(8 + 4) + 2$	$8 + (4 + 2)$
$= 12 + 2$	$= 8 + 6$
$= 14$	$= 14$

The result is the same therefore the ***associative law DOES apply for addition***

Subtraction

Use an example to see if the associative law applies for subtraction:

Example $8 - 4 - 2$

These can be grouped two ways: $(8 - 4) - 2$ or $8 - (4 - 2)$

$(8 - 4) - 2$	$8 - (4 - 2)$
$= 4 - 2$	$= 8 - 2$
$= 2$	$= 6$

The result is ***not*** the same therefore the ***associative law DOES NOT apply for subtraction***

Multiplication

Use an example to see if the associative law applies for multiplication:

Example $8 \times 4 \times 2$

These can be grouped two ways: $(8 \times 4) \times 2$ or $8 \times (4 \times 2)$

$(8 \times 4) \times 2$	$8 \times (4 \times 2)$
$= 32 \times 2$	$= 8 \times 8$
$= \mathbf{64}$	$= \mathbf{64}$

The result is the same therefore the
associative law DOES apply for multiplication

Division

Use an example to see if the associative law applies for division:

Example $8 \div 4 \div 2$

These can be grouped two ways: $(8 \div 4) \div 2$ or $8 \div (4 \div 2)$

$(8 \div 4) \div 2$	$8 \div (4 \div 2)$
$= 2 \div 2$	$= 8 \div 2$
$= \mathbf{1}$	$= \mathbf{4}$

The result is **not** the same therefore the
associative law DOES NOT apply for division

EXERCISE 2N

1. State whether the following statements are **true (T)** or **false (F)**.

(a) $(a + b) + c = a + (b + c)$	(b) $(a - b) - c = a - (b - c)$
(c) $(a \times b) \times c = a \times (b \times c)$	(d) $(a \div b) \div c = a \div (b \div c)$
2. Evaluate the following to verify the associative law for each operation.

(a)(i) $(12 + 6) + 2$	(ii) $12 + (6 + 2)$
(b)(i) $(12 - 6) - 2$	(ii) $12 - (6 - 2)$
(c)(i) $(12 \times 6) \times 2$	(ii) $12 \times (6 \times 2)$
(d)(i) $(12 \div 6) \div 2$	(ii) $12 \div (6 \div 2)$

Distributive Law

The **distributive law** states that if a bracket is multiplied by a term then each term in the bracket is multiplied by the term.

Examples

$$\begin{array}{ll}
 \text{1. } 6(5 + 4) & 6(5 + 4) \\
 = 6 \times (5 + 4) & \text{or} \quad = 6 \times (5 + 4) \\
 = (6 \times 5) + (6 \times 4) & = 6 \times 9 \\
 = 30 + 24 & = \mathbf{54} \\
 = \mathbf{54} &
 \end{array}$$

$$\begin{array}{ll}
 \text{2. } 8(12 - 3) & 8(12 - 3) \\
 = 8 \times (12 - 3) & \text{or} \quad = 8 \times (12 - 3) \\
 = (8 \times 12) - (8 \times 3) & = 8 \times 9 \\
 = 96 - 24 & = \mathbf{72} \\
 = \mathbf{72} &
 \end{array}$$

EXERCISE 20

1. Evaluate the following problems two ways (as in the example) to verify the distributive law.

- | | | |
|-----------------|----------------|----------------|
| (a) $5(6 + 3)$ | (b) $7(8 + 4)$ | (c) $6(9 - 2)$ |
| (d) $12(8 - 5)$ | (e) $8(5 + 6)$ | (f) $9(8 - 6)$ |

2. Use the distributive law to simplify these problems before solving.

Example 8×27

This can be rewritten as: $8 \times (20 + 7)$ or $8 \times (30 - 3)$

$$\begin{array}{ll}
 8 \times (20 + 7) & 8 \times (30 - 3) \\
 = (8 \times 20) + (8 \times 7) & = (8 \times 30) - (8 \times 3) \\
 = 160 + 56 & = 240 - 24 \\
 = \mathbf{216} & = \mathbf{216}
 \end{array}$$

- | | | |
|-------------------|--------------------|---------------------|
| (a) 7×32 | (b) 8×53 | (c) 9×89 |
| (d) 6×78 | (e) 11×71 | (f) 13×102 |
| (g) 8×97 | (h) 7×67 | (i) 15×103 |

Using the Number Laws

EXERCISE 2P

1. Use the *associative law* to find which *three* of the calculations below (**A - D**) would give the same answer as 15×36 .

Example $15 \times 36 = 3 \times 5 \times 6 \times 6$

- | | |
|--|--|
| A $5 \times 3 \times 36$ | B $15 \times 3 \times 12$ |
| C $5 \times 3 \times 30 \times 6$ | D $3 \times 5 \times 2 \times 18$ |

2. Use the *associative law* to find which of the calculations below (**A - F**) would give the same answer as 24×72 .

- | | |
|---|--|
| A $8 \times 3 \times 36 \times 2$ | B $6 \times 4 \times 12 \times 6$ |
| C $2 \times 12 \times 8 \times 9$ | D $3 \times 8 \times 2 \times 36$ |
| E $12 \times 2 \times 4 \times 18$ | F $4 \times 6 \times 24 \times 3$ |

3. Use the *associative law* and *commutative law* to find which of the calculations below (**A - D**) would give the same answer as 12×24 .

- | | |
|---------------------------------|----------------------------------|
| A $3 \times 12 \times 8$ | B $6 \times 24 \times 2$ |
| C $4 \times 24 \times 8$ | D $2 \times 12 \times 12$ |

4. Use the *associative law* and *commutative law* to find which of the calculations below (**A - F**) would give the same answer as 36×48 .

- | | |
|---|--|
| A $3 \times 4 \times 12 \times 12$ | B $6 \times 4 \times 12 \times 6$ |
| C $2 \times 16 \times 18 \times 2$ | D $9 \times 8 \times 6 \times 4$ |
| E $12 \times 2 \times 4 \times 18$ | F $4 \times 16 \times 9 \times 3$ |

5. From the list below find calculations that would give the same answer as the following:

- | | |
|----------------------------|----------------------------|
| (a) $3 \times 4 \times 8$ | (b) $2 \times 6 \times 5$ |
| (c) $8 \times 12 \times 4$ | (d) $8 \times 7 \times 9$ |
| (e) $6 \times 8 \times 9$ | (f) $8 \times 10 \times 9$ |

- | | |
|---------------------------------|----------------------------------|
| A $3 \times 21 \times 8$ | B $3 \times 18 \times 8$ |
| C $3 \times 30 \times 8$ | D $3 \times 5 \times 4$ |
| E $2 \times 12 \times 4$ | F $2 \times 16 \times 12$ |

6. Some problems can be made simpler by using the **associative** and **commutative laws**.

Use this technique to make the following problems easier to solve without using a calculator.

Example

$$\begin{aligned} 18 \times 50 \\ &= 9 \times 2 \times 10 \times 5 \\ &= 9 \times 2 \times 5 \times 10 \\ &= 9 \times 10 \times 10 \\ &= \mathbf{900} \end{aligned}$$

- | | | |
|--------------------|--------------------|--------------------|
| (a) 16×50 | (b) 18×25 | (c) 14×35 |
| (d) 45×24 | (e) 55×14 | (f) 66×15 |
| (g) 25×62 | (h) 35×22 | (i) 15×84 |

7. Bertie's calculator had the **6** button broken. To solve any problem that involved a **6** he needed to find another way of writing the problem.

Example 36×66 could be solved by entering $9 \times 4 \times 2 \times 33$

Give one example of how Bertie could solve the following problems on his calculator.

- | | | |
|--------------------|----------------------|----------------------|
| (a) 36×6 | (b) 16×64 | (c) 6×106 |
| (d) 60×26 | (e) 56×96 | (f) 63×162 |
| (g) 69×46 | (h) 164×346 | (i) 256×686 |

Other problems required Bertie to use the **distributive law**.

Example 23×67 could be solved by using the following steps:

$$\begin{aligned} 23 \times 67 \\ &= 23 \times (59 + 8) \\ &= (23 \times 59) + (23 \times 8) \end{aligned}$$

8. Give one example of how Bertie could solve the following problems on his calculator.

- | | | |
|---------------------|----------------------|---------------------|
| (a) 35×61 | (b) 19×163 | (c) 32×263 |
| (d) 617×29 | (e) 56×61 | (f) 63×269 |
| (g) 163×46 | (h) 643×346 | (i) 76×601 |

PROBLEM SOLVING

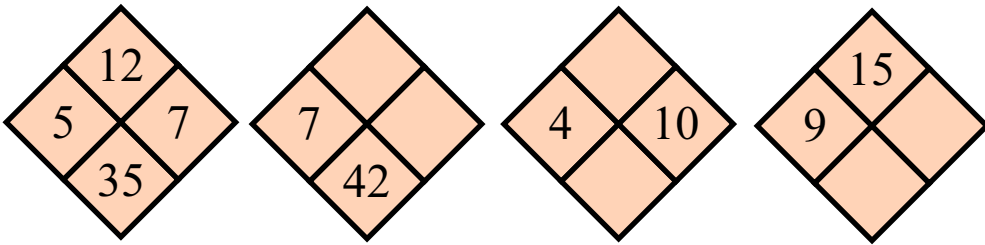
1. The sum of the first two odd numbers $= 1 + 3 = 4 = 2^2$
 The sum of the first three odd numbers $= 1 + 3 + 5 = 9 = 3^2$
 The sum of the first four odd numbers $= 1 + 3 + 5 + 7 = 16 = 4^2$

(a) Explain any pattern that is developing from the above information.

(b) Use this pattern to find the following sums.

- (i) The sum of the first 10 odd numbers.
- (ii) The sum of the first 30 odd numbers.
- (iii) The sum of the first 127 odd numbers.

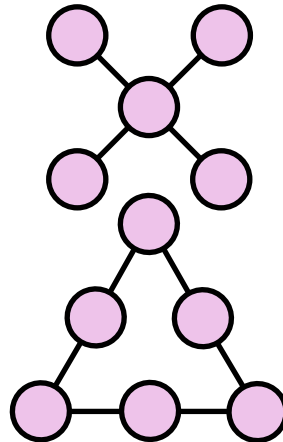
2. Using links between the numbers in the first diamond copy and complete the other diamonds.



3. Find the following numbers.

- (a) This number is the sum of a pair of twin primes.
It is a square number and is between 100 and 200.
- (b) This number is the product of a pair of twin primes.
It is a number between 0 and 1000.
It would be the same number if it was written forwards or backwards.

4. (a) Copy this diagram and place the numbers 1, 2, 3, 4 and 5 into the circles so that all the lines add to the same number.



- (b) Copy this diagram and place the numbers 1, 2, 3, 4, 5 and 6 into the circles so that all the lines add to the same number.

5. (a) Find the name given to a number (or word) that is the same if written forwards or backwards.

Examples 12321 1098901 DAD MINIM GLENELG

- (b) Find the first prime number (greater than 2) that would be the same if written forwards or backwards.
6. Use 5 twos and any of the operations (+, -, $\times$ or $\div$) or brackets to make the numbers 0 to 10.

Example $(2 - 2) \times (2 + 2 + 2) = 0$

7. The 'Licks' Licorice Factory has three machines for cutting long pieces of licorice into smaller lengths.
 The 'Bite-Size' machine cuts the licorice into 4 cm lengths.
 The 'Lick-a-Lot' machine cuts it into 15 cm lengths.
 The 'Snake-Size' machine cuts it into 40 cm lengths.
 What is the shortest length of licorice that could be fed into any of the machines without any licorice being wasted?

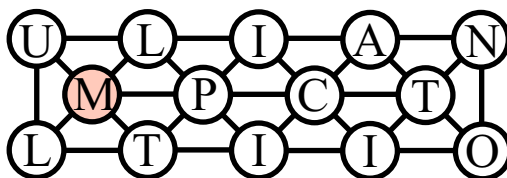
PUZZLES

1. Find words from this chapter 'hidden' in these sentences.

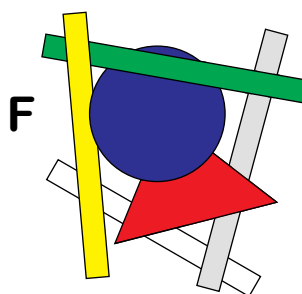
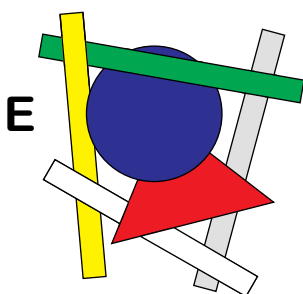
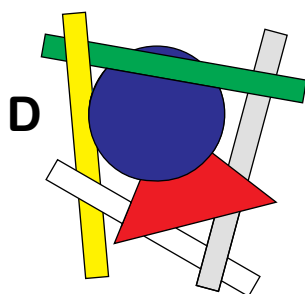
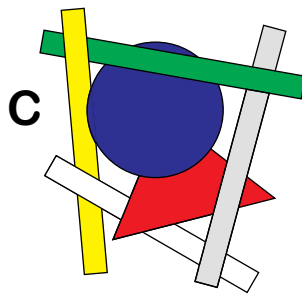
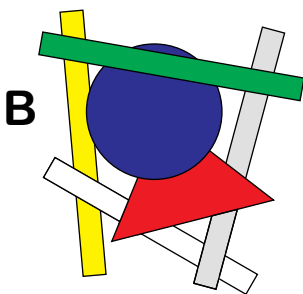
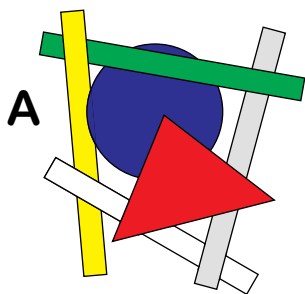
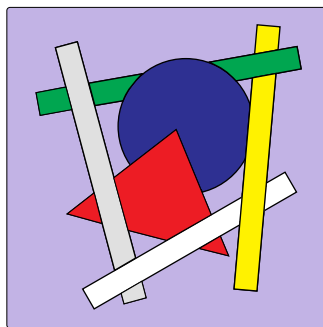
Example Dad drives to work each day.

- (a) Are UFOs fact or fiction?
 (b) When solving these riddles assume nothing.
 (c) To move a tree you must dig it up first.
 (d) When her hand went numb Erin couldn't write.
 (e) Steve needs to do more homework.
 (f) The College performed a production of Romeo and Juliet.
 (g) The hot rod drove very fast.

2. Starting with the shaded letter, follow the lines and use all the letters to spell a word from this chapter.



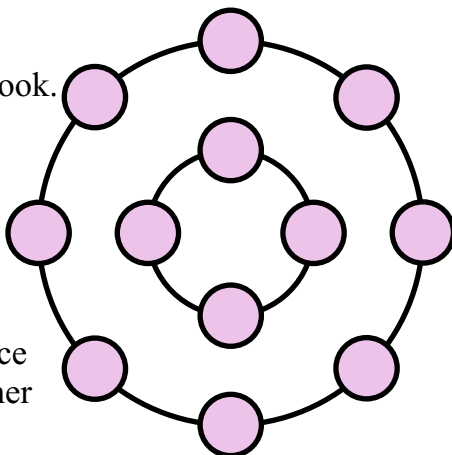
3. Which of the alternatives below shows what these objects would look like if viewed from behind?



4. (a) What number has two factors that add to 12 with one factor twice the other?
 (b) Find three consecutive numbers that add to 72.
 (c) Find two numbers that have a sum of 8 and product of 15.
 (d) What number has two factors that have a sum of 17 and a difference of 1.

5. Copy this diagram twice into your book.

- (a) Place the numbers 1-12 in the circles so that the sum of the numbers in the outer circle is the same as in the inner circle.
 (b) Place the numbers 1-12 in the circles so that the sum of the numbers in the outer circle is twice the sum of the numbers in the inner circle.



CHAPTER REVIEW

1. Find the answers to the following problems using mental arithmetic.
(a) $8 + 7$ (b) $25 + 32$ (c) $119 + 64$ (d) $18 - 14$ (e) $237 - 218$
2. Find the answers to the following problems without using a calculator.
(a) $2739 + 857 + 216$ (b) $3923 + 489 - 506 - 79$
3. (a) The sum of the weights of three dogs is 56 kg. If two of the dogs weigh 19 kg and 28 kg find the weight of the third dog.
(b) The total population of three towns is 2900.
Swansey has twice as many people as Duckton.
Heronville has 300 less people than Duckton.
What is the population of each town?
4. (a) $3 \times 6 = \underline{\quad}$ (b) $8 \times 9 = \underline{\quad}$ (c) $5 \times 7 = \underline{\quad}$ (d) $6 \times 8 = \underline{\quad}$
(e) $4 \times \underline{\quad} = 28$ (f) $\underline{\quad} \times 12 = 96$ (g) $\underline{\quad} \times 6 = 54$ (h) $11 \times \underline{\quad} = 99$
5. Sarah was planting an orchard. She planted four trees an hour for eight hours a day every day for three weeks.
How many trees did she plant?
6. Find the highest common factor of the following pairs of numbers.
(a) 20, 24 (b) 15, 20 (c) 27, 30 (d) 18, 36 (e) 56, 42
7. A carpenter has two lengths of wood. One is 72 cm, the other is 42 cm. He wants to cut the two lengths into smaller pieces that are all the same length. What is the length of the longest pieces he could cut with no waste wood?
8. List the five prime numbers between 63 and 84.
9. Use the tests for divisibility to find which of the following numbers are prime numbers.
132 245 137 171 351 205 852 337 500
10. (i) Write a factor tree for each of the following numbers.
(ii) Write each number as a product of prime numbers in index form.
(a) 36 (b) 84 (c) 80 (d) 124 (e) 333

11. Between which two consecutive whole numbers is $\sqrt{84}$.
12. Find the lowest common multiple of the following pairs of numbers.
(a) 5, 8 (b) 6, 9 (c) 10, 12 (d) 3, 12 (e) 6, 8
13. Find the following products without using a calculator.
(a) 259×6 (b) 798×21 (c) 307×245
14. Find the following products without using a calculator.
(a) 4×100 (b) 57×1000 (c) 200×100
15. (a) A person wanted to measure their heartbeat. They counted 23 beats in 15 seconds. What is their heartbeat in beats per minute?
(b) The water flow to a small town was measured to be 3500 litres every minute. How many litres would the town use in a day?
16. Find the following products without using a calculator.
(a) 40×70 (b) 300×20 (c) $60 \times 20 \times 300$
17. Find the approximate answers to the following calculations by rounding the numbers to the nearest 10 before finding the answers without using a calculator.
(a) 29×41 (b) 38×138 (c) 87×253
18. Find the following divisions without using a calculator.
(a) $84 \div 4$ (b) $963 \div 3$ (c) $2570 \div 5$ (d) $20\,000 \div 200$
(e) $3000 \div 500$ (f) $884\,400 \div 400$ (g) $573\,000 \div 3000$
19. Solve the following calculations without using a calculator.
(a) $2 + 5 \times 6$ (b) $\frac{1}{2}$ of $8 + 2 \times 4$ (c) $3 \times 8 \div 2 \times 3 \div 9$
(d) $(3 + 2) \times (7 - 5)$ (e) $72 \div 12 + 3 \times 2 + 5$
20. Use the distributive law to make the following problem easier and solve it: 8×36
21. Which of the following alternatives would give the same answer as 12×20 ? (More than one answer)
A $3 \times 4 \times 5 \times 4$ **B** $6 \times 10 \times 2 \times 2$ **C** $2 \times 2 \times 3 \times 4 \times 5$
D $6 \times 3 \times 4 \times 2$ **E** $3 \times 5 \times 4 \times 4$ **F** $2 \times 2 \times 2 \times 2 \times 3 \times 5$

Integers

Positive and Negative Numbers

Positive numbers are those numbers *greater than zero*.
Negative numbers are those numbers *less than zero*.

Integers are defined as all *positive numbers, negative numbers* and *zero*.

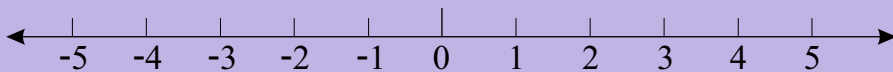
Negative numbers are written with a negative sign (-) in front of them.

Example negative 3 = -3

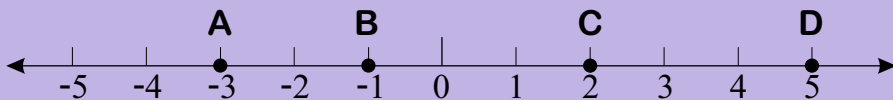
Positive numbers can be written with a positive sign (+) in front of them but this sign is not needed.

Example positive 6 = +6 = 6

Integers can be represented on a number line as shown below.



Examples



The points shown on this number line are:

A = -3

B = -1

C = 2 (or +2)

D = 5 (or +5)

EXERCISE 3A

1. Which points on the number line below correspond to the following numbers.

(a) -5

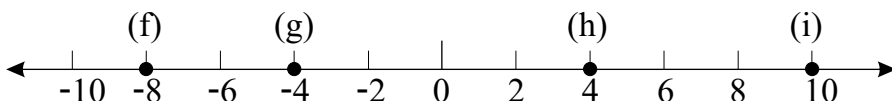
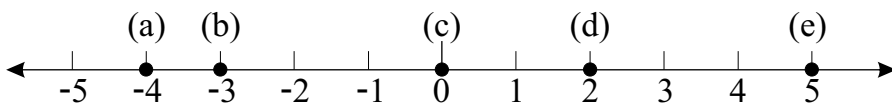
(b) 4

(c) 3

(d) -1



2. Write the number shown by each of the points on the number lines below.



3. Complete the following patterns.

(a) -9, -8, -7, -6, -5, __, __, __, __.

(b) -12, -10, -8, -6, -4, __, __, __, __.

(c) -35, -30, -25, -20, -15, __, __, __, __.

(d) 6, 5, 4, 3, 2, __, __, __, __.

(e) 60, 50, 40, 30, 20, __, __, __, __.

(f) -37, -32, -27, -22, -17, __, __, __, __.

(g) 34, 28, 22, 16, 10, __, __, __, __.

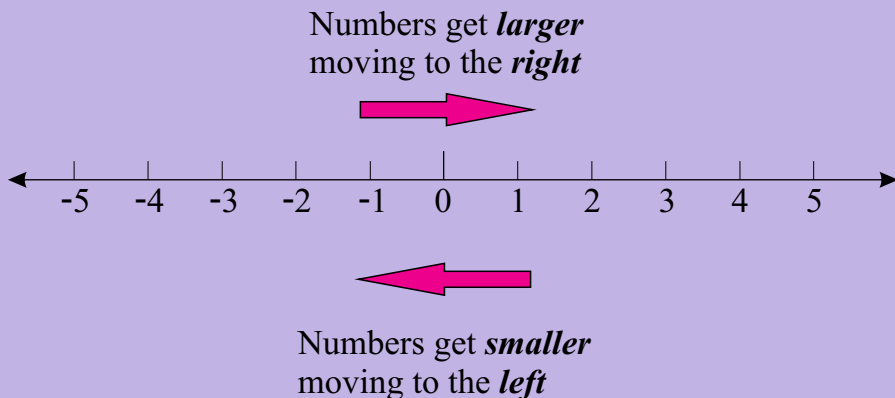
(h) 20, 19, 17, 14, 10, __, __, __, __.

(i) 100, 95, 85, 70, 50, __, __, __, __.

(j) -50, -48, -44, -38, -30, __, __, __, __.

(k) -50, -47, -41, -32, -20, __, __, __, __.

Numbers get larger as you move to the right along a number line.
Numbers get smaller as you move to the left along a number line.



Examples

1. Arrange these numbers from *smallest* to *largest*.

-6, 8, -10, 0, 3, -2

Answer: -10, -6, -2, 0, 3, 8

2. Which of the following numbers is the *largest*?

-12, 0, 8, 5, -4, -9, 2

Answer: 8

EXERCISE 3B

1. Arrange the following numbers from *smallest* to *largest*.

(a) 6, -5, 9, 3, -4, 4, -8

(b) -14, -23, 0, 21, -11, 17, 9, -5

(c) -12, -8, -3, 0, -4, -10

(d) 8, 7, -6, 5, -4, 3, -2, -1, 0, 1

2. Which of the following numbers is the *largest*?

(a) -21, -45, 33, 18, -64, 27

(b) -8, -17, -23, 0, -4, -7, -35

3. Which of the following numbers is the *smallest*?

(a) 8, -13, 14, 0, -11, 7, 2, -3

(b) 15, 9, 7, 1, 0, 8, 12, -3, -1

4. Choose the correct symbol ($<$ or $>$) to place between the pairs of numbers below.

Examples Remember the crocodile wants to eat the **larger** number.

1. -6 4



Answer: $-6 < 4$

2. -3 -8



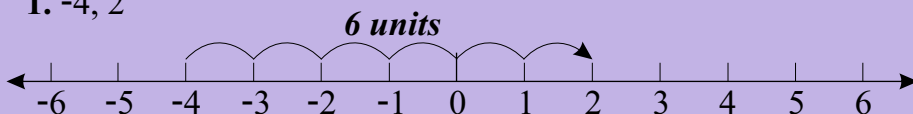
Answer: $-3 > -8$

- (a) -5 1 (b) -4 -7 (c) 9 4 (d) 4 -9 (e) 0 7
 (f) 7 12 (g) -8 -5 (h) 0 -7 (i) 6 -1 (j) -12 -1

5. Draw a number line between -6 and 6 in your books and use it to find the gap between the following pairs of numbers.

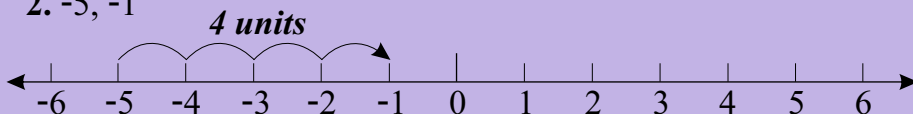
Examples

1. -4 , 2



The gap between -4 and 2 is **6**.

2. -5 , -1



The gap between -5 and -1 is **4**.

- (a) -2 , 3 (b) -5 , -2 (c) -4 , 3 (d) -3 , 0 (e) 2 , 6
 (f) -4 , 5 (g) -3 , 6 (h) -6 , -1 (i) -1 , 5 (j) -5 , -4

6. Find the gap between the following pairs of numbers.

- (a) -8 , 3 (b) -9 , 5 (c) -10 , 8 (d) -8 , 23 (e) -18 , -7
 (f) -15 , 25 (g) -13 , 29 (h) -34 , -8 (i) -19 , 37 (j) -23 , 45
 (k) -34 , -16 (l) -46 , -7 (m) -29 , 17 (n) -56 , -17 (o) -78 , -23

7. The highest temperature variation recorded on one day was in Browning, Montana, USA on 23rd January 1916. The temperature dropped 58°C . If the maximum temperature was 7°C , what was the lowest temperature on that day?

8. The temperature on the moon varies from 117°C to -163°C . What is the temperature variation on the moon?



9. This table shows the maximum temperature (max. temp.) and minimum temperature (min. temp.) for three cities on a particular day.

City	Min. Temp.	Max. Temp.
New York	-2°C	15°C
Paris	-5°C	9°C
Moscow	-12°C	-4°C

- (a) Find the difference between the maximum and minimum temperatures for: (i) New York (ii) Paris (iii) Moscow
- (b) Which city had the largest increase in temperature?
- (c) Which city had the highest maximum?
- (d) Which city had the lowest minimum?
- (e) What is the difference between the lowest minimum and highest maximum for the three cities?
- (f) The minimum temperature on the next day in New York was three degrees lower than on the day shown. What was the minimum temperature on the next day?
- (g) The maximum temperature on the next day in Moscow was seven degrees higher than on the day shown. What was the maximum temperature on the next day?



10. Use the information below to find the maximum and minimum temperatures for the three cities.

- * The three minimum temperatures were -3°C , -10°C and -5°C
- * Warsaw had the lowest minimum
- * Warsaw's maximum was nine degrees higher than its minimum
- * Stockholm's minimum was two degrees lower than Warsaw's maximum
- * Berlin's maximum was 18 degrees higher than Warsaw's minimum
- * The difference in Stockholm's temperatures was 15 degrees

Addition and Subtraction

EXERCISE 3C

1. Find the answers to the problems below by following these steps.

Step 1 Use the first number as the starting point on a number line.

Step 2 Move to the right or left by the amount shown.

Remember: $+$ means move to the *right*
 $-$ means move to the *left*

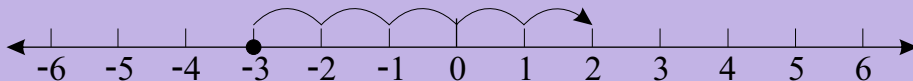
Examples

1. $-3 + 5$

$-3 + 5$

-3 is the starting
number

$+ 5$ means move to the *right*
by 5 units



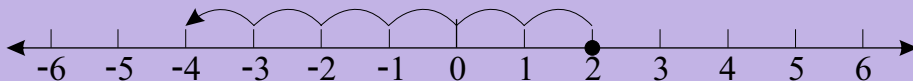
Answer = 2

2. $2 - 6$

$2 - 6$

2 is the starting
number

$- 6$ means move to the *left*
by 6 units



Answer = -4

(a) $-2 + 3$

(b) $3 - 4$

(c) $-3 + 1$

(d) $-1 - 3$

(e) $4 - 7$

(f) $5 - 6$

(g) $-6 + 8$

(h) $-3 + 9$

(i) $-5 + 3$

(j) $-4 + 4$

(k) $5 - 4$

(l) $4 - 8$

(m) $5 - 11$

(n) $-2 - 4$

(o) $4 - 5$

(p) $-4 + 7$

2. Find the answers to the following problems.

- | | | | |
|---------------|------------------|------------------|-------------------|
| (a) $-8 + 10$ | (b) $1 - 10$ | (c) $-12 + 7$ | (d) $-9 - 3$ |
| (e) $-5 - 15$ | (f) $7 - 20$ | (g) $-8 + 14$ | (h) $12 - 26$ |
| (i) $4 - 22$ | (j) $-17 + 11$ | (k) $-21 - 13$ | (l) $-25 + 24$ |
| (m) $-9 + 16$ | (n) $-8 - 7 - 5$ | (o) $9 - 12 + 7$ | (p) $-8 + 5 - 10$ |

3. Find the missing number in each of these equations.

- | | | |
|------------------------|-------------------------|--------------------------|
| (a) $3 - \square = 1$ | (b) $4 - \square = -2$ | (c) $-2 + \square = 3$ |
| (d) $1 - \square = -4$ | (e) $-8 + \square = -5$ | (f) $-3 - \square = -10$ |
| (g) $-5 + \square = 7$ | (h) $5 - \square = -4$ | (i) $-4 - \square = -12$ |
| (j) $\square - 2 = -5$ | (k) $\square + 4 = 1$ | (l) $\square - 5 = -2$ |
| (m) $\square + 3 = -3$ | (n) $\square - 4 = -10$ | (o) $\square + 7 = 2$ |

4. Where two signs occur together, replace them with one sign before finding the answer. Use the guide and examples below.

Note: Some positive (+) signs are shown on positive numbers in this question. This is not usually necessary.

$+ + = +$	$+ - = -$
$- - = +$	$- + = -$

Examples

1. $2 + +3$

$2 \overset{+}{+} 3$
 Replace with +
 $= 2 + 3$
 $= 5$

2. $-2 + -4$

$-2 \overset{+}{-} 4$
 Replace with -
 $= -2 - 4$
 $= -6$

3. $5 - +3$

$5 \overset{+}{-} 3$
 Replace with -
 $= 5 - 3$
 $= 2$

4. $-3 - -4$

$-3 \overset{-}{-} 4$
 Replace with +
 $= -3 + 4$
 $= 1$

- | | | | |
|---------------|---------------|---------------|----------------|
| (a) $3 - -4$ | (b) $5 + -2$ | (c) $7 - +5$ | (d) $-2 + +6$ |
| (e) $-3 - -5$ | (f) $4 - +7$ | (g) $-4 + -5$ | (h) $4 - -6$ |
| (i) $-1 + +5$ | (j) $-5 + -6$ | (k) $-6 - -6$ | (l) $-8 + -4$ |
| (m) $7 + +5$ | (n) $4 + -10$ | (o) $-7 + +9$ | (p) $-8 - -10$ |

5. Find the answers to the following problems.

- (a) $-12 - -20$ (b) $15 - +25$ (c) $16 - -18$ (d) $-30 + -20$
 (e) $-27 + -23$ (f) $35 + -45$ (g) $-20 + +25$ (h) $-37 - -23$
 (i) $38 + -23$ (j) $-24 - -27$ (k) $-58 - -42$ (l) $63 + -85$

6. Find the missing number in each of the following equations.

- (a) $5 + \square = 3$ (b) $-2 - \square = -6$ (c) $-3 - \square = -1$
 (d) $4 - \square = 7$ (e) $-2 + \square = -8$ (f) $-9 + \square = -3$
 (g) $\square + 2 = -2$ (h) $\square + 7 = 0$ (i) $\square - 5 = -11$
 (j) $\square + 1 = -5$ (k) $-6 - \square = 0$ (l) $-10 - \square = -5$

7. Find the missing operation + or - in the following equations.

Examples

1. $-5 \square -2 = -3$

$-5 - -2 = -3$

$-5 + 2 = -3$

Answer: -

2. $4 \square -5 = -1$

$4 + -5 = -1$

$4 - 5 = -1$

Answer: +

- (a) $5 \square -2 = 3$ (b) $-2 \square 7 = 5$ (c) $-4 \square 3 = -7$
 (d) $1 \square -5 = 6$ (e) $-2 \square -2 = 0$ (f) $-9 \square 6 = -3$
 (g) $-8 \square -5 = -3$ (h) $5 \square -6 = 11$ (i) $6 \square 11 = -5$

8. Place +, - and = in the following equations to make them correct.

Examples

1. $3 \square -2 \square -1 \square 6$

Answer: $3 - -2 = -1 + 6$

Both sides = 5

2. $2 \square -3 \square 4 \square -5$

Answer: $2 + -3 - 4 = -5$

Both sides = -5

3. $-1 \square 4 \square -2 \square 3$

Answer: $-1 = 4 + -2 - 3$

Both sides = -1

4. $-3 \square -4 \square -1 \square 6$

Answer: $-3 + -4 = -1 - 6$

Both sides = -7

- (a) $2 \square -4 \square 5 \square -7$ (b) $-3 \square 5 \square -2 \square -4$
 (c) $6 \square 5 \square -1 \square -2$ (d) $1 \square 7 \square -9 \square 3$
 (e) $-3 \square 4 \square -8 \square 9$ (f) $-5 \square 6 \square -3 \square 8$

PROBLEM SOLVING

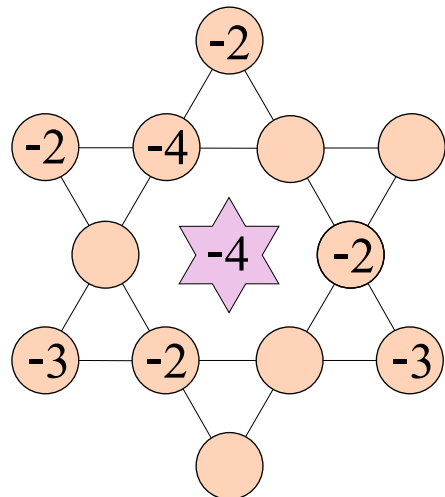
Maynard and Wade are camping on the Murray River. They decide to go fishing in a small boat. After a while they notice they have drifted 900 metres downstream because of the current.

They start paddling back to their campsite. They estimate they can paddle 100 metres in 10 minutes but then need to rest for 2 minutes. They drift back downstream 20 metres while they are resting. How long will it take them to paddle back to their campsite?



PUZZLE

Find the missing numbers from this diagram. The sum of the numbers in each line is equal to the number in the centre.



CHAPTER REVIEW

1. What numbers are shown by the points on this number line?



2. Complete the following patterns.

(a) -34, -28, -22, -16, __, __, __. (b) 15, 14, 12, 9, __, __, __.

3. List the following numbers from the smallest to the largest.

9 -26 5 -15 13 -19 -6 -21

4. Place the correct symbol ($<$ or $>$) between these numbers.

(a) -1 7 (b) -9 -15 (c) 0 -5 (d) -6 -1

5. Find the gap between the following pairs of numbers.

(a) -3, 7 (b) -10, -2 (c) -13, 21 (d) -36, -9

6. The temperature at Hobart dropped from 17°C to -6°C .

By how many degrees did the temperature drop?

7. Find the answers to the following problems.

(a) $3 - 9$ (b) $-6 + 10$ (c) $-12 - 3$ (d) $8 - -5$
 (e) $-10 + -6$ (f) $-5 - -13$ (g) $-18 + 7$ (h) $12 - -10$
 (i) $-6 - -4$ (j) $8 + -3$ (k) $-6 - -6$ (l) $-3 - -10$

8. Find the missing number in each of the following equations.

(a) $8 - \square = 10$ (b) $-4 + \square = -7$ (c) $7 - \square = -3$
 (d) $\square - 2 = -6$ (e) $\square + -1 = -5$ (f) $\square - 4 = -3$

9. Find the missing operation $+$ or $-$ in each of the following equations.

(a) $3 \square -1 = 4$ (b) $-2 \square 5 = 3$ (c) $6 \square -8 = -2$
 (d) $7 \square -1 = 8$ (e) $-4 \square -1 = -3$ (f) $-4 \square -9 = 5$

10. Place $+$, $-$ and $=$ in the following equations to make them correct.

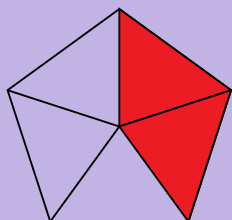
(a) $-6 \square 2 \square 1 \square -9$ (b) $5 \square 2 \square -3 \square -6$

Fractions

If a shape is divided into a number of equal parts these parts are referred to as a fraction of the shape.

Examples

1. (a) What fraction of this shape is shaded?
(b) What fraction of this shape is **not** shaded?



This shape is divided into five equal parts.

Each part is one fifth ($\frac{1}{5}$) of the shape.

(a) The fraction that is shaded is two fifths or $\frac{2}{5}$.

(b) The fraction that is not shaded is three fifths or $\frac{3}{5}$.

2. Vaughan had a seven hour plane flight. He slept for three hours of the flight.
(a) What fraction of the flight did Vaughan sleep?
(b) What fraction of the flight was he awake?

Answers (a) He slept for three out of the seven hours: $\frac{3}{7}$ of the flight

(b) He was awake for four out of seven hours: $\frac{4}{7}$ of the flight

3. Polly mixed two litres of white paint with one litre of green paint.
(a) What fraction of the mixture is white?
(b) What fraction of the mixture is green?

Answers (a) There is a total of three litres of paint in the mixture.

Two of the three are white: $\frac{2}{3}$ of the mixture is white

(b) One of the three litres is green: $\frac{1}{3}$ of the mixture is green

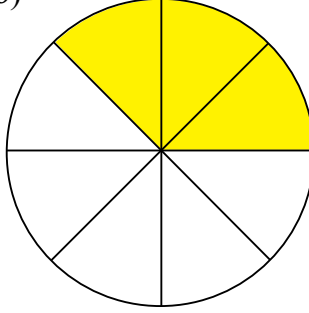
EXERCISE 4A

1. What fraction of each of the following diagrams is shaded?

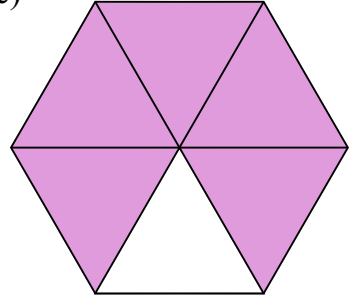
(a)



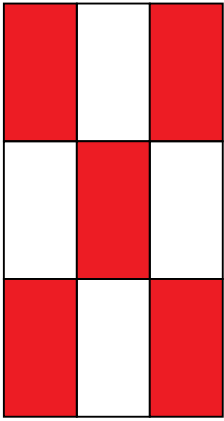
(b)



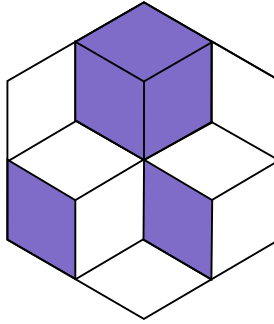
(c)



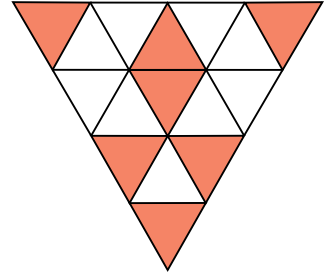
(d)



(e)



(f)



2. What fraction of each of the shapes in question 1 is **not** shaded.

3. A pizza is divided equally between five friends.

What fraction of the pizza should each person get?



4. Seven people shared equally in the price of a raffle ticket.

If they win a prize, what fraction should they each get?

5. In a class of 25 students, 13 are boys.

What fraction of the class is: (a) boys? (b) girls?

6. In January one year there were eight days when the temperature was 35°C or hotter.

(a) What fraction of the month had temperatures 35°C or hotter?

(b) What fraction of the month had temperatures below 35°C ?

7. A baker made three dozen pies. Twenty-three were meat pies and the rest were vegetable.

- (a) What fraction of the pies were meat?
(b) What fraction of the pies were vegetable?

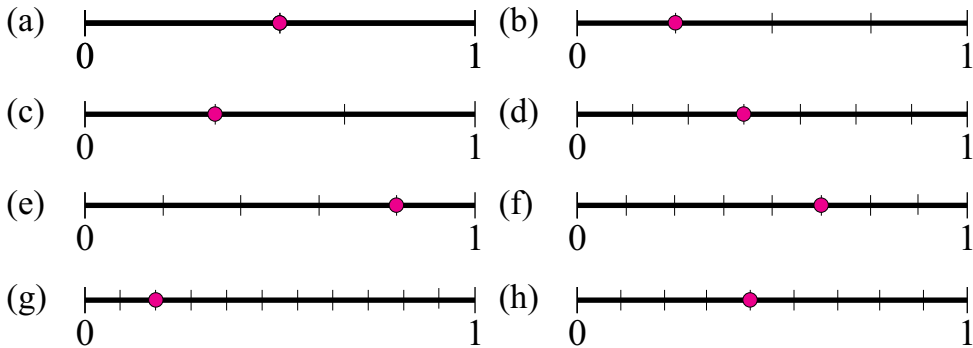


8. Larissa went to sleep at 10.00 pm and woke up at 5.00 am.

- (a) What fraction of the day did she sleep?
(b) What fraction of the day was she awake?

9. (a) What fraction of an hour is a minute?
(b) What fraction of a day is an hour?
(c) What fraction of the alphabet is vowels?

10. Write the fraction indicated by the dot on the number lines below.



11. For the fraction shown here:

- (a) state the numerator
(b) state the denominator

$$\frac{2}{3}$$

The **numerator** is the **top number**
The **denominator** is the **bottom number**

12. (a) Write the fraction that has 1 as its numerator and 7 as its denominator.
(b) Write the fraction that has 5 as its denominator and 3 as its numerator.

13. Write the following fractions in numeral form.

- (a) five-sevenths (b) three-eighths (c) two-fifths
(d) seven-tenths (e) three-quarters (f) eleven-twentieths

14. Write the following fractions in words.

- (a) $\frac{2}{9}$ (b) $\frac{7}{8}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{11}{13}$ (f) $\frac{23}{30}$

Equivalent Fractions

EXERCISE 4B

1. (a) State the fraction that is shaded in each of these shapes.
- (i) 
- (ii) 
- (b) Comment on the amount of each shape that is shaded.
- (iii) 
- (iv) 
- (c) What can you say about the fractions in part (a)?

2. Complete these equivalent fractions by filling in the boxes.

Example

The numerator and denominator must be multiplied by the same amount.

$$\frac{3}{4} = \frac{\boxed{}}{8}$$

$$\frac{3}{4} = \frac{\boxed{}}{8}$$

↗
×2

$$\frac{3}{4} = \frac{\boxed{}}{8}$$

↖
×2

$$\frac{3}{4} = \frac{6}{8}$$

- (a) $\frac{2}{5} = \frac{\boxed{}}{10}$ (b) $\frac{1}{3} = \frac{\boxed{}}{6}$ (c) $\frac{1}{4} = \frac{\boxed{}}{12}$ (d) $\frac{4}{5} = \frac{\boxed{}}{20}$
- (e) $\frac{5}{6} = \frac{\boxed{}}{18}$ (f) $\frac{3}{4} = \frac{\boxed{}}{24}$ (g) $\frac{3}{5} = \frac{\boxed{}}{35}$ (h) $\frac{3}{10} = \frac{\boxed{}}{50}$
- (i) $\frac{5}{9} = \frac{25}{\boxed{}}$ (j) $\frac{9}{11} = \frac{72}{\boxed{}}$ (k) $\frac{7}{8} = \frac{42}{\boxed{}}$ (l) $\frac{6}{7} = \frac{66}{\boxed{}}$

3. Which of the following fractions is **not** equivalent to $\frac{4}{7}$?

A $\frac{25}{28}$

B $\frac{32}{56}$

C $\frac{44}{77}$

D $\frac{48}{84}$

4. Which of the following fractions is **not** equivalent to $\frac{5}{9}$?

A $\frac{10}{18}$

B $\frac{30}{54}$

C $\frac{50}{90}$

D $\frac{110}{180}$

5. Which of the following fractions is **not** equivalent to $\frac{3}{13}$?

A $\frac{6}{26}$

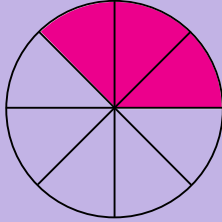
B $\frac{15}{65}$

C $\frac{30}{130}$

D $\frac{9}{169}$

6. Find the fraction of each of the following shapes that is shaded and write the fraction **three** different ways.

Example

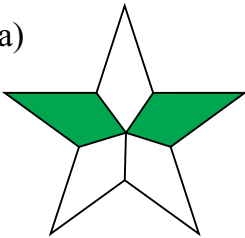


The fraction shaded is $\frac{3}{8}$.

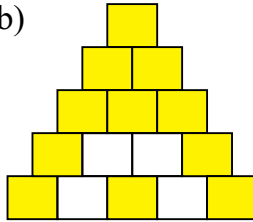
This could be written as:

$$\frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \frac{15}{40}, \text{ etc.}$$

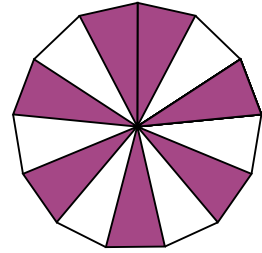
(a)



(b)



(c)



Simplest Form

EXERCISE 4C

1. Write the following fractions in their simplest form.

Example

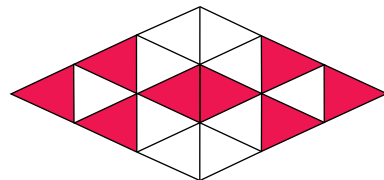
$$\frac{8}{12}$$

Find the largest number that divides into 8 and 12:- **4**.
Divide each number by 4.

$$\frac{8}{12} \xrightarrow{\div 4} \frac{2}{3}$$

- (a) $\frac{4}{8}$ (b) $\frac{10}{12}$ (c) $\frac{14}{16}$ (d) $\frac{15}{18}$ (e) $\frac{20}{25}$ (f) $\frac{24}{30}$
(g) $\frac{14}{49}$ (h) $\frac{36}{42}$ (i) $\frac{100}{200}$ (j) $\frac{18}{27}$ (k) $\frac{27}{27}$ (l) $\frac{48}{96}$

2. Find the fraction of this shape that is shaded and write the fraction in its simplest form.



3. Write the following fractions in their simplest form.

- (a) James plays basketball. In one game he scored 18 points out of the team's total of 30 points.

What fraction of the team's score did James get?



- (b) Michael plays tennis. In one set he had 36 first serves and he got 20 in.

What fraction of his first serves did he get in?

- (c) William enjoys fishing. On one fishing trip he caught 20 fish and kept 5 of them.

(i) What fraction of the fish did he keep?

(ii) What fraction did he throw back?

- (d) Jess recorded a 30 minute program onto a 3 hour DVD.

What fraction of the DVD did she use?

- (e) Emma earned \$50. She spent \$40 and saved the rest.

(i) What fraction of her earnings did she spend?

(ii) What fraction of her earnings did she save?

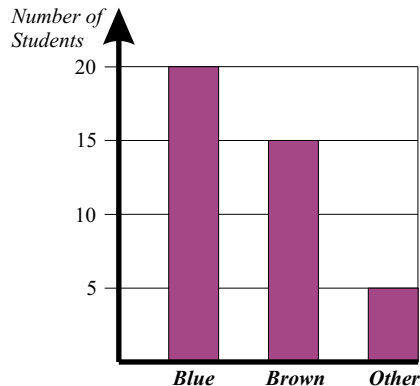
- (f) In a karate class there were 6 boys and 8 girls.

(i) What fraction of the class was boys?

(ii) What fraction of the class was girls?

- (g) In a netball game Natasha shot 18 goals, Melissa shot 12 goals and Carley shot 10 goals for one team. If no-one else in their team scored any goals, what fraction of the team's score did each of the three girls get?

- (h) The colour of the eyes of a group of students was recorded and shown on this graph. What fraction of the people in the group had blue eyes?



- (i) What fraction of the week is the weekend?
- (j) Jacinta made some cordial drink. She used 2 litres of cordial and 9 litres of water. What fraction of the drink is cordial?
- (k) In a chess club there were twice as many boys as girls.
- (i) What fraction of the club was boys?
- (ii) What fraction of the club was girls?

<i>Person</i>	<i>Number of Votes</i>
Belinda	22
Adrian	23
Emma	30
Nick	25

- (l) An election was held for the President of a rowing club. The result of the election is shown in this table.
Who won the election and what fraction of the total vote did they get?
- (m) In the town of Dotcom the population was 2700. A study was conducted and it was found that 2025 of the people in Dotcom had shopped online and 1350 people had used online banking.
- (i) What fraction of Dotcom's population had shopped online?
- (ii) What fraction of the people who had used online banking?

Comparing Fractions Using Fraction Strips

One way to compare the size of fractions is to draw fraction strips of the same length.

Example Draw fraction strips for the following two fractions and state which fraction is larger.

$$\frac{1}{3}, \frac{2}{5}$$



It can be easily seen that $\frac{2}{5}$ is the larger fraction.

EXERCISE 4D

Example Which of the following fractions is larger?

$$\frac{3}{4}, \frac{4}{5}$$

Step 1 Find a number the two denominators will divide into .
The two denominators, 4 and 5, will both divide into **20**.

Step 2 Draw two strips that are this number, in cm, long.

20 cm



Step 3 Shade $\frac{3}{4}$ of the first strip and $\frac{4}{5}$ of the second.

Each section of the first strip will be 5 cm long.
Each section of the second strip will be 4 cm long.



Step 4 By comparing the two fraction strips it can be seen that

$\frac{4}{5}$ is the larger.

State which of the following fractions is larger by using the steps shown in the example above.

1. $\frac{2}{3}, \frac{3}{4}$

2. $\frac{1}{2}, \frac{3}{7}$

3. $\frac{5}{7}, \frac{2}{3}$

4. $\frac{2}{3}, \frac{5}{6}$

Comparing Fractions Using Fraction Walls

The size of fractions can be compared by using fractions walls or rectangular arrays.

Example Which of the following fractions is larger?

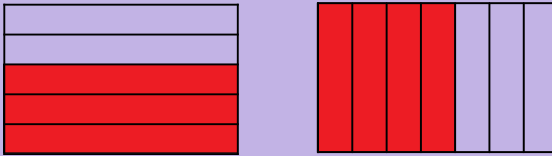
$$\frac{3}{5}, \frac{4}{7}$$

Step 1 Draw two boxes. *They don't have to be drawn accurately.*

Shade $\frac{3}{5}$ of the first box using horizontal strips.

Shade $\frac{4}{7}$ of the second box using vertical strips.

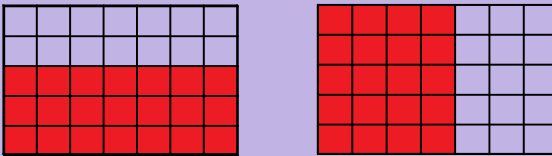
The strips don't have to be measured or drawn accurately.



Step 2 Divide the first box into the same number of vertical strips as the second box.

Divide the second box into the same number of horizontal strips as the first box.

The strips don't have to be measured or drawn accurately.



Step 3 Find the fraction of small rectangles that is shaded in each box. There are 35 small rectangles in each box.

21 of the 35 are shaded in the first box ($\frac{21}{35}$).

20 of the 35 are shaded in the second box ($\frac{20}{35}$).

Step 4 The larger fraction is that shaded in the first box $\frac{3}{5}$

EXERCISE 4E

State which of the following fractions is larger by using fraction walls.

1. $\frac{2}{5}$, $\frac{3}{8}$
2. $\frac{3}{4}$, $\frac{5}{7}$
3. $\frac{1}{4}$, $\frac{2}{9}$
4. $\frac{2}{3}$, $\frac{7}{10}$
5. $\frac{5}{7}$, $\frac{3}{5}$
6. $\frac{5}{6}$, $\frac{6}{7}$
7. $\frac{3}{5}$, $\frac{5}{8}$
8. $\frac{3}{7}$, $\frac{4}{9}$

Comparing Fractions Using Lowest Common Denominator

EXERCISE 4F

1. Find the lowest common denominator for the following pairs of fractions and write the fractions with this common denominator.

Example $\frac{2}{3}$, $\frac{3}{4}$

The lowest common denominator is the smallest number that both denominators divide into.

The lowest common denominator of 3 and 4 is **12**.

$$\frac{2}{3} = \frac{\boxed{}}{12}$$

$$\frac{3}{4} = \frac{\boxed{}}{12}$$

*Use equivalent fractions
- Exercise 2.*

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

- (a) $\frac{3}{5}$, $\frac{1}{2}$
- (b) $\frac{1}{3}$, $\frac{1}{2}$
- (c) $\frac{3}{4}$, $\frac{1}{5}$
- (d) $\frac{3}{5}$, $\frac{2}{3}$
- (e) $\frac{2}{5}$, $\frac{3}{10}$
- (f) $\frac{5}{6}$, $\frac{2}{5}$
- (g) $\frac{2}{7}$, $\frac{5}{8}$
- (h) $\frac{5}{12}$, $\frac{3}{8}$

2. Write the pairs of fractions from question 1 with the symbol $<$ or $>$ between them to indicate which fraction is larger.

Example $\frac{2}{3}, \frac{3}{4}$ Look at the fractions with a common denominator.

$$\frac{2}{3} = \frac{8}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

$$\frac{8}{12} < \frac{9}{12}$$

$$\frac{2}{3} < \frac{3}{4}$$

3. Justin caught a fish that weighed $\frac{3}{5}$ of a kg.
Magnus caught a fish that weighed $\frac{4}{7}$ of a kg.
Who caught the larger fish?
4. Find the fraction which is in the middle of each of the following pairs of fractions. Write the fraction in its simplest form.
- (a) $\frac{5}{8}, \frac{7}{8}$ (b) $\frac{2}{3}, \frac{4}{5}$ (c) $0, \frac{2}{13}$ (d) $\frac{5}{6}, 1$
5. Write the following fractions in order from the smallest to the largest.
- (a) $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}$ (b) $\frac{2}{3}, \frac{5}{6}, \frac{1}{2}$ (c) $\frac{7}{8}, \frac{5}{6}, \frac{3}{4}$ (d) $\frac{3}{4}, \frac{5}{8}, \frac{13}{24}, \frac{7}{12}$
6. Damien, Chris and Brendan are all cricketers who play for different teams. In one round they made the following scores.
Damien scored 28 runs out of his team's score of 84.
Chris scored 45 runs out of his team's score of 108.
Brendan scored 36 runs out of his team's score of 96.
Which cricketer scored the largest fraction of his team's score?



7. Alice, Megan and Danae share the workload in running a clothes shop. They calculated that Alice did $\frac{2}{7}$ of the work, Megan did $\frac{4}{11}$ of the work and Danae did the rest.
- (a) Who did the most work in the shop?
- (b) Who did the least amount of work?

Proper and Improper Fractions/Mixed Numbers

A **proper fraction** has the numerator smaller than the denominator.

Examples $\frac{5}{7}$, $\frac{13}{25}$, $\frac{1}{37}$, etc

An **improper fraction** has the numerator larger than, or equal to, the denominator.

Examples $\frac{8}{3}$, $\frac{27}{19}$, $\frac{9}{9}$, etc

A **mixed number** is a combination of a whole number and a fraction.

Examples $7\frac{3}{4}$, $15\frac{12}{31}$, etc

Improper fractions and **mixed numbers** can be used to represent the same amounts.

Example Write the shaded portion below as:

(a) a mixed number

(b) an improper fraction



(a) There are two whole rectangles shaded and three-fifths of the third.

The total portion shaded, as a **mixed number**, is: $2\frac{3}{5}$

(b) The rectangles are all divided into fifths. There is a total of 13 fifths.

The total portion shaded, as an **improper fraction**, is: $\frac{13}{5}$

EXERCISE 4G

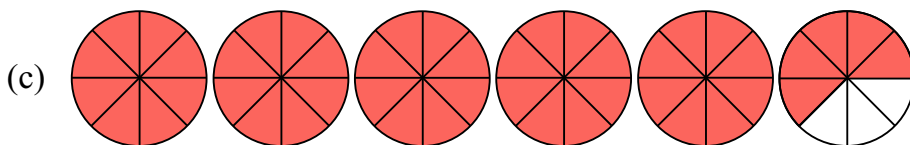
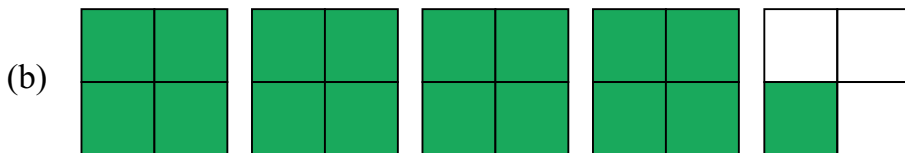
1. For the fractions shown:

- State which are proper fractions.
- State which are improper fractions.
- State which are mixed numbers.

$$\frac{37}{10}, \frac{3}{2}, \frac{8}{9}, 9\frac{15}{16}, \frac{1}{2}, 3\frac{7}{13}, \frac{13}{7}, 1\frac{5}{7}, \frac{3}{4}$$

2. State the shaded portions of the diagrams below as:

- mixed numbers
- improper fractions



- If three pizzas were each cut into six equal sections how many sections would there be altogether?
 - Richard had six lengths of wood that were each 2000 mm long. He cut all the lengths into five equal pieces.
 - How long was each of the smaller pieces?
 - How many of the smaller pieces were there?
 - If four blocks of chocolate were each cut into eighths, how many small pieces would there be?
 - If 10 oranges are cut into quarters, how many would there be?
- How many thirds are in 2?
 - How many fifths are in 10?
 - How many halves are in $5\frac{1}{2}$?
 - How many quarters are in $3\frac{3}{4}$?

5. Change the following mixed numbers to improper fractions.

A simple way to obtain the numerator of the improper fraction is to multiply the denominator of the fraction in the mixed number by the whole number and add the numerator.

The denominator of the improper fraction is the same as the denominator of the fraction in the mixed number.

Example 1 $3\frac{2}{5}$

The numerator of the improper fraction will be $5 \times 3 + 2 = 17$

The denominator of the improper fraction = 5

$$3\frac{2}{5} = \frac{17}{5}$$

Example 2 $8\frac{5}{6}$

The numerator of improper fraction = $6 \times 8 + 5 = 53$

$$8\frac{5}{6} = \frac{53}{6}$$

- (a) $2\frac{1}{2}$ (b) $3\frac{3}{4}$ (c) $5\frac{1}{6}$ (d) $4\frac{3}{7}$ (e) $3\frac{2}{5}$
 (f) $8\frac{2}{7}$ (g) $10\frac{3}{4}$ (h) $6\frac{2}{9}$ (i) $15\frac{2}{11}$ (j) $23\frac{7}{18}$

6. Change the following improper fractions to mixed numbers.

A simple way to change an improper fraction to a mixed number is to divide the denominator into the numerator. The answer (quotient) is the whole number and the remainder is the numerator of the fraction in the mixed number.

Example 1 $\frac{32}{5}$

$32 \div 5 = 6$ and a remainder of 2

$$\frac{32}{5} = 6\frac{2}{5}$$

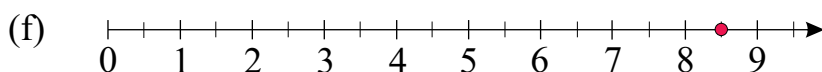
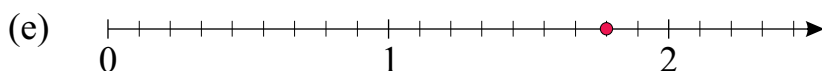
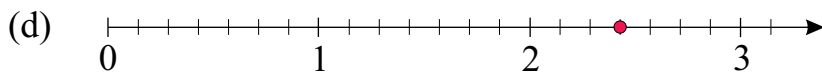
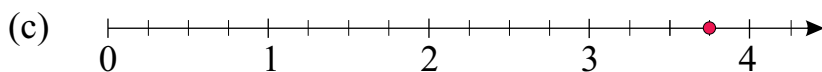
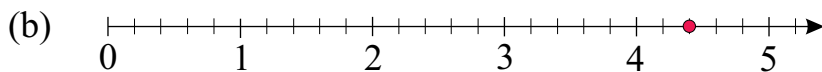
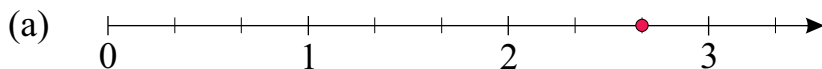
Example 2 $\frac{75}{8}$

$75 \div 8 = 9$ and a remainder of 3

$$\frac{75}{8} = 9\frac{3}{8}$$

- (a) $\frac{7}{3}$ (b) $\frac{11}{2}$ (c) $\frac{9}{7}$ (d) $\frac{37}{10}$ (e) $\frac{73}{8}$
 (f) $\frac{27}{5}$ (g) $\frac{31}{4}$ (h) $\frac{41}{6}$ (i) $\frac{199}{15}$ (j) $\frac{611}{20}$

7. State the number shown by the dot on each of the following number lines. Write each number as a mixed number.



8. Write the following fractions in their simplest form.

- (a) $2\frac{4}{8}$ (b) $\frac{28}{12}$ (c) $15\frac{8}{10}$ (d) $\frac{106}{56}$ (e) $8\frac{18}{24}$
 (f) $\frac{36}{18}$ (g) $7\frac{32}{64}$ (h) $3\frac{72}{84}$ (i) $\frac{55}{44}$ (j) $9\frac{112}{168}$

PROBLEM SOLVING

1. Three banks are offering the following interest rates.

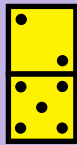
Bank A	$7\frac{3}{8}\%$	Bank B	$7\frac{4}{11}\%$	Bank C	$7\frac{5}{13}\%$
--------	------------------	--------	-------------------	--------	-------------------

Which bank is offering the highest interest rate?

2. (a) Find a fraction that if 2 is added to the numerator the fraction doubles in size.
 (b) Find a fraction that if 2 is added to the denominator the fraction halves in size.
 (c) Find a fraction that if 3 is added to the numerator and the denominator the fraction doubles in size.

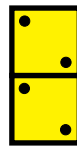
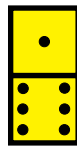
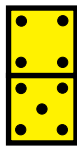
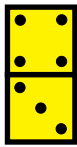
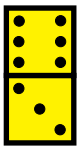
3. Consider a domino as a fraction.

Example



This domino would be $\frac{2}{5}$.
 It could also be turned upside down. The fraction would then be $\frac{5}{2}$.

- (a) List all the possible fractions that could be formed by the dominoes below in order from smallest to largest.
 (Don't forget to turn them upside down)



- (b) If dominoes are any combination of numbers from 1 to 6, find all possible pairs of dominoes where the fraction formed by one domino is twice that formed on the other domino.
4. (a) What type of fraction would become **larger** if 1 was added to the numerator and the denominator?
 (b) What type of fraction would become **smaller** if 1 was added to the numerator and the denominator?
5. Find the fraction that is half way between $\frac{3}{7}$ and $\frac{4}{9}$.

PUZZLES

1. Four words from this chapter have been broken up into their syllables and rearranged. Find the four words and there will be two syllables left that form the name of a famous French mathematician who lived in the 1600's.

A TOR NU
DE TION CAL FRAC E
NOM TOR IN A MER A
QUIV PAS LENT A

2. Patrick is my mother's mother's son's son.
What relation is Patrick to me?



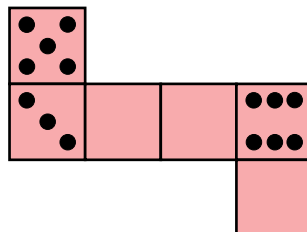
3. Sean, whose flag is the same as Shane's, has Stan on his left and Zane on his right. Zane's flag is the same as Dan's.
Find which flag is held by each boy.



4. In this addition problem the three symbols each represent a different number.
Find which number each of the symbols represents to make the addition correct.

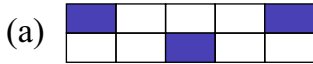
$$\begin{array}{cccc}
 \text{Smiley Face} & \text{Yin-Yang} & \text{Sad Face} & \text{Sad Face} \\
 + & & & \\
 1 & \text{Smiley Face} & \text{Yin-Yang} & \text{Sad Face} \\
 \text{Yin-Yang} & \text{Smiley Face} & \text{Sad Face} & 0
 \end{array}$$

5. If the opposite sides of a die add to 7, complete this diagram so it could be folded to form a die.



CHAPTER REVIEW

1. What fraction of the following shapes is shaded?

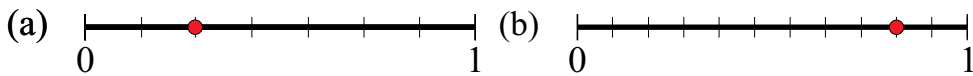


2. In a riding club there are 21 members. 14 of them are girls.

(a) What fraction of the riding club members are girls?

(b) What fraction of the riding club members are boys?

3. What fraction is indicated by the dot on these number lines?



4. State the fraction that has a denominator of 13 and numerator of 3.

5. Write $\frac{7}{11}$ in words.

6. Complete the following equivalent fractions.

(a) $\frac{3}{5} = \frac{\square}{15}$ (b) $\frac{2}{7} = \frac{8}{\square}$ (c) $\frac{3}{10} = \frac{\square}{90}$ (d) $\frac{4}{9} = \frac{\square}{36}$

7. Write the following fractions in their simplest form.

(a) $\frac{3}{6}$ (b) $\frac{24}{28}$ (c) $\frac{20}{30}$ (d) $\frac{56}{64}$ (e) $\frac{100}{300}$ (f) $\frac{112}{168}$

8. State which of the following fractions is larger.

(a) $\frac{2}{7}$, $\frac{1}{10}$ (b) $\frac{3}{8}$, $\frac{4}{7}$ (c) $\frac{1}{5}$, $\frac{2}{9}$ (d) $\frac{2}{5}$, $\frac{5}{12}$

9. Write the following pairs of fractions with a common denominator.

(a) $\frac{1}{2}$, $\frac{1}{3}$ (b) $\frac{3}{5}$, $\frac{1}{2}$ (c) $\frac{3}{7}$, $\frac{5}{8}$ (d) $\frac{3}{10}$, $\frac{5}{12}$

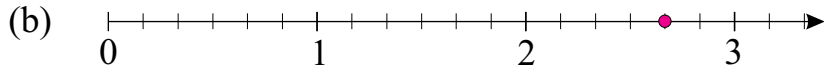
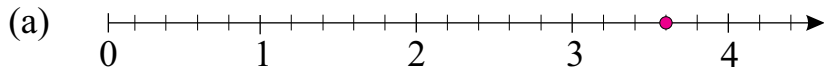
10. Change the following mixed numbers to improper fractions.

(a) $3\frac{1}{2}$ (b) $2\frac{3}{4}$ (c) $5\frac{7}{8}$ (d) $10\frac{7}{10}$ (e) $9\frac{5}{7}$

11. Change the following improper fractions to mixed numbers.

(a) $\frac{8}{3}$ (b) $\frac{12}{5}$ (c) $\frac{59}{6}$ (d) $\frac{18}{11}$ (e) $\frac{72}{7}$

12. State the number shown by the dot on each of the following number lines. Write each number as a mixed number in its simplest form.



13. Write the following fractions in their simplest form.

(a) $3 \frac{8}{12}$

(b) $\frac{42}{18}$

(c) $12 \frac{6}{9}$

(d) $\frac{104}{40}$

14. A mining truck and its load weighed 168 tonnes. If the load weighed 56 tonnes, what fraction of the total weight was the load?
Give the answer in its simplest form.



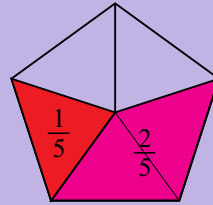
15. If 8 bread rolls were each cut into thirds how many would there be?

Fraction Operations

Addition and Subtraction

EXERCISE 5A

When adding or subtracting fractions if the denominators are the same then simply add or subtract the numerators. This can easily be seen in a diagram.



$$\begin{aligned}\frac{1}{5} + \frac{2}{5} \\ = \frac{3}{5}\end{aligned}$$

Examples

$$\begin{aligned}1. \quad \frac{2}{7} + \frac{3}{7} \\ = \frac{5}{7}\end{aligned}$$

$$\begin{aligned}2. \quad \frac{5}{9} - \frac{4}{9} \\ = \frac{1}{9}\end{aligned}$$

$$\begin{aligned}3. \quad \frac{2}{11} + \frac{4}{11} + \frac{3}{11} \\ = \frac{9}{11}\end{aligned}$$

$$\begin{aligned}4. \quad \frac{3}{13} + \frac{11}{13} - \frac{8}{13} \\ = \frac{6}{13}\end{aligned}$$

1. Complete the following calculations.

(a) $\frac{1}{3} + \frac{1}{3}$

(b) $\frac{5}{9} + \frac{2}{9}$

(c) $\frac{6}{7} - \frac{2}{7}$

(d) $\frac{7}{11} - \frac{6}{11}$

(e) $\frac{7}{15} + \frac{4}{15}$

(f) $\frac{18}{23} - \frac{12}{23}$

(g) $\frac{2}{9} + \frac{2}{9} + \frac{1}{9}$

(h) $\frac{14}{17} - \frac{6}{17} - \frac{2}{17}$

(i) $\frac{9}{19} + \frac{12}{19} - \frac{4}{19}$

(j) $\frac{11}{25} - \frac{6}{25} + \frac{14}{25}$

(k) $\frac{6}{7} - \frac{2}{7} + \frac{1}{7}$

(l) $\frac{18}{29} + \frac{23}{29} - \frac{15}{29}$

2. Complete the following calculations. Write answers in their simplest form.

Examples

$$\begin{aligned} 1. \quad & \frac{3}{10} + \frac{1}{10} \\ &= \frac{4}{10} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{11}{12} - \frac{5}{12} \\ &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

(a) $\frac{1}{8} + \frac{5}{8}$

(b) $\frac{4}{9} + \frac{2}{9}$

(c) $\frac{7}{8} - \frac{3}{8}$

(d) $\frac{7}{15} - \frac{2}{15}$

(e) $\frac{5}{16} + \frac{7}{16}$

(f) $\frac{19}{24} - \frac{1}{24}$

(g) $\frac{2}{21} + \frac{4}{21} + \frac{8}{21}$

(h) $\frac{23}{27} - \frac{4}{27} - \frac{7}{27}$

(i) $\frac{9}{25} + \frac{19}{25} - \frac{8}{25}$

(j) $\frac{23}{36} - \frac{7}{36} + \frac{5}{36}$

(k) $\frac{23}{42} - \frac{11}{42} + \frac{23}{42}$

(l) $\frac{19}{56} + \frac{33}{56} - \frac{17}{56}$

3. Complete the following calculations. Write answers as mixed numbers.

Examples

$$\begin{aligned} 1. \quad & \frac{6}{7} + \frac{5}{7} \\ &= \frac{11}{7} \\ &= 1\frac{4}{7} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{7}{9} + \frac{4}{9} + \frac{8}{9} + \frac{7}{9} \\ &= \frac{26}{9} \\ &= 2\frac{8}{9} \end{aligned}$$

(a) $\frac{3}{5} + \frac{4}{5}$

(b) $\frac{5}{9} + \frac{8}{9}$

(c) $\frac{7}{8} + \frac{5}{8} + \frac{3}{8}$

(d) $\frac{6}{7} + \frac{6}{7} + \frac{6}{7}$

(e) $\frac{8}{9} + \frac{7}{9} + \frac{8}{9} + \frac{5}{9}$

(f) $\frac{6}{7} + \frac{5}{7} - \frac{2}{7} + \frac{4}{7}$

(g) $\frac{4}{5} - \frac{2}{5} + \frac{4}{5} + \frac{3}{5}$

(h) $\frac{5}{9} + \frac{7}{9} + \frac{4}{9} - \frac{2}{9} + \frac{7}{9} - \frac{1}{9}$

(i) $\frac{10}{11} - \frac{2}{11} + \frac{9}{11} - \frac{1}{11} + \frac{6}{11} + \frac{8}{11} + \frac{9}{11}$

4. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

Examples

$$1. \quad \frac{9}{4} + \frac{13}{4}$$

$$= \frac{22}{4}$$

$$= 5\frac{2}{4}$$

$$= 5\frac{1}{2}$$

or

$$\frac{9}{4} + \frac{13}{4}$$

$$= \frac{22}{4}$$

$$= \frac{11}{2}$$

$$= 5\frac{1}{2}$$

$$2. \quad \frac{39}{6} - \frac{11}{6}$$

$$= \frac{28}{6}$$

$$= 4\frac{4}{6}$$

$$= 4\frac{2}{3}$$

or

$$\frac{39}{6} - \frac{11}{6}$$

$$= \frac{28}{6}$$

$$= \frac{14}{3}$$

$$= 4\frac{2}{3}$$

$$(a) \quad \frac{7}{8} + \frac{13}{8}$$

$$(b) \quad \frac{17}{9} + \frac{13}{9}$$

$$(c) \quad \frac{27}{4} - \frac{9}{4}$$

$$(d) \quad \frac{33}{6} - \frac{13}{6}$$

$$(e) \quad \frac{31}{12} + \frac{19}{12}$$

$$(f) \quad \frac{25}{8} - \frac{9}{8}$$

$$(g) \quad \frac{11}{9} + \frac{26}{9} + \frac{14}{9}$$

$$(h) \quad \frac{37}{6} - \frac{5}{6} - \frac{17}{6}$$

$$(i) \quad \frac{27}{8} + \frac{33}{8} - \frac{13}{8} + \frac{17}{8}$$

$$(j) \quad \frac{39}{10} + \frac{21}{10} + \frac{17}{10} + \frac{13}{10}$$

$$(k) \quad \frac{93}{12} - \frac{23}{12} - \frac{13}{12} + \frac{7}{12}$$

5. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

Examples

$$1. \quad 5\frac{3}{8} + 2\frac{7}{8}$$

$$= \frac{43}{8} + \frac{23}{8}$$

$$= \frac{66}{8}$$

$$= \frac{33}{4}$$

$$= 8\frac{1}{4}$$

$$2. \quad 7\frac{2}{9} - 3\frac{8}{9}$$

$$= \frac{65}{9} - \frac{35}{9}$$

$$= \frac{30}{9}$$

$$= \frac{10}{3}$$

$$= 3\frac{1}{3}$$

$$(a) \quad 4\frac{3}{5} + 6\frac{4}{5}$$

$$(b) \quad 3\frac{5}{9} + 4\frac{7}{9}$$

$$(c) \quad 8\frac{3}{8} - 2\frac{7}{8}$$

$$(d) \quad 2\frac{5}{6} + 5\frac{5}{6}$$

$$(e) \quad 9\frac{1}{4} - 4\frac{3}{4}$$

$$(f) \quad 8\frac{3}{10} - 3\frac{7}{10}$$

$$(g) \quad 3\frac{3}{4} + 4\frac{3}{4} + 2\frac{1}{4}$$

$$(h) \quad 8\frac{1}{6} - 5\frac{5}{6} + 3\frac{1}{6}$$

$$(i) \quad 8\frac{8}{9} - 2\frac{2}{9} - 3\frac{5}{9}$$

6. Complete the following calculations.

Change the fractions to have a common denominator first.

Write answers in their simplest form.

Examples

$$\begin{aligned} 1. \quad & \frac{2}{3} + \frac{1}{6} \\ &= \frac{4}{6} + \frac{1}{6} \\ &= \frac{5}{6} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{2}{3} - \frac{1}{4} \\ &= \frac{8}{12} - \frac{3}{12} \\ &= \frac{5}{12} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{3}{5} + \frac{1}{4} - \frac{1}{10} \\ &= \frac{12}{20} + \frac{5}{20} - \frac{2}{20} \\ &= \frac{15}{20} \\ &= \frac{3}{4} \end{aligned}$$

(a) $\frac{3}{8} + \frac{1}{4}$

(b) $\frac{5}{12} - \frac{1}{6}$

(c) $\frac{3}{10} + \frac{2}{5}$

(d) $\frac{1}{2} + \frac{3}{8}$

(e) $\frac{5}{6} - \frac{2}{3}$

(f) $\frac{5}{9} - \frac{1}{3}$

(g) $\frac{3}{4} - \frac{1}{3}$

(h) $\frac{3}{5} + \frac{1}{4}$

(i) $\frac{2}{3} + \frac{1}{5}$

(j) $\frac{7}{8} - \frac{1}{6}$

(k) $\frac{3}{4} + \frac{1}{10}$

(l) $\frac{7}{9} - \frac{5}{12}$

(m) $\frac{2}{3} + \frac{1}{4} - \frac{1}{2}$

(n) $\frac{3}{5} + \frac{3}{4} - \frac{1}{2}$

(o) $\frac{2}{3} + \frac{2}{5} - \frac{5}{6}$

(p) $\frac{3}{4} + \frac{1}{12} + \frac{1}{6}$

7. Complete the following calculations.

Change the fractions to have a common denominator first.

Write answers as mixed numbers in their simplest form.

Examples

$$\begin{aligned} 1. \quad & \frac{4}{5} + \frac{7}{10} \\ &= \frac{8}{10} + \frac{7}{10} \\ &= \frac{15}{10} \\ &= \frac{3}{2} \\ &= 1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{5}{6} + \frac{4}{5} \\ &= \frac{25}{30} + \frac{24}{30} \\ &= \frac{49}{30} \\ &= 1\frac{19}{30} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{5}{8} + \frac{2}{3} - \frac{1}{4} \\ &= \frac{15}{24} + \frac{16}{24} - \frac{6}{24} \\ &= \frac{25}{24} \\ &= 1\frac{1}{24} \end{aligned}$$

(a) $\frac{5}{6} + \frac{2}{3}$

(b) $\frac{7}{8} + \frac{3}{4}$

(c) $\frac{4}{5} + \frac{2}{3}$

(d) $\frac{3}{5} + \frac{3}{4}$

(e) $\frac{2}{3} + \frac{5}{8}$

(f) $\frac{7}{8} + \frac{4}{5}$

(g) $\frac{5}{6} + \frac{4}{5}$

(h) $\frac{7}{9} + \frac{5}{8}$

(i) $\frac{3}{4} + \frac{7}{12} + \frac{1}{6}$

(j) $\frac{3}{5} + \frac{3}{4} + \frac{1}{2}$

(k) $\frac{2}{3} + \frac{4}{5} - \frac{1}{6}$

(l) $\frac{2}{3} + \frac{3}{4} - \frac{1}{6}$

8. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

When adding mixed numbers there are two methods that can be used.

Example $4\frac{2}{3} + 5\frac{3}{5}$

Method 1

$$\begin{aligned}
 &4\frac{2}{3} + 5\frac{3}{5} \\
 = &\frac{14}{3} + \frac{28}{5} && \text{Convert to improper fractions} \\
 = &\frac{70}{15} + \frac{84}{15} && \text{Convert to a common denominator} \\
 = &\frac{154}{15} \\
 = &10\frac{4}{15}
 \end{aligned}$$

Method 2

$$\begin{aligned}
 &4\frac{2}{3} + 5\frac{3}{5} \\
 = &(4 + 5) + \left(\frac{2}{3} + \frac{3}{5}\right) && \text{Add the whole numbers and} \\
 & && \text{the fractions separately} \\
 = &9 + \left(\frac{10}{15} + \frac{9}{15}\right) \\
 = &9 + \frac{19}{15} \\
 = &9 + 1\frac{4}{15} \\
 = &10\frac{4}{15}
 \end{aligned}$$

(a) $2\frac{2}{5} + 3\frac{3}{4}$ (b) $6\frac{1}{2} + 4\frac{2}{3}$ (c) $7\frac{2}{3} + 5\frac{3}{4}$

(d) $1\frac{7}{8} + 3\frac{2}{3}$ (e) $8\frac{1}{2} + 5\frac{3}{5}$ (f) $7\frac{3}{4} + 2\frac{5}{8}$

(g) $2\frac{1}{7} + 3\frac{3}{5}$ (h) $5\frac{2}{3} + 4\frac{5}{6}$ (i) $9\frac{1}{2} + 5\frac{1}{3}$

(j) $2\frac{2}{3} + 1\frac{3}{4} + 2\frac{1}{6}$ (k) $3\frac{1}{2} + 7\frac{3}{4} + 4\frac{7}{8}$

9. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

When subtracting mixed numbers method 2 shown in question 8 can only be used when the first fraction is bigger than the second. Method 1 can be used for all subtraction problems.

Example $5\frac{3}{4} - 2\frac{1}{3}$

Note: $\frac{3}{4}$ is bigger than $\frac{1}{3}$

Method 1

$$\begin{aligned} & 5\frac{3}{4} - 2\frac{1}{3} \\ &= \frac{23}{4} - \frac{7}{3} \quad \text{Convert to improper fractions} \\ &= \frac{69}{12} - \frac{28}{12} \quad \text{Convert to a common denominator} \\ &= \frac{41}{12} \\ &= 3\frac{5}{12} \end{aligned}$$

Method 2

$$\begin{aligned} & 5\frac{3}{4} - 2\frac{1}{3} \\ &= (5 - 2) + \left(\frac{3}{4} - \frac{1}{3}\right) \quad \text{Subtract the whole numbers and} \\ &= 3 + \left(\frac{9}{12} - \frac{4}{12}\right) \quad \text{the fractions separately} \\ &= 3 + \frac{5}{12} \\ &= 3\frac{5}{12} \end{aligned}$$

Remember to check which of the fractions is bigger.

If the first is bigger than the second then use either method 1 or 2.

If the second is bigger than the first then only method 1 can be used.

(a) $6\frac{4}{5} - 2\frac{1}{2}$

(b) $4\frac{1}{3} - 2\frac{3}{4}$

(c) $7\frac{5}{6} - 4\frac{2}{5}$

(d) $3\frac{7}{8} - 1\frac{1}{5}$

(e) $4\frac{1}{5} - 2\frac{2}{3}$

(f) $9\frac{5}{8} - 2\frac{2}{7}$

10. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

Remember:

When adding mixed numbers methods 1 and 2 shown in questions 8 and 9 can be used.

When subtracting mixed numbers method 2 can only be used when the first fraction is bigger than the second fraction. Method 1 can be used in all subtraction problems.

(a) $3\frac{3}{5} + 2\frac{1}{4}$ (b) $4\frac{1}{2} - 3\frac{2}{3}$ (c) $7\frac{2}{3} - 5\frac{1}{5}$

(d) $1\frac{5}{8} + 3\frac{1}{3}$ (e) $4\frac{5}{6} - 2\frac{3}{10}$ (f) $9\frac{3}{4} + 5\frac{3}{11}$

(g) $3\frac{1}{5} - 1\frac{5}{8}$ (h) $5\frac{7}{10} - 2\frac{1}{4}$ (i) $9\frac{1}{2} + 4\frac{2}{9}$

(j) $2\frac{2}{3} + 3\frac{3}{4} + 2\frac{1}{8}$ (k) $3\frac{1}{2} + 7\frac{3}{4} - 4\frac{5}{6}$

(l) $4\frac{5}{6} + 2\frac{1}{12} + 3\frac{3}{4}$ (m) $4\frac{3}{4} + 3\frac{3}{8} - 2\frac{3}{10}$

11. Aaron owns chooks. He had 10 dozen eggs to sell.

He sold $2\frac{1}{2}$ dozen to Tara, $3\frac{1}{2}$ dozen to Chris and $1\frac{1}{2}$ dozen to Elle.

- How many dozen eggs did he sell?
- How many dozen eggs did he have left?
- How many eggs did he have left?
- If he sold the eggs for \$3 a dozen, how much money did he make?



12. Fletcher had to make a car trip that would take 12 hours of travelling time. He left at 8 a.m. and drove for $1\frac{3}{4}$ hours before having a rest for a quarter of an hour. He then drove for $2\frac{3}{4}$ hours before stopping for three quarters of an hour to have lunch.

He then drove for $3\frac{1}{2}$ hours before stopping at a motel for the night.

- At what time did he arrive at the motel?
- How many more hours of travel time would he need to do to complete his trip?

13. Dehne used the following recipe to make a bowl of fruit punch for his birthday party.

$7\frac{1}{2}$ cups of soda water

$4\frac{3}{4}$ cups of orange juice

$3\frac{3}{4}$ cups of apple juice

$2\frac{1}{4}$ cups of pineapple juice

$1\frac{3}{4}$ cups of grapefruit juice



- How many cups of fruit punch would be made with this recipe?
- If there were 10 people at the party how many cups of fruit punch should each person get?
- Dehne's sister Desiree is having a party but wants to make twice the amount of fruit punch. How many cups of each of the ingredients should she use?

14. Alana made a cake at school. She gave $\frac{1}{3}$ to her dad, $\frac{1}{4}$ to her mum and $\frac{1}{6}$ to her sister. What fraction of the cake was left?

15. Ishmael sold rice at his delicatessen shop. He sold bags that held $1\frac{3}{4}$ kg and small boxes that held $5\frac{2}{3}$ kg. If someone bought 2 bags and 2 boxes, how many kilograms of rice did they buy?

16. Calem caught two fish.
One was 1 kg heavier than the other and the total weight of the two fish was 8 kg.
What did each fish weigh?



17. Elke found two lengths of material. One was twice the length of the other and the combined length of the two was 10 metres.
How long was each length of material?

18. Complete the following patterns.

(a) $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, —, —, —.

(b) $\frac{2}{3}$, $\frac{5}{6}$, 1, $1\frac{1}{6}$, —, —, —.

(c) $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{3}{4}$, —, —, —.

(d) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$, —, —, —.

(e) $\frac{5}{8}$, $\frac{13}{24}$, $\frac{11}{24}$, $\frac{3}{8}$, —, —, —.

(f) 1, $\frac{23}{24}$, $\frac{11}{12}$, $\frac{7}{8}$, —, —, —.

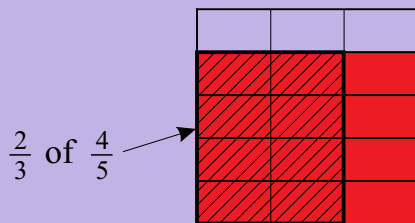
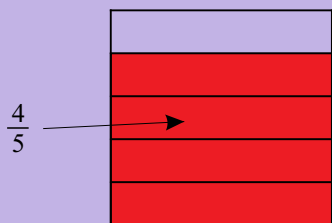
Multiplication

EXERCISE 5B

Example $\frac{2}{3} \times \frac{4}{5}$

Note $\frac{2}{3} \times \frac{4}{5}$ is the same as $\frac{2}{3}$ of $\frac{4}{5}$

The answer can be found by using a fraction wall.



By adding the number of small rectangles it can be seen that $\frac{2}{3}$ of $\frac{4}{5}$ is 8 of the 15 small rectangles.

$$\frac{2}{3} \times \frac{4}{5} = \frac{2}{3} \text{ of } \frac{4}{5} = \frac{8}{15}$$

A quick way to find the answer to the multiplication of fractions is to multiply the numerators and multiply the denominators.

$$\begin{array}{c} \times \\ \frac{2}{3} \times \frac{4}{5} \\ \times \end{array} = \frac{8}{15}$$

Examples

1. $\frac{1}{5} \times \frac{3}{8}$	2. $\frac{4}{7} \times \frac{3}{11}$	3. $\frac{2}{3} \times \frac{4}{5} \times \frac{1}{7}$
$= \frac{3}{40}$	$= \frac{12}{77}$	$= \frac{8}{105}$

1. Complete the following multiplications.

(a) $\frac{1}{3} \times \frac{1}{2}$

(b) $\frac{2}{5} \times \frac{3}{7}$

(c) $\frac{5}{6} \times \frac{5}{7}$

(d) $\frac{2}{3} \times \frac{5}{9}$

(e) $\frac{3}{10} \times \frac{3}{10}$

(f) $\frac{11}{12} \times \frac{1}{2}$

(g) $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$

(h) $\frac{3}{4} \times \frac{5}{7} \times \frac{1}{2}$

2. Cancel the fractions in the following problems to their simplest form before solving. Check that the answers are in their simplest form.

Example

$$\begin{aligned}
 & \frac{9}{14} \times \frac{7}{12} \\
 &= \frac{\overset{3}{\cancel{9}}}{14} \times \frac{7}{\underset{4}{\cancel{12}}} \quad (3 \text{ is the highest common factor of } 9 \text{ and } 12) \\
 &= \frac{\underset{2}{\cancel{3}}}{14} \times \frac{\cancel{7}^1}{4} \quad (7 \text{ is the highest common factor of } 7 \text{ and } 14) \\
 &= \frac{3}{2} \times \frac{1}{4} \\
 &= \frac{3}{8}
 \end{aligned}$$

(a) $\frac{3}{5} \times \frac{5}{6}$

(b) $\frac{3}{4} \times \frac{8}{9}$

(c) $\frac{4}{5} \times \frac{15}{16}$

(d) $\frac{2}{9} \times \frac{9}{10}$

(e) $\frac{24}{35} \times \frac{15}{32}$

(f) $\frac{30}{49} \times \frac{14}{15}$

(g) $\frac{28}{45} \times \frac{18}{35}$

(h) $\frac{10}{21} \times \frac{14}{25}$

3. Complete the following multiplications.

Write answers as mixed numbers in their simplest form.

Note Whole numbers can be written as fractions: *example* $7 = \frac{7}{1}$

Examples

$$\begin{aligned}
 1. \quad & 8 \times \frac{3}{5} \\
 &= \frac{8}{1} \times \frac{3}{5} \\
 &= \frac{24}{5} \\
 &= 4\frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{5}{6} \times 9 \\
 &= \frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{1} \\
 &= \frac{15}{2} \\
 &= 7\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & 8 \times \frac{3}{4} \\
 &= \frac{\overset{2}{\cancel{8}}}{1} \times \frac{3}{\cancel{4}_1} \\
 &= \frac{6}{1} \\
 &= 6
 \end{aligned}$$

(a) $5 \times \frac{3}{4}$

(b) $\frac{2}{3} \times 8$

(c) $6 \times \frac{5}{8}$

(d) $\frac{2}{7} \times 9$

(e) $12 \times \frac{4}{9}$

(f) $18 \times \frac{5}{6}$

(g) $\frac{1}{2} \times 11$

(h) $16 \times \frac{7}{8}$

(i) $15 \times \frac{2}{3}$

(j) $\frac{3}{10} \times 25$

(k) $18 \times \frac{7}{12}$

(l) $\frac{4}{5} \times 35$

4. Complete the following multiplications.

Write answers as mixed numbers in their simplest form.

Examples	1. $4 \times 2\frac{4}{5}$ $= \frac{4}{1} \times \frac{14}{5}$ $= \frac{56}{5}$ $= 11\frac{1}{5}$	2. $3\frac{3}{4} \times 6$ $= \frac{15}{4} \times \frac{6}{1}$ $= \frac{15}{2\cancel{4}} \times \frac{\cancel{6}^3}{1}$ $= \frac{45}{2}$ $= 22\frac{1}{2}$	3. $8 \times 4\frac{1}{4}$ $= \frac{8}{1} \times \frac{17}{4}$ $= \frac{\cancel{8}^2}{1} \times \frac{17}{\cancel{4}_1}$ $= \frac{34}{1}$ $= 34$
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- (a) $5 \times 2\frac{1}{2}$ (b) $4\frac{2}{5} \times 2$ (c) $3 \times 5\frac{1}{6}$ (d) $6\frac{1}{2} \times 4$
 (e) $4 \times 3\frac{5}{8}$ (f) $7 \times 3\frac{2}{3}$ (g) $6\frac{3}{10} \times 5$ (h) $8 \times 5\frac{3}{4}$
 (i) $6 \times 4\frac{2}{5}$ (j) $7\frac{2}{3} \times 9$ (k) $3 \times 6\frac{3}{4}$ (l) $5\frac{5}{6} \times 12$

5. Complete the following multiplications.

Write answers as mixed numbers in their simplest form.

Examples	1. $3\frac{1}{2} \times 2\frac{1}{3}$ $= \frac{7}{2} \times \frac{7}{3}$ $= \frac{49}{6}$ $= 8\frac{1}{6}$	2. $4\frac{3}{4} \times 3\frac{3}{5}$ $= \frac{19}{4} \times \frac{18}{5}$ $= \frac{19}{2\cancel{4}} \times \frac{\cancel{18}^9}{5}$ $= \frac{171}{10}$ $= 17\frac{1}{10}$	3. $5\frac{1}{4} \times 3\frac{1}{9}$ $= \frac{21}{4} \times \frac{28}{9}$ $= \frac{7\cancel{21}}{1\cancel{4}} \times \frac{\cancel{28}^7}{\cancel{9}_3}$ $= \frac{49}{3}$ $= 16\frac{1}{3}$
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- (a) $3\frac{1}{4} \times 1\frac{1}{3}$ (b) $4\frac{1}{2} \times 2\frac{2}{3}$ (c) $5\frac{2}{5} \times 2\frac{2}{3}$ (d) $3\frac{3}{4} \times 4\frac{2}{5}$
 (e) $6\frac{2}{3} \times 4\frac{5}{6}$ (f) $4\frac{4}{5} \times 1\frac{5}{12}$ (g) $3\frac{3}{4} \times 7\frac{1}{5}$ (h) $5\frac{5}{8} \times 2\frac{2}{9}$
 (i) $7\frac{1}{7} \times 4\frac{9}{10}$ (j) $4\frac{2}{3} \times 4\frac{1}{8}$ (k) $5\frac{1}{9} \times 3\frac{3}{4}$ (l) $6\frac{7}{8} \times 1\frac{7}{15}$

6. Complete the following calculations.

Remember: *Of* is the same as $\times$

- (a) $\frac{1}{2}$ of 20 (b) $\frac{1}{5}$ of 40 (c) $\frac{1}{4}$ of 16 (d) $\frac{3}{4}$ of 16
 (e) $\frac{1}{2} \times 24$ (f) $\frac{1}{3} \times 18$ (g) $\frac{2}{3} \times 18$ (h) $\frac{2}{5} \times 30$
 (i) $\frac{3}{4}$ of 40 (j) $\frac{2}{3}$ of 66 (k) $\frac{2}{7}$ of 77 (l) $\frac{3}{5}$ of 25
 (m) $\frac{3}{10}$ of $\frac{5}{9}$ (n) $\frac{2}{3}$ of $\frac{3}{5}$ (o) $\frac{3}{4}$ of $\frac{8}{9}$ (p) $\frac{5}{6}$ of $\frac{12}{13}$
 (q) $\frac{1}{2}$ of $4\frac{2}{3}$ (r) $\frac{2}{3}$ of $9\frac{1}{2}$ (s) $\frac{3}{5}$ of $8\frac{1}{3}$ (t) $\frac{7}{10}$ of $9\frac{2}{7}$

7. Complete the following calculations.

- (a) $\frac{1}{2}$ of 8 m (b) $\frac{3}{4}$ of 20 kg (c) $\frac{5}{8}$ of \$80 (d) $\frac{2}{3}$ of 300 mm
 (e) $\frac{2}{5}$ of 45 cm (f) $\frac{3}{10}$ of \$250 (g) $\frac{3}{8}$ of 400 m (h) $\frac{4}{5}$ of 1000 kg
 (i) $\frac{3}{4}$ of $9\frac{1}{3}$ g (j) $\frac{3}{5}$ of $2\frac{1}{2}$ m (k) $\frac{2}{3}$ of $2\frac{1}{4}$ m (l) $\frac{9}{10}$ of $6\frac{2}{3}$ kg

8. Complete the following calculations.

- (a) If an orange weighs $\frac{1}{4}$ kg, find the total weight of the following number of oranges.
 (i) 12 oranges (ii) 18 oranges (iii) 25 oranges
 (b) It takes 15 cups of water to fill a jug. If the cup held one-third of a litre, how much water is in the jug?
 (c) Lily measured her step to be $\frac{3}{4}$ of a metre when she walked.
 How far has she walked if she takes the following number of steps?
 (i) 16 steps (ii) 30 steps (iii) 50 steps
 (d) A developer bought 15 blocks of land that were each $\frac{2}{5}$ of a hectare.
 What is the total area of land the developer bought?
 (e) A concrete paving block weighed $3\frac{1}{2}$ kg. What is the total weight of the following number of blocks?
 (i) 6 blocks (ii) 10 blocks (iii) 25 blocks
 (f) It took Jack $2\frac{1}{2}$ minutes to run around a park. How long would it take him to run the following number of laps of the park?
 (i) 6 laps (ii) 16 laps (iii) 25 laps

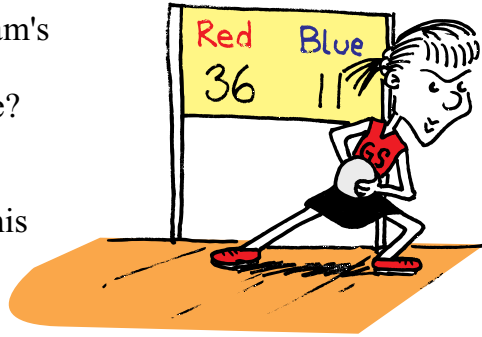
9. Kristie scored $\frac{3}{4}$ of her netball team's score of 36 goals.

How many goals did Kristie score?

10. Joel earned \$45. He spent $\frac{3}{5}$ of his earnings and saved the rest.

(a) How much did he spend?

(b) How much did he save?



11. A crowd of 4000 people attended a baseball game between the Devils and the Saints. Five-eighths of the crowd barracked for the Devils.

How many people barracked for each team?

12. Belinda sells skin care products.

She is paid $\frac{1}{6}$ of the value of what she sells.

If Belinda sells \$3600 worth of products how much will she be paid?

13. Ian is using a 3 hour DVD to record some of his favourite programs.

He has recorded one $\frac{3}{4}$ hour show, one $\frac{1}{2}$ hour show and another show that went for 50 minutes.

How many minutes are left on his DVD?

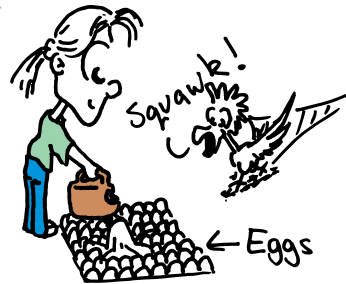
14. Keira wanted to grow chickens.

She bought 2400 eggs and placed them in an incubator.

She was told that $\frac{5}{6}$ of the eggs should hatch and of the ones that hatch $\frac{1}{5}$ will be black and the rest white.

(a) How many black chickens should she expect to hatch?

(b) How many white chickens should she expect to hatch?



15. Boris bought $2\frac{1}{2}$ tonnes of wood. He sold $\frac{2}{5}$ of the load to his brother Morris and $\frac{3}{10}$ to his sister Doris.

(a) How many tonnes of wood did Morris get?

(b) How many tonnes of wood did Doris get?

(c) How many tonnes of wood did Boris have left?

16. Muddlehead Max went into a shop and asked for $\frac{3}{4}$ of $\frac{2}{3}$ of a dozen eggs.

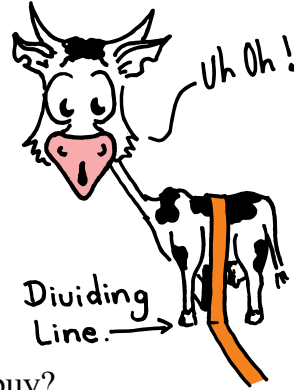
How many eggs did he want?

17. What fraction is a $\frac{1}{2}$ of a $\frac{1}{2}$ of a $\frac{1}{2}$ of a $\frac{1}{2}$?

18. Morry is a farmer who decided to divide his 3000 hectare farm equally between his five children.

One of his children, Rory, divides his section into four equal parts and sells one to his friend Corey.

- What **fraction** of Morry's farm did each of his children receive?
- What **fraction** of Rory's land did Corey buy?
- What **fraction** of Morry's farm did Corey buy?
- How many hectares did Corey buy?
- How many hectares did Rory have left after he sold the section to Corey?



19. Callum is a pastry cook. It takes him $2\frac{1}{2}$ minutes to decorate a cake. How long will it take Callum to decorate the following number of cakes?

- 6 cakes
- 15 cakes
- 35 cakes

20. Mischel is a potter. She made six bowls from an 8 kg block of clay.

- What is the weight of each bowl? Give answer as a mixed number in its simplest form.
- How many kilograms of clay would be needed to make the following number of bowls? Give answers as mixed numbers in their simplest form.
 - 10 bowls
 - 15 bowls
 - 25 bowls

Division

EXERCISE 5C

1. Complete these calculations by following the steps shown below.
Where necessary write answers as mixed numbers in their simplest form.

The following steps can be used to solve problems where one fraction is divided by another.

- Step 1** Change the $\div$ to a $\times$ and turn the second fraction upside down.
Step 2 Cancel the problem to its simplest form.
Step 3 Multiply the fractions.
Step 4 Write the answer as a mixed number.

Examples

$$\begin{aligned} 1. \quad & \frac{3}{5} \div \frac{2}{3} \\ &= \frac{3}{5} \times \frac{3}{2} \\ &= \frac{9}{10} \end{aligned}$$

$$\begin{aligned} 2. \quad & \frac{4}{5} \div \frac{6}{7} \\ &= \frac{4}{5} \times \frac{7}{6} \\ &= \frac{\cancel{2}^1 \cancel{4}^2}{5} \times \frac{7}{\cancel{6}_3} \\ &= \frac{14}{15} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{5}{8} \div \frac{3}{10} \\ &= \frac{5}{8} \times \frac{10}{3} \\ &= \frac{5}{\cancel{4}_2} \times \frac{\cancel{10}^5}{3} \\ &= \frac{25}{12} \\ &= 2\frac{1}{12} \end{aligned}$$

- | | | | |
|--------------------------------------|--------------------------------------|---------------------------------------|--------------------------------------|
| (a) $\frac{2}{3} \div \frac{3}{4}$ | (b) $\frac{3}{7} \div \frac{5}{8}$ | (c) $\frac{1}{3} \div \frac{1}{2}$ | (d) $\frac{3}{10} \div \frac{5}{7}$ |
| (e) $\frac{4}{5} \div \frac{8}{9}$ | (f) $\frac{3}{10} \div \frac{3}{5}$ | (g) $\frac{2}{3} \div \frac{11}{12}$ | (h) $\frac{7}{15} \div \frac{3}{5}$ |
| (i) $\frac{8}{9} \div \frac{16}{27}$ | (j) $\frac{4}{5} \div \frac{2}{3}$ | (k) $\frac{9}{10} \div \frac{3}{5}$ | (l) $\frac{13}{15} \div \frac{4}{5}$ |
| (m) $\frac{8}{9} \div \frac{4}{21}$ | (n) $\frac{9}{16} \div \frac{3}{40}$ | (o) $\frac{8}{15} \div \frac{12}{55}$ | (p) $\frac{9}{14} \div \frac{6}{49}$ |

2. Convert the mixed numbers in the following problems to improper fractions before solving.

Use the steps shown in question 1 and write answers as mixed numbers where possible.

Examples	1. $4\frac{1}{6} \div 1\frac{2}{3}$	2. $2\frac{5}{8} \div 5\frac{1}{4}$
	$= \frac{25}{6} \div \frac{5}{3}$	$= \frac{21}{8} \div \frac{21}{4}$
	$= \frac{25}{6} \times \frac{3}{5}$	$= \frac{21}{8} \times \frac{4}{21}$
	$= \frac{\cancel{5}^{25}}{\cancel{2}^6} \times \frac{\cancel{3}^1}{\cancel{5}_1}$	$= \frac{\cancel{1}^{21}}{\cancel{2}^8} \times \frac{\cancel{4}^1}{\cancel{21}_1}$
	$= \frac{5}{2}$	$= \frac{1}{2}$
	$= 2\frac{1}{2}$	

(a) $2\frac{1}{2} \div 1\frac{3}{4}$ (b) $3\frac{1}{3} \div 4\frac{4}{9}$ (c) $1\frac{5}{7} \div 5\frac{1}{7}$ (d) $8\frac{2}{5} \div 2\frac{1}{10}$
 (e) $8\frac{1}{2} \div 6\frac{3}{8}$ (f) $7\frac{1}{3} \div 1\frac{2}{9}$ (g) $2\frac{1}{5} \div 9\frac{9}{10}$ (h) $2\frac{5}{8} \div 15\frac{3}{4}$

3. Write the answers to the following divisions as fractions in their simplest form.

Convert to mixed numbers where necessary.

Examples	
1. $5 \div 8$	2. $14 \div 6$
$= \frac{5}{8}$	$= \frac{14}{6}$
	$= \frac{7\cancel{14}}{3\cancel{6}}$
	$= \frac{7}{3}$
	$= 2\frac{1}{3}$

(a) $3 \div 5$ (b) $17 \div 22$ (c) $16 \div 18$ (d) $5 \div 30$
 (e) $35 \div 20$ (f) $27 \div 4$ (g) $39 \div 10$ (h) $82 \div 12$

4. Write the answers to the following problems as fractions.
Where possible write as mixed numbers in their simplest form.

Example

How many 4's are in 11?

This can be written as $11 \div 4$

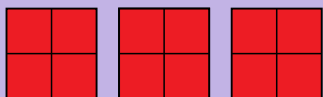
This can be written as $\frac{11}{4} = 2\frac{3}{4}$

- | | |
|-----------------------------|------------------------------|
| (a) How many 3's are in 31? | (b) How many 5's are in 32? |
| (c) How many 8's are in 3? | (d) How many 10's are in 7? |
| (e) How many 7's are in 85? | (f) How many 6's are in 4? |
| (g) How many 4's are in 34? | (h) How many 18's are in 10? |
| (i) How many 15's are in 9? | (j) How many 36's are in 12? |

5. The following problems can be solved by using diagrams.

Examples

1. How many $\frac{1}{4}$'s are in 3?



It can be seen there are
12 $\frac{1}{4}$'s in 3.

Answer = 12

2. How many $\frac{1}{6}$'s are in $2\frac{1}{6}$?



It can be seen there are
13 $\frac{1}{6}$'s in 3.

Answer = 13

- | | |
|---|---|
| (a) How many $\frac{1}{3}$'s are in 1? | (b) How many $\frac{1}{5}$'s are in 2? |
| (c) How many $\frac{1}{4}$'s are in 4? | (d) How many $\frac{1}{10}$'s are in 3? |
| (e) How many $\frac{1}{2}$'s are in $3\frac{1}{2}$? | (f) How many $\frac{1}{5}$'s are in $2\frac{1}{5}$? |
| (g) How many $\frac{1}{10}$'s are in $7\frac{3}{10}$? | (h) How many $\frac{1}{6}$'s are in $4\frac{5}{6}$? |
| (i) How many $\frac{1}{8}$'s are in $6\frac{5}{8}$? | (j) How many $\frac{1}{7}$'s are in $9\frac{6}{7}$? |

6. Solve the following problems.

Diagrams could be used but it is easier to use division of fractions.

Examples1. How many $\frac{3}{4}$'s are in 12?

This can be written as $12 \div \frac{3}{4}$.

$$\begin{aligned}
 & 12 \div \frac{3}{4} \\
 &= \frac{12}{1} \div \frac{3}{4} \\
 &= \frac{12}{1} \times \frac{4}{3} \\
 &= \frac{4\cancel{12}}{1} \times \frac{4}{\cancel{3}_1} \\
 &= \frac{16}{1} \\
 &= \mathbf{16}
 \end{aligned}$$

2. How many $\frac{2}{3}$'s are in $8\frac{5}{6}$?

This can be written as $8\frac{5}{6} \div \frac{2}{3}$.

$$\begin{aligned}
 & 8\frac{5}{6} \div \frac{2}{3} \\
 &= \frac{53}{6} \div \frac{2}{3} \\
 &= \frac{53}{6} \times \frac{3}{2} \\
 &= \frac{53}{2\cancel{6}} \times \frac{\cancel{3}^1}{2} \\
 &= \frac{53}{4} \\
 &= \mathbf{13\frac{1}{4}}
 \end{aligned}$$

3. How many 6's are in $9\frac{3}{4}$?

This can be written as $9\frac{3}{4} \div 6$.

$$\begin{aligned}
 & 9\frac{3}{4} \div 6 \\
 &= \frac{39}{4} \div \frac{6}{1} \\
 &= \frac{39}{4} \times \frac{1}{6} \\
 &= \frac{13\cancel{39}}{4} \times \frac{1}{\cancel{6}_2} \\
 &= \frac{13}{8} \\
 &= \mathbf{1\frac{5}{8}}
 \end{aligned}$$

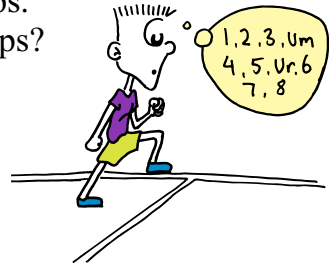
4. How many $9\frac{1}{6}$'s are in $5\frac{5}{9}$?

This can be written as $5\frac{5}{9} \div 9\frac{1}{6}$.

$$\begin{aligned}
 & 5\frac{5}{9} \div 9\frac{1}{6} \\
 &= \frac{50}{9} \div \frac{55}{6} \\
 &= \frac{50}{9} \times \frac{6}{55} \\
 &= \frac{10\cancel{50}}{3\cancel{9}} \times \frac{\cancel{6}^2}{\cancel{55}_{11}} \\
 &= \mathbf{\frac{20}{33}}
 \end{aligned}$$

(a) How many $\frac{2}{5}$'s are in 6?(b) How many $\frac{3}{4}$'s are in $4\frac{7}{8}$?(c) How many 4's are in $13\frac{1}{3}$?(d) How many $6\frac{1}{2}$'s are in $4\frac{1}{3}$?(e) How many $\frac{3}{8}$'s are in $2\frac{7}{16}$?(f) How many $\frac{8}{9}$'s are in $4\frac{2}{3}$?(g) How many 7's are in $51\frac{4}{5}$?(h) How many $8\frac{3}{4}$'s are in $7\frac{7}{9}$?(i) How many $2\frac{3}{5}$'s are in $8\frac{9}{20}$?(j) How many $4\frac{5}{8}$'s are in $43\frac{1}{6}$?

7. How many pieces of wood $\frac{2}{3}$ of a metre long could be cut from a 6 metre length of timber?
8. If a dozen donuts were divided equally between nine people how many would each person get?
9. (a) Matthew wanted to measure how long his steps were when he walked. He walked the length of the basketball court (24 m) at his school and he counted 32 of his steps.
What fraction of a metre is each of his steps?
Give the fraction in its simplest form.
- (b) How many steps would Matthew need to walk (i) 60 metres?
(ii) 330 metres?
(iii) 3 kilometres?



10. Marion owns 30 hectares of land. She is going to have her land divided into $1\frac{1}{2}$ hectare blocks.
How many of the smaller blocks will there be?
11. An old measure of land area was an acre.
There are about $2\frac{1}{2}$ acres in a hectare.
(a) How many hectares would there be in 50 acres?
(b) How many acres would there be in 50 hectares?
12. Anastasia had a 90 litre drinking trough for her pet calf. She had to fill it using a $4\frac{1}{2}$ litre bucket.
How many full buckets would it take to fill the drinking trough?
13. Anneke wanted to make some blackberry jam.
The recipe required a kilogram of sugar for every kilogram of blackberries.
Anneke had 9 kilograms of blackberries.
She had to measure the amount of sugar using a container that holds $\frac{3}{4}$ of a kilogram of sugar.
How many of these containers of sugar would be needed?



BODMAS

EXERCISE 5D

Remember **BODMAS** which shows the order that operations are performed.

One example of **BODMAS** is shown below. Other examples are shown in the chapter on **Number Operations**.

BRACKETS

OF

{ DIVISION

{ MULTIPLICATION

{ ADDITION

{ SUBTRACTION

Example $\frac{1}{3} \times (\frac{2}{5} + 3\frac{1}{2}) + \frac{1}{3} \text{ of } 9$

$$\frac{2}{5} + \frac{7}{2}$$

$$\frac{4}{10} + \frac{35}{10}$$

Brackets

$$= \frac{1}{3} \times \frac{39}{10} + \frac{1}{3} \text{ of } 9$$

Of

$$= \frac{1}{3} \times \frac{39}{10} + 3$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{39}^{13}}{10}$$

Multiplication

$$= \frac{13}{10} + 3$$

Addition

$$= 1\frac{3}{10} + 3$$

$$= 4\frac{3}{10}$$

Give answers as mixed numbers in their simplest form where appropriate.

1. $\frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$

2. $\frac{1}{2} \times (\frac{1}{2} + \frac{1}{2})$

3. $\frac{1}{4} \text{ of } 10 + \frac{1}{2}$

4. $\frac{1}{2} \times \frac{1}{3} + \frac{2}{3}$

5. $1\frac{1}{2} \times (1\frac{1}{3} - \frac{4}{9})$

6. $\frac{3}{5} - \frac{4}{15} \div \frac{2}{3}$

7. $\frac{2}{5} \times 2\frac{1}{2} + \frac{4}{7} \times 1\frac{5}{16}$

8. $\frac{2}{9} \times 1\frac{2}{7} + \frac{4}{5} \div 1\frac{4}{5}$

9. $\frac{1}{3} + \frac{1}{2} \text{ of } 2\frac{1}{3} - 2 \div 1\frac{1}{3}$

Calculator Problems



EXERCISE 5E

Use a calculator that has a fraction key to help solve the following problems.

1. $\frac{3}{4} + \frac{3}{8} \times \frac{3}{5}$
2. $3\frac{2}{5} \times 2\frac{3}{7}$
3. $\frac{4}{7} + \frac{3}{5} - \frac{2}{9}$
4. $3\frac{1}{8} \times (\frac{2}{3} + \frac{1}{5})$
5. $3\frac{7}{9} + 2\frac{5}{8} \times 2\frac{1}{7}$
6. $(\frac{1}{4} + 3\frac{1}{3}) \times (2\frac{1}{5} - 1\frac{5}{6})$
7. $\frac{1}{4}$ of 63 + $\frac{2}{3}$ of 71
8. $\frac{3}{4} \div (2\frac{1}{2} + 3\frac{1}{6}) \div \frac{3}{5}$
9. $2\frac{1}{2} + \frac{2}{3}$ of $(3\frac{1}{3} - 2\frac{5}{7})$

Worded Problems

EXERCISE 5F

1. Fujeko is a pigeon breeder. He makes his own pigeon food using pellets, seeds and corn.
 A bag of pellets weighs $5\frac{1}{2}$ kg.
 A bag of seed weighs $1\frac{1}{4}$ kg.
 A bag of corn weighs $\frac{3}{4}$ kg.
 Fujeko mixes together 2 bags of pellets, 6 bags of seeds and 2 bags of corn.
 (a) What is the total weight of this mix of pigeon food?
 (b) If he uses $2\frac{1}{2}$ kg of the mixed pigeon food each day, how many days will he be able to feed his pigeons with this amount of food?
 (c) How many bags of each ingredient would be required if Fujeko wants to feed his pigeons for 24 days?
2. A 2 metre tall eucalyptus tree is planted and its height measured every year. It was found that at the end of each year the tree was $1\frac{1}{2}$ times its height at the start of that year.
 What is the tree's height at the end of:
 (a) 1 year?
 (b) 2 years?
 (c) 5 years?

3. In a science experiment a ball is pushed off the end of a $1\frac{1}{2}$ m high bench and the height reached by the ball after each bounce is measured.

It was calculated that the height reached by the ball after each bounce was $\frac{2}{3}$ of the height reached before that bounce.

What was the height reached after:

- (a) the first bounce? (b) the fourth bounce?

4. Shelana plans to ride her bicycle 180 km over three days.

She decides to ride $\frac{1}{3}$ of the trip on the first day.

On the second day she plans to ride $\frac{1}{3}$ of the remaining distance.

On the third day she will ride the rest of her trip.

- (a) How many kilometres will she ride each day?

- (b) What ***fraction*** of the trip will she ride each day?

5. Wasim owns a fruit shop.

- (a) He had 36 kg of apples that he packed equally into 24 bags.

What was the weight of each bag of apples?

- (b) He had 35 kg of oranges that he packed equally into 20 bags.

What was the weight of each bag of oranges?

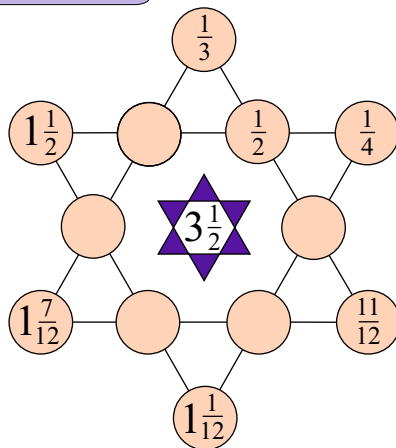
Sabrina wants to make 36 bowls of fruit salad for a party. She goes to Wasim's fruit shop and buys 3 bags of apples and 3 bags of oranges. She also buys 3 pineapples that each weigh $\frac{3}{4}$ of a kg. She cuts all the fruit into small pieces and divides them equally into the 36 bowls.

- (c) What will be the weight of fruit salad in each bowl?



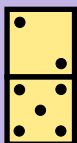
PROBLEM SOLVING

1. Copy this diagram into your books.
Fill in the circles so that each line adds to the number in the centre.



2. Consider a domino as a fraction.

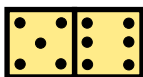
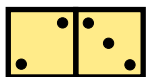
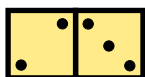
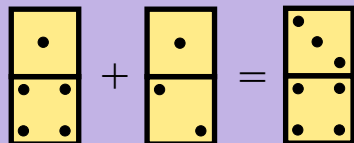
Example



This domino would be $\frac{2}{5}$.
It could also be turned upside down. The fraction would then be $\frac{5}{2}$.

Use the dominoes below and any of the operations $+$, $-$, $\times$ or $\div$ to form an equation.

Example



3. A group of students went on an excursion to a water activities resort. Half of the students went on the water slide. One third of the students went on the paddle boats. The remaining five students went on the canoes. How many students went to the resort?



4. Lark wanted to make a pen for her guinea pigs. She had 480 cm of wire mesh that she used for the edges of the rectangular pen. The width of the pen was one third its length. Find the width and length of Lark's guinea pig pen.

PUZZLES

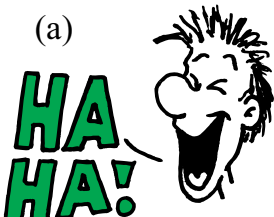
1. Change **HALF** into **FULL** in three steps by changing one letter at a time to form a new word.

Example
CAT can be changed to **DOG** in three steps

CAT
COT
DOT
DOG

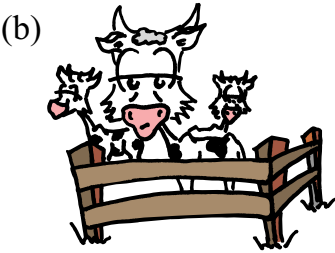
2. Find the fraction that rhymes with the word missing from the caption under each of the following pictures.

(a)



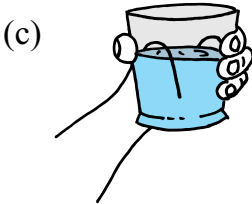
That joke made me _____!

(b)



A _____ of cattle.

(c)



It is important to drink a lot of _____.

3. A sentence describing the equation shown is written in code below.
Find the code and use it to solve the riddle.

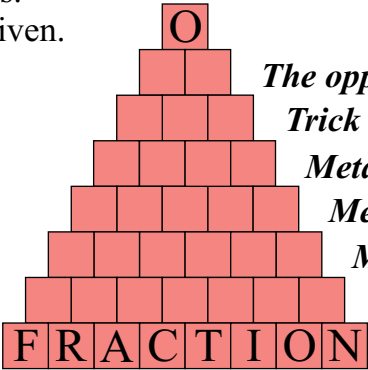
$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

B IBMG PG B IBMG JT B RVBSUFS

What did the flowers say to the gardener?

TUPQ QJDLJOH PO VT

4. Complete the diagram below by finding the word on each line.
Each word can be found by adding one letter to the line above it and rearranging the letters.
A clue for each word is given.



The opposite of YES

Trick someone

Metal money

Medicine

Movement

Small political group

CHAPTER REVIEW

1. Complete the following calculations.

Write answers in their simplest form.

(a) $\frac{3}{5} + \frac{1}{5}$ (b) $\frac{5}{7} - \frac{2}{7}$ (c) $2\frac{3}{11} + 3\frac{4}{11}$ (d) $5\frac{6}{7} - 2\frac{3}{7}$

(e) $1 - \frac{3}{8}$ (f) $\frac{1}{2} + \frac{1}{4}$ (g) $\frac{7}{8} - \frac{1}{4}$ (h) $\frac{3}{5} + \frac{2}{3}$

(i) $\frac{5}{6} - \frac{3}{4}$ (j) $\frac{3}{7} + \frac{3}{8}$ (k) $\frac{1}{2} + \frac{1}{3} - \frac{1}{5}$ (l) $\frac{3}{7} - \frac{1}{10} + \frac{3}{5}$

2. Complete the following calculations.

Write answers as mixed numbers in their simplest form.

(a) $2\frac{3}{5} + 1\frac{1}{4}$ (b) $3\frac{2}{3} - 1\frac{3}{4}$ (c) $3 - 1\frac{2}{7}$ (d) $6\frac{2}{7} + 3\frac{3}{4}$

3. Robin makes muesli using the following recipe.

$2\frac{1}{3}$ cups of wheat flakes

$1\frac{1}{2}$ cups of oats

$\frac{2}{3}$ of a cup of sultanas

$\frac{1}{4}$ of a cup of sunflower seeds

$\frac{1}{4}$ of a cup of crushed nuts

(a) How many cups of muesli did Robin make?

(b) If Robin eats $\frac{1}{2}$ of a cup of muesli a day, how many days will this mix last?

(c) If he wants to make a batch of muesli that will last twice as long, how many cups of each ingredient should Robin use?

4. Complete the following calculations.

Write answers in their simplest form.

(a) $\frac{1}{4} \times \frac{1}{3}$ (b) $\frac{2}{5} \times \frac{3}{4}$ (c) $\frac{3}{7} \times \frac{14}{27}$ (d) $\frac{9}{10} \times \frac{14}{27}$

5. Complete the following calculations

Write answers as mixed numbers in their simplest form.

(a) $2\frac{1}{4} \times 1\frac{7}{9}$ (b) $8\frac{1}{3} \times 1\frac{1}{5}$ (c) $1\frac{9}{10} \times 6\frac{2}{3}$ (d) $12\frac{3}{5} \times 1\frac{1}{14}$

6. Complete the following calculations

(a) $\frac{1}{4}$ of \$80 (b) $\frac{2}{5}$ of 25 kg (c) $\frac{3}{4}$ of \$240 (d) $\frac{7}{10}$ of 120 m

7. At the Etol College there were 800 students. Two-fifths of students at the school could speak a second language.

(a) How many students could speak a second language?

(b) What fraction of the students at the school could **not** speak a second language?

8. What fraction is a third of a third of a third?

9. Complete the following calculations.

Write answers in their simplest form.

(a) $\frac{1}{4} \div \frac{1}{3}$ (b) $\frac{2}{5} \div \frac{7}{10}$ (c) $\frac{2}{9} \div \frac{14}{15}$ (d) $\frac{9}{20} \div \frac{15}{16}$

10. Complete the following calculations

Write answers as mixed numbers in their simplest form.

(a) $2\frac{4}{7} \div \frac{9}{14}$ (b) $1\frac{1}{14} \div 4\frac{2}{7}$ (c) $2\frac{7}{10} \div 1\frac{4}{5}$ (d) $2\frac{2}{15} \div 1\frac{5}{9}$

11. (a) How many $\frac{1}{4}$'s are in 5?

(b) How many 3's are in 5?

(c) How many $\frac{1}{5}$'s are in $3\frac{4}{5}$?

(d) How many $\frac{2}{3}$'s are in 8?

12. Five people bought three pizzas to share equally.

What fraction of a pizza should each person get?

13. Remember BODMAS for these problems.

(a) $\frac{3}{4} \times (\frac{1}{2} + \frac{2}{5})$ (b) $\frac{2}{3} + \frac{1}{4} \times \frac{5}{6}$ (c) $\frac{5}{6} \div \frac{5}{8} - \frac{1}{6} \times 2$

14. Dale makes ceramics models. He makes his own clay mix using the following ingredients.

$10\frac{1}{2}$ kg of clay, $5\frac{3}{4}$ kg of sand and $1\frac{3}{4}$ kg of cement.

(a) If Dale makes 24 models what does each model weigh?

(b) If Dale wants to make 48 models, how much of each ingredient will he need in the clay mix?

Decimals

Place Value

The digits in a decimal number represent multiples of 10.

The **place value** defines the multiple of 10 that applies to that place in the decimal number.

Moving to the right, each place is equal to the previous place divided by 10.

Moving to the left, each place is equal to the previous place multiplied by 10.

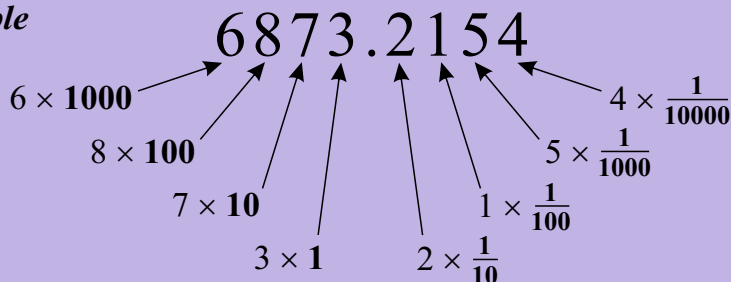
The decimal point shows the change from whole numbers to parts of numbers.

Summary of the four place values either side of the decimal point.

1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten - Thousandths

↑
Decimal point

Example



This number could be written as:

$$6 \times 1000 + 8 \times 100 + 7 \times 10 + 3 \times 1 + 2 \times \frac{1}{10} + 1 \times \frac{1}{100} + 5 \times \frac{1}{1000} + 4 \times \frac{1}{10000}$$

OR

6 thousands + 8 hundreds + 7 tens + 3 units + 2 tenths + 1 hundredth + 5 thousandths + 4 ten-thousandths

OR

$$6000 + 800 + 70 + 3 + 0.2 + 0.01 + 0.005 + 0.0004$$

EXERCISE 6A

1. Copy and complete the following table.

10 000	1000		10		$\frac{1}{10}$		$\frac{1}{1000}$	
Ten Thousands		Hundreds		Units		Hundredths		Ten Thousandths

2. Write the following decimals as:

(i) words (ii) fractions

- (a) 0.008

(b) 0.5

(c) 0.06
- (d) 0.00009

(e) 0.0001

Example

0.03

(i) three hundredths

(ii) $\frac{3}{100}$

3. What number is represented on each spike abacus below?
The place value of one of the spikes is given for each abacus.

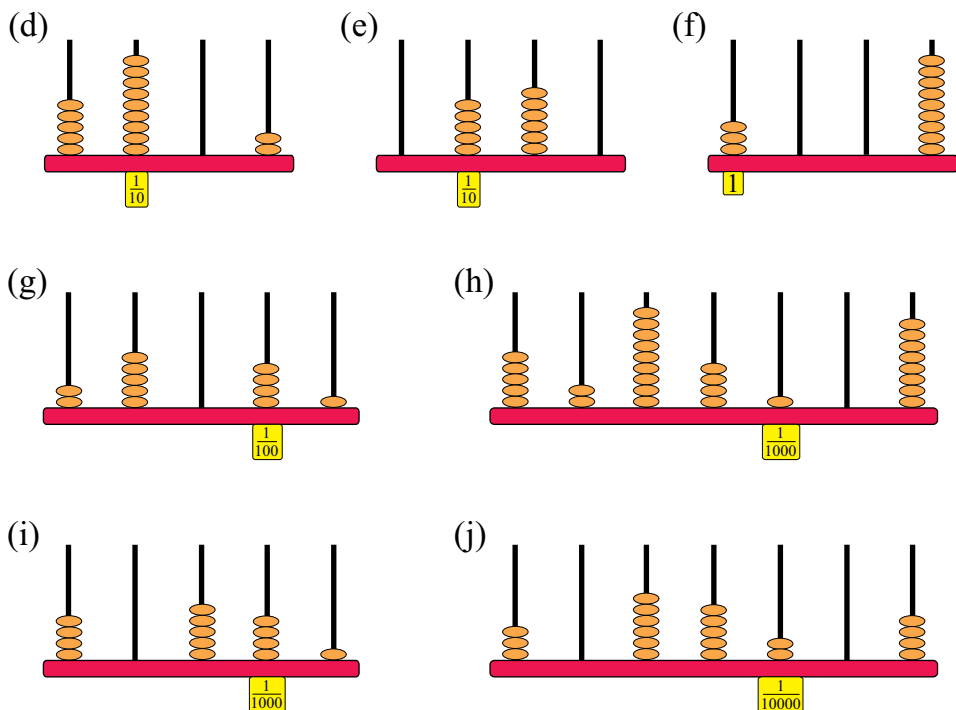
Example

To find the number represented on this spike abacus:

- use the place value given to find the place value of each spike
- the number of beads on each spike equals the digit in each place value

The number shown on this spike abacus is **953.6**

- (a)
- (b)
- (c)



4. State the place value of the 3 in each of the following decimals.

Write the answers as: (i) words

(ii) numbers or fractions

Examples

1. 635.8

(i) three tens

(ii) 30

2. 7.0931

(i) three thousandths

(ii) $\frac{3}{1000}$

(a) 57.34

(b) 1347.8

(c) 2.5308

(d) 83.06

(e) 0.04937

(f) 3208.9

(g) 38.572

(h) 39577.2

(i) 7.205643

(j) 99.02387

5. Complete the following patterns.

(a) 700, 70, 7, 0.7, __, __, __.

(b) 0.0002, 0.002, 0.02, 0.2, __, __, __.

6. Write the following as decimal numbers.

(a) $\frac{7}{100}$ (b) $\frac{2}{1000}$ (c) $\frac{9}{10}$ (d) $\frac{4}{10000}$

(e) five thousandths

(f) eight hundred-thousandths

(g) six hundredths

Examples

1. $\frac{1}{10} = 0.1$

2. three hundredths = **0.03**

7. Write the following numbers in decimal form.

Examples

1. $\frac{13}{1000}$
= **0.013**

2. $6\frac{21}{100}$
= **6.21**

3. $\frac{7215}{100}$
= **72.15**

(a) $\frac{18}{100}$

(b) $\frac{87}{1000}$

(c) $\frac{206}{1000}$

(d) $\frac{39}{10000}$

(e) $\frac{400}{1000}$

(f) $2\frac{7}{10}$

(g) $15\frac{18}{100}$

(h) $6\frac{279}{1000}$

(i) $123\frac{82}{1000}$

(j) $7\frac{20}{100}$

(k) $\frac{87}{10}$

(l) $\frac{163}{100}$

(m) $\frac{205}{10}$

(n) $\frac{7099}{1000}$

(o) $\frac{80501}{10}$

8. Write as decimal numbers.

Examples

1. $3 + \frac{2}{10} + \frac{5}{100}$
= **3.25**

2. $6 + \frac{9}{1000}$
= **6.009**

(a) $5 + \frac{6}{10} + \frac{7}{100}$

(b) $4 + \frac{1}{10} + \frac{2}{100} + \frac{9}{1000}$

(c) $\frac{8}{10} + \frac{7}{100} + \frac{4}{1000}$

(d) $6 + \frac{2}{100}$

(e) $9 + \frac{4}{10} + \frac{2}{1000}$

(f) $30 + \frac{7}{100} + \frac{1}{10000}$

(g) $\frac{8}{10} + \frac{7}{100} + \frac{4}{1000}$

(h) $6 + \frac{3}{1000} + \frac{9}{10}$

(i) $800 + 9 + \frac{5}{100} + \frac{1}{10}$

(j) $\frac{3}{100} + 100 + \frac{6}{10} + 2$

(k) $\frac{5}{1000} + 50 + \frac{5}{100000} + \frac{5}{10} + 5000$

9. List the following numbers from the smallest to the largest.

Hint: Write them all as decimal numbers first.

(a) $0.5, \frac{4}{10}, \frac{63}{100}, \frac{310}{1000}, 0.9$

(b) $2.02, 0.22, 2.2, 0.202, 0.022$

(c) $0.91, \frac{9}{10}, 0.089, \frac{8}{100}, 1, 0.01$

(d) $6.5621, 6.6531, 6.5361, 6.5163, 6.6351$

10. Choose the correct symbol ($<$ or $>$) to place between the pairs of numbers below.

Remember - the arrow points towards the *smaller* number

Examples $1.8 < 2.8$ $0.37 > 0.309$

- (a) 0.3 0.4 (b) 1.8 2.1 (c) 0.305 0.31
(d) 0.721 0.719 (e) 3.605 3.596 (f) 7.136 7.141

11. **Add** one tenth to each of the following numbers.

- (a) 0.6 (b) 1.8 (c) 0.27 (d) 4 (e) 307
(f) 6.032 (g) 0.222 (h) 0.001 (i) 0.9 (j) 3.89
(k) 15.92 (l) 20.9 (m) 209.9 (n) 59.9 (o) 999.9

12. **Add** one thousandth to each of the following numbers.

- (a) 3.6215 (b) 0.2048 (c) 2.54 (d) 18 (e) 3.07
(f) 6.032 (g) 0.289 (h) 0.00001 (i) 0.9999 (j) 19.999

13. Complete the following patterns.

- (a) 0.1, 0.2, 0.3, 0.4, __, __, __, __.
(b) 0.70, 0.71, 0.72, 0.73, __, __, __, __.
(c) 3.0, 3.2, 3.4, 3.6, __, __, __, __.
(d) 0.25, 0.26, 0.27, 0.28, __, __, __, __.
(e) 5.95, 5.96, 5.97, 5.98, __, __, __, __.

14. Find two numbers between the following pairs of numbers.

- (a) 5 and 10 (b) 4 and 5 (c) 0.2 and 0.3 (d) 5.67 and 5.68

15. Find the number **midway** between the following pairs of numbers.

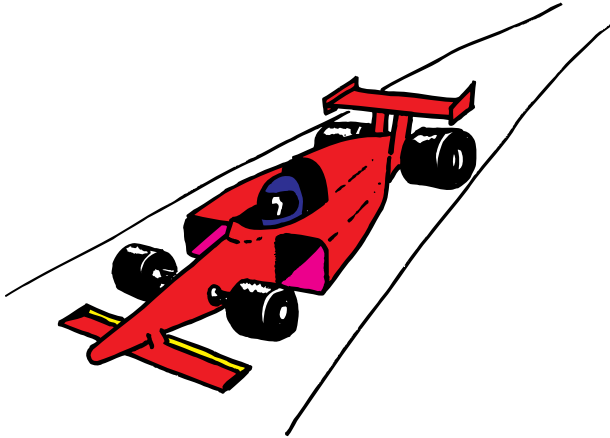
- (a) 12 and 18 (b) 8 and 9 (c) 2.1 and 2.2 (d) 7.84 and 7.87

16. (a) How many tenths are in one unit?

- (b) How many hundredths are in two units?
(c) How many hundredths are in three tenths?
(d) How many thousandths are in five tenths?

17. The body temperature of a healthy person is 36.9°C . Toekie was not feeling well and her temperature was 0.5° above normal. What was Toekie's temperature?

18. In the 100 metre race at a school athletics meeting Sharon ran a time of 12.2 seconds.
Yvonne was 0.4 seconds slower than Sharon.
Sharon was 0.3 seconds slower than Charley.
If these three girls were the place winners, state their places and times.
19. In a car race Michael Shoemaker had a lap time of 1 minute 31.295 seconds. Damian Dill had a lap time which was three thousandths of a second faster than Michael Shoemaker. Jack Badman had a lap time which was two hundredths of a second slower than Michael Shoemaker.
Find the lap times of Damian Dill and Jack Badman.



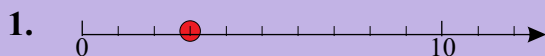
20. Bradon is a high-jumper. His best jump is 1.83 metres.
Brendan is also a high-jumper whose best jump is 1.79 metres.
Who has jumped the highest and by how many metres higher than the other jumper?
21. Mr. Potts is a farmer. On his property he has five dams that all have names. Their names and capacities are listed below.
Long Lake - 1.659 Megalitres
Deep Dam - 1.579 Megalitres
Peaceful Pond - 1.665 Megalitres
Wide Waters - 1.576 Megalitres
Rocky Reservoir - 1.678 Megalitres
List the dams in order from the largest to the smallest.

Reading Scales

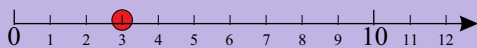
When reading scales it is necessary to work out the size of the divisions on the scales when not all the amounts are shown.

Examples

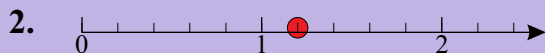
Write the number shown by the dot on each of the scales below.



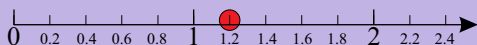
There are ten equal divisions between 0 and 10. These must all be one unit.



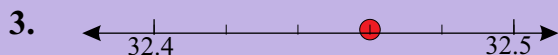
The number shown by the dot is **3**.



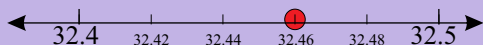
There are 5 divisions between 0 and 1. These must all be 0.2.



The number shown by the dot is **1.2**.



There are 5 divisions between 32.4 and 32.5. These must all be 0.02.

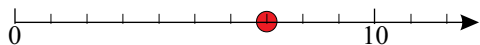


The number shown by the dot is **32.46**.

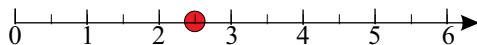
EXERCISE 6B

1. Write the number shown by the dot on each of the scales below.

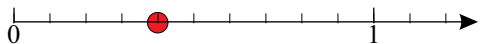
(a)



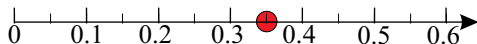
(b)



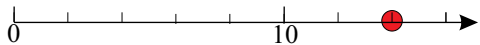
(c)



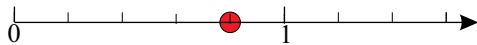
(d)



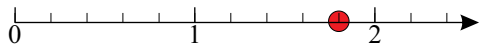
(e)



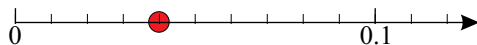
(f)



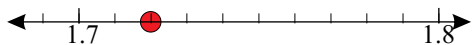
(g)



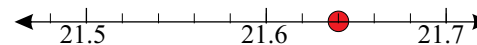
(h)



(i)

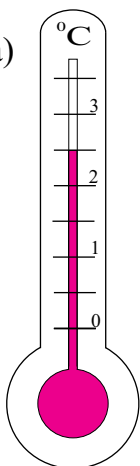


(j)

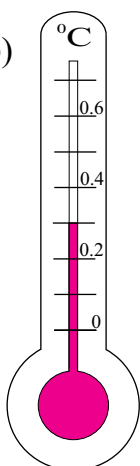


2. Read the temperature shown on each thermometer below.

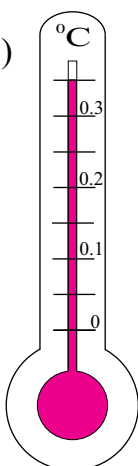
(a)



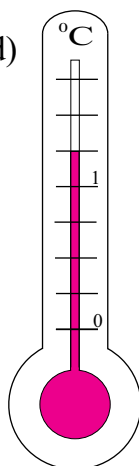
(b)



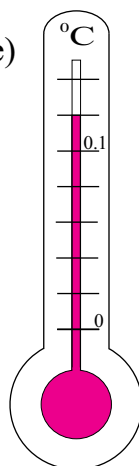
(c)



(d)

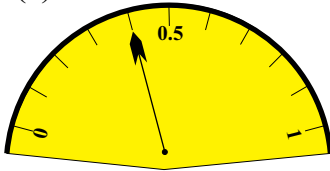


(e)

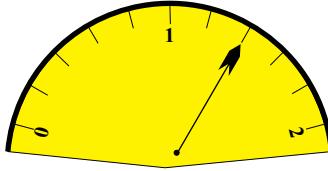


3. What numbers are shown on the meters below?

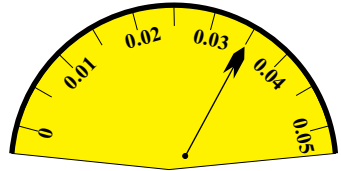
(a)



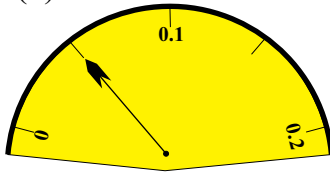
(b)



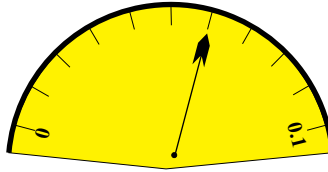
(c)



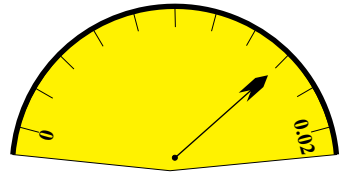
(d)



(e)



(f)



Rounding

EXERCISE 6C

1. Round the following prices to the nearest 5 cents.

To round a price to the nearest 5 cents follow the steps below.

Step 1 Find the multiples of 5 cents below and above the price.

Step 2 Choose the multiple that the price is closer to.

Example \$4.78

The multiple of 5 cents below this price is \$4.75

The multiple of 5 cents above this price is \$4.80

\$4.78 is closer to \$4.80

Answer: \$4.80

- | | | | | |
|-------------|-------------|-------------|-------------|-------------|
| (a) \$8.51 | (b) \$10.78 | (c) \$15.33 | (d) \$68.16 | (e) \$22.22 |
| (f) \$21.98 | (g) \$6.07 | (h) \$1.02 | (i) \$0.66 | (j) \$0.33 |

2. Round the following prices to the nearest 10 cents.

To round a price to the nearest 10 cents follow the steps below.

Step 1 Find the multiples of 10 cents below and above the price.

Step 2 Choose the multiple that the price is closer to.

Example \$12.64

The multiple of 10 cents below this price is \$12.60

The multiple of 10 cents above this price is \$12.70

\$12.64 is closer to \$12.60

Answer: \$12.60

Note: for a price midway between two multiples of 10 cents round up to the price above

Example \$5.35 would be rounded up to \$5.40

- | | | | | |
|-------------|-------------|-------------|-------------|-------------|
| (a) \$8.51 | (b) \$10.78 | (c) \$15.33 | (d) \$68.16 | (e) \$22.22 |
| (f) \$21.98 | (g) \$6.07 | (h) \$1.02 | (i) \$0.66 | (j) \$0.33 |
| (k) \$8.35 | (l) \$59.96 | (m) \$8.95 | (n) \$99.97 | (o) \$76.77 |

3. Round the prices in question 2 to the nearest dollar.

4. How many *decimal places* are in the following numbers?

The number of decimal places is defined as the number of digits after (to the right of) the decimal point.

Examples

75.621 has **three** decimal places (75.**621**)

8.60175 has **five** decimal places (8.**60175**)

5609.3 has **one** decimal place (5609.**3**)

- | | | | | |
|-------------|-------------|-------------|------------|--------------|
| (a) 3.7 | (b) 18.971 | (c) 0.0731 | (d) 45.301 | (e) 5.01856 |
| (f) 34.6692 | (g) 5.66666 | (h) 100.001 | (i) 52890 | (j) 0.000009 |

To round a decimal number off to a certain number of decimal places follow the steps below.

Step 1 Identify the number of decimal places being rounded off to.

Step 2 Look at the digit *after* this number of decimal places.

Step 3 - If this digit is *less than 5* round down to the existing number.
 - If this digit is *greater than or equal to 5* round up to the next number.

Examples

1. Round 56.7834 to *two* decimal places.

Step 1 Two decimal places means the two digits after the decimal point.

56.7834

Step 2 The next digit is 3 which is less than 5.

56.7834

Step 3 Round *down* to the existing number.

Answer: 56.78

2. Round 8.0976 to *three* decimal places.

Step 1 Three decimal places means the three digits after the decimal point.

8.0976

Step 2 The next digit is 6 which is greater than 5.

8.0976

Step 3 Round *up* to the next number.

Answer: 8.098

Examples continued

3. Round 12.7513 to **one** decimal place.

Step 1 One decimal place means the one digit after the decimal point.

12.7513

Step 2 The next digit is 5.

12.7**5**13

Step 3 Round **up** to the next number.

Answer: 12.8

4. Round 2.569973 to **four** decimal places.

Step 1 Four decimal places means the four digits after the decimal point.

2.569973

Step 2 The next digit is 7 which is greater than 5.

2.5699**7**3

Step 3 Round **up** to the next number.

Answer: 2.5700

5. Round the following numbers to the number of decimal places shown in brackets.

- (a) 56.713 (two decimal places)
- (b) 8.1632 (one decimal place)
- (c) 8.450651 (four decimal places)
- (d) 123.603 (one decimal place)
- (e) 0.5691 (two decimal places)
- (f) 45.32238 (three decimal places)
- (g) 11.4962 (two decimal places)
- (h) 781.973 (one decimal place)
- (i) 47.4552 (two decimal places)
- (j) 0.007 (two decimal places)
- (k) 14.0900953 (five decimal places)
- (l) 245.6995 (three decimal places)

6. Round the following numbers to the nearest *tenth*.

Remember: the tenths is the first digit after the decimal point. Rounding to the nearest tenth is the same as rounding to one decimal place.

Example 67.842

Rounding 67.842 to the nearest tenth is the same as rounding to one decimal place.

Answer: 67.8

- | | | | |
|-----------|-------------|-------------|------------|
| (a) 45.78 | (b) 610.815 | (c) 93.0518 | (d) 0.2736 |
| (e) 8.962 | (f) 13.45 | (g) 100.98 | (h) 0.0729 |

7. Complete the following sentences by filling in the blanks.

- (a) Rounding to the nearest hundredth is the same as rounding to _____ decimal places.
- (b) Rounding to the nearest _____ is the same as rounding to three decimal places.
- (c) Rounding to the nearest ten-thousandth is the same as rounding to _____ decimal places.

8. Round the following numbers to the nearest *hundredth*.

- | | | | |
|--------------|---------------|-------------|--------------|
| (a) 567.0267 | (b) 9.45652 | (c) 17.8926 | (d) 23.5973 |
| (e) 6.09549 | (f) 102.30253 | (g) 0.56789 | (h) 7.654321 |

9. Round the numbers in question 8 to the nearest *thousandth*.

10. Round the following lengths to the nearest tenth of a metre.

- | | | | |
|---------------|-------------|--------------|--------------|
| (a) 68.73 m | (b) 9.653 m | (c) 35.062 m | (d) 0.8725 m |
| (e) 120.583 m | (f) 34.96 m | (g) 9.008 m | (h) 59.973 m |

11. Round the following times to the nearest thousandth of a second.

- | | | |
|------------------|------------------|------------------|
| (a) 5.6739 sec | (b) 0.04829 sec | (c) 19.20853 sec |
| (d) 59.13864 sec | (e) 107.9692 sec | (f) 53.8996 sec |

12. Round the following masses to the nearest hundredth of a kilogram.

- | | | |
|-----------------|---------------|-----------------|
| (a) 67.85493 kg | (b) 7.9735 kg | (c) 41.77629 kg |
| (d) 108.9874 kg | (e) 0.6734 kg | (f) 0.09875 kg |

13. Round the following masses to the nearest kilogram.

- (a) 6.7 kg (b) 12.3 kg (c) 19.9 kg (d) 0.8 kg
 (e) 9.9 kg (f) 7.09 kg (g) 28.71 kg (h) 3.5 kg

14. Giaan owns a strawberry farm. Her pickers collect the strawberries in small buckets, weigh the buckets then empty them to be packed. Giaan pays the pickers \$2 per kilogram of strawberries they pick. A list of the weights of the buckets picked by one picker is shown below.

5.8 kg 4.6 kg 5.3 kg 7.5 kg 4.9 kg 5.2 kg
 6.6 kg 7.2 kg 5.3 kg 6.5 kg 4.4 kg 5.9 kg

To make it quicker to find the total weight of strawberries picked, Giaan rounds the weights to the nearest kilogram then adds them to get the total weight.

- (a) For the list of weights shown above use Giaan's method to find the total weight of strawberries picked by that picker.
 (b) What will Giaan pay that picker?
 (c) Use a calculator to add the exact weights as shown above.
 (d) Round this to the nearest kilogram.
 (e) What would the picker have been paid if Giaan had done this?

15. Glenn and Linda go shopping and buy the following items.

A loaf of bread \$2.35	2 litres of milk \$3.20
1 can of baked beans \$1.35	500 grams of butter \$1.65
3 kg of apples at \$2.69 per kg	5 kg of potatoes at \$0.87 per kg

They both estimated the total cost of the shopping list. Glenn rounded the prices to the nearest 50 cents before adding them. Linda rounded the prices to the nearest dollar before adding them.

- (a) Find both Glenn and Linda's estimate of the total cost.
 (b) Use a calculator to find the actual total cost of the shopping list.
 (c) Round the total cost to the nearest 5 cents and calculate the change Glenn and Linda should receive from \$50.

Converting Fractions to Decimals

EXERCISE 6D

1. Convert the following fractions to decimals.

Examples

$$1. \quad \frac{7}{10} = 0.7$$

$$2. \quad \frac{9}{100} = 0.09$$

$$3. \quad \frac{7}{1000} = 0.007$$

$$4. \quad \frac{53}{100} = 0.53$$

$$5. \quad \frac{29}{1000} = 0.0029$$

$$6. \quad \frac{159}{1000} = 0.159$$

(a) $\frac{1}{10}$ (b) $\frac{3}{100}$ (c) $\frac{27}{100}$ (d) $\frac{9}{1000}$ (e) $\frac{81}{1000}$ (f) $\frac{451}{1000}$

2. Convert the following fractions to decimals.

Examples

$$1. \quad \frac{1}{4}$$

$$= 1 \div 4$$

$$= 4 \overline{) 1.00}$$

$$= 0.25$$

$$2. \quad \frac{3}{8}$$

$$= 3 \div 8$$

$$= 8 \overline{) 3.000}$$

$$= 0.375$$

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{8}$ (e) $\frac{9}{20}$ (f) $\frac{11}{4}$

3. Use the answer for question 2(c) to help convert the following fractions to decimals. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$

4. Convert the following numbers to decimals.
(Try to find the decimals using your memory)

(a) $3\frac{1}{2}$ (b) $8\frac{1}{4}$ (c) $10\frac{1}{5}$ (d) $6\frac{3}{4}$ (e) $25\frac{3}{5}$ (f) $17\frac{4}{5}$

Recurring Decimals

EXERCISE 6E

Many decimal numbers are a recurring or repeating number or series of numbers.

Examples 1. 0.44444..... 2. 0.235235235... 3. 56.3787878...

These types of recurring decimals are represented by a dot over a single number that is repeated or a bar over several numbers that are repeated. The above examples would be represented as follows.

1. $0.\dot{4}$

2. $0.\overline{235}$

3. $56.\overline{378}$

1. Use a dot or bar to indicate which numbers are recurring (repeating) in the following decimals.

(a) 0.7777.... (b) 0.252525... (c) 0.61111.... (d) 0.9712451245....
(e) 6.474747... (f) 23.8888.... (g) 17.96212121.. (h) 3.04595959...

2. Change the following fractions to decimals using a dot or bar as shown in question 1 to indicate the repeating numbers.

(a) $\frac{1}{3}$ (b) $\frac{5}{6}$ (c) $\frac{7}{9}$ (d) $\frac{5}{12}$ (e) $\frac{3}{7}$ (f) $5\frac{11}{12}$

3. Find the fractions that when converted would give the following decimals (use the answers to question 2 to help).

(a) $0.\dot{6}$ (b) $0.\dot{1}$ (c) $0.\dot{8}$ (d) $0.1\dot{6}$ (e) $0.\overline{142857}$

PROBLEM SOLVING

1. Match the pairs from the following quantities and find the common connection between the pairs.

1 litre

1 metre

1 dollar

10 kg

10 cm³

10 mm

1 hectare

100 m²

1 year

1 tonne

1 cent

1 century

2. Find the following number.

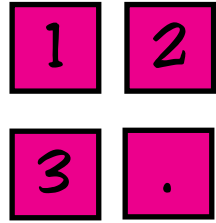
It has five digits. It has twice as many tenths as tens.

It has two decimal places. The sum of all its digits is 12.

It has twice as many hundreds as hundredths.

It has three times as many tenths as hundreds.

3. (a) How many different numbers can be formed using these four cards?
 (b) How many of the numbers are less than one?
 (c) How many of the numbers are greater than two?
 (d) What is the largest number?
 (e) What is the smallest number?



4. Convert $\frac{1}{81}$ to a decimal showing the numbers that are repeated.

PUZZLE

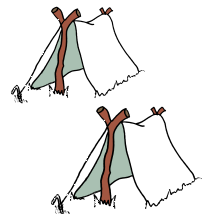
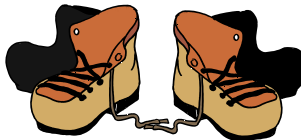
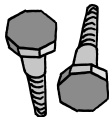
Change **TENTH** into **BOOKS** in seven steps by changing one letter at a time to form a new word. Use the clues below to find the words in between. (They are not in order)

Example
CAT can be changed to **DOG** in three steps

CAT
COT
DOT
DOG



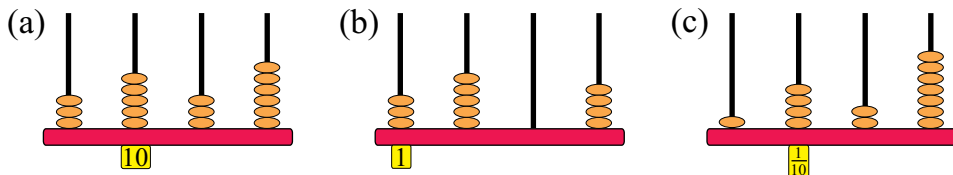
Celtic People



CHAPTER REVIEW

1. What number is represented on each spike abacus below?

The place value of one of the spikes is given for each abacus.



2. Write the following fractions as: (i) words (ii) decimals

(a) $\frac{5}{100}$

(b) $\frac{6}{10}$

(c) $7\frac{28}{1000}$

(d) $23\frac{17}{10000}$

(e) $8\frac{204}{1000}$

3. Write the following additions as decimals.

(a) $3 + \frac{7}{10} + \frac{9}{100}$

(b) $60 + \frac{3}{10} + \frac{2}{100}$

(c) $7 + \frac{2}{10} + 50 + \frac{7}{1000}$

4. List the following numbers from smallest to largest.

(a) 0.4 , $\frac{7}{100}$, $\frac{8}{10}$, 0.09 , $\frac{63}{100}$, $\frac{714}{1000}$.

(b) 0.6215 , 0.6125 , 0.6251 , 0.6321 , 0.6152 , 0.6312 , 0.6123

5. (a) How many tenths are in 2 units?

(b) How many thousandths are in 3 tenths?

6. Add one hundredth to each of the following numbers.

(a) 3.478

(b) 0.215

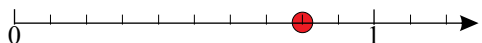
(c) 6.3

(d) 8.1973

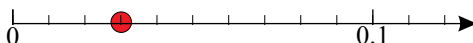
(e) 0.0001

7. Write the number shown by the dot on each of the scales below.

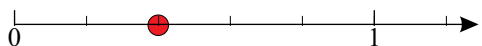
(a)



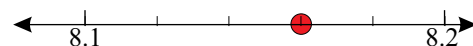
(b)



(c)



(d)



8. Round the following amounts to the nearest: (i) 10 cents (ii) dollar

(a) \$0.89

(b) \$3.42

(c) \$15.76

(d) \$10.49

(e) \$101.62

9. Convert the following fractions to decimals.

(a) $\frac{3}{4}$

(b) $\frac{4}{5}$

(c) $8\frac{1}{2}$

(d) $13\frac{1}{3}$

(e) $65\frac{2}{9}$

10. Aidan, Luke and Andrew filled the first three places (not necessarily in that order) of a 100 m backstroke race.

Aidan swam two hundredths of a second slower than Luke.

Andrew swam one tenth of a second faster than Aidan.

The winner of the race swam a time of 58.96 seconds.

Find the time swum by each of the swimmers.

Decimal Operations

Addition

EXERCISE 7A

1. Find the answers to the following additions.

Examples

Remember these steps:

Step 1 Line up all the decimal points.

Step 2 Add zeros where necessary.

1. $3.7 + 2.8$

$$\begin{array}{r} 3.7 \\ + 2.8 \\ \hline 6.5 \end{array}$$

2. $16.7 + 4.85$

$$\begin{array}{r} 16.70 \\ + 4.85 \\ \hline 21.55 \end{array}$$

3. $25.6 + 0.804 + 8.57$

$$\begin{array}{r} 25.600 \\ + 0.804 \\ + 8.570 \\ \hline 34.974 \end{array}$$

(a) $5.6 + 4.3$

(b) $16.4 + 3.1$

(c) $27.63 + 2.25$

(d) $8.3 + 6.4$

(e) $28.7 + 9.2$

(f) $0.783 + 0.116$

(g) $326.6 + 67.6$

(h) $3.6 + 9.5$

(i) $8.4 + 9.7 + 6.5$

(j) $2.56 + 13.8$

(k) $0.6 + 12.08$

(l) $87.09 + 12.608$

(m) $5.28 + 8.073$

(n) $4.99 + 0.086$

(o) $15.6 + 2.75 + 28.468$

2. Kim bought a magazine that cost \$3.75, a book that cost \$8.50 and a birthday card that cost \$ 2.65.
What is the total cost of these items?

3. Complete these patterns.

(a) 0.95, 1.25, 1.55, 1.85, __, __, __, __.

(b) 0.78, 0.81, 0.84, 0.87, __, __, __, __.

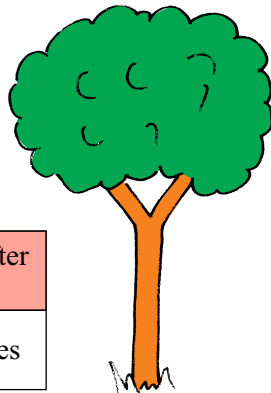
(c) 1.4, 1.6, 2.0, 2.6, __, __, __, __.

(d) 10.5, 12, 15, 19.5, __, __, __, __.

4. David loaded four programs onto his computer. The size of the programs were 4.62 MB, 1.3 MB, 0.635 MB and 10.5 MB.
What is the total number of megabytes (MB) used by the programs?

5. A 3 metre tall tree is planted and its height recorded at the end of each year. The heights after the first four years are shown in the table below.

Height when planted	Height after 1 year	Height after 2 years	Height after 3 years	Height after 4 years
3.0 metres	3.1 metres	3.3 metres	3.6 metres	4.0 metres



- (a) Austin is a forester studying the growth of trees. He observed that the tree's growth followed a pattern. Find the pattern and use it to find the height of the tree after:
- (i) 5 years (ii) 10 years
- (b) How much did the tree grow in the twelfth year after planting?
- (c) After how many years is the tree 15 metres tall?
- (d) In what year after planting did the tree grow 3 metres?

Subtraction

EXERCISE 7B

1. Find the answers to the following subtractions.

Examples

Remember these steps:

Step 1 Line up all the decimal points.

Step 2 Add zeros where necessary.

1. $3.8 - 2.6$

$$\begin{array}{r} 3.8 \\ - 2.6 \\ \hline 1.2 \end{array}$$

2. $16.9 - 4.65$

$$\begin{array}{r} 16.90 \\ - 4.65 \\ \hline 12.25 \end{array}$$

3. $25.624 - 9.85$

$$\begin{array}{r} 25.624 \\ - 9.850 \\ \hline 15.774 \end{array}$$

(a) $5.6 - 4.3$

(b) $16.4 - 3.1$

(c) $27.68 - 2.25$

(d) $8.3 - 6.4$

(e) $28.2 - 9.6$

(f) $0.783 - 0.116$

(g) $326.6 - 67.6$

(h) $13 - 9.5$

(i) $9.87 - 6.5$

(j) $12.56 - 3.8$

(k) $35.6 - 12.08$

(l) $487.09 - 12.608$

(m) $8.28 - 0.373$

(n) $4.99 - 0.086$

(o) $7815.624 - 209.7685$

2. Joe is 1.56 metres tall. His sister, Mary, is 1.72 metres tall. Joe's brother, Eric is 0.18 metres shorter than Joe.

- (a) How tall is Eric? (b) How much taller than Joe is Mary?
- (c) How much taller than Eric is Mary?

Addition and Subtraction Problems

EXERCISE 7C

- Hatsue bought her lunch at school and purchased a salad roll, donut and orange juice.
The salad roll cost twice as much as the donut.
The orange juice cost 20 cents more than the donut.
Her lunch cost exactly \$3.00.
Find the cost of the salad roll, the donut and the orange juice.
- Thomas wanted to find the weight of his cat so he could give it some tablets.
While holding his cat, he stood on bathroom scales and found their combined weight to be 72.3 kg.
He then weighed himself and found that he weighed 68.8 kg.
What did Thomas' cat weigh?
- The sum of two numbers is 4. The difference between them is 1.
Find the two numbers.
- The sum of two numbers is 11. The difference between them is 5.4.
Find the two numbers.
- Graeme went shopping and bought the following items.
Bread \$1.85, Cheese \$2.65, Jam \$2.75, Milk \$3.25, Butter \$1.95 and a tub of ice-cream.
If Graeme received \$4.00 change from \$20 how much did the ice-cream cost?
- In a chemistry experiment Ased mixed the following three chemicals:
0.84 litres of HCl
1.32 litres of H_2SO_4
2.56 litres of H_2O
Ased then used 1.85 litres of the mixture in the experiment.
How much of the mixture was remaining?



7. Nurse Judy had to give medicine to a patient in the following amounts.
7.2 mL (millilitres) in the first hour and then reduce the amount by
1.2 mL every hour until the patient requires no further medication.

(7.2 mL in the first hour, 6.0 mL in the second hour,
4.8 mL in the third hour, etc)

Nurse Judy obtained the medicine from a container
that held 30 mL of the medicine.

- List the amounts of medicine required each hour.
- What is the total amount of medicine required?
- How much of the medicine is remaining in the container after the patient has received all he needed?



8. Suni had solar panels installed on the roof of his house. They generated electricity and Suni was paid 12.5 cents for every kWh (kilowatt-hour) of electricity his panels produced.

Suni recorded the amount of electricity his panels produced each day for a week and these amounts are shown here:

Monday - 9.45 kWh, Tuesday - 8.78 kWh, Wednesday - 6.53 kWh,
Thursday - 11.66 kWh, Friday - 7.90 kWh, Saturday - 12.25 kWh,
Sunday - 7.83 kWh

How much would Suni have been paid for the electricity his solar panels produced that week? Give answer in dollars and cents.

9. The temperature of the seawater near a coastal town was recorded on the first day of each month for a year and the 1st of January of the following year. The temperatures are shown in the table below.

Date	Jan 1	Feb 1	Mar 1	Apr 1	May 1	Jun 1	Jul 1	Aug 1	Sep 1	Oct 1	Nov 1	Dec 1	Jan 1
Temp ($^{\circ}\text{C}$)	18.2	19.7	21.4	21.8	21.2	19.6	17.7	15.4	14.3	14.6	15.4	16.5	17.9

- In which months did the temperature increase?
- What was the maximum temperature recorded?
- What was the minimum temperature recorded?
- What is the difference between the maximum and minimum recorded temperatures?
- What was the change in water temperature in the May? State if the change was an increase or decrease.
- What was the change in water temperature in the October? State if the change was an increase or decrease.
- During which month did the water temperature increase the most?
- During which month did the water temperature decrease the most?

Multiplication - Decimal by Whole Number

EXERCISE 7D

1. State the number of decimal places in the following numbers.

Remember - the number of decimal places is equal to the number of digits after the decimal point.

Examples 57.309 has three decimal places (57.**309**)
105.6 has one decimal place (105.**6**)

- (a) 6.8 (b) 0.25 (c) 185.3 (d) 4.578
(e) 5.004 (f) 56.509 (g) 53 (h) 10.0506

2. Find the answer to each of the following multiplications.

Example 23.46×51

$$\begin{array}{r} 23.46 \quad \text{2 decimal places} \\ \times 51 \quad \text{0 decimal places} \\ \hline 2346 \\ 117300 \\ \hline 1196.46 \end{array}$$

Remember:

The number of decimal places in the answer is the same as the total number of decimal places in the question.

Total of 2 decimal places in the question.
There must be 2 decimal places in the answer.

- (a) 73.5×8 (b) 6.05×9 (c) 13.43×6
(d) 1.783×7 (e) 2.7×16 (f) 5.21×28
(g) 28.302×40 (h) 47.721×15 (i) 8.796×315

3. If an apple weighs 0.15 kg find the weight of:

(a) 5 apples (b) 20 apples (c) three dozen apples

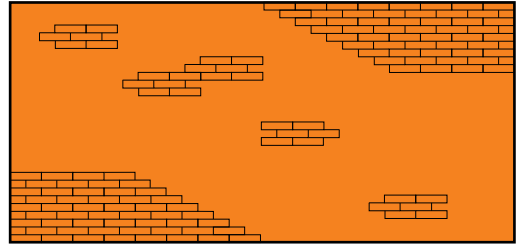
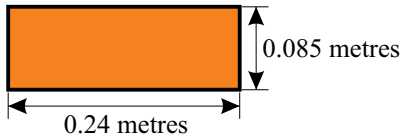
4. A surf shop is selling shirts for \$8.75. Find the cost of buying 8 shirts.

5. The solar panels on a house produce an average of 8.45 kWh (kilowatt-hours) of energy per day.

How many kWh would be produced in:

(a) a week? (b) a year (365 days)?

6. Ahmed wanted to calculate the size of a brick wall. He measured the size of one brick to be 0.24 metres long and 0.085 metres high.



Ahmed counted the bricks along the length and up the wall. He found the wall to be 15 bricks long and 32 bricks high. Use this information to calculate the length and height of the wall.

Multiplication - Decimal by Decimal

EXERCISE 7E

1. Find the answer to each of the following multiplications.

Example 23.46×7.5

$$\begin{array}{r}
 23.46 \quad \begin{array}{l} 2 \text{ decimal places} \\ 1 \text{ decimal place} \end{array} \\
 \times 7.5 \\
 \hline
 11730 \\
 164220 \\
 \hline
 175.950
 \end{array}$$

Remember:

The number of decimal places in the answer is the same as the total number of decimal places in the question.

Total of 3 decimal places in the question.
There must be 3 decimal places in the answer.

- | | | |
|--------------------------|-------------------------|------------------------------------|
| (a) 3.5×8.1 | (b) 6.5×9.3 | (c) 13.4×6.2 |
| (d) 0.34×7.6 | (e) 2.7×0.16 | (f) 0.52×0.8 |
| (g) 28.3×0.04 | (h) 1.001×1.5 | (i) 8.006×0.05 |
| (j) 0.005×2.01 | (k) 3.07×0.046 | (l) 12.05×2.04 |
| (m) 0.004×0.005 | (n) 0.1×0.1 | (o) $0.08 \times 0.08 \times 0.01$ |

2. A stock feed supplier was selling birdseed for \$2.35 per kilogram.

Find the price of the following amounts of birdseed.

- (a) 0.8 kg (b) 5.6 kg (c) 24.2 kg

Division by Whole Numbers

EXERCISE 7F

1. Find the answers to the following divisions.

Examples

1. $120.5 \div 5$

$$\begin{array}{r} 24.1 \\ 5 \overline{)120.5} \\ \underline{100} \\ 20 \\ \underline{20} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

= 24.1

2. $2.8 \div 4$

$$\begin{array}{r} 0.7 \\ 4 \overline{)2.8} \\ \underline{28} \\ 0 \end{array}$$

= 0.7

(a) $6.5 \div 5$

(b) $12.8 \div 4$

(c) $73.8 \div 6$

(d) $146.7 \div 3$

(e) $90.3 \div 7$

(f) $123.75 \div 5$

(g) $290.16 \div 6$

(h) $229.14 \div 9$

(i) $10.494 \div 2$

(j) $7.524 \div 6$

(k) $0.182 \div 7$

(l) $0.0384 \div 8$

2. Find the answers to the following divisions.

Example

$9 \div 4$

$$\begin{array}{r} 2.25 \\ 4 \overline{)9.00} \\ \underline{8} \\ 10 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

= 2.25

Add enough zeros
to make it possible
to find the answer.

(a) $6 \div 4$

(b) $18 \div 4$

(c) $17 \div 5$

(d) $1.2 \div 8$

(e) $124 \div 8$

(f) $30 \div 8$

(g) $0.1 \div 8$

(h) $3.44 \div 5$

(i) $31.1 \div 8$

(j) $45.31 \div 5$

(k) $23.73 \div 4$

(l) $100.01 \div 4$

3. Jackson bought 6 kg of nails from a hardware shop. The total cost of the nails was \$27.30. What was the cost of 1 kg of the nails?

4. Eight friends hired a boat for a day. The total cost was \$274.
How much should each person pay if they share the cost equally?

5. A farmer used 165.2 kL of water in a week. How much water did he use each day if he used the same amount each day?

6. A tree grew from 3.64 m to 7.96 m in eight years. How much did it grow each year?

Division by Decimals

EXERCISE 7G

1. Find the answers to the following divisions.

Examples

Remember these steps:

Step 1 Move the decimal point in the divisor to make it a whole number.

Step 2 Move the decimal point in the dividend the same number of places.

Step 3 Add zeros if necessary.

1. Dividend Divisor

$$2.48 \div 0.2$$

Decimal point needs to move 1 place to make the divisor a whole number. Move the decimal point 1 place in the divisor and dividend.

$$\begin{array}{r} 2.48 \div 0.2 \\ = 24.8 \div 2 \\ = 12.4 \end{array}$$

2. $14.2 \div 0.04$

Decimal point needs to move 2 places to make the divisor a whole number. Move the decimal point 2 places in the divisor and dividend. Add 1 zero to the dividend.

$$\begin{array}{r} 14.2 \div 0.04 \\ = 1420 \div 4 \\ = 355 \end{array}$$

3. $48 \div 1.2$

Decimal point needs to move 1 place to make the divisor a whole number. Move the decimal point 1 place in the divisor and dividend. Add 1 zero to the dividend.

$$\begin{array}{r} 48. \div 1.2 \\ = 480 \div 12 \\ = 40 \end{array}$$

(a) $3.6 \div 0.9$

(b) $2.7 \div 0.3$

(c) $0.24 \div 0.8$

(d) $5.4 \div 0.06$

(e) $12.3 \div 0.03$

(f) $0.168 \div 0.4$

(g) $0.008 \div 0.004$

(h) $5.05 \div 0.5$

(i) $0.24 \div 0.006$

(j) $2 \div 0.05$

(k) $7.38 \div 0.6$

(l) $4.8 \div 0.06$

(m) $24.15 \div 0.007$

(n) $0.1 \div 0.01$

(o) $20 \div 0.04$

2. How many 0.2 kg oranges would be in a 3 kg bag?
3. How many pieces of wood 0.4 metres long could be cut from a 4.8 metre length?
4. If a sheet of paper is 0.01 cm thick how many sheets would be in a book which is 5 cm thick?
5. A red blood cell is 0.008 mm across. How many cells would there be across a blood vessel which is 0.32 mm in diameter?

Division Problems

EXERCISE 7H

1. Four people had a meal at a restaurant and the total bill was \$125. If they share the cost equally how much should each person pay?
2. Thierry bought a 6 kg bag of potatoes for \$5.10. What would 1 kg of the potatoes cost?
3. In the town of Raynor it rained every day for a week. The weather bureau measured 178.5 mm of rain for the week. What was the average rainfall each day for that week?
4. The annual sales for a clothes company was 6.816 million dollars. What was the average sales per month?
5. How many 0.3 litre drink bottles could be filled from 15 litres?
6. If a dozen eggs weighs 0.66 kg, what does each egg weigh?
7. If 8 books weigh 2.8 kg, what does each book weigh?
8. If a bolt weighed 0.06 kg, how many bolts would be in 3 kg?



Multiplication and Division by Powers of 10

EXERCISE 7I

1. Find the answers to the following multiplications.

Examples

When multiplying by a power of 10:

Step 1 Move the decimal point to the *right* the same number of places as the number of zeros.

Step 2 Add zeros if necessary.

$$1. \quad 8.057 \times 100$$

$$\begin{array}{r} 8.057 \\ \hline = 805.7 \end{array}$$

2 zeros—
move the decimal
point 2 places to
the right.

$$2. \quad 3.65 \times 1000$$

$$\begin{array}{r} 3.65 \\ \hline = 3650 \end{array}$$

3 zeros—
move the decimal
point 3 places.
Add 1 zero in
the blank space.

$$(a) \quad 7.83 \times 10$$

$$(b) \quad 62.08 \times 100$$

$$(c) \quad 0.214 \times 100$$

$$(d) \quad 31 \times 1000$$

$$(e) \quad 0.0008 \times 100$$

$$(f) \quad 0.55 \times 10\,000$$

$$(g) \quad 0.0278 \times 1000$$

$$(h) \quad 82.7 \times 100\,000$$

$$(i) \quad 0.05 \times 100 \times 1000$$

$$(j) \quad 0.0056 \times 100$$

$$(k) \quad 65.175 \times 10$$

$$(l) \quad 2.967 \times 10 \times 100$$

2. Find the answers to the following divisions.

Examples

When dividing by a power of 10:

Step 1 Move the decimal point to the *left* the same number of places as the number of zeros.

Step 2 Add zeros if necessary.

$$1. \quad 284.7 \div 100$$

$$\begin{array}{r} 284.7 \\ \hline = 2.847 \end{array}$$

2 zeros—
move the decimal
point 2 places to
the left.

$$2. \quad 94.6 \div 1000$$

$$\begin{array}{r} 94.6 \\ \hline = 0.0946 \end{array}$$

3 zeros—
move the decimal
point 3 places.
Add 1 zero in
the blank space.

$$(a) \quad 35.2 \div 10$$

$$(b) \quad 657.3 \div 1000$$

$$(c) \quad 0.6 \div 10$$

$$(d) \quad 5000 \div 100$$

$$(e) \quad 793.5 \div 10\,000$$

$$(f) \quad 891.3 \div 10$$

$$(g) \quad 22\,000 \div 10\,000$$

$$(h) \quad 3.8 \div 100\,000$$

$$(i) \quad 5.9 \div 100 \div 10$$

$$(j) \quad 75.145 \div 1000$$

$$(k) \quad 452.78 \div 100$$

$$(l) \quad 540.01 \div 10 \div 1000$$

3. Ask a friend to race you to find the answers to the following problems. Tell your friend to use a calculator!

- | | | |
|-----------------------------|-------------------------|--------------------------------|
| (a) 9.83×10 | (b) $162.8 \div 100$ | (c) 0.804×100 |
| (d) $457 \div 1000$ | (e) 0.005×1000 | (f) $843.9 \div 100$ |
| (g) 0.0448×1000 | (h) $8270.2 \div 1000$ | (i) $0.05 \div 100$ |
| (j) $0.0016 \times 10\ 000$ | (k) 95.075×10 | (l) $2.967 \times 10 \div 100$ |

Prefixes and Conversions

Table of Common Prefixes

Prefix	Symbol	Conversion
giga	G	$\times 1\ 000\ 000\ 000$
mega	M	$\times 1\ 000\ 000$
kilo	k	$\times 1000$
centi	c	$\div 100$
milli	m	$\div 1000$
micro	μ	$\div 1\ 000\ 000$

Examples

Complete the following conversions.

1. 6.89 kilograms = __ grams

The prefix in this example is **kilo**.

From the table above the conversion for kilo is $\times 1000$.

Replace the prefix with the conversion.

$$\begin{aligned}
 6.89 \text{ kilograms} &= 6.89 \times 1000 \text{ grams} \\
 &= \mathbf{6890 \text{ grams}}
 \end{aligned}$$

Examples continued

2. 4500 millimetres = __ metres

The prefix in this example is *milli*.

From the table above the conversion for kilo is $\div 1000$.

Replace the prefix with the conversion.

$$\begin{aligned} 4500 \text{ millimetres} &= 4500 \div 1000 \text{ metres} \\ &= \mathbf{4.5 \text{ metres}} \end{aligned}$$

3. 9.3 ML = __ L

The symbol in this example is *M (mega)*.

From the table above the conversion for mega is $\times 1\,000\,000$.

Replace the symbol with the conversion.

$$\begin{aligned} 9.3 \text{ ML} &= 9.3 \times 1\,000\,000 \text{ L} \\ &= \mathbf{9\,300\,000 \text{ L}} \end{aligned}$$

4. 621 μm = __ m

The symbol in this example is μ (*micro*).

From the table above the conversion for micro is $\div 1\,000\,000$.

Replace the symbol with the conversion.

$$\begin{aligned} 621 \mu\text{m} &= 621 \div 1\,000\,000 \text{ m} \\ &= \mathbf{0.000\,621 \text{ m}} \end{aligned}$$

EXERCISE 7J

1. Complete the following conversions.

- | | |
|----------------------------------|------------------------------------|
| (a) 3 kilometres = __ metres | (b) 5.64 kilometres = __ metres |
| (c) 200 centimetres = __ metres | (d) 2540 centimetres = __ metres |
| (e) 2 megalitres = __ litres | (f) 0.056 megalitres = __ litres |
| (g) 21.5 kilograms = __ grams | (h) 0.785 kilograms = __ grams |
| (i) 5000 millimetres = __ metres | (j) 78 600 millimetres = __ metres |
| (k) 0.4 gegalitres = __ litres | (l) 256 milligrams = __ grams |
| (m) 4370 micrometres = __ metres | (n) 0.89 milligrams = __ grams |

2. Complete the following conversions.

Example 62 mg = __ g

The symbol **m** represents the prefix **milli**.

The conversion for milli is $\div 1000$.

Replace the symbol with the conversion.

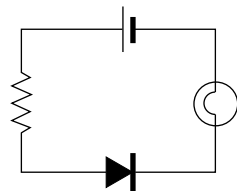
$$62 \text{ mg} = 62 \div 1000 \text{ g}$$

$$= \mathbf{0.062 \text{ g}}$$

- | | | |
|--------------------|---------------------------------|-------------------------------|
| (a) 7.5 kg = __ g | (b) 250 cm = __ m | (c) 350 mm = __ m |
| (d) 3.65 km = __ m | (e) 35 000 μm = __ m | (f) 0.6785 ML = __ L |
| (g) 250 mg = __ g | (h) 0.8 cm = __ m | (i) 6.25 mm = __ m |
| (j) 0.23 GL = __ L | (k) 0.019 MW = __ W | (l) 45.4 μg = __ g |

3. A power station was producing 0.895 Gigawatts (GW) of electricity. How many watts (W) is this?

4. In an electric circuit there are 7.5 milliamps (mA) of electricity.
How many amps (A) is this?



5. A chemist mixed 65 mL of hydrochloric acid with 25 mL of water.
Find the volume of mixed solution in

- (a) millilitres (mL)
- (b) litres (L)

6. A dam on a farm could hold 3 megalitres (ML) of water.
How many litres of water does the dam hold?

7. A sample of salty water contained 0.058 milligrams (mg) of salt.
How many grams of salt were in the water?

8. A layer of oil was measured as being 0.35 mm thick.
Convert this measurement to:

- (a) metres
- (b) micrometres

Averages

EXERCISE 7K

To find the average of a group of numbers add them all and divide by how many numbers there are.

Example

Find the average height of five trees if their heights are 3.3 m, 3.6 m, 4.2 m, 3.1 m and 3.8 m.

$$\begin{aligned}
 \text{Average height} &= \frac{\text{all heights added}}{\text{number of trees}} \\
 &= \frac{3.3 + 3.6 + 4.2 + 3.1 + 3.8}{5} \\
 &= \frac{18}{5} \\
 &= 3.6 \text{ m}
 \end{aligned}$$

1. A high-jumper cleared the following heights at a school athletics meeting: 1.72 m, 1.78 m, 1.69 m, 1.77 m
What was the average of the four jumps?
2. Lena grew 6 pumpkins.
They weighed 20.6 kg, 24.8 kg, 23.5 kg, 28.4 kg, 30.7 kg and 25.6 kg.
What is the average weight of her pumpkins?
3. Olga plays basketball. In 10 games she scored the following points:
12, 8, 15, 21, 11, 6, 13, 15, 10, 13.
What is Olga's average score per game?
4. In a 4×100 m freestyle relay race, the times swum by each swimmer were 56.9 sec, 57.2 sec, 56.6 sec and 55.6 sec.
What was the average time taken by the swimmers?
5. In 9 innings of cricket Cain has scored a total of 441 runs.
 - (a) What is his average score per innings?
 - (b) If Cain wants to have an average of 50 after the next innings how many runs must he score?



Calculator Problems

EXERCISE 7L

Use a calculator to solve the following problems.

1. (a) $(9.3 + 4.7) \times 2.8$ (b) $(20.18 + 12.77 - 5.11) \div 3.2$
 (c) $(6.3 + 12.9) \times (28.7 - 5.9)$ (d) $(487.388 - 56.21) \div (45.09 + 20.24)$
 (e)
$$\frac{(4.3 \times 8.7) + (1.6 \times 5.9) - (4.0 \times 8.29)}{(8.9 + 2.7 - 11.4)}$$

 (f)
$$\frac{(6.8 + 2.3)}{(8.7 - 6.1)} + \frac{(9.5 - 5.6)}{(7.2 - 5.7)} + \frac{(4.7 + 9.9)}{(8.3 - 7.8)}$$
2. A load of 250 bricks weighed 587.5 kg. Find the weight of each brick.
3. 45 planks of timber were loaded onto a truck. Each plank weighed 8.4 kg. Find the total weight of the timber loaded onto the truck.
4. Taylor had to walk up 36 steps to get from the ground to his seat in a football stadium. Each step was 0.28 m high. How high was his seat above the ground?
5. Rhiannon ran eight laps of an oval and her brother recorded the time for each lap. The eight times are listed below.
 54.7 sec, 52.1 sec, 58.0 sec, 56.4 sec, 48.3 sec, 50.9 sec, 59.3 sec, 50.7 sec
 What is the average time for each lap?
6. Six friends are organising a party. They plan to buy the list of food and drinks shown below. The prices are shown. The friends are going to share the total cost of the party equally.
 Calculate how much each will need to pay.

12 bottles of soft drink each costing \$2.66
 5 cartons of fruit juice each costing \$3.76
 8 packets of potato chips each costing \$2.48
 6 boxes of dry biscuits each costing \$1.89
 5 tubs of dip each costing \$3.86
 6 packets of sweet biscuits each costing \$3.25
 4 blocks of chocolate each costing \$4.55

Rounding and Estimating

Rounding decimal numbers can be used to estimate answers.

Examples

1. 14.6×6.2

- (a) For the above multiplication round the numbers to the nearest whole number to find an estimate of the answer.

Remember the rules for rounding from the section on **Numbers**.

If the digit in the first decimal place is less than 5 round down to the existing whole number.

If the digit in the first decimal place is equal to, or greater than, 5 round up to the next whole number.

- (b) Use a calculator to find the exact answer.

- (a) Round the decimal numbers

$$\begin{aligned} 15 \times 6 \\ = \mathbf{90} \end{aligned}$$

- (b) Using a calculator

$$\begin{aligned} 14.6 \times 6.2 \\ = \mathbf{90.52} \end{aligned}$$

2. Clara picked 8.2 kg of olives off her tree at home.

She sold them for \$13.85 per kg.

- (a) Round the numbers to the nearest whole number to find an estimate of what she was paid for her olives.

- (b) Use a calculator to find the exact amount she was paid.

- (c) Round this payment to the nearest 5 cents.

- (a) Round the numbers:

$$\begin{aligned} &8 \text{ kg of olives} \\ &\$14 \text{ per kg} \\ &\text{Estimate of what Clara} \\ &\text{was paid} = 8 \times 14 = \mathbf{\$112} \end{aligned}$$

- (b) Exact payment

$$\begin{aligned} &= 8.2 \times \$13.85 \\ &= \mathbf{\$113.57} \end{aligned}$$

- (c) Rounded to the nearest 5 cents

$$= \mathbf{\$113.60}$$

EXERCISE 7M

1. (i) Round the numbers in the following questions to the nearest whole number then use these to find an estimate of the answer.

(ii) Use a calculator to find the exact answer.

(a) 6.8×9.2

(b) $48.36 \div 6.2$

(c) 19.7×6.1

(d) $71.76 \div 7.8$

(e) $(5.9 + 2.3) \times 7.3$

(f) $79.79 \div (15.7 - 7.8)$

(g) $(15.8 - 6.9) \times (23.7 - 17.3)$

(h) $(6.7 + 8.4 + 9.1) \times (6.2 + 3.6)$

2. Jeremy bought a box of tomatoes that weighed 8.8 kg.

The cost of tomatoes was \$3.15 per kg.

- (a) Round these figures to the nearest whole numbers to find an estimate of the cost of the tomatoes Jeremy bought.

- (b) Use a calculator to find the exact cost of the tomatoes.

3. Alisha picked cherries during the school holidays.

She was paid \$2.85 for every kilogram of cherries she picked.

In one morning she picked bags with the following weights of cherries.



- (a) Round all these figures to the nearest whole numbers to find an estimate of what Alisha will be paid.

- (b) Use a calculator to find her exact pay.

- (c) Round her exact pay to the nearest 10 cents.

4. The area of a rectangle can be found by multiplying the length by the width.

For the rectangle shown here:

- (a) Find an estimate of the area by rounding the length and width to the nearest metre first.

- (b) Use a calculator to find the exact area.



4.9 m

12.1 m

PROBLEM SOLVING

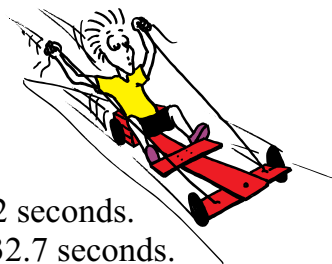
1. Find the following numbers.
 - (a) When this number is multiplied by 10 the result is 5 less than 40.
 - (b) This number is the same amount above 5 as it is below 16.
 - (c) When 2 is added to this number and the result multiplied by 10 the result is mid way between 90 and 95.

2. There are about 1.6 km in a mile (an old unit of measuring distance).
 - (a) How many kilometres are in:
 - (i) 2 miles (ii) 5 miles (iii) 10 miles (iv) 250 miles
 - (b) How many miles are in:
 - (i) 3.2 km (ii) 16 km (iii) 32 km (iv) 480 km

3. The water level of a river was indicated on a marker that showed zero when the river started to flood. In one night the river level went from -2.6 m to 1.5 m.
By how much did the river rise?

4. (a) Yossarian competed in a go-cart race that went for 3 laps.
He took 1 minute 28.6 seconds for the first lap and 1 minute 34.8 seconds for the second lap.
Yossarian took 4 minutes 29.9 seconds for the complete race.
What was his time for the third lap?
 (b) Frieda competed in the same go-cart race.
Her time for the first lap was 1 minute 29.2 seconds.
Her time for the second lap was 1 minute 32.7 seconds.
Frieda averaged 1 minute 31.2 seconds per lap for the race.
What was her time for the third lap?

5. Zena measured her school bag with its contents to weigh 6.84 kg.
A list of most of Zena's schoolbooks and their weights is given below.
Find out which items Zena had in her bag.

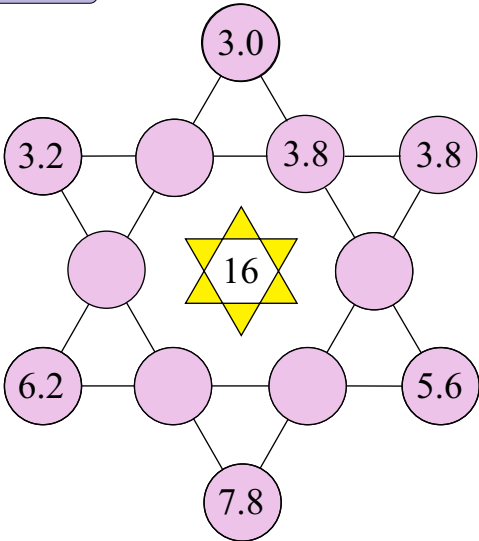


Empty school bag - 1.84 kg
 Mathematics Textbook - 1.08 kg
 3 novels - each 0.13 kg

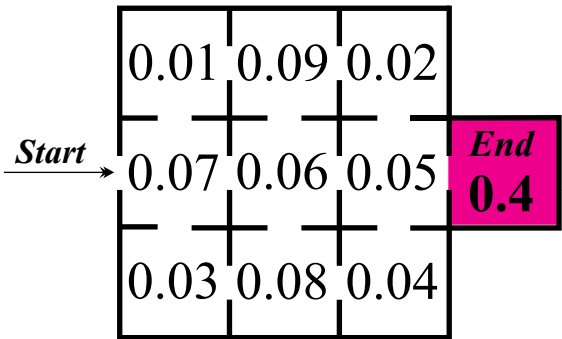
English Textbook - 0.89 kg
 Science Textbook - 1.21 kg
 3 folders - 1.32 kg each

PUZZLES

1. Copy this diagram into your books.
Fill in the circles so that each line adds to the number in the centre.

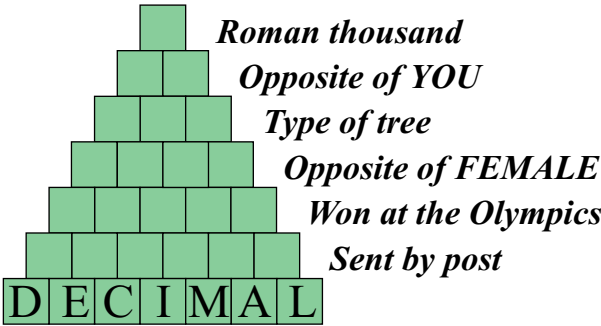


2.



Copy this maze into your book.
Pass through the rooms from the start to the end adding the numbers as you go.
Find a path that has the total shown at the end.

3. Complete the diagram below by finding the word on each line.
Each word can be found by adding one letter to the line above it and rearranging the letters.
A clue for each word is given.



CHAPTER REVIEW

1. Find the answers to the following additions.

- | | | |
|-------------------|------------------|---------------------|
| (a) $2.3 + 4.5$ | (b) $12.5 + 7.1$ | (c) $6.7 + 8.5$ |
| (d) $23.1 + 80.9$ | (e) $6.3 + 2.56$ | (f) $18.39 + 9.704$ |

2. Find the answers to the following subtractions.

- | | | |
|-------------------|---------------------|-------------------|
| (a) $8.9 - 6.2$ | (b) $18.58 - 6.26$ | (c) $7.39 - 2.62$ |
| (d) $18.35 - 9.4$ | (e) $256.8 - 29.83$ | (f) $28 - 9.28$ |

3. Find the answers to the following multiplications.

- | | | |
|----------------------|-----------------------|------------------------|
| (a) 6.2×3 | (b) 18.5×7 | (c) 26.4×37 |
| (d) 8.2×3.1 | (e) 17.4×0.6 | (f) 6.05×2.17 |

4. Find the answers to the following divisions.

- | | | |
|-------------------|-------------------|-------------------|
| (a) $13.5 \div 5$ | (b) $60.3 \div 9$ | (c) $2.16 \div 8$ |
| (d) $13 \div 4$ | (e) $7.2 \div 5$ | (f) $51.1 \div 8$ |

5. Find the answers to the following divisions.

- | | | |
|--------------------|-----------------------|-----------------------|
| (a) $4.5 \div 0.5$ | (b) $0.208 \div 0.02$ | (c) $12.86 \div 0.04$ |
|--------------------|-----------------------|-----------------------|

6. Find the answers to the following problems.

- | | | |
|----------------------------|----------------------|--------------------|
| (a) 8.65×100 | (b) 0.37×10 | (c) $68.3 \div 10$ |
| (d) $0.056 \times 10\ 000$ | (e) $89.4 \div 1000$ | (f) $0.08 \div 10$ |

7. Complete the following conversions.

- | | | |
|-------------------------------------|--------------------------------------|--------------------------------------|
| (a) $7\text{ km} = _\text{ m}$ | (b) $65\text{ mg} = _\text{ g}$ | (c) $8300\text{ mm} = _\text{ m}$ |
| (d) $683.2\text{ cm} = _\text{ m}$ | (e) $0.0056\text{ ML} = _\text{ L}$ | (f) $0.0578\text{ kg} = _\text{ g}$ |

8. If a cherry weighs 0.09 kg how many cherries would be in 1.8 kg?

9. Indira ran 3.26 km a day. How far would she run in a week?

10. The heights of three brothers are 1.72 m, 1.85 m and 1.68 m.
What is the average height of the brothers?

11. Find an estimate for the cost of buying 4.8 kg of oil that costs \$9.15 per kg by rounding the numbers to the nearest whole number.

Percentages

Percentages

Percentages are often used as a convenient way of representing a fraction or proportion of an object. The word **percent** originates from the Latin term **per centum** which means **per hundred**.
The symbol for percentage is %.

Example 65% means **65 per 100** or **65 out of 100**

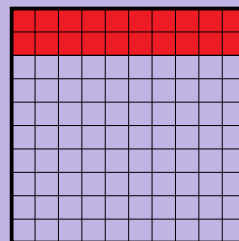
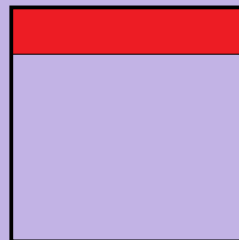
Examples

1. Find the percentage of this shape that is shaded.

This can be found by dividing the shape into 100 equal parts. Then find out how many of these parts are shaded.

It can be seen that 20 of the 100 equal parts are shaded. Therefore **20 percent** of the shape is shaded.

This can be written as **20%**.



2. What percentage of the above shape is **not** shaded?

The total number of equal parts is 100 so the number that is not shaded is $100 - 20 = 80$.

Therefore the percentage of the shape that is not shaded is **80%**.

Examples continued

3. George had 100 attempts at goal at basketball training and he scored 74 goals.

- (a) What percentage of his attempts scored a goal?
 (b) What percentage of his attempts missed?

Answers:

- (a) He scored goals with 74 out of 100 attempts = **74%**
 (b) He missed 26 of the 100 attempts = **26%**

4. Marvey made a tropical drink with 60% pineapple juice, 30% mango juice and the remainder coconut milk.
 What percentage of the drink was coconut milk?

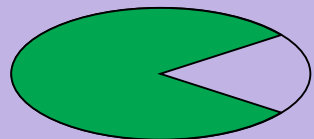
Answer: The total percentage must be 100%.
 90% of the juice is pineapple and mango juice so the remainder was $100 - 90 = 10$.
10% of the drink was coconut milk.

5. Brad and Hayley get paid for a job. Brad did 20% more work so will get paid 20% more than Hayley.
 What percentage of the pay should each receive?

Answer: The total of the percentages will add to 100%.
 The answer can be found by trial and error (guessing).
Brad - 60% of the pay
Hayley - 40% of the pay

6. Which of the following percentages is the best estimate of the shaded area of this shape?

A 20% B 50% C 80% D 100%

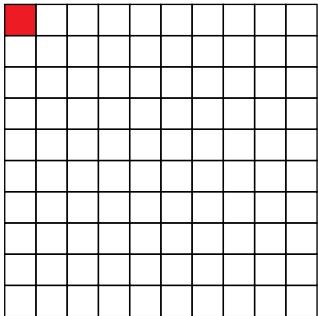


Answer: C

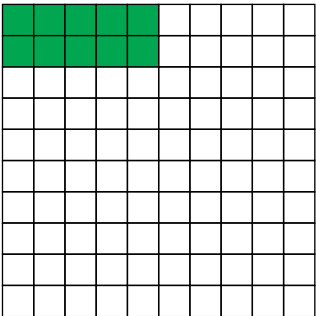
EXERCISE 8A

1. Each of the diagrams below consists of 100 small squares.
For each of the diagrams state:
(i) the percentage that is shaded.
(ii) the percentage that is not shaded.

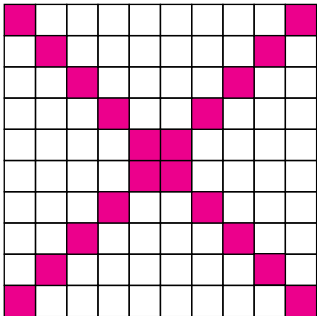
(a)



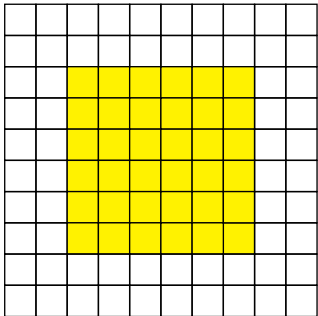
(b)



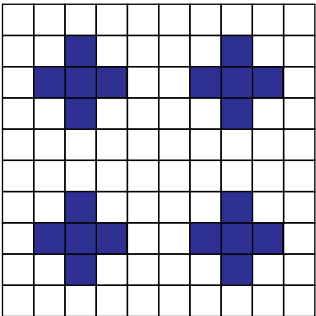
(c)



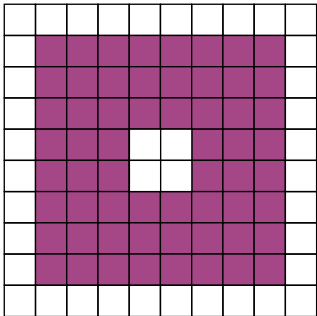
(d)



(e)



(f)

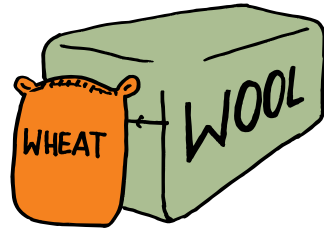


2. In a fly fishing club, 63% of the members were more than 30 years old.
What percentage of the members were 30 years old or younger?



3. A fruit juice drink contained 15% orange juice.
What percentage of the drink was not orange juice?
4. Maxine, Jean-Rene and Bobbie ate a cake. Maxine ate 25% and Jean-Rene ate 35% of the cake.
What percentage of the cake did Bobbie eat?
5. Milo, Helena and Cherie shared in the profits of their business.
Milo got 10% more than Helena. Cherie got 30% more than Helena.
What percentage of the profits did each person receive?
6. Washington achieved 82 out of 100 for a Mathematics test.
What percentage is this?
7. Russell and Iain were running for the position of president of the local rugby club. Russell received 52% of the votes.
(a) What percentage of the votes did Iain receive?
(b) Who won the election?

8. Youssef is a farmer. He produces wool, wheat and oats.
In a particular year, 63.5% of his income came from his wool sales and 22.5% of his income came from wheat.
What percentage of his income came from oats?



9. Approximately what percentage of this flag is blue?

A 10 **B** 30 **C** 50 **D** 70



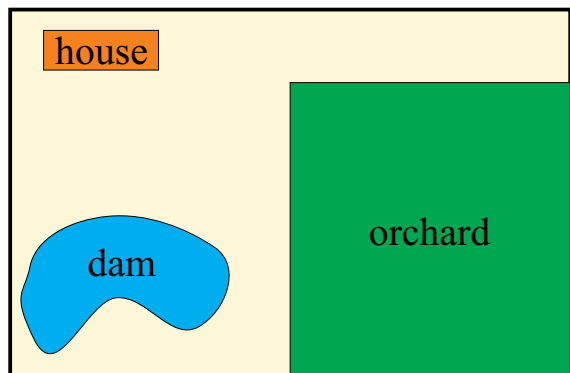
10. The plan of a farm is shown here.

- (a) Approximately what percentage of the farm is the orchard?

A 10 **B** 40 **C** 60 **D** 90

- (b) Approximately what percentage of the farm is the dam?

A 10 **B** 40 **C** 60 **D** 90



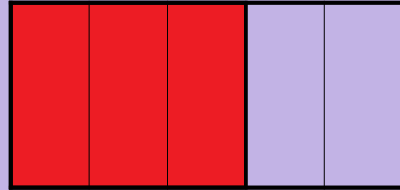
11. Georgia made 10 litres of fruit punch for her party.
 She mixed 10% apple juice, 15% grape juice and 25% orange juice.
 The remainder of the drink was soda water.
- What percentage of the drink was soda water?
 - If she wanted to make 20 litres of drink what percentage of the drink would be soda water?

Converting Fractions to Percentages

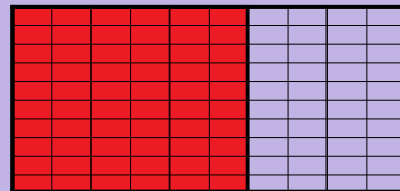
Any fraction can be converted to a percentage.

Example 1

It can be seen that $\frac{3}{5}$ of this shape is shaded.



To find what **percentage** of the shape is shaded divide the shape into 100 equal parts.
 It can be seen that 60% is shaded.



So $\frac{3}{5} = 60\%$

An easier way of converting a fraction into a percentage is to multiply the fraction by 100.

Example 1 $\frac{3}{5}$

$$\begin{aligned}
 & \frac{3}{5} \times \frac{100}{1} \\
 &= \frac{3}{\cancel{5}^1} \times \frac{100^2}{1} \\
 &= \frac{60}{1} \\
 &= 60\%
 \end{aligned}$$

Example 2 Convert $\frac{11}{20}$ to a percentage.

To convert to a percentage multiply by 100.

$$\begin{aligned} & \frac{11}{20} \times \frac{100}{1} \\ &= \frac{11}{20} \times \frac{100}{1} \\ &= \frac{55}{1} \\ &= \mathbf{55\%} \end{aligned}$$

Example 3 Convert $\frac{24}{40}$ to a percentage.

To convert to a percentage multiply by 100.

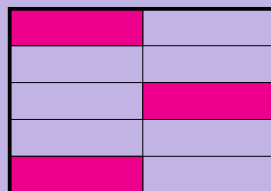
$$\begin{aligned} & \frac{24}{40} \times \frac{100}{1} \\ &= \frac{24}{40} \times \frac{100}{1} \quad \text{Simplify the fraction} \\ &= \frac{3}{5} \times \frac{100}{1} \\ &= \frac{60}{1} \\ &= \mathbf{60\%} \end{aligned}$$

Example 4 What percentage of this shape is shaded?

There is a total of 10 equal rectangles.
3 of the 10 are shaded.

So $\frac{3}{10}$ of the shape is shaded.

= **30%**



Example 5 There are 25 students in a swimming class and 16 of them are girls.

- (a) What percentage of the class is girls?
- (b) What percentage of the class is boys?

(a) 16 out of 25 can be written as a fraction: $\frac{16}{25}$

Convert this to a percentage: **64%**

(b) The percentage of boys is $100 - 64 = \mathbf{36\%}$

Example 6 An organic gardener made a soil mix using 18 barrow loads of manure and 12 barrow loads of compost and 10 barrow loads of mushroom mulch .

- (a) What percentage of the soil was manure?
- (b) What percentage of the soil was compost?
- (c) What percentage of the soil is mushroom mulch?

(a) There was a total of 40 barrow loads used.
18 of these were manure.

This can be written as a fraction: $\frac{18}{40}$

Convert this to a percentage: **45%**

(b) The fraction of compost is $\frac{12}{40}$

Convert this to a percentage: **30%**

(c) The fraction of mushroom mulch is $\frac{10}{40}$

Convert this to a percentage: **25%**

Example 7 The heights (in cm) of the players in a squad of basketball players is shown below.

178 185 197 201 174 183 192 191 176 182 189 182

What percentage of the players is taller than 190 cm?

There are 12 players and 3 are taller than 190 cm.

178 185 197 201 174 183 192 181 176 182 189 182

3 out of 12 can be written as a fraction $\frac{3}{12}$

Convert this to a percentage: **25%**

EXERCISE 8B

1. Convert the following fractions to percentages.

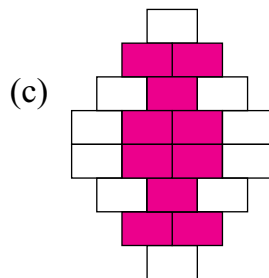
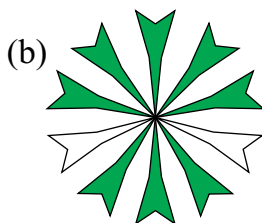
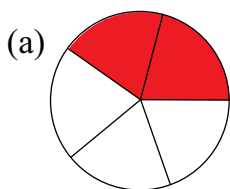
- (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{9}{10}$ (d) $\frac{1}{2}$ (e) $\frac{1}{5}$
 (f) $\frac{3}{4}$ (g) $\frac{7}{20}$ (h) $\frac{13}{20}$ (i) $\frac{12}{25}$ (j) $\frac{21}{50}$

2. Celine has used $\frac{1}{4}$ of the hard drive on her computer.

- (a) What percentage has she used?
 (b) What percentage has she not used?

3. For each of the following diagrams state:

- (i) the percentage that is shaded.
 (ii) the percentage that is not shaded.



4. Mia achieved the following marks in mathematics tests.

Fractions $\frac{17}{20}$

Decimals $\frac{48}{60}$

Numbers $\frac{36}{40}$

Graphs $\frac{22}{25}$

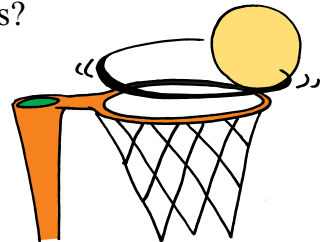
- (a) Change each of the marks to a percentage.
 (b) In which test did she achieve the best mark?

5. (a) Gabrielle, at netball training, had 45 shots at goal and scored 36 goals.
 What percentage of her goal attempts were goals?

(b) Kelsey also was practising her goal shooting
 at netball training.

She scored 28 goals but missed 12.

What percentage of her goal attempts
 were goals?



6. A town council were deciding whether to build a tennis court or a bike track.
 They surveyed 450 people. 180 voted for the tennis court and 270 voted for
 the bike track.

- (a) What percentage of the people surveyed voted for the tennis court?
 (b) What percentage of the people surveyed voted for the bike track?

7. A school council decided to ask all the students in the school if they
 wanted a wall in the undercover lunch area to be painted blue, green
 or orange.

The votes are shown here.

- (a) How many students
 were at the school?


 225 votes


 180 votes


 495 votes

- (b) What percentage of the student population voted for each colour?

8. The ages of the students in an apprenticeship's class for electricians is
 shown below.

17 19 23 18 22 19 21 18 17 21 18 19 18 19 17 19

- (a) How many students were in the class?
 (b) What percentage of the students were younger than 20?
 (c) If there were four girls in the class, what percentage of the class was girls?

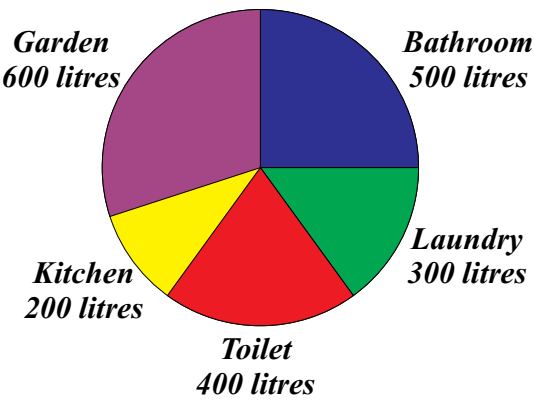
9. The nationalities of the people on a tour to Central Australia are shown in the table here.

- (a) How many people were on the tour?
- (b) What percentage of the people were of each nationality?
- (c) What percentage of the people were European?
- (d) What percentage of the people were Asian?

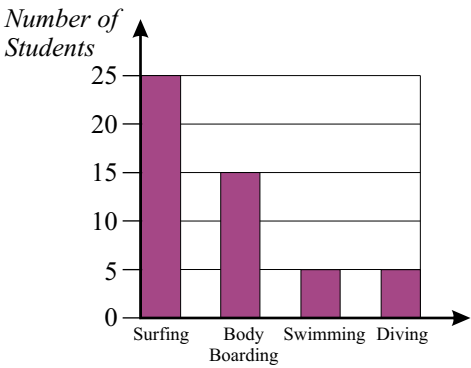
Nationality	Number of People
American	9
German	7
French	4
English	6
Japanese	8
Chinese	3
Australian	13

10. Jackson wanted to calculate the percentage of water used in each area of his house. Using different methods he estimated the amount of water used in each area of the house and displayed it in the pie chart shown.

What percentage of the water was used in each area of the house?



11. A number of students were asked to state their favourite water sport. The results are shown on this graph. What percentage of the students chose each of the sports?



Converting Percentages to Fractions

EXERCISE 8C

1. Convert the following percentages to fractions.

Example 23% $23 \div 100$
 $= \frac{23}{100}$

To convert a percentage to a fraction - divide by 100.

- (a) 1% (b) 11% (c) 19% (d) 73% (e) 99%

2. Convert the following percentages to fractions.

Write answers in
their simplest form.

Example 45% $45 \div 100$
 $= \frac{45}{100}$ ⁹
 $= \frac{9}{20}$

- (a) 5% (b) 12% (c) 18% (d) 25% (e) 35%
(f) 50% (g) 60% (h) 76% (i) 95% (j) 100%

Write the fractions in the following questions in their simplest form.

3. The air we breath is made up of approximately 78% nitrogen and 21% oxygen.

- What fraction of the air is nitrogen?
- What fraction of the air is oxygen?
- What fraction of the air is made up of other gases?

4. (a) Approximately 85% of Australia's population live within 50 km of the coast. What fraction of Australia's population does this represent?

(b) Approximately 22% of Australia's population were born overseas. What fraction of Australia's population does this represent?

5. The approximate percentage of Australia's population that live in each state and territory are shown in the table below.

State/Territory	ACT	NSW	NT	Qld	SA	Tas	Vic	WA
Approx. % of Pop.	2%	32%	1%	20%	8%	2%	25%	10%

What fraction of Australia's population live in each state or territory?

Converting Decimals to Percentages

EXERCISE 8D

Convert the following decimals to percentages.

*To convert a decimal to a percentage - multiply by 100.
Move the decimal point 2 places to the right.*

Examples

1. 0.21

$$= 0.21 \times 100$$

$$= 0.21 \rightarrow$$

$$= 21\%$$

2. 0.9

$$= 0.9 \times 100$$

$$= 0.9 \rightarrow$$

$$= 90\%$$

3. 0.0753

$$= 0.0753 \times 100$$

$$= 0.0753 \rightarrow$$

$$= 7.53\%$$

1. 0.33

2. 0.19

3. 0.87

4. 0.06

5. 0.4

6. 0.295

7. 0.056

8. 0.008

9. 2

10. 0.1057

Converting Percentages to Decimals

EXERCISE 8E

Convert the following percentages to decimals.

*To convert a percentage to a decimal - divide by 100.
Move the decimal point 2 places to the left.*

Examples

1. 37%

$$= 37 \div 100$$

$$= 0.37$$

$$= 0.37$$

2. 6%

$$= 6 \div 100$$

$$= 0.06$$

$$= 0.06$$

3. 8.92%

$$= 8.92 \div 100$$

$$= 0.0892$$

$$= 0.0892$$

1. 59%

2. 24%

3. 76%

4. 30%

5. 8%

6. 10%

7. 100%

8. 65.4%

9. 5.8%

10. 0.25%

Conversions

EXERCISE 8F

1. Copy and complete this table.

Write all fractions in their simplest form.

One line has been completed as an example.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
10%		
	$\frac{1}{4}$	
		0.2
	$\frac{9}{10}$	
66%		
	$\frac{9}{50}$	
		0.32
	$\frac{17}{20}$	
9%		
	$\frac{3}{100}$	
		0.04

2. From the quantities in the box below find one that is equivalent to each of the following amounts.

- (a) 90% (b) $\frac{4}{5}$ (c) $\frac{1}{100}$ (d) 0.1 (e) $\frac{9}{100}$
 (f) $\frac{2}{25}$ (g) 60% (h) 0.06 (i) 1 (j) 0.001

80% 10% 0.08 0.9 $\frac{3}{5}$ 0.09
 6% 0.01 100% 0.1%

3. In a class of 25 students, 20 kept pets.

- (a) What percentage of the class kept pets?
 (b) What fraction of the class did not keep pets?
 Of the students who kept pets, 12 had dogs.
 (c) What percentage of those students who kept pets had dogs?



Common Percentages

EXERCISE 8G

Copy and complete this table. You should remember all these!

Percentage	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%
Fraction							
Decimal							

Percentages of Quantities

EXERCISE 8H

1. Find the following quantities.

Try to solve these problems using mental arithmetic.

Example 50% of \$20
 $= \frac{1}{2}$ of \$20
 $= \$10$

- | | | |
|-------------------------------|-------------------------------|---------------------------------|
| (a) 50% of 8 m | (b) 25% of \$40 | (c) 10% of \$50 |
| (d) $33\frac{1}{3}\%$ of 9 kg | (e) $66\frac{2}{3}\%$ of 9 kg | (f) 75% of 12 m |
| (g) 50% of \$120 | (h) 25% of \$400 | (i) $33\frac{1}{3}\%$ of 30 km |
| (j) 10% of 250 kg | (k) 50% of \$8000 | (l) 20% of 500 kg |
| (m) 50% of \$5 | (n) 10% of \$3 | (o) $66\frac{2}{3}\%$ of \$3.90 |

2. Find the following quantities.

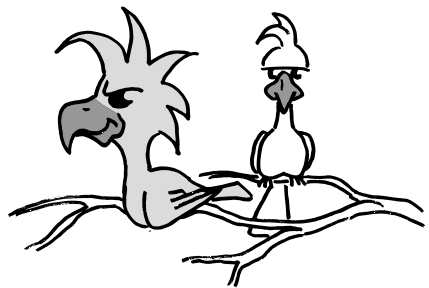
Example 20% of \$480 *Remember 'of' is the same as $\times$*
 $= \frac{20}{100} \times \frac{480}{1}$
 $= \$96$

- | | | |
|-------------------|-------------------|-------------------|
| (a) 60% of 200 kg | (b) 15% of 600 m | (c) 80% of \$120 |
| (d) 30% of \$700 | (e) 45% of 3000 m | (f) 55% of 80 m |
| (g) 85% of \$60 | (h) 5% of \$420 | (i) 8% of 350 km |
| (j) 16% of 250 kg | (k) 35% of \$90 | (l) 24% of 560 kg |
| (m) 2% of \$5 | (n) 19% of \$12 | (o) 12% of \$358 |

3. Approximately 10% of the population is left-handed. How many left-handed students would there be expected to be in:
- (a) a class of 20 students?
 - (b) a school of 800 students?
4. Con the fruiterer found that 5% of the mangoes he bought were bruised. If he bought 80 mangoes how many could he expect to be bruised?
5. Rose is a keen tennis player. She has worked out that on average she gets 60% of her first serves in play. If Rose has 30 first serves in a match, how many of these could she expect to hit in play?

6. The population of cockatoos in a national park was found to be increasing by 10% each year.
3000 cockatoos were counted at the start of a year.

- (a) How many would there be at the end of the year?
- (b) How many would there be at the end of the *next* year?



7. Krisha earns \$45 000 a year. She is given a 5% pay rise.
- (a) What is the size of her pay rise?
 - (b) What will be her new yearly salary?
8. Henry makes and sells pottery. He makes 40% profit on what he sells. If he sells 50 vases for \$80 each, how much profit will he make?
9. A baby weighed 4 kg at birth. If its weight increased by 30% in the first year how much would it weigh?
10. A farmer decided to plant 15% of his farm with canola, 45% with wheat and the remainder with oats. His farm was 40 000 hectares.
- (a) What percentage of his farm will he plant with oats?
 - (b) How many hectares will he plant with each crop?

Discounts

EXERCISE 8I

Example A 25% discount is offered on a \$40 shirt.
 What will you pay for the shirt?
 25% of \$40 is \$10
 You will pay \$40 - \$10 = **\$30**

Find the price that you will pay for the following items after they have been discounted.

Item and Price	Discount
1. Fishing Rod - \$80	20%
2. Book - \$30	10%
3. Electric Guitar - \$600	25%
4. Car - \$25 000	5%
5. Computer - \$350	30%
6. Dress - \$220	40%

Calculator Problems

EXERCISE 8J

1. Use a calculator to find the following quantities.

Example 57% of \$620

$$= \frac{57}{100} \times \$620$$

$$= 57 \div 100 \times \$620$$

$$= \mathbf{\$353.40}$$

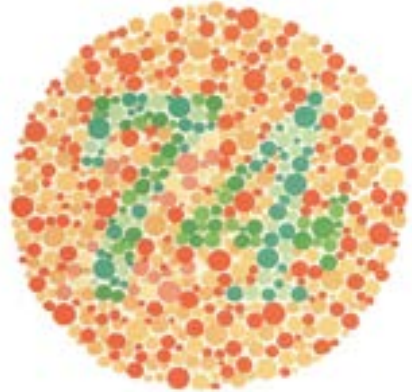
- (a) 62% of 450 kg
 (d) 47% of \$2400
 (g) 6.5% of \$750

- (b) 19% of 850 m
 (e) 82% of 3675 m
 (h) 8.5% of \$42000

- (c) 23% of \$2580
 (f) 11% of 3.5 m
 (i) 12.75% of \$84700

Use a calculator to help solve the following problems.

2. Novak put \$2500 into a bank account that earned 4.5% interest per year. How much interest would he earn in one year?
3. A scientific study showed that there were 350 rare marsupials on an island at the beginning of a particular year. It was expected that the population would increase by 6% that year. How many of the marsupials would be expected to be on the island at the end of the year?
4. Colour blindness affects about 8% of males and 0.5% of females. In the most common type of colour blindness people cannot distinguish between red and green. One test for this type of colour blindness is shown here. If you can see the number 74 in the pattern of dots then you have not got this type of colour blindness.
In a school of 1200 students:
(a) how many boys would be expected to be colour blind?
(b) how many girls would be expected to be colour blind?
5. Sea water contains about 3.5% salt. How many kilograms of salt would be found in 50 kg of sea water?
6. In a particular set of data from the Australian Bureau of Statistics it was reported that there were 12 562 500 people in the Australian workforce. It was also reported that of these people 703 500 were unemployed.
(a) What percentage of the workforce was unemployed?
(b) The Australian Government wanted the unemployment rate to be 5%. How many people would be unemployed for the unemployment rate to be 5%?
(c) How many jobs would need to be created?



PROBLEM SOLVING

1. In a netball game Sigrid, Stella and Karen scored all the goals for their team. Sigrid scored 50% of the team's score, Stella scored one fifth of the team's score and Karen scored 12 goals.
What was the team's score?

2. 80 students were asked if they had been to a concert or to the theatre.
20% of the students had not been to either.
70% of the students had been to a concert.
40% of the students had been to the theatre.
How many students had been to both a concert and the theatre?

3. Kelvin bought a skateboard that had been discounted by 20%.
He paid \$48.
What was the full price of the skateboard?

4. What percentage is ten percent of ten percent?

5. Giovanna earned some money by cleaning neighbours' houses.
She put 50% of her earnings in the bank and spent 20% of the remainder on a pair of sunglasses that cost \$15.
How much did she earn?

6. $\alpha + \beta + \sigma = 350$ α is 50% of β β is 50% of σ
Find α , β and σ .

PUZZLE

Abel thought up a code that he could use in his diary so no-one could read what he was writing.

In his code his name, **ABEL**, could be **YABBY EELS**.

See if you can work out his code and use it to decode the following words. Hint - the words can be found in this chapter.

**SPIES ROCKET NOT TANGLED
SHOUT NO DART ENDS
ADD IT SO COOK UP NUTS**

Use Abel's code to write your name.

CHAPTER REVIEW

1. (a) Barbara achieved 68 out of 100 for a science test.
What percentage is this?
(b) Quinton achieved 40 out of 50 for a mathematics test.
What percentage is this?
2. Denzel made some two stroke fuel by mixing petrol and oil.
95% of the fuel is petrol.
(a) What percentage of the fuel is oil?
(b) What fraction of the fuel is oil?
3. Teresa is a painter. She is going to paint a room with a paint mix that contains red, yellow and white.
10% of the mix is red and 20% is yellow.
(a) What percentage of the paint mix is white?
(b) If she wants to make 20 litres of paint how much of each colour should she use?

4. Copy and complete this table.

Percentage	10%				1%		
Fraction		$\frac{1}{4}$		$\frac{1}{5}$		$\frac{1}{3}$	
Decimal			0.5				0.7

5. Find the following quantities using mental arithmetic.

(a) 10% of \$80	(b) 25% of 200 m	(c) $33\frac{1}{3}\%$ of 90 kg
(d) 50% of \$180	(e) $66\frac{2}{3}\%$ of 3 m	(f) 75% of 16 kg
6. Find the following quantities.

(a) 20% of \$300	(b) 70% of 580 m	(c) 85% of 300 kg
(d) 7% of \$8	(e) 23% of 300 m	(f) 55% of \$75
7. The frog population of Shady Creek was decreasing at the rate of 10% each year. The population was estimated to be 5000.
What would be the expected population a year later?
8. Find the price paid for the following items.

(a) \$150 wetsuit with a 20% discount.
(b) \$80 jumper with a 10% discount.

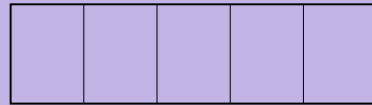
Ratio

Ratio

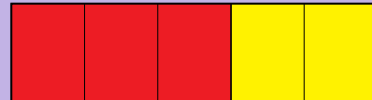
If an object is divided into a number of equal parts and those parts are grouped, the comparison of the size of those groups is called the *ratio* of those group sizes.

Example 1

This rectangle is divided into five equal parts.



Three of the parts are coloured red and two coloured yellow.



The ratio of those coloured red to those coloured yellow is 3 to 2.

This is written **3:2**

Example 2

\$70 is divided into two portions in the ratio of 4:3.
Find the size of each portion.

Step 1 The ratio of 4:3 means there are 7 equal parts ($4 + 3$).

Step 2 If \$70 is divided into 7 equal parts there are \$10 in each part.

Step 3 The first portion will have $4 \times \$10 = \$40$.

The second portion will have $3 \times \$10 = \$30$.

Answer: \$40, \$30

Example 3

The ratio of teachers to students on a school committee was to be 1:6. How many teachers and students should be on a committee of 14 people?

Note: the order the groups are given in the information is the same as the order of the ratio.

Therefore teachers to students will be 1:6.

(1 part teacher to 6 parts students)

Step 1 If 14 people are divided into 7 equal parts
there are 2 people in each part.

Step 2 1 part is teachers = 2 teachers
6 parts are students = 12 students

Answer: 2 teachers and 12 students

EXERCISE 9A

1. If the following quantities are divided into the given number of equal parts find the size of each part.

Example

\$24 is divided into 6 equal parts

Each part would be $\$24 \div 6 = \$4$

- (a) \$80 divided into 8 parts (b) 60 kg divided into 10 parts
(c) 48 m divided into 4 parts (d) \$20 divided into 5 parts
(e) \$300 divided into 3 parts (f) 72 litres divided into 9 parts
2. (a) \$36 was divided up between Iris and Lily so that Iris received 1 part and Lily received 2 parts (ratio of 1:2).
(i) What was the total number of parts the \$36 was divided into?
(ii) How much money was in each part?
(iii) How much money did each person receive?
(b) How much would Iris and Lily each receive if the \$36 was divided up in the following ratios.
(i) 2:1 (ii) 1:3 (iii) 3:1 (iv) 1:5 (v) 8:1 (vi) 7:2 (vii) 5:4

3. Copy and complete the following table.

The amount in the first column is divided into two amounts using the given ratio.

The first line has been completed as an example.

Amount to be Divided	Ratio	Total Number of Parts	Size of each Part	Size of First Amount	Size of Second Amount
\$24	3:1	4	\$6	\$18	\$6
\$30	1:4				
28 m	4:3				
35 kg	2:3				
100 m	7:3				
80 km	3:5				

4. Sachin and Amir are partners in a gardening business. They get paid \$90 for a job and divide it in the ratio of 4:5.
How much does each of the partners receive?
5. Wayne owns a horse and feeds it a mixture of 2 parts oats and 5 parts lucerne.
How much of each feed will he need to make up 35 boxes of feed mix?
6. Ernie and Carla buy 15 cubic metres of wood and share it in the ratio 2:3. How much should each person get?
7. As part of an inheritance, 40 hectares of land is to be divided between two brothers, Sebastian and Christian, in the ratio 5:3.
How many hectares will each brother receive?
8. Pauline feeds her cats a mix of dry food and fresh meat in the ratio of 3:2.
She feeds her cats 200 grams of food a day.
How much dry food and fresh meat will she need to feed her cats for a *week*?
9. A crowd of 10 000 people watched a rugby game between the Sharks and the Dolphins. The ratio of Sharks supporters to Dolphin supporters was 2:3.
How many of the crowd supported each team?

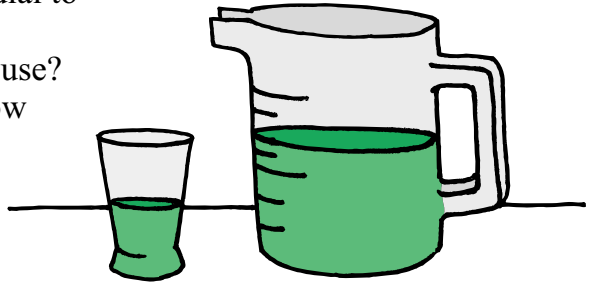


10. Vincent was a plasterer and made plaster paste using plaster powder and water in the ratio 4:1.

How many cups of each would he need to make 20 cups of plaster paste?

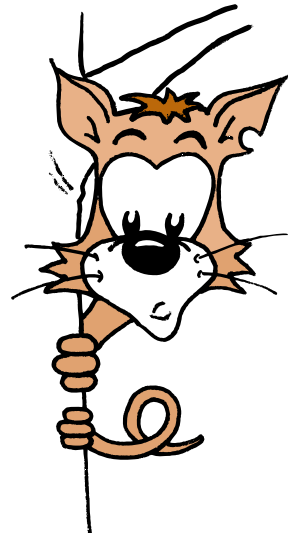
11. Venus wanted to make 30 litres of cordial drink using the ratio of 1 part cordial to 5 parts water.

- (a) How much cordial should she use?
(b) If she had 1 litre of cordial, how much drink could she make?



12. A prize was divided in the ratio of 2:3 between Petr and Amy.
If Petr received \$4000, how much did Amy receive?
13. A length of timber was to be cut into two pieces in the ratio 5:2.
If the timber was 1750 mm long how long were the two pieces?
14. The Ube Ute Car Sales sold 24 vehicles in January.
These sales consisted of sedans and station wagons.
They sold twice as many sedans as station wagons.
(a) What is the ratio of sedans to station wagons sold?
(b) How many sedans did they sell?
(c) How many station wagons did they sell?

15. Scientists found that the ratio of births to deaths in a colony of Pygmy Possums was 8:7.
- (a) Is the population of the colony increasing or decreasing?
(b) If 56 Pygmy Possums were born in one year, how many died?



Ratios are usually given in their simplest form.
This is done in a similar way to cancelling down fractions.

Example 1

Write the following ratios in their simplest form.

1. $8:6 = 4:3$ 2. $25:15 = 5:3$ 3. $10:30 = 1:3$

Write the following amounts as ratios in their simplest form.

4. $\$24 : \$60$ 5. $32 \text{ kg} : 28 \text{ kg}$ 6. $21 \text{ m} : 33 \text{ m}$
 $= 24:60$ $= 32:28$ $= 21:33$
 $= 2:5$ $= 8:7$ $= 7:11$

Example 2

8 cups of salt is dissolved in 20 cups of water.
What is the ratio of salt to water?

8 cups of salt:20 cups of water
 $= 8:20$
 $= 2:5$

EXERCISE 9B

- Write the following ratios in their simplest form.
 (a) $8:4$ (b) $6:24$ (c) $10:8$ (d) $12:18$ (e) $36:28$ (f) $54:72$
- Write the following amounts as ratios in their simplest form.
 (a) $36 \text{ m} : 9 \text{ m}$ (b) $\$8 : \$48$ (c) $72 \text{ kg} : 48 \text{ kg}$
 (d) $56 \text{ g} : 72 \text{ g}$ (e) $24 \text{ L} : 84 \text{ L}$ (f) $63 \text{ mm} : 49 \text{ mm}$
 (g) $180 \text{ kg} : 270 \text{ kg}$ (h) $\$2000 : \$1500$ (i) $6400 \text{ km} : 7200 \text{ km}$

Write the following ratios in their simplest form

- A painter mixes 85 mL of green paint and 25 mL of red paint.
What is the ratio of green paint to red paint?
- In a university 3600 students were from Australia and 800 were from overseas. What is the ratio of Australian students to those from overseas?

Example

Tom mixed 12 buckets of pellets to 8 buckets of wheat as a feed for his chickens.

- What is the ratio of pellets to wheat?
- If he has a feed bin that holds 30 buckets of feed, how many buckets of pellets and grain would he need to fill the bin with feed in this ratio?
- If he uses 6 buckets of pellets, how many buckets of wheat should he add to make feed with this ratio?
- If he uses 10 buckets of wheat, how many buckets of pellets should he add to make feed with this ratio?

Answers:

- (a) 12 buckets : 8 buckets
= 12:8
= **3:2**

- (b) The ratio of 3:2 means 5 equal parts
 - 3 parts pellets and 2 parts wheat.
 30 buckets divided into 5 equal parts means 6 buckets in each part.
 $3 \times 6 = \mathbf{18 \text{ buckets of pellets}}$
 $2 \times 6 = \mathbf{12 \text{ buckets of wheat}}$

- (c) Find what the ratio is multiplied by to get to the amount required.

Ratio: 3:2

3 parts pellets:2 parts wheat

Amount required: 6 buckets of pellets : ? buckets of wheat

The ratio has been multiplied by 2

$\times 2$
 $\swarrow$
 3 parts pellets : 2 parts wheat
 6 buckets of pellets : ? buckets of wheat

Therefore **4 buckets of wheat** are required.

Example continued:

(d) Use the same method used in part (c).

Find what the ratio is multiplied by to get to the amount required.

Ratio: 3:2

3 parts pellets:2 parts wheat

Amount required: ? buckets of pellets : 10 buckets of wheat

The ratio has been multiplied by 5

3 parts pellets : 2 parts wheat

$\downarrow \times 5$
 ? buckets of pellets : 10 buckets of wheat

Therefore **15 buckets of pellets** are required.

EXERCISE 9C

Write the ratios in the following questions in their simplest form

1. Max the baker made chicken and vegetable pies. He used a mix of 4 kg of chicken meat and 6 kg of vegetables.
 - (a) What is the ratio of chicken meat to vegetables?
 - (b) He bought 10 kg of chicken meat. How many kg of vegetables would he need?
 - (c) He was asked to cater for a function and would need 50 kg of chicken/vegetable mix to make all the pies. How many kg of each ingredient would he need?
2. A two stroke fuel required 10 mL of oil to be mixed with 240 mL of petrol.
 - (a) What is the ratio of oil to petrol?
 - (b) How many mL of petrol would be needed to mix with 25 mL of oil?
 - (c) How many mL of oil would be needed to add to 1200 mL of petrol?
 - (d) How many mL of oil and petrol would be needed to make 2000 mL of the two stroke fuel?

3. A bricklayer is mixing mortar using cement and sand.

He uses a mix of 2 parts cement to 8 parts sand.

- What is the ratio of cement to sand?
- If he puts 4 shovels full of cement in the mixer, how many shovels full of sand should he add?
- If he puts 12 shovels full of sand in the mixer, how many shovels full of cement should he add?
- If a wheelbarrow holds 25 shovels full, how many shovels full each of cement and sand should he put in the mixer?



4. Costa was mixing some liquid fertilizer for his garden.

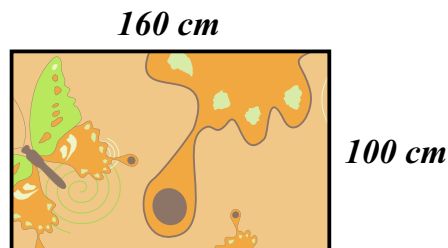
30 mL of the fertilizer needed to be mixed with 600 mL of water.

- What is the ratio of fertilizer to water?
- How much water would be needed to mix with 75 mL of fertilizer?
- How much fertilizer would be needed to be added to 2000 mL of water?
- A container held 10.5 L (10 500 mL). How much fertilizer and water would be needed to fill this container?

5. An artist's painting is shown here.

The length of the painting is 160 cm and the width is 100 cm.

The artist wanted to paint a series of paintings with the same ratio of length to width.



- What is the ratio of the length to the width?
- What would be the width of a painting that was 200 cm long?
- What would be the length of a painting that was 150 cm wide?

6. A flower grower sold in one season 5600 roses and 4000 carnations.

- What is the ratio of roses to carnations that he sold?
- He wanted to increase his sales in the next year to 12 000 flowers. How many of each flower type should he grow to keep this ratio?
- If he grew 7000 carnations, how many roses should he grow to keep this ratio?

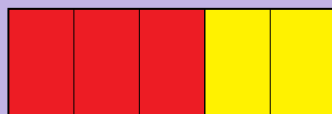
7. A petrol company was looking into mixing ethanol and petrol. They found that mixing 300 mL of ethanol with 750 mL of petrol gave best results.

- What was the ratio of ethanol to petrol?
- How much petrol should they add to 900 mL of ethanol?
- How much ethanol should they add to 6000 mL of petrol?
- How much of each fuel would they need to produce 9100 mL of the mix?

Ratios and Fractions

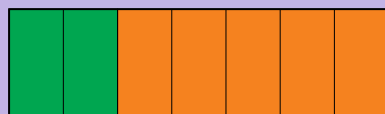
Example 1 $\frac{3}{5}$ of a rectangle is coloured red and $\frac{2}{5}$ is coloured yellow.
What is the ratio of red to yellow?

Answer: It can be easily seen using a diagram the ratio of red to yellow is **3:2**.



Example 2 A rectangle is coloured green and orange in the ratio of 2:5.
What fraction of the rectangle is each colour?

Answer: Using a diagram it can be seen that the rectangle will be divided into 7 sections and that:



$\frac{2}{7}$ is green and $\frac{5}{7}$ is orange

Example 3 In the town of Soggiton in the month of June there were 24 days when it rained and 6 days when it didn't rain.

- (a) Find the fraction of days in June that it rained. Write the fraction in its simplest form.
- (b) What is the ratio of rainy days to dry days?

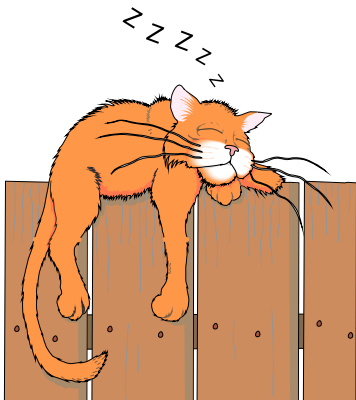
Answer: (a) 24 rainy days out of 30 as a fraction is:

in its simplest form $\frac{24}{30} = \frac{4}{5}$

(b) $\frac{4}{5}$ rainy : $\frac{1}{5}$ dry
= **4:1**

EXERCISE 9D

1. Georgie earned some money at an after school job. She saved one third of what she earned and spent the rest.
What is the ratio of the money she saved to the money she spent?
2. Harry also had an after school job. He earned \$60 a week and saved \$45 of it.
(a) What fraction of his earnings did he save? (Write the fraction in its simplest form)
(b) What is the ratio of the money Harry saved to the money he spent?
3. At a concert the ratio of the number of people sitting to those standing was 1:5.
What fraction of the people at the concert were sitting?
4. From a group of 35 students 14 had been to a concert.
(a) What fraction of the students in the group had been to a concert? (Write the fraction in its simplest form)
(b) What is the ratio of those who had been to a concert to those who hadn't?
5. The number of hours a day that several animals sleep is shown in this table.
(a) Find the fraction of the day that each animal sleeps.
(b) Find the ratio of hours asleep to hours awake for each animal.
(c) Find the ratio of hours asleep for the following pairs of animals:
(i) pig / tiger
(ii) horse / koala
(iii) koala / cat



Animal	Hours of Sleep Each Day
Horse	3
Elephant	4
Pig	8
Sloth	10
Dog	11
Cat	12
Platypus	14
Tiger	16
Koala	18
Brown Bat	20

Ratios and Percentages

Example 1 The ratio of boys to girls in a karate class was 3:2. What percentage of the class was boys and what percentage was girls?

Answer: To find the percentage of each group divide 100 in the same ratio as the question.
Divide 100 in the ratio 3:2.
Use the method from Exercise 9A.
100 in the ratio of 3:2 is 60:40.
So **60% of the class is boys and 40% was girls.**

Example 2 The recipe for a salad dressing consisted of 25% vinegar and the remainder olive oil. What is the ratio of vinegar to olive oil? Give the ratio in its simplest form.

Answer: 25% vinegar and 75% olive oil
This is a ratio of 25:75
In its simplest form **1:3**

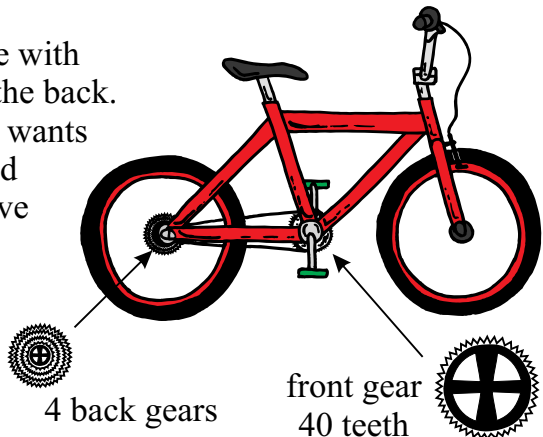
EXERCISE 9E

- The ratio of adults who owned their own homes to those who rented in a city was 4:1.
What percentage of the adults in the town owned their own homes?
- The ratio of pickers to packers in a fruit farm was 7:3.
What percentage of the workers were pickers?
- An acid solution consisted of 10% acid and the remainder water.
What is the ratio of acid to water? Give the ratio in its simplest form.
- Angela ran regularly to keep fit. She would sprint for 15% of the run and jog for the remainder of the time.
What is the ratio of the time Angela sprints to the time she jogs?
Give the ratio in its simplest form.

Challenging Problems

EXERCISE 9F

- Concrete is to be made using cement, sand and stones in the ratio 1:3:6.
 - What percentage of the concrete is each component?
 - What fraction of the concrete is each component?
 - How many kg of each component would be needed to make:
 - 40 kg of concrete?
 - 75 kg of concrete?
- Felix and Trent own a plumbing business. For a particular job, Felix did 90 hours work and Trent did 60 hours work. They get paid \$6000 for the job.
 - What is the ratio of hours worked by Felix to those worked by Trent?
 - How much should *each* plumber get paid?
- A farmer had 600 metres of fencing. He wanted to use all this fencing to make a paddock that had a length to width ratio of 2:1. What would be the dimensions of the paddock?
- A nurse had 600 mL of disinfectant and water mixed in the ratio of 1:5. She wanted to add water so that the disinfectant to water ratio was 1:8. How much water should she add?
- An artist wanted to make a coloured paint by mixing blue and yellow paint in the ratio 1:3. She mixed 30 mL of blue and 120 mL of yellow. Her friend, who was studying ratios at school, told her she added too much yellow.
 - How much extra yellow did she add above what she should have?
 - How much blue should she now add to make the correct ratio?
- Cadel was going to make a bicycle with one gear on the front and four on the back. The front gear has 40 teeth and he wants to work out how many teeth should be on each of the back gears to have front to back gear ratios of 4:1, 8:3, 5:4 and 10:11. How many teeth should be on each of the back gears?



PROBLEM SOLVING

An olive oil supplier used a mixture of local and imported olives in his olive oil.

He had two containers that he was going to combine.

The first container had 60 litres of oil made from local and imported olives in the ratio of 5:1.

The second container had 40 litres of oil made from local and imported olives in the ratio of 3:1.

He combined the oil from both these containers.

What is the ratio of local to imported olives in the combined oil?

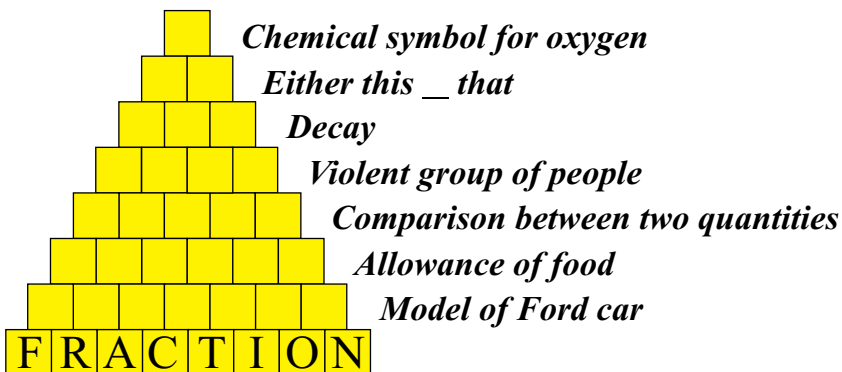


PUZZLE

Complete the diagram below by finding the word on each line.

Each word can be found by adding one letter to the line above it and rearranging the letters.

A clue for each word is given.



CHAPTER REVIEW

1. (a) Divide \$48 in the ratio of 2:1.
(b) Divide 72 kg in the ratio of 1:7.
(c) Divide 90 m in the ratio of 4:5.
(d) Divide 160 L in the ratio of 5:3.
2. Gary and Melanie divide 90 kg of tomatoes in the ratio of 2:3.
How many kg of tomatoes will they each receive?
3. Write the following ratios in their simplest form.
(a) 16:24 (b) 36:9 (c) 72:27
4. (a) A bird breeder mixed 40 kg of sunflower seeds and 24 kg of grain as a feed for his birds. What is the ratio of sunflower seeds to grain in its simplest form?
(b) If he had 15 kg of sunflower seeds, how many kg of grain would he need to keep the same ratio?
(c) If he wanted to make 160 kg of feed, how many kg of sunflower seeds and grain would he need?
5. In a class of 30 students there were 18 boys.
(a) What fraction of the class is boys? Give answer in its simplest form.
(b) What is the ratio of boys to girls? Give answer in its simplest form.
6. Three quarters of a pie was eaten. What is the ratio of the amount eaten to the amount not eaten?
7. A drink is made with 30% orange juice and the rest soda water.
What is the ratio of orange juice to soda water? Give answer in its simplest form.
8. The ratio of water to milk in a hot chocolate was 1:9.
(a) What percentage of the drink is water?
(b) If a glass held 300 mL of drink how many mL of milk is required?
9. A liquid fertilizer needs to be mixed with water in the ratio of 1:20.
Don mixes 12 mL of the fertilizer with 200 mL of water. How much extra water needs to be added to made the correct ratio?

Rates

Rates

Example

Eve took 30 minutes to make 5 Christmas cards.

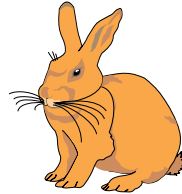
- (a) How long did it take her to make one card?
- (b) How long would it take her to make 12 cards?

Answer:

- (a) Each card took $30 \div 5 = \mathbf{6 \text{ minutes}}$
- (b) To make 12 cards would take $12 \times 6 = \mathbf{72 \text{ minutes}}$

EXERCISE 10A

1. Jasmin fed her rabbits 800 grams of rabbit food every 4 days.
 - (a) How many grams of the food did they eat each day?
 - (b) How many grams of the food would they eat in a week?
 - (c) How many grams of the food would they eat in 10 days?
2. Jay worked 4 hours at a shop and was paid \$60.
 - (a) What was he paid each hour?
 - (b) How much would he be paid if he worked 7 hours?
3. Hailee bought 6 metres of material that cost \$24.
 - (a) What was the cost of one metre of the material?
 - (b) What would 20 metres of the material cost?
4. Stephanie swam 6 laps of a pool in 3 minutes.
 - (a) How long did it take her to swim one lap?
 - (b) How long would it take her to swim 40 laps?



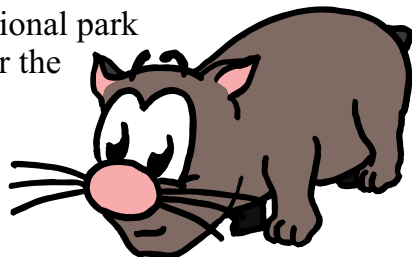
Example

Jack took 12 minutes to jog 4 laps of an oval.
How long would it take him to jog 15 laps of the oval?

Answer:

First find how long it takes him to jog one lap of the oval.
One lap took him $12 \div 4 = 3$ minutes
15 laps would take $15 \times 3 = \mathbf{45 \text{ minutes}}$

5. If 5 bricks weighed 15 kg, what would 12 bricks weigh?
6. Bob the builder bought 6 metres of roofing that cost \$54.
What would be the cost of 10 metres of the roofing?
7. Dwayne drove 320 km and used 16 litres of fuel.
 - (a) How much fuel would his car use in driving 400 km?
 - (b) Dwayne's car's fuel tank held 40 litres. How far could he drive on a full tank of fuel?
8. A water pump could pump 500 litres of water in 10 minutes.
 - (a) How long would it take to pump 1200 litres of water?
 - (b) How many litres would it pump in an hour?
9. Mitch used a hose to fill a 15 litre bucket with water and it took 3 minutes. How long would it take to fill a 350 litre fish pond?
10. Harriot bought 300 kg of fertilizer and it cost \$600. Find the cost of the following amounts of fertilizer:
 - (a) 200 kg (b) 500 kg (c) 2000 kg
11. A dam was leaking at the rate of 600 litres every 12 hours.
How long would it take for 20 000 litres to leak from the dam?
12. The population of wombats in a national park was estimated at 500. Five years later the population was estimated to be 550.
If the population keeps increasing at the same rate, after how many more years will the population be 700?



Best Buys

EXERCISE 10B

Example

Which of the following packets of rice is the best value?

A 500 g for 80c **B** 2 kg for \$3.10 **C** 5 kg for \$7.85

One method of doing this is to calculate the cost of buying 1 kg of each of the alternatives.

A 500 g for 80c - 1 kg would cost **\$1.60** (remember 1 kg = 1000 g)

B 2 kg for \$3.10 - 1 kg would cost **\$1.55** ($\$3.10 \div 2$)

C 5 kg for \$7.85 - 1 kg would cost **\$1.57** ($\$7.85 \div 5$)

The best value is alternative **B** (the cheapest per kg)

- Which of the following packets of cereal is the best value?
A 500 g for 90c **B** 2 kg for \$3.80 **C** 5 kg for \$9.25
- Which of the following packets of pasta is the best value?
A 500 g for 45c **B** 5 kg for \$4.10 **C** 8 kg for \$6.80
- Which of the following packets of nuts is the best value?
A 200 g for \$1.80 **B** 500 g for \$4.60 **C** 2 kg for \$17.80
- Which of the following bags of dog food is the best value?
A 2 kg for \$2.70 **B** 5 kg for \$7.00 **C** 20 kg for \$26.40

You may need to use a calculator for the following problems.

- Which of the following boxes of soap powder is the best value?
A 1.2 kg for \$4.02 **B** 2.6 kg for \$8.58 **C** 5.5 kg for \$18.04
- Which of the following bags of frozen peas is the best value?
A 500 g for \$1.12 **B** 1.8 kg for \$4.05 **C** 3.5 kg for \$7.77

EXERCISE 10C

It is sometimes easier to compare prices by calculating the price for 100 g.

Example

Which of the following packets of seeds is the best value?

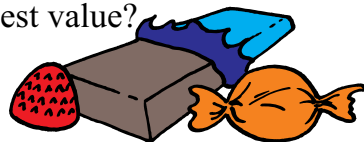
- A** 50 g for 33c **B** 200 g for \$1.36 **C** 500 g for \$3.35

Calculate the price for 100 g of each alternative.

- A** 50 g for 33c - 100 g would cost **66c**
B 200 g for \$1.36 - 100 g would cost **68c**
C 500 g for \$3.35 - 100 g would cost **67c**

The best value is alternative **A**

- Which of the following packets of spices is the best value?
A 50 g for 85c **B** 200 g for \$3.44 **C** 500 g for \$8.40
- Which of the following boxes of biscuits is the best value?
A 50 g for \$1.15 **B** 500 g for \$11.25 **C** 800 g for \$17.60
- Which of the following cans of tomatoes is the best value?
A 500 g for 75c **B** 600 g for 96c **C** 700 g for \$1.19
- Which of the following tubs of yoghurt is the best value?
A 600 g for \$1.98 **B** 800 g for \$2.56 **C** 1 kg for \$3.40
- Which of the following packets of sweets is the best value?
A 50 g for 78c **B** 200 g for \$3.04
C 500 g for \$7.10 **D** 1 kg for \$14.80



You may need to use a calculator for the following problem.

- Which of the following cans of fruit is the best value?
A 250 g for \$2.75 **B** 420 g for \$4.83 **C** 550 g for \$6.16

Speed

Speed can be calculated by using the following formula.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Example 1

Find the speed of a car that travels 500 metres in 25 seconds.

The distance travelled = 500 m

The time taken = 25 s

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{500}{25}$$

$$\text{Speed} = 20 \text{ metres per second (m/s)}$$

Example 2

Find the speed of a bird that flies 40 kilometres in 20 minutes.

The distance travelled = 40 km

The time taken = 20 min

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{40}{20}$$

$$\text{Speed} = 2 \text{ kilometres per minute (km/min)}$$

Example 3

Find the speed of an aeroplane that flies 2000 kilometres in 4 hours.

The distance travelled = 2000 km

The time taken = 4 hours

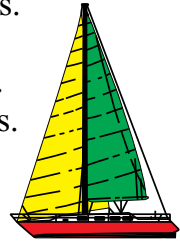
$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$= \frac{2000}{4}$$

$$\text{Speed} = 500 \text{ kilometres per hour (km/h)}$$

EXERCISE 10D

- Find the speed of a person who ran 200 metres in 40 seconds.
- The Sydney to Hobart yacht race is approximately 1200 km.
Find the speed of a yacht that completed the race in 60 hours.
- An aeroplane flew from Melbourne to Los Angeles.
The distance is approximately 12 600 km and the flight took 14 hours. At what speed was the plane flying?
- A marathon is 42 km. Jobe completed a marathon in 3 hours.
At what speed did Jobe run?
- Kaylee and Cayden are in Europe on holidays. They decide to cycle from Paris to Rome a distance of 1620 km. They plan to ride for 5 hours every day and hope to travel 90 km every day.
 - How many days will it take them to complete their ride?
 - At what speed will they be riding?
 - How many metres will they ride each day?
 - How many minutes will they be riding for each day?
 - What is their speed in metres per minute (m/min)?
 - What is their speed in metres per second (m/sec)?
- The maximum speed of several animals was being studied. The animals were observed travelling at their fastest. Distances and times were recorded to calculate their speeds. These measurements are shown in the table below. For each animal calculate the speed and list the animals in order from the fastest to the slowest.



<i>Animal</i>	<i>Distance (metres)</i>	<i>Time (seconds)</i>
Kangaroo	108	6
Ostrich	182	7
Black marlin	720	20
Horse	192	12
Cheetah	90	3
Peregrine falcon	525	5
Southern giant dragonfly	12	0.5
Greyhound	84	4

If the speed of an object is known, the distance it will travel in a certain time can be calculated using the following formula.

$$\text{Distance travelled} = \text{Speed} \times \text{Time taken}$$

If the speed of an object is known, the time taken to travel a certain distance can be calculated using the following formula.

$$\text{Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$$

Example 1

A person is jogging at a speed of 3 metres per second (m/sec). How far will they jog in 40 seconds?

$$\begin{aligned}\text{Distance travelled} &= \text{Speed} \times \text{Time taken} \\ &= 3 \times 40 \\ \text{Distance travelled} &= 120 \text{ metres}\end{aligned}$$

Example 2

How long would it take a person jogging at 3 m/sec to jog 600 metres?

$$\begin{aligned}\text{Time taken} &= \frac{\text{Distance travelled}}{\text{Speed}} \\ &= \frac{600}{3}\end{aligned}$$

$$\text{Time taken} = 200 \text{ seconds (3 minutes and 20 seconds)}$$

The units of speed must be the same as the units of distance travelled and time.

Example: If the units of *speed* are *metres per second* (m/sec) then *distance travelled* will be in *metres* (m) and *time* will be in *seconds* (sec).

If other units for distance or time are stated they need to be converted first.

Example 3

A cyclist is riding at 8 m/sec.

- How far will he ride in 30 seconds?
- How far will he ride in 10 minutes?
- How long will it take him to ride 800 metres?
- How long will it take him to ride 40 kilometres?

The units of speed are *m/sec* so the units of distance travelled must be *metres* and the units of time taken must be *seconds*.

Answers:

$$(a) \text{ Distance travelled} = \text{Speed} \times \text{Time taken} \\ = 8 \times 30$$

$$\text{Distance travelled} = 240 \text{ metres}$$

- (b) The time of 10 minutes needs to be converted to seconds.
10 minutes = 600 seconds

$$\text{Distance travelled} = \text{Speed} \times \text{Time taken} \\ = 8 \times 600$$

$$\text{Distance travelled} = 4800 \text{ metres}$$

$$(c) \text{ Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$$

$$= \frac{800}{8}$$

$$\text{Time taken} = 100 \text{ seconds (1 minute and 40 seconds)}$$

- (d) The distance of 40 kilometres needs to be converted to metres.
 40 km = 40 000 m

$$\text{Time taken} = \frac{\text{Distance travelled}}{\text{Speed}}$$

$$= \frac{40\,000}{8}$$

$$\text{Time taken} = 5000 \text{ seconds}$$

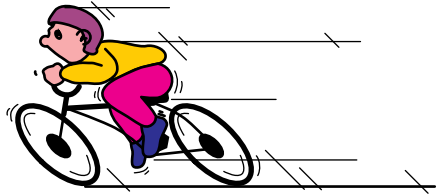
(1 hour 23 minutes and 20 seconds)

EXERCISE 10E

- Cheryl can jog at a speed of 4 m/sec.
 - How far would she jog in 50 seconds?
 - How far would she jog in 2 minutes?
 - How long would it take her to jog 500 metres?
 - How long would it take her to jog 2 kilometres?
- A space probe is travelling at a speed of 15 km/sec.
 - How far will it travel in 5 seconds?
 - How far will it travel in 2 minutes?
 - How far will it travel in 2 hours?
- Amilie is riding her horse at a speed of 12 m/sec.
 - How far will she ride in 40 seconds?
 - How far will she ride in 10 minutes?
 - How long will it take her to ride 300 m?
 - How long will it take her to ride 6 km?
- Charlie is flying his hang glider at a speed of 20 m/sec.
 - How far will he fly in 50 seconds?
 - How far will he fly in 5 minutes?
 - How far will he fly in 2 hours?
 - How long will he take to fly 800 metres?
 - How long will he take to fly 50 kilometres?



5. Cluffy is running in a 5 km fun run. He noted that it took him 80 seconds to run the first 400 metres.
- At what speed is he running?
 - How long will it take him to run the 5 km?
 - Convert this time to minutes and seconds.
 - How far would he run in 2 hours at this speed?
6. Shane is planning to do a charity bicycle ride from Melbourne to Perth - a distance of 3420 km. In training he rode 150 km in 5 hours.
- At what speed was Shane riding his bike while training?
 - If he rode 6 hours a day for 5 days how far would he ride?
 - How long would it take him to ride from Melbourne to Perth?
 - If he rode for 6 hours a day, how many days would it take him to ride from Melbourne to Perth?



You may need to use a calculator for the following problems.

7. Susie enjoys competing in ocean swims. Her last swim was 12 km and it took her 2 hours and 40 minutes.
- At what speed did she swim in this race? Give answer in m/min.
 - She wants to see if she could break the record for swimming the English Channel - a distance of 33.9 km. The record for a woman is 7 hours and 25 minutes. Will she break this record if she swims at the same speed that she did in her last swim?
8. Dennis plays cricket and is a bowler.
A batter is standing 20 metres from Dennis when he bowls the cricket ball.
How long would it take the ball to reach the batter if Dennis bowled it at 120 km/h?
9. The speed of light is 300 000 000 m/sec.
The Earth is 150 000 000 km from the Sun.
How long does it take light to get from the Sun to the Earth?

PROBLEM SOLVING

1. Jasmin jogs regularly. Her jogging speed is 4 m/sec.

One day she passes the house of her friend Heath. One minute after Jasmin has passed his house Heath starts jogging along the same path as Jasmin. Heath's jogging speed is 5 m/sec.

- (a) How long after Heath leaves his house will he catch up to Jasmin?
- (b) How far has Heath run when he catches Jasmin?

2. Florence loves skating. It takes her 1 minute and 45 seconds to skate around a track that is a distance of 840 metres.

Harry runs around the same track and it takes him 2 minutes and 20 seconds. One day they start at the same spot and go in opposite directions around the track.

- (a) After how long will they meet for the first time?
- (b) How far have they each travelled when they meet for the first time?
- (c) If they continued skating and running for half an hour, how many times would they meet?



PUZZLE

Change **JOG** into **RUN** in three steps by changing one letter at a time to form a new word.

Example
CAT can be changed
 to **DOG** in three steps

CAT
COT
DOT
DOG

CHAPTER REVIEW

1. It took Holly 30 minutes to pot 6 plants.
 - (a) How long did it take her to pot one plant?
 - (b) How long would it take her to pot 10 plants?
2. Holly bought 4 kg of potting mix for \$8.
 - (a) What is the cost of 1 kg of potting mix?
 - (b) What would 25 kg of the potting mix cost?
3. Paddy took 40 minutes to sort 8 boxes of mail. How long would it take him to sort 25 boxes of mail?
4. Find the cost per kg of each of the following packets of cereal and calculate which is the best value.
A 2 kg for \$4.90 **B** 3 kg for \$7.65 **C** 5 kg for \$12.60
5. Find the cost per 100 g of each of the following jars of chutney and calculate which is the best value.
A 200 g for \$2.92 **B** 500 g for \$7.10 **C** 600 g for \$8.40
6. Which of the following packets of nuts is the best value?
A 50 g for \$1.12 **B** 200 g for \$4.44 **C** 500 g for \$11.05
7. Pedro and Lucia are planning to paddle a canoe down the Murray River which has a total length of 2400 km. They know they can paddle 30 km in 5 hours.
 - (a) At what speed can they paddle their canoe?
 - (b) If they paddle for 8 hours a day how far will they travel each day?
 - (c) How many days will it take for them to complete paddling down the Murray River?
8. Jack rode his bicycle 20 km in 30 minutes.
 - (a) At what speed (in km/h) does he ride?
 - (b) How long would it take him to ride 160 km?
 - (c) How far would he ride in 24 hours?

Patterns and Rules

Patterns

EXERCISE 11A

1. Complete the following number patterns.

- | | |
|----------------------------------|--|
| (a) 2, 4, 6, 8, __, __, __. | (b) 20, 17, 14, 11, __, __, __. |
| (c) 1, 3, 9, 27, __, __, __. | (d) 0, 4, 8, 12, __, __, __. |
| (e) 100, 90, 80, 70, __, __, __. | (f) 64, 32, 16, 8, __, __, __. |
| (g) 3, 6, 12, 24, __, __, __. | (h) 100 000, 10 000, 1000, __, __, __. |
| (i) 0.2, 2, 20, 200, __, __, __. | (j) 5, 10, 15, 20, __, __, __. |

2. Explain how to find a number in each of the number patterns in question 1.

Examples

1. 1, 3, 5, 7, __, __, __.

In this pattern a number is found by **adding 2** to the number before it.

Answer: +2

2. 1, 4, 16, 64, __, __, __.

In this pattern a number is found by **multiplying by 4** the number before it.

Answer: $\times 4$

3. Find the first five numbers in the number patterns that have the following starting numbers and method of finding the other numbers.

Examples

1. 3 (+4)

Starting number = 3

Each number is found by **adding 4** to the number before it.

First five numbers are:

3, 7, 11, 15, 19

2. 2 ($\times 3$)

Starting number = 2

Each number is found by **multiplying by 3** the number before it.

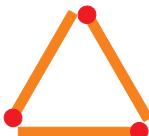
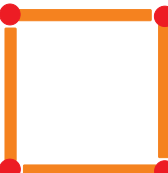
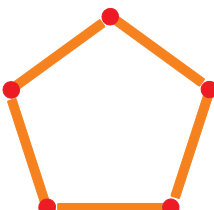
First five numbers are:

2, 6, 18, 54, 162

- (a) 7 (+3) (b) 50 (-5) (c) 1 ($\times 10$) (d) 10 (+10) (e) 2187 ($\div 3$)

4. Find the eighth number in each of the patterns in question 3.

5. The table below shows several patterns that can be made with matches.

	Step 1	Step 2	Step 3
A			
B			
C			
D			
E			

(a) Describe how the step number can be used to find the number of matches needed for each step of the patterns.

Example In pattern A the number of matches can be found by **multiplying the step number by 3**.

(b) Find the number of matches needed to form **step 8** of each pattern.

Example For pattern A at step 8 there would need to be:
 $8 \times 3 = 24$ matches

Symbols and Rules

EXERCISE 11B

1. Refer to the patterns in question 5 Exercise 11A.

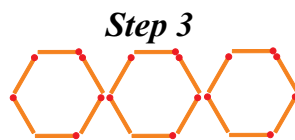
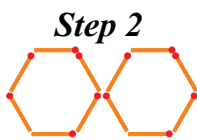
Let the symbol m represent the number of matches and s represent the step number.

Use these symbols to find a rule for each pattern that can be used to find the number of matches needed for a step number.

Example In pattern A the number of matches, m , is equal to 3 times the step number, s .

$$m = 3 \times s$$

2. (a) For the pattern below, find how many matches are needed to form: (i) step 1 (ii) step 2 (iii) step 3 (iv) step 4 (v) step 6



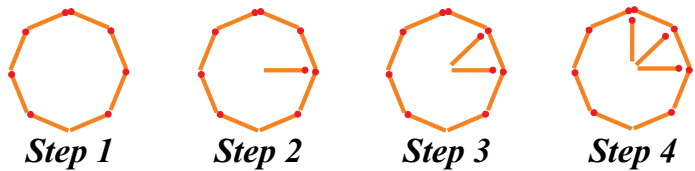
- (b) Copy and complete this table showing the number of matches, m , needed for the different steps, s , of the pattern.
- (c) How many matches would be needed to form step 20 of the pattern?
- (d) What step of the pattern would need:
- (i) 60 matches?
- (ii) 72 matches?

s	m
1	
2	
3	
4	
5	
6	

- (e) Copy and complete the following rule that can be used to find the number of matches, m , for a step number, s .

$$m = _ \times s$$

3. (a) For the pattern below, copy and complete the table showing the number of matches, m , and the step number, s .



s	m
1	
2	
3	
4	
5	
6	

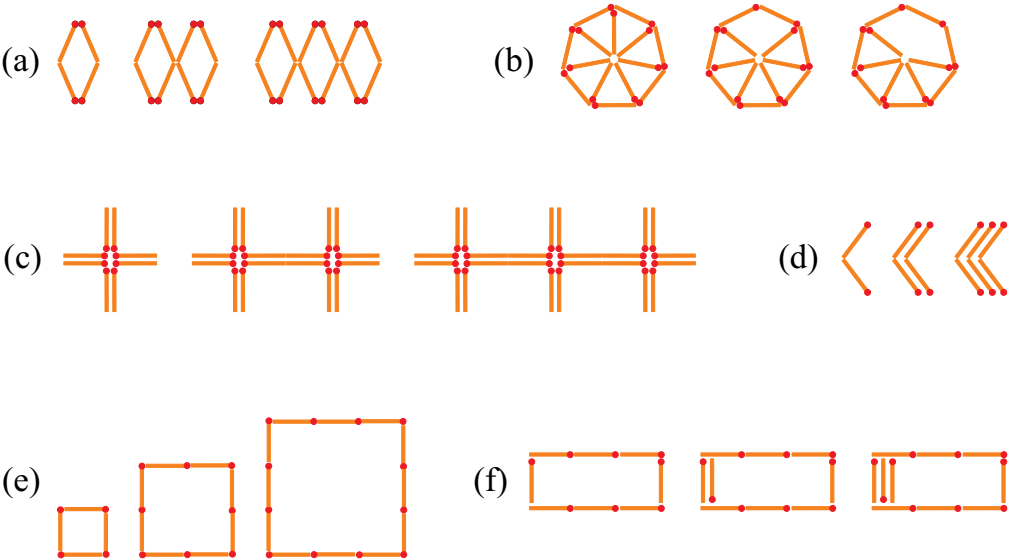
(b) Copy and complete the following rule:

$m = s + \underline{\hspace{1cm}}$

4. For each of the patterns below:

- (i) Copy and complete the table showing the number of matches, m , and the step number, s .
- (ii) Write a rule that could be used to find the number of matches for a step number.

s	m
1	
2	
3	
4	
5	
6	



5. Find the rules for the symbols shown in the following tables.

Examples

s	m
1	2
2	4
3	6
4	8
5	10
6	12

$$m = 2 \times s$$

a	b
1	3
2	4
3	5
4	6
5	7
6	8

$$b = a + 2$$

M	N
11	10
12	11
13	12
14	13
15	14
16	15

$$N = M - 1$$

(a)

s	m
1	5
2	10
3	15
4	20
5	25
6	30

$$m =$$

(b)

A	B
1	5
2	6
3	7
4	8
5	9
6	10

$$B =$$

(c)

p	q
1	0
2	1
3	2
4	3
5	4
6	5

$$q =$$

(d)

c	t
1	10
2	20
3	30
4	40
5	50
6	60

$$t =$$

(e)

x	y
1	21
2	22
3	23
4	24
5	25
6	26

$$y =$$

(f)

F	T
20	10
18	9
16	8
14	7
12	6
10	5

$$T =$$

(g)

W	T
15	9
14	8
13	7
12	6
11	5
10	4

$$T =$$

(h)

h	k
50	10
40	8
30	6
20	4
10	2
0	0

$$k =$$

(i)

α	β
0	8
6	14
11	19
20	28
34	42
55	63

$$\beta =$$

6. For each of the following rules, copy and complete the table shown.

- (a) $n = m + 5$ (b) $n = 11 \times m$ (c) $n = m + 15$
 (d) $n = m \div 10$ (e) $n = m - 6$ (f) $n = m \div 2$
 (g) n is found by multiplying m by 2.
 (h) n is found by adding 10 to m .
 (i) n is found by subtracting 1 from m .

m	n
10	
20	
30	
40	
50	
60	

7. Use the rule shown to answer the following questions.

$$y = x + 6$$

- (a) Find y if: (i) $x = 6$ (ii) $x = 10$ (iii) $x = 22$ (iv) $x = 38$
 (b) Find x if: (i) $y = 7$ (ii) $y = 10$ (iii) $y = 26$ (iv) $y = 43$

8. For each of the following rules, copy and complete the table shown.

- (a) $y = x + 5$ (b) $y = 3 \times x$ (c) $y = x + 15$
 (d) $y = x \div 3$ (e) $y = x - 2$ (f) $y = 2 \times x$
 (g) y is found by multiplying x by 10.
 (h) y is found by adding 10 to x .
 (i) y is found by subtracting 1 from x .

x	y
3	
6	
	30
12	
21	
	60

9. A sweets shop was selling bags of chocolates. There were 10 chocolates in each bag.

- (a) How many chocolates would there be in:

- (i) 2 bags? (ii) 5 bags?
 (iii) 10 bags? (iv) 20 bags?

- (b) Which of the following rules could be used to find the number of chocolates, c , in a certain number of bags, b ?

- A** $c = 10 \times b$ **B** $c = b + 10$ **C** $c = b - 10$ **D** $c = b \div 10$

- (c) How many bags would be needed for:

- (i) 30 chocolates? (ii) 70 chocolates? (iii) 500 chocolates?

- (d) Which of the following rules could be used to find the number of bags, b , needed for a certain number of chocolates, c ?

- A** $b = 10 \times c$ **B** $b = c + 10$ **C** $b = c - 10$ **D** $b = c \div 10$

10. Alisha is 6 years older than George.

(a) How old will Alisha be when George is:

(i) 10 years old? (ii) 18 years old? (iii) 39 years old?

(b) Which of the following rules could be used to find Alisha's age, A , given George's age, G ?

A $A = 6 \times G$ **B** $A = G + 6$ **C** $A = G - 6$ **D** $A = G \div 6$

(c) How old will George be when Alisha is:

(i) 26 years old? (ii) 38 years old? (iii) 53 years old?

(d) Which of the following rules could be used to find George's age, G , given Alisha's age, A ?

A $G = 6 \times A$ **B** $G = A + 6$ **C** $G = A - 6$ **D** $G = A \div 6$

11. Glenda can walk 4 kilometres every hour when bush-walking.

(a) How far would she walk in:

(i) 3 hours? (ii) 7 hours?

(b) Which of the following rules could be used to find the distance, d , Glenda would walk in h hours.

A $d = 4 \times h$ **B** $d = h + 4$

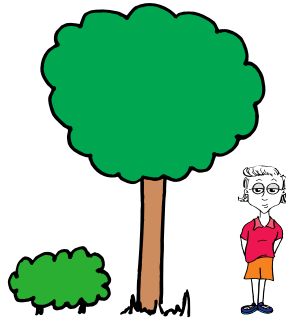
C $d = h - 4$ **D** $d = h \div 4$

(c) How many hours would it take Glenda to walk:

(i) 16 km? (ii) 24 km?

(d) Which of the following rules could be used to find how many hours, h , it would take Glenda to walk d kilometres?

A $h = 4 \times d$ **B** $h = d + 4$ **C** $h = d - 4$ **D** $h = d \div 4$



Number Machines

EXERCISE 11C

1. Copy and complete the tables for each of the number machines below.

In a number machine a number that is put into the machine is called the **input** (*i*).

This number is processed and the number that comes out of the machine is called the **result** (*r*).

The operation stated on the machine is how the number is processed.

In the number machine below the process is $\times 10$. This means that each input number is multiplied by 10 to find the result.

Example

Input (*i*) →  → Result (*r*)

<i>i</i>	1	3	12	20
<i>r</i>				

Answer

<i>i</i>	1	3	12	20
<i>r</i>	10	30	120	200

(a)

i →  → *r*

<i>i</i>	1	4	9	15
<i>r</i>				

(b)

i →  → *r*

<i>i</i>	2	3	9	11
<i>r</i>				

(c)

i →  → *r*

<i>i</i>	8	13	19	26
<i>r</i>				

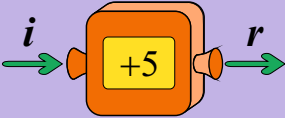
(d)

i →  → *r*

<i>i</i>	3	9	21	36
<i>r</i>				

2. Copy and complete the tables for each of the number machines below.

Example



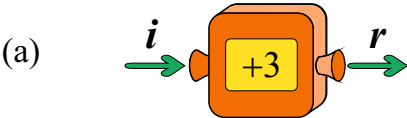
<i>i</i>				
<i>r</i>	8	11	17	22

To find the input knowing the result you need to work ‘backwards’.

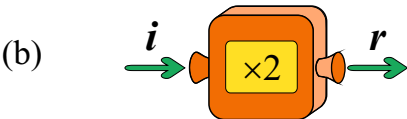
$$\begin{aligned}i + 5 &= r \\ \text{For } r &= 8 \\ i + 5 &= 8 \\ i &= 8 - 5 \\ i &= 3\end{aligned}$$

Answer

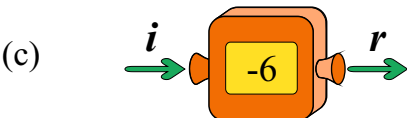
<i>i</i>	3	6	12	17
<i>r</i>	8	11	17	22



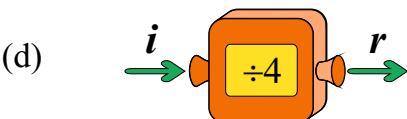
<i>i</i>				
<i>r</i>	5	9	13	19



<i>i</i>				
<i>r</i>	6	8	14	20



<i>i</i>				
<i>r</i>	2	5	9	13



<i>i</i>				
<i>r</i>	2	5	8	11

3. Copy and complete the tables for each of the number machines below.

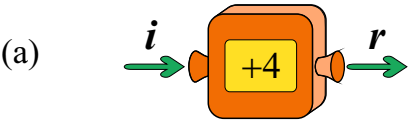
Example



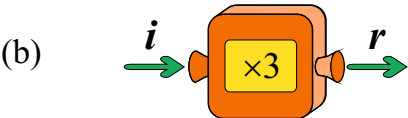
<i>i</i>	2	6		
<i>r</i>			56	70

Answer

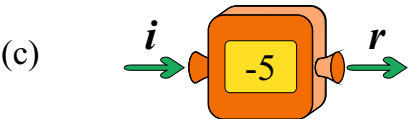
<i>i</i>	2	6	8	10
<i>r</i>	14	42	56	70



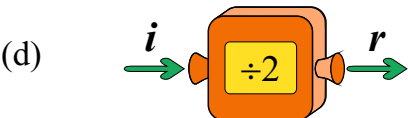
<i>i</i>	3	5		
<i>r</i>			13	17



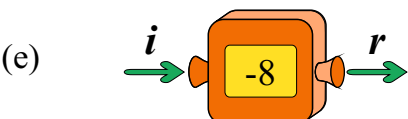
<i>i</i>	2	4		
<i>r</i>			18	24



<i>i</i>	8		15	
<i>r</i>		6		17



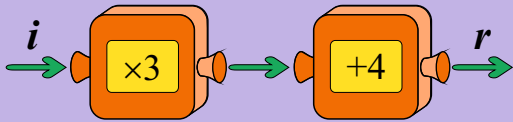
<i>i</i>	6			22
<i>r</i>		4	7	



<i>i</i>		13		26
<i>r</i>	0		9	

4. Copy and complete the tables for the number machines below.

Example



i	2	4	6	9
r				

In these problems there are two steps. The input is processed by the first number machine then this result is processed by the second number machine to give the final result.

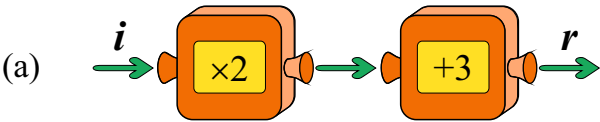
For $i = 2$

The input is processed by the first number machine: $2 \times 3 = 6$

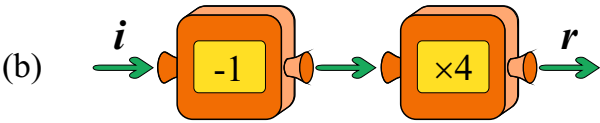
This result is then processed by the second number machine: $6 + 4 = 10$

Answer

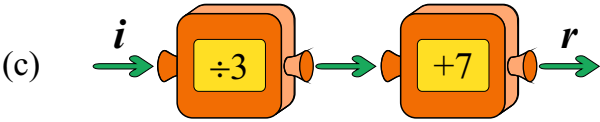
i	2	4	6	9
r	10	16	22	31



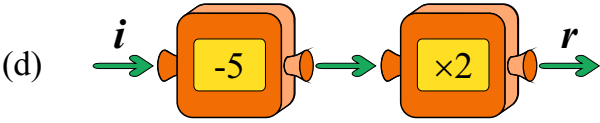
i	1	3	7	9
r				



i	3	5	6	10
r				



i	6	9	15	21
r				



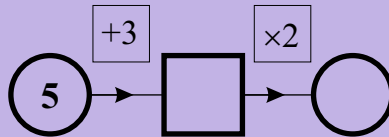
i	8	10	13	17
r				

Flowcharts and Rules

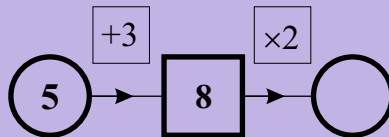
EXERCISE 11D

1. Complete the following flowcharts.

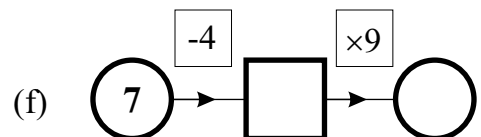
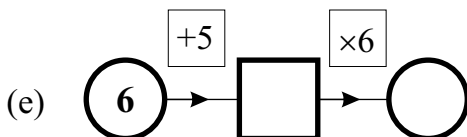
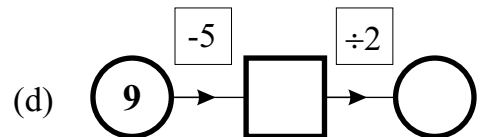
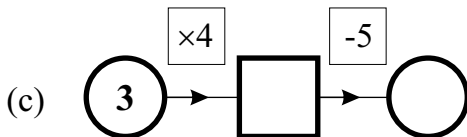
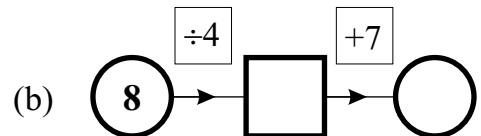
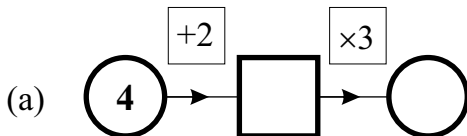
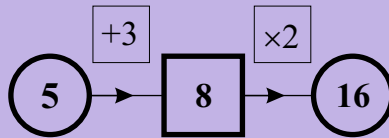
Example



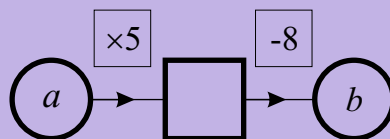
Step 1



Step 2

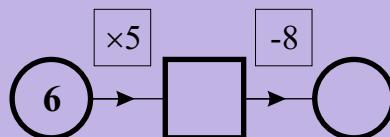


Example For the following flowchart find b when $a = 6$.

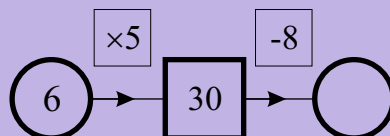


Step 1

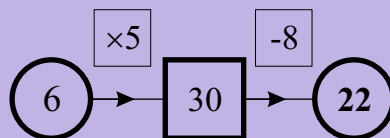
Let $a = 6$



Step 2

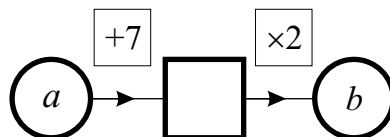


Step 3



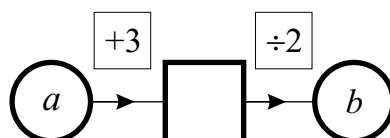
Answer: $b = 22$

2. For the following flowchart find b when $a = 3$.

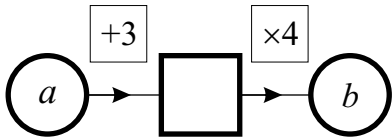


3. For the flowchart shown below find b for the following values of a .

(a) $a = 5$ (b) $a = 7$ (c) $a = 11$ (d) $a = 27$



4. For the following flowchart copy and complete the table by finding the values of b for the values of a shown.



a	b
1	
2	
3	
4	

5. Match the tables of values below with their correct flowcharts.
Check that the flowchart works for all pairs of numbers in the table.

(a)

a	b
1	6
2	10
3	14
4	18

(b)

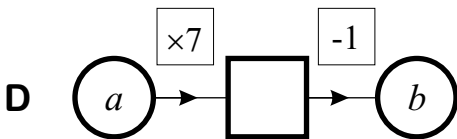
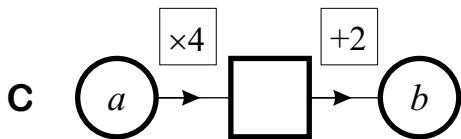
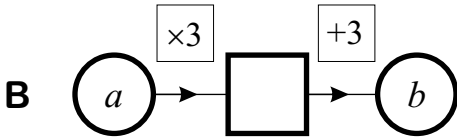
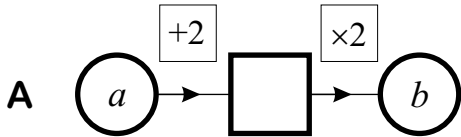
a	b
1	6
2	8
3	10
4	12

(c)

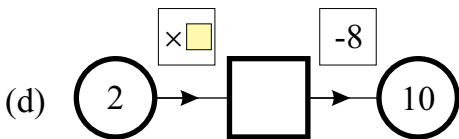
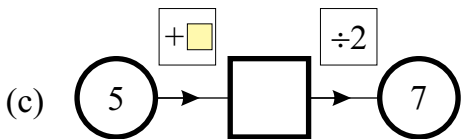
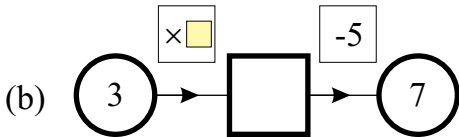
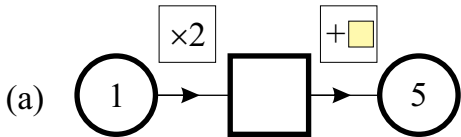
a	b
1	6
2	13
3	20
4	27

(d)

a	b
1	6
2	9
3	12
4	15

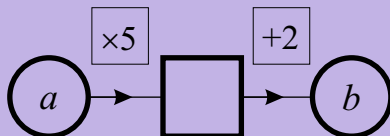


6. Complete the following flowcharts by finding the missing numbers.



In algebra, mathematical rules are called *equations*.
Write the following flowchart as an equation.

Example 1



The value of b is found by multiplying a by 5 then adding 2 to the result.
This can be written as a mathematical equation:

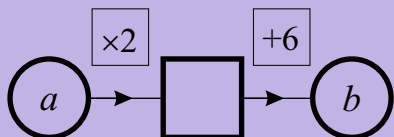
$$b = a \times 5 + 2$$

This is written as: $b = 5 \times a + 2$

In algebra it is not necessary to show the multiplication sign ($\times$) and the number is always written before the symbol when multiplying.
The above equation would be written as shown below.

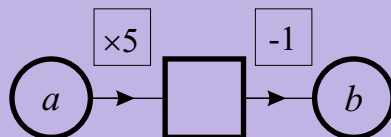
Answer: $b = 5a + 2$

Example 2



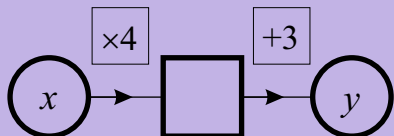
Answer: $b = 2a + 6$

Example 3



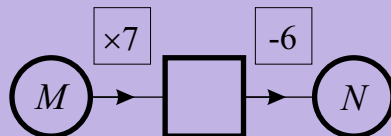
Answer: $b = 5a - 1$

Example 4



Answer: $y = 4x + 3$

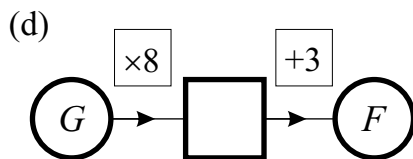
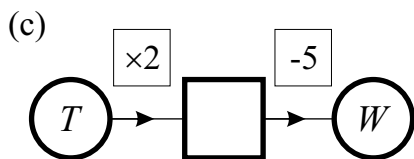
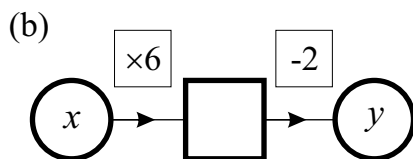
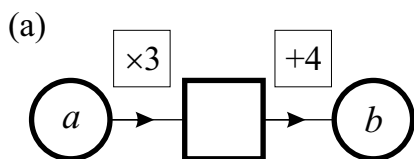
Example 5



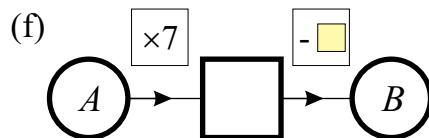
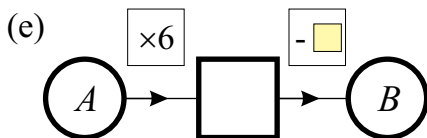
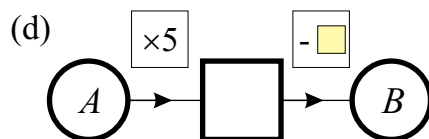
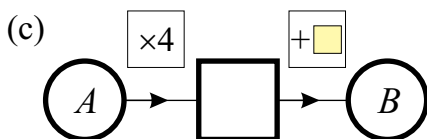
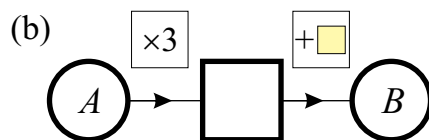
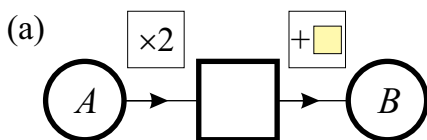
Answer: $N = 7M - 6$

EXERCISE 11E

1. Write the following flowcharts as equations.



2. Copy and complete the following flowcharts for $A = 2$ and $B = 9$.



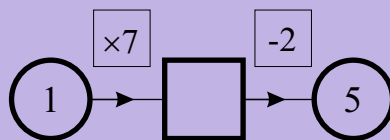
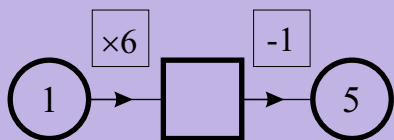
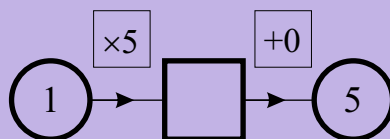
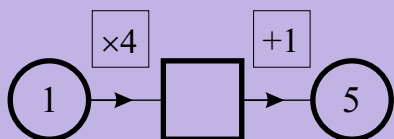
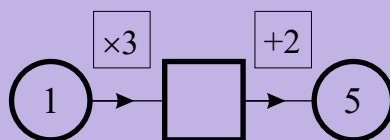
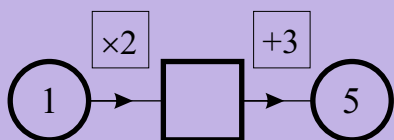
3. Write the equations for each of the flowcharts in question 2.

Example

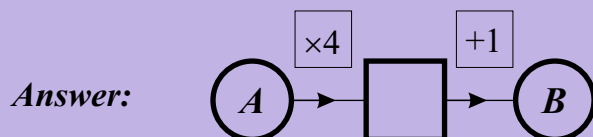
Find the flowchart and equation for the table of values shown.

<i>A</i>	<i>B</i>
1	5
2	9
3	13
4	17

Step 1 Using the first pair of numbers the flowchart could be one of many. Several of these are shown below.



Step 2 By trial and error find which of these flowcharts works for all the other pairs of numbers.



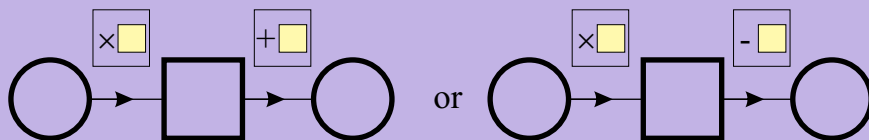
Equation: $B = 4A + 1$

Always check that the equation works for all the pairs of numbers.

4. For the following tables of values:

- (i) draw the flowchart.
(ii) find the equation.

Note: the flowcharts will be one of the following.



(a)

a	b
1	3
2	5
3	7
4	9

(b)

c	d
1	2
2	5
3	8
4	11

(c)

T	S
1	5
2	7
3	9
4	11

(d)

v	w
1	5
2	8
3	11
4	14

(e)

G	H
1	5
2	9
3	13
4	17

(f)

j	k
1	1
2	5
3	9
4	13

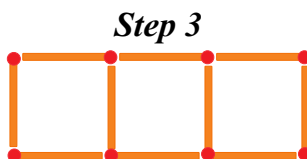
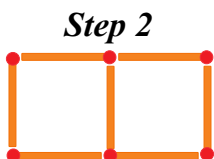
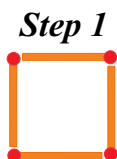
(g)

m	n
1	7
2	9
3	11
4	13

(h)

P	Q
1	0
2	3
3	6
4	9

5. (a) Copy and complete this table showing the number of matches, m , needed for each step number, s .



s	m
1	4
2	
3	
4	

(b) Draw the flowchart that could be used to find the number of matches (m) for a certain step number (s).

(c) Find the equation for this flowchart.

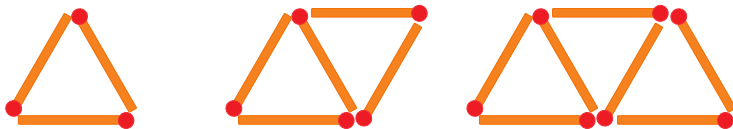
(d) How many matches would be needed for: (i) step 6? (ii) step 10?

6. For each of the following patterns:

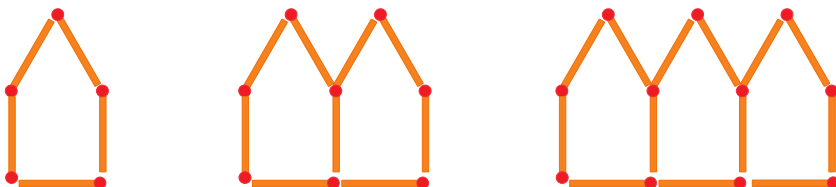
- copy and complete the table showing the number of matches, m , needed for each step number, s .
- draw the flowchart.
- write the equation.
- find the number of matches needed for step 10.

s	m
1	
2	
3	
4	

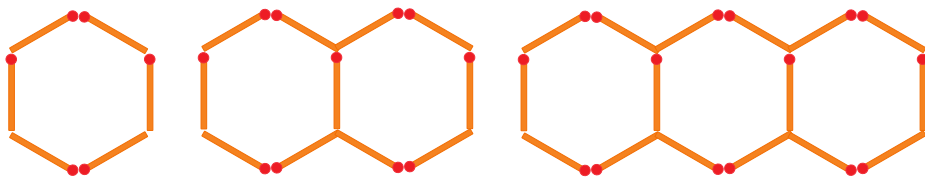
(a)



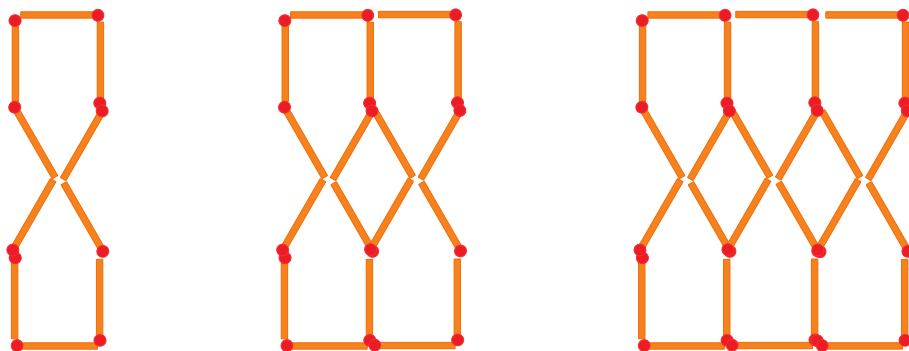
(b)



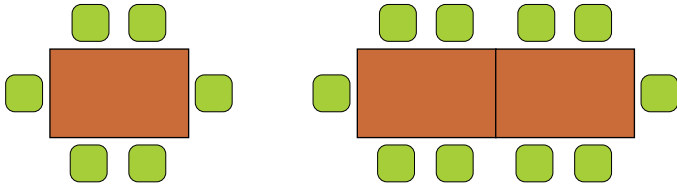
(c)



(d)



7. Jane and Hamish are getting married. They are organising the tables for the reception and can join any number of tables, end to end, as shown below. The number of chairs around each set of tables is shown. Six chairs can be placed around one table, 10 chairs around two tables, etc.



<i>t</i>	<i>c</i>
1	6
2	10
3	
4	

- (a) Copy and complete this table showing the number of chairs, *c*, that could be placed around *t* tables.
- (b) Find a rule that could be used to find the number of chairs that could be placed around a given number of tables.
- (c) How many chairs could be placed around 6 joined tables?

There are 50 guests coming to the wedding. Jane and Hamish have decided to have the tables arranged so there are no more than four tables joined together.

- (d) Copy and complete the table below showing all the possible ways that the tables could be arranged to seat exactly 50 guests. Show the number of groups of 1, 2, 3 or 4 tables. There are *eight* different combinations.

Example

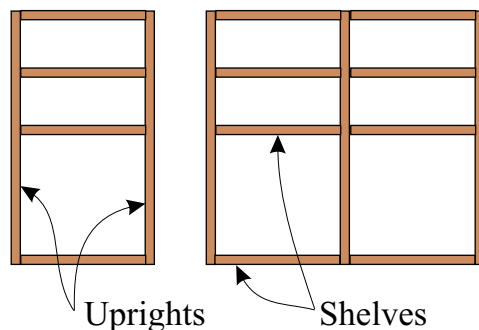
6 single tables + 1 group of 3 tables
= $(6 \times 6) + (1 \times 14)$
= $36 + 14$
= 50

Number of Groups	Number of Tables in each Group			
	1	2	3	4
	6		1	

- (e) Which combinations require the least number of tables?

PROBLEM SOLVING

1. Roland is a cabinet maker who builds bookcases as shown here. The bookcases are made using four shelves fixed in between uprights. Roland can make the bookcases any size by adding more uprights and shelves.



- (a) Copy and complete this table showing the number of shelves, s , that would be needed for a given number of uprights u .

u	s
2	4
3	8
4	
5	

- (b) Write a rule that could be used to find the number of shelves, s , needed for u uprights.
- (c) How many shelves would be needed if there are 8 uprights?
- (d) How many uprights would be needed if the bookcase is to have 20 shelves?
- (e) Roland has orders for 8 bookcases - two with 8 shelves, three with 12 shelves and three with 16 shelves. How many uprights will he need to cut to make these bookcases?

2. A greengrocer bought a full box of avocados. A customer bought half the avocados in the box.

Another customer bought half of the remaining avocados.

Another customer bought three avocados.

Another customer bought the remaining two avocados.

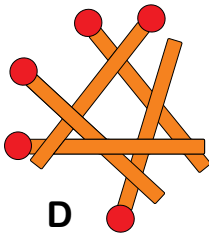
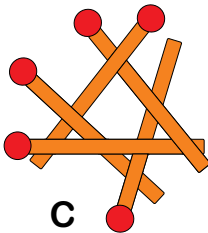
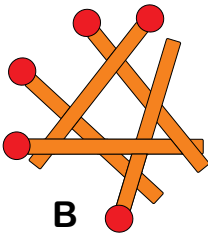
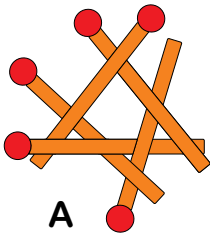
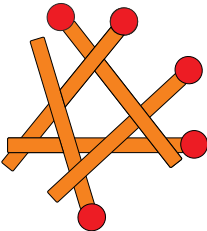
How many avocados were in the full box?



3. Iona is five years younger than her brother Byron. Karen is their mother and is three times Iona's age. Marlene is Iona's grandmother and is five times Iona's age. The sum of all their ages is 125. How old is Iona?

PUZZLES

1. Five matches are dropped onto a glass table. This diagram shows how they landed. Which of the diagrams below show what the matches would look like if viewed from under the glass table?



2. All the words that are needed to complete this puzzle can be found in this chapter. When you have found all the words the letters in boxes spell the answer to the following riddle.

	A			I		G
M		C			N	
				U		B
	Y	M			L	
					E	P
		P			T	E
F		O		C		R

What did the prehistoric carpenter call one of his tools?

3. Complete this diagram below by finding the word on each line. Each word can be found by adding one letter to the line above it and rearranging the letters. A clue for each word is given.

Roman 50

Chemical symbol for aluminium

Drop behind

Strong wind

Royal

Speak in confused manner

The mathematics of using symbols

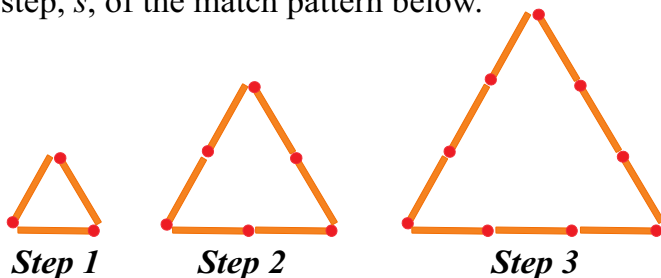
CHAPTER REVIEW

1. Complete the following number patterns.

(a) 1, 4, 7, 10, __, __, __.

(b) 10, 20, 40, 80, __, __, __.

2. (a) Copy and complete the table showing the number of matches, m , needed for each step, s , of the match pattern below.



s	m
1	
2	
3	
4	
5	
6	

(b) Draw the flowchart.

(c) Write the rule.

(d) How many matches would be needed for step 8 of the pattern?

(e) What step of the pattern would need 30 matches?

3. Find the rules for the symbols shown in the following tables.

(a)

s	m
1	5
2	6
3	7
4	8
5	9
6	10

(b)

A	B
1	5
2	10
3	15
4	20
5	25
6	30

(c)

p	q
20	10
21	11
22	12
23	13
24	14
25	15

4. Use the rule shown to answer the following questions.

$$y = 10 \times x$$

(a) Find y if: (i) $x = 1$ (ii) $x = 4$ (iii) $x = 7$

(b) Find x if: (i) $y = 20$ (ii) $y = 50$ (iii) $y = 100$

5. Copy and complete the table for the rule below.

$$B = A + 6$$

A	B
1	
3	
	13
9	
	25

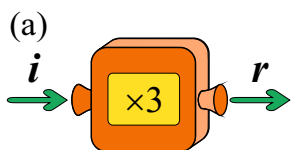
6. Sunil was making a fruit drink using 3 cups of orange juice to 1 cup of pineapple juice.

(a) Copy and complete this table showing the number of cups of orange juice, o , needed for p cups of pineapple juice.

p	o
1	
2	
3	
4	

(b) Write a rule that can be used to find the number of cups of orange juice, o , needed for p cups of pineapple juice.

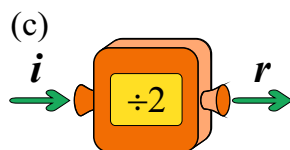
7. Copy and complete the tables for the number machines below.



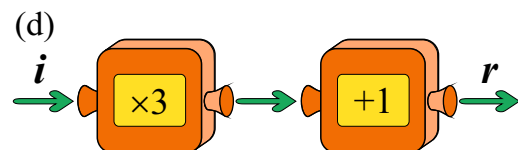
i	1	5		
r			21	33



i	2	6		
r			15	21



i	2	8		
r			9	12



i	1	5	7		11	
r				31		37

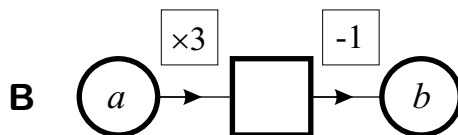
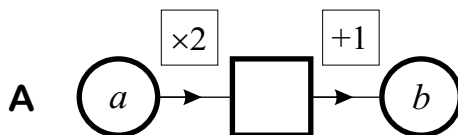
8. Match the flowchart with the table of values.

(a)

a	b
1	3
2	5
3	7
4	9

(b)

a	b
1	2
2	5
3	8
4	11



9. For the following tables:

(i) draw the flowchart.

(ii) find the equation.

(a)

x	y
1	5
2	7
3	9
4	11

(b)

N	M
1	1
2	4
3	7
4	10

Equations

Writing Equations

EXERCISE 12A

1. Write the following relationships as equations.

Remember

Operation	Product, Of (Multiplication)	Quotient (Division)	Sum (Addition)	Difference (Subtraction)
Symbol	$\times$	$\div$	$+$	$-$

Examples

1. x equals 6 added to y .
 $x = y + 6$

2. A is equal to B multiplied by 3.
 $A = 3 \times B$
 $A = 3B$

(**Note:** in equations with multiplication, the number comes before the letter and it is not necessary to include the multiplication symbol.)

3. F is equal to the product of J and K .
 $F = J \times K$
 $F = JK$

4. M is equal to Q divided by 7.
 $M = Q \div 7$
 $M = \frac{Q}{7}$

5. T is equal to 12 subtracted from V .
 $T = V - 12$

- c is equal to 8 added to d .
- L is equal to 3 subtracted from M .
- a is equal to b multiplied by 7.
- f is equal to g divided by 5.
- W equals the product of X and Y .
- q is equal to the sum of p and r .
- z is equal to p divided by t .
- W equals the product of I , m and t .
- G equals a fifth of H .
- A is equal to 5 more than B .
- C equals 6 less than D .

Algebra is the mathematics of using letters or symbols to represent objects or physical properties.

Equations can be written using these symbols to simply show the relationships between the objects.

Example 1

The total cost of a purchasing an item online is equal to the sum of the price of the item and the delivery charge.

Symbols can be given to represent the parts of this information.

The total cost could be C .

The price of the item could be P .

The delivery charge could be D .

This information can be written as an equation.

$$C = P + D$$

Example 2

The density, D , of an object is equal to its mass, m , divided by its volume, V .

$$D = \frac{m}{V}$$

Example 3

Voltage, V , is equal to the product of current, I , and resistance, R .

$$V = IR$$

Example 4

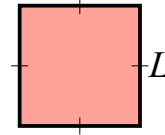
Final velocity, v , is equal to the sum of initial speed, u , and the product of acceleration, a , and time, t .

$$v = u + at$$

2. Write the following relationships as equations.

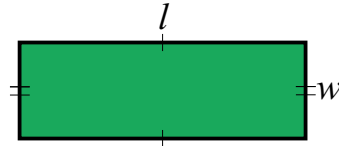
(a) The total number of students in a class, T , is equal to the sum of the number of boys, B , and the number of girls, G .

(b) The perimeter, P , of a square equals the length of one side, L , multiplied by 4.



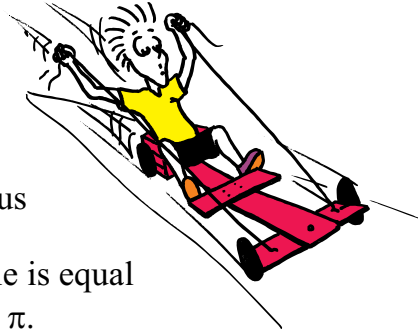
(c) The side length of a square, L , is equal to the perimeter, P , divided by 4.

(d) The area of a rectangle, A , is equal to its length, l , multiplied by its width, w .



(e) The power output, P , of a machine is equal to the work it does, W , divided by the time it took, t .

(f) Speed, S , is equal to the distance travelled, d , divided by time taken, t .



(g) Profit, P , equals income, I , minus expenses, E .

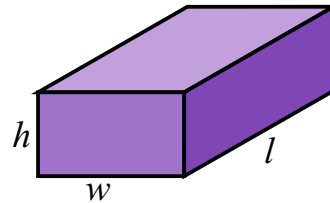
(h) The circumference, C , of a circle is equal to its diameter, D , multiplied by π .

(i) Momentum, p , is equal to the product of mass, m , and velocity, v .

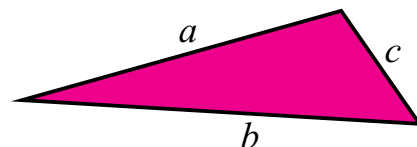
(j) Interest earned, I , is equal to the product of principal, P , rate, R , and time, t .

(k) The area of a square, A , is equal the side length, l , squared.

(l) The volume, V , of a prism is equal to the product of its length, l , its width, w , and its height, h .



(m) The perimeter of a triangle, P , is equal to the sum of its three side lengths, a , b and c .



(n) Albert Einstein's famous equation states that energy, E , is equal to the product of mass, m , and the speed of light, c , squared.

Substitution

EXERCISE 12B

1. Find the value of a in the following equations if $b = 12$.

Substitute b for 12.

Examples

$$\begin{aligned} 1. \quad a &= b + 7 \\ &= 12 + 7 \\ \mathbf{a} &= \mathbf{19} \end{aligned}$$

$$\begin{aligned} 2. \quad a &= 5b - 3 \\ &= 5 \times b - 3 \\ &= 5 \times 12 - 3 \\ &= 60 - 3 \\ \mathbf{a} &= \mathbf{57} \end{aligned}$$

$$\begin{aligned} 3. \quad a &= \frac{2b}{3} \\ &= \frac{2 \times b}{3} \\ &= \frac{2 \times 12}{3} \\ &= \frac{24}{3} \\ \mathbf{a} &= \mathbf{8} \end{aligned}$$

$$\begin{aligned} 4. \quad a &= 3(2b + 1) \\ &= 3 \times (2 \times 12 + 1) \\ &= 3 \times (24 + 1) \\ &= 3 \times 25 \\ \mathbf{a} &= \mathbf{75} \end{aligned}$$

(a) $a = b + 2$

(b) $a = b - 1$

(c) $a = 3b$

(d) $a = \frac{b}{2}$

(e) $a = b \div 4$

(f) $a = 20 - b$

(g) $a = b^2$

(h) $a = 2b + 3$

(i) $a = 4b - 10$

(j) $a = \frac{3b}{4}$

(k) $a = 3(b + 1)$

(l) $a = 2(3b - 10)$

2. Find the value of m in the following equations if $p = 4$ and $q = 5$.

Example

$$\begin{aligned} m &= 3p + 2q \\ &= 3 \times p + 2 \times q \\ &= 3 \times 4 + 2 \times 5 \\ &= 12 + 10 \\ \mathbf{m} &= \mathbf{22} \end{aligned}$$

(a) $m = 4p + 3q$

(b) $m = 6p - 2q$

(c) $m = 3(p + q)$

(d) $m = 2(3p - q)$

(e) $m = \frac{3p + 4q}{2}$

(f) $m = \frac{7p}{2} - \frac{4q}{10}$

3. Find the value of m in the following equations for the values given.

Examples

1. $m = 2n + 7$ ($n = 5$)

$$m = 2 \times 5 + 7$$

$$m = 10 + 7$$

$$m = 17$$

2. $m = 5b - 2c$ ($b = 3, c = 1$)

$$m = 5 \times 3 - 2 \times 1$$

$$m = 15 - 2$$

$$m = 13$$

(a) $m = n + 1$

($n = 9$)

(b) $m = 7P$

($P = 4$)

(c) $m = \frac{Q}{6}$

($Q = 24$)

(d) $m = c - d$

($c = 15, d = 4$)

(e) $m = 5T + 5$

($T = 2$)

(f) $m = 20 - b$

($b = 11$)

(g) $m = x^2 + 7$

($x = 3$)

(h) $m = 5w + 3y$

($w = 2, y = 4$)

(i) $m = 14 - 10h$

($h = 1$)

(j) $m = \frac{b}{4} + \frac{c}{5}$

($b = 16, c = 25$)

(k) $m = 5(b - 4)$

($b = 11$)

(l) $m = 2(3q + 4p)$

($q = 1, p = 2$)

(m) $m = 2(10 - 3f)$

($f = 3$)

(n) $m = abc$

($a = 2, b = 3, c = 10$)

4. Solve the following equations if $a = 4, b = 10$ and $c = 1$.

Example

$$B = 2a + 3b - 4c$$

$$= 2 \times a + 3 \times b - 4 \times c$$

$$= 2 \times 4 + 3 \times 10 - 4 \times 1$$

$$= 8 + 30 - 4$$

$$B = 34$$

(a) $R = a + b$

(b) $Q = b - c$

(c) $n = 3b + c$

(d) $X = \frac{b}{2} - 5c$

(e) $D = 3a + 4c$

(f) $j = \frac{3a + 2b - 4c}{7}$

(g) $V = a + b^2$

(h) $H = 2a + 3b - c$

(i) $N = ab + c$

(j) $y = 3ab$

(k) $Z = 3(a + b)$

(l) $d = 2(abc - 10)$

Solving Equations

- Trial and Error (Guess, Check and Improve)

EXERCISE 12C

- State if the following equations are correct(C) or incorrect(I)?
 (a) $2 \times 5 = 15$ (b) $8 \div 2 = 4$ (c) $8 + 6 = 15$
 (d) $9 - 7 = 2$ (e) $10 \times 4 = 40$ (f) $5 + 7 = 6 \times 2$
 (g) $2 \times (3 + 4) = 10$ (h) $14 - 8 = 36 \div 6$ (i) $5 \times 7 = 42 - 7$
- Find the missing number that would make these equations correct.
 (a) $_ \times 6 = 24$ (b) $8 \times _ = 32$ (c) $7 + _ = 13$
 (d) $25 - _ = 14$ (e) $_ - 5 = 22$ (f) $30 \div _ = 6$
- Change one number on the **left side** of the equals sign (=) in the following equations to make them correct.
 (a) $8 \times 3 = 16$ (b) $6 \times 9 = 60$ (c) $24 \div 5 = 5$
- Use trial and error to find the value of x in the following equations.

Examples

The left side of the equals sign(=) must be equal to the right side.

1. $x + 6 = 10$

Guess for x	Left side $x + 6$	Right side 10
5	$5 + 6 = 11$	10
3	$3 + 6 = 9$	10
4	$4 + 6 = 10$	10

Answer $x = 4$

2. $2x + 7 = 19$

Guess for x	Left side $2x + 7$	Right side 19
3	$2 \times 3 + 7 = 13$	19
4	$2 \times 4 + 7 = 15$	19
5	$2 \times 5 + 7 = 17$	19
6	$2 \times 6 + 7 = 19$	19

Answer $x = 6$

- | | | |
|---------------------------|---------------------------|----------------------|
| (a) $x + 7 = 9$ | (b) $x + 9 = 23$ | (c) $x - 5 = 2$ |
| (d) $x - 8 = 15$ | (e) $3x = 12$ | (f) $5x = 35$ |
| (g) $\frac{x}{2} = 8$ | (h) $\frac{x}{7} = 9$ | (i) $3x + 1 = 7$ |
| (j) $2x + 5 = 17$ | (k) $5x - 3 = 7$ | (l) $4x - 1 = 23$ |
| (m) $\frac{x}{4} + 1 = 5$ | (n) $\frac{x}{3} - 5 = 0$ | (o) $2(x + 1) = 12$ |
| (p) $3(x + 2) = 36$ | (q) $5(x - 1) = 30$ | (r) $3(2x + 3) = 27$ |

5. Use trial and error to find the value of n in the following equations.

Example

The left side of the equals sign(=) must be equal to the right side.

$$5n + 1 = 3(n + 3)$$

Guess for n	Left side $5n + 1$	Right side $3(n + 3)$
1	$5 \times 1 + 1 = 6$	$3 \times (1 + 3) = 12$
2	$5 \times 2 + 1 = 11$	$3 \times (2 + 3) = 15$
5	$5 \times 5 + 1 = 26$	$3 \times (5 + 3) = 24$
4	$5 \times 4 + 1 = 21$	$3 \times (4 + 3) = 21$

Answer $n = 4$

(a) $4n = 2n + 6$

(b) $\frac{n}{3} = n - 2$

(c) $\frac{n}{5} = 2n - 18$

(d) $n + 6 = \frac{n}{3} + 12$

(e) $3(n - 2) = 2n + 1$

(f) $\frac{n}{3} + 5 = n + 1$

(g) $n(n + 2) = 4n + 3$

(h) $n(n + 4) = 7n + 10$

(i) $(n + 2) \times (n + 3) = 11n + 13$

(j) $3n + 6 = 4n - 4$

6. Solve the following problems using trial and error.

(a) Mitchell is four years older than his brother Lachlan.

The sum of their ages is 30. How old is Mitchell?

(b) The product of which two consecutive numbers is 30?

(c) Find the two numbers that add to give 11 and multiply to give 24.

(d) Katie wanted to buy 50 kg of mulch. The mulch could be bought in either 11 kg or 6 kg bags. How many of each bag should she buy?

(e) Olivia bought seven paint brushes and four pencils. The prices were all rounded to the nearest dollar and she spent a total of \$29. What was the price of each paint brush and pencil?

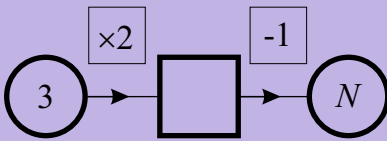
(f) In a class of 24 students there were five times as many right-handers as left-handers. How many left-handers were there?

Solving Equations - Backtracking

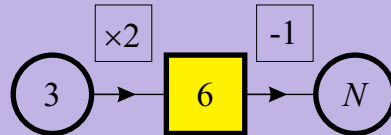
EXERCISE 12D

1. Find the value of N in each of the following flowcharts.

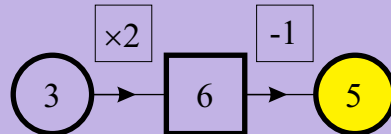
Example



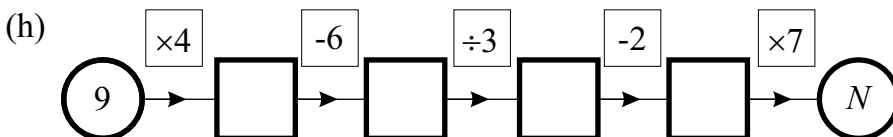
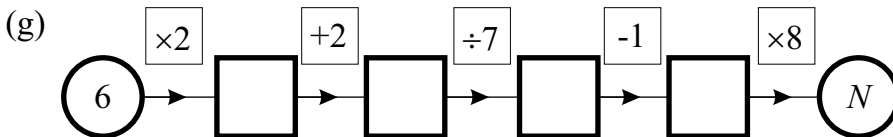
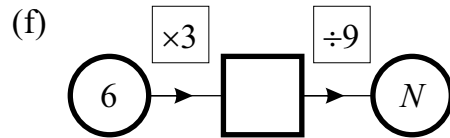
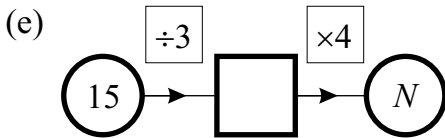
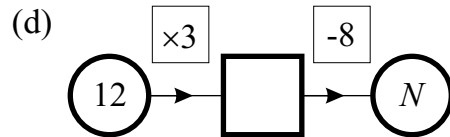
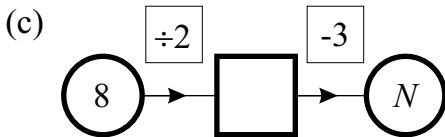
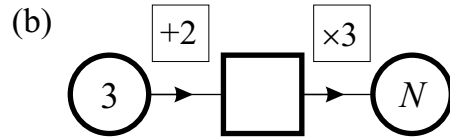
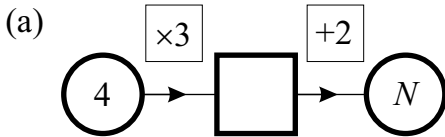
Step 1



Step 2



Answer $N = 5$



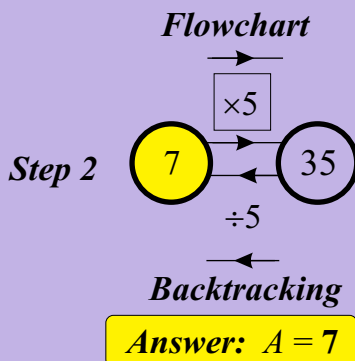
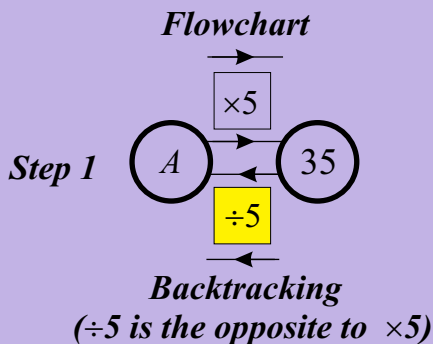
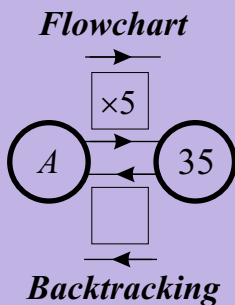
Backtracking is a technique that can be used to find the unknown quantity in an equation or flowchart.

Backtracking starts at the ‘answer’ and works back to the unknown quantity using **opposite operations** to those used in the **flowchart**.

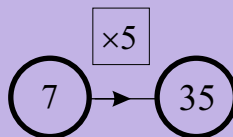
Operation	Opposite
Addition (+)	Subtraction (-)
Subtraction (-)	Addition (+)
Multiplication ($\times$)	Division ($\div$)
Division ($\div$)	Multiplication ($\times$)

Example 1

Find the value of A in the following flowchart.

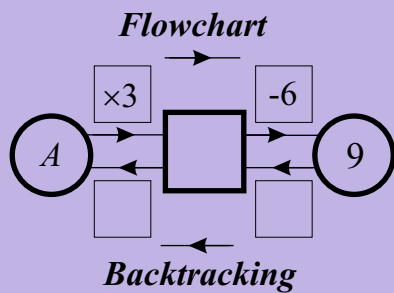
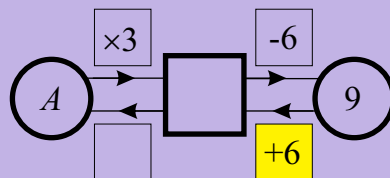
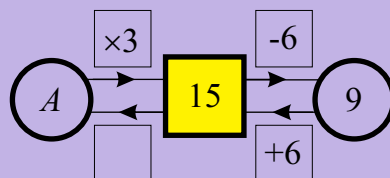
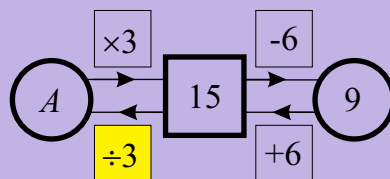
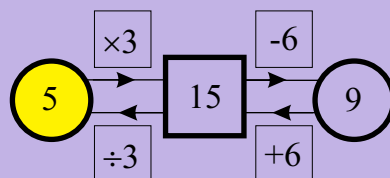


Step 3 Always check that the answer is correct in the flowchart.



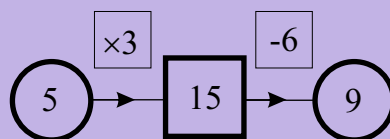
Example 2

Find the value of A in the following flowchart.

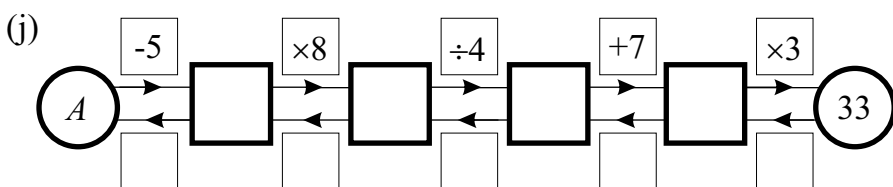
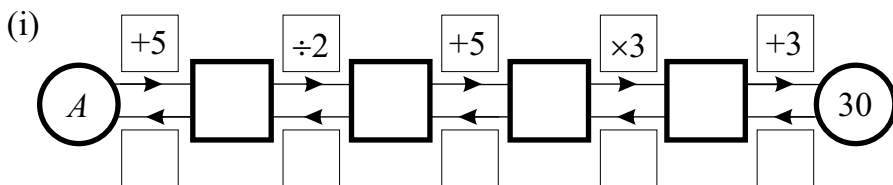
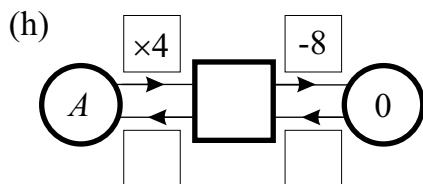
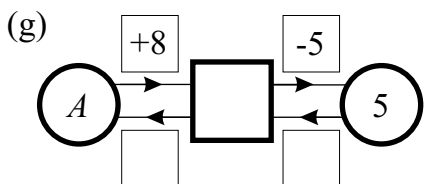
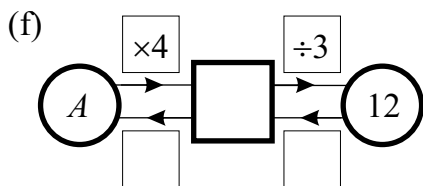
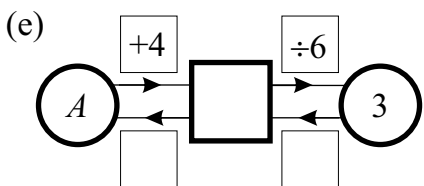
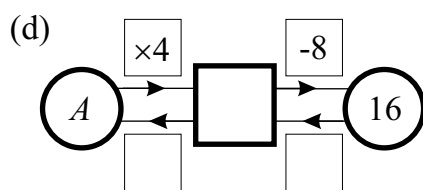
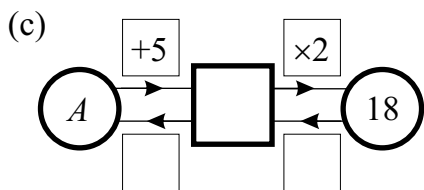
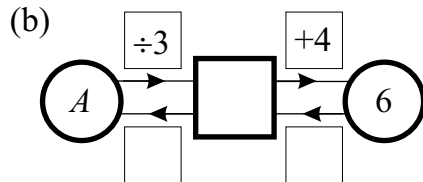
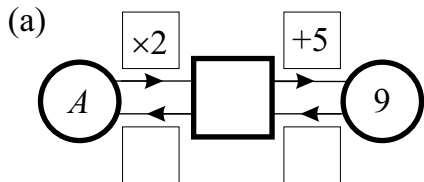
**Step 1****Step 2****Step 3****Step 4**

Answer $A = 5$

Step 5 Check the answer.



2. Copy and complete the following flowcharts using backtracking to find the value of A . Remember to check the answers.

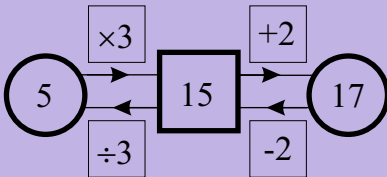
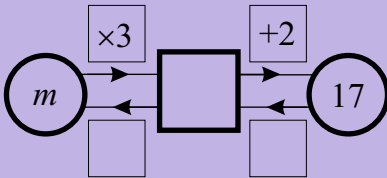


3. Change the following equations to flowcharts and use backtracking to find each value of m . Remember to check the answers.

Examples

1. $3m + 2 = 17$

Flowchart

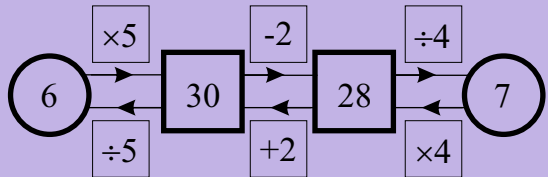
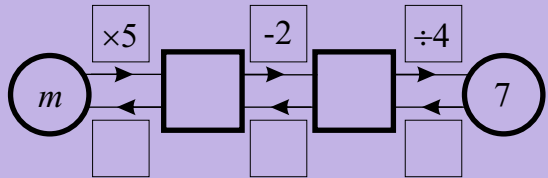


Backtracking

Answer $m = 5$

2. $\frac{5m - 2}{4} = 7$

Flowchart



Backtracking

Answer $m = 6$

(a) $2m + 6 = 14$

(b) $6m - 4 = 26$

(c) $3m - 12 = 24$

(d) $\frac{m}{5} - 1 = 2$

(e) $\frac{m + 4}{7} = 2$

(f) $\frac{m - 7}{4} = 1$

(g) $5(m + 3) = 20$

(h) $2(m - 3) = 12$

(i) $\frac{3m - 2}{5} = 2$

(j) $\frac{4m}{3} = 12$

(k) $\frac{7m}{2} = 14$

(l) $\frac{2m}{3} + 4 = 10$

(m) $\frac{m - 4}{2} + 5 = 7$

(n) $\frac{m + 3}{5} - 3 = 2$

(o) $3(2m + 5) = 21$

(p) $10(3m - 7) = 20$

(q) $\frac{4(m - 2)}{3} + 7 = 11$

(r) $\frac{6m - 10}{8} + 3 = 10$

4. (i) Write the following worded problems as equations.
 (ii) Use backtracking to find the value of the unknown letter.
 (a) When x is multiplied by 2 and 7 is added the result is 25.
 (b) If 6 is added to t and the result is multiplied by 3 the answer is 33.
 (c) If H is divided by 7 and 11 added to the result the answer is 15.
 (d) If 8 is subtracted from A and the result divided by 5 the answer is 3.
 (e) w is multiplied by 2, 9 is added to the result and then 3 is divided into the resultant number to give an answer of 7.

Solving Equations

- Doing the Same to Both Sides

Another way to solve equations is by using a technique called *doing the same to both sides*.

In this technique it is important to note that each side of the equals sign (=) must stay the same for the two sides to remain equal.

What happens to one side of the equation must also happen to the other side for the two sides to remain equal.

It is also important to remember the *opposite operations*.

<i>Operation</i>	<i>Opposite</i>
Addition (+)	Subtraction (-)
Subtraction (-)	Addition (+)
Multiplication (×)	Division (÷)
Division (÷)	Multiplication (×)

This technique of solving equations involves eliminating everything from the left side of the equation except the unknown quantity being solved for.

Example 1

Find the value of x in the following equation.

$$x - 5 = 7$$

Step 1 To eliminate $- 5$ from the left side of the equation the opposite operation ($+ 5$) is used.

If $+ 5$ is used on the left side of the equation it must also be used on the right side for the two sides to remain equal. Then simplify.

$$\begin{aligned} x - 5 + 5 &= 7 + 5 \\ x &= 12 \end{aligned}$$

Answer: $x = 12$

Step 2 Substitute the answer into the original equation to check that it is correct.

$$\text{Left side of the equation} = 12 - 5$$

$$= 7$$

$$= \text{right side of the equation}$$

Example 2

Find the value of x in the following equation.

$$2x + 3 = 11$$

Step 1 When there is more than one operation note the order of operations on x .

$$x \times 2 + 3$$

Step 2 Eliminate these numbers by working in the opposite order and using the opposite operations.

+ 3 first by doing - 3 to both sides of the equation.

$\times 2$ next by doing $\div 2$ to both sides of the equation.

Step 3 Perform these eliminations.

(a) Subtract 3 from both sides of the equation.

$$2x + 3 - 3 = 11 - 3$$

$$2x = 8$$

(b) Divide both sides of this equation by 2.

$$\frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

Answer: $x = 4$

Step 4 Always check that this answer is correct by substituting it into the left side of the original equation.

$$2x + 3 = 11$$

$$\text{Left side} = 2x + 3$$

$$\text{Substitute } x = 4$$

$$\text{Left side} = 2 \times 4 + 3$$

$$= 8 + 3$$

$$= 11$$

$$= \text{right side of the equation}$$

So the answer is correct

EXERCISE 12E

1. Find the value of x in the following equations.

(a) $x + 5 = 8$

(b) $x - 7 = 9$

(c) $3x = 21$

(d) $\frac{x}{4} = 5$

(e) $2x + 3 = 9$

(f) $3x - 4 = 2$

(g) $4x + 2 = 22$

(h) $5x - 6 = 9$

(i) $2x + 5 = 19$

(j) $\frac{x}{2} + 3 = 7$

(k) $\frac{x}{5} - 4 = 1$

(l) $\frac{x}{3} + 7 = 11$

(m) $\frac{x+4}{2} = 5$

(n) $\frac{x-3}{4} = 2$

(o) $\frac{x+5}{3} = 8$

(p) $\frac{3x}{4} = 6$

(q) $\frac{2x}{5} = 4$

(r) $\frac{3x+2}{4} = 5$

(s) $\frac{4x-1}{3} = 5$

(t) $\frac{4x}{3} + 1 = 9$

(u) $\frac{5x}{2} - 3 = 7$

2. (i) Write the following worded problems as equations.

(ii) Find the value of the unknown letter.

(a) When m is multiplied by 3 and 5 added to the result the answer is 26.

(b) When a is divided by 4 and 7 added to the result the answer is 12.

(c) When 6 is added to x and the result is divided by 5 the answer is 3.

(d) When n is multiplied by 4 and the result is divided by 3 the answer is 12.

3. Six bricks weighed 24 kg.

(a) Let b represent the weight of a brick. Write the above information as an equation.

(b) Solve this equation to find the weight of a brick.

4. Mickey took 24 minutes to run three laps of a park.

(a) Let t represent the time it takes for Mickey to run one lap. Write the above information as an equation.

(b) Solve this equation to find the time it takes Mickey to run one lap.

5. A drum contained 50 litres of water. Hermione filled a bucket six times from the drum and noted there were two litres of water remaining in the drum.

(a) Let b represent the amount of water the bucket could hold. Write the above information as an equation.

(b) Solve this equation to find the amount of water the bucket could hold.

PROBLEM SOLVING

1. The cost of buying tickets to go to a rugby game was \$15 per ticket plus a \$10 booking fee.

This could be written as the following equation:

$$C = 15 \times n + 10$$

or $C = 15n + 10$

Where C represents the total cost of buying the tickets and n represents the number of tickets bought.

- (a) What would be the cost of buying:

- (i) 2 tickets
- (ii) 6 tickets
- (iii) 20 tickets

- (b) How many tickets could be bought for:

- (i) \$70
- (ii) \$160
- (iii) \$235



2. Martina is six years older than her brother, Scott.

Martina's mother, Tanya, is three times Martina's age.

Martina's father, Cameron, is four years younger than Tanya.

The sum of all their ages is the same as Martina's great-grandmother, Beryl, who is seven times Martina's age.

- (a) Copy the table below.

- (b) Using m to represent Martina's age write symbols to represent the ages of the other members of her family.

As an example: Scott's age would be $m - 6$.

- (c) Use trial and error to find the age of each of the family members.

An example is given trying

15 as Martina's age.

	Symbol	Age of each person using a guess for Martina's age					
Martina's age	m	15					
Scott's age	$m - 6$	9					
Tanya's age		45					
Cameron's age		41					
	Total	110					
Beryl's age		105					

3. Copy this diagram into your books.

Use the numbers 1 - 9 to fill in the blanks so that the rows and columns add to give the numbers shown.

	+		+		=	
+		+		+		
	+		+		=	16
+		+		+		
	+		+		=	23
=		=		=		
16		13		16		

PUZZLES

1. The clues below can be used to find a word from this chapter.

It contains all the vowels.

The fifth letter is the twentieth letter of the alphabet.

Its last letter is the same as the last letter of the number on which the decimal system is based.

It is an eight letter word.

The second letter is usually followed by 'u'.

2. Copy and complete the following sentences.

- The process of working from the end to the start of a flowchart is called _ _ C _ T _ AC _ _ N _.
- One way to solve an equation is to ' _ U _ S _ , check and improve'.
- When we solve a problem we find the _ N _ W _ _.
- The answer to puzzle question 1 is _ _ U _ T _ _ _.
- The opposite of subtraction is _ _ D _ _ I _ _.
- Mathematical R _ L _ T _ O _ _ H _ _ S can be written as equations.
- The _ _ F _ side of an equation must equal the right side.

3. Rearrange the first letters of the missing words from puzzle question 2 to form a word that describes an area of mathematics.

CHAPTER REVIEW

1. Write the following relationships as equations.
 - (a) A equals the product of M and Q .
 - (b) B equals the sum of T and W .
 - (c) C is equal to 12 added to the product of M and U .
 - (d) D is equal to 5 more than E .

2. Find the value of n in the following equations if $m = 2$ and $p = 7$.
 - (a) $n = 2m$
 - (b) $n = m + p$
 - (c) $n = 3m + 2p$
 - (d) $n = 5(p - m)$
 - (e) $n = \frac{m + p}{3}$
 - (f) $n = \frac{6m}{4} + 8 - p$

3. Use trial and error to find the value of y in the following equations.
 - (a) $y + 7 = 10$
 - (b) $3y = 15$
 - (c) $\frac{y}{2} = 6$
 - (d) $2y + 1 = 11$
 - (e) $3(y - 1) = 18$
 - (f) $\frac{y}{3} - 2 = 2$
 - (g) $4y + 1 = 5y - 4$
 - (h) $4y + 6 = 6y$
 - (i) $2(y + 2) = 3y$

4. Find the value of A in the following flowcharts by using backtracking.

(a)

(b)

5. Change the following equations to flowcharts and use backtracking to find each value of k .
 - (a) $3k + 7 = 19$
 - (b) $4k - 3 = 9$
 - (c) $\frac{k}{2} + 5 = 11$
 - (d) $\frac{2k + 1}{3} = 5$
 - (e) $3(4k - 5) = 33$
 - (f) $\frac{k + 4}{3} - 2 = 4$

6. Solve the equations in question 5 by the *doing the same to both sides* method.

7. (a) Kelly has two dogs. Boofer is 2 kg heavier than Woofer. The total weight of the dogs is 10 kg. What is the weight of each dog?
 (b) Noah is 5 years older than his sister Lisa. In 4 years time Noah will be twice Lisa's age. How old are Noah and Lisa?

Simplifying

Addition and Subtraction

Algebra is the mathematics of using letters or symbols to make arithmetic simpler when dealing with physical quantities.

Example 1

Let the cost of an apple be represented by the letter a .
What would be the cost of 6 apples?

Answer: $6 \times a$
 $= 6a$ (*Remember the $\times$ sign does not need to be shown in algebra*)

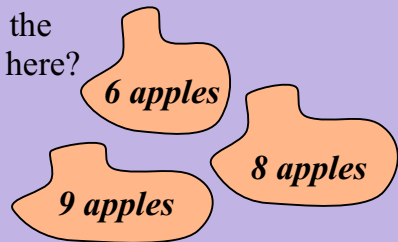
Example 2

If the cost of an apple was a what would be the total cost of the three bags of apples shown here?

The total cost would be:

$$6a + 8a + 9a$$
$$= 23a$$

Answer: $23a$



Example 3

The weight of bale of hay was represented by b .

(a) 25 bales of hay are loaded on a truck.

What is the total weight of the hay on the truck?

Answer: $25b$

(b) 8 bales of hay are taken off the truck.

What is the weight of the remaining bales of hay on the truck?

The remaining load would be:

$$25b - 8b$$
$$= 17b$$

Answer: $17b$

EXERCISE 13A

1. Let b represent the cost of a banana. What is the cost of 9 bananas?
2. It took Gemima w minutes to clean a window. How many minutes would it take her to clean 15 windows of the same size?
3. Let c represent the weight of a coconut. What would be the weight of a dozen coconuts?
4. Let s represent the length of rope needed to make a skipping rope. What length of rope would be needed to make 25 skipping ropes?
5. Let n represent the cost of a kilogram of mixed nuts. What would be the total cost of the three boxes of nuts shown below?

Mixed Nuts
3 kg

Mixed Nuts
4 kg

Mixed Nuts
2 kg

6. Let p represent the time to make a pizza. A pizza shop had four orders. One for two pizzas, one for five pizzas, one for three pizzas and one for six pizzas. In terms of p how long will it take to make all these pizzas?
7. A brick weighs b kg. A builder puts 12 bricks into a wheelbarrow.
 - (a) What is the total weight of the bricks in the wheelbarrow?
 - (b) If the builder uses 5 of the bricks, what is the weight of the remaining bricks in the wheelbarrow?
8. It takes a potter v minutes to make a vase. He has an order for 15 of these vases.
 - (a) How long will it take him to make all these vases?
 - (b) He makes 11 of the vases. How much longer will it take him to complete the 15 vases?
9. Two builders are loading bags of cement onto a truck. Each bag of cement weighs c kg. One builder loads 15 bags and the other loads 18 onto the truck. They take the truck to a work site and use 12 bags of the cement on a job. What is the total weight of the bags of cement remaining on the truck?

EXERCISE 13B

1. Simplify the following.

Examples

Note: x means $1x$ - the number 1 does not need to be shown.

$$\begin{aligned} 1. \quad & 3x + x \\ &= 3x + 1x \\ &= \mathbf{4x} \end{aligned}$$

$$\begin{aligned} 2. \quad & m + m \\ &= 1m + 1m \\ &= \mathbf{2m} \end{aligned}$$

$$\begin{aligned} 3. \quad & p + 7p \\ &= \mathbf{8p} \end{aligned}$$

(a) $6n + n$ (b) $y + 5y$ (c) $7t + t$ (d) $k + k$

2. Simplify the following.

Examples

$$\begin{aligned} 1. \quad & a + a + a + a \\ &= \mathbf{4a} \end{aligned}$$

$$\begin{aligned} 2. \quad & 3w + w + 2w \\ &= \mathbf{6w} \end{aligned}$$

$$\begin{aligned} 3. \quad & 8x + 3x + x + 5x \\ &= \mathbf{17x} \end{aligned}$$

(a) $c + c + c$ (b) $2u + 5u + u$ (c) $3g + g + 6g$
 (d) $e + e + e + 7e$ (e) $h + 4h + h + 9h$ (f) $4d + 7d + 3d + 2d$
 (g) $8u + 7u + 5u + u$ (h) $4s + 9s + s + s$ (i) $7A + 6A + A + 6A$

3. Simplify the following.

Examples

$$\begin{aligned} 1. \quad & 8p - 5p \\ &= \mathbf{3p} \end{aligned}$$

$$\begin{aligned} 2. \quad & 7x - x \\ &= \mathbf{6x} \end{aligned}$$

$$\begin{aligned} 3. \quad & 5n - 5n \\ &= \mathbf{0} \end{aligned}$$

(a) $4y - y$ (b) $9m - 4m$ (c) $6v - 5v$ (d) $8f - 8f$

4. Simplify the following.

Examples

$$\begin{aligned} 1. \quad & 6x + 4x + x - 3x \\ &= \mathbf{8x} \end{aligned}$$

$$\begin{aligned} 2. \quad & 4a + 2a - 3a + 7a - a \\ &= \mathbf{9a} \end{aligned}$$

(a) $2n + 5n + n - 3n$ (b) $4p + 6p + 3p - 5p$
 (c) $7y + 3y - 2y + 4y$ (d) $5a + 2a - 6a + 8a - a$

5. Simplify the following.

- (a) $7m - 2m + 8m - m + 4m$ (b) $9B + B - 6B + 4B - 8B$
 (c) $2K - K - K + 12K + 3K$ (d) $12P + 5P - 9P - 3P + 2P$
 (e) $8q + 7q - q - 2q - 3q + q$ (f) $12d + 6d - d + 4d - 15d - 5d$
 (g) $3x + x - 4x + 6x - x - x$ (h) $11y + 7y - y + 2y - 8y + 3y - 12y$

6. Simplify the following.

***Different letters or symbols cannot be combined when adding or subtracting.
Only like letters or symbols can be added to or subtracted from each other.***

Examples

$$\begin{aligned} 1. \quad & 3a + 2a + 4b + 7b \\ & = 5a + 11b \end{aligned}$$

$$\begin{aligned} 2. \quad & 3x + x + 5y + 2y \\ & = 4x + 7y \end{aligned}$$

- (a) $4a + 5a + 3b + 2b$ (b) $5x + 2x + 3y + 6y$
 (c) $2p + p + 5q + 2q$ (d) $3m + m + n + n$
 (e) $k + 5k + 6h + h$ (f) $5T + 6T + 7U + U$

7. Simplify the following.

- (a) $4a + 2a + a + 6b + b + b$ (b) $3g + 5g + 4g + 2h + h + h + 5h$
 (c) $2m + m + 3n + 6n + 2n$ (d) $7x + 4x + x + 3x + 2y + 2y + y$
 (e) $4N + N + 2M + M + 3M$ (f) $2A + 3A + B + 3B + 5B + 12B$
 (g) $5e + 2e + e + e + 5f$ (h) $6c + 4d + d + 5d + 2d$
 (i) $2p + 4p + 6p + p + q$ (j) $2w + w + 2v + v + 3v + 4v + 5v$

8. Simplify the following.

Example
$$\begin{aligned} & 2x + 3x + x + 2y + 4y + 5y + 2z + z \\ & = 6x + 11y + 3z \end{aligned}$$

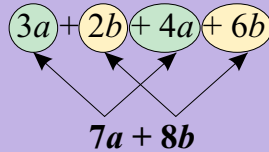
- (a) $a + 2a + 2b + 3b + 5c + 4c$
 (b) $2x + 6x + 2y + y + 4y + 2y + 2z + 2z + 3z$
 (c) $A + 4A + 6A + 2B + 8B + 4B + C + C + 5C + 6C$
 (d) $4m + 2m + m + m + 2n + 3n + 9n + p + 3p + 2p$
 (e) $2f + f + 2g + g + 6g + 7g + 3h + 2h + h + 9h + 5h$

9. Simplify the following.

Example

$$3a + 2b + 4a + 6b$$

Remember: only like letters can be added



Another method of solving these types of problems is to rearrange the terms so like terms are next to each other.

$$\begin{aligned} 3a + 2b + 4a + 6b \\ = 3a + 4a + 2b + 6b \\ = 7a + 8b \end{aligned}$$

(a) $2x + 3y + x + 4y$

(c) $5a + b + 5a + 4b$

(e) $2p + 6q + 4q + p$

(g) $3r + t + 8t + 5r$

(b) $5m + 2n + 4m + n$

(d) $2C + 4D + 3C + 8D$

(f) $2M + 6N + 3N + 9M$

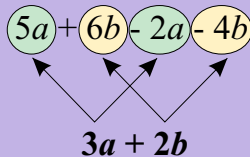
(h) $7J + 3K + 11K + 5J$

10. Simplify the following.

Example

$$5a + 6b - 2a - 4b$$

Make sure that subtraction signs are included when simplifying.



Rearranging the terms:

$$\begin{aligned} 5a + 6b - 2a - 4b \\ = 5a - 2a + 6b - 4b \\ = 3a + 2b \end{aligned}$$

(a) $6x + 5y - 4x - 2y$

(c) $9p + 3q - 6p - q$

(e) $7y + 11z - 6y - 9z$

(g) $8A + 6B - 7A - 6B$

(b) $7m + 8n - 2m - 3n$

(d) $5g + 7h - g - h$

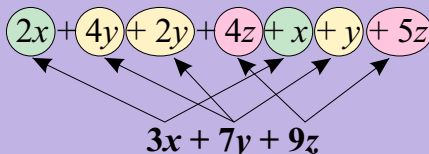
(f) $11M + 12N - M - 11N$

(h) $6h + 9k - 6h - 9k$

11. Simplify the following.

Example

$$2x + 4y + 2y + 4z + x + y + 5z$$



Rearranging the terms:

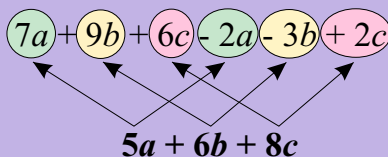
$$\begin{aligned} & 2x + 4y + 2y + 4z + x + y + 5z \\ &= 2x + x + 4y + 2y + y + 4z + 5z \\ &= 3x + 7y + 9z \end{aligned}$$

- (a) $2a + 4b + c + 2c + 3a + 2b + 2c$
- (b) $6m + 2n + 3m + 2q + n + q + 2m + m$
- (c) $2A + 3B + 5A + C + B + 2A + 7C + 8B$
- (d) $x + y + x + y + z + 2z + 2y + 3z + 3x + 2y$
- (e) $g + 2h + 3k + 4k + 5h + 6g + 7h + 8k + 9g$
- (f) $10P + 8Q + 12R + 8P + 7Q + 6P + 4Q + 7P$
- (g) $2L + 3W + 8G + 2W + G + 5L + W + 6G$

12. Simplify the following.

Example

$$7a + 9b + 6c - 2a - 3b + 2c$$



Rearranging the terms:

$$\begin{aligned} & 7a + 9b + 6c - 2a - 3b + 2c \\ &= 7a - 2a + 9b - 3b + 6c + 2c \\ &= 5a + 6b + 8c \end{aligned}$$

- (a) $7m + 4n + 6p - 3m - 2n - 4p$
- (b) $8x + 6y + 3z - 5x - 2y + z$
- (c) $5c + 6d + 8e - 2c + 4d - 5e$
- (d) $4a + 7b + 9c + 3a - 6b - c$
- (e) $9p + 11q + 4r - 6p + 4q - 4r$
- (f) $3X + 8Y + 12Z - 5Y + 8X + Z$
- (g) $5a + b + 4c + b - 4a - 3c$
- (h) $2m + 7n + 11p - m - p - 6n$
- (i) $2x + 3y + 4z - 2y - 3z - x$
- (j) $4A + 5B + 3C - 3C - 3A - 5B$

Multiplication

Algebraic terms consist of a number and one or more letters or symbols. When algebraic terms are multiplied follow these steps:

Step 1 Multiply the numbers.

Step 2 Write the letters in alphabetical order. (This is preferred but other combinations are not incorrect: $ab = ba$)

Remember - in algebra the multiplication sign ($\times$) does **not** need to be shown.

Examples

$$\begin{aligned} 1. \quad & 4 \times 3x \\ & = 12x \end{aligned}$$

$$\begin{aligned} 2. \quad & 5a \times 6b \\ & = 5 \times 6 \times ab \\ & = 30ab \end{aligned}$$

$$\begin{aligned} 3. \quad & 4n \times 8m \\ & = 4 \times 8 \times mn \\ & = 32mn \end{aligned}$$

$$\begin{aligned} 4. \quad & 2y \times x \times 5z \\ & = 2y \times 1x \times 5z \\ & = 2 \times 1 \times 5 \times xyz \\ & = 10xyz \end{aligned}$$

$$\begin{aligned} 5. \quad & 4r \times 2c \times 3w \\ & = 4 \times 2 \times 3 \times crw \\ & = 24crw \end{aligned}$$

$$\begin{aligned} 6. \quad & q \times 2m \times 4 \times 3p \\ & = 2 \times 4 \times 3 \times mpq \\ & = 24mpq \end{aligned}$$

EXERCISE 13C

1. Simplify the following.

(a) $5 \times 3a$

(b) $2 \times 7b$

(c) $6y \times 7$

(d) $4f \times 6$

(e) $8 \times 5m$

(f) $g \times 5 \times 9$

(g) $3 \times n \times 7$

(h) $5 \times 2 \times c \times 2$

2. Simplify the following.

(a) $2x \times y$

(b) $g \times 7f$

(c) $3k \times 4l$

(d) $2d \times 8c$

(e) $5m \times 6n$

(f) $4 \times a \times b \times 3$

(g) $4h \times 7g$

(h) $9q \times 3p$

(i) $2k \times 4 \times 2j$

(j) $y \times 4 \times 7z$

(k) $r \times 3 \times t \times 8$

(l) $4M \times 2 \times 5P$

3. Simplify the following.

(a) $3b \times a \times c$

(b) $h \times 7g \times i$

(c) $2p \times r \times 3q$

(d) $4A \times 2B \times 2C$

(e) $4Z \times 6X \times Y$

(f) $3t \times 5 \times 2u \times 2s$

(g) $5h \times 2j \times 5k$

(h) $2d \times 2f \times 2e$

(i) $3N \times 2 \times 4P \times 2Q$

(j) $7f \times 3d \times 2e$

(k) $6u \times 2w \times 7v$

(l) $6 \times 3K \times 2M \times 4H$

Example 1

$$a \times a \times a \\ = a^3$$

The 3 means that there are three a 's multiplied.
This number is called an **index**.
(The plural of index is **indices**)

Indices are used to show when the same letters are multiplied.

Example 2

$$m \times m \times m \times m \\ = m^4$$

Example 3

$$4x \times 5x \\ = 4 \times 5 \times x \times x \\ = 20x^2$$

EXERCISE 13D

1. Simplify the following.

- | | | |
|---|--------------------------------------|--|
| (a) $a \times a \times a \times a \times a$ | (b) $p \times p \times p$ | (c) $m \times m \times m \times m \times m \times m$ |
| (d) $4 \times b \times b \times b \times b$ | (e) $2 \times w \times 6 \times w$ | (f) $8 \times n \times 2 \times n \times n \times n$ |
| (g) $2 \times v \times 2 \times v \times 2$ | (h) $6t \times 5t$ | (i) $3m \times 2m \times m \times m$ |
| (j) $2k \times 5k \times 6k$ | (k) $3r \times 2r \times r \times 8$ | (l) $2x \times 2x \times 2x \times 2x \times 2x$ |

2. Simplify the following.

Example 1

$$x \times x \times y \times y \times y \times y \\ = x^2 \times y^4 \\ = x^2y^4$$

Example 2

$$g \times h \times g \times h \times g \times g \times g \times h \\ = g \times g \times g \times g \times g \times h \times h \times h \\ = g^5h^3$$

Example 3

$$2 \times m \times 3 \times n \times m \times n \times n \times 4 \\ = 2 \times 3 \times 4 \times m \times m \times n \times n \times n \\ = 24m^2n^3$$

Example 4

$$3a \times a \times 2b \times a \times 5b \times a \\ = 3 \times 2 \times 5 \times a \times a \times a \times a \times b \times b \\ = 30a^4b^2$$

- | | |
|--|---|
| (a) $a \times a \times b \times b \times b \times b \times b$ | (b) $m \times m \times m \times m \times m \times n \times n \times n$ |
| (c) $h \times g \times h \times h \times g \times h \times h \times g$ | (d) $x \times x \times y \times y \times x \times y \times x \times y \times x \times y$ |
| (e) $2 \times c \times 3 \times c \times 2 \times d \times c \times d$ | (f) $k \times 2 \times j \times 5 \times j \times k \times 2 \times k \times j \times j \times j$ |
| (g) $3v \times 3w \times 2v \times w \times w \times v \times 2w$ | (h) $4p \times 2q \times 2q \times 4q \times p \times p \times q \times p$ |
| (i) $3a \times b \times 2a \times 2a \times 2a \times a \times 2a$ | (j) $2x \times 4x \times 5x \times y \times x \times x \times x \times x \times 2x$ |

Division

EXERCISE 13E

1. Simplify the following.

Cancel the numbers in the fraction by dividing the top and bottom numbers by the highest common factor.

Example 1 $\frac{8a}{4}$ In this example the highest common factor is 4.

$$= \frac{\overset{2}{\cancel{8}}a}{\underset{1}{\cancel{4}}}$$

$$= \frac{2a}{1}$$

$$= 2a$$

Example 2 $\frac{18m}{12}$ In this example the highest common factor is 6.

$$= \frac{\overset{3}{\cancel{18}}m}{\underset{2}{\cancel{12}}}$$

$$= \frac{3m}{2}$$

(a) $\frac{10n}{5}$ (b) $\frac{16a}{4}$ (c) $\frac{24b}{3}$ (d) $\frac{30c}{10}$ (e) $\frac{28u}{7}$

(f) $\frac{16h}{12}$ (g) $\frac{24x}{18}$ (h) $\frac{24}{30m}$ (i) $\frac{36v}{32}$ (j) $\frac{45a}{72}$

(k) $\frac{8i}{24}$ (l) $\frac{15}{25a}$ (m) $\frac{6}{42n}$ (n) $\frac{8c}{40}$ (o) $\frac{28}{56p}$

2. Simplify the following.

If there are the **same letters** on the top and bottom of a fraction they can be cancelled.

Example 1

$$\begin{aligned}
 & \frac{12a}{6a} \\
 = & \frac{\overset{2}{\cancel{12}}a}{\underset{1}{\cancel{6}}a} && \text{First cancel the numbers.} \\
 = & \frac{\overset{2}{\cancel{2}}\cancel{a}}{\cancel{a}} && \text{Then cancel the letters.} \\
 = & \frac{2}{1} \\
 = & \mathbf{2}
 \end{aligned}$$

Example 2

$$\begin{aligned}
 & \frac{16a}{24ab} \\
 = & \frac{\overset{2}{\cancel{16}}a}{\underset{3}{\cancel{24}}ab} && \begin{array}{l} \text{First cancel the numbers.} \\ \text{(Highest common factor = 8)} \end{array} \\
 = & \frac{\overset{2}{\cancel{2}}\cancel{a}}{\underset{3}{\cancel{3}}\cancel{a}b} && \text{Then cancel the same letters on the} \\
 & && \text{top and the bottom.} \\
 = & \frac{\mathbf{2}}{\mathbf{3b}}
 \end{aligned}$$

(a) $\frac{18n}{3n}$

(b) $\frac{20a}{5a}$

(c) $\frac{24b}{8b}$

(d) $\frac{9c}{18c}$

(e) $\frac{7u}{35u}$

(f) $\frac{4ab}{24a}$

(g) $\frac{2x}{12xy}$

(h) $\frac{24mn}{36m}$

(i) $\frac{40v}{32vw}$

(j) $\frac{12k}{48kl}$

(k) $\frac{56fh}{8h}$

(l) $\frac{5a}{25ab}$

(m) $\frac{6mn}{42n}$

(n) $\frac{18c}{40cd}$

(o) $\frac{35uv}{56v}$

3. Simplify the following.

Example 1

$$\frac{8ab}{24abc}$$

$$= \frac{\overset{1}{8}ab}{\underset{3}{24}abc}$$

First cancel the numbers.
(Highest common factor = 8)

$$= \frac{\cancel{a}\cancel{b}}{3\cancel{a}bc}$$

Then cancel all the letters that are
the same on the top and bottom.

$$= \frac{1}{3c}$$

Example 2

$$\frac{24cde}{18cef}$$

$$= \frac{\overset{4}{24}cde}{\underset{3}{18}cef}$$

First cancel the numbers
(Highest common factor = 6)

$$= \frac{4\cancel{c}de}{3\cancel{c}ef}$$

Then cancel all the letters that are
the same on the top and bottom.

$$= \frac{4d}{3f}$$

(a) $\frac{6xy}{18xyz}$

(b) $\frac{28abc}{14ac}$

(c) $\frac{16pr}{18pqr}$

(d) $\frac{21cdk}{7dk}$

(e) $\frac{8uw}{32uvw}$

(f) $\frac{4abd}{28acd}$

(g) $\frac{8uxz}{12xyz}$

(h) $\frac{28mrt}{36mtx}$

(i) $\frac{9kmn}{27kmn}$

(j) $\frac{72hkt}{81htw}$

(k) $\frac{32ach}{36ahg}$

(l) $\frac{42uwx}{36wxy}$

(m) $\frac{12prt}{48pqv}$

(n) $\frac{72ace}{36cdf}$

(o) $\frac{64aht}{16bgs}$

Example 1

$$\frac{6a^2}{9a^5}$$

$$= \frac{\overset{2}{\cancel{6}}a^2}{\underset{3}{\cancel{9}}a^5}$$

First cancel the numbers
(Highest common factor = 3)

$$= \frac{2a^2}{3a^5}$$

$$= \frac{2 \times \cancel{a} \times \cancel{a}}{3 \times a \times a \times a \times \cancel{a} \times \cancel{a}}$$

If the indices are expanded
it can be seen that the same
number of the letters can be
cancelled from the top and
bottom of the fraction.

$$= \frac{2}{3 \times a \times a \times a}$$

$$= \frac{2}{3a^3}$$

Example 2

$$\frac{15x^3z}{12x^7yz}$$

$$= \frac{\overset{5}{\cancel{15}}x^3z}{\underset{4}{\cancel{12}}x^7yz}$$

First cancel the numbers
(Highest common factor = 3)

$$= \frac{5x^3z}{4x^7yz}$$

$$= \frac{\overset{1}{\cancel{5}}x^{\cancel{3}}z}{\underset{x^4}{\cancel{4}}x^7yz}$$

Cancel the letter with indices.

$$= \frac{5z}{4x^4yz}$$

$$= \frac{\cancel{5}z}{4x^4\cancel{y}z}$$

Cancel any other letters that are
the same on the top and the
bottom of the fraction.

$$= \frac{5}{4x^4y}$$

4. Simplify the following.

$$\begin{array}{lllll} \text{(a)} \frac{8x^4}{4x^2} & \text{(b)} \frac{3a^2}{15a^6} & \text{(c)} \frac{20n^4}{12n} & \text{(d)} \frac{12c}{16c^5} & \text{(e)} \frac{18v^3}{9v} \\ \text{(f)} \frac{6m^3}{24m^7} & \text{(g)} \frac{21x^5}{28x^6} & \text{(h)} \frac{18v^6}{24v^8} & \text{(i)} \frac{48r^8}{36r} & \text{(j)} \frac{60s}{72s^5} \end{array}$$

5. Simplify the following.

$$\begin{array}{lllll} \text{(a)} \frac{15x^6y}{18x^4yz} & \text{(b)} \frac{24a^3bc}{16a^8c} & \text{(c)} \frac{22pr^5}{18pqr^4} & \text{(d)} \frac{21ce^4k}{27e^4k} & \text{(e)} \frac{30uw^7}{32uvw} \\ \text{(f)} \frac{14a^7bd}{28a^4cd} & \text{(g)} \frac{48uxz^9}{12xyz^4} & \text{(h)} \frac{9m^{10}rt}{36m^6tx} & \text{(i)} \frac{90km^7n}{27km^4n} & \text{(j)} \frac{72hkv^{11}}{54hv^{14}w} \end{array}$$

6. Simplify the following.

Example 1 $6m \div 26$

$$= \frac{6m}{26}$$

$$= \frac{3m}{13}$$

Example 2 $45xy \div 9x^3y$

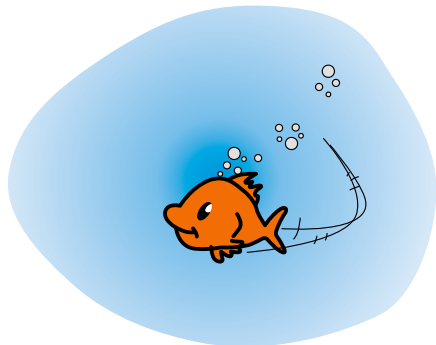
$$= \frac{45xy}{9x^3y}$$

$$= \frac{5}{x^2}$$

$$\begin{array}{lll} \text{(a)} 8a \div 12 & \text{(b)} 24 \div 6m & \text{(c)} 9x \div 27 \\ \text{(d)} 15v \div 30v & \text{(e)} 36u \div 24u & \text{(f)} 8w \div 48w \\ \text{(g)} 16ab \div 24a & \text{(h)} 9xy \div 9y & \text{(i)} 7m \div 28mn \\ \text{(j)} 15abc \div 18ac & \text{(k)} 9mnp \div 14p & \text{(l)} 8vw \div 32vwy \\ \text{(m)} 8x^2 \div 20x & \text{(n)} 42m^5 \div 36m^3 & \text{(o)} 56a^3bc \div 32ac \end{array}$$

PROBLEM SOLVING

1. Hervey has a new fish tank and wants to buy some fish. He went to the local pet shop and decides to buy some goldfish, guppies and tetras. From the following information find the cost of each type of fish.



Two goldfish, three guppies and four tetras would cost a total of \$43.
 Three goldfish, two guppies and four tetras would cost a total of \$46.
 Four goldfish, three guppies and four tetras would cost a total of \$59.

2. Simplify the following.

$$\frac{3a \times 2b \times 2c \times 3a^2 \times b^3 \times 4c^3 \times 2a^4 \times 2b \times 5c^3}{6a^2 \times b^3 \times c^2 \times 4a \times 3b^2 \times 4a^4 \times 10c^5}$$

PUZZLE

Copy and complete this puzzle by adding **A**, **B**, **C**, **D** or **E** to each of the squares. There must be only one of each letter in each column and row.

A				
	B			
		C		
			D	
				E

CHAPTER REVIEW

1. The cost of potatoes is p dollars per kilogram.
What is the cost of 6 kilograms of potatoes?
2. Simplify the following.
 (a) $a + 5a$ (b) $4d + 3d + d$ (c) $7u - 2u$ (d) $4x + 2x - x + 3x$
3. Simplify the following.
 (a) $3n + 4n + 2p + 3p$ (b) $6a + 4b + 3b + a$
 (c) $2m + 4m + 6n - 2n$ (d) $3y + 7z - 2y - 3z$
4. Simplify the following.
 (a) $2a + 3b + c + 4a + 5b + 2c$ (b) $3x + 7y + 8z + 2x - 3y - 5z$
5. Simplify the following.
 (a) $5 \times 4a$ (b) $2 \times n \times 7$ (c) $3 \times t \times u$ (d) $3x \times 5y$
6. Simplify the following.
 (a) $2 \times a \times 3b \times 4 \times c$ (b) $5f \times 2h \times 7 \times g$
7. Simplify the following.
 (a) $c \times c \times c \times c \times c$ (b) $2n \times n \times 3m \times m \times n \times 2m \times n$
8. Simplify the following.
 (a) $\frac{18n}{6}$ (b) $\frac{20}{12a}$ (c) $\frac{28b}{12b}$ (d) $\frac{8cd}{18c}$ (e) $\frac{24u}{36uv}$
 (f) $\frac{3uvx}{12ux}$ (g) $\frac{45adf}{36abf}$ (h) $\frac{32a^6}{18a^2}$ (i) $\frac{16a}{48a^4}$ (j) $\frac{8m^2n^5}{18m^6n^3}$
9. Simplify the following.
 (a) $6 \div 15n$ (b) $24ab \div 16a$ (c) $21m^2n \div 15mn$ (d) $6g^4h^5 \div 20g^7h^2$

Graphs

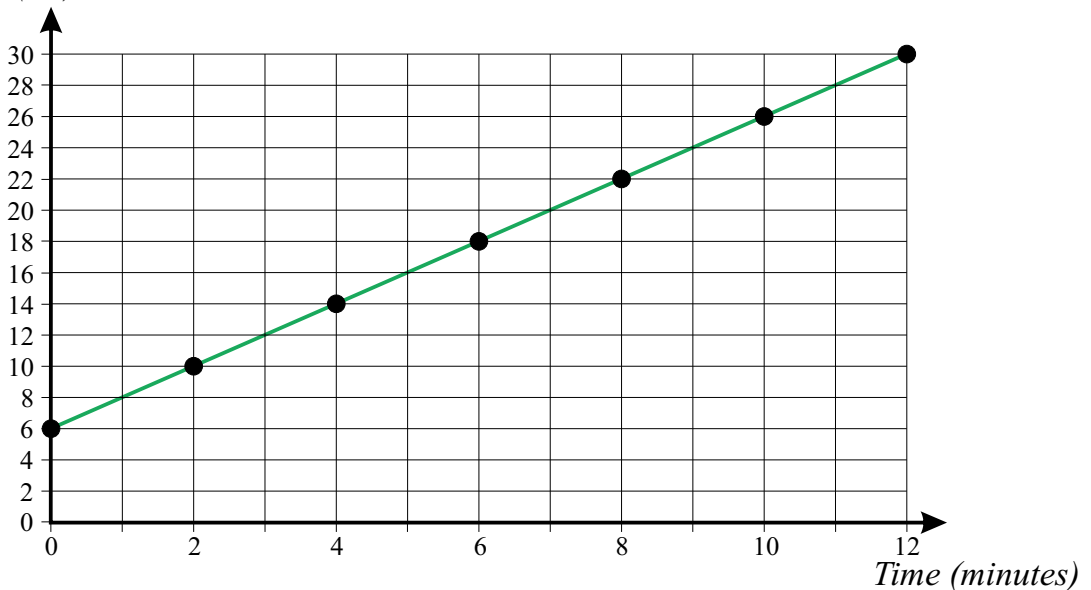
Interpreting Graphs

EXERCISE 14A

1. In a science class a beaker of cold water was heated and the temperature was recorded every two minutes. The results are shown in the table and graph below. The points have been connected with a line.

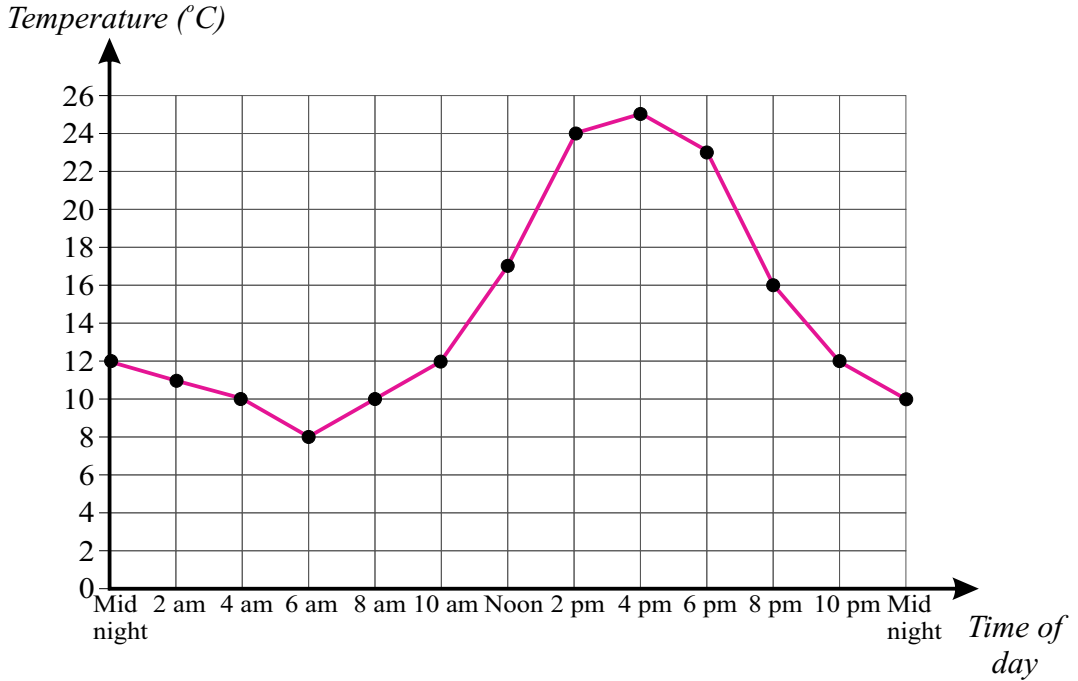
Time (mins)	0	2	4	6	8	10	12
Temperature ($^{\circ}\text{C}$)	6	10	14	18	22	26	30

Temperature
($^{\circ}\text{C}$)



- What was the water temperature before the heating started?
- What was the water temperature after 12 minutes of heating?
- What was the water temperature after 6 minutes of heating?
- What was the water temperature after 9 minutes of heating?

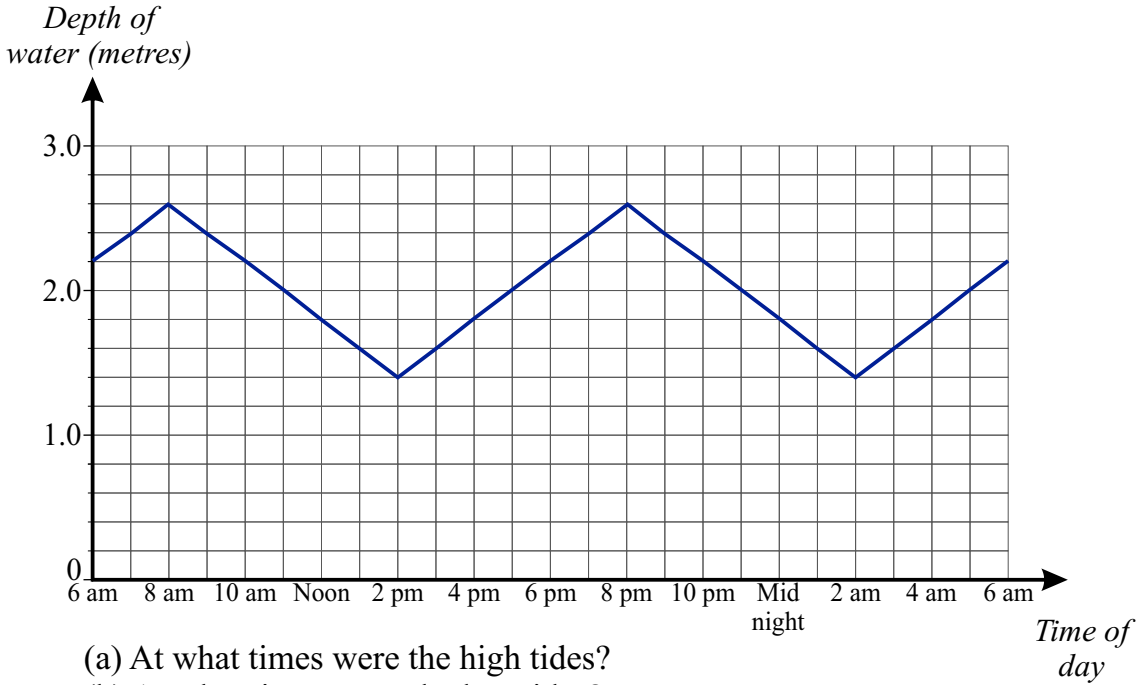
2. The temperatures(in $^{\circ}\text{C}$) in a town were recorded every two hours over a 24 hour period and represented on the graph shown. The points have been connected with a line.



Use the graph to answer the following questions.

- What was the temperature at the following times?
(i) 10 am (ii) noon (iii) 5 pm (iv) 9 pm
- At what times was the temperature 10°C ?
- What was the maximum temperature during the 24 hour period?
- What was the minimum temperature during the 24 hour period?
- At what time did the maximum temperature occur?
- At what time did the minimum temperature occur?
- What is the difference between the maximum and minimum temperatures?
- What is the difference between the temperatures at the start and end of the 24 hour period?

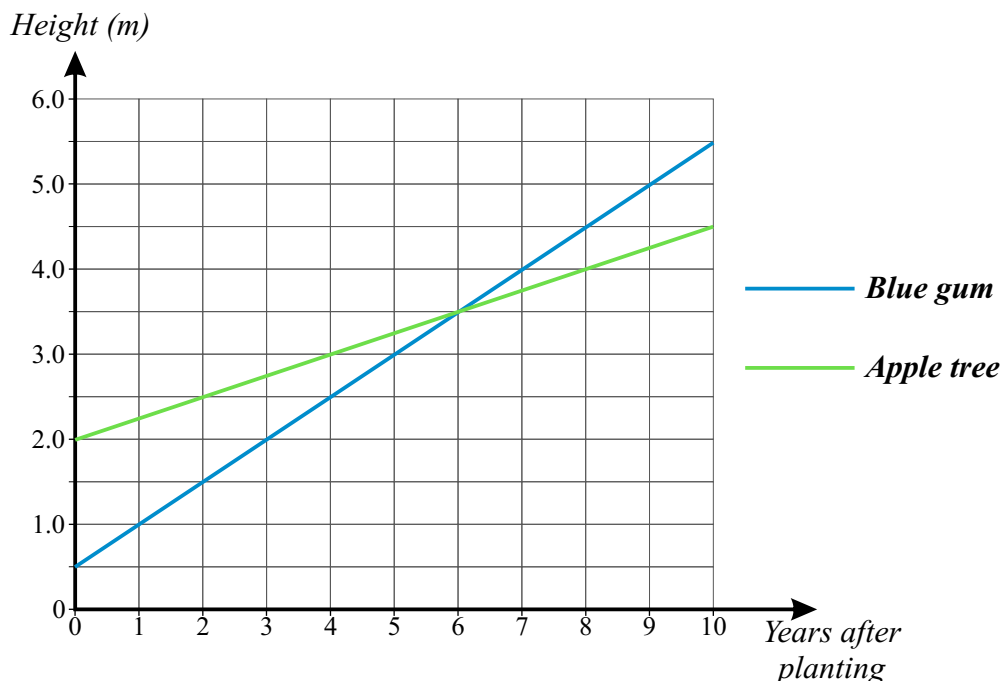
3. Rowan wanted to study the tides at a seaside town. He measured the depth of the water at the end of a pier every hour for 24 hours. The measurements are shown on the graph below.



- At what times were the high tides?
- At what times were the low tides?
- How many hours are there between high tides?
- What is the depth of the water at high tide?
- What is the depth of the water at low tide?
- What is the difference in the depth of the water between low and high tide?
- Rowan heard that a good time to go fishing was two hours after a low tide. If he wants to go fishing in the afternoon, what time should he do this?



4. Katrina planted two trees in her garden - a blue gum and an apple tree. She measured the height of the trees when she planted them and each year for the next ten years. The heights of the trees are shown on the graph below.



- What was the height of each tree when they were planted?
- What was the height of each tree 10 years after being planted?
- How many years after planting were the trees the same height?
- How tall were the trees when they were the same height?
- How tall was each tree 4 years after being planted?
- How many years after planting was the blue gum 4 metres tall?
- How many years after planting was the apple tree 4 metres tall?
- How many years after planting was the apple tree 1 metre taller than the blue gum?
- How many years after planting was the blue gum 1 metre taller than the apple tree?
- How many metres did each tree grow each year?
- Which tree grew at the faster rate?
- If the trees continue growing at the rate they did in the first 10 years, calculate the height of each tree:
 - 12 years after they were planted.
 - 20 years after they were planted.

5. Ivan was studying the population of frogs, fish and ducks in a lake after it was formed into a state park.

He recorded the approximate numbers of the animals each year for 10 years after the formation of the park. His records are shown on the table below.

	Number of years after formation of park										
	0	1	2	3	4	5	6	7	8	9	10
Number of frogs	300	310	320	330	340	350	360	370	380	390	400
Number of fish	100	125	150	175	200	225	250	275	300	325	350
Number of ducks	50	100	150	200	250	300	350	400	450	500	550

- (a) On graph paper neatly draw the axes as shown here.

(b) On these axes plot the information that is given in the table showing the population of frogs, fish and ducks in the lake for each year after the state park was formed.

(c) Connect the points for each animal with a straight line.

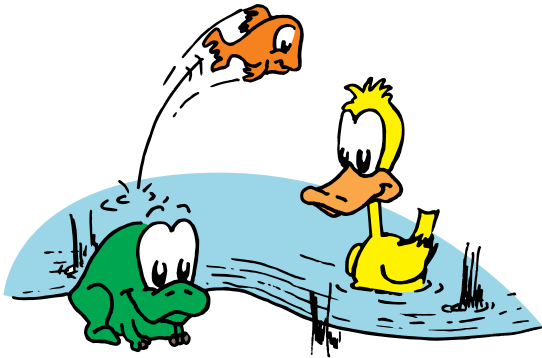
(d) Which animal's population is increasing at the fastest rate?

(e) Which animal's population is increasing at the slowest rate?

(f) Find the population of each of the animals 15 years after the state park was formed (assume the populations increase at the same rate that they did in the 10 years after the state park was formed).

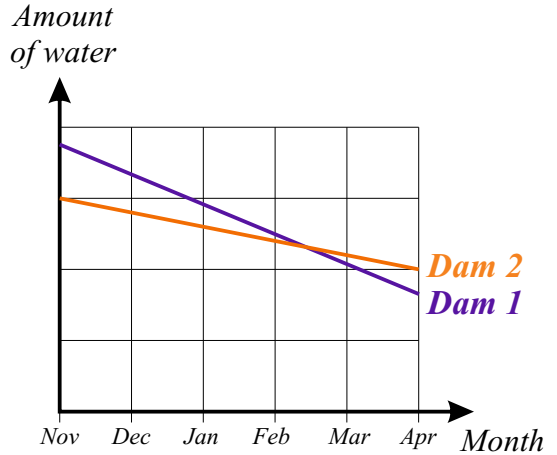
Number of animals

Years after formation of state park

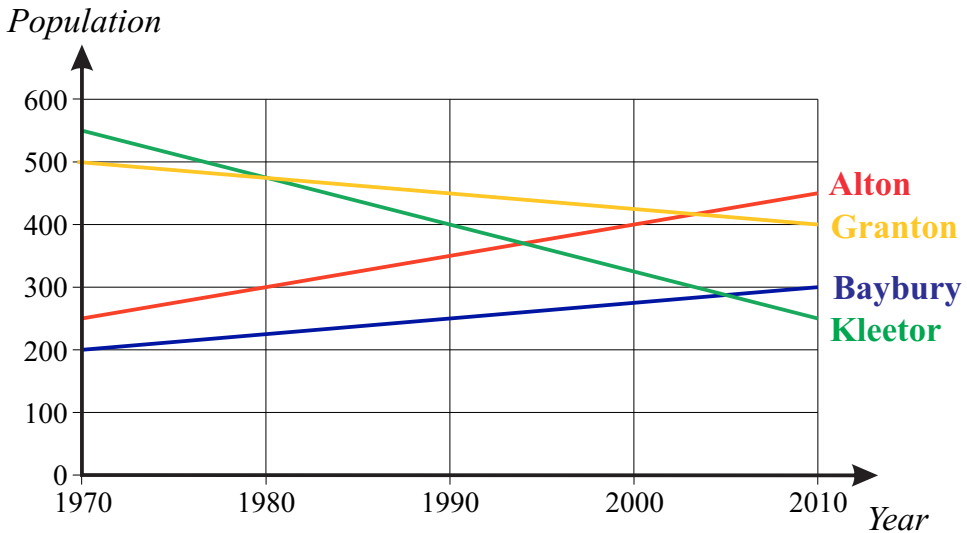


6. The graph here shows the amount of water in two dams on a farm over a period of time.

- Is the amount of water in dam 1 increasing or decreasing in the period shown?
- Is the amount of water in dam 2 increasing or decreasing in the period shown?
- Which dam had the most water in November?
- Which dam had the most water in April?
- Which dam is losing water at the faster rate?



7. The graph below shows the population of four towns between 1970 and 2010.



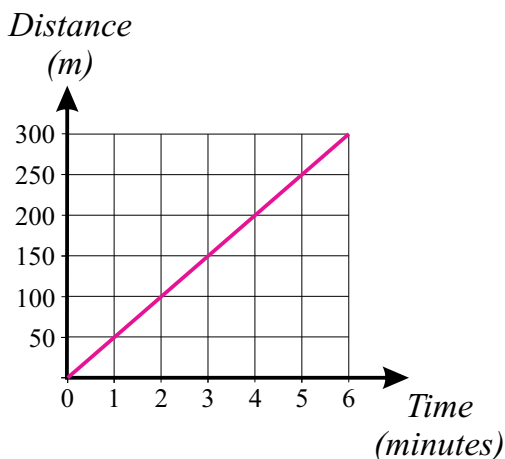
- What was the approximate population of each town in:
 - 1970?
 - 1990?
 - 2010?
- In which towns is the population increasing?
- In which towns is the population decreasing?
- In which town is the population increasing at the greater rate?
- In which town is the population decreasing at the greater rate?

Distance - Time Graphs

EXERCISE 14B

1. Bella walked to her friend's house and using a trundle wheel to measure the distance she walked, she recorded the time taken every 50 metres from her home.

The graph below shows this information.

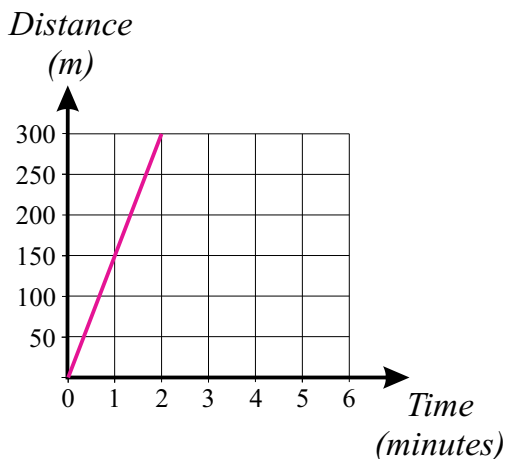


- How far is her friend's house from Bella's house?
- How long did it take Bella to get there?
- At what speed (metres/min) was Bella walking?

Speed in metres/minute (m/min) is found by calculating how far she walks every minute.

2. The next day it was raining so Bella ran to her friend's house. The graph shown here represents distance versus time for her run to her friend's house.

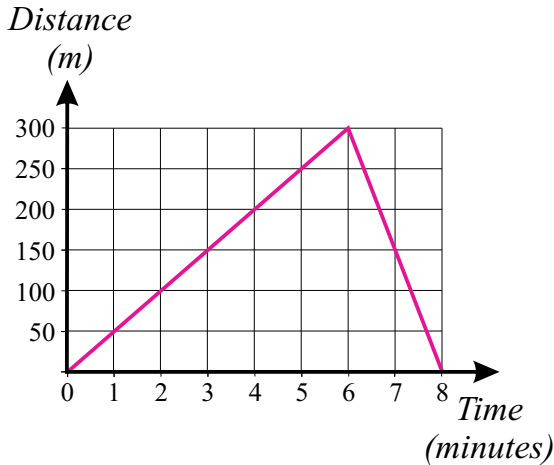
- How long did it take Bella to run to her friend's house?
- At what speed (m/min) was she running?



3. The next day Bella walked to her friend's house, dropped a letter in her letter box and then ran home.

The graph below represents her travels on this day.

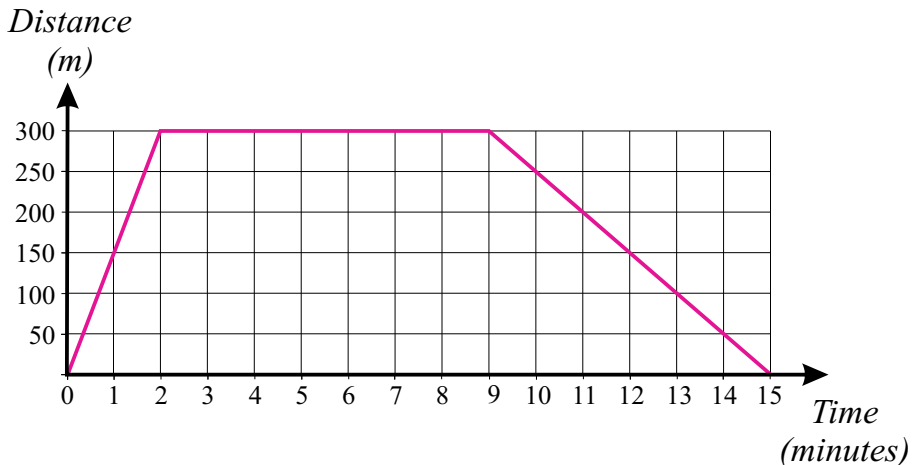
How long was she away from her home?



4. The next day Bella ran to her friend's house, stayed for a few minutes and then walked home.

The graph below represents her travels on this day.

How long did she stay at her friend's house?

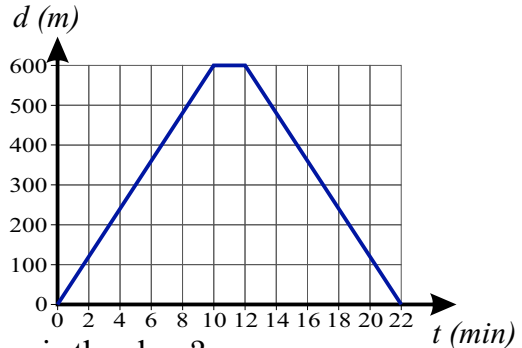


5. Draw graphs of the following situations.

- Bella runs to her friend's house, stays for 2 minutes then runs home.
- Bella walks to her friend's house, stays for 10 minutes then walks home.
- Bella walks halfway to her friend's house, realises she has forgotten something, then runs home.

6. Alanna walked from her home to a shop, bought some groceries and walked home again.

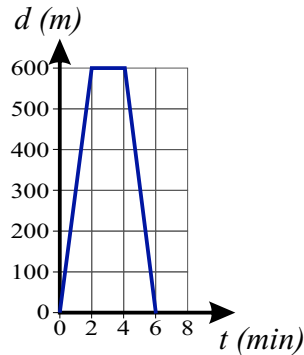
This graph shows the distance, d , (in metres) she was from her home and the time taken, t , (in minutes).



- (a) Use the information represented on the graph to answer the following questions.

- How far from Alanna's home is the shop?
- How many minutes did it take Alanna to walk to the shop?
- How many minutes was Alanna in the shop?
- How many metres did Alanna walk every minute?

- (b) The next time Alanna needed groceries she decided to run to the shop to keep fit. This graph shows the distance Alanna is from her home and time taken when she ran to the shop and home again.



- How long did it take Alanna to run to the shop?
- How far did she run every minute?

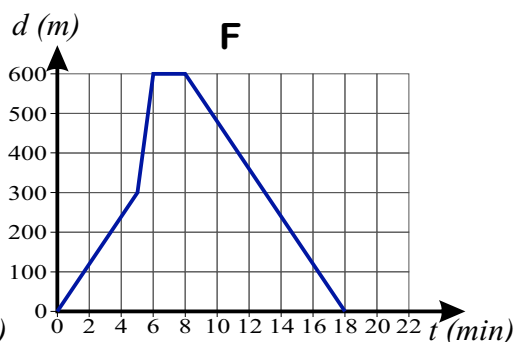
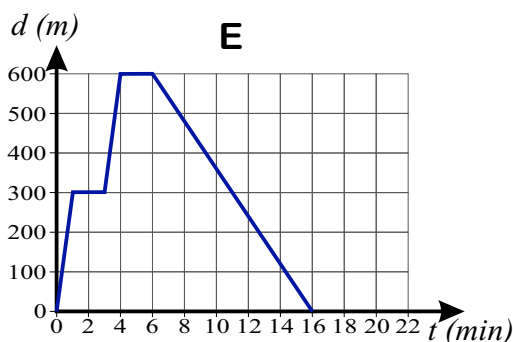
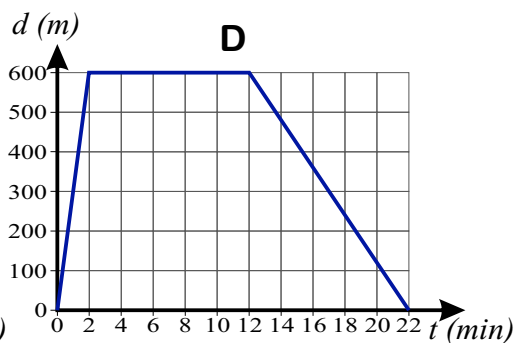
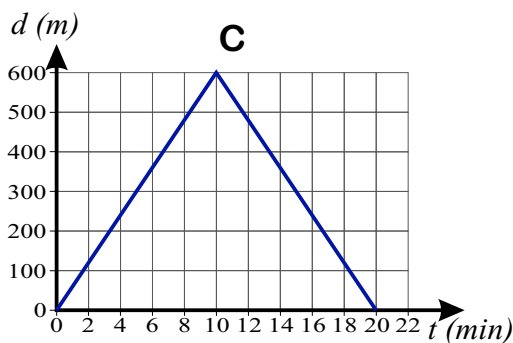
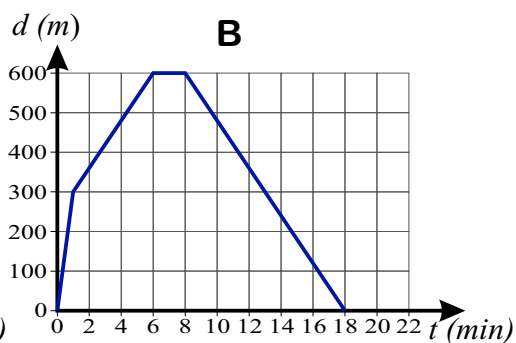
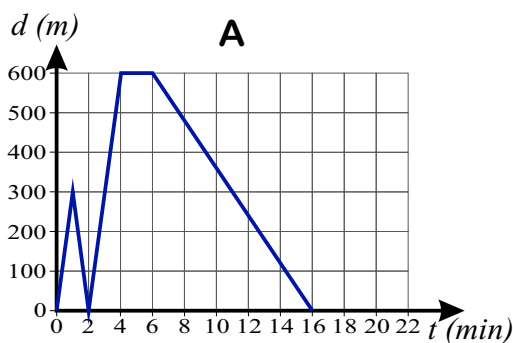
- (c) The graph shown here also represents a visit to the shop by Alanna. Copy and complete the following sentence that describes how Alanna went to and back from the shop. Use the words **walked** and/or **ran** to complete the sentence.



Alanna ____ to the shop, bought some groceries and ____ home.

(d) Match the graphs below with the following descriptions.

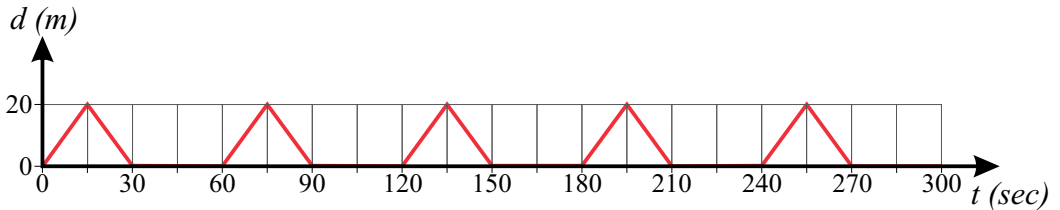
- (i) Alanna walks to the shop, finds that it is closed and walks home.
- (ii) Alanna walks half way to the shop, runs the rest of the way, buys some groceries and walks home.
- (iii) Alanna runs half way to the shop, walks the rest of the way, buys some groceries and walks home.
- (iv) Alanna runs half way to the shop, meets a friend and talks for two minutes. She then runs the rest of the way to the shop, buys some groceries and walks home.
- (v) Alanna runs half way to the shop and realises that she has left her money at home. She then runs home to get the money, runs to the shop, buys the groceries and walks home.
- (vi) Alanna runs to the shop, meets a friend while she is buying groceries and talks for ten minutes. She then walks home.



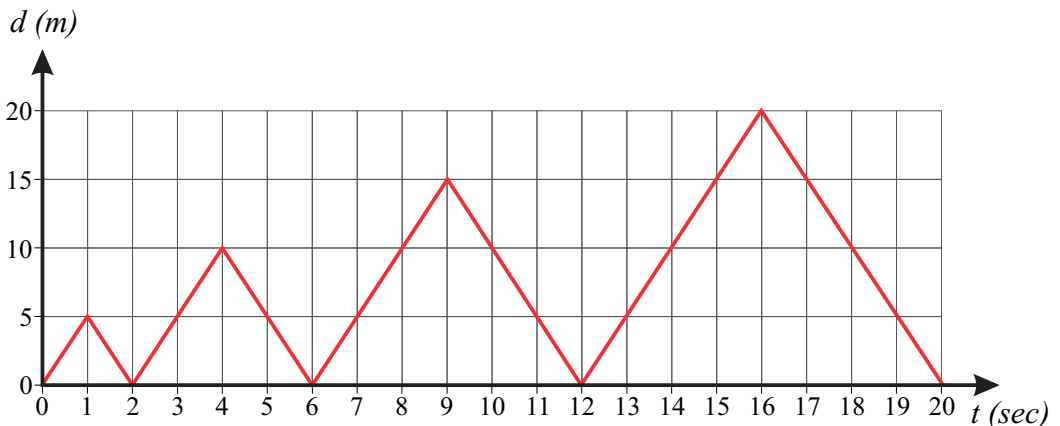
- (e) Using graph paper, draw the graphs for the following situations. Assume that Alanna runs and walks at the same rates as in the other parts of this question and she takes two minutes to buy her groceries.
- Alanna walks to the shop, buys her groceries and then runs home.
 - Alanna runs to the shop, buys some groceries, walks half way home, stops and talks to a friend for five minutes, then walks home.
 - Alanna runs to the shop, buys some groceries, walks half way home and realises she forgot to buy something. She then walks back to the shop, buys the other groceries, then runs home.

7. Each of the graphs below describes the motion of a person performing a particular activity. Describe, in words, the motion in each graph and give an example of what the person might be doing.
- In each graph the distance, d , is in metres and the time, t , is in seconds.

(a)



(b)

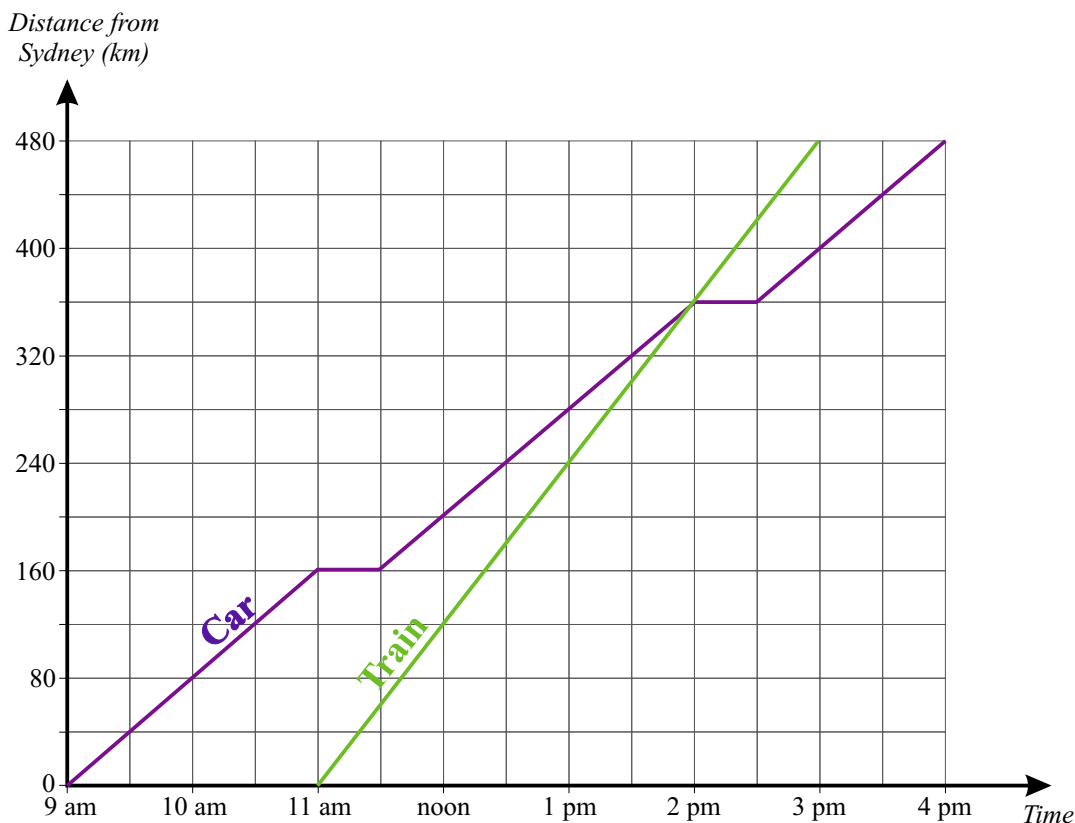


8. Ruth and Fagan travel separately from Sydney to Eden, a distance of 480 km, for a holiday.

Ruth drives her car and stops twice on the way for rests.

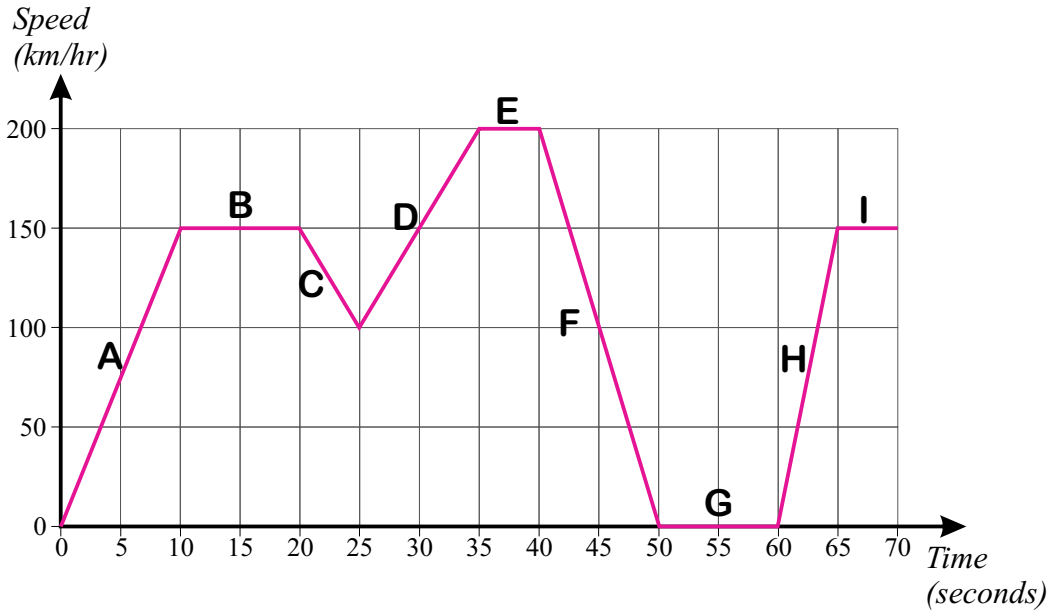
Fagan catches a train which travels directly from Sydney to Eden without stopping.

The graph below shows the distance, d , in kilometres from Sydney and the time of day.



- What time did Ruth leave Sydney in her car?
- What time did Fagan leave Sydney on the train?
- At what times did Ruth stop for rests?
- For how long did Ruth rest at each stop?
- At what time did the train pass Ruth's car?
- How far from Sydney were they when the train passed the car?
- How far from Eden were they when the train passed the car?
- At what time did each person arrive in Eden?
- How long did each person take to get from Sydney to Eden?
- What was the travel speed, in km/hr, of the: (i) train? (ii) car?

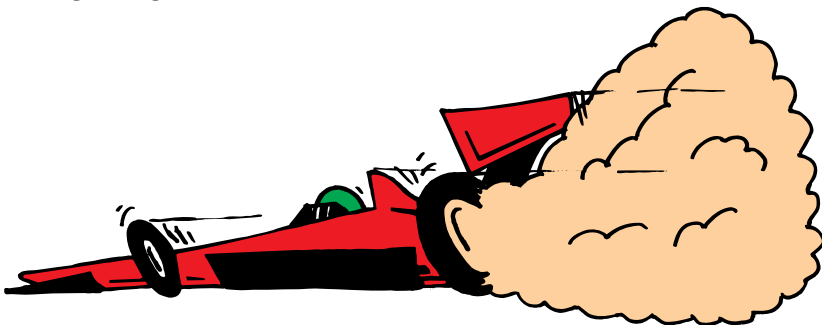
9. The speed of a racing car, in kilometres per hour, during part of a race is recorded on the graph below.



The different sections of the graph have been labelled (A-I).

Use these letters to answer the following questions.

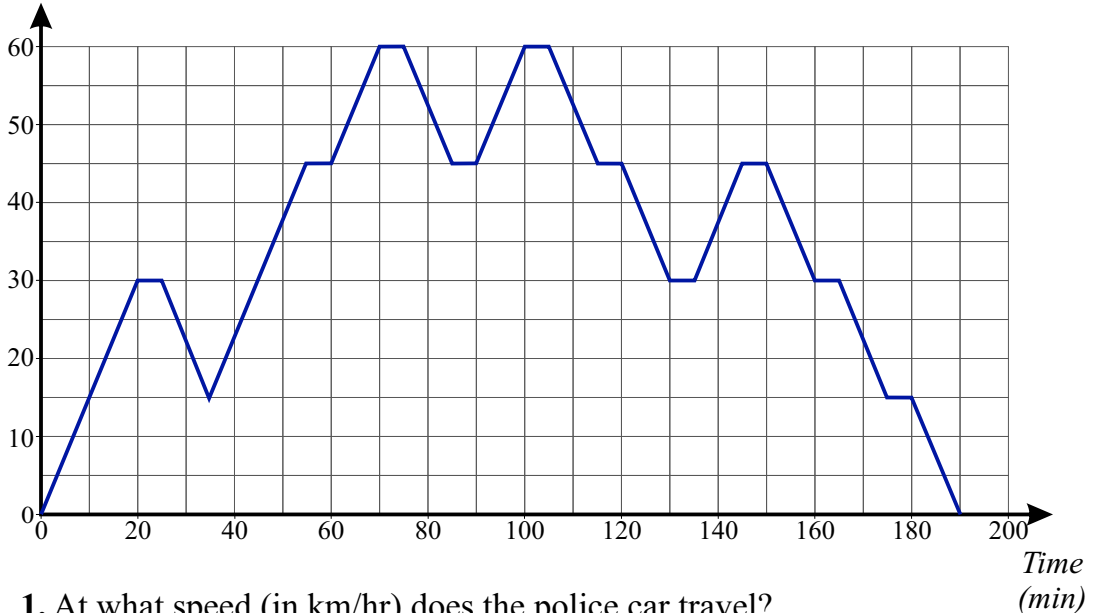
- In which sections of the graph was the car:
 - getting faster?
 - getting slower?
 - travelling at a constant speed?
- In which section of the graph did the car stop to have its tyres changed?
- For how long was the car stopped?
- What was the fastest speed reached by the car?
- For how long did the car travel at this speed?
- Rearrange the letters RATGINCLEACE to spell a word that means getting faster?
- Rearrange the letters RATGINCLEEDE to spell a word that means getting slower?



PROBLEM SOLVING

The graph below shows the movements of a police car. On the vertical axis is the distance, in kilometres, the police car is from the police station. The horizontal axis is time, in minutes.

Distance from station (km)



1. At what speed (in km/hr) does the police car travel?
2. All the stops made by the police car in the time period shown were to talk to motorists. To how many motorists did the police talk?
3. How long did they talk to each motorist?
4. What is the total distance travelled by the police in the period shown?

PUZZLE

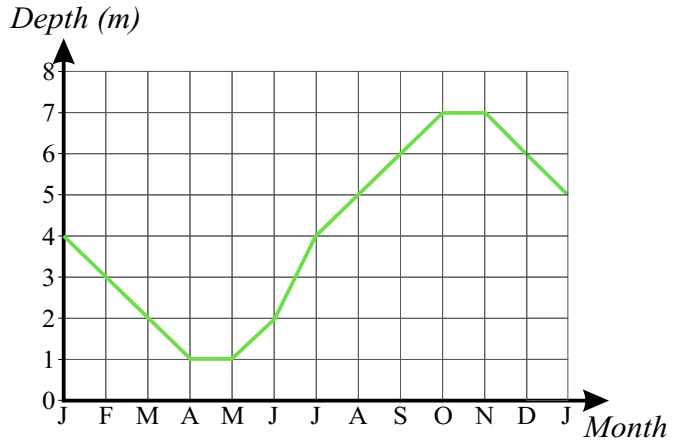
Find words from this chapter hidden in these sentences.

Example Dad drives to work each day.

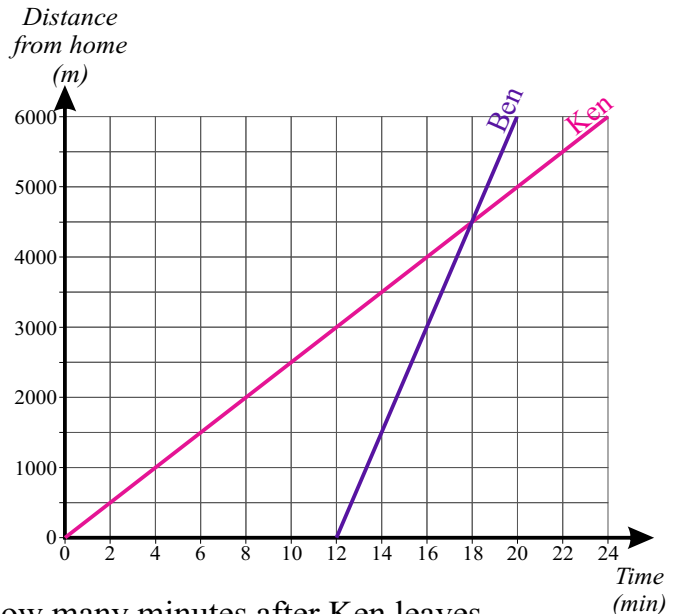
1. Jeremy saw his friend Greg rap his hand with a hammer.
2. After she ran a marathon we heard Alicia admit 'I'm exhausted'.
3. At school Jeremy enjoys P.E., Edward enjoys mathematics and Henriette enjoys English.
4. Dale has three dogs - Osmond is tan, Celeste is black and Jaxson is white.

CHAPTER REVIEW

1. The depth of water, in metres, in a reservoir was recorded at the **start** of each month for a year. The depths are shown on this graph.



- (a) What was the depth of water at the start of the year?
 - (b) By how many metres did the depth of water change in March?
 - (c) In which months did the water level **not** change?
 - (d) In which months did the water level decrease?
 - (e) In which month did the water level change the most?
 - (f) What was the change in water level over the year?
2. Ken and Ben are brothers who wanted to visit a friend. Ken ran from their home to their friend's home. Ben left 12 minutes after Ken left and rode his bike. A graph showing their distance from home, in metres, and time taken, in minutes, is shown below.



- (a) How long did it take each brother to get to their friend's home?
- (b) Who got there first and how many minutes before his brother?
- (c) How far is it to their friend's home?
- (d) Calculate the speed, in metres per minute, of each brother.
- (e) How far from their home were they when Ben passed Ken?
- (f) The next time they go to their friend's home how many minutes after Ken leaves should Ben leave so they arrive at the same time?

The Cartesian Plane

The Cartesian Plane

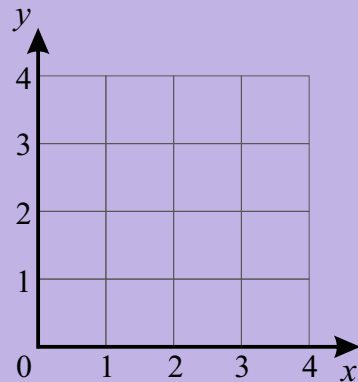
The Cartesian plane is formed by two perpendicular number lines called **axes**.

The horizontal axis is called the x -axis.

The vertical axis is called the y -axis.

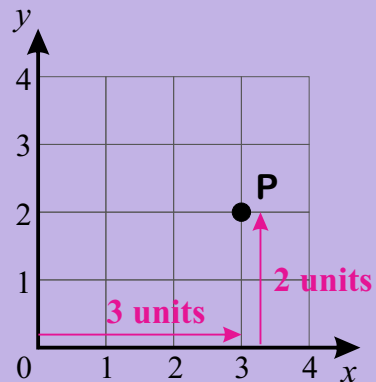
The two axes join at zero called the **origin**.

An example of a Cartesian plane is shown here with each axis going to 4 units.



A point can be located on a Cartesian plane by stating how far it is along in the direction of the x -axis then the y -axis.

For example point **P** on this Cartesian plane is found by moving 3 units from the origin in the direction of the x -axis then 2 units in the direction of the y -axis.



To describe the location of a point these two amounts are called the **coordinates** and written in brackets.

The coordinates of point **P** are (3,2).

The x -coordinate is always written first then the y -coordinate.

Examples

The coordinates of the points shown on this Cartesian plane are:

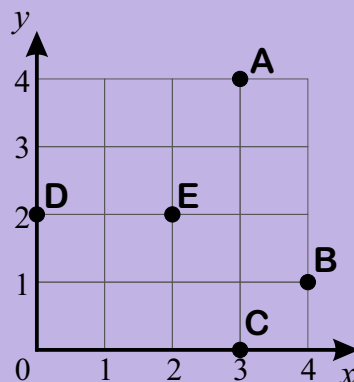
$$A = (3, 4)$$

$$B = (4, 1)$$

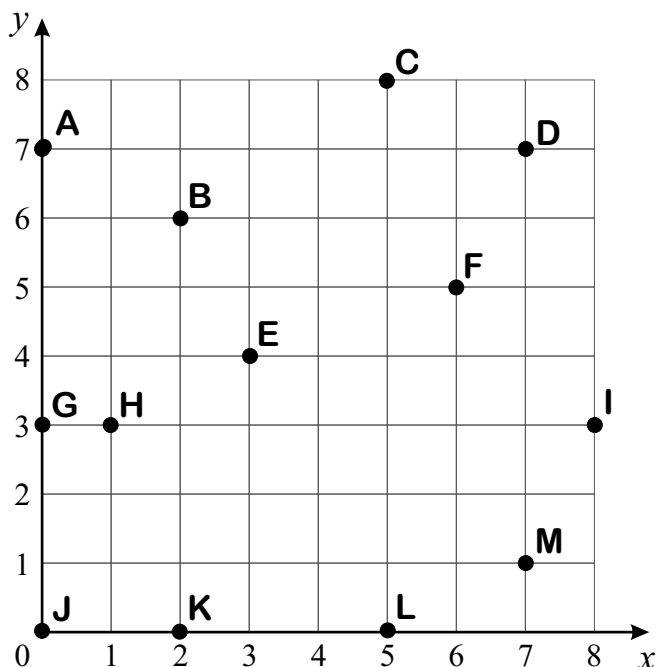
$$C = (3, 0)$$

$$D = (0, 2)$$

$$E = (2, 2)$$

**EXERCISE 15A**

1. Write the coordinates of the points shown on this Cartesian plane.



2. On graph paper draw a Cartesian plane with both axes numbered up to 6 units. Plot the following points and connect them with straight lines in the order the points are listed.
 (2, 1) (3, 3) (1, 2) (1, 5) (3, 4) (2, 6) (5, 6) (4, 4) (6, 5) (6, 2) (4, 3) (5, 1) (2, 1)

Cartesian planes can be extended in the negative direction in both the x -axis and y -axis.

Remember: the x -coordinate is stated first

Examples

$$A = (-4, 4)$$

$$B = (-3, 2)$$

$$C = (0, 2)$$

$$D = (2, 2)$$

$$E = (4, 1)$$

$$F = (-1, 0)$$

$$G = (3, 0)$$

$$H = (1, -1)$$

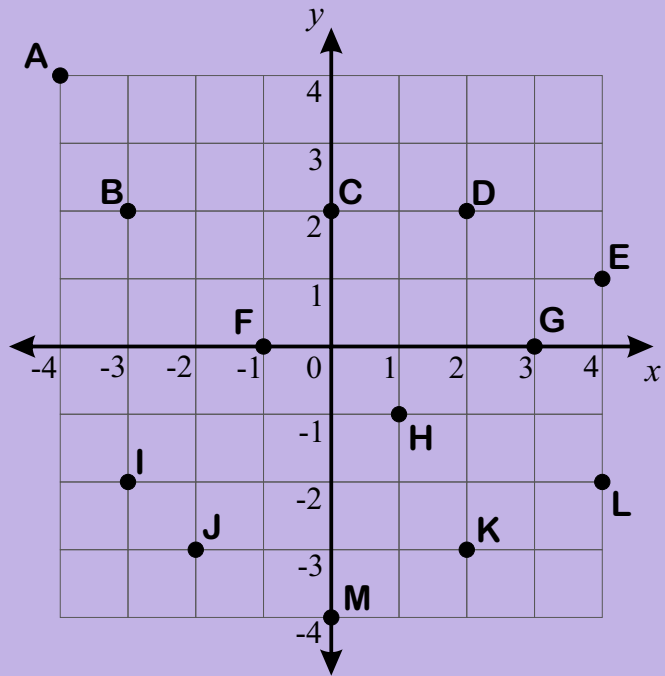
$$I = (-3, -2)$$

$$J = (-2, -3)$$

$$K = (2, -3)$$

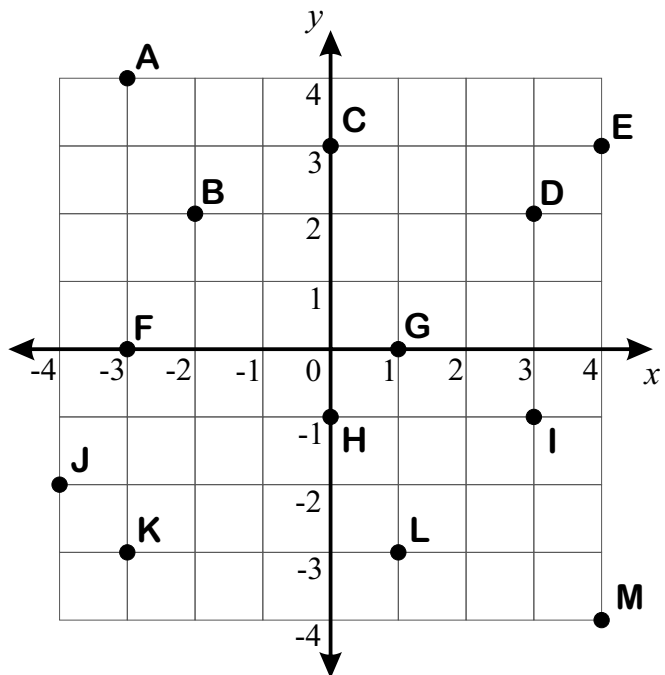
$$L = (4, -2)$$

$$M = (0, -4)$$



EXERCISE 15B

- Write the coordinates of the points shown on this Cartesian plane.



2. On graph paper draw a Cartesian plane with both axes numbered from -5 to 5. Plot the following points and connect them with straight lines in the order the points are listed.
 (5,0) (4,1) (3,1) (-2,5) (0,1) (-3,1) (-5,2) (-4,0) (-5,-2) (-3,-1)
 (0,-1) (-2,-5) (3,-1) (4,-1) (5,0)
3. (a) On graph paper draw a Cartesian plane with both axes numbered from -5 to 5.
 (b) Plot the following points and connect them with straight lines in the order the points are listed.
 (0,5) (-1,3) (0,0) (-3,1) (-5,0) (-3,-1) (0,0) (-1,-3) (0,-5)
 (c) Draw a reflection of this shape on the other side of the y -axis.
 (d) List the coordinates of the points needed to make this symmetrical shape.

Straight Line Graphs

Another term used for pairs of coordinates is *ordered pairs*.

Example

1. List all the ordered pairs for x values from 0 to 5 where the y value is twice the x value.

Answer: (0,0) (1,2) (2,4) (3,6) (4,8) (5,10)

2. Write this relationship between x and y as an equation.

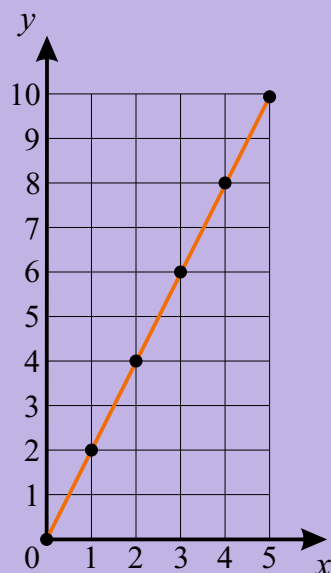
Answer: $y = 2x$

3. Write these ordered pairs in a table.

x	0	1	2	3	4	5
y	0	2	4	6	8	10

x	0	1	2	3	4	5
y	0	2	4	6	8	10

4. Plot these points on a Cartesian plane that has the x -axis from 0 to 5 and the y -axis from 0 to 10. Connect the points with a straight line.



EXERCISE 15C

1. (a) List all the ordered pairs for x values from 0 to 5 where the y value is three times the x value.
 (b) Complete the equation below for this relationship between x and y .

$$y =$$

- (c) Copy and complete the table below showing these ordered pairs.

x	0	1	2	3	4	5
y						

- (d) Plot these points on a Cartesian plane that has the x -axis from 0 to 5 and the y -axis from 0 to 15. Connect them with a straight line.

2. (a) List all the ordered pairs for x values from 0 to 5 where the y value is 1 more than the x value.
 (b) Complete the equation below for this relationship between x and y .

$$y =$$

- (c) Copy and complete the table below showing these ordered pairs.

x	0	1	2	3	4	5
y						

- (d) Plot these points on a Cartesian plane that has the x -axis from 0 to 5 and the y -axis from 0 to 6. Connect them with a straight line.

3. For relationship $y = 2x + 1$ complete the following.

- (a) Copy and complete this table of ordered pairs.

x	0	1	2	3	4	5
y						

- (b) Plot these points on a Cartesian plane that has the x -axis from 0 to 5 and the y -axis from 0 to 12. Connect them with a straight line.

4. Repeat the steps from question 3 for the following relationships.
 Plot the graphs on the same Cartesian plane.

(a) $y = x + 6$ (b) $y = 8 - x$ (c) $y = 12 - 2x$

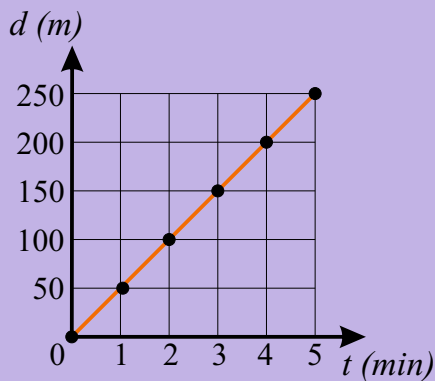
Linear Relationships

Example

The table below shows the distance (d), in metres, a person walking is from their home versus the time (t), in minutes, taken.

t	0	1	2	3	4	5
d	0	50	100	150	200	250

A graph can be drawn to represent this information.



An equation can be written for this relationship:

$$d = 50t$$

This equation can be used to find the distance travelled after any give time.

- Find the distance travelled after 8 minutes.

$$\begin{aligned}
 d &= 50t \\
 &= 50 \times t \\
 &= 50 \times 8 \\
 d &= 400 \text{ m}
 \end{aligned}$$

- Find the distance travelled after 1 hour.

It is important to use the same units as in the original information.

1 hour needs to be changed to minutes = 60 minutes

$$\begin{aligned}
 d &= 50 \times 60 \\
 d &= 3000 \text{ m}
 \end{aligned}$$

When transferring information from a table of values to a graph the first quantity is on the horizontal axis.

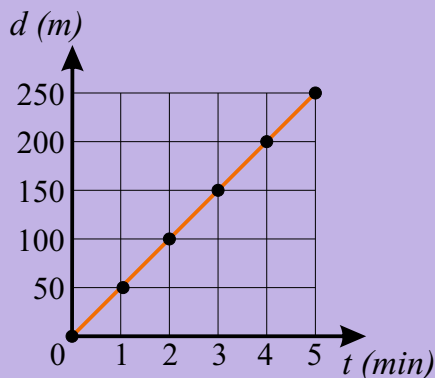
horizontal axis

vertical axis

t	0	1	2	3	4	5
d	0	50	100	150	200	250

When an equation is written from a table of values the second quantity is written on the left of the equation.

$$d = 50t$$



EXERCISE 15D

1. The table of values shown below records distance (d), in metres, versus time (t), in seconds, of a person running.

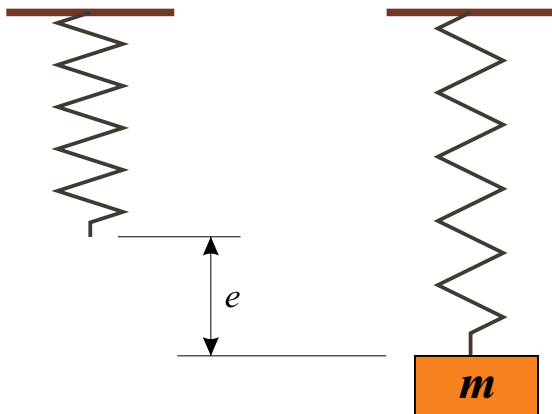
t	0	1	2	3	4	5
d	0	5	10	15	20	25

- Draw a graph to represent this information.
 - Find the equation for this relationship.
 - Find the distance this person would run in:
 - 20 seconds
 - 2 minutes
2. The pressure (P), in kilopascal (kPa), at a depth (d), in metres, in water is shown in the table of values below.

d	0	1	2	3	4	5
P	0	10	20	30	40	50

- Draw a graph to represent this information.
- Find the equation for this relationship.
- Find the pressure at a depth of 8 metres.

3. In a science class, students measured the amount a spring extended (e), in centimetres (cm), when different masses (m), in kilograms (kg), were attached to the spring. (See diagram)



The table below shows the results.

m	0	1	2	3	4	5
e	0	2	4	6	8	10

- Draw a graph to represent this information.
 - Find the equation for this relationship.
 - Find the extension if 9 kg was attached to the spring.
4. In another science experiment the voltage drop (V), measured in volts (V) across a resistor was measured for different currents (I), measured in amps (A).

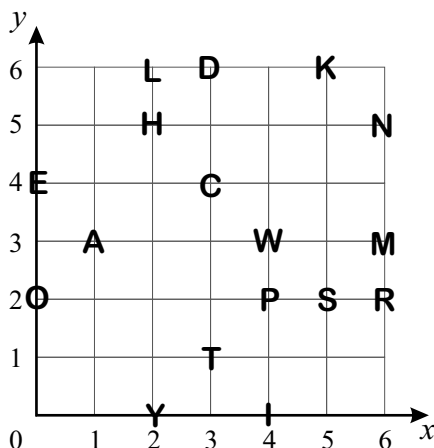
The results are shown in the table below.

I	0	1	2	3	4	5
V	0	20	40	60	80	100

- Draw a graph to represent this information.
- Find the equation for this relationship.
- Find the voltage drop across the resistor for the following currents:
 - 6 amps
 - 15 amps
 - 25 amps

PROBLEM SOLVING

Use the letters on this Cartesian plane to solve the question and riddles below. The coordinates of the points with the letters for the answers are given.



1. The mathematician who the Cartesian plane is named after is:
 (6,2) (0,4) (6,5) (0,4) (3,6) (0,4) (5,2) (3,4) (1,3) (6,2) (3,1) (0,4) (5,2)
2. What is a kangaroo's favourite time?
 (1,3) (2,6) (0,4) (1,3) (4,2) (2,0) (0,4) (1,3) (6,2)
3. How deep can a frog swim?
 (5,6) (6,5) (0,4) (0,4) (3,6) (0,4) (0,4) (4,2)
 (5,6) (6,5) (0,4) (0,4) (3,6) (0,4) (0,4) (4,2)
4. How does Santa take photos?
 (4,3) (4,0) (3,1) (2,5) (1,3) (6,5) (0,2) (6,2) (3,1) (2,5)
 (4,2) (0,2) (2,6) (0,4) (1,3) (6,2) (0,2) (4,0) (3,6)
 (3,4) (1,3) (6,3) (0,4) (6,2) (1,3)

PUZZLE

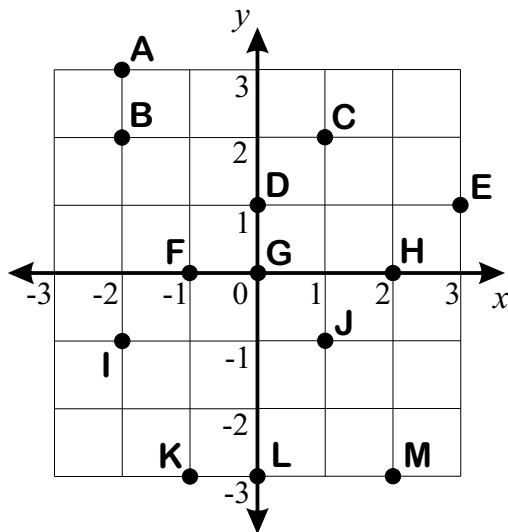
Find words from this chapter hidden in these sentences.

Example Dad drives to work each day.

1. The coach said to stick to the game plan even if it's not working.
2. I had to wave several times before the taxi stopped to pick me up.
3. We went to Africa and saw a giraffe, lion and hippo in the Kenyan National Park.
4. Jasmine waxes her surfboard every time she goes to the beach.

CHAPTER REVIEW

1. Write the coordinates of the points shown on this Cartesian plane.



2. For relationship $y = 2x + 3$ complete the following.

(a) Copy and complete this table of ordered pairs.

x	0	1	2	3	4	5
y						

(b) Plot these points on a Cartesian plane that has the x -axis from 0 to 5 and the y -axis from 0 to 14. Connect them with a straight line.

3. The table of values shown below records distance (d), in metres, versus time (t), in minutes, of a person cycling.

t	0	1	2	3	4	5
d	0	300	600	900	1200	1500

- (a) Draw a graph to represent this information.
 (b) Find the equation for this relationship.
 (c) Find the distance this person would cycle in:
 (i) 10 minutes (ii) 1 hour

Length and Perimeter

Length and Units

The standard unit of length is the *metre*.

The abbreviation for metre is *m*.

Other common units used are either multiples or fractions of a metre.

The most common are *kilometres*, *centimetres* and *millimetres*.

1 kilometre (km) = 1000 metres

1 metre = 100 centimetres (cm)

1 metre = 1000 millimetres (mm)

EXERCISE 16A

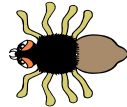
1. Which of the following is *closest* to **1 metre**.

A the length of a car

B the length of a spider

C the height of your desk

D the distance to the Sun



2. Which of the following units would be best to use when making the measurements below.

A millimetres

B centimetres

C metres

D kilometres

(a) The length of a person's hair.

(b) The thickness of a guitar string.

(c) The size of a television screen.

(d) The length of an insect.

(e) The height of a tree.

(f) The distance a football is kicked.

(g) The dimensions of your classroom.

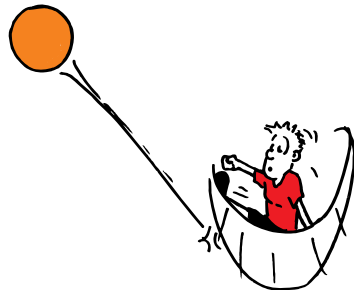
(h) The distance from Perth to Cairns.

(i) The diameter of a drop of water.

(j) The dimensions of a book.

(k) The monthly rainfall.

(l) The thickness of the Earth's atmosphere.



3. Choose which of the alternatives is the best estimate of the following lengths.

(a) The length of the Murray River.

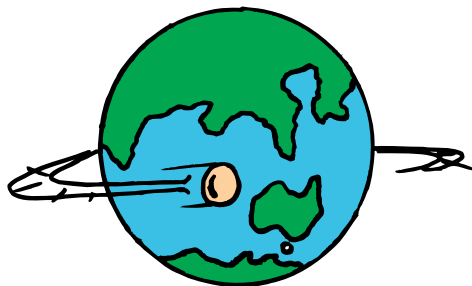
- A** 4000 m **B** 40 000 m **C** 4000 km **D** 40 000 km

(b) The thickness of a piece of cardboard.

- A** 0.2 mm **B** 2 mm **C** 2 cm **D** 2 m

(c) The distance from the Earth to the Moon.

- A** 400 km
B 4000 km
C 40 000 km
D 400 000 km



(d) The length of a lawn bowls green.

- A** 36 m **B** 360 m **C** 3600 m **D** 36 000 m

(e) The diameter of a tennis ball.

- A** 0.7 mm **B** 7 mm **C** 70 mm **D** 700 mm

(f) The thickness of a human hair.

- A** 0.01 mm **B** 0.1 mm **C** 1 mm **D** 10 mm

(g) The height of your mathematics teacher.

- A** 17 mm **B** 170 mm **C** 17 cm **D** 170 cm

(h) The height of an adult giraffe.

- A** 60 cm **B** 600 cm **C** 6000 cm **D** 60 000 cm

(i) The length of the longest snake ever recorded.

- A** 2 m **B** 10 m **C** 25 m **D** 35 m

(j) The distance run in a marathon.

- A** 12 km **B** 22 km **C** 32 km **D** 42 km

4. Estimate the following measurements.

(a) The diameter of a volleyball (in **cm**).

(b) The diameter of a DVD (in **cm**).

(c) The distance from Darwin to Alice Springs (in **km**).

(d) The length of a bee (in **mm**).

(e) The diameter of a 20 cent piece (in **mm**).

(f) The length of the average car (in **m**).

5. Find which of the units in the following story are incorrect and state what they should be.

Michael and William decided to go fishing at a nearby reservoir. "We'll have to ride our bikes because the reservoir is 10 cm away," said Michael.

"It should only take us half an hour to get there. We can ride at 20 metres per hour," said William.

The reservoir was almost full because the previous week there had been 60 metres of rain.

When they got there William had to wind new line onto his fishing reel. He had bought 100 km of extra strong line. It was a new brand that was only half a centimetre thick.

Michael had his line in the water first with a good cast of 17 mm. "That's a pretty good cast," said William, "but I reckon I can cast further." He proceeded to cast his line 18 km.

Michael got a bite and as he was reeling the line in it got caught on a stump in the water. He didn't want to break his line so he decided to wade in to retrieve it. He had to be careful as he waded into the cold water because he was 175 mm tall and the water was one millimetre deep.

William caught two fish that were 20 km long and weighed half a gram each. Michael caught one fish that was 30 cm long and weighed 1000 grams.

That night they enjoyed baked fish for dinner.

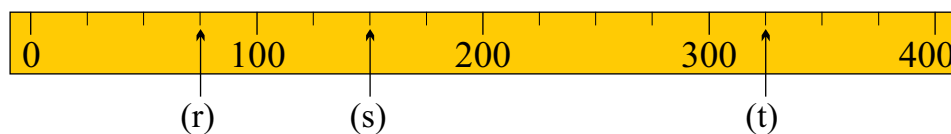
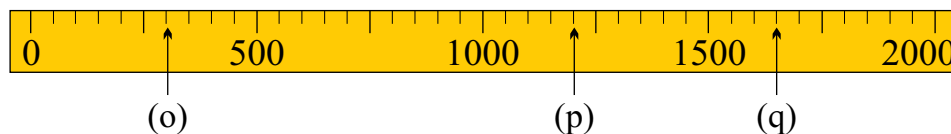
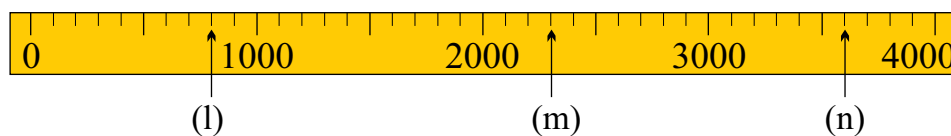
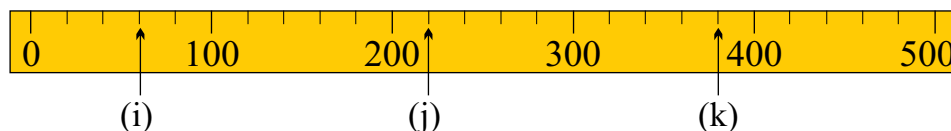
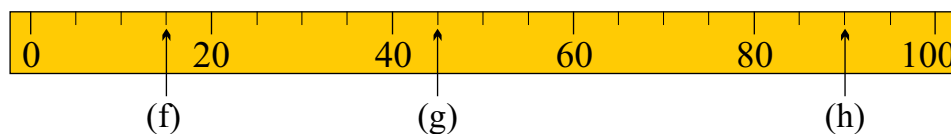
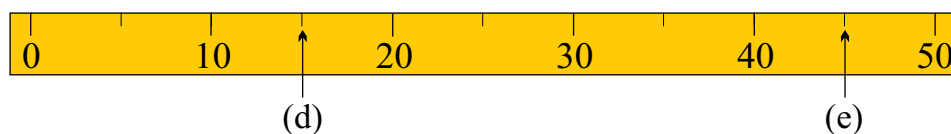
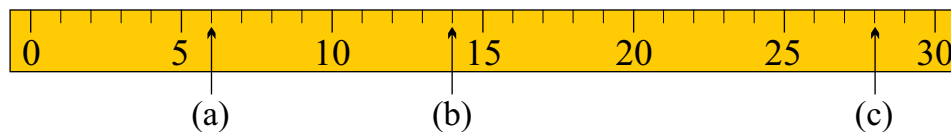


Write your own 'silly' story.

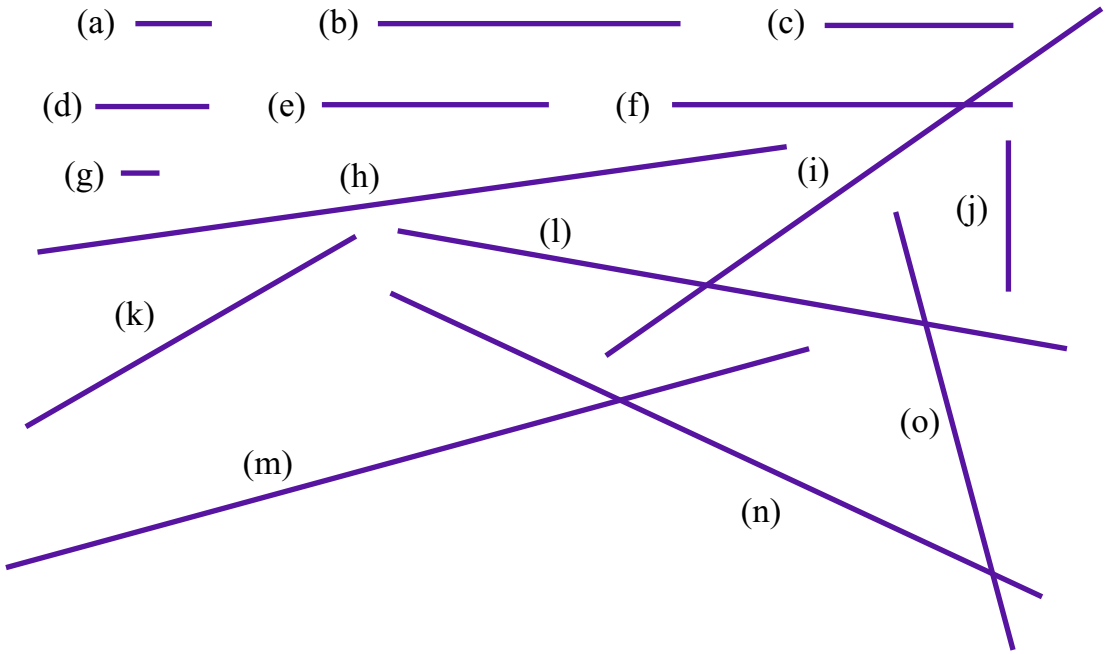
Measuring

EXERCISE 16B

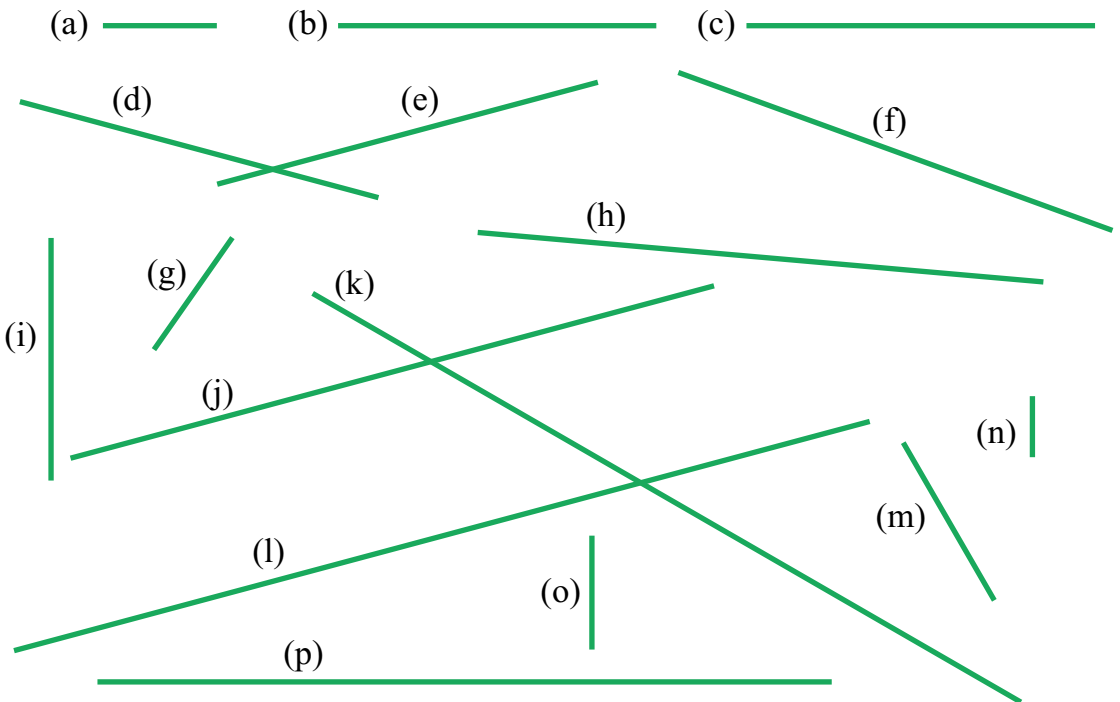
1. Read the measurements shown on these scales.



2. Use a ruler to measure the following lines.
Give lengths in centimetres.

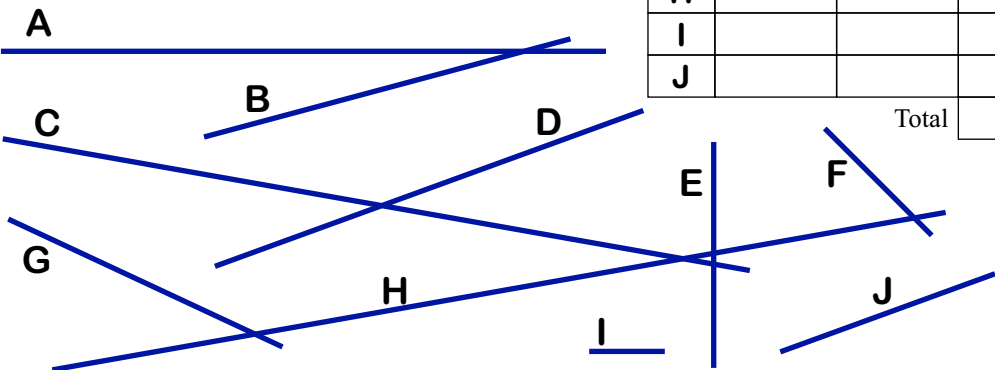


3. Use a ruler to measure the following lines.
Give lengths in millimetres.



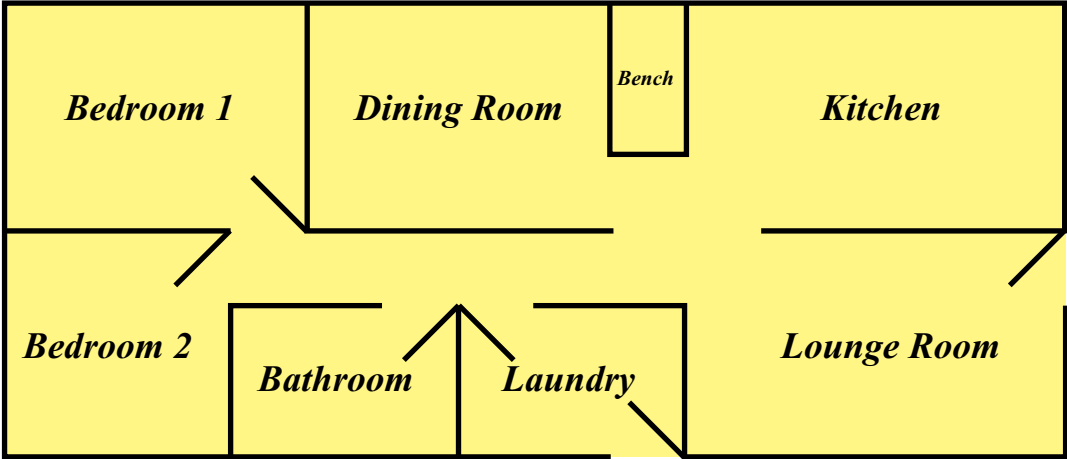
4. Copy the table shown into your books.
For each of the lines below, guess their length (in cm) and record your guesses in the second column.
Then measure the lengths and record them in the third column.
Find the differences between your guesses and the actual lengths.
Find the total of the differences.
If this is less than 20 - well done.

Line	Guess of the Length	Actual Length	Difference
A			
B			
C			
D			
E			
F			
G			
H			
I			
J			
Total			



5. In the house plan below 1 cm represents 1 m.
Use this conversion and a ruler to find the dimensions of each room and the overall dimensions of the house.

Example The dimensions of the bathroom on the plan are 3 cm × 2 cm.
This means the bathroom is 3 m × 2 m.

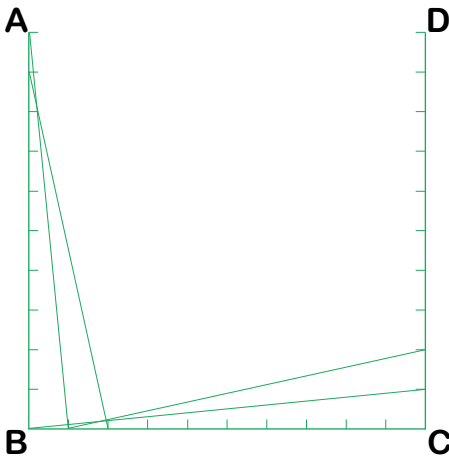


6. Using a ruler and pencil follow these instructions.

- Step 1.** Draw a rectangle that has side lengths of 6 cm and 8 cm.
Step 2. Accurately mark the mid point of each side of the rectangle.
Step 3. Connect these mid points to form a quadrilateral.
Step 4. Measure the length of the sides in this quadrilateral.
Step 5. State the name of this type of quadrilateral.
Step 6. Mark the mid point of each side in this quadrilateral.
Step 7. Connect these mid points to form another quadrilateral.
Step 8. Measure the length of each side in this quadrilateral.
Step 9. Make a comment on what you have found.

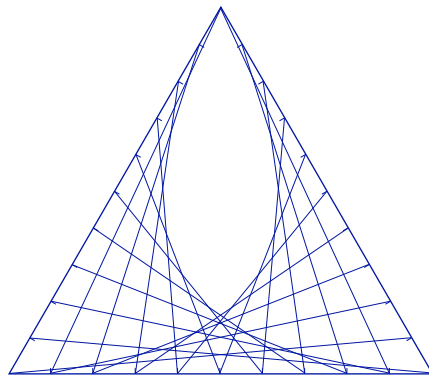
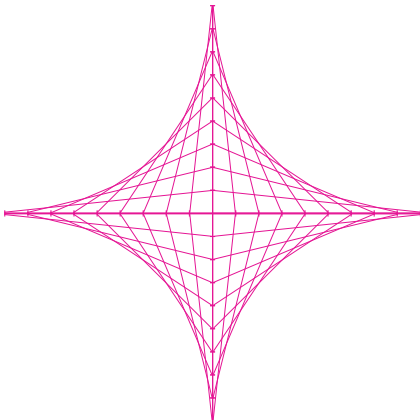
7. Copy the diagram below and complete it by connecting the points as shown.

Be careful to use the lengths given.



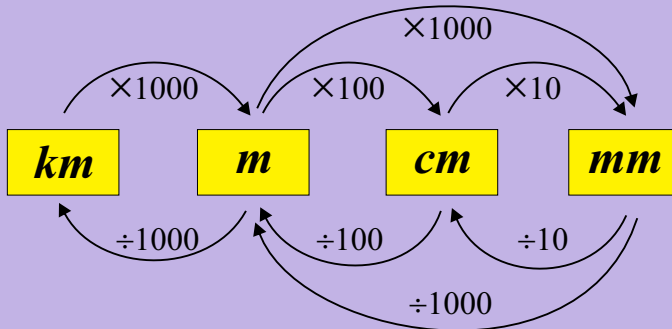
The lines in the starting shape ABCD are each to be 10 cm long.
 The points along these lines are 1 cm apart.

Use a similar technique to construct the designs below.
 Choose your own lengths.



Conversions

This diagram can be used to find the conversion factor when changing from one unit to another.

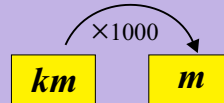


Examples

Change the following lengths to the units shown in the brackets.
Use the diagram above to find the conversion factors.

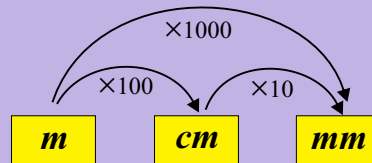
1. 3 km (m)

To change from km to m: $\times 1000$
 $= 3 \times 1000$
 $= \mathbf{3000\ m}$



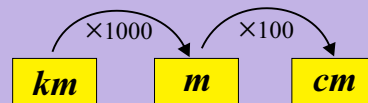
2. 26 m (mm)

To change from m to mm: $\times 1000$
 $= 26 \times 1000$
 $= \mathbf{26\ 000\ mm}$



3. 4 km (cm)

To change from km to cm: $\times 1000$ then $\times 100 = \times 100\ 000$
 $= 4 \times 100\ 000$
 $= \mathbf{400\ 000\ cm}$



Examples continued

Remember - When **multiplying** by a power of 10 move the decimal point to the **right** the same number of decimal places as the number of zeros.

4. 9.705 m (cm)

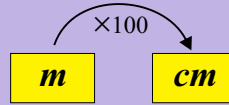
To change from m to cm: $\times 100$

$$= 9.705 \times 100$$

Move the decimal point two places to the right (100 has two zeros)

$$= 9.705$$

$$= \mathbf{970.5 \text{ cm}}$$

**5. 37.8 km (m)**

To change from km to m: $\times 1000$

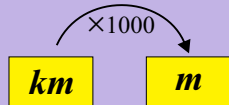
$$= 37.8 \times 1000$$

Move the decimal point three places to the right (1000 has three zeros)

Add zeros where necessary.

$$= 37.8$$

$$= \mathbf{37\ 800 \text{ m}}$$

**EXERCISE 16C**

1. Change the following lengths to the units shown in the brackets.

(a) 6 cm (mm)

(b) 8 m (cm)

(c) 20 km (m)

(d) 23 m (mm)

(e) 40 cm (mm)

(f) 59 km (m)

(g) 8.9 cm (mm)

(h) 3.75 m (cm)

(i) 2.3 km (m)

(j) 0.1 m (mm)

(k) 0.07 km (m)

(l) 27.6 cm (mm)

(m) 0.003 m (cm)

(n) 2.357 cm (mm)

(o) 0.578 km (m)

(p) 0.3 km (cm)

(q) 0.43 km (mm)

(r) 0.0017 km (m)

Examples

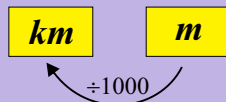
Change the following lengths to the units shown in the brackets.

1. 9000 m (km)

To change from m to km: $\div 1000$

$$= 9000 \div 1000$$

$$= \mathbf{9 \text{ km}}$$



Remember - When *dividing* by a power of 10 move the decimal point to the **left** the same number of decimal places as the number of zeros.

2. 138.5 cm (m)

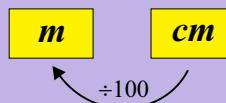
To change from cm to m: $\div 100$

$$= 138.5 \div 100$$

Move the decimal point two places to the left (100 has two zeros)

$$= \overset{\wedge}{\overset{\wedge}{138.5}}$$

$$= \mathbf{1.385 \text{ m}}$$



3. 37.8 mm (m)

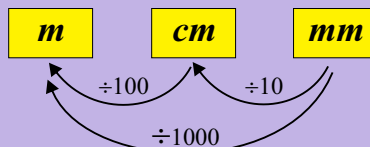
To change from mm to m: $\div 1000$

$$= 37.8 \div 1000$$

Move the decimal point three places to the left (1000 has three zeros)
Add zeros where necessary.

$$= \overset{\wedge}{\overset{\wedge}{\overset{\wedge}{37.8}}} \div 1000$$

$$= \mathbf{0.0378 \text{ m}}$$



2. Change the following lengths to the units shown in the brackets.

- | | | |
|------------------|--------------------|--------------------|
| (a) 50 mm (cm) | (b) 200 cm (m) | (c) 7000 m (km) |
| (d) 9000 mm (m) | (e) 800 mm (cm) | (f) 930 000 m (km) |
| (g) 6 mm (cm) | (h) 33 cm (m) | (i) 58 m (km) |
| (j) 0.3 cm (m) | (k) 58 mm (cm) | (l) 380 cm (m) |
| (m) 350.2 mm (m) | (n) 785.1 cm (m) | (o) 3.7 mm (m) |
| (p) 500 cm (km) | (q) 30 000 mm (km) | (r) 872 100 mm (m) |

3. Change the following lengths to the units shown in the brackets.

- | | | |
|-----------------|------------------|--------------------|
| (a) 10 mm (cm) | (b) 3.2 m (cm) | (c) 82 cm (mm) |
| (d) 35 cm (m) | (e) 0.09 m (mm) | (f) 0.0475 km (m) |
| (g) 0.4 mm (cm) | (h) 2.35 cm (m) | (i) 123.456 m (km) |
| (j) 2.2 cm (mm) | (k) 12.5 mm (cm) | (l) 0.008 m (cm) |
| (m) 0.03 mm (m) | (n) 0.5 m (km) | (o) 2.35 mm (m) |

4. Change the following lengths to the units shown in the brackets.

Example

$$\begin{aligned}
 &3 \text{ m } 2 \text{ cm (m)} \\
 &= 3 \text{ m} + 2 \text{ cm} \\
 &= 3 \text{ m} + 0.02 \text{ m (Change both lengths to units in the brackets)} \\
 &= \mathbf{3.02 \text{ m}}
 \end{aligned}$$

- | | |
|--------------------|------------------------|
| (a) 2 m 11 cm (m) | (b) 3 cm 2 mm (cm) |
| (c) 6 m 6 cm (m) | (d) 20 m 20 cm (m) |
| (e) 5 km 20 m (km) | (f) 18 m 5 cm (m) |
| (g) 5 m 8 cm (m) | (h) 6 m 23 cm 8 mm (m) |
| (i) 4 m 7 mm (m) | (j) 20 cm 4 mm (cm) |

5. Arrange the following lengths in order from the smallest to the largest.

Example

Arrange the following lengths in order from the smallest to the largest.

578 mm, 65 cm, 0.055 m

The easiest method is to convert the lengths to the same unit and choose the smallest unit from the given lengths.

In this example the smallest unit from the three is **mm**.

Convert the three lengths to mm.

$$578 \text{ mm, } 65 \text{ cm} = 650 \text{ mm, } 0.055 \text{ m} = 55 \text{ mm}$$

Arranging the converted lengths in order from smallest to largest:

55 mm, 578 mm, 650 mm

Answer: 0.055 m, 578 mm, 65 cm

- | | |
|-----------------------------|----------------------------|
| (a) 83 mm, 7.5 cm, 0.063 m | (b) 74 cm, 620 mm, 0.812 m |
| (c) 1.32 m, 178 cm, 953 mm | (d) 345 cm, 2.8 m, 3921 mm |
| (e) 8 mm, 0.0076 m, 0.82 cm | (f) 4.77 m, 598 cm, 796 mm |

6. At a school athletics competition Liam threw the shot putt 8 m 47 cm. Keiran threw it 850 cm. Who threw it the furthest?
7. Zita is 147 cm tall. She walked under a tree branch that was 2.09 m above the ground.
(a) What was the gap (in cm) between her head and the branch?
A possum was sitting one and a half metres above the branch.
(b) How far above the ground (in cm) was the possum?
8. Imogen wanted to calculate the distance from her home to her friend's home. She measured the length around the wheel of her bike to be 200 cm. She put a counter on the wheel and then rode to her friend's home.
The counter showed that the wheel had turned 1250 times.
(a) How far (in km) is it from Imogen's to her friend's home?
(b) If Imogen rode at 10 km/hour, how long did it take her?
9. Calista wanted to wallpaper a wall in her bedroom. She measured the length of the wall to be 3.85 metres. The width of the wallpaper is 55 cm.
(a) How many times will the wallpaper fit across the wall?
(b) If the wall is 2.9 m high what length of wallpaper is required?
10. Yevgeny wanted to see how many people could be seated in the local church. The width of the seating area was 16 metres. He estimated that the width of each person was 800 mm when seated.
There were 20 rows of pews.
How many people could be seated in the church?



11. Hayden trained for his gymnastics by running up stairs in a building. One day he decided to count the number of stairs to work out the height of the building.
Each step was 20 cm high and he counted 300 steps to get from the ground to the roof.
(a) How tall (in metres) was the building?
(b) If there were 15 floors in the building, what was the height of each floor?



Rounding

EXERCISE 16D

1. Round the following lengths to the nearest metre.

If the number after the decimal point is **5 or greater** then **round up** to the next metre.

If the number after the decimal point is **less than 5** then **round down**.

Examples

1. 41.82 m

41.**8**2

The number after the decimal point is greater than 5 so round up.

Answer: 42 m

2. 9.5 m

9.**5**

The number after the decimal point is equal to 5 so round up.

Answer: 10 m

3. 132.39 m

132.**3**9

The number after the decimal point is less than 5 so round down.

Answer: 132 m

(a) 2.8 m

(b) 66.2 m

(c) 14.09 m

(d) 29.7 m

(e) 15.56 m

(f) 20.199 m

2. Change the following lengths to decimal numbers then round to the nearest metre.

(a) 3 m 15 cm

(b) 2 m 80 cm

(c) 15 m 56 cm

(d) 4 m 4 cm

(e) 8 m 9 cm

(f) 10 m 89 mm

(g) 11 m 256 mm

(h) 5 m 780 mm

(i) 48 m 458 mm

3. Round the following lengths to the nearest centimetre.

(a) 3.4 cm

(b) 2.8 cm

(c) 15.9 cm

(d) 1 cm 1 mm

(e) 13 cm 8 mm

(f) 25 cm 6 mm

(g) 9 mm

(h) 7 cm 2 mm

(i) 19 cm 9 mm

Perimeter

EXERCISE 16E

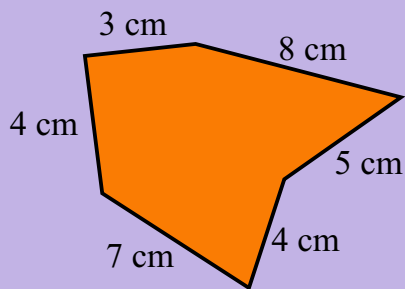
The *perimeter* of a shape is equal to the sum of all the side lengths.

Example

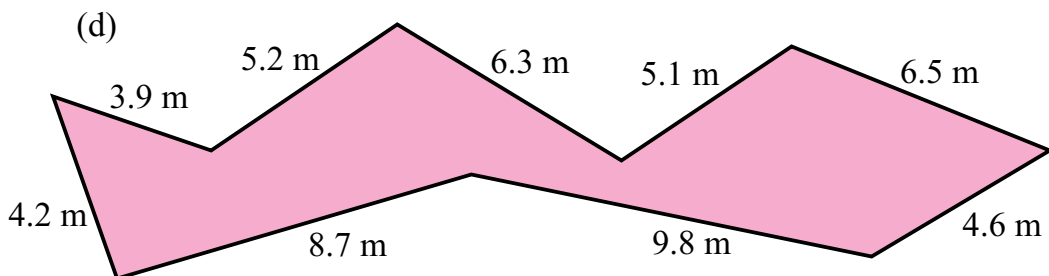
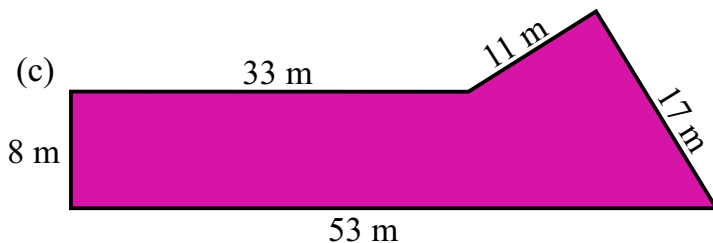
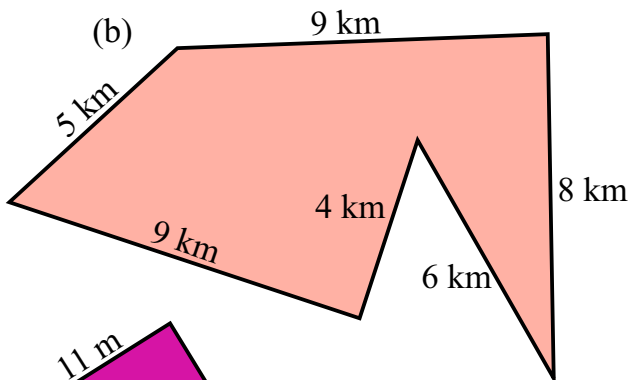
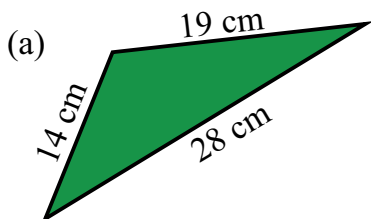
Find the perimeter of this shape.

Perimeter = the sum of all
the side lengths
 $= 3 + 8 + 5 + 4 + 7 + 4$

Perimeter = 31 cm



1. Use the measurements shown to find the perimeter of each of the following shapes.

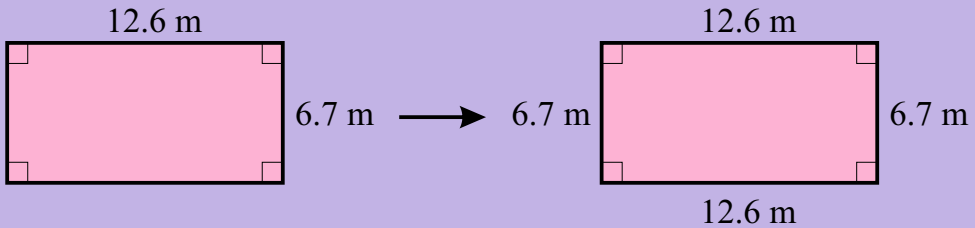


Find the perimeter of the following shapes.

Examples

Use logic and knowledge of rectangles and symbols to fill in all the side lengths that are not given then calculate the perimeter.

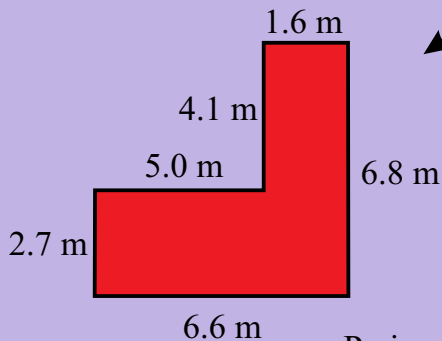
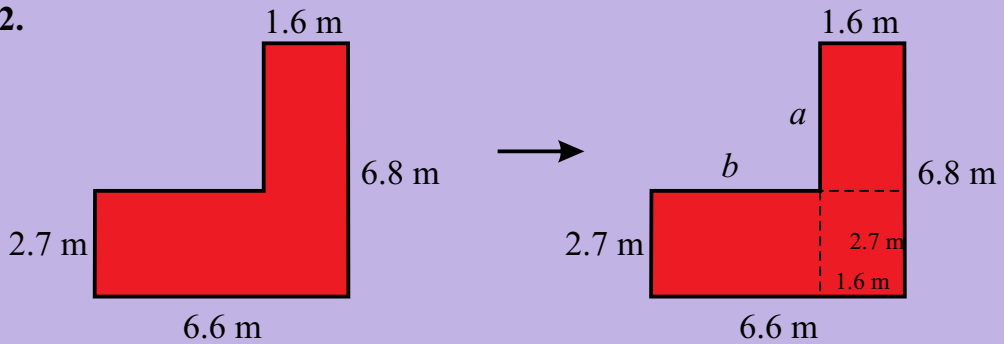
1.



Opposite sides of a rectangle are the same length.

$$\begin{aligned}\text{Perimeter} &= 6.7 + 12.6 + 6.7 + 12.6 \\ &= \mathbf{38.6 \text{ m}}\end{aligned}$$

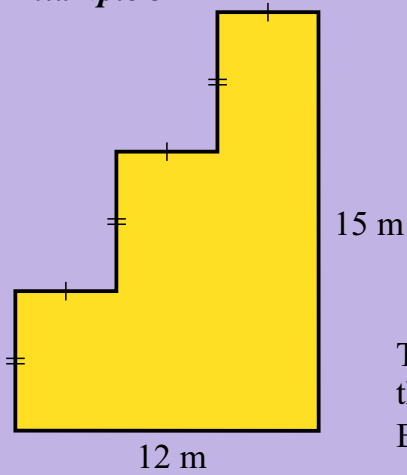
2.



$$\begin{aligned}a &= 6.8 - 2.7 \\ &= 4.1 \text{ m}\end{aligned}$$

$$\begin{aligned}b &= 6.6 - 1.6 \\ &= 5.0 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Perimeter} &= 2.7 + 5.0 + 4.1 + 1.6 + 6.8 + 6.6 \\ &= \mathbf{26.8 \text{ m}}\end{aligned}$$

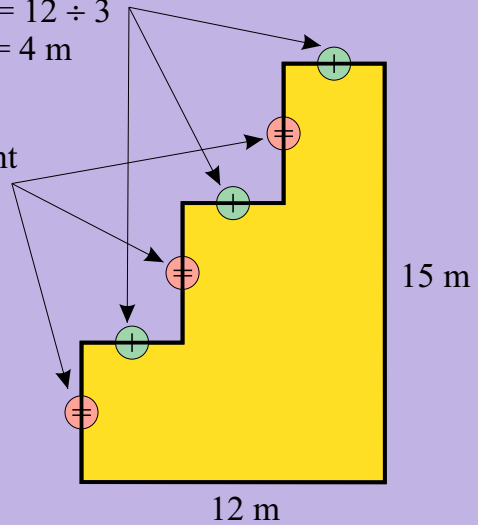
Example 3

These symbols mean that these three sides are the same length.

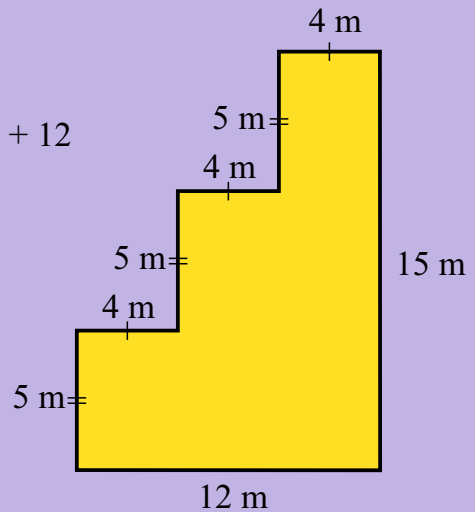
$$\text{Each side} = 12 \div 3 \\ = 4 \text{ m}$$

These three sides are the same length as each other but different to the other three side lengths.

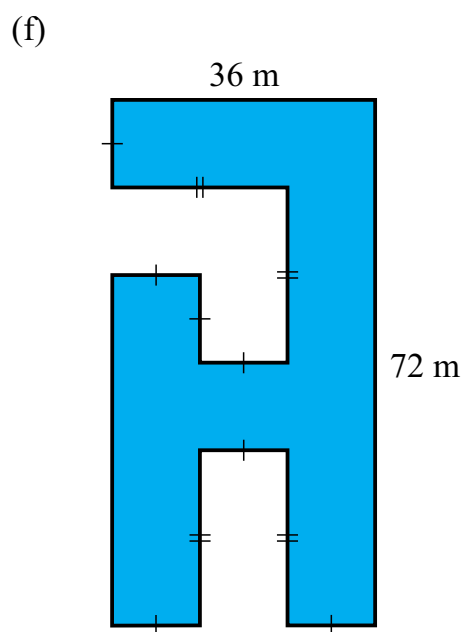
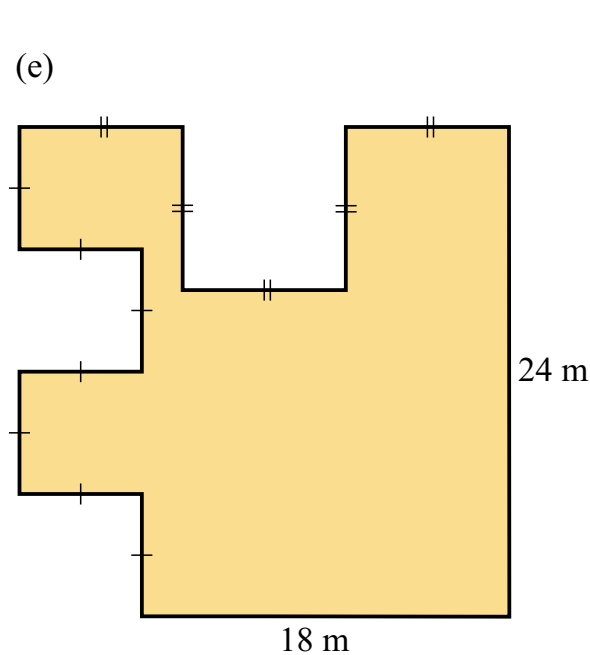
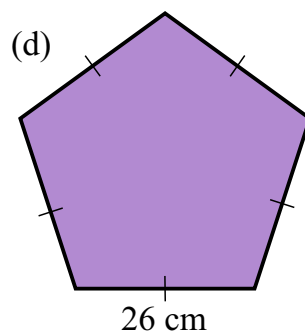
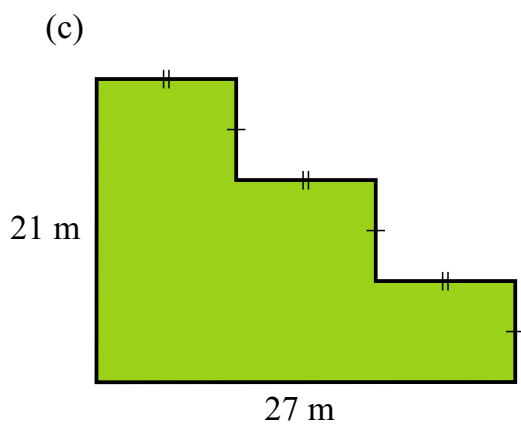
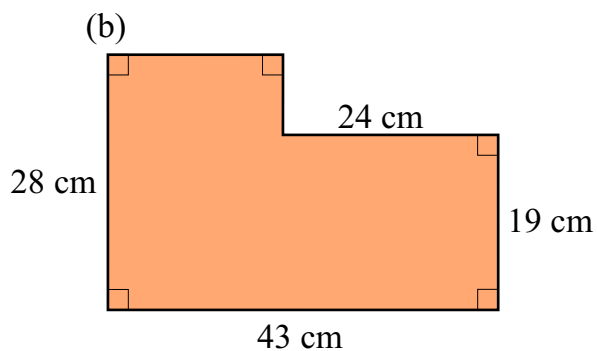
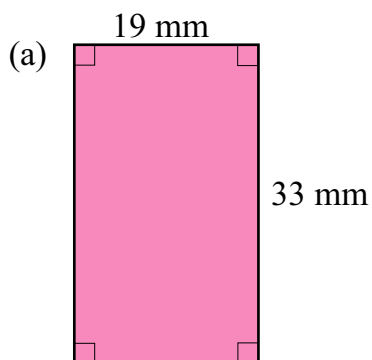
$$\text{Each side} = 15 \div 3 \\ = 5 \text{ m}$$



$$\text{Perimeter} = 5 + 4 + 5 + 4 + 5 + 4 + 15 + 12 \\ = 54 \text{ m}$$

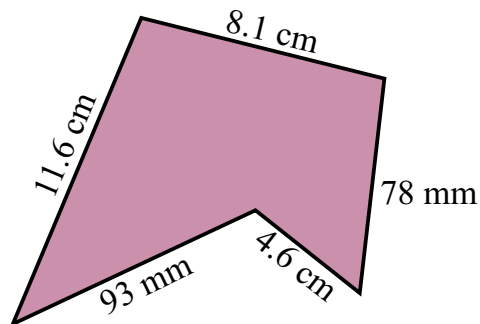


2. Find the perimeter of the following shapes.

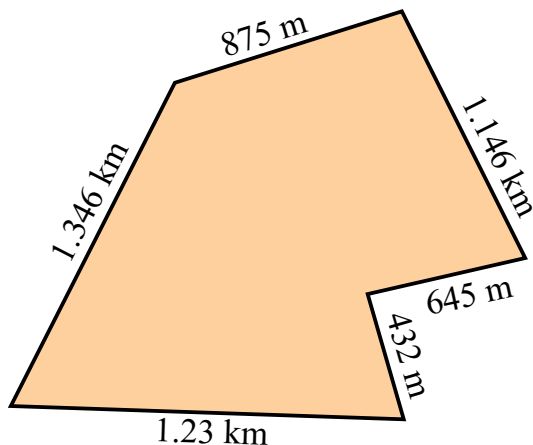


3. Convert the side lengths in the following shapes to the same units then find the perimeter of each shape.

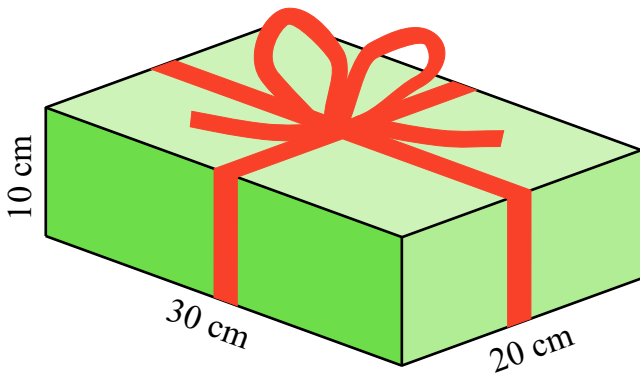
(a)



(b)

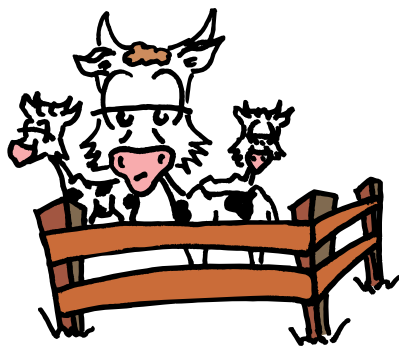


4. Find the perimeter (in cm) of an A4 sheet of paper.
5. (a) What is the side length of a square that has a perimeter of 64 cm?
 (b) What are the side lengths of a rectangle that has a perimeter of 42 cm and the long side is twice the length of the shorter side?
6. The diamond in a game of baseball has side lengths that are all equal and are each 27 metres long.
 How far does a batter run if he hits a home run and runs in a straight line between all the bases?
7. The gift shown has ribbon wrapped around it in both directions. Find the length (in cm) of ribbon required to tie up the gift (allow 50 cm for the bow).



PROBLEM SOLVING

1. How many centimetres are in a megametre?
2. A square and an equilateral triangle have the same perimeter.
The side length of the triangle is 3 cm longer than the side length of the square.
Find the side length of each shape.
3. The perimeter of a rectangular paddock is 500 metres.
The length of one of the sides is 10 metres longer than the other.
(a) Find the side lengths of the paddock.
(b) The owner wanted to fence the paddock. He had to put a post every 5 metres.
How many posts did he need?



4. Bertie, Barney, Barbie, Betty and Benny raced each other in a cross-country running course.
The average speeds of the runners over the course were:
Bertie - 10 km per hour.
Barney - 8000 m per hour.
Barbie - 200 m per minute.
Betty - 3 m per second.
Benny - 2500 mm per second.
If they started running at the same time, list the runners in the order they finished the course.



PUZZLES

1. Find words from this chapter hidden in these sentences.

Example Dad drives to work each day.

- (a) For supper I met Eric at the restaurant.
- (b) I also met Rex who was Eric's friend.
- (c) At the mill I met Reagan who was the baker.
- (d) After practising my French accent I met Renee.
- (e) Sandi's tan certainly was dark.
- (f) Glen G. Thornton was the owner of a computer company.
- (g) At the races Pete the punter gave me a sure bet.

2. Change METRE into METER by changing one letter at a time to form a new word. The letters may need to be rearranged. A clue for each word is given.

M	E	T	R	E
M	E	T	E	R

- Single digit number.
- You write on one, you sleep on one.
- Farm animal.
- A square is one, a circle is one,  is one.
- 'The final frontier' or a gap.
- Used to dig a hole.
- One possibility when a coin is tossed.
- Conceals from view.
- Swims under water.
- Where bees live.
- Blood vessels.
- Number of days in a week.
- People between the ages of 12 and 20 are in their ____.
- Control the direction of a vehicle.

CHAPTER REVIEW

1. Choose the alternative which is the best estimate of the following lengths.

(a) The diameter of an orange.

A 7 mm **B** 70 mm **C** 700 mm **D** 7000 mm

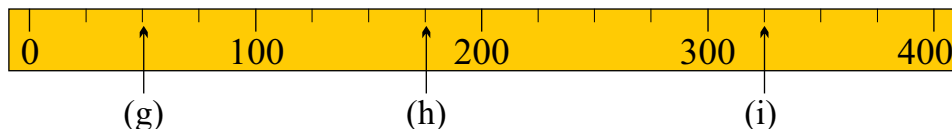
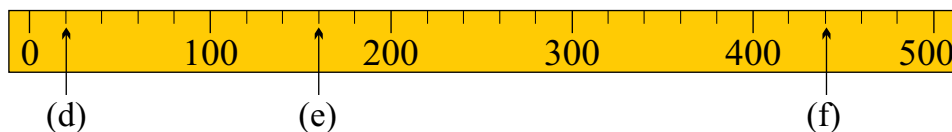
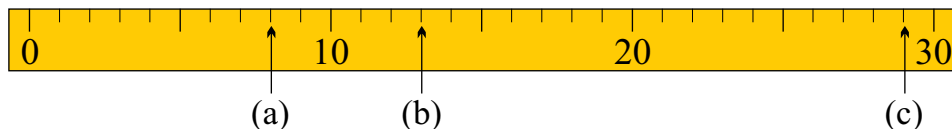
(b) The height of a soccer goal.

A 250 mm **B** 250 cm **C** 250 m **D** 250 km

(c) The distance from Brisbane to Sydney.

A 1000 mm **B** 1000 cm **C** 1000 m **D** 1000 km

2. Read the measurements shown on these scales.



3. Change the following lengths to the units shown in the brackets.

(a) 7 cm (mm)	(b) 1 km (m)	(c) 20 m (cm)
(d) 6000 cm (m)	(e) 850 mm (cm)	(f) 6 mm (cm)
(g) 970 cm (m)	(h) 0.03 m (cm)	(i) 0.5 mm (cm)
(j) 4 cm (m)	(k) 0.7105 km (m)	(l) 210 mm (m)

4. Change the following lengths to the units shown in the brackets.

(a) 3 m 67 cm (m)	(b) 8 cm 9 mm (cm)
(c) 10 m 7 cm (m)	(d) 5 km 300 m (km)
(e) 1 km 50 m (km)	(f) 7 m 6 cm (m)
(g) 1 cm 1 mm (cm)	(h) 8 m 34 cm 2 mm (m)

5. Round the following lengths to the nearest metre.

- | | | |
|---------------|----------------|--------------|
| (a) 6.2 m | (b) 37.8 m | (c) 99.7 m |
| (d) 1 m 25 cm | (e) 14 m 80 cm | (f) 5 m 8 cm |

6. Round the following lengths to the nearest centimetre.

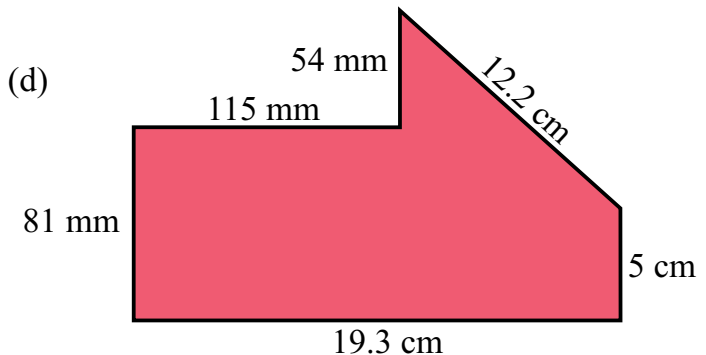
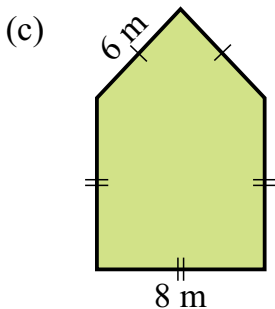
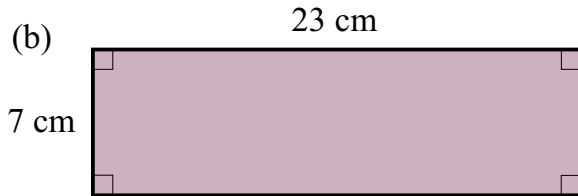
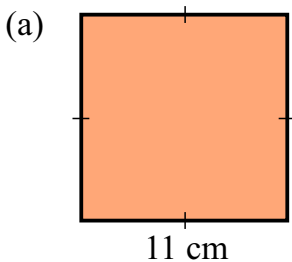
- | | | |
|---------------|---------------|----------------|
| (a) 6.2 cm | (b) 10.8 cm | (c) 9.09 cm |
| (d) 1 cm 8 mm | (e) 3 cm 2 mm | (f) 35 cm 7 mm |

7. Find the length (in cm) of a roll of 100 stamps if each stamp is 30 mm wide?

8. Phaedra is a runner. She measured each of her strides while sprinting to be 50 cm.

How many strides would she take to sprint 100 metres?

9. Find the perimeter of the each of the following shapes.



10. A rectangle has a perimeter of 44 cm. Its length is 12 cm.
Find the width of the rectangle.

11. The perimeter of a square paddock is 1.2 km.
Find the side length (in metres) of the paddock.

Area

The size of a surface or the amount of space of a flat or two-dimensional shape is called its **area**.

Common units of area are:

- square millimetres (mm^2)
- square centimetres (cm^2)
- square metres (m^2)
- hectares
- square kilometres (km^2)

The unit used would depend on the size of the area.

1 mm^2 is the area equal to a square with a side length of 1 mm.

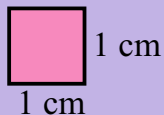
1 cm^2 is the area equal to a square with a side length of 1 cm.

1 m^2 is the area equal to a square with a side length of 1 m.

1 hectare is the area equal to a square with a side length of 100 m.

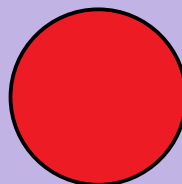
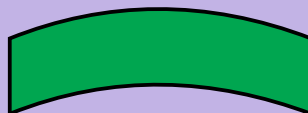
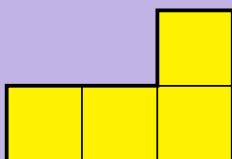
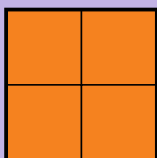
1 km^2 is the area equal to a square with a side length of 1 km.

This square has an area of 1 cm^2 .



But many shapes would have the same area.

Examples All the shapes below have an area of 4 cm^2 .

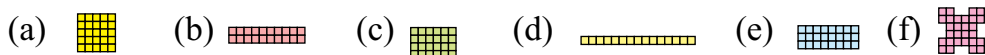


Area

EXERCISE 17A

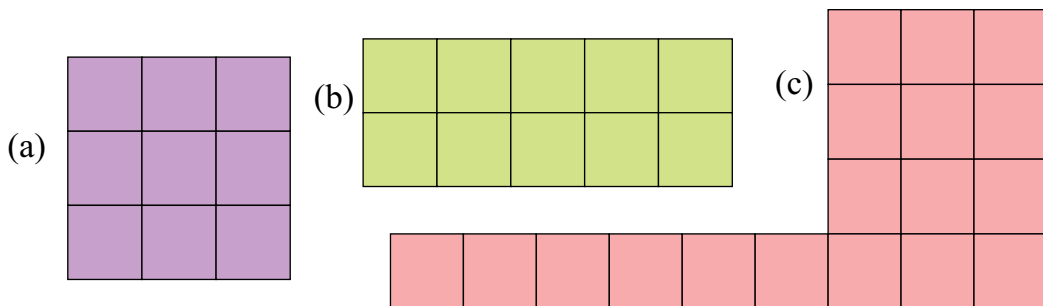
1. Each of the small squares in the following shapes is 1 mm^2 .

How many mm^2 are in each shape?



2. Each of the small squares in the following shapes is 1 cm^2 .

How many cm^2 are in each shape?



3. How many mm^2 are in one cm^2 ?



4. Choose the alternative which is the best estimate of the area of each of the following objects.

(a) A postage stamp.

A 1 mm^2

B 10 mm^2

C 10 cm^2

D 100 cm^2

(b) A page of a textbook.

A 4 cm^2

B 40 cm^2

C 400 cm^2

D 4000 cm^2

(c) A netball court.

A 40 m^2

B 400 m^2

C 4000 m^2

D $40\,000 \text{ m}^2$

(d) A single bed sheet.

A 400 mm^2

B 400 cm^2

C 4 m^2

D 40 m^2

(e) Tasmania.

A 70 km^2

B 700 km^2

C 7000 km^2

D $70\,000 \text{ km}^2$

(f) Your bedroom.

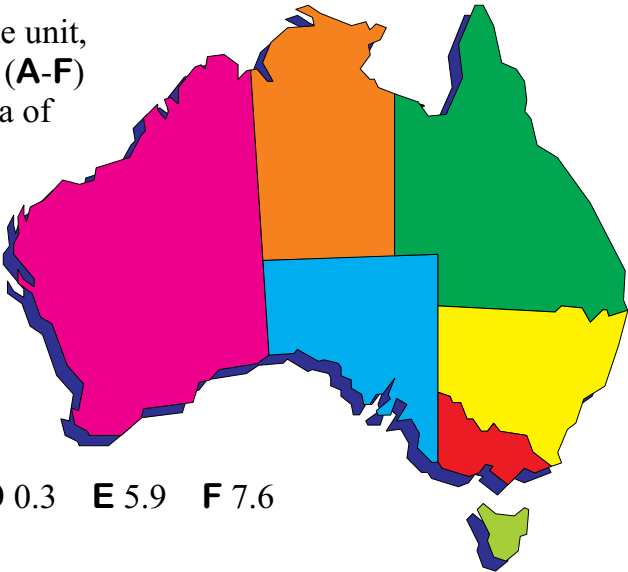
A 10 m^2

B 100 m^2

C 1000 cm^2

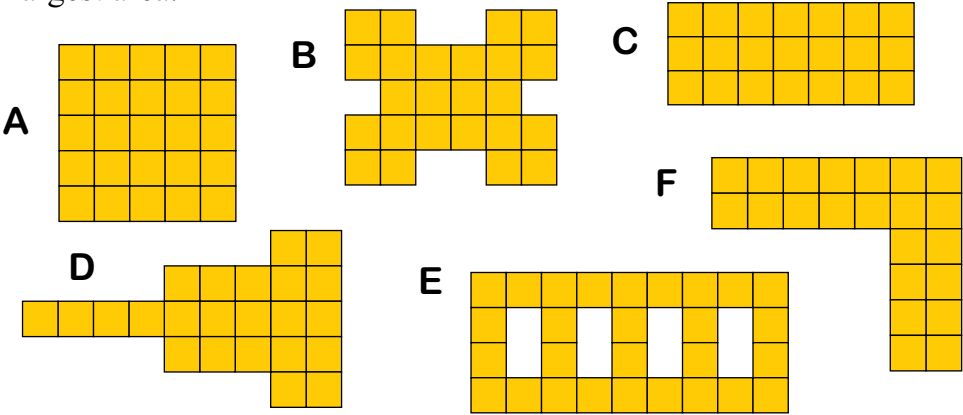
D $10\,000 \text{ cm}^2$

5. If the area of Victoria is one unit, choose from the list below (A-F) the best estimate of the area of each of the states and the Northern Territory.
- (a) New South Wales
 - (b) Queensland
 - (c) Tasmania
 - (d) South Australia
 - (e) Western Australia
 - (f) Northern Territory

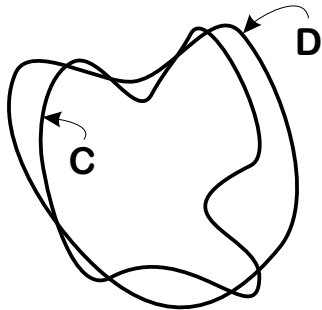
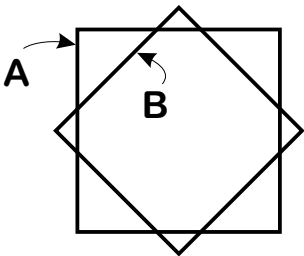
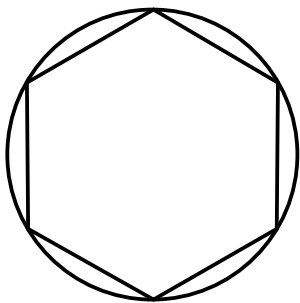


A 4.3 **B** 3.5 **C** 11.1 **D** 0.3 **E** 5.9 **F** 7.6

6. If the small squares in the following shapes are all the same size, list the shapes from the one with the smallest area to the one with the largest area.



7. State which one of the following pairs of objects has the larger area.
- (a) A square of side length 10 cm or a circle of diameter 10 cm.
 - (b) The circle or hexagon shown below.
 - (c) Square **A** or **B** shown below.
 - (d) Shape **C** or **D** shown below.

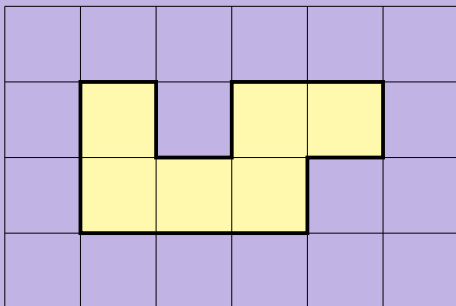


Using a Grid

Use the 1 cm square grid to find the area of the shapes below.

Examples

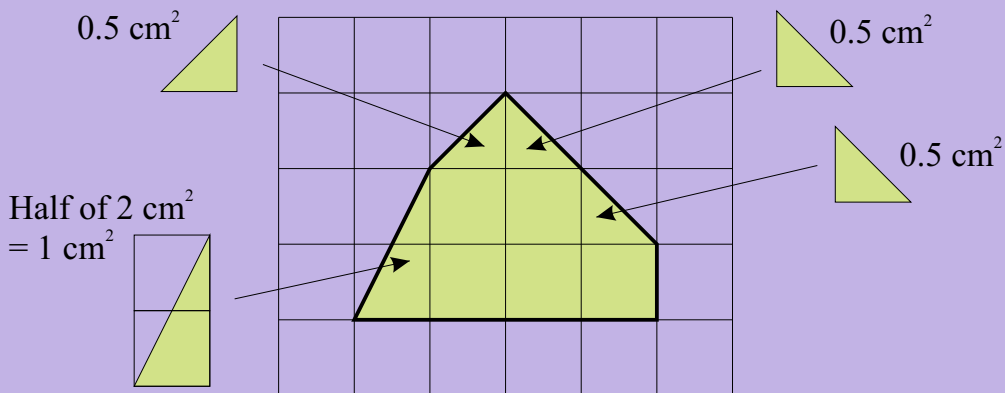
1.



To find the area add all the 1 cm^2 squares in the shape.

$$\text{Area} = 6 \text{ cm}^2$$

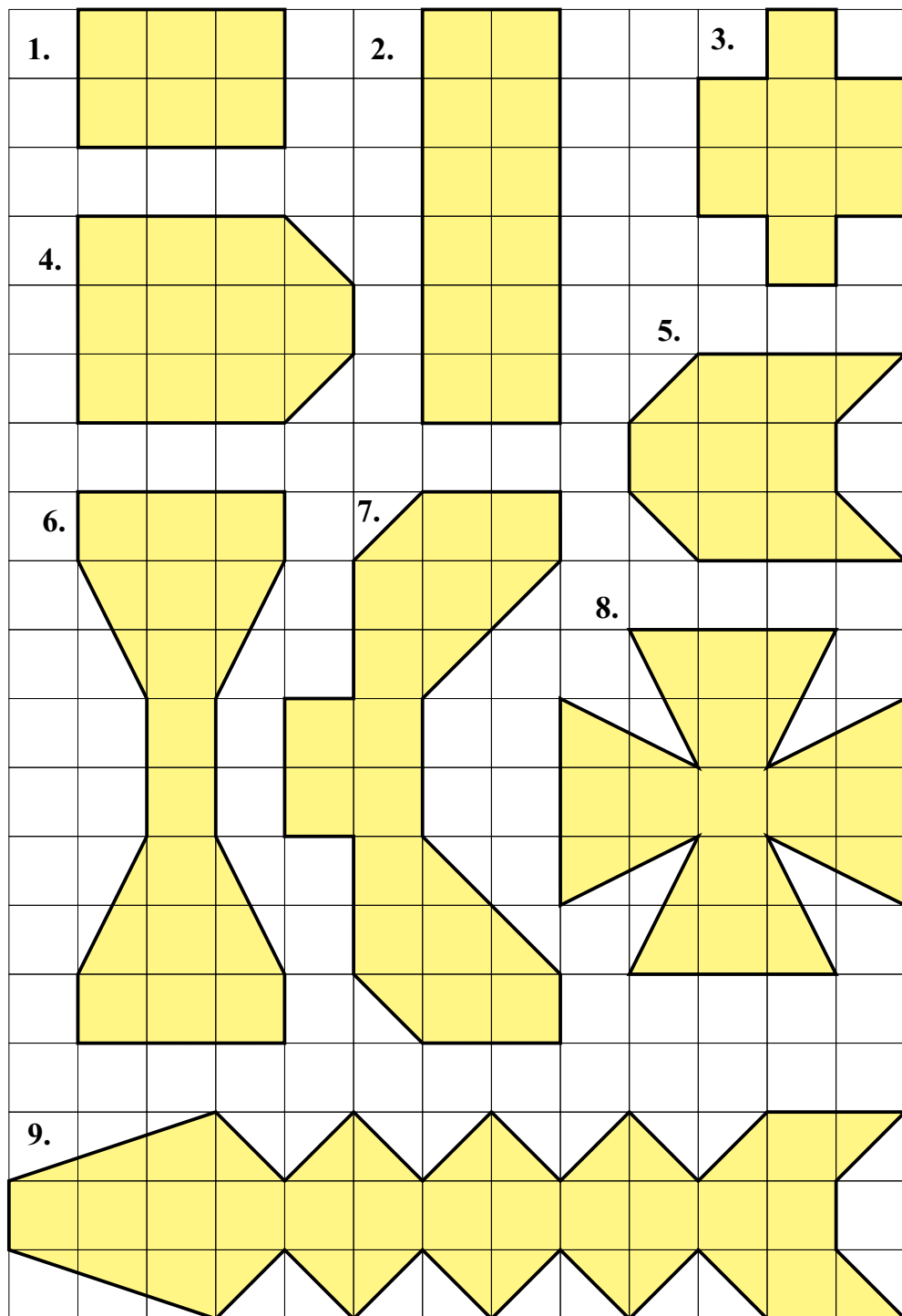
2. In this example there are several parts of squares and 5 whole 1 cm^2 squares to be added to find the total area.



$$\begin{aligned} \text{Area} &= 5 \text{ squares} + 0.5 + 0.5 + 0.5 + 1 \\ \text{Area} &= 7.5 \text{ cm}^2 \end{aligned}$$

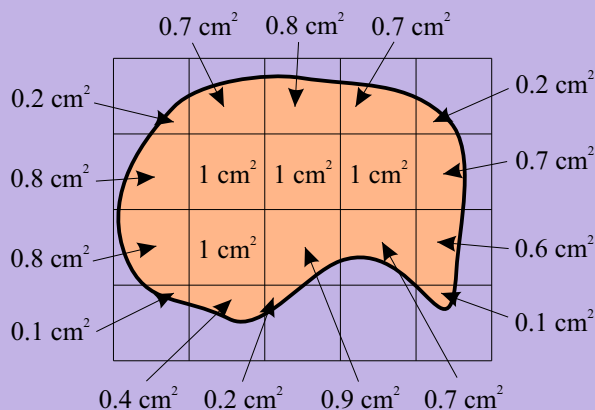
EXERCISE 17B

Use the 1 cm square grid to find the area of the shapes below.



Example

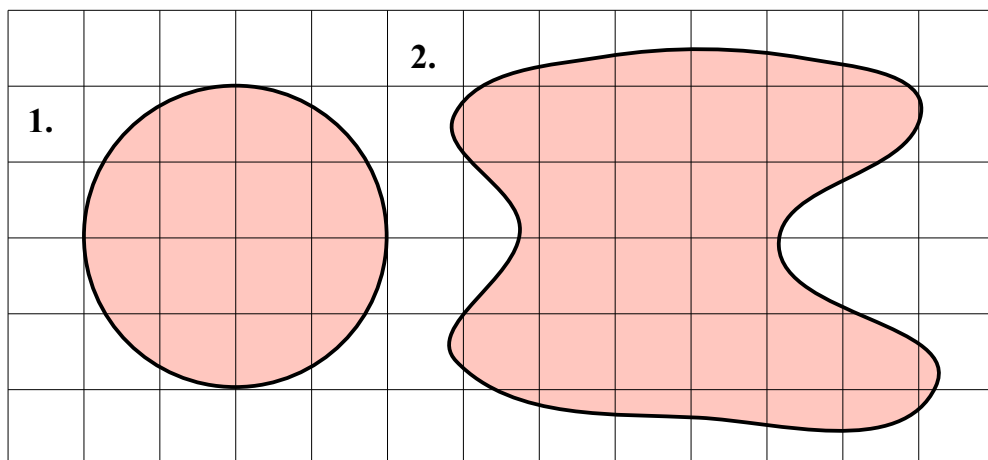
It is only possible to find an approximate area of a shape with curves when using a grid. Estimate the area of the shape in each square then add all these to find the approximate total area.



$$\begin{aligned} \text{Area} &= 0.1 + 0.8 + 0.8 + 0.2 + 0.7 + 0.8 + 0.7 + 0.2 + 0.7 \\ &\quad + 0.6 + 0.1 + 0.7 + 0.9 + 0.2 + 0.4 + 1 + 1 + 1 + 1 \\ \text{Area} &= 11.9 \text{ cm}^2 \end{aligned}$$

EXERCISE 17C

Use the 1 cm grid to find the approximate area of the shapes below.

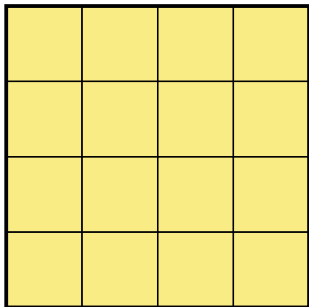


Squares and Rectangles

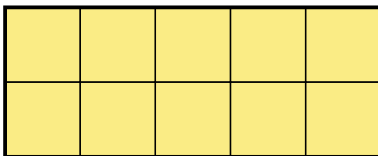
EXERCISE 17D

1. Find the area of each of the following shapes if each small square is 1 cm^2 .

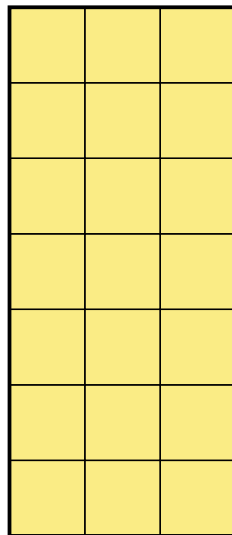
(a)



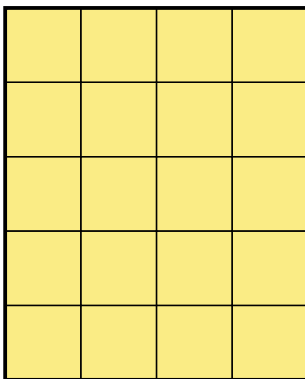
(b)



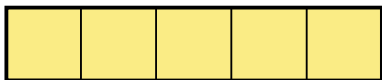
(c)



(d)



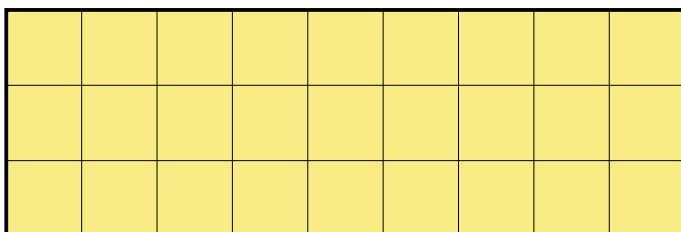
(e)



2. (i) State the length and width of each of the shapes in question 1.
 (ii) Find a relationship between the length and width and the area of each shape.
 (iii) Complete the following rule by placing one of the arithmetic operations $+$, $-$, $\div$ or $\times$ in the box.

Area = length width

3. Use this rule to find the area of the rectangle below then check the answer by counting the 1 cm^2 squares.



4. Use the dimensions shown to find the area of each of the following rectangles and squares.

Example

Area of a rectangle = length $\times$ width



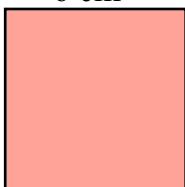
6 m

8 m

$$\begin{aligned}\text{Area} &= 8 \times 6 \\ &= \mathbf{48 \text{ m}^2}\end{aligned}$$

(a) 6 cm

6 cm



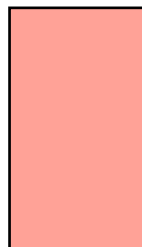
(b) 10 m

4 m



(c) 3 mm

9 mm



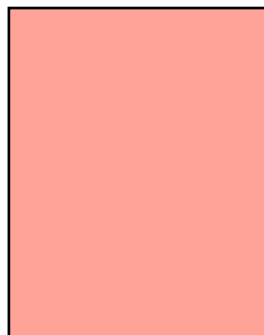
(d) 20 m

4 m



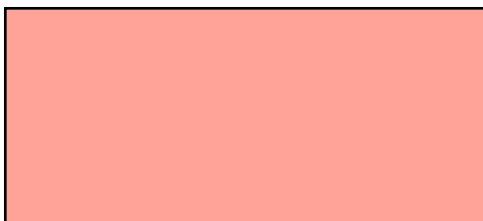
(e) 12 mm

15 mm



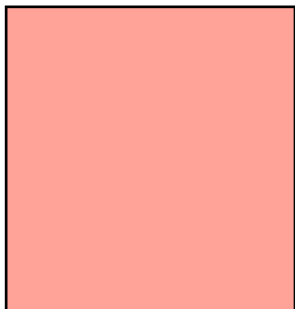
(f) 42 cm

24 cm



(g) 8.5 mm

12.5 mm

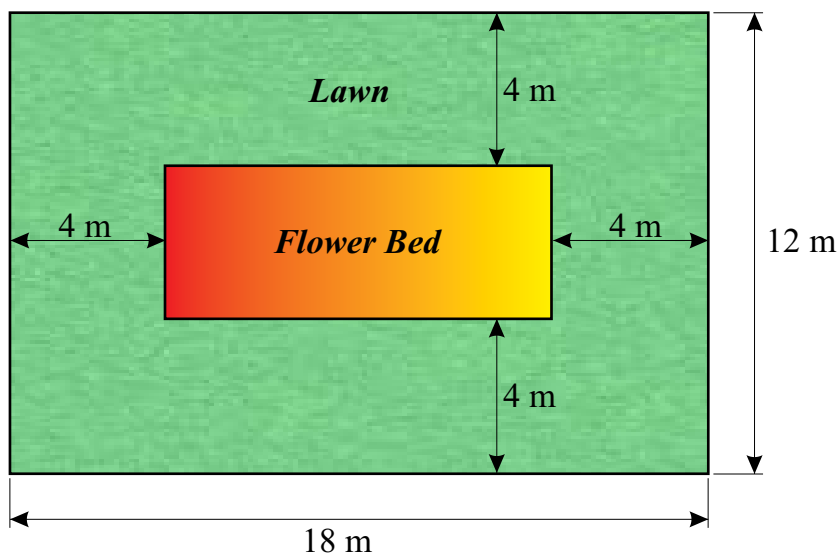


(h) 200 m

350 m



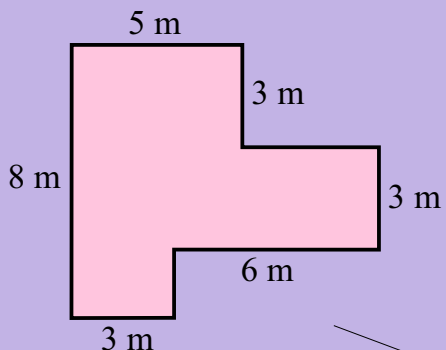
5. (a) Find the area of an 8 metre square.
(b) Find the area of a $9\text{ cm} \times 11\text{ cm}$ rectangle.
6. Winona wanted to paint the floor of her lounge room. The dimensions of the floor are $4.5\text{ metres} \times 6\text{ metres}$.
What is the area of floor to be painted?
7. The perimeter of a paddock is 300 metres. The length of the paddock is 30 metres longer than the width.
What is the area of the paddock?
8. (a) List the dimensions (whole numbers only) of all the rectangles that can be formed having perimeters of 16 metres.
(b) Find the area of each of these rectangles.
(c) What are the dimensions of the rectangle with the largest area?
(d) What is the name given to this type of rectangle?
(e) Use this information to find the area of the largest rectangle that could be formed having a perimeter of 40 metres.
9. Isolde had a rectangular garden of 18 metres by 12 metres. She planned to have a central flower bed surrounded by 4 metres of lawn as shown in the diagram below.
(a) Find the area of the flower bed.
(b) Find the area of lawn.



Composite Shapes - Squares and Rectangles

Find the area of the shaded region in each of the following shapes.

Example 1



To find the area of this shape:

Step 1 - Find all the unknown lengths.

Step 2 - Divide the shape into squares and/or rectangles.

Step 3 - Find the area of all these and add them to find the total area.

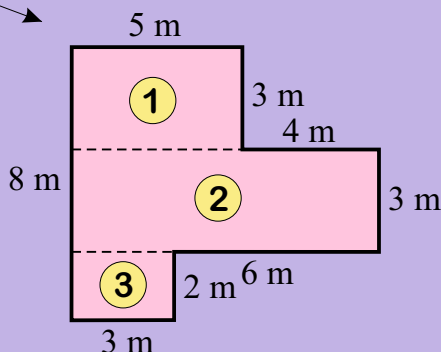
$$\text{Area 1} = 5 \times 3 = 15 \text{ m}^2$$

$$\text{Area 2} = 9 \times 3 = 27 \text{ m}^2$$

$$\text{Area 3} = 3 \times 2 = 6 \text{ m}^2$$

$$\text{Total area} = 15 + 27 + 6$$

$$\text{Total area} = 48 \text{ m}^2$$



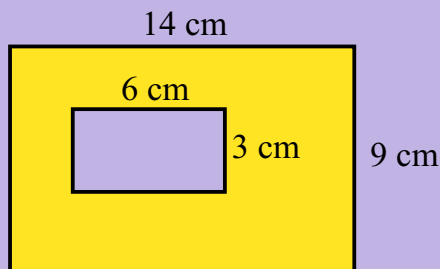
Example 2

The area of the shaded region is equal to the area of the larger rectangle minus the area of the smaller rectangle.

$$\text{Area} = (14 \times 9) - (6 \times 3)$$

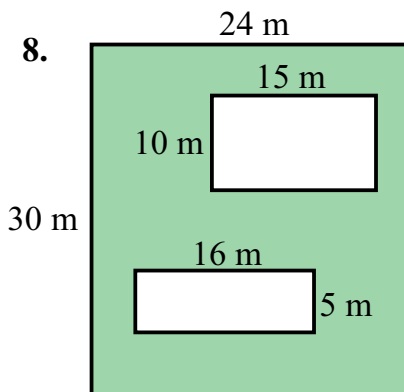
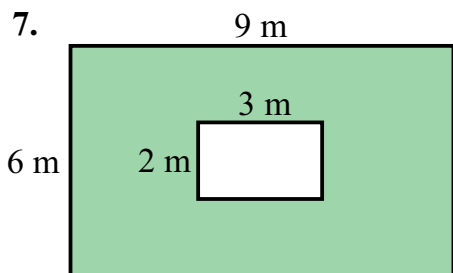
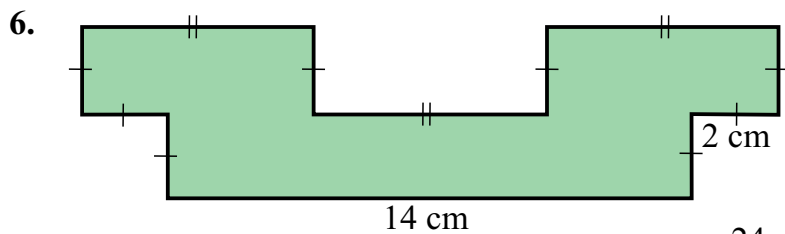
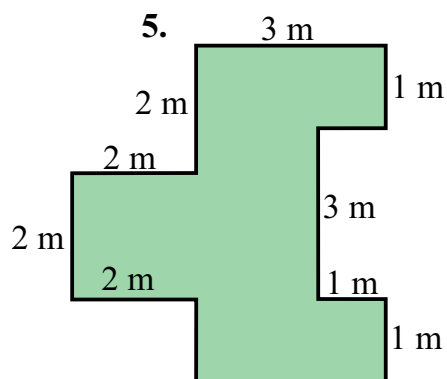
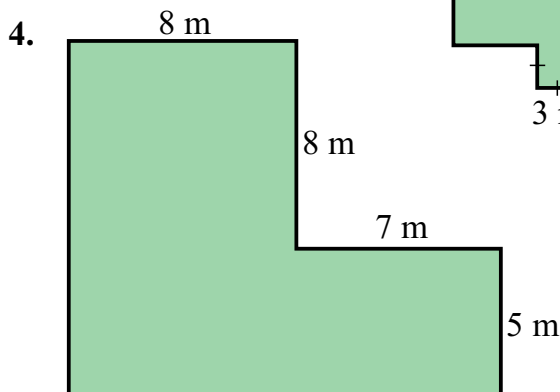
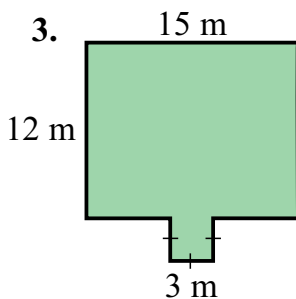
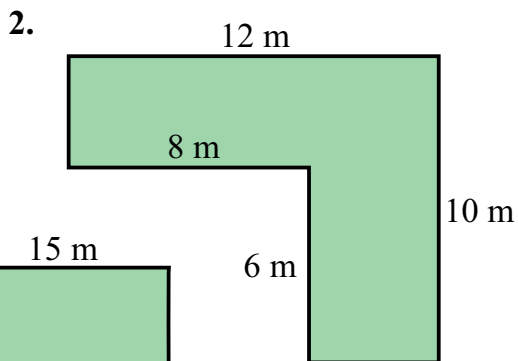
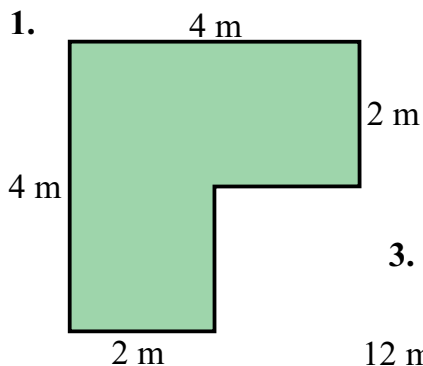
$$= 126 - 18$$

$$\text{Area} = 108 \text{ cm}^2$$



EXERCISE 17E

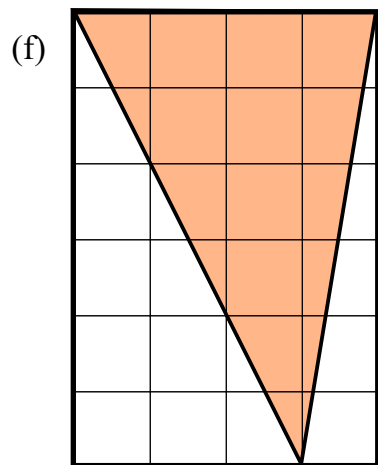
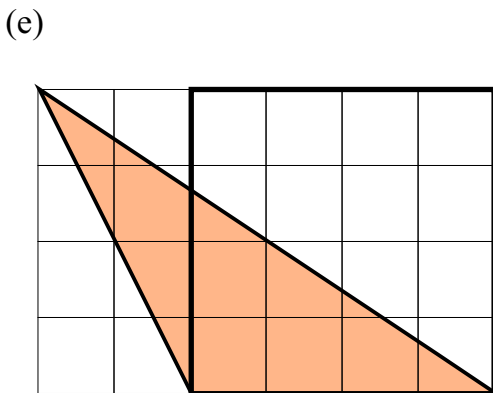
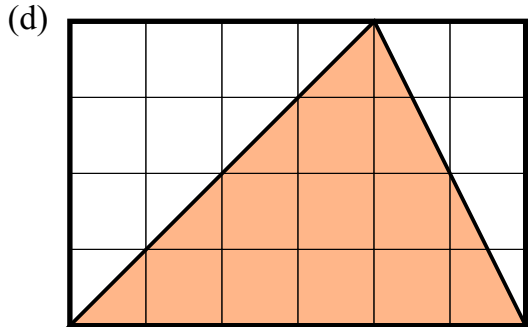
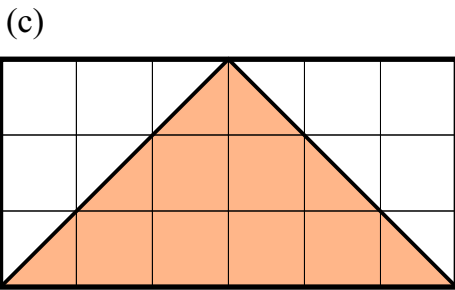
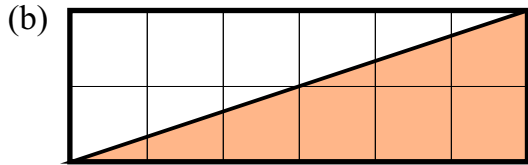
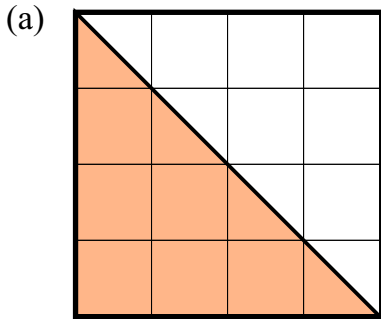
Find the area of each of the shaded regions below.



Triangles

EXERCISE 17F

1. Use the 1 cm square grids in each of the following diagrams to:
- find the area of the rectangle.
 - find the area of the shaded triangle.

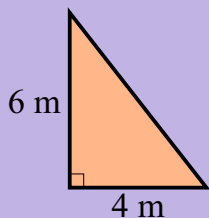


2. Is there any relationship between the area of the rectangle and the area of the triangle in each diagram?
If there is, state this relationship.

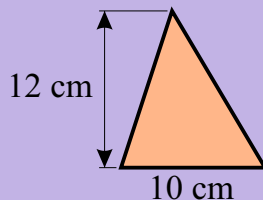
3. Find the area of each of the following triangles.

Examples

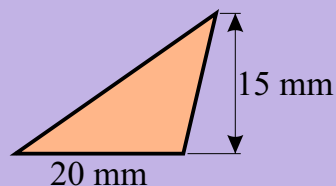
$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$



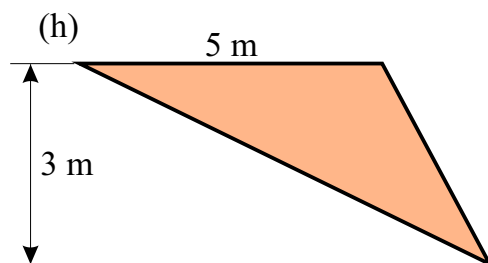
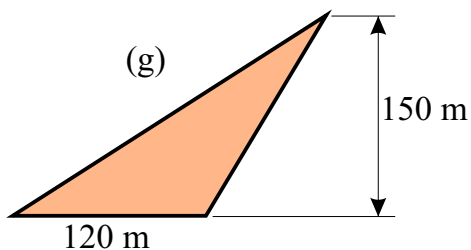
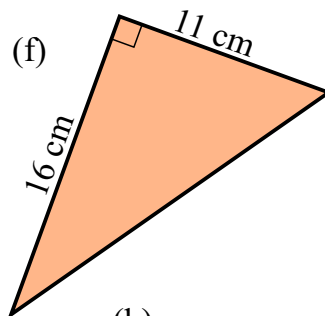
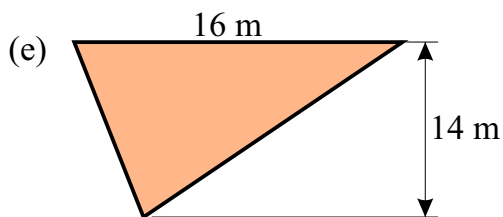
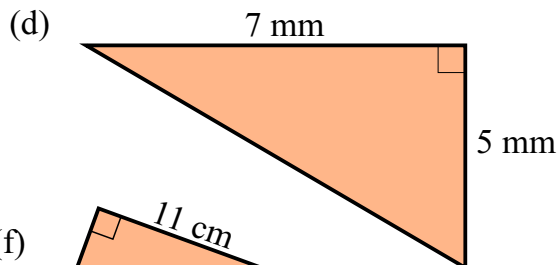
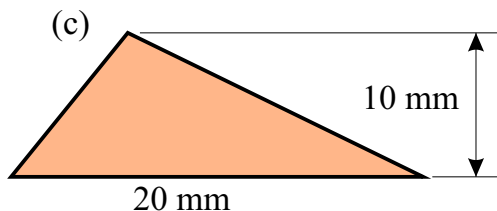
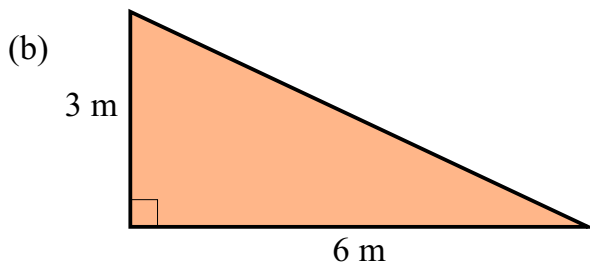
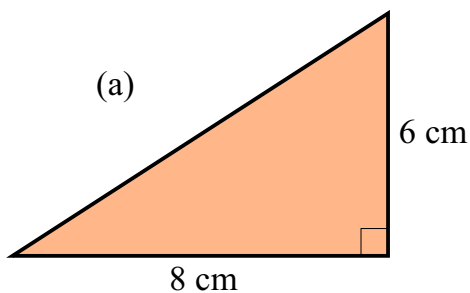
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 4 \times 6 \\ &= 12 \text{ m}^2 \end{aligned}$$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 10 \times 12 \\ &= 60 \text{ cm}^2 \end{aligned}$$



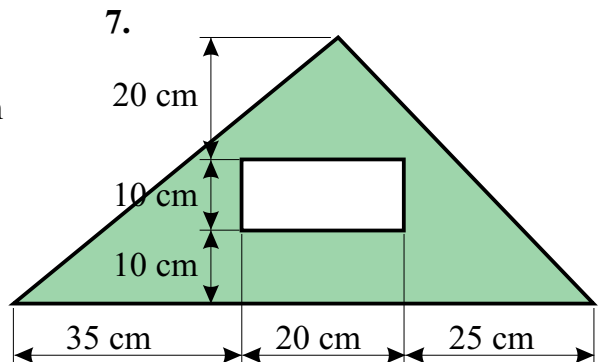
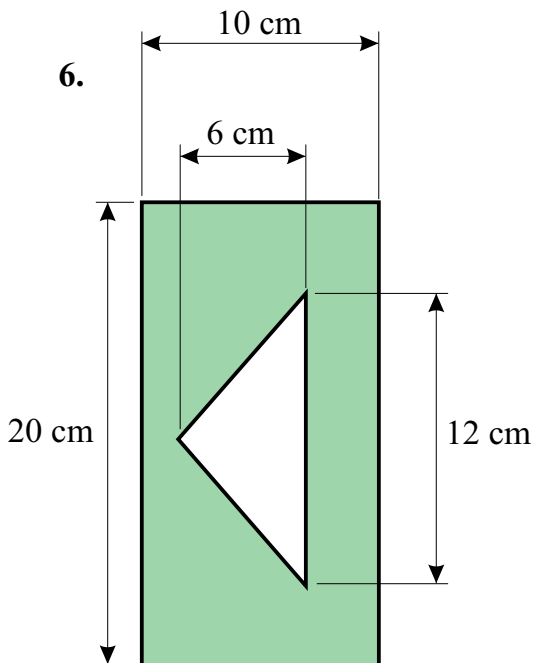
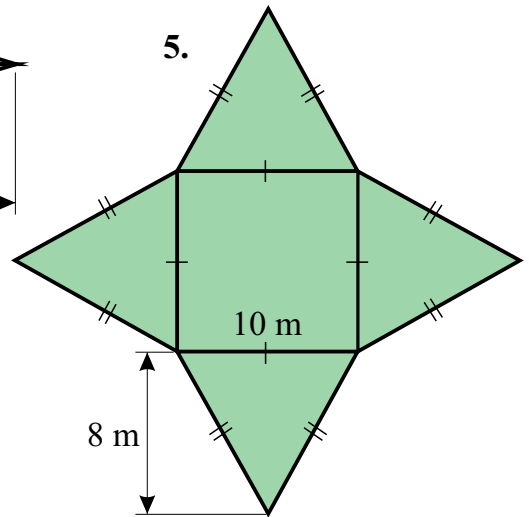
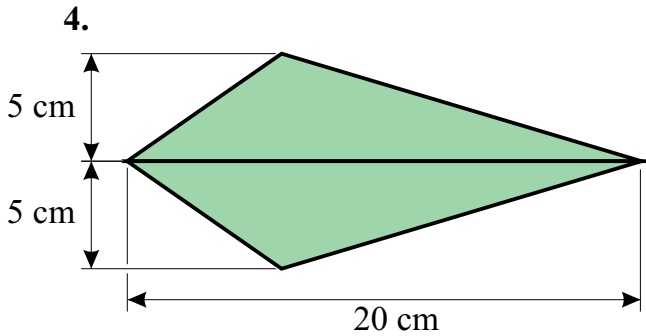
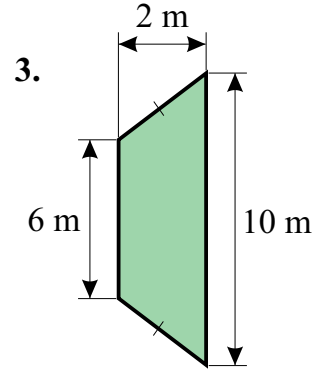
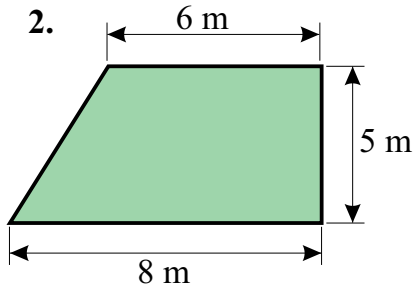
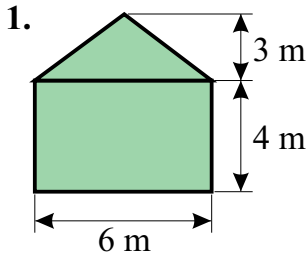
$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 20 \times 15 \\ &= 150 \text{ mm}^2 \end{aligned}$$



Composite Shapes - Triangles and Rectangles

EXERCISE 17G

Find the area of each of the shaded regions below.



Practical Problems

EXERCISE 17H

1. A newspaper was charging \$2 per cm^2 for advertising space.

What would be the cost to place the following advertisements in the newspaper?

(a)

**TIM'S
MOWING
SERVICE**

No job too small
You grow it
I'll mow it
Contact Tim
52531269

3 cm

5 cm

(b)

**HELP WANTED
TO MAKE TOYS**

Must be short and be
prepared to travel.
Contact S. Claus
The North Pole

4 cm

3 cm

(c)

**F r Sale
Sec ndhand
C mputer
Keyb ard
and
M use**

In g d
c nditi n.
C uld be
used f r
w rd
pr cessing

C ntact Sc tt
Ph ne
9 5 67 2

2 cm

7 cm

2. A hectare (ha) is an area equal to $10\,000\text{ m}^2$.

(a) Convert the following areas to hectares.

(i) $40\,000\text{ m}^2$ (ii) $80\,000\text{ m}^2$ (iii) $300\,000\text{ m}^2$ (iv) $2\,500\,000\text{ m}^2$

(b) Find the area, in hectares, of land with the following dimensions.

(i) $100\text{ m} \times 200\text{ m}$ (ii) $1000\text{ m} \times 50\text{ m}$ (iii) $300\text{ m} \times 400\text{ m}$
(iv) $1\text{ km} \times 1\text{ km}$

(c) Angela wanted to buy a 10 hectare rectangular block of land. She measured the front of the block to be 200 metres.

What is the other dimension of the block of land?

(d) Jeff had a **square** block of land with an area of one hectare that he wanted to fence. What length of fencing is required?

3. (a) Kirsten wanted to paint a room. She worked out that area to be painted was 60 m^2 . She was going to use paint that covered 20 m^2 per litre. How many litres of paint will she need?

(b) Ling wanted to paint a floor that was $10\text{ m} \times 4\text{ m}$. She was using the same paint that Kirsten used.
How many litres of paint will Ling need?

4. Find the areas of the following countries and list them from smallest to largest.

BRAZIL, CHINA, AUSTRALIA, GERMANY, INDIA, CANADA

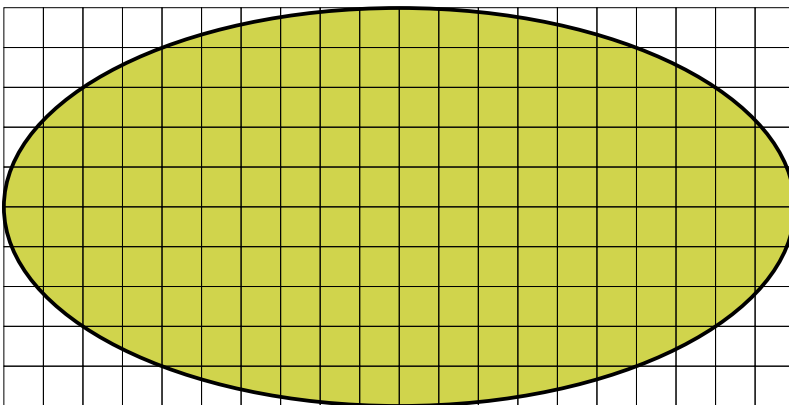
PROBLEM SOLVING

1. (a) Find the dimensions (whole numbers only) of all the rectangles (including squares) that have a perimeter of 16 cm.
 (b) Calculate the area of each of these rectangles.
 (c) State which one has the largest area.
 (d) Use the answer to part (c) to find the dimensions of the rectangle with perimeter of 100 cm that has the largest area.

2. Vladimir can paint a $4\text{ m} \times 2\text{ m}$ wall in 3 hours.
 How long will it take Vladimir to paint an $8\text{ m} \times 4\text{ m}$ wall?

3. A square and rectangle have the same area.
 The length of the rectangle is 3 metres longer than its width.
 The side length of the square is 1 metre longer than the shorter side of the rectangle.
 Find the dimensions of the square and rectangle.

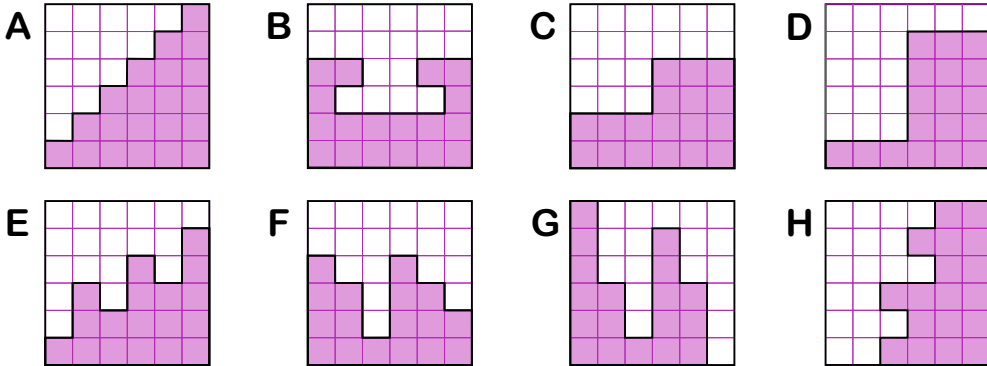
4. John wanted to calculate the area of the local football ground so he could buy enough soil to cover it and plant new grass.
 He measured it to be 200 metres long and 100 metres wide.
 He then drew a 20×10 grid of squares and sketched the ground on the grid as shown below.



- (a) How long (in metres) does the side length of each square on the grid represent?
- (b) What area (in m^2) is represented by each square on the grid?
- (c) Use the grid to find the approximate area of the football ground.
- (d) Rearrange the letters PELLIES to find a word that describes the shape of the football ground.

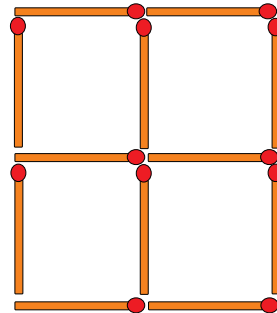
PUZZLES

1. Which of the following squares, if cut into the two shapes shown, would form two identical shapes (except for the colour).
More than one answer.



2. Draw a 6 cm square on a centimetre square grid.
Divide the square into two identical shapes that when rearranged and joined together would form a 9 cm $\times$ 4 cm rectangle.

3. The pattern shown can be formed using matches.
- Remove two matches from this pattern to leave two squares.
 - Move four matches in the pattern to form three squares.



4. Find words from this chapter hidden in these sentences.

Example Dad drives to work each day.

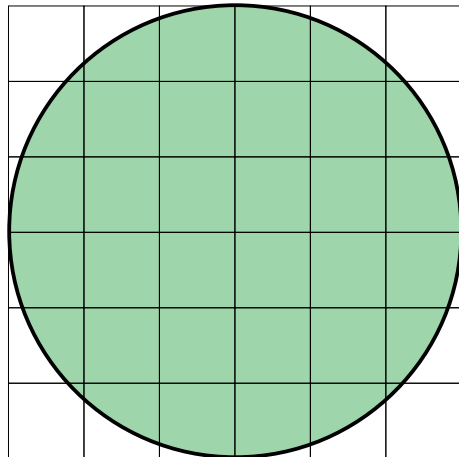
- Are aliens really out there?
- Greg rides his horse every weekend.
- Zoe has three goldfish, a pet snake and two cats.
- One of my mates, Tim, ate seven sausages at the BBQ.
- On a spelling test Rachel misspelt QUARREL as QUAREL.
- To shoot the target you must aim at the correct angle.
- The letters AMTMSIHECTA rearranged form MATHEMATICS.
- The Bondi Mens Ionosphere Group studies the Earth's atmosphere.

CHAPTER REVIEW

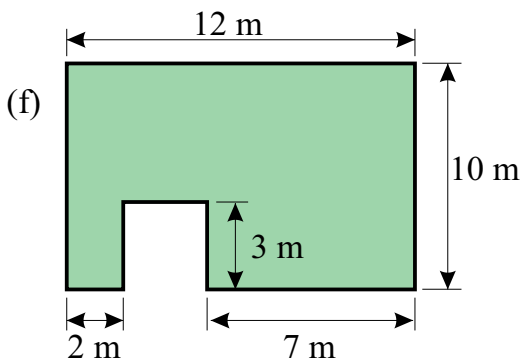
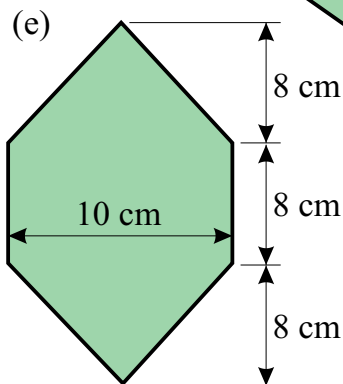
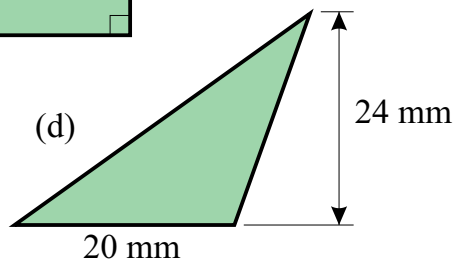
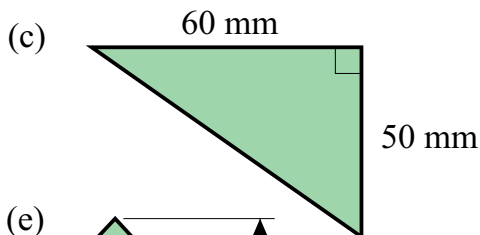
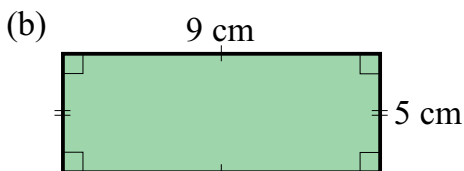
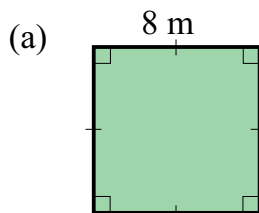
1. Choose the best estimate of the area of a computer screen.

- A** 60 mm^2 **B** 600 mm^2
C 60 cm^2 **D** 600 cm^2

2. Use the centimetre square grid shown to find the approximate area of a circle with radius 3 cm.



3. Find the area of the following shapes using the dimensions given.



4. Find the area of a rectangle that has a perimeter of 32 metres and its longer side is 2 metres longer than the shorter side.

5. How many hectares is a $500 \text{ m} \times 600 \text{ m}$ property?

Volume

Volume

The amount of space occupied by a three-dimensional solid object is called its **volume**.

Common units of volume are: cubic millimetres (mm^3)
 cubic centimetres (cm^3)
 litres
 cubic metres (m^3)

The unit used would depend on the size of the volume.

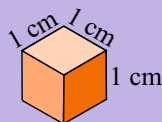
1 mm^3 is the volume of a cube with side lengths of 1 mm.

1 cm^3 is the volume of a cube with side lengths of 1 cm.

1 litre is the volume of a cube with side lengths of 10 cm.

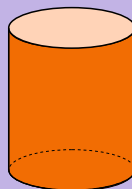
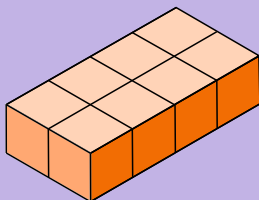
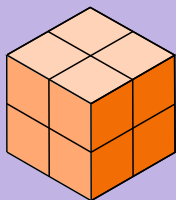
1 m^3 is the volume of a cube with side lengths of 1 m.

This cube has a volume of 1 cm^3 .



But many shapes would have the same volume.

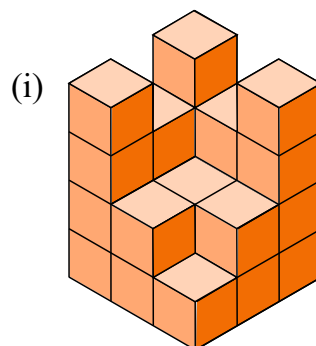
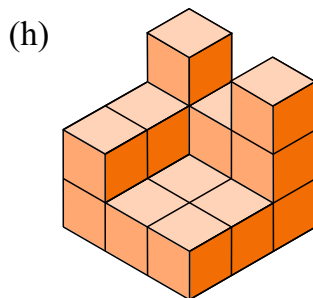
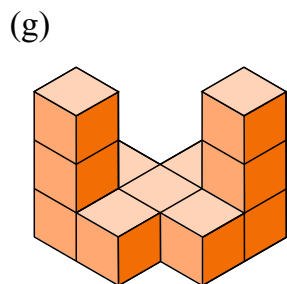
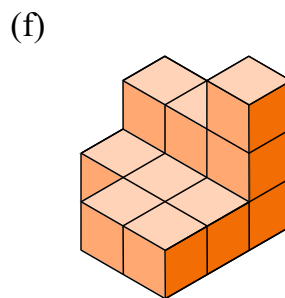
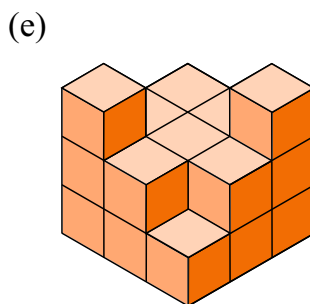
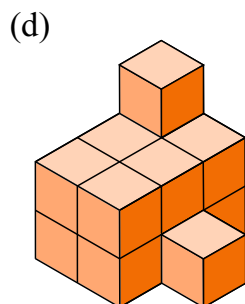
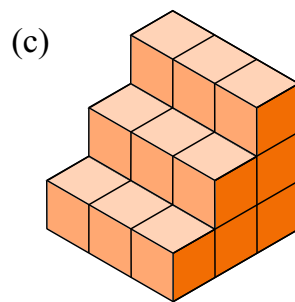
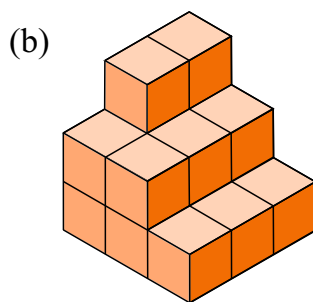
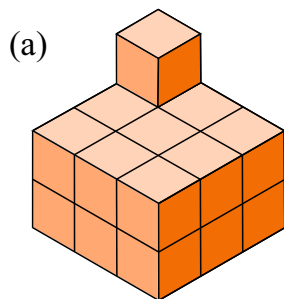
Examples All the objects below have a volume of 8 cm^3 .



Rectangular Prisms

EXERCISE 18A

1. How many small cubes are in each of the following objects?



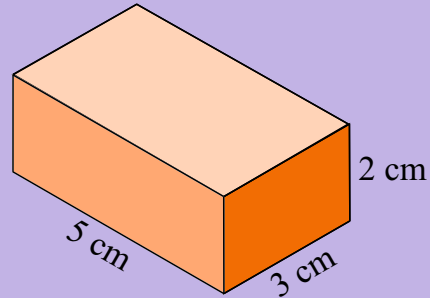
2. If each of the small cubes in the objects in question 1 have a volume of 1 cm^3 , state the volume of each object.

A **rectangular prism** is a three-dimensional object that has rectangles on all three faces.

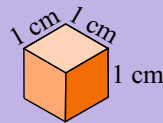
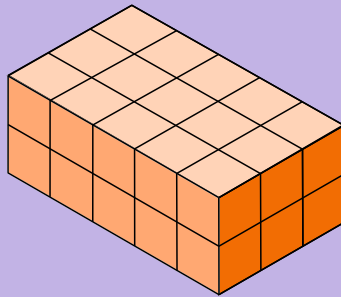
Example

The rectangular prism shown here has the following dimensions:

length = 5 cm
width = 3 cm
height = 2 cm



Its volume can be found by seeing how many one centimetre cubes would be needed to make this object.

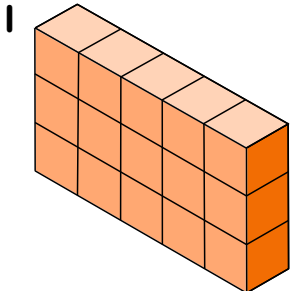
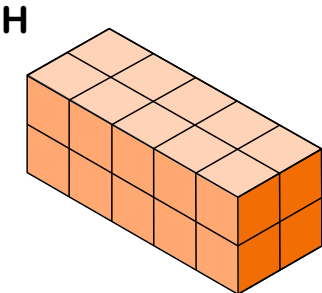
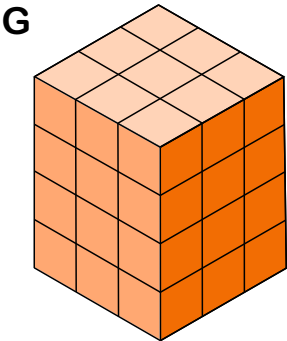
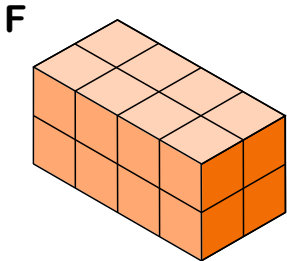
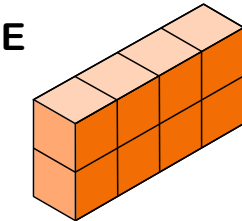
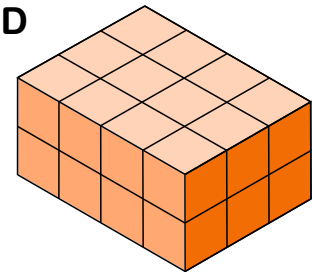
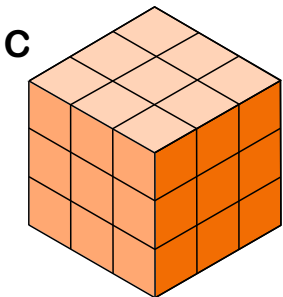
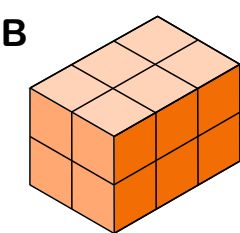
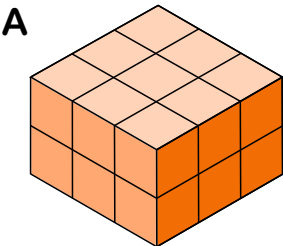


It would take 30 one centimetre cubes to make this object.
Each one centimetre cube has a volume of one cubic centimetre (1 cm^3).

$$\text{Volume} = 30 \text{ cm}^3$$

3. Copy the table below into your books.
Complete it by filling in the side lengths and the volume of each of the objects shown below.
Each of the small cubes in the rectangular prisms is one cubic centimetre (1 cm^3).

Object	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
A	3	3	2	18
B				
C				
D				
E				
F				
G				
H				
I				



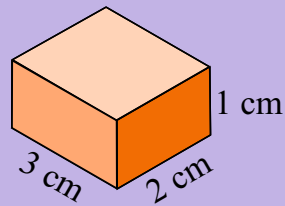
4. (a) Using the answers from question 3 as examples describe how the volume of a rectangular prism can be found.
 (b) Complete the following rule by placing the correct arithmetic operations $+$, $-$, $\div$ or $\times$ in the boxes.

Volume = length width height

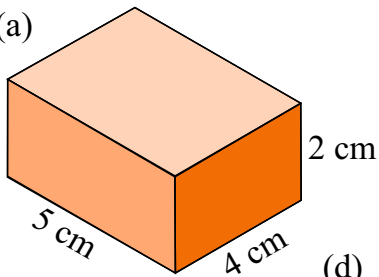
5. Find the volume of each of the rectangular prisms below.

Example

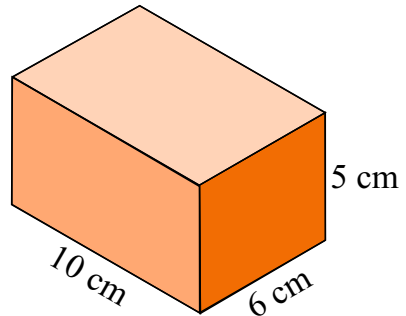
$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 3 \times 2 \times 1 \\ &= 6 \text{ cm}^3\end{aligned}$$



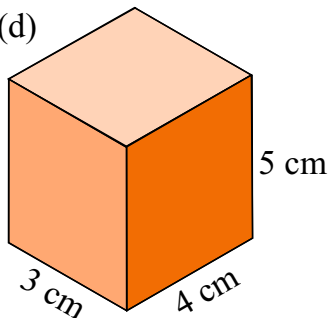
(a)



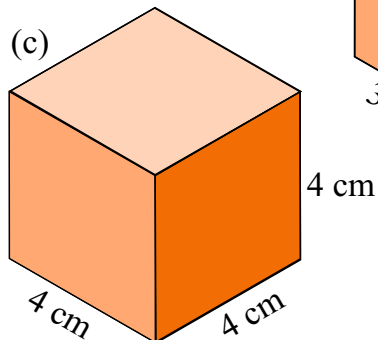
(b)



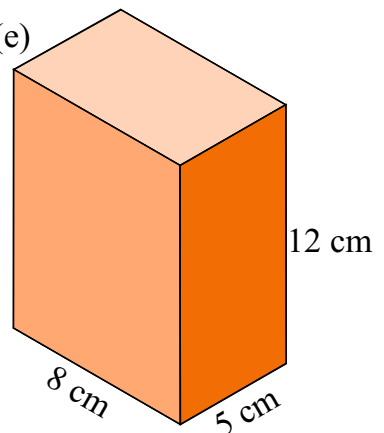
(d)



(c)



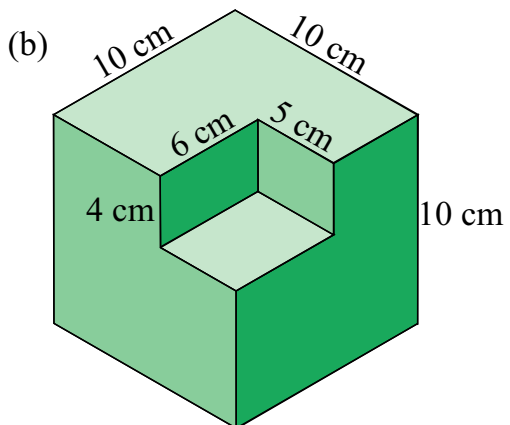
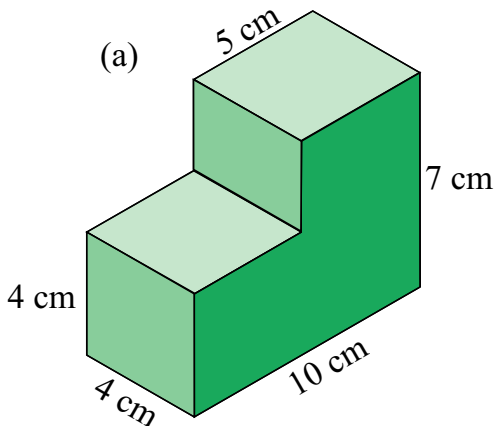
(e)



6. Find the volume of the rectangular prisms with the dimensions shown in the table below.

	Length (cm)	Width (cm)	Height (cm)
(a)	3	5	2
(b)	10	10	4
(c)	12	5	2
(d)	20	30	10
(e)	8	9	6

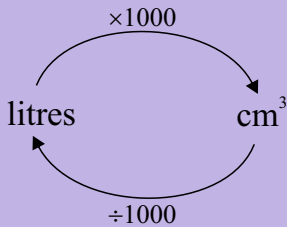
7. Find the volume of a cube with side length 10 cm.
8. (a) List the dimensions of all the different rectangular prisms that could be formed using 24 one cubic centimetre cubes.
 (b) Find the height of a rectangular prism formed using 24 one cubic centimetre cubes that has a length and width of 2 cm.
 (c) State the dimensions of a rectangular prism formed using 24 one centimetre cubes that has a length which is 1 cm longer than its width and 2 cm longer than its height.
9. A rectangular prism has a volume of 100 cm^3 . Its height is 1 cm less than its length and its width is equal to its length.
 Find its height if all the dimensions are whole numbers.
10. Find the volume of the following objects.



Litres

EXERCISE 18B

$$1 \text{ litre} = 1000 \text{ cm}^3$$



Remember -

When **multiplying** by 1000 move the decimal point 3 places to the **right**.

When **dividing** by 1000 move the decimal point 3 places to the **left**.

Add zeros where necessary.

1. Convert the following volumes to litres.

Examples

$$\begin{aligned} 1. \quad 3000 \text{ cm}^3 \\ &= 3000 \div 1000 \\ &= \overbrace{3000}^{\div 1000} \\ &= \mathbf{3 \text{ litres}} \end{aligned}$$

$$\begin{aligned} 2. \quad 28.5 \text{ cm}^3 \\ &= 28.5 \div 1000 \\ &= \overbrace{28.5}^{\div 1000} \\ &= \mathbf{0.0285 \text{ litres}} \end{aligned}$$

(a) 8000 cm^3

(b) $12\,000 \text{ cm}^3$

(c) $200\,000 \text{ cm}^3$

(d) 500 cm^3

(e) 9600 cm^3

(f) $35\,700 \text{ cm}^3$

(g) 410 cm^3

(h) 20 cm^3

(i) 85 cm^3

(j) 3976.5 cm^3

(k) 13.4 cm^3

(l) 258.7 cm^3

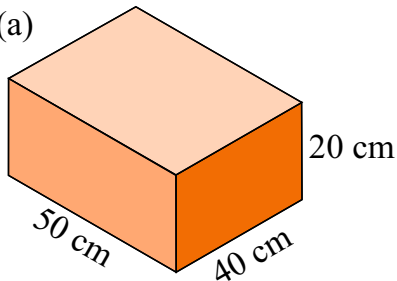
(m) 1 cm^3

(n) 8.5 cm^3

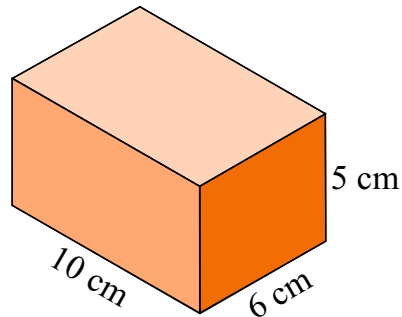
(o) 0.7 cm^3

2. Find the volume (in litres) of the following rectangular prisms.

(a)



(b)



3. Declan made a go-cart and used a petrol tank that was in the shape of a rectangular prism with dimensions $20 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}$.

(a) How many litres of petrol will the tank hold?

(b) Declan's go-cart will travel 5 km per litre of petrol. How far will the go-cart travel on a full tank of petrol?

4. Convert the following volumes to cubic centimetres.

Examples

1. 5 litres

$$= 5 \times 1000$$

$$= 5 \text{ m}^3$$

$$= 5000 \text{ cm}^3$$

2. 0.6875 litres

$$= 0.6875 \times 1000$$

$$= 687.5$$

$$= 687.5 \text{ cm}^3$$

(a) 2 litres

(b) 7 litres

(c) 13 litres

(d) 80 litres

(e) 4.1 litres

(f) 7.15 litres

(g) 12.685 litres

(h) 10.08 litres

(i) 5.5555 litres

(j) 0.9 litres

(k) 0.6752 litres

(l) 5.0008 litres

(m) 0.0052 litres

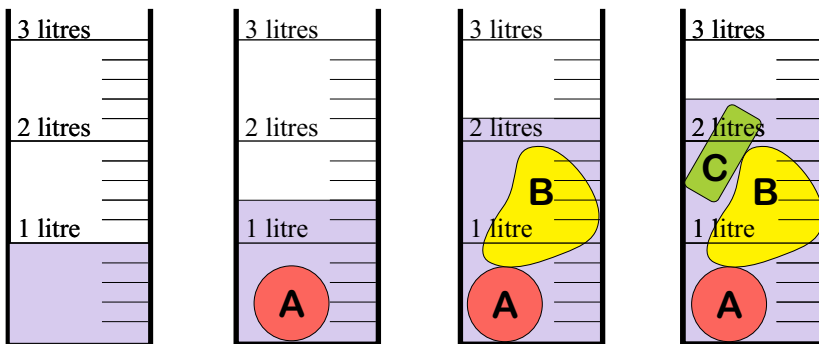
(n) 0.01 litres

(o) 0.00007 litres

5. How many 200 cm^3 glasses could be filled from a 2 litre bottle?

6. A container in the shape of a rectangular prism holds half a litre of drink. The base of the container is a square with side length 5 cm. What is the height of the drink container?

7.



A glass cylinder with volume markings on the side, as shown above, has an amount of water in it. Three objects, A, B and C, are dropped into the cylinder and the volume recorded after each one is dropped in.

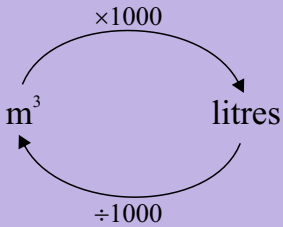
(a) What volume of water was in the cylinder before the objects were dropped in?

(b) What is the volume (in cm^3) of each of the objects?

8. A fish tank has the dimensions of 50 cm long, 30 cm wide and 40 cm deep. How many 10 litre buckets filled with water will it take to fill the fish tank?

9. Convert the following volumes to litres.

$$1 \text{ m}^3 = 1000 \text{ litres}$$



Remember -

When **multiplying** by 1000 move the decimal point 3 places to the **right**.
When **dividing** by 1000 move the decimal point 3 places to the **left**.
Add zeros where necessary.

- | | | | |
|-----------------------|------------------------|------------------------|--------------------------|
| (a) 4 m^3 | (b) 12 m^3 | (c) 50 m^3 | (d) 0.5 m^3 |
| (e) 1.2 m^3 | (f) 0.01 m^3 | (g) 5.08 m^3 | (h) 6.9875 m^3 |

10. Find the volume (in litres) of rectangular prisms with the following dimensions.

- | | |
|---|---|
| (a) $2 \text{ m} \times 3 \text{ m} \times 1 \text{ m}$ | (b) $2 \text{ m} \times 2 \text{ m} \times 2 \text{ m}$ |
| (c) $3 \text{ m} \times 4 \text{ m} \times 5 \text{ m}$ | (d) $4 \text{ m} \times 5 \text{ m} \times 0.5 \text{ m}$ |
| (e) $2 \text{ m} \times 1 \text{ m} \times 1.6 \text{ m}$ | (f) $2.4 \text{ m} \times 3 \text{ m} \times 1.5 \text{ m}$ |
| (g) $0.8 \text{ m} \times 0.6 \text{ m} \times 1 \text{ m}$ | (h) $1.2 \text{ m} \times 0.5 \text{ m} \times 0.9 \text{ m}$ |
| (i) $2 \text{ m} \times 100 \text{ cm} \times 50 \text{ cm}$ | (j) $1 \text{ m} \times 80 \text{ cm} \times 70 \text{ cm}$ |
| (k) $180 \text{ cm} \times 150 \text{ cm} \times 20 \text{ cm}$ | (l) $200 \text{ cm} \times 250 \text{ cm} \times 30 \text{ cm}$ |

11. (a) A container for transporting goods by ship was 6 metres long, 3 metres wide and 2.5 metres high.

What is the capacity of the container in: (i) m^3 (ii) litres

(b) Another container was in the shape of a cube and had a capacity of 27 000 litres. Find the side length (in metres) of the container.

12. A drinking water trough for horses was 2 metres long, 1 metre wide and 50 centimetres deep.

(a) How many litres of water does the trough hold?

(b) The trough is filled using a hose. The water flows from the hose at the rate of 100 litres per minute. How long would it take to fill the trough using this hose?

(c) There are 5 horses using the drinking trough and each horse drinks about 40 litres of water a day.

For how many days would this trough supply drinking water for the 5 horses?

PROBLEM SOLVING

1. A tray is to be made to hold 30 children's blocks. The blocks are cubes with side length 6 cm. The tray is to be as close to square shaped as possible. The blocks are to be two deep in the tray.
Find the dimensions of the tray.
2. Explain how a 9 litre and 4 litre container can be used to measure the following volumes.
(a) 3 litres (b) 5 litres (c) 6 litres (d) 7 litres

Example

1 litre can be obtained by following these steps:

- fill the 9 litre container
- fill the 4 litre container from the 9 litre container
- empty the 4 litre container
- fill the 4 litre container from the 9 litre again
- there is 1 litre remaining in the 9 litre container

PUZZLE

Change EMPTY into LITRE by changing one letter at a time and rearranging the letters to form a new word. A clue for each word is given.

E	M	P	T	Y
L	I	T	R	E

Uses a keyboard

Paces

Used at the end of sentences - full ____

Tennis, hockey, football, netball, cricket, etc.

Used to catch animals

Birthday, New Year's Eve, etc.

Thick at one end and thin at the other end

King of Macedonia (356-323 BC) Alexander the ____

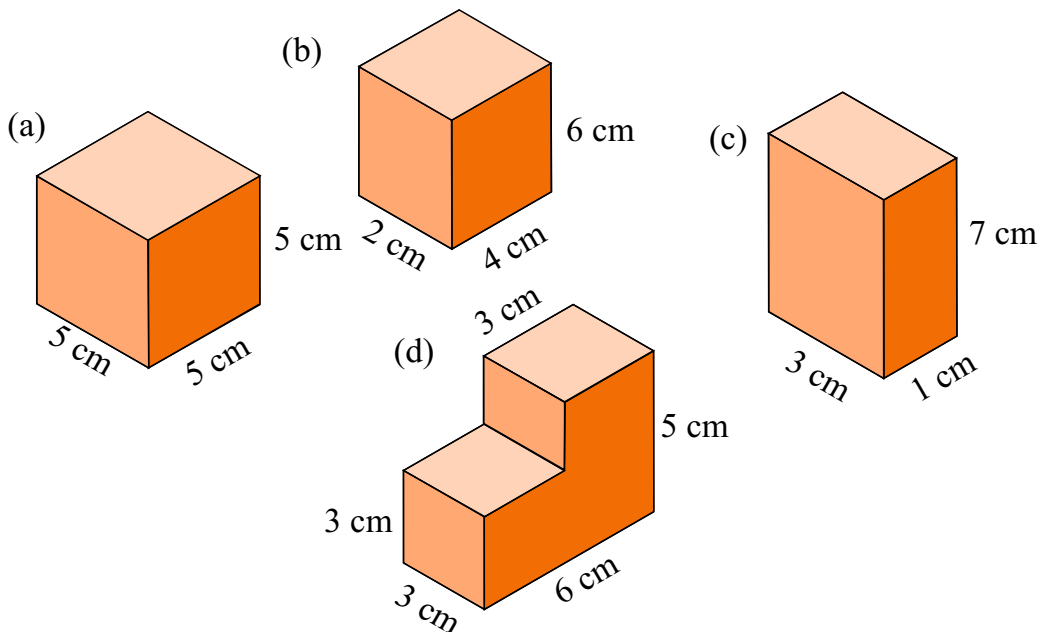
One of the felines

The square root of sixty-four

Often misspelt THERE

CHAPTER REVIEW

1. Find the volume (in cm^3) of the rectangular prisms below.



2. Find the volume (in litres) of the rectangular prisms with the following dimensions.

- | | |
|---|---|
| (a) $4 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$ | (b) $3 \text{ m} \times 2 \text{ m} \times 3 \text{ m}$ |
| (c) $1 \text{ m} \times 2 \text{ m} \times 5 \text{ m}$ | (d) $2 \text{ m} \times 5 \text{ m} \times 0.5 \text{ m}$ |
| (e) $2 \text{ m} \times 2 \text{ m} \times 2.5 \text{ m}$ | (f) $0.4 \text{ m} \times 3 \text{ m} \times 1.5 \text{ m}$ |
| (g) $80 \text{ cm} \times 30 \text{ cm} \times 10 \text{ cm}$ | (h) $20 \text{ cm} \times 50 \text{ cm} \times 16 \text{ cm}$ |
| (i) $20 \text{ cm} \times 10 \text{ cm} \times 5 \text{ cm}$ | (j) $30 \text{ cm} \times 80 \text{ cm} \times 20 \text{ cm}$ |
| (k) $180 \text{ cm} \times 150 \text{ cm} \times 20 \text{ cm}$ | (l) $200 \text{ cm} \times 250 \text{ cm} \times 30 \text{ cm}$ |

3. A rectangular prism has a volume of 12 cm^3 . How high is the prism if it is 3 cm long and 2 cm wide?

4. A rectangular prism is made using cubes of side length 1 cm. The prism is 4 cm long, 4 cm wide and 2 cm high. If all the centimetre cubes are removed and stacked, one on top of the other, find the height of the stack?

5. How many litres of petrol would be in a tank that is 25 cm long, 30 cm wide and 24 cm high?

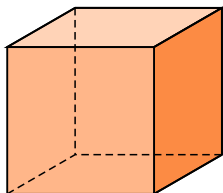
Three Dimensions

Three-dimensional Objects

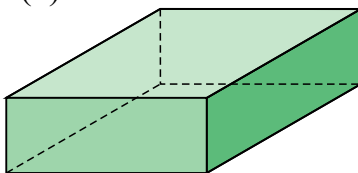
EXERCISE 19A

1. From the list below choose the correct name for each of these objects.

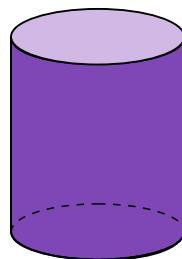
(a)



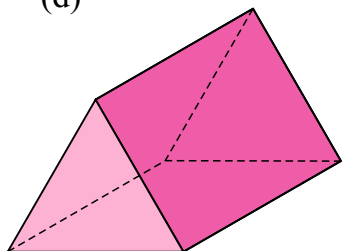
(b)



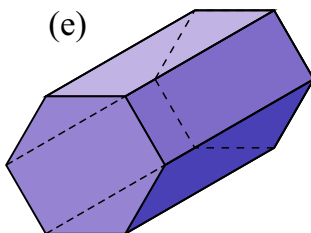
(c)



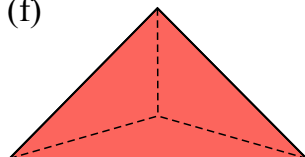
(d)



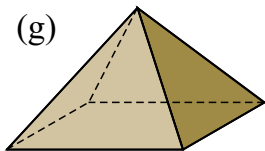
(e)



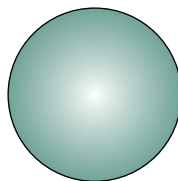
(f)



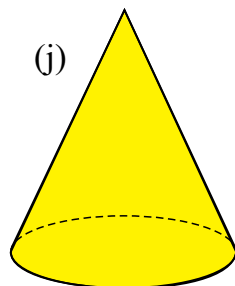
(g)



(i)

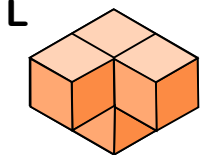
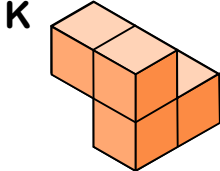
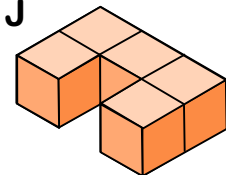
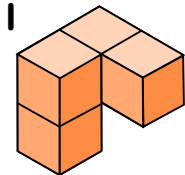
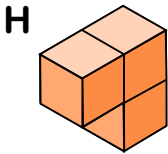
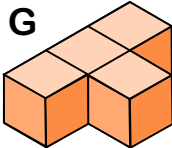
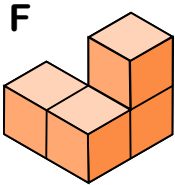
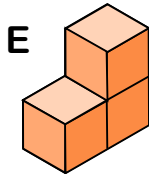
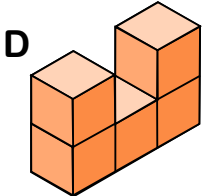
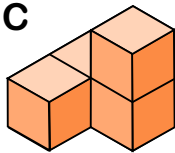
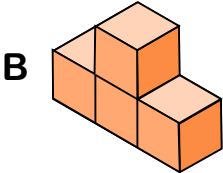
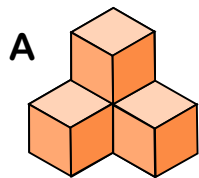


(j)



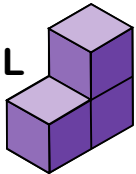
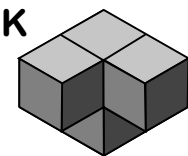
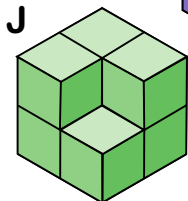
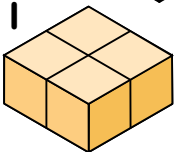
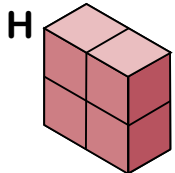
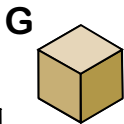
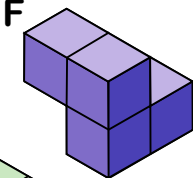
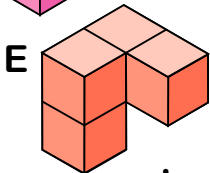
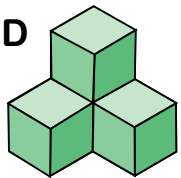
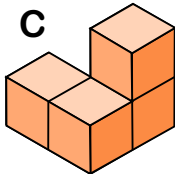
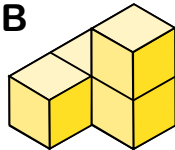
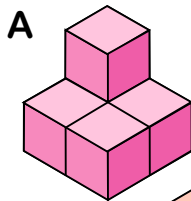
***SPHERE CONE TRIANGULAR PRISM
RECTANGULAR PRISM PYRAMID HEXAGONAL PRISM
TETRAHEDRON CUBE CYLINDER
TRAPEZOIDAL PRISM***

2. Match the *pairs* of *identical* objects from the following.
Note: they have been rotated.

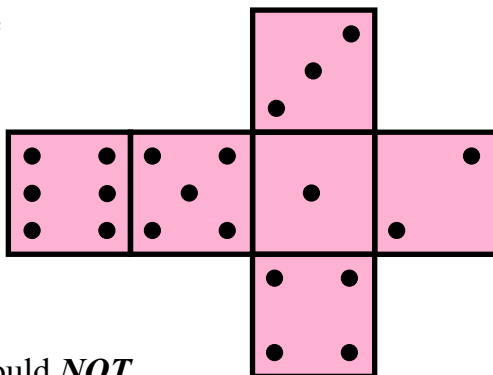


3. Find the pairs from the following objects that would form a 2×2 cube if joined together.

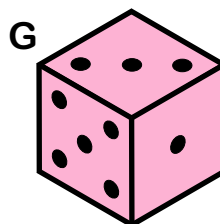
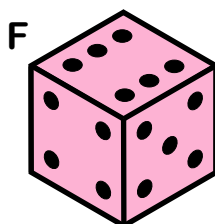
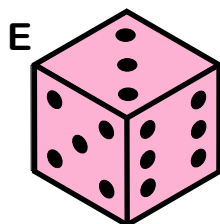
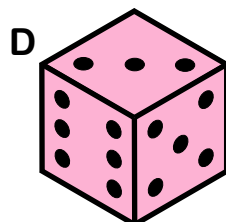
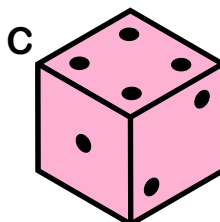
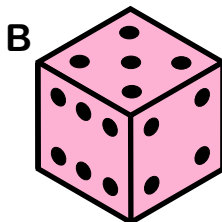
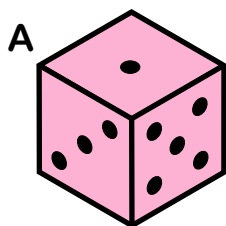
Example



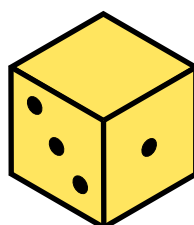
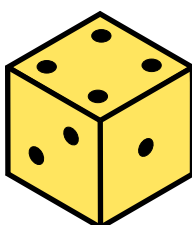
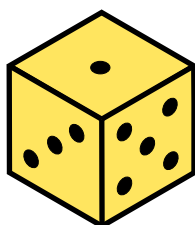
4. The piece of board shown here is folded to form a die.



Which of the following dice could *NOT* be formed from this board?
(More than one answer)



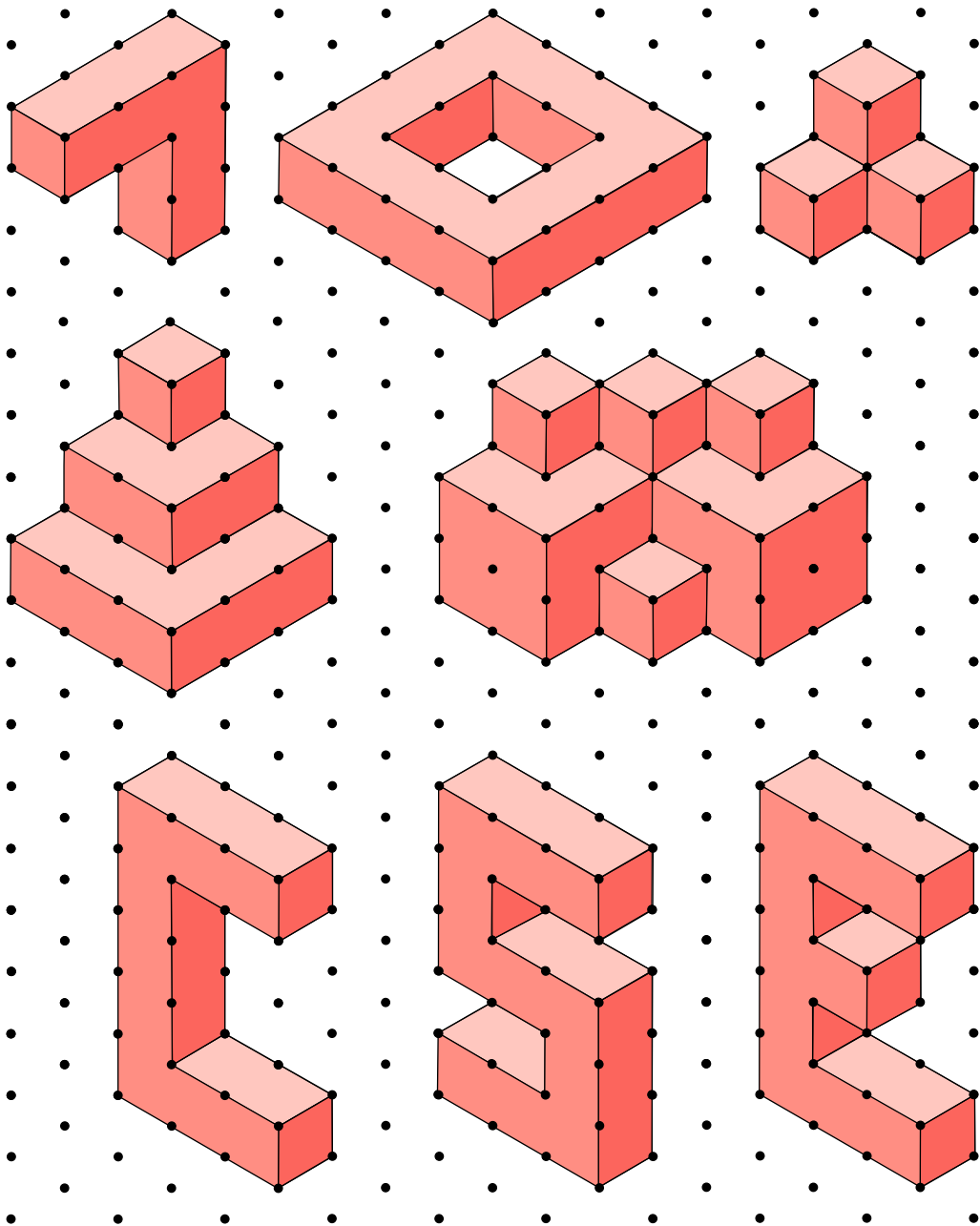
5. Three views of a die are shown below. Draw the third view of the die and complete it by showing the correct number of dots on the blank face.



Isometric Drawings

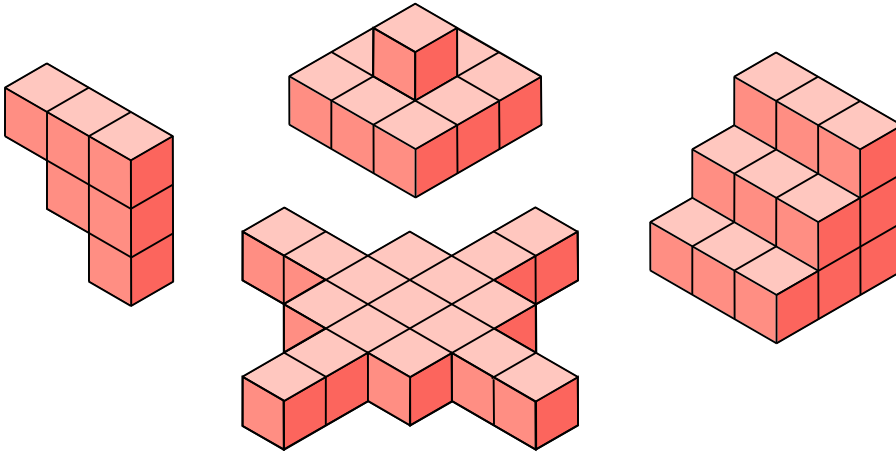
EXERCISE 19B

1. On isometric dot paper copy the following objects.



2. See if you can write your name or another word on isometric dot paper.

3. Draw the following objects on isometric dot paper.



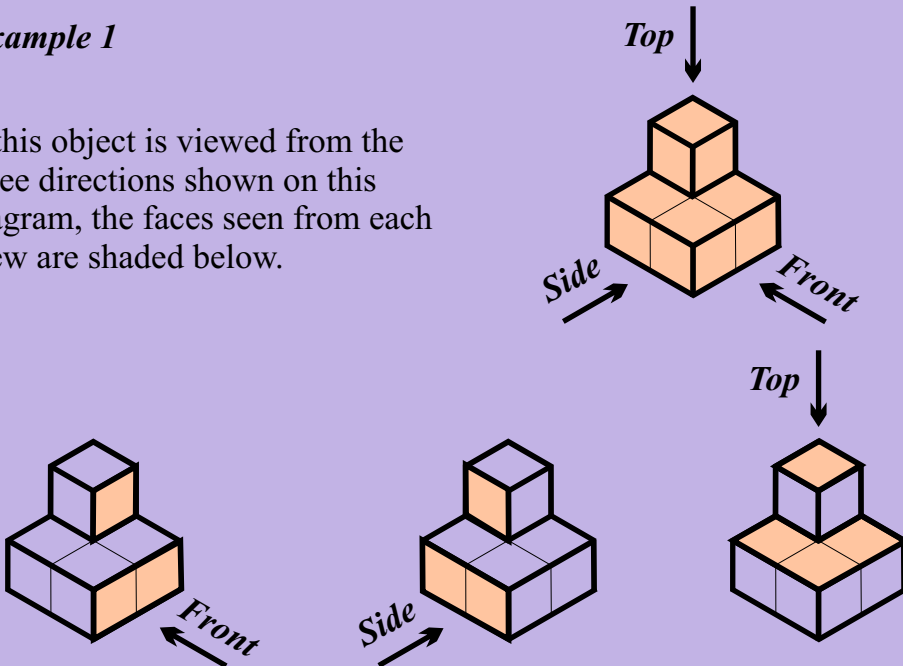
Views on 3D Objects

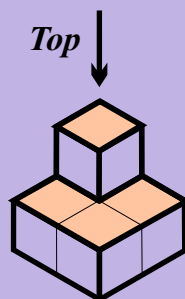
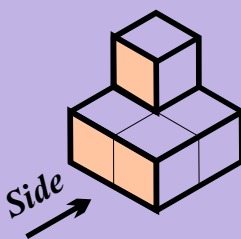
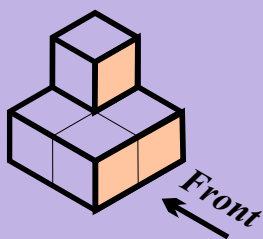
Three-dimensional objects can be represented by the views looking at the objects from different directions.

Three views often used are from the *front*, *side* and *top* of the object.

Example 1

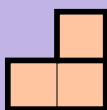
If this object is viewed from the three directions shown on this diagram, the faces seen from each view are shaded below.



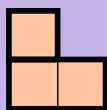
Example 1 continued

These views can then be shown as two-dimensional diagrams.

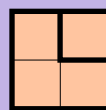
Front View



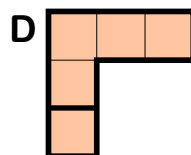
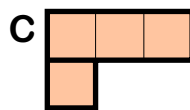
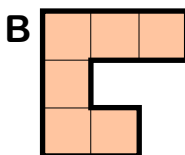
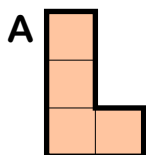
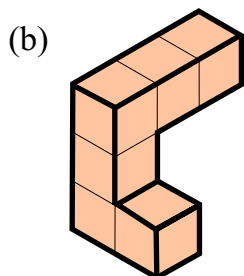
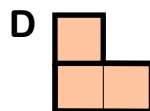
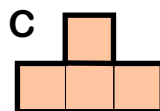
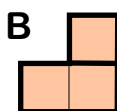
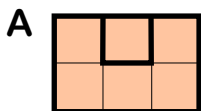
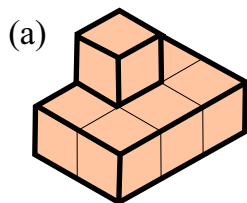
Side View



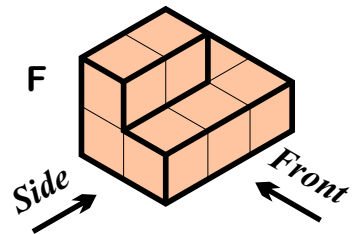
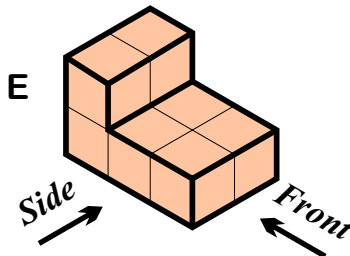
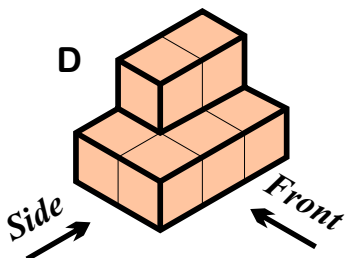
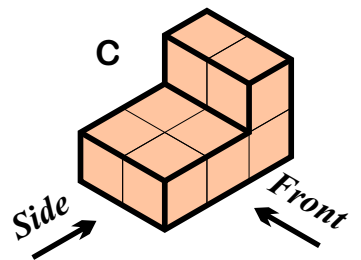
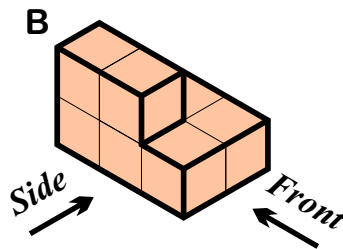
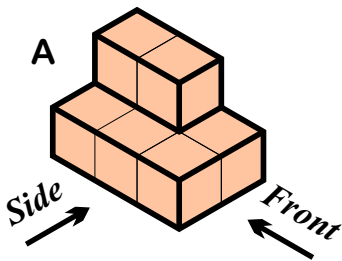
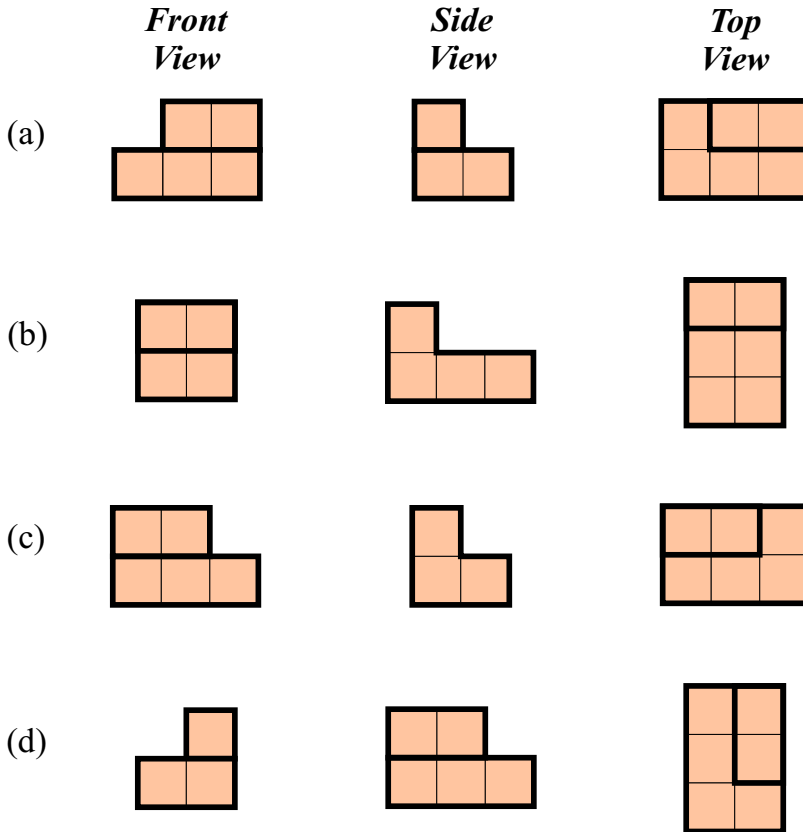
Top View

**EXERCISE 19C**

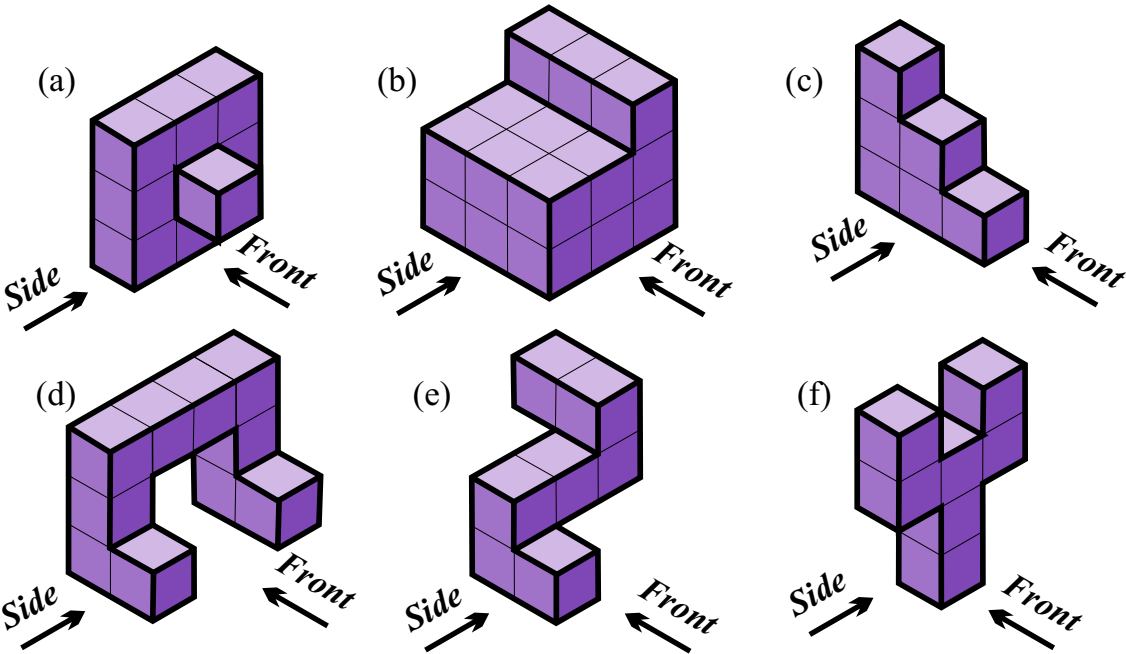
1. For the objects below choose the diagram that correctly shows the: (i) front view (ii) side view (iii) top view



2. Which of the objects below (A-F) are shown by the following front, side and top views.



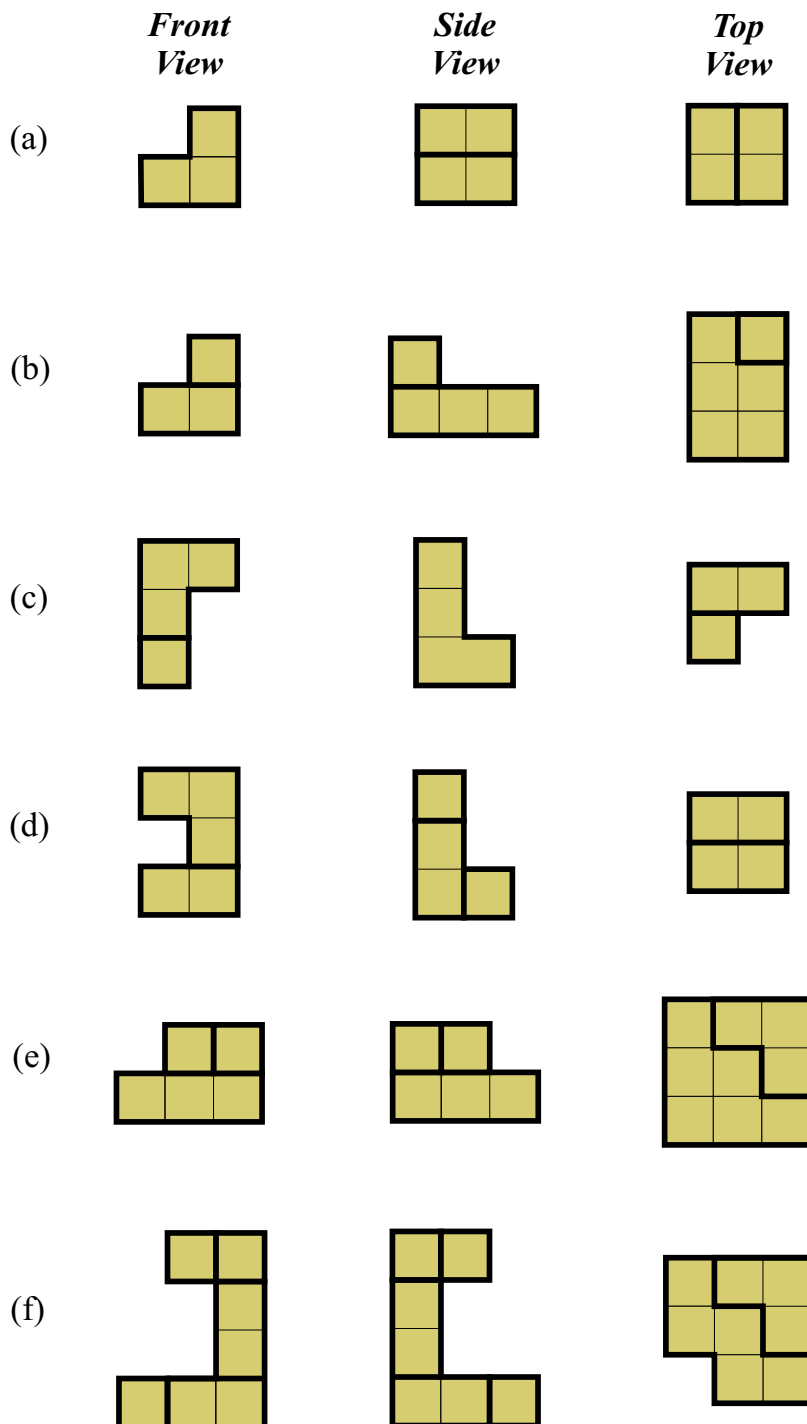
3. Draw the front, side and top views of these objects.



4. The front, side and top views of several objects are given below.
Write the **names** of these objects.

	Front View	Side View	Top View		Front View	Side View	Top View
(a)				(e)			
(b)				(f)			
(c)				(g)			
(d)				(h)			

5. The front, side and top views of several objects are given below.
Draw these objects on isometric dot paper.

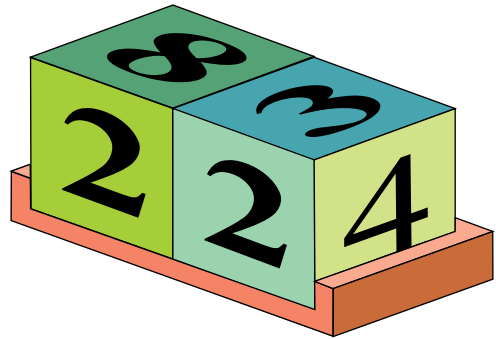


PROBLEM SOLVING

Graham wants to make a calendar using two cubes. When the two cubes are placed together the numbers on the front are to show the date.

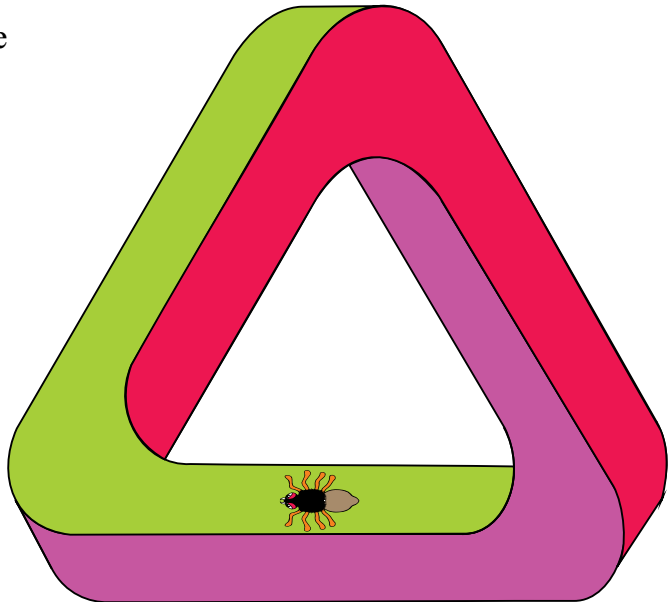
(*Example:* the date shown is the 22nd)

What numbers should Graham write on each cube so that every day in each month can be represented?



PUZZLE

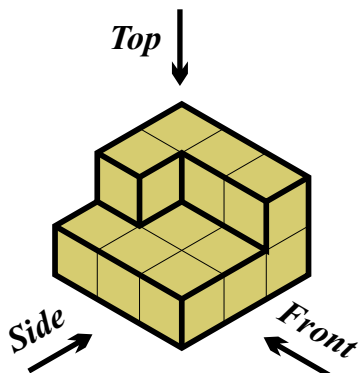
How many times would the spider crawl around this object before it returned to its starting point?



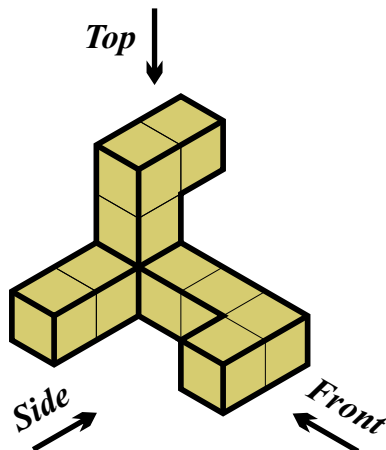
CHAPTER REVIEW

1. Draw the front, side and top views of the objects below.

(a)



(b)



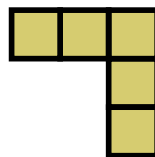
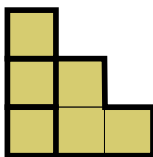
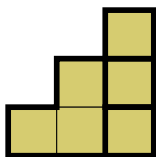
2. The front, side and top views of two objects are given below.
Draw these objects on isometric dot paper.

*Front
View*

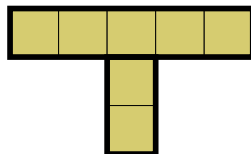
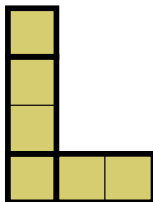
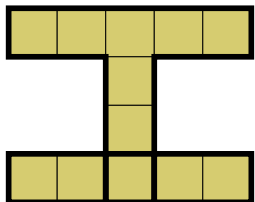
*Side
View*

*Top
View*

(a)



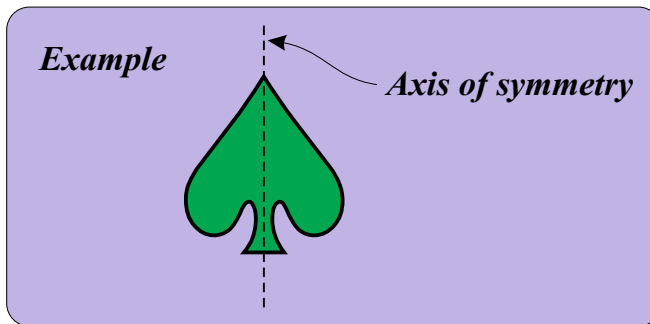
(b)



Transformations

Symmetry

A shape is ***symmetrical*** if half of it is a mirror image of the other half.
The line about which the two halves reflect is called the ***axis of symmetry***.



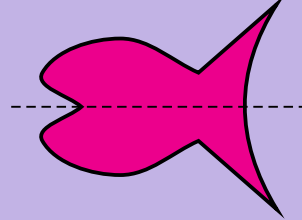
Shapes may have ***vertical axes of symmetry***, ***horizontal axes of symmetry*** or many ***axes of symmetry***.

Examples

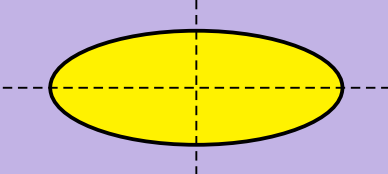
This shape has a ***vertical axis of symmetry***.



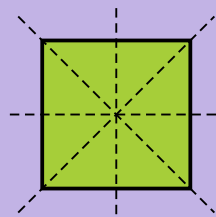
This shape has a ***horizontal axis of symmetry***.



This shape has a ***vertical and horizontal axis of symmetry***.



A square has ***four axes of symmetry***.



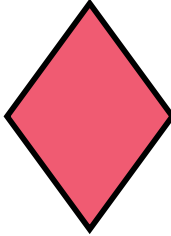
EXERCISE 20A

1. Copy the following shapes and show all their axes of symmetry.

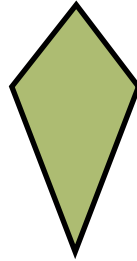
(a)



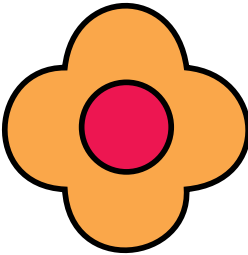
(b)



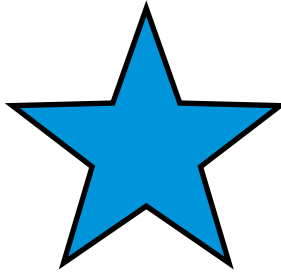
(c)



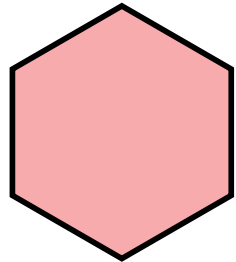
(d)



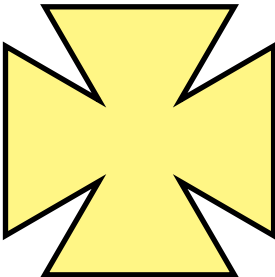
(e)



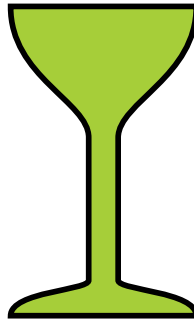
(f)



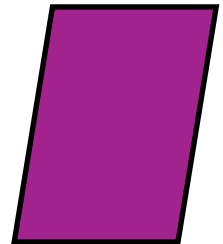
(g)



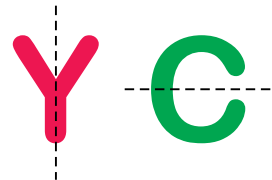
(h)



(i)



2. The letter **Y** has a vertical axis of symmetry.
The letter **C** has a horizontal axis of symmetry.



- List five other letters that have a vertical axis of symmetry.
- List five other letters that have a horizontal axis of symmetry.
- List the letters that have **both** vertical and horizontal axes of symmetry.

3. (a) Complete the following words written vertically by drawing the half letters shown and their reflections.



- (b) How many words can you find that, when written vertically, have a vertical line of symmetry?

4. (a) Complete the words written horizontally below by drawing the half letters shown and their reflections.

BOY

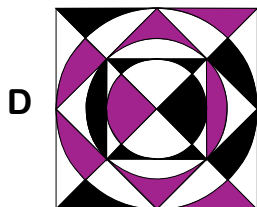
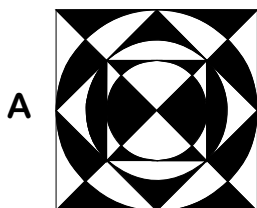
CHECK

DICE

- (b) How many words can you find that, when written horizontally, have a horizontal line of symmetry?
- (c) How many words can you find that, when written horizontally, have a vertical line of symmetry?

5. Which of the patterns below have:

- (a) both a horizontal and a vertical line of symmetry?
- (b) only a horizontal line of symmetry?
- (c) only a vertical line of symmetry?



Reflections

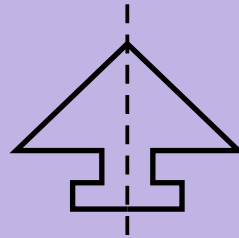
EXERCISE 20B

1. Copy these half-shapes into your book and complete each symmetrical shape by drawing the other half.

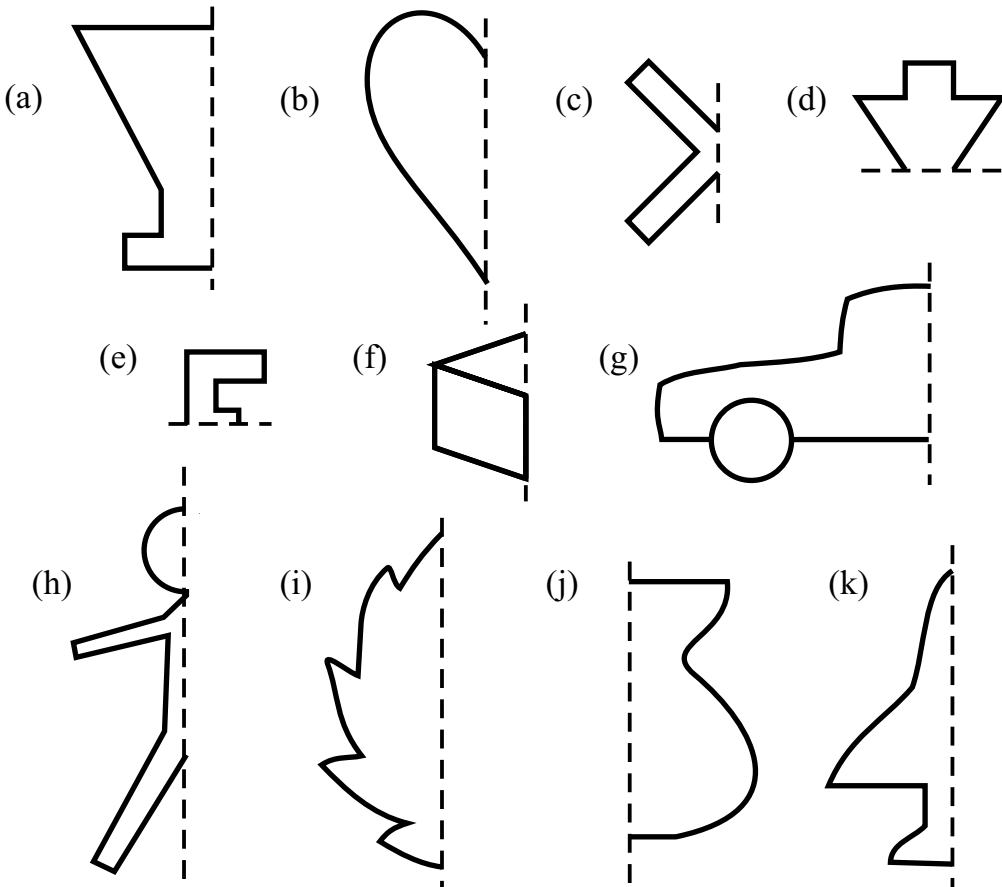
Example



Half-shape

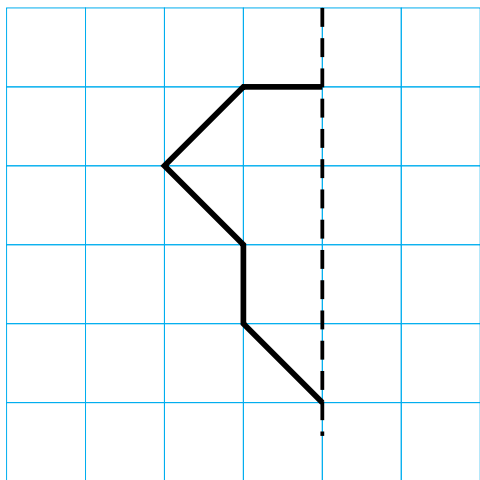


Complete shape

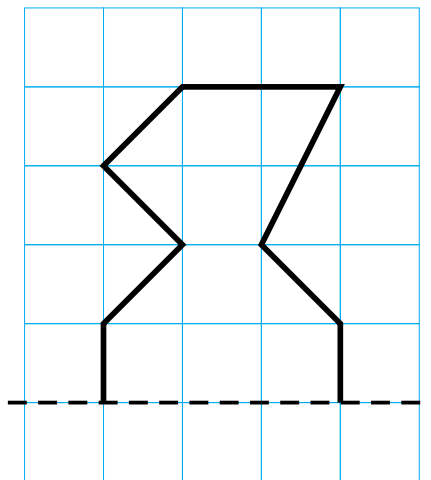


2. Copy these half-shapes onto graph paper and accurately complete each symmetrical shape by drawing the other half.

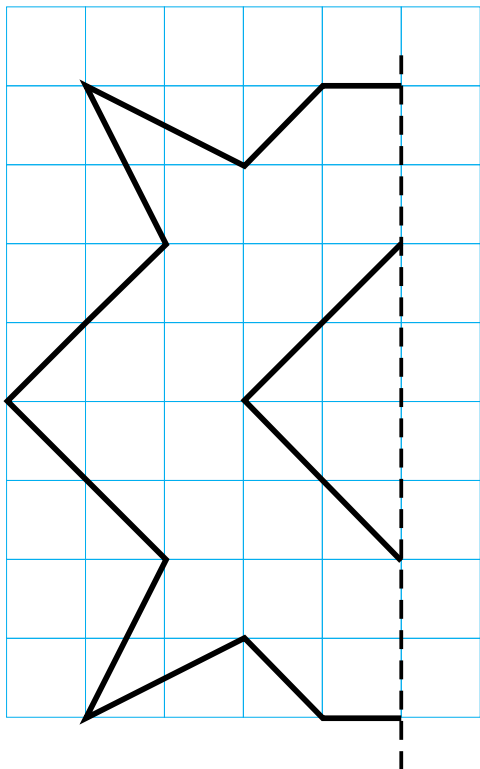
(a)



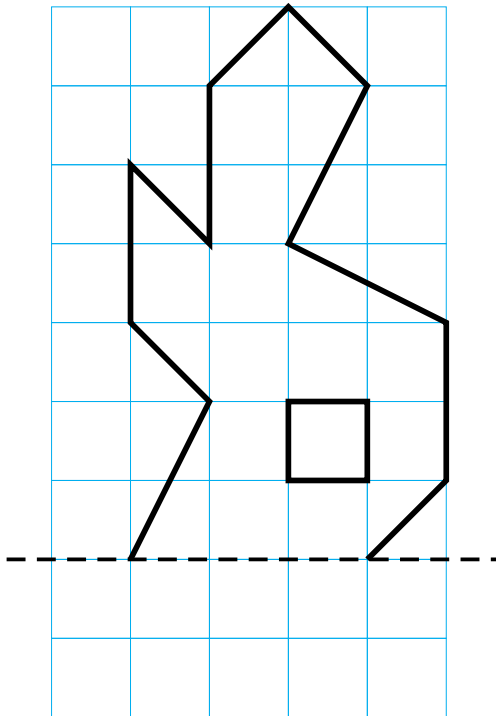
(b)



(c)

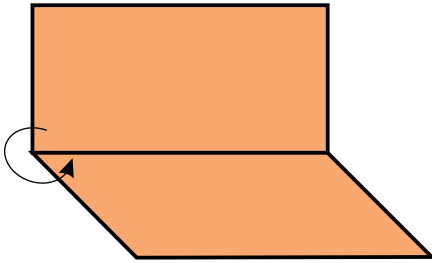


(d)

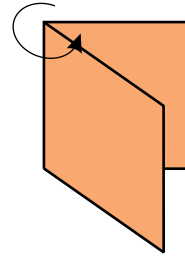


3. A design that has a vertical and horizontal line of symmetry can be created by folding a piece of paper in half and then folding it in half again. A design can then be cut out of the folded paper and then the paper unfolded.

An example is shown below.



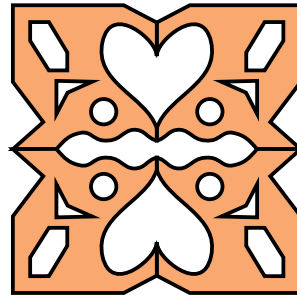
Square piece of paper folded in half.



Folded in half again.

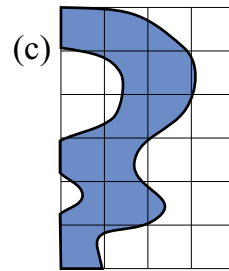
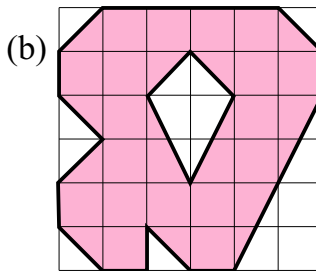
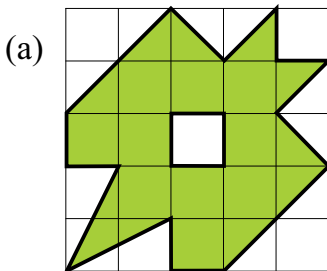


Cut-out design



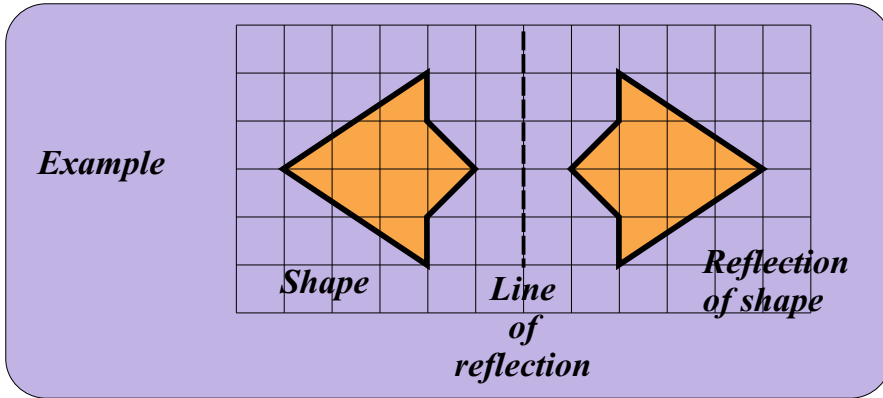
Unfolded design

Draw the unfolded design that would be created by the following cut-out designs (the cut-out designs can be drawn on graph paper).

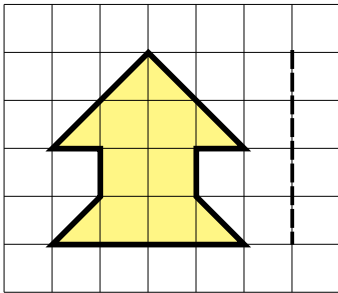


Try folding and cutting pieces of paper to check your answers.
Try making some designs of your own.

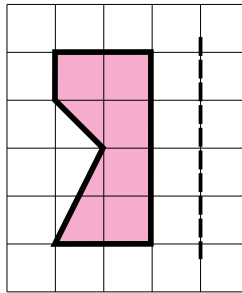
4. Draw the reflection of the shapes below about the line shown.
Use graph paper so the shapes can be drawn accurately.



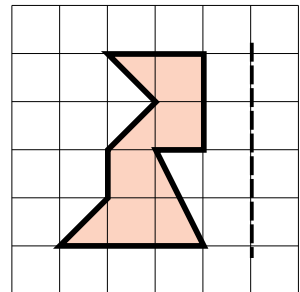
(a)



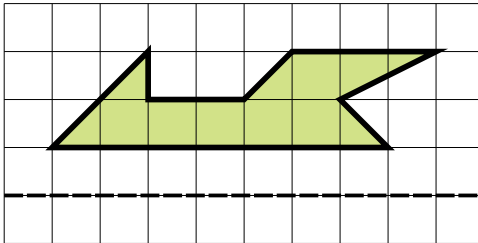
(b)



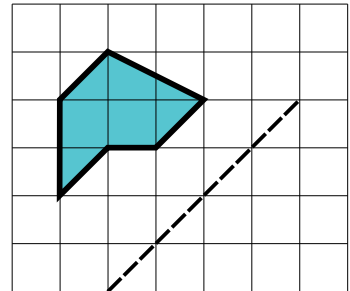
(c)



(d)



(e)



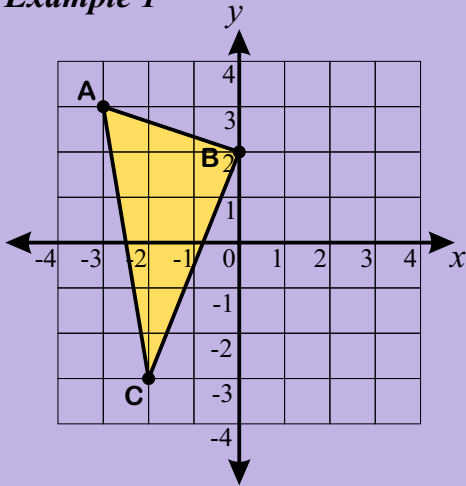
5. (a) Which vehicle would this word be written on?
(b) Why is it written this way?

AMBULANCE

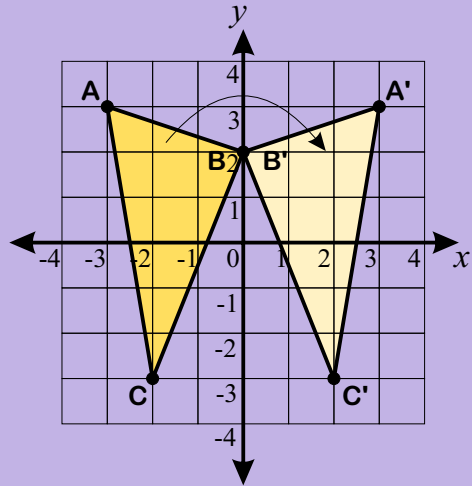
6. Anita has a digital clock in her bedroom next to a mirror.
(a) What is the actual time if the reflection of the clock in the mirror shows the digits **505**?
(b) Anita looked in the mirror one morning and saw the digits **802**. Thinking she was late for school she jumped out of bed to find it still dark outside. How much earlier was it than she thought?

EXERCISE 20C

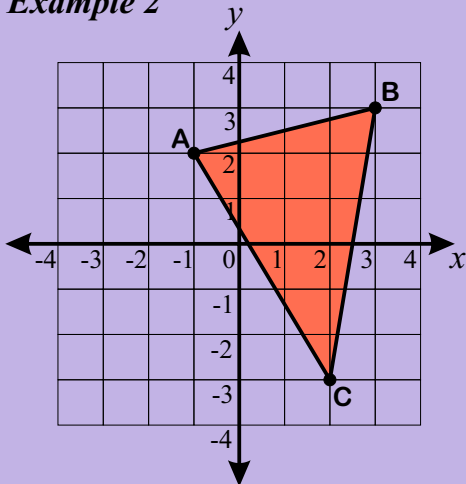
1. List the coordinates of the vertices of the following shapes.
2. Draw the reflection of each shape about the **y-axis** and list the coordinates of the vertices of each reflected shape.

Example 1

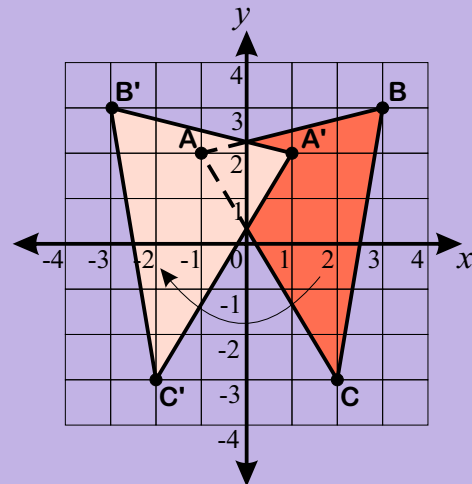
Coordinates of the original shape:
A = $(-3, 3)$
B = $(0, 2)$
C = $(-2, -3)$



Coordinates of the reflected shape:
A' = $(3, 3)$
B' = $(0, 2)$
C' = $(2, -3)$

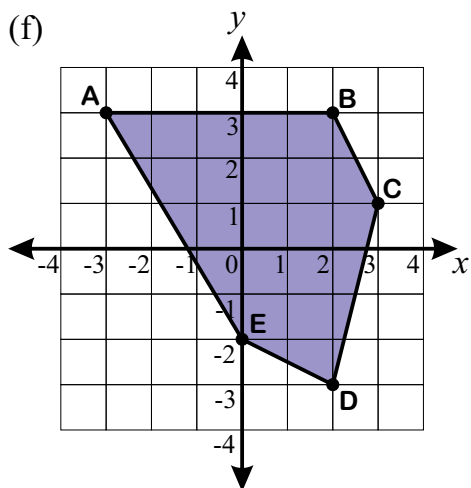
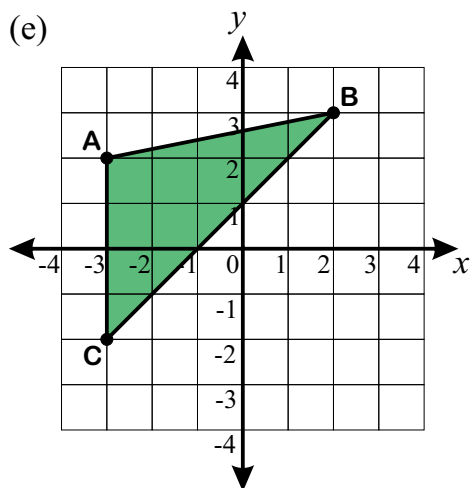
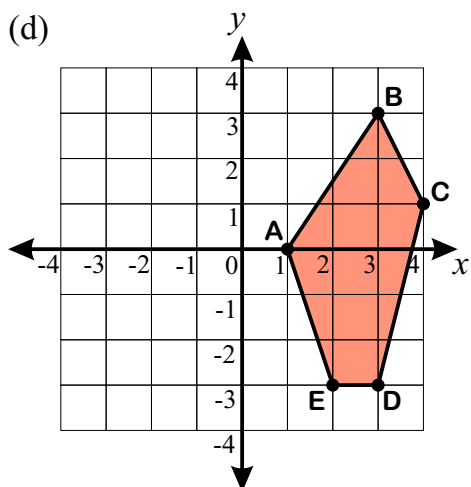
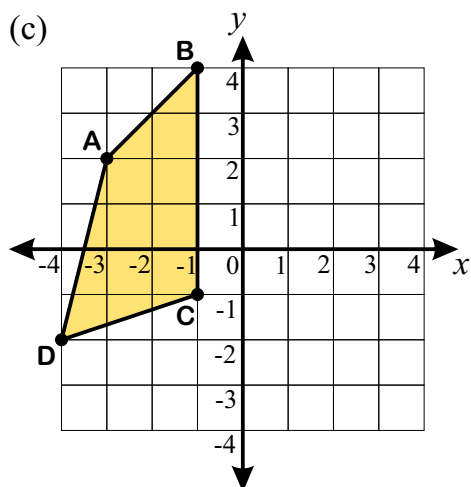
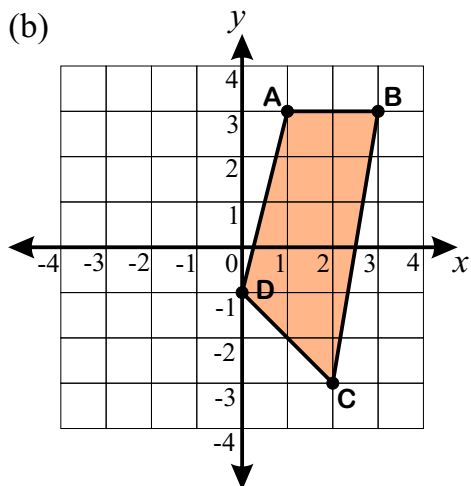
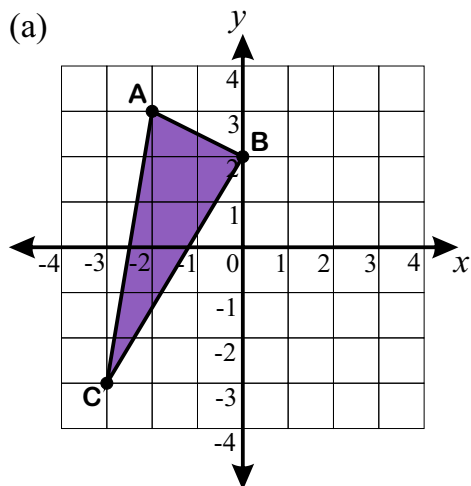
Example 2

Coordinates of the original shape:
A = $(-1, 2)$
B = $(3, 3)$
C = $(2, -3)$

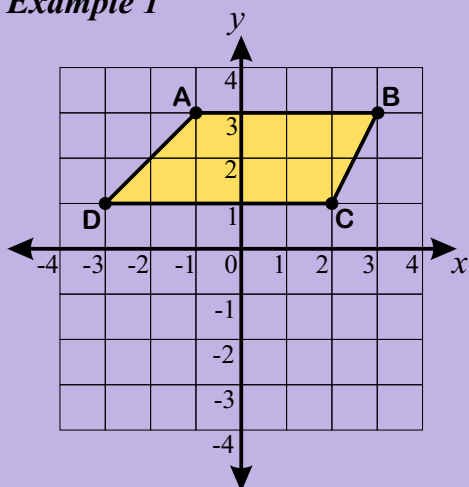


Coordinates of the reflected shape:
A' = $(1, 2)$
B' = $(-3, 3)$
C' = $(-2, -3)$

1. (i) List the coordinates of the vertices of the following shapes.
 (ii) Draw the reflection of each shape about the **y-axis** and list the coordinates of the vertices of each reflected shape.



1. List the coordinates of the vertices of the following shapes.
2. Draw the reflection of each shape about the x -axis and list the coordinates of the vertices of each reflected shape.

Example 1

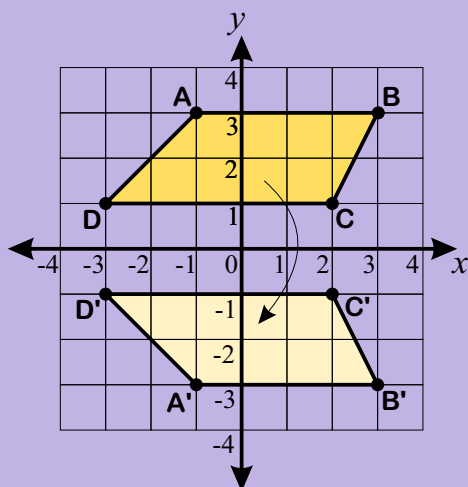
Coordinates of the original shape:

$$A = (-1, 3)$$

$$B = (3, 3)$$

$$C = (2, 1)$$

$$D = (-3, 1)$$



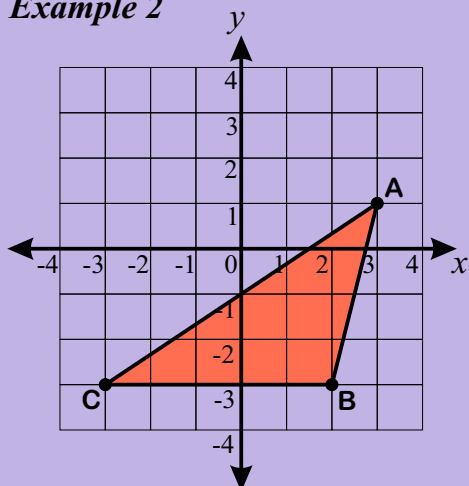
Coordinates of the reflected shape:

$$A' = (-1, -3)$$

$$B' = (3, -3)$$

$$C' = (2, -1)$$

$$D' = (-3, -1)$$

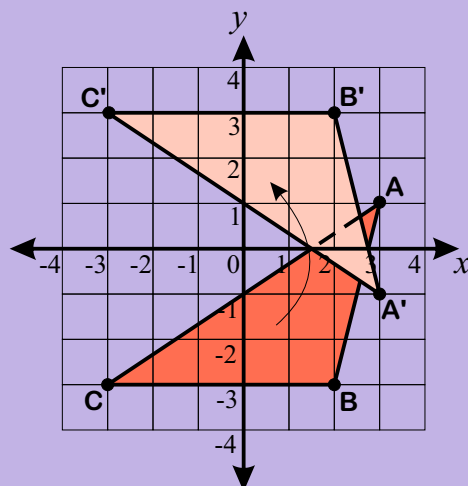
Example 2

Coordinates of the original shape:

$$A = (3, 1)$$

$$B = (2, -3)$$

$$C = (-3, -3)$$



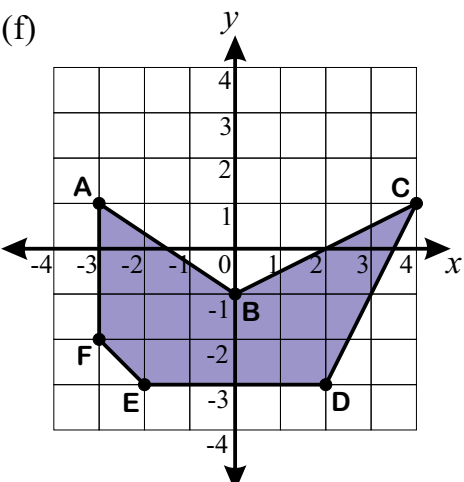
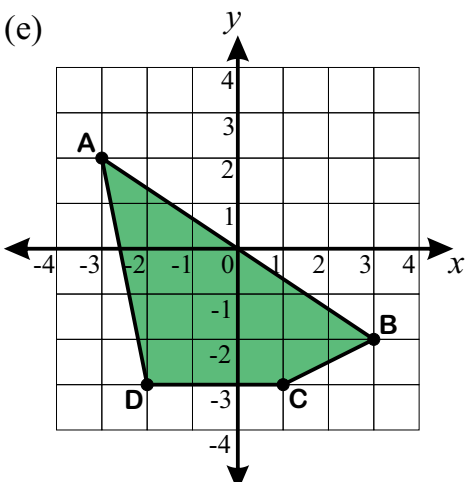
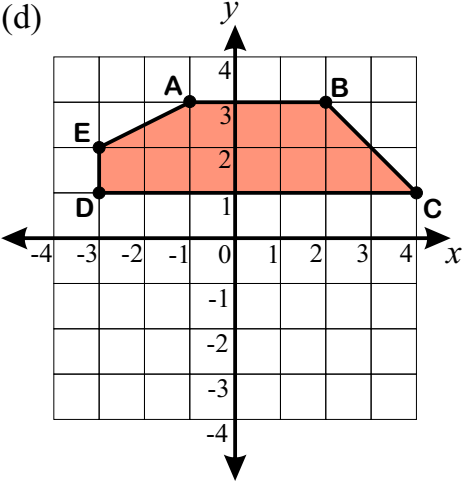
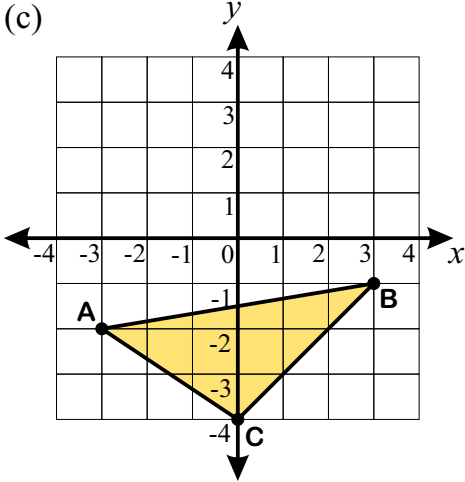
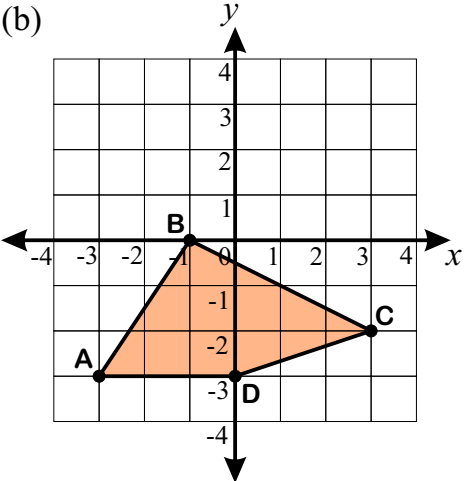
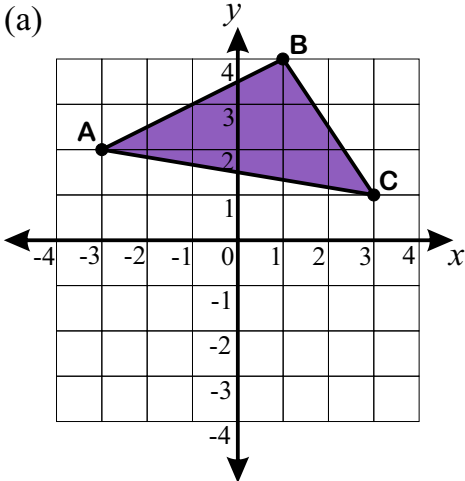
Coordinates of the reflected shape:

$$A' = (3, -1)$$

$$B' = (2, 3)$$

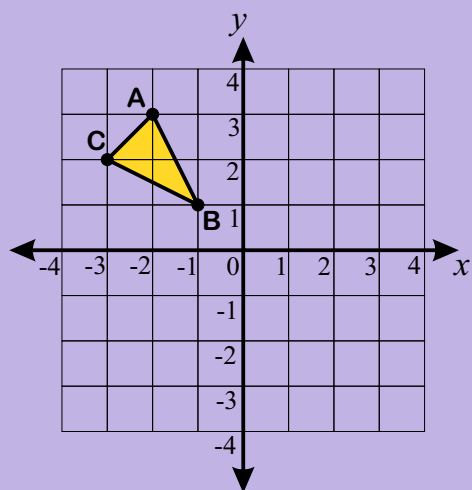
$$C' = (-3, 3)$$

2. (i) List the coordinates of the vertices of the following shapes.
 (ii) Draw the reflection of each shape about the x -axis and list the coordinates of the vertices of each reflected shape.



3. (i) List the coordinates of the vertices of the following shapes.
 (ii) Draw the reflection of each shape after it has been reflected about the **y-axis** and then reflected about the **x-axis**. List the coordinates of the vertices of each reflected shape.

Example

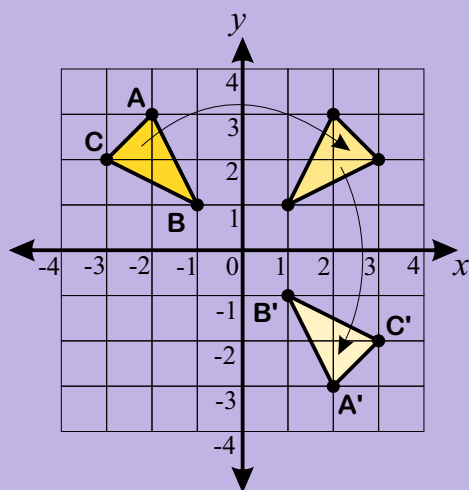


Coordinates of the original shape:

$$\mathbf{A} = (-2, 3)$$

$$\mathbf{B} = (-1, 1)$$

$$\mathbf{C} = (-3, 2)$$

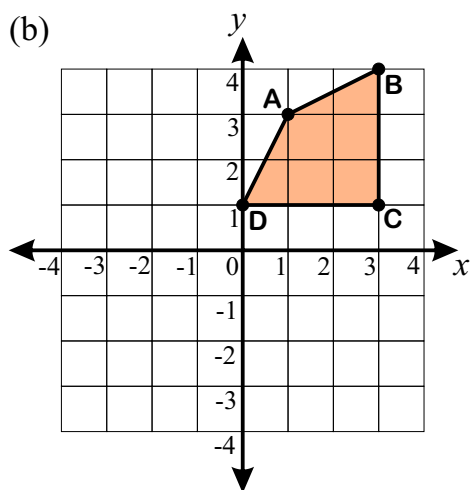
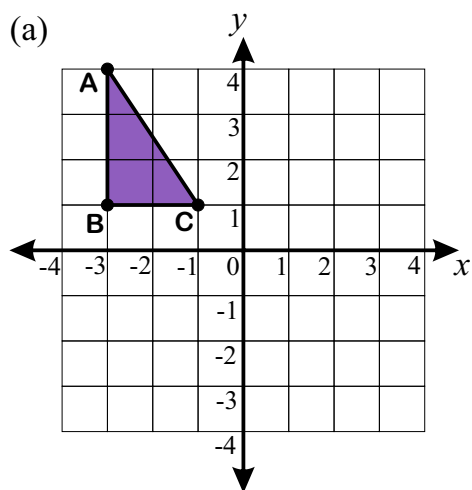


Coordinates of the reflected shape:

$$\mathbf{A'} = (2, -3)$$

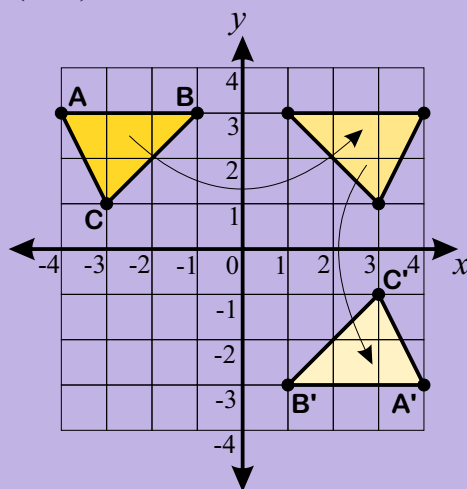
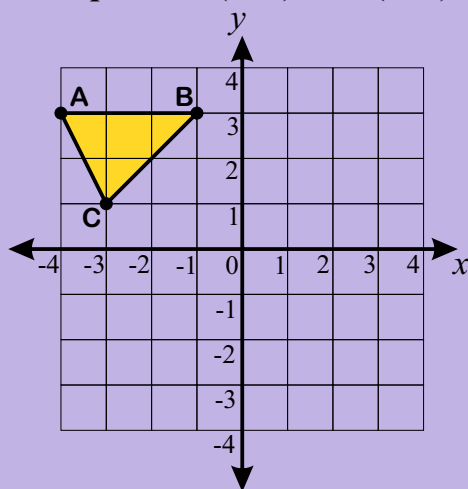
$$\mathbf{B'} = (1, -1)$$

$$\mathbf{C'} = (3, -2)$$



4. (i) Draw the shapes with the following points as vertices.
 (ii) Draw the reflection of each shape after it has been reflected about the **y-axis** and then reflected about the **x-axis**. List the coordinates of the vertices of each reflected shape.

Example $A = (-4,3)$, $B = (-1,3)$, $C = (-3,1)$



Coordinates of the reflected shape:

$$A' = (4, -3)$$

$$B' = (1, -3)$$

$$C' = (3, -1)$$

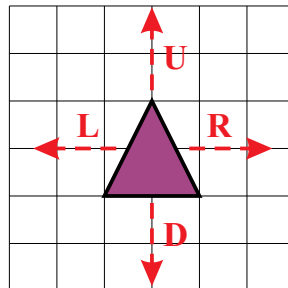
- (a) $A = (-1,4)$, $B = (-1,1)$, $C = (-3,1)$
 (b) $A = (1,3)$, $B = (4,2)$, $C = (2,0)$
 (c) $A = (1,1)$, $B = (3,-1)$, $C = (4,-3)$, $D = (1,-4)$
 (d) $A = (-2,4)$, $B = (-1,3)$, $C = (-1,1)$, $D = (-4,2)$
5. (a) Reflect the shapes from question 4. (a) and (b) about the **x-axis** then about the **y-axis**.
 (b) Is reflecting a shape about the **y-axis** then about the **x-axis** the same as reflecting it about the **x-axis** then about the **y-axis**?
6. Find a rule that connects the coordinates of the vertices of the shapes and the reflected shapes in question 4.
 Use this rule to find the coordinates of the vertices of the following shapes after they have been reflected about the **y-axis** then about the **x-axis**.
 (a) $A = (-2,4)$, $B = (0,1)$, $C = (-4,2)$
 (b) $A = (-1,0)$, $B = (-1,-3)$, $C = (-4,-2)$, $D = (-3,-1)$

Translations

Shapes can be translated (moved) on a grid to the right (R), left (L), up (U) or down (D).

The size of a translation is defined by the number of grid squares it is moved.

Abbreviations can be used to show the direction and size of a translation.

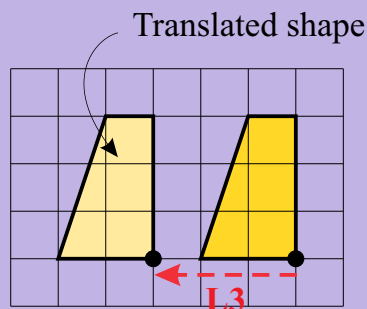
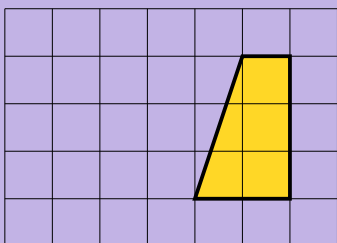


Examples

1. A translation to the right of four units = R4
2. A translation up of six units = U6
3. Translation to the left of three units followed by down two units = L3, D2

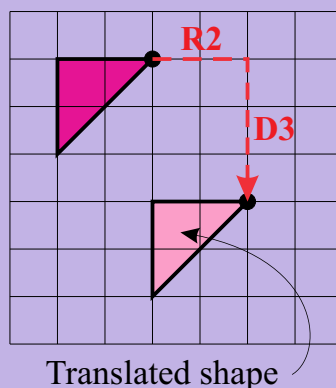
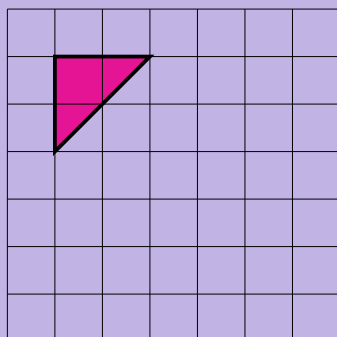
Examples

1. Translate the shape below left three units (L3).



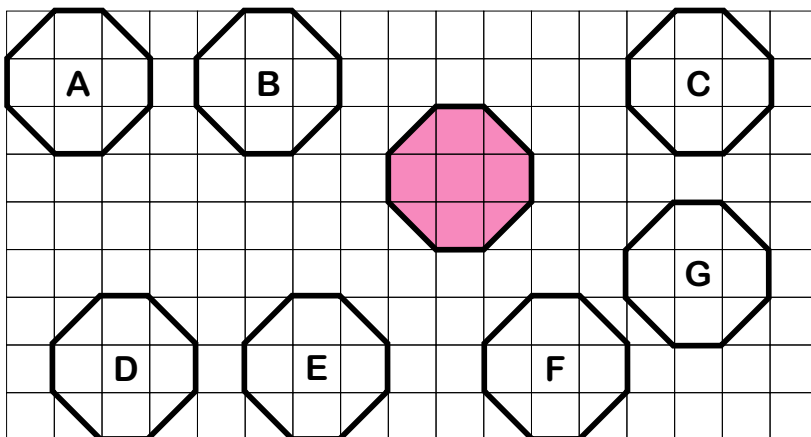
Any point on the shape can be used to translate the shape.

2. Translate the shape below right two units then down three units (R2, D3).



EXERCISE 20D

1. (a) The shaded shape on the grid below is translated to the left three units and down four units (L3, D4). Which of the shapes (**A-G**) would be the position of the shaded shape after it has been translated?



- (b) State the translation required to get from the shaded shape to each of the other shapes.
2. Copy the shapes shown below onto a grid.
Translate the shapes by the amounts shown to form something that is used at the start of mathematics classes.

Shape **A** - R5, D1

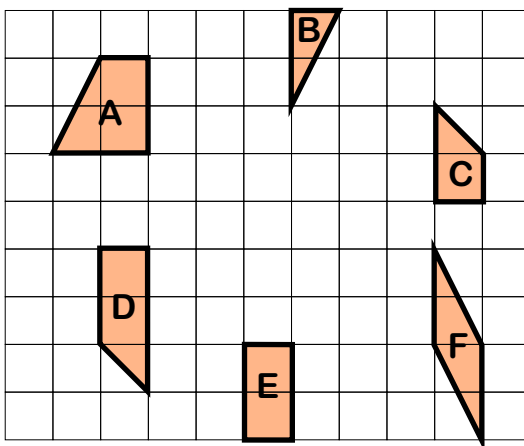
Shape **B** - D4

Shape **C** - L5, D2

Shape **D** - R2, U3

Shape **E** - R2, U3

Shape **F** - L4, U3



3. Copy the shapes shown below onto a grid.

Translate the shapes by the amounts shown to form an Australian icon.

Shape **A** - R4, D7

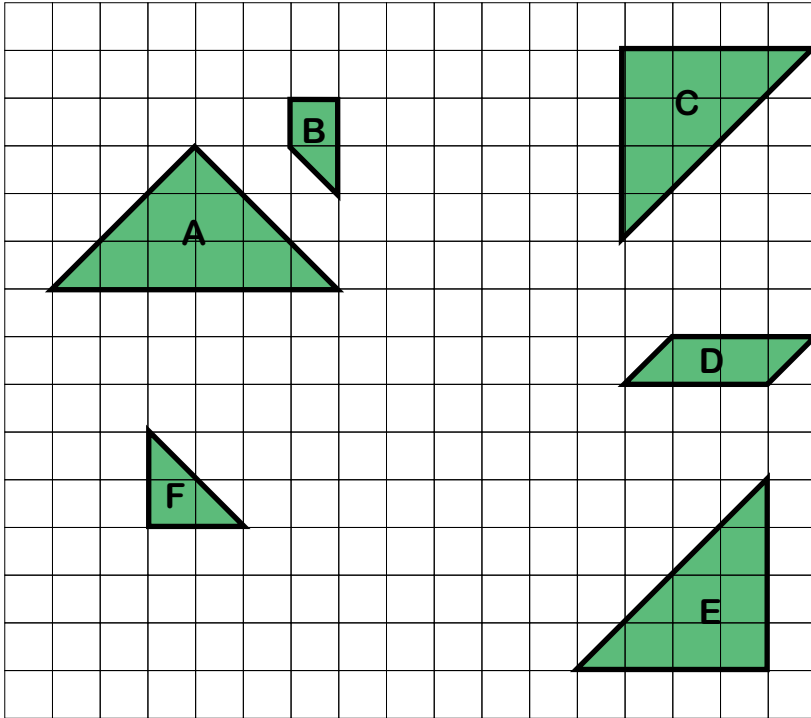
Shape **B** - R4, D5

Shape **C** - L7, D7

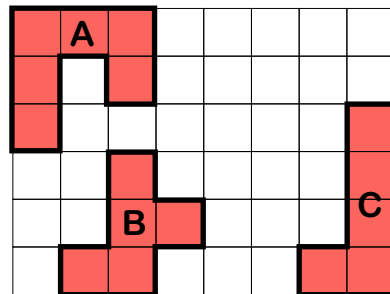
Shape **D** - L11, D5

Shape **E** - L6, U6

Shape **F** - R7, U6



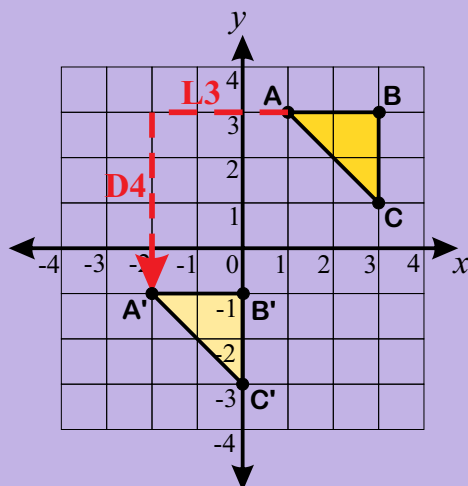
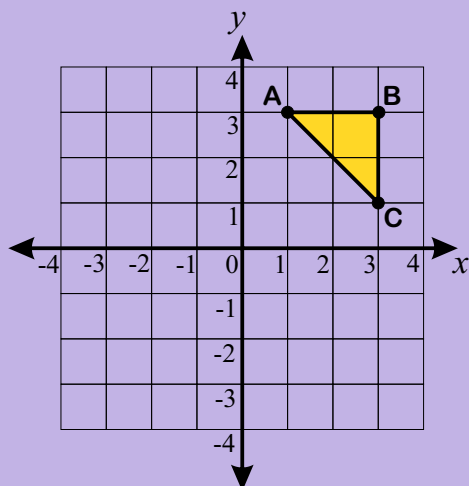
4. Describe how the shapes on this grid could be translated to form a **square** that is symmetrical in the grid.



EXERCISE 20E

Example

Find the coordinates of the vertices of the shape shown below after it has been translated to the left three units and down four units (L3, D4)



Coordinates of the vertices of the translated shape:

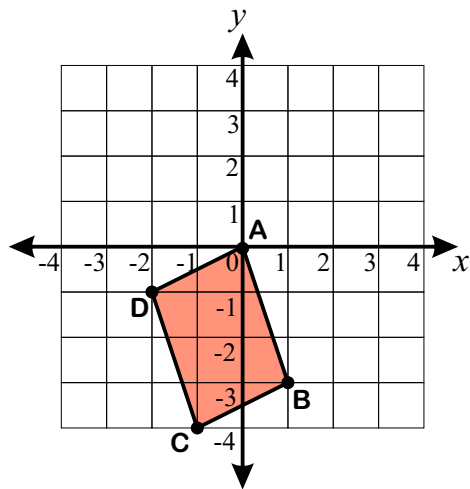
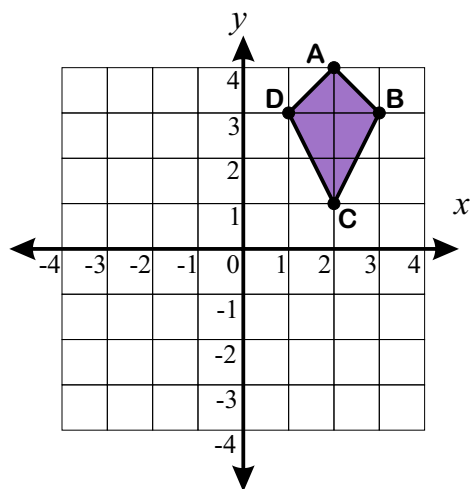
$$A' = (-2, -1)$$

$$B' = (0, -1)$$

$$C' = (0, -3)$$

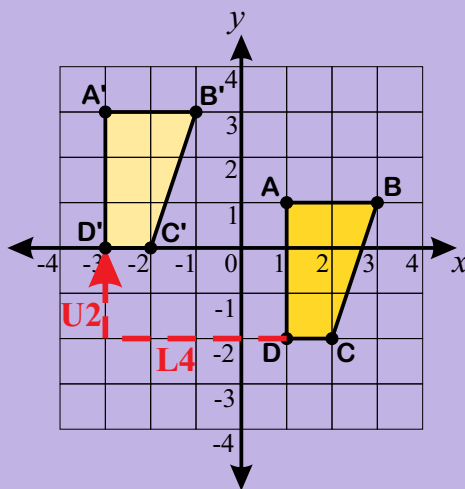
1. Find the coordinates of the shapes below after they have been translated by the amounts shown.

(a) This shape is translated L4, D3. (b) This shape is translated R2, U4.



Example

1. Draw the shape with the following coordinates as vertices:
A = (1,1), **B** = (3,1), **C** = (2,-2), **D** = (1,-2)
2. On the same axes draw the shape after it has been translated left four units and up two units (L4, U2).
3. State the coordinates of the vertices of the translated shape: **A'**, **B'**, **C'**, **D'**



The coordinates of the vertices of the translated shape are:

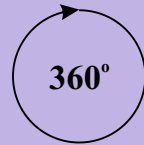
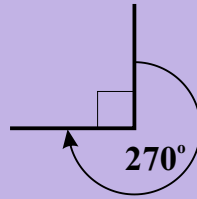
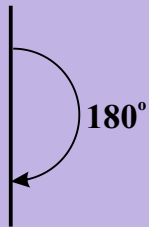
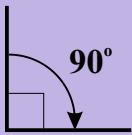
$$\begin{aligned} \mathbf{A'} &= (-3,3) \\ \mathbf{B'} &= (-1,3) \\ \mathbf{C'} &= (-2,0) \\ \mathbf{D'} &= (-3,0) \end{aligned}$$

For the following questions draw grids with the x -axis and y -axis from -4 to 4.

2. (a) Draw the shape with the following coordinates as vertices:
A = (-3,3), **B** = (0,3), **C** = (-1,1), **D** = (-2,1)
 - (b) On the same axes draw the shape after it has been translated right four units and down two units (R4, D2).
 - (c) State the coordinates of the vertices of the translated shape: **A'**, **B'**, **C'**, **D'**
3. (a) Draw the shape with the following coordinates as vertices:
A = (2,4), **B** = (3,4), **C** = (2,3), **D** = (3,2), **E** = (2,2), **F** = (1,3)
 - (b) On the same axes draw the shape after it has been translated left four units and down five units (L4, D5).
 - (c) State the coordinates of the vertices of the translated shape: **A'**, **B'**, **C'**, **D'**, **E'**, **F'**

Rotations

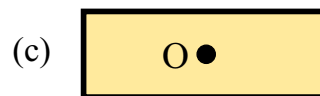
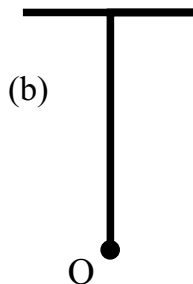
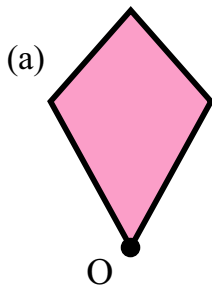
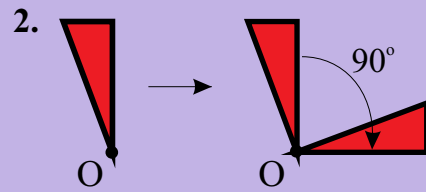
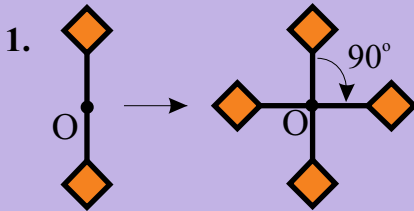
Remember

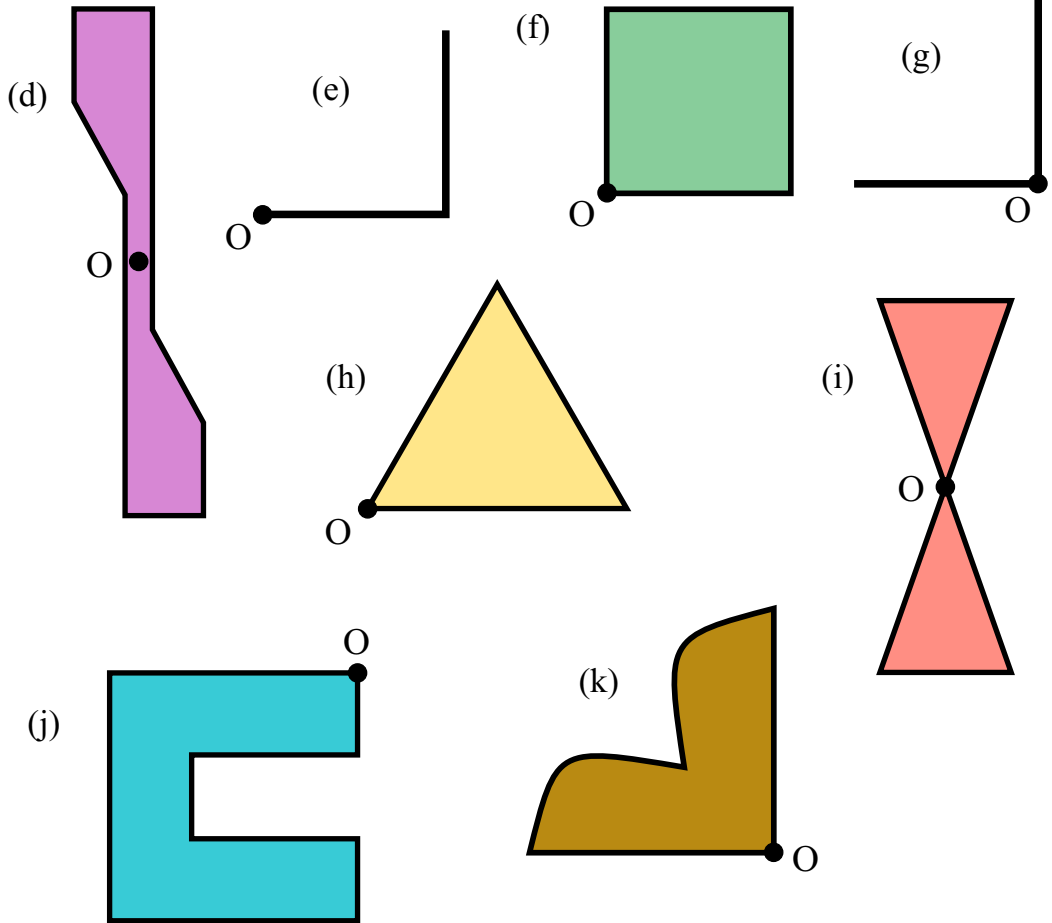


EXERCISE 20F

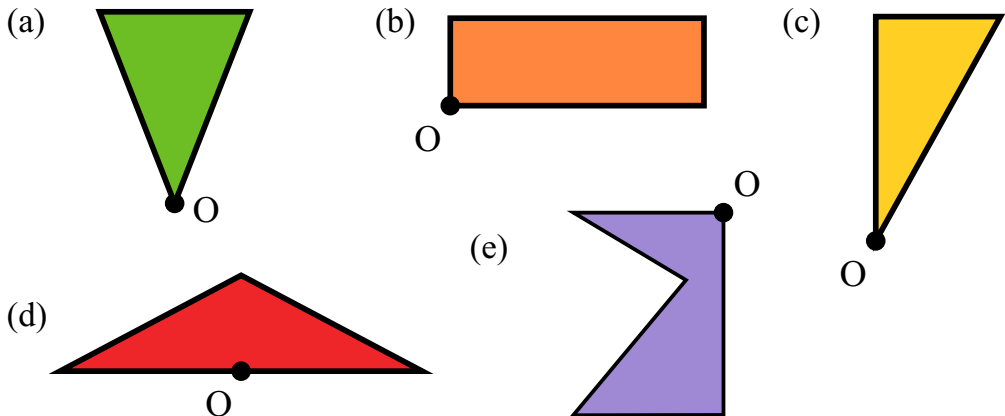
- Copy the following shapes into your books or onto graph paper and redraw them after they have been rotated **90° clockwise** about O.

Examples

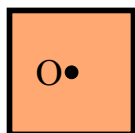




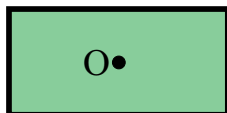
2. Copy the following shapes into your books or onto graph paper and redraw them after they have been rotated 180° *clockwise* about O.



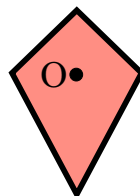
3. Which of the following shapes would appear the same after being rotated by 90° in a clockwise direction about O?



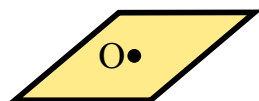
Square



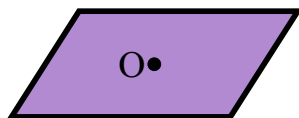
Rectangle



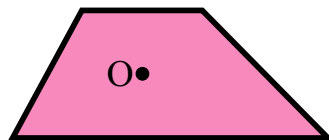
Kite



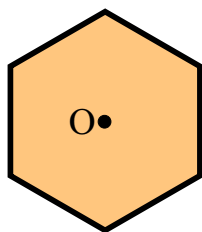
Rhombus



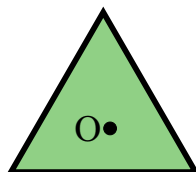
Parallelogram



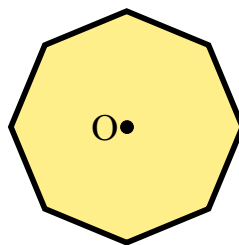
Trapezium



Regular Hexagon



Equilateral Triangle



Regular Octagon

4. Which of the shapes in question 3 would appear the same after being rotated 180° about O?

5. Choose the correct answer for the following statements.

- (a) A clockwise rotation of 180° is the same as an anticlockwise rotation of:

A 90° **B** 180° **C** 270° **D** 360°

- (b) A clockwise rotation of 90° is the same as an anticlockwise rotation of:

A 90° **B** 180° **C** 270° **D** 360°

- (c) A clockwise rotation of 45° is the same as an anticlockwise rotation of:

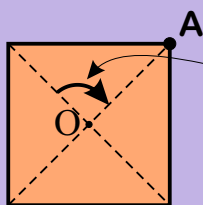
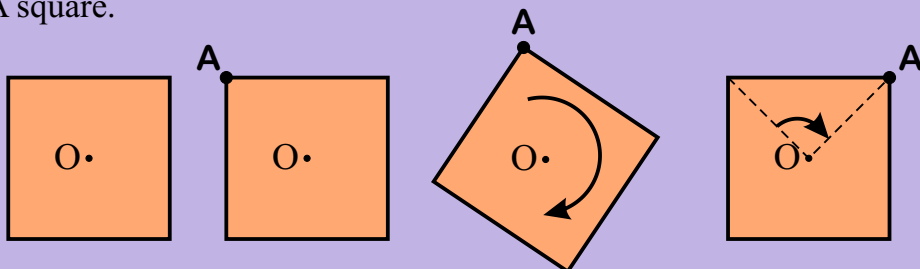
A 45° **B** 135° **C** 225° **D** 315°

Find the **minimum** angle of clockwise rotation about O for these shapes to appear the same.

Examples

To find the minimum angle of clockwise rotation about O for a shape to appear the same, choose a point on the shape and find the angle through which this point rotates. In the examples below this is point A.

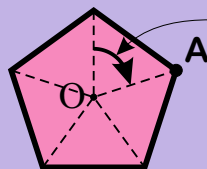
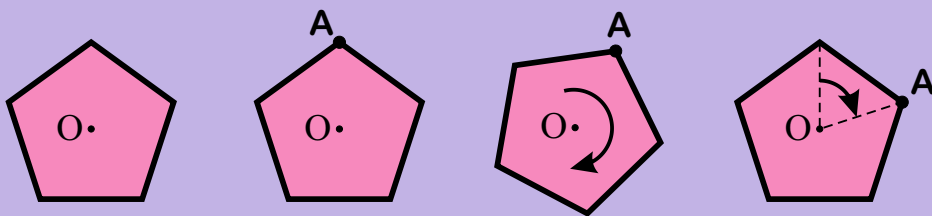
1. A square.



The minimum angle of clockwise rotation about O for a square to appear the same is a quarter of the square = $360^\circ \div 4$

$$= 90^\circ$$

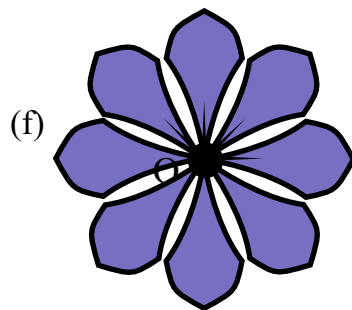
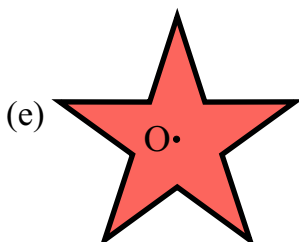
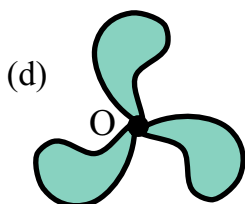
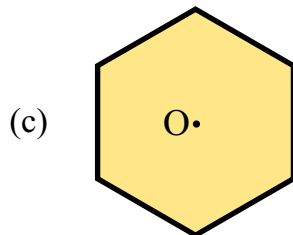
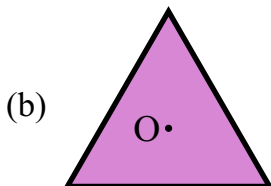
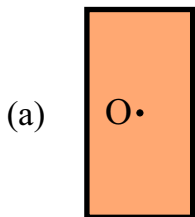
2. A regular pentagon.



The minimum angle of clockwise rotation about O for a pentagon to appear the same is one fifth of the pentagon = $360^\circ \div 5$

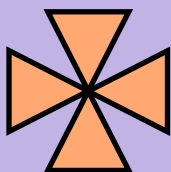
$$= 72^\circ$$

6. Find the **minimum** angle of clockwise rotation about O for these shapes to appear the same.



Many shapes can be constructed by using a starting shape that has been rotated and copied a number of times.

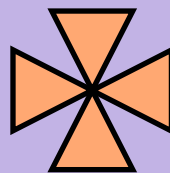
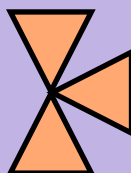
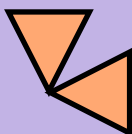
Example



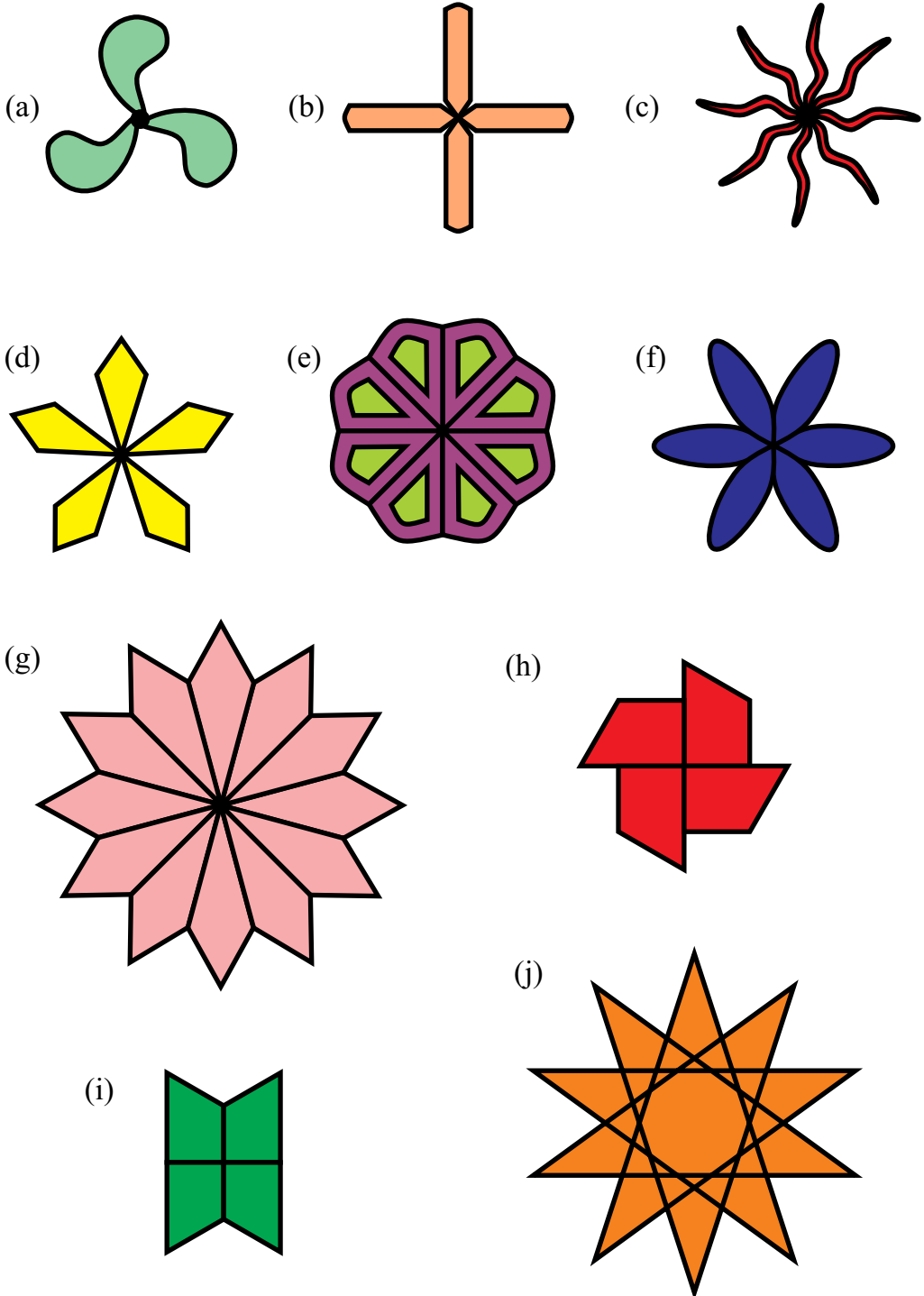
This shape can be constructed by following these steps. The starting shape is rotated clockwise 90° each time and redrawn.



Starting Shape



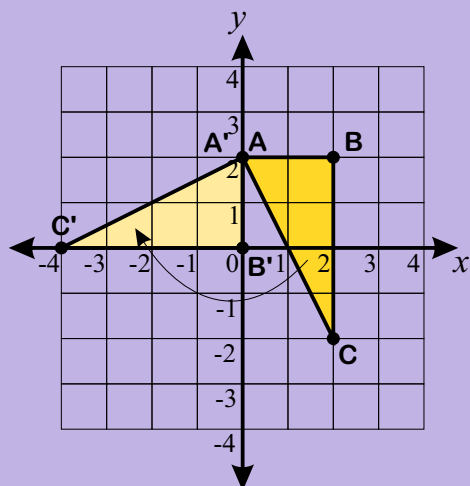
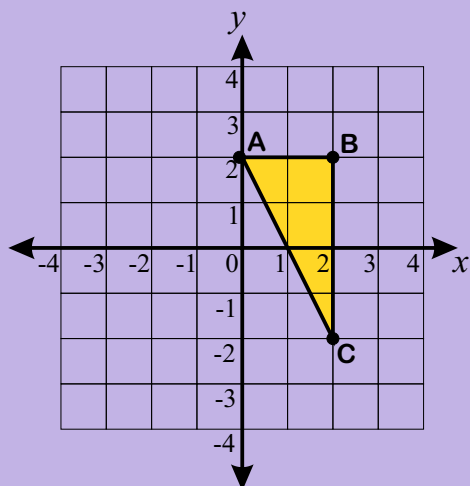
7. Draw the starting shape for each of the following shapes and state the angle of clockwise rotation for the starting shape to be repeatedly redrawn.



EXERCISE 20G

Example

Find the coordinates of the vertices of the shape shown below after it has been rotated 90° *clockwise* about **A**.



Coordinates of the vertices of the rotated shape:

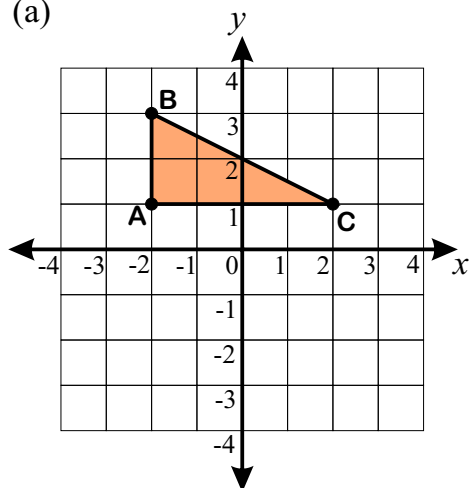
$$\mathbf{A'} = (0, 2)$$

$$\mathbf{B'} = (0, 0)$$

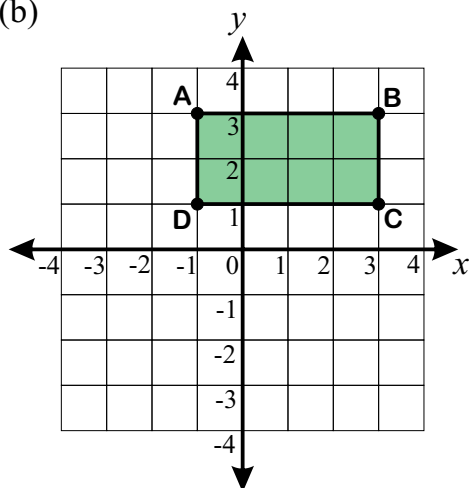
$$\mathbf{C'} = (-4, 0)$$

1. Find the coordinates of the vertices of the shapes shown below after they have been rotated 90° *clockwise* about **A**.

(a)

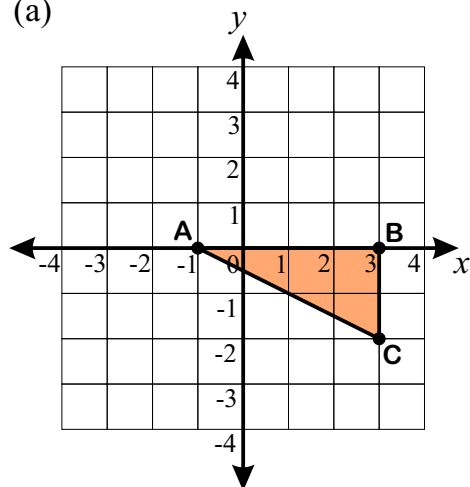


(b)

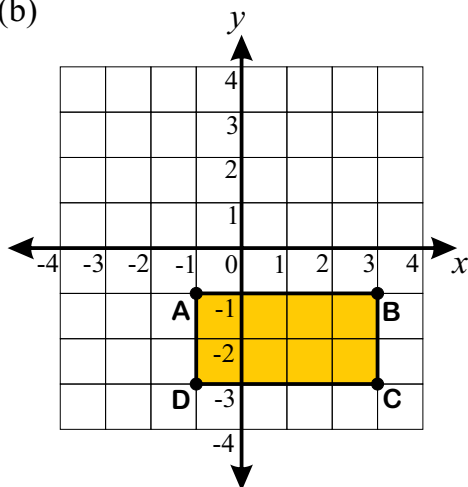


2. Find the coordinates of the vertices of the shapes shown below after they have been rotated 90° *anticlockwise* about **A**.

(a)

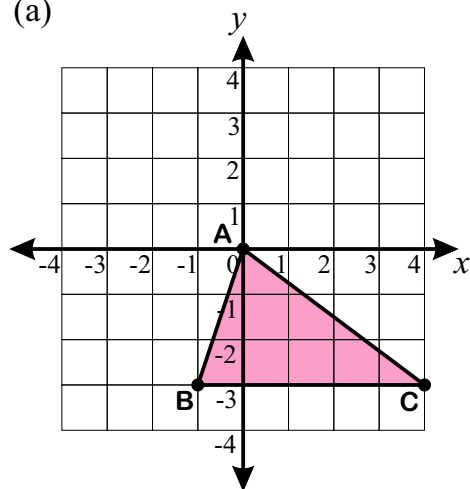


(b)

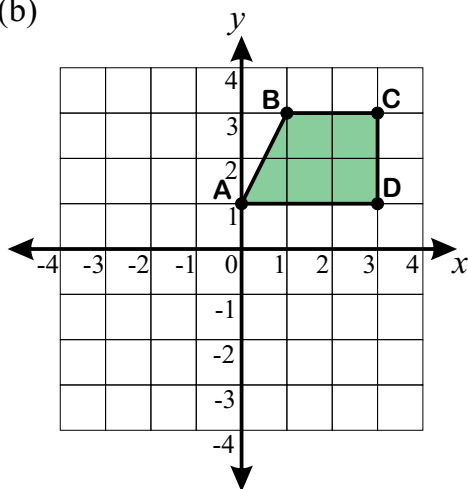


3. Find the coordinates of the vertices of the shapes shown below after they have been rotated 180° about **A**.

(a)



(b)



4. (a) Draw a grid with the x -axis and y -axis from -5 to 5.
 (b) Draw the shape with the following coordinates as vertices:
A = (1,0), **B** = (2,0), **C** = (2,2), **D** = (3,2), **E** = (3,3),
F = (2,3), **G** = (2,4), **H** = (4,4), **I** = (4,5), **J** = (1,5)
 (c) On the same axes draw the shape after it has been rotated
 by 180° about **A**.
 (d) List the coordinates of the vertices of the rotated shape:
A', **B'**, **C'**, **D'**, **E'**, **F'**, **G'**, **H'**, **I'**, **J'**

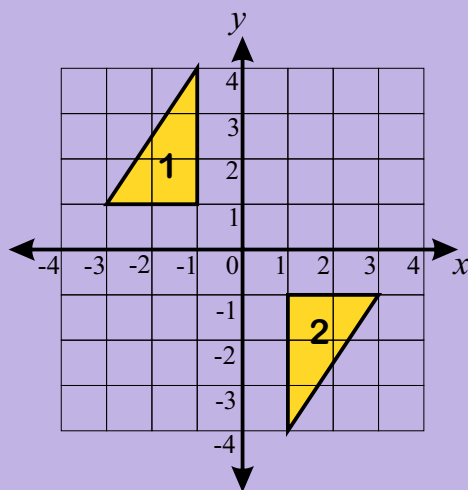
Combinations

Shapes can be moved from one position to another using a combination of the different types of transformations:

- reflections, translations and/or rotations.

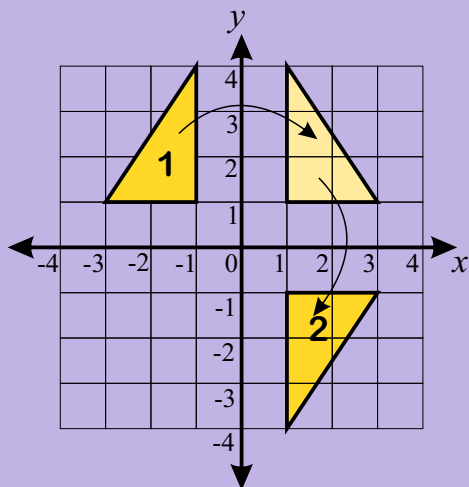
Example 1

Using reflections, translations and/or rotations state two methods that could be used to move the shape shown here from position **1** to position **2**.



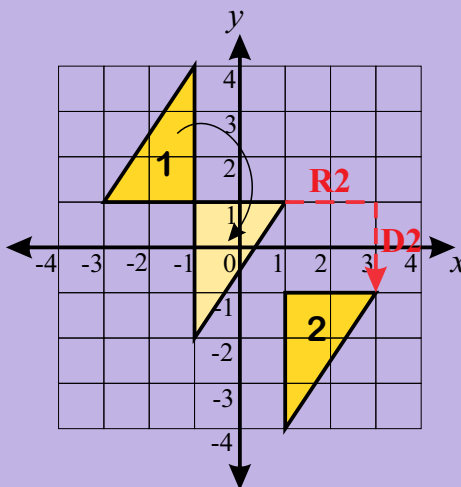
Method 1

Reflection about the y -axis then reflection about the x -axis.



Method 2

Rotation of 180° about the point $(-1, 1)$ then translation to the right two units then down two units (R_2 , D_2).

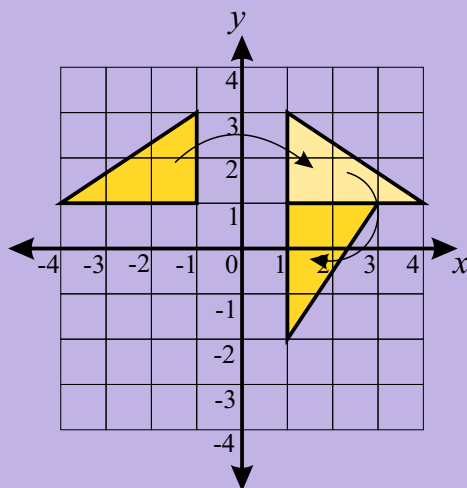
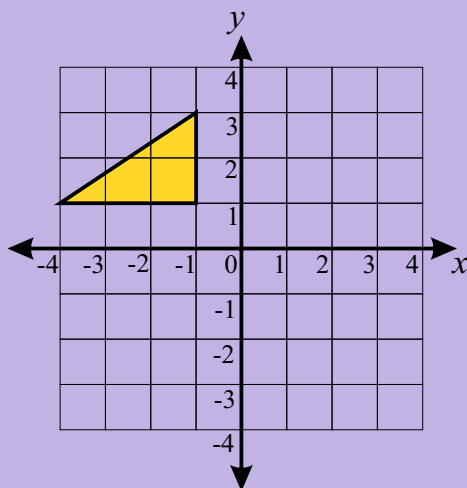


Example 2

Perform the following transformations on the shape shown below.

Step 1 Reflect the shape about the y -axis.

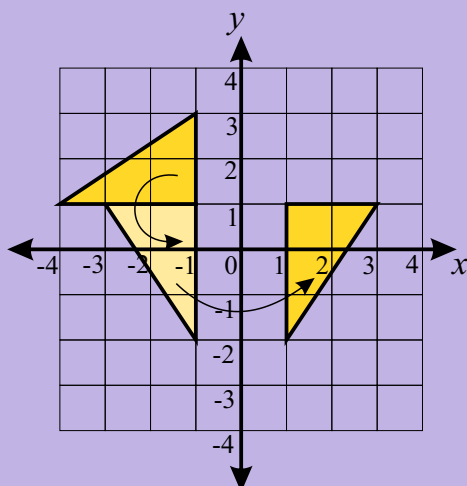
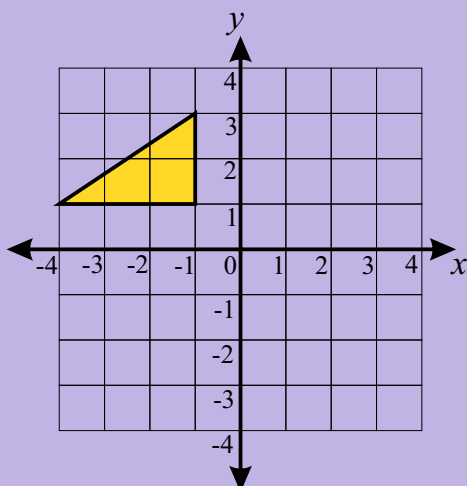
Step 2 Rotate it 90° clockwise about the point $(1,1)$.



This transformation could also be achieved the following way:

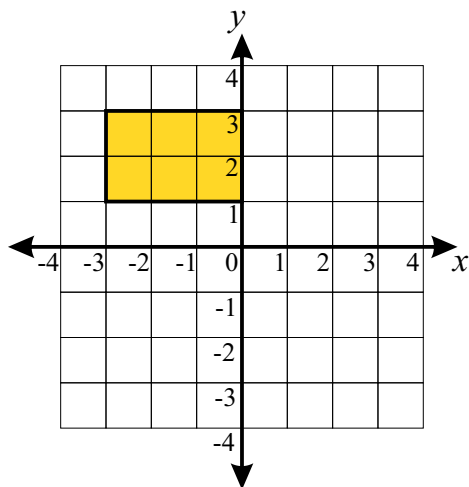
Step 1 Rotate it 90° anticlockwise about the point $(-1,1)$.

Step 2 Reflect it about the y -axis.

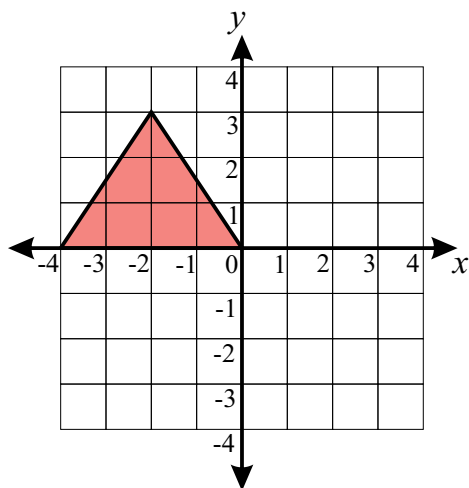


EXERCISE 20H

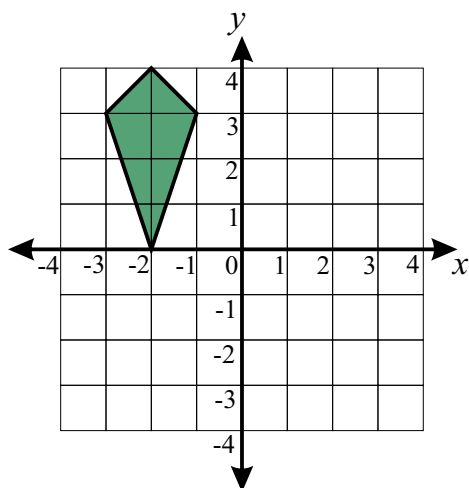
1. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Reflect it about the y -axis.
Step 2 Reflect it about the x -axis.
 (c) Find another way of achieving this transformation.



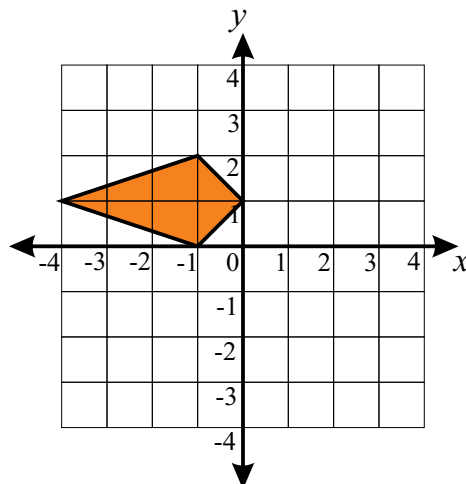
2. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Rotate it 180° about $(0,0)$.
Step 2 Translate it down one unit (D1).
 (c) Find another way of achieving this transformation.



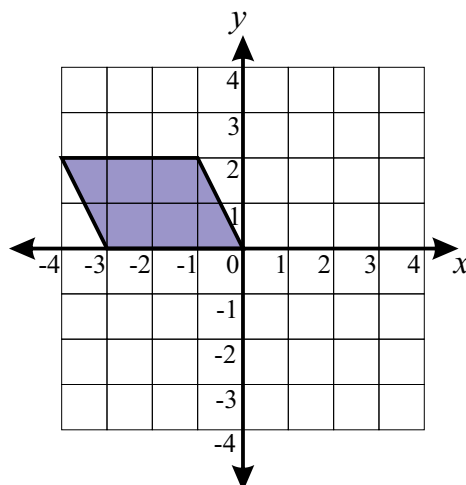
3. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Reflect it about the x -axis.
Step 2 Translate it to the right one unit and up three units (R1, U3).
 (c) Find another way of achieving this transformation.



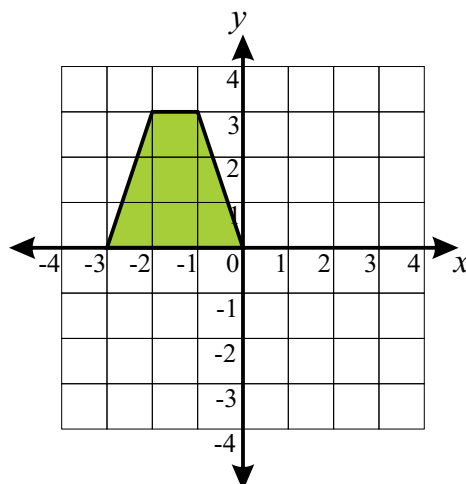
4. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Rotate it 180° about $(0,1)$.
Step 2 Translate it left one unit and up one unit (L1, U1).
 (c) Find another way of achieving this transformation.



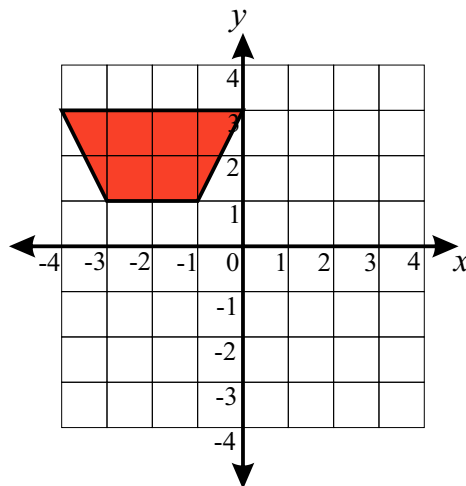
5. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Reflect it about the x -axis.
Step 2 Translate it up four units (U4).
 (c) Find another way of achieving this transformation.



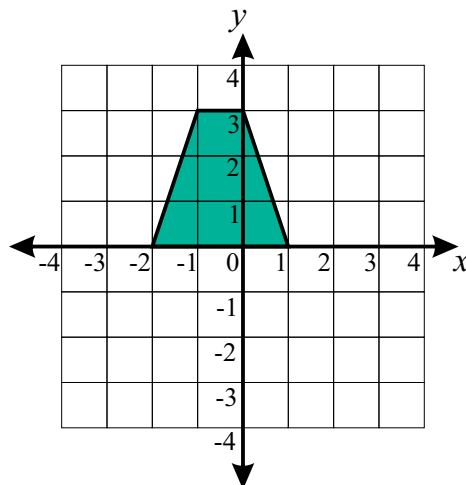
6. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Reflect it about the y -axis.
Step 2 Reflect it about the x -axis.
 (c) Find another way of achieving this transformation.



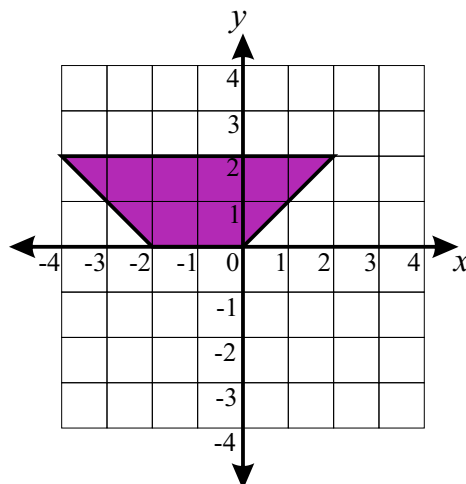
7. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Reflect it about the x -axis.
Step 2 Translate it to the right three units and up four units (R3, U4).
 (c) Find another way of achieving this transformation.



8. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Rotate it 180° about $(1, 0)$.
Step 2 Translate it to the left one unit and up three units (L1, U3).
 (c) Find another way of achieving this transformation.



9. (a) Draw this shape.
 (b) Redraw it after the following transformations:
Step 1 Rotate it 90° anticlockwise about $(0, 0)$.
Step 2 Reflect it about the y -axis.
 (c) Find another way of achieving this transformation.



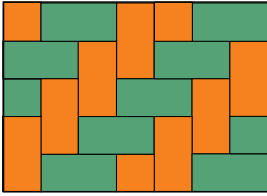
Tessellations

EXERCISE 20I

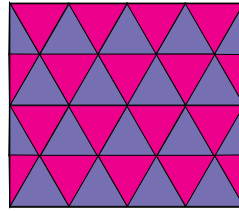
A tessellation is a pattern of shapes that fit together perfectly with no gaps. A tessellation can be made using one or more shapes that are reflected, translated and/or rotated.

1. State the shapes that form the following tessellations.

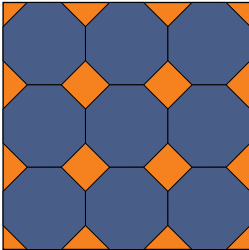
(a)



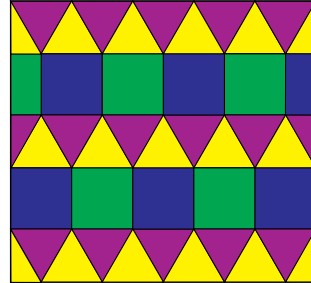
(b)



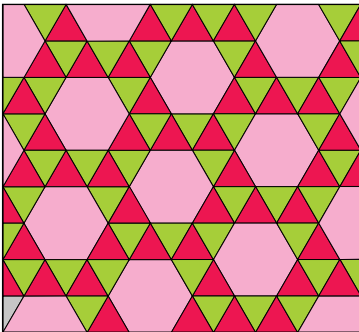
(c)



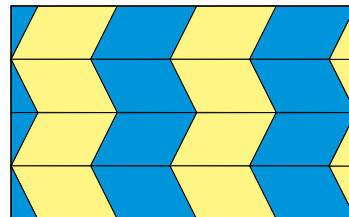
(d)



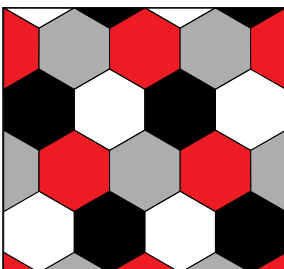
(e)



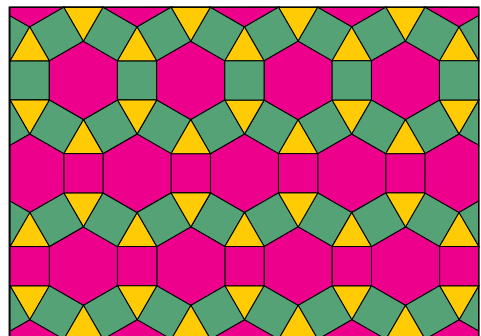
(f)



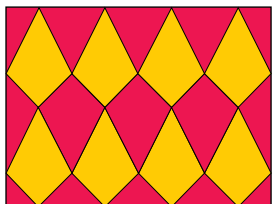
(g)



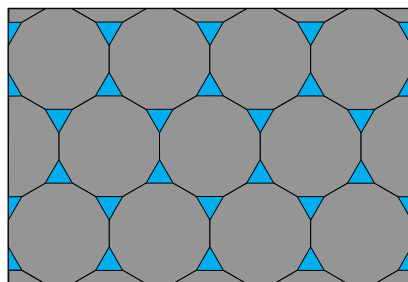
(h)



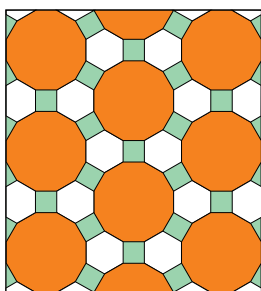
(i)



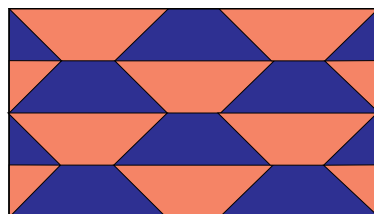
(j)



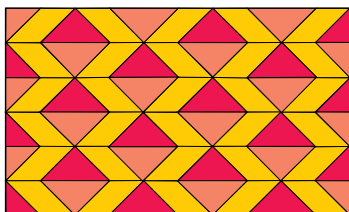
(k)



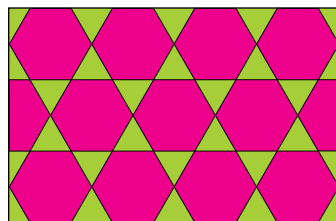
(l)



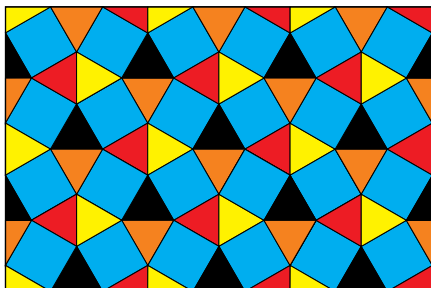
(m)



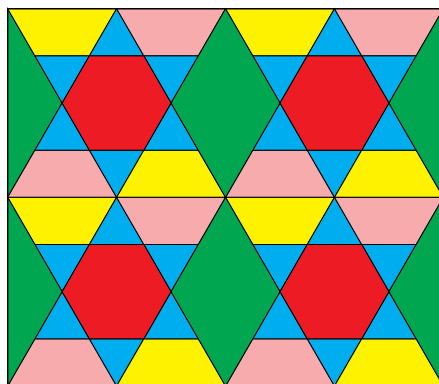
(n)



(o)



(p)

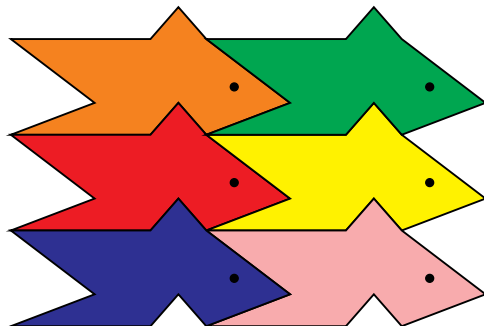
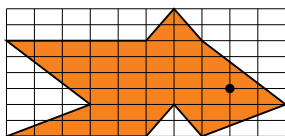
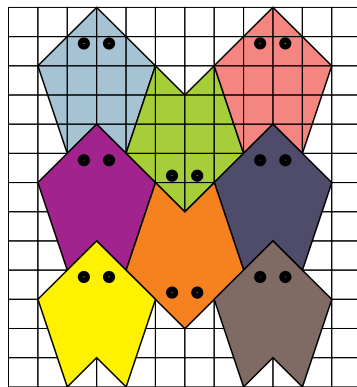
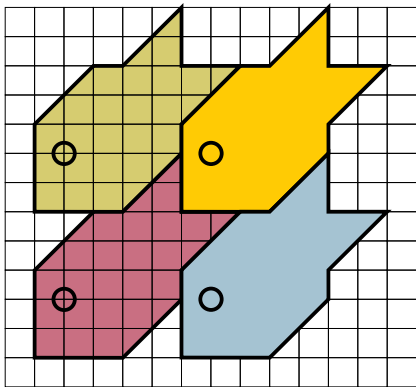
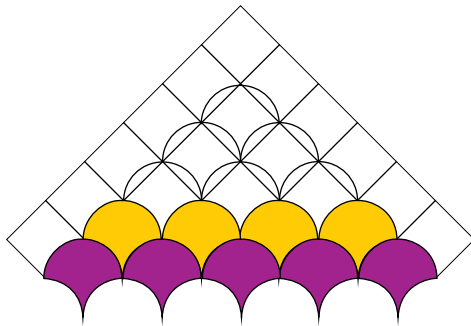
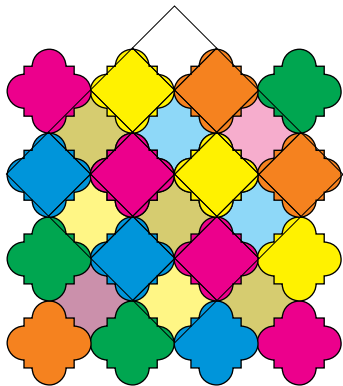


2. Tessellations can also be created using other shapes.

Some examples are shown below.

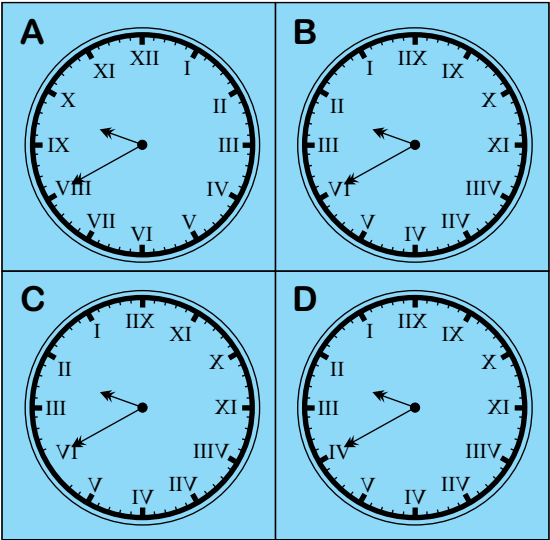
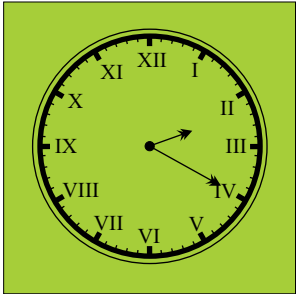
Try drawing some of these, create some of your own or find other tessellations to draw.

An artist famous for his tessellations is M. C Escher. Research Escher's work.



PROBLEM SOLVING

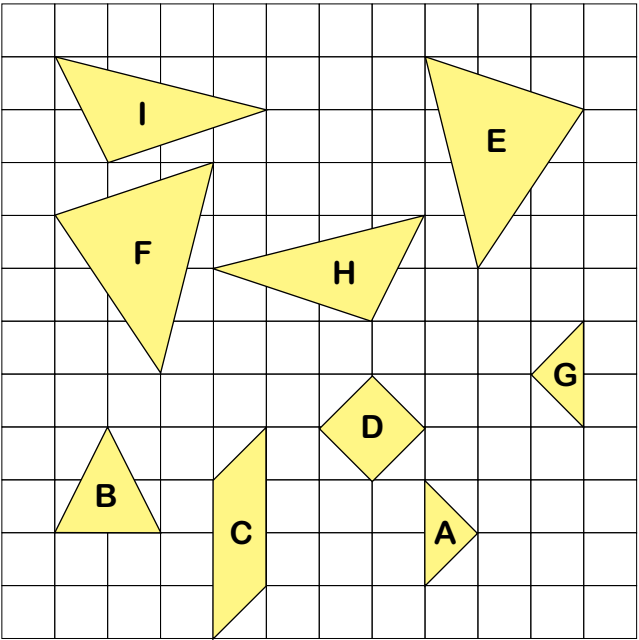
Which of the clocks (A - D) would be a reflection of the clock shown below.



PUZZLE

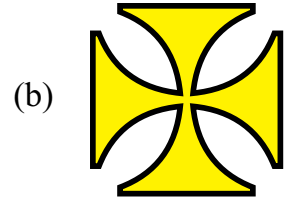
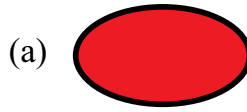
Draw a grid and translate the shapes shown here by the amounts shown below. The resultant shape is what someone with *ailurophobia* fears.

Shape	Movement
A	L7, U8
B	R4, U5
C	R6, U8
D	L5, U5
E	L6, D3
F	R6, D1
G	L8, U5
H	L2, U1
I	R5, D2

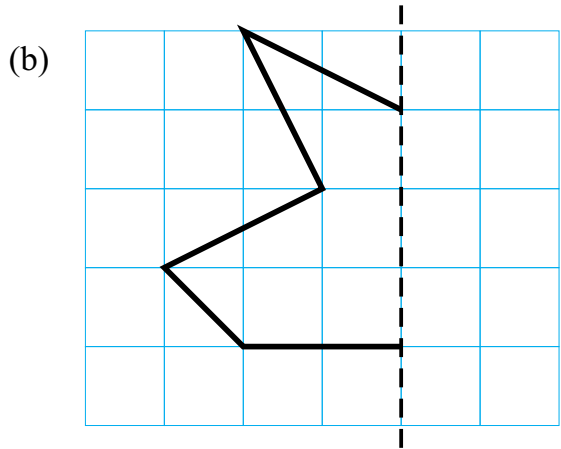
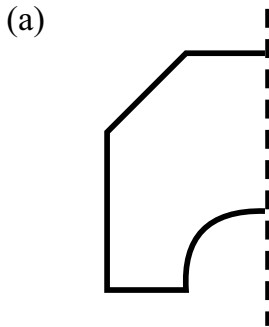


CHAPTER REVIEW

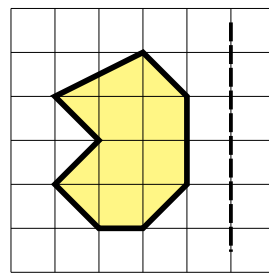
1. Copy the following shapes and show all axes of symmetry.



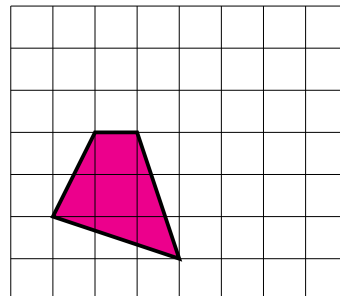
2. Copy the following half-shapes and complete each symmetrical shape by drawing the other half.



3. Draw this shape on graph paper and draw its reflection about the dotted line.

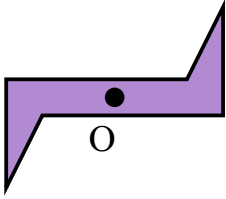


4. Copy the shape shown here and redraw it after it has been translated right three units and up two units (R3, U2).

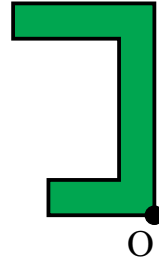


5. Copy the following shapes and redraw them after they have been rotated 90° *clockwise* about O.

(a)



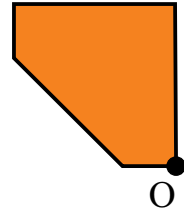
(b)



6. Copy this shape and redraw it after:

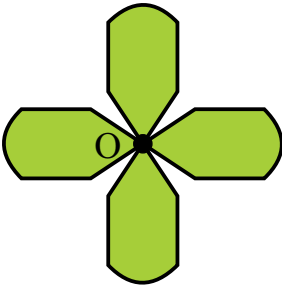
(a) it has been rotated 180° about O.

(b) it has been rotated 90° *anticlockwise* about O.

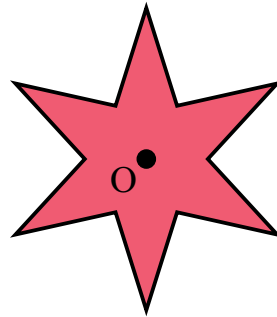


7. What is the minimum angle of clockwise rotation about O for the following shapes to appear the same?

(a)

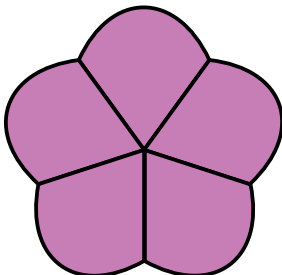


(b)

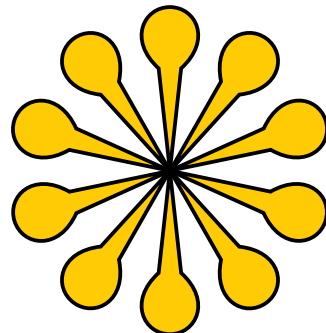


8. Draw the starting shape for the following shapes and state the angle of clockwise rotation for the starting shape to be repeatedly redrawn.

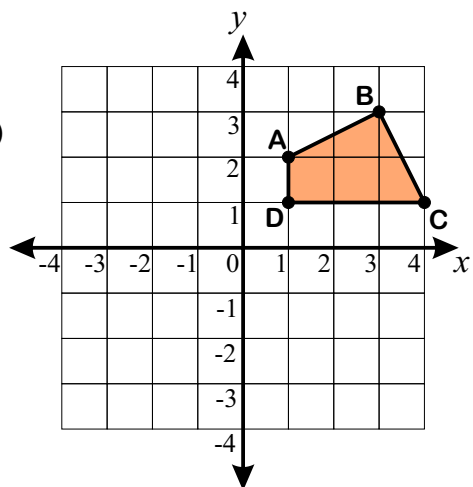
(a)



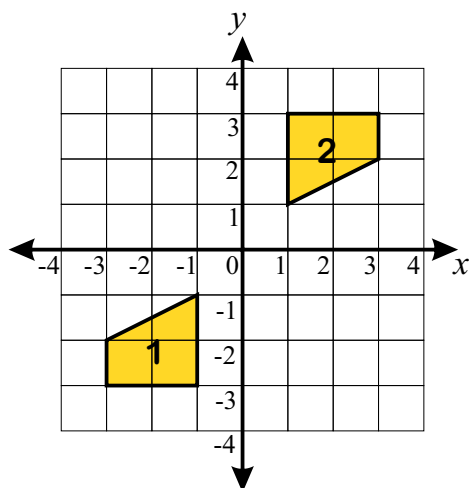
(b)



9. (a) Find the coordinates of the vertices of the shape shown on the grid below.
- (b) Find the coordinates of the vertices of the shape after it has been reflected about the **y-axis**.
- (c) Find the coordinates of the vertices of the shape after it has been reflected about the **x-axis**.
- (d) Find the coordinates of the vertices of the shape after it has been translated to the left three units and down four units (L3, D4).
- (e) Find the coordinates of the vertices of the shape after it has been rotated 90° **clockwise** about the point (1,1).
- (f) Find the coordinates of the vertices of the shape after it has been reflected about the **y-axis** then translated to the right one unit and down three units (R1, D3).
- (g) Find the coordinates of the shape after it has been rotated 90° **anticlockwise** about the point (1,1) then translated to the right two units and down four units (R2, D4).



10. Using reflections, translations and/or rotations state two methods that could be used to move the object shown here from position **1** to position **2**.

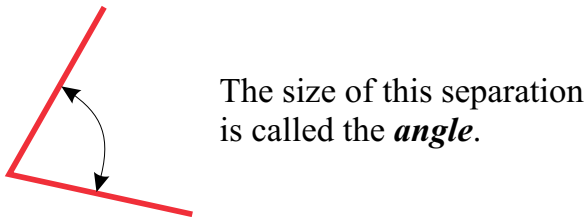
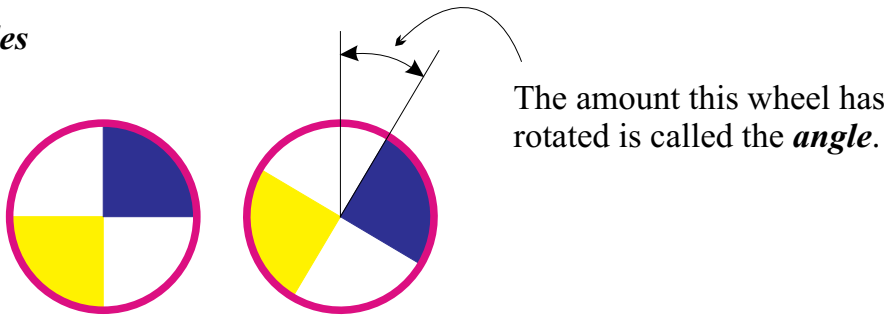


Angles

Angles

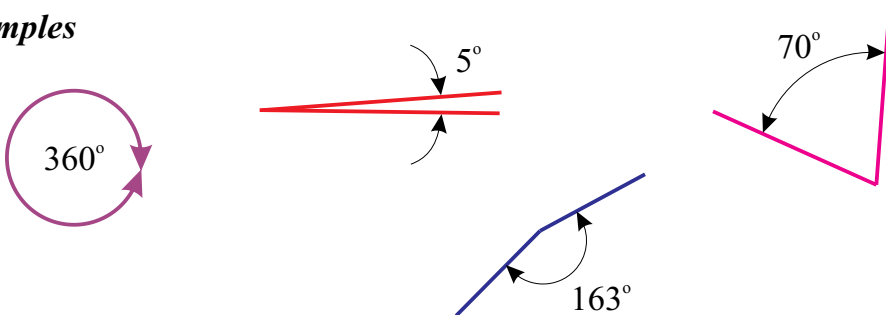
An **angle** is defined as the amount that an object rotates or the size of the separation between two lines that meet at a point.

Examples



A common unit for measuring angles is a degree (symbol $^{\circ}$). There are 360 degrees (360°) in a revolution or a full circle.

Examples



Measuring Angles Between 0° and 180°

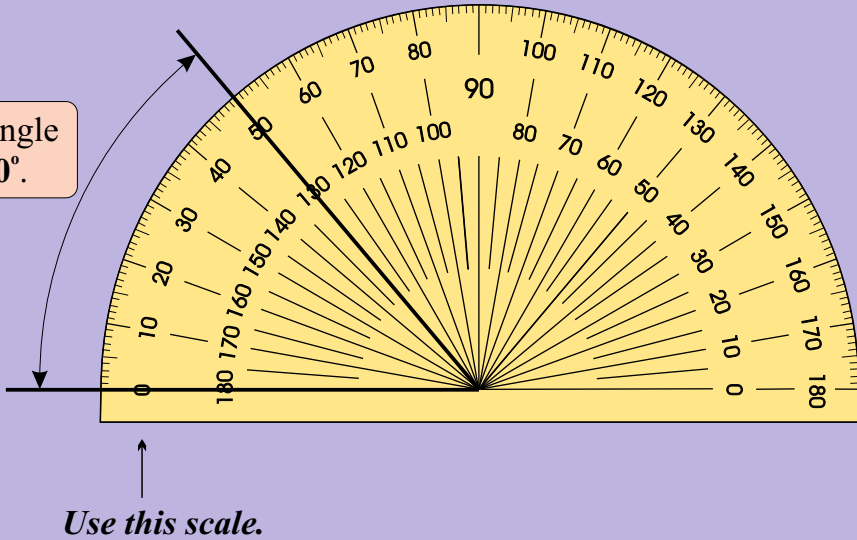
A protractor is one instrument that is used to measure angles.

Examples

Read the size of the angles shown below using the scale on the protractor. Use the scale that starts at 0° .

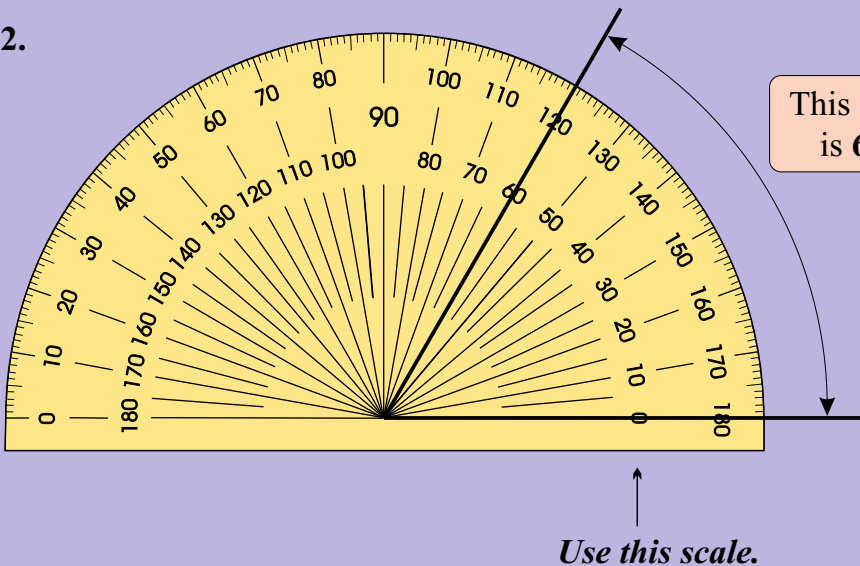
1.

This angle
is 50° .



2.

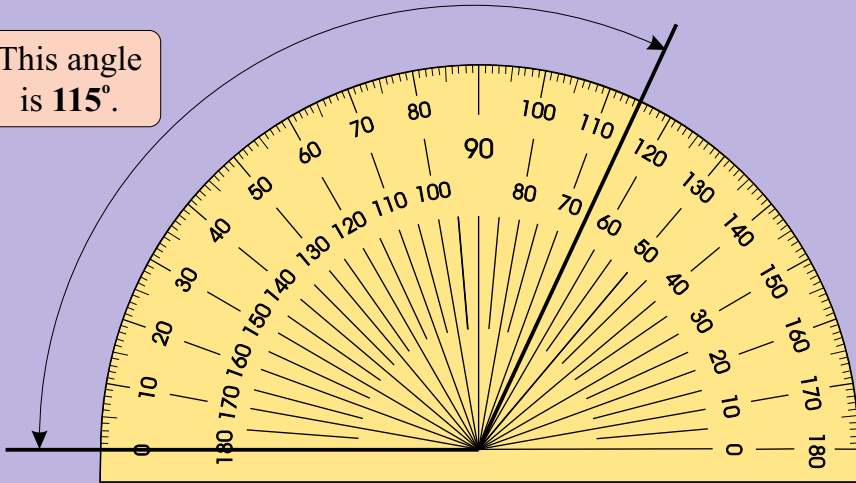
This angle
is 60° .



Examples continued

3.

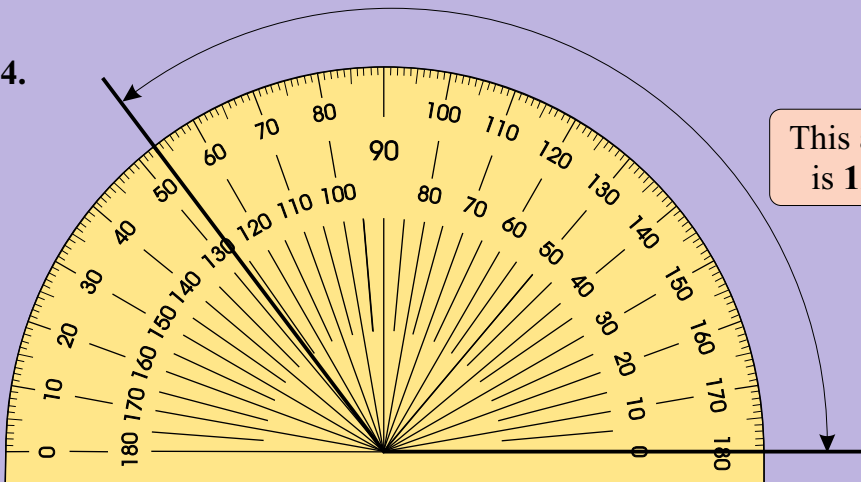
This angle
is 115° .



Use this scale.

4.

This angle
is 127° .

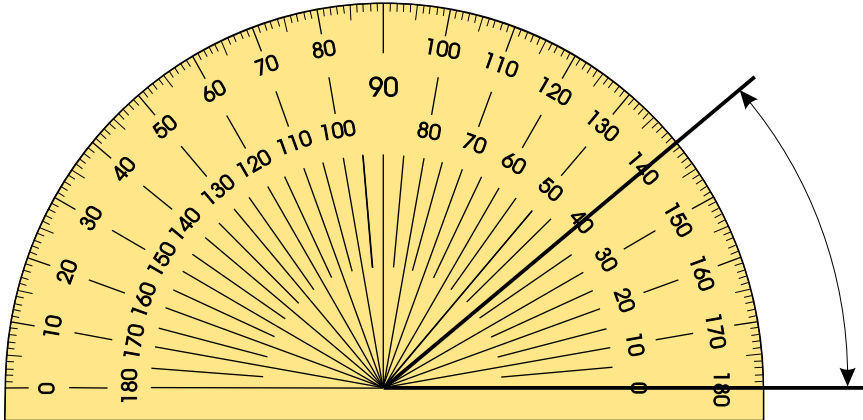


Use this scale.

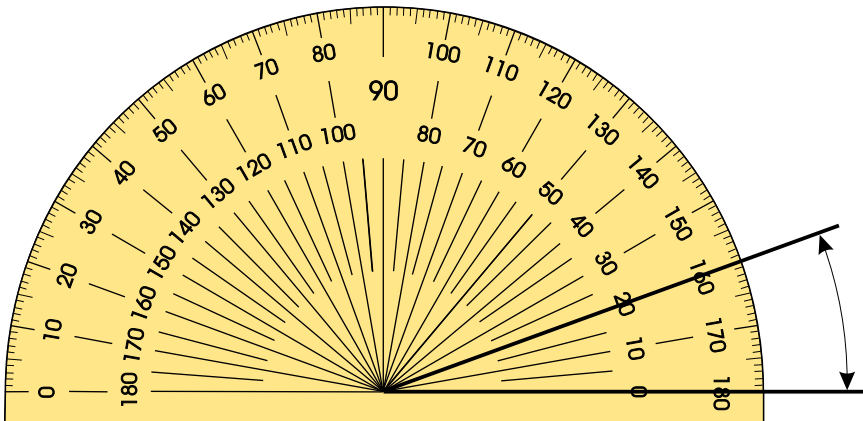
EXERCISE 21A

1. Read the size of the angles shown below using the scale on the protractor. Remember to use the scale that starts at 0° .

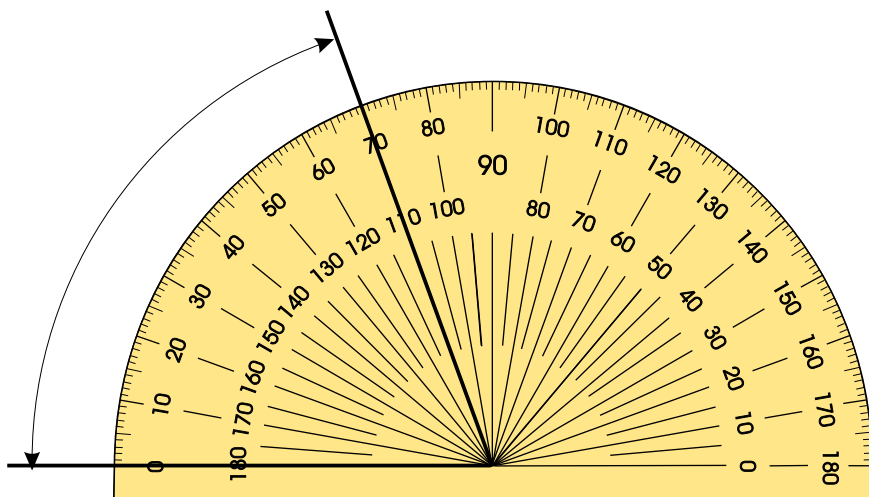
(a)



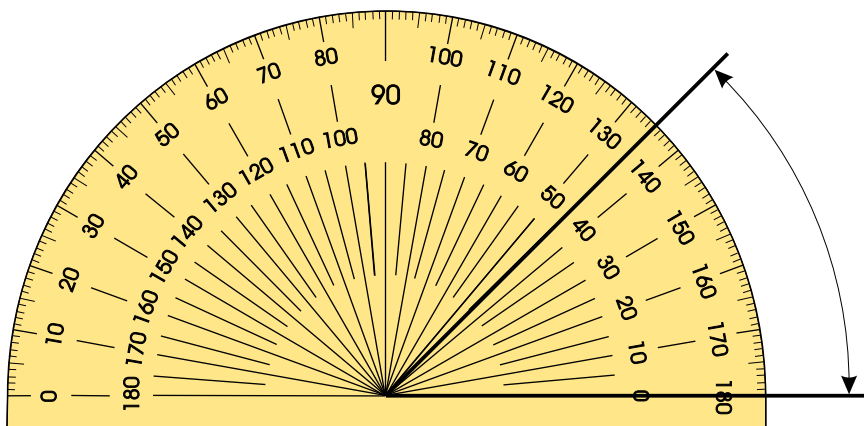
(b)



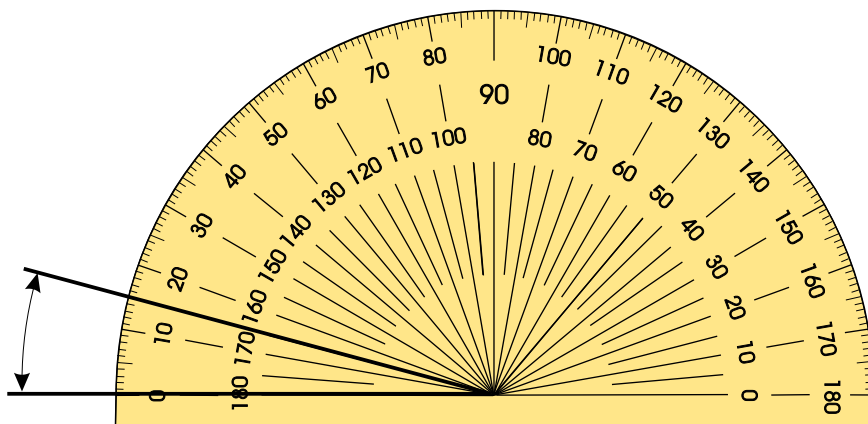
(c)



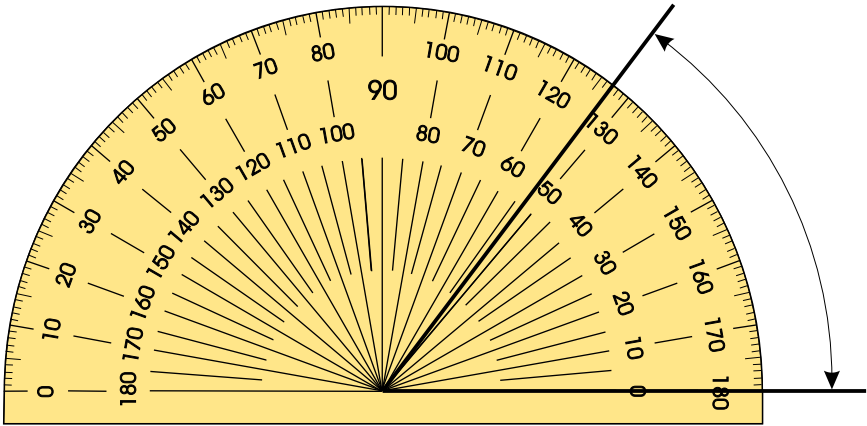
(d)



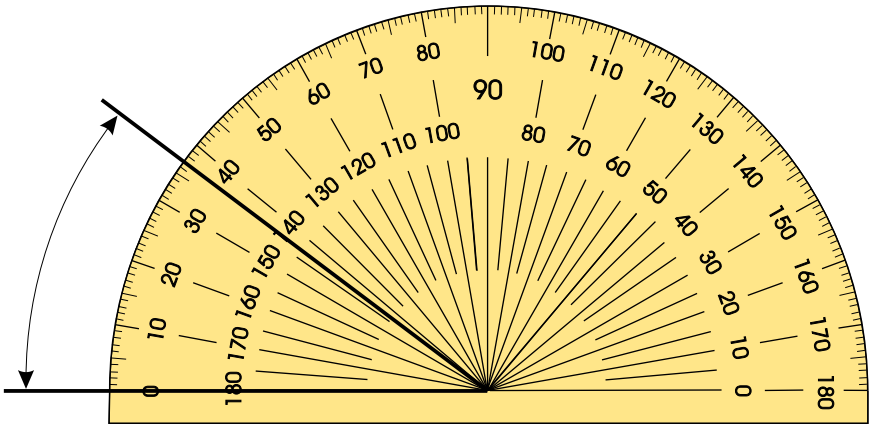
(e)



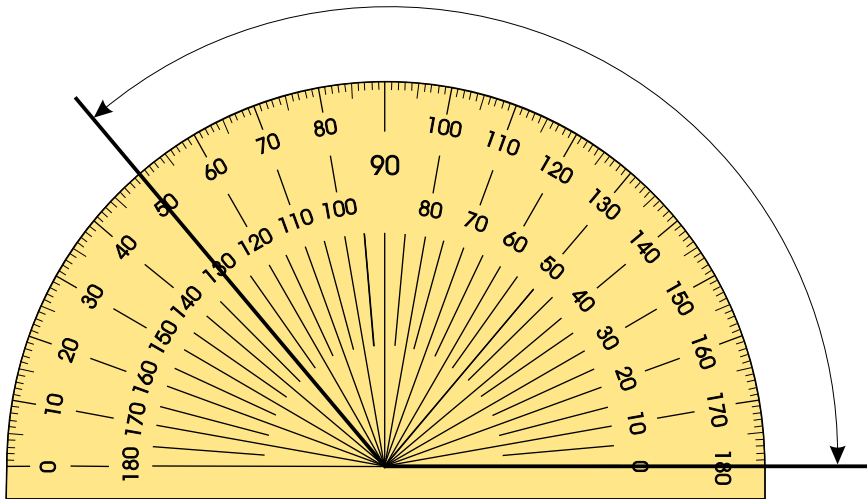
(f)



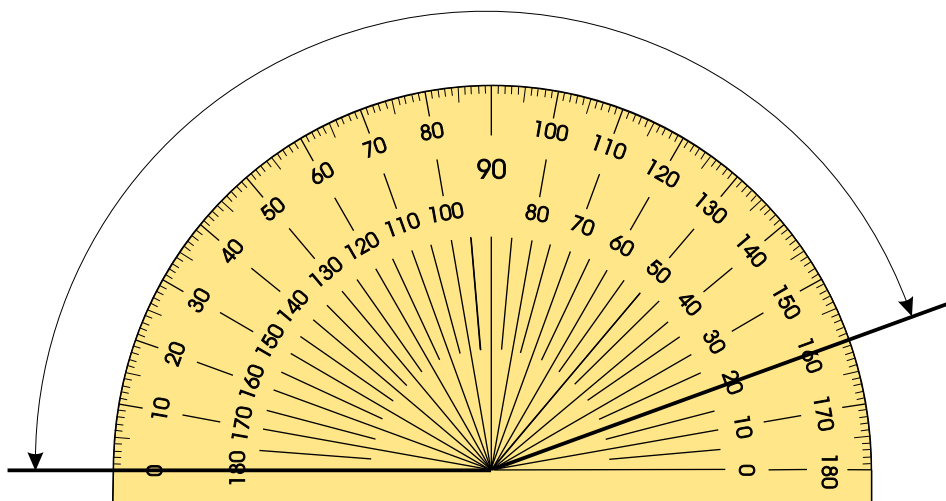
(g)



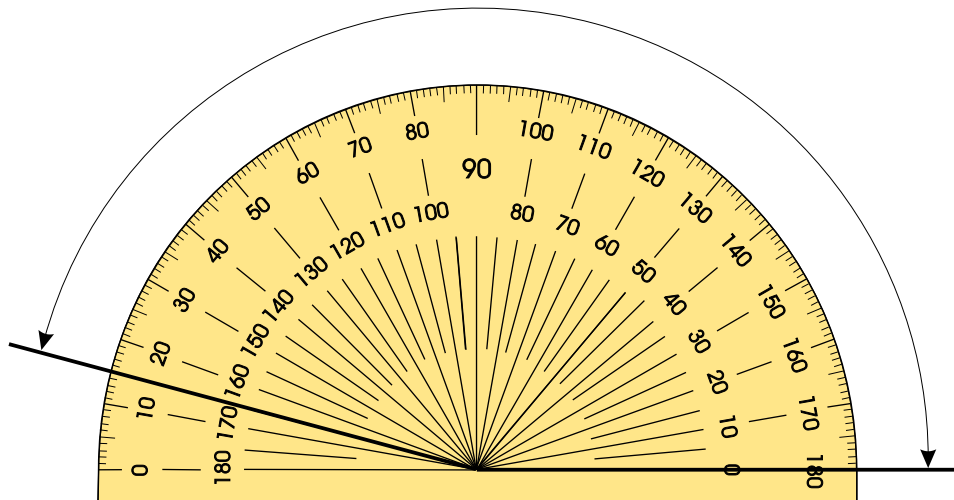
(h)



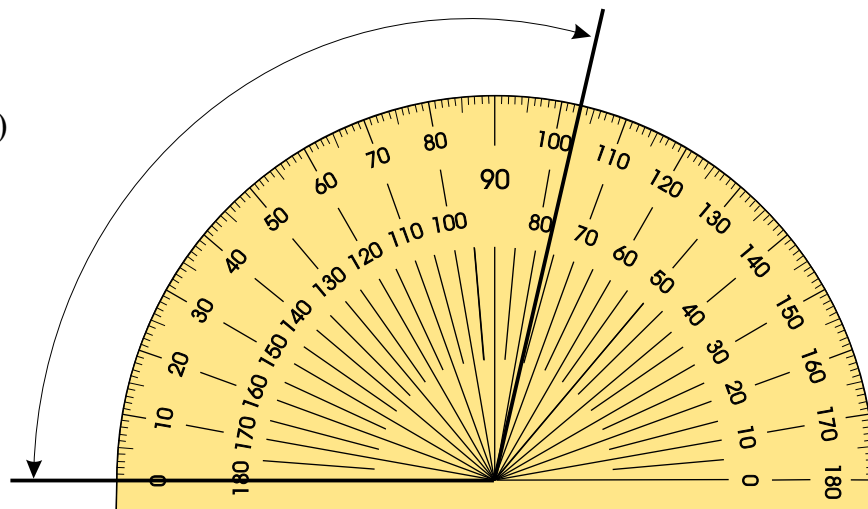
(i)



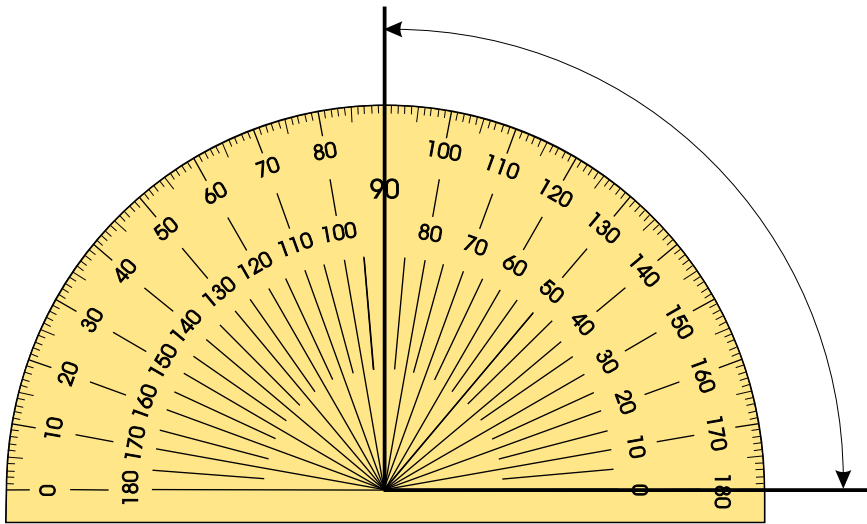
(j)



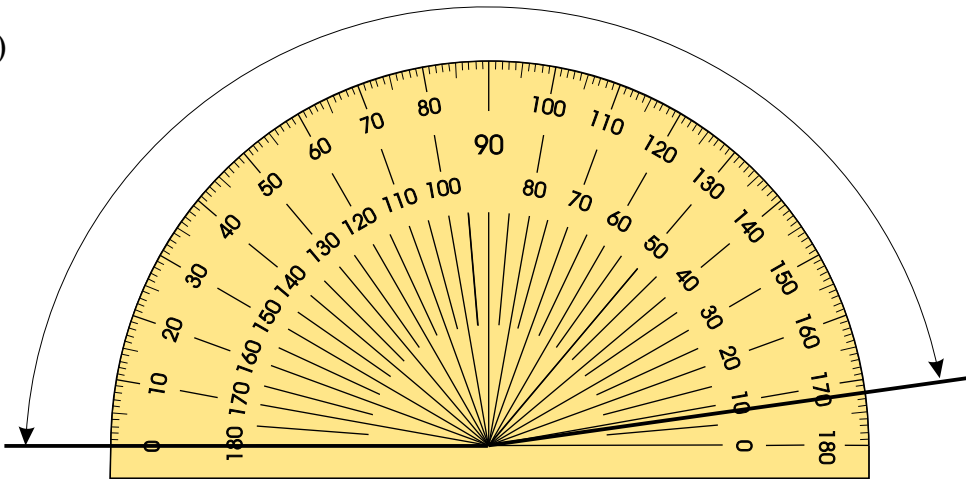
(k)



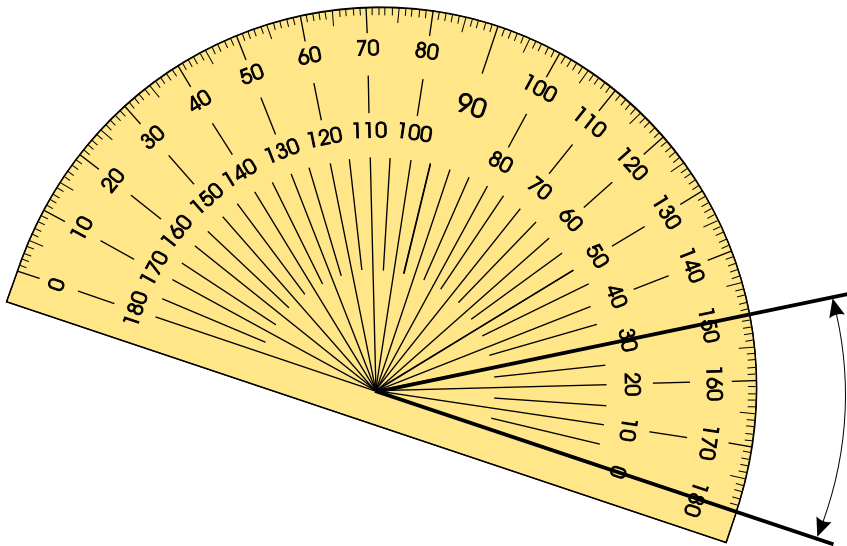
(l)

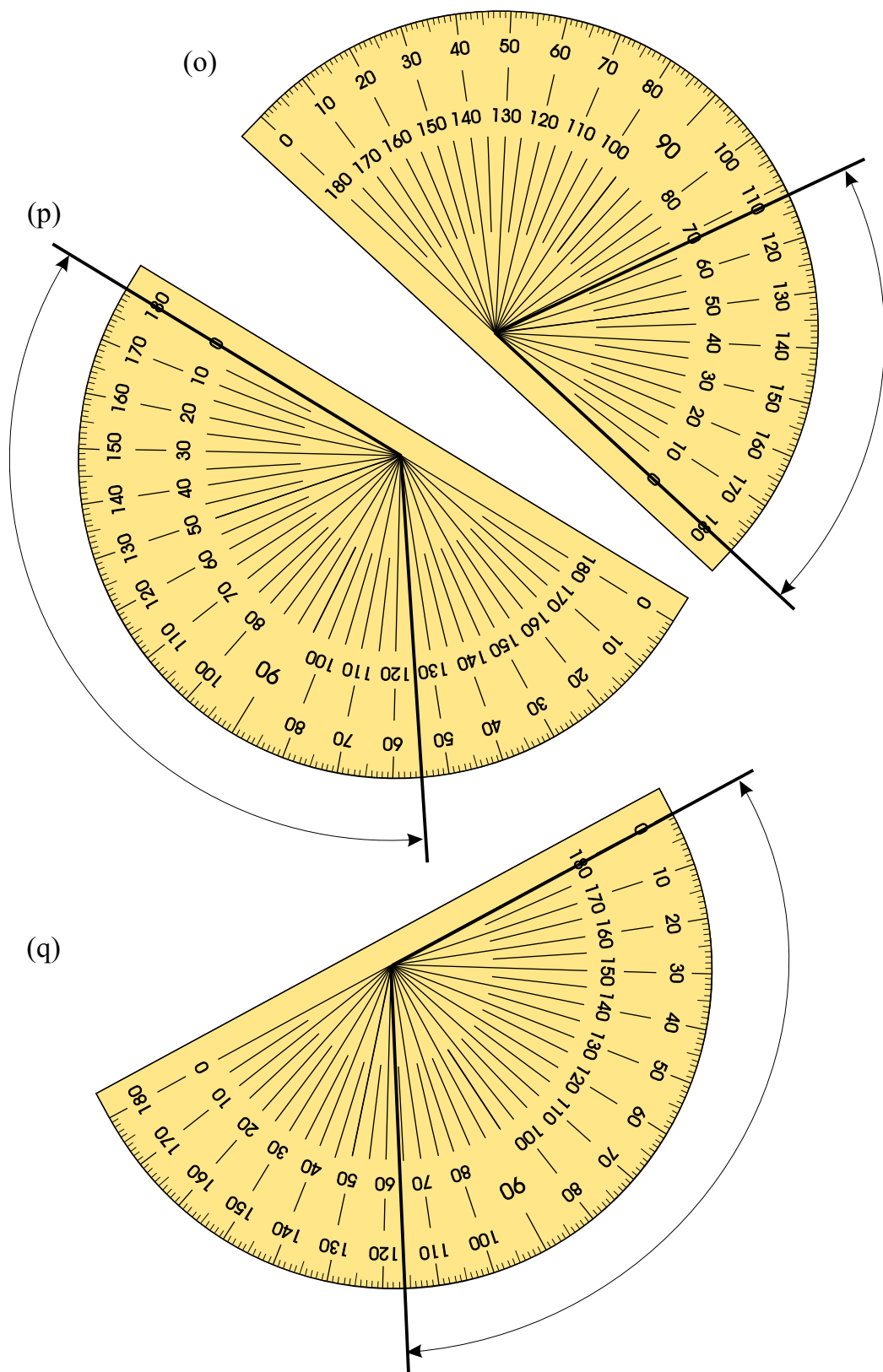


(m)

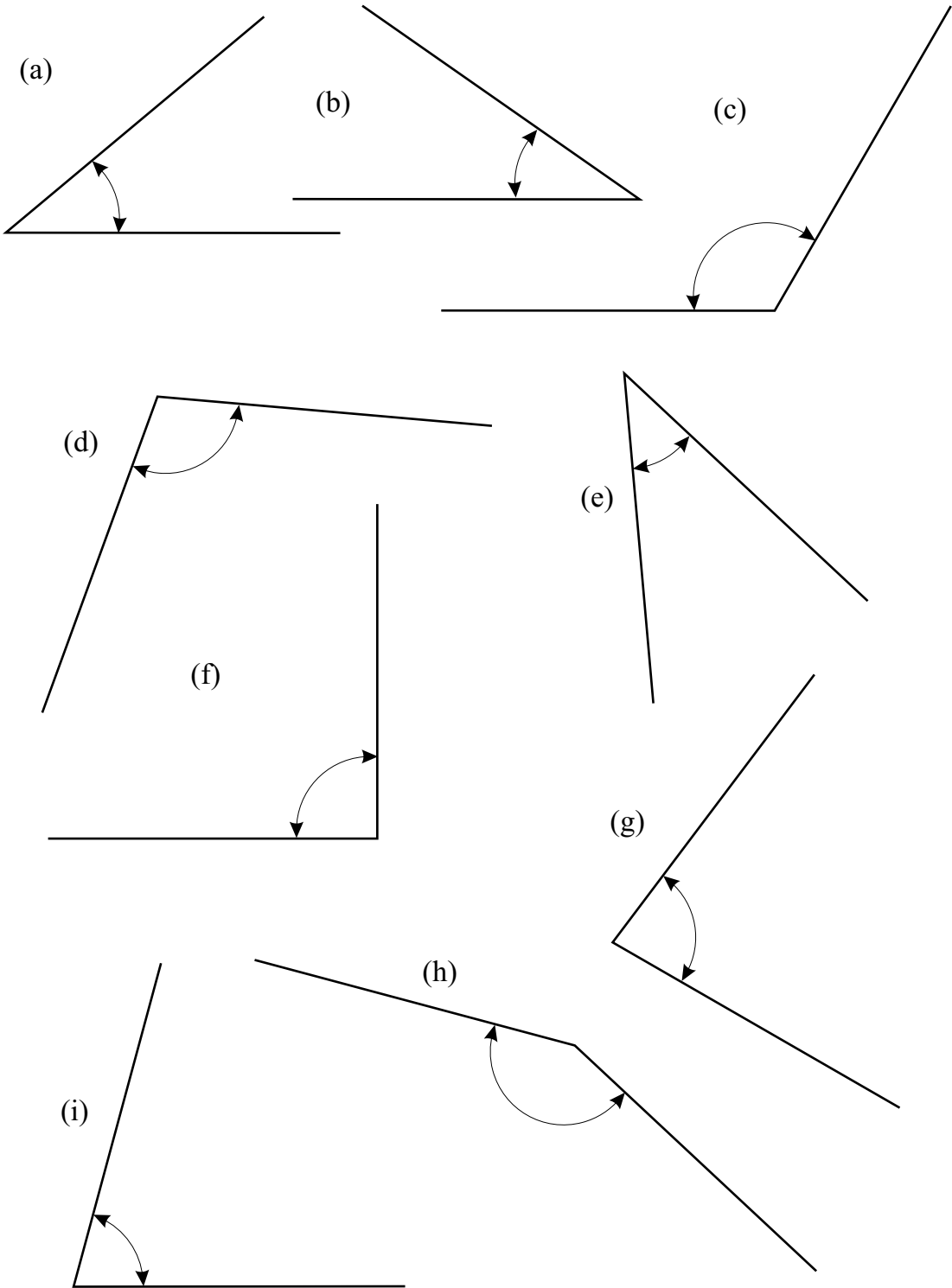


(n)





2. Use a protractor to measure the size of the angles below.

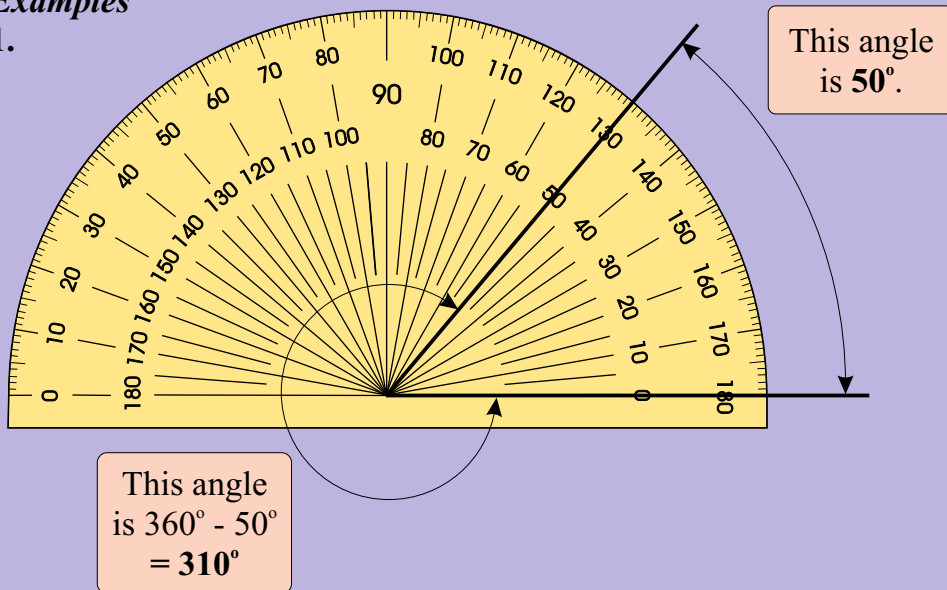


Measuring Angles Between 180° and 360°

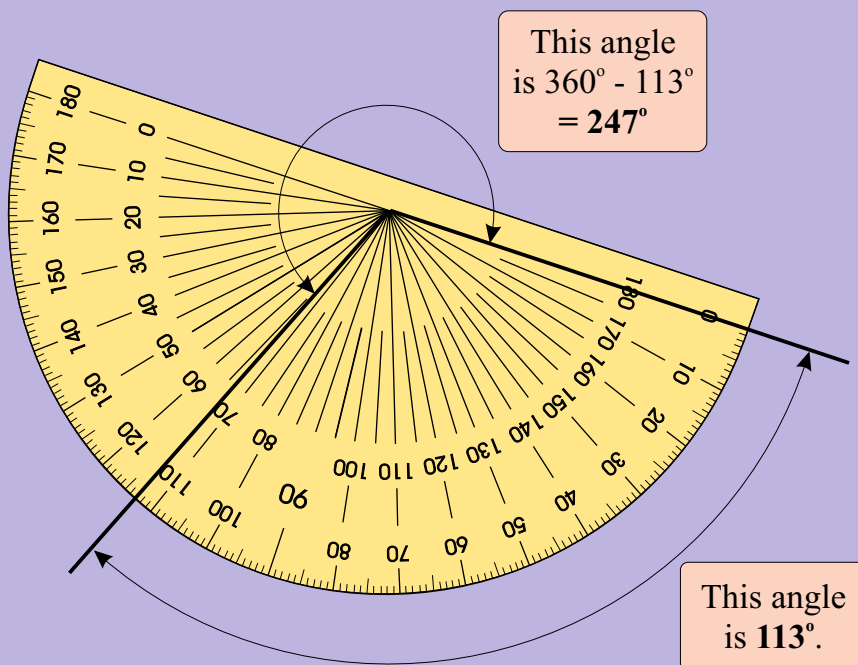
To find the larger angle formed by two joined lines first measure the smaller angle then subtract this from 360° .

Examples

1.



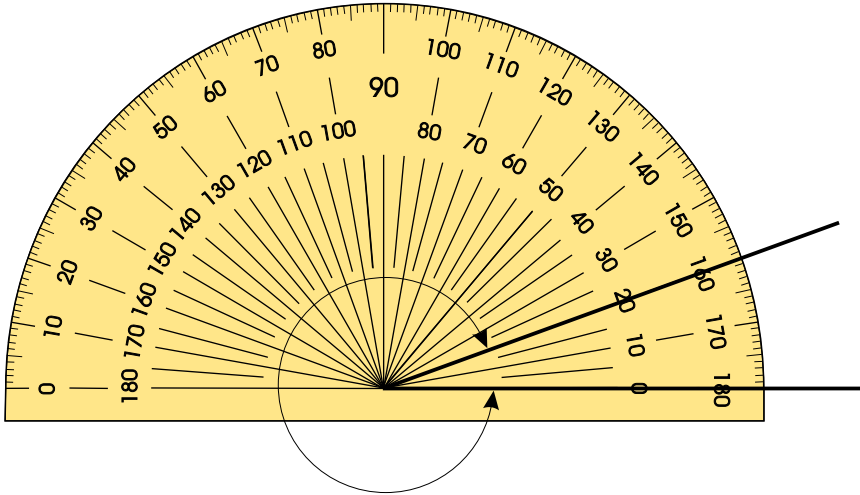
2.



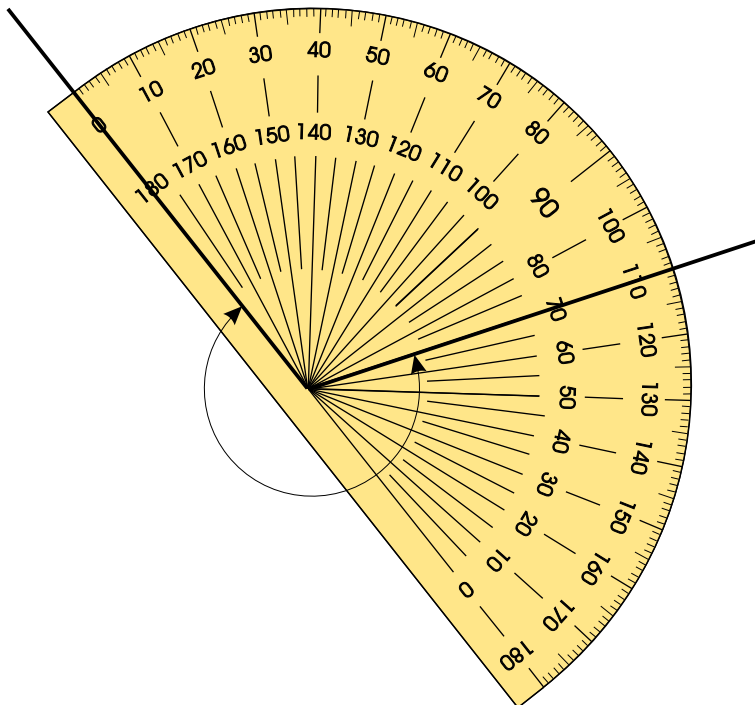
EXERCISE 21B

1. Read the size of the angles shown below.

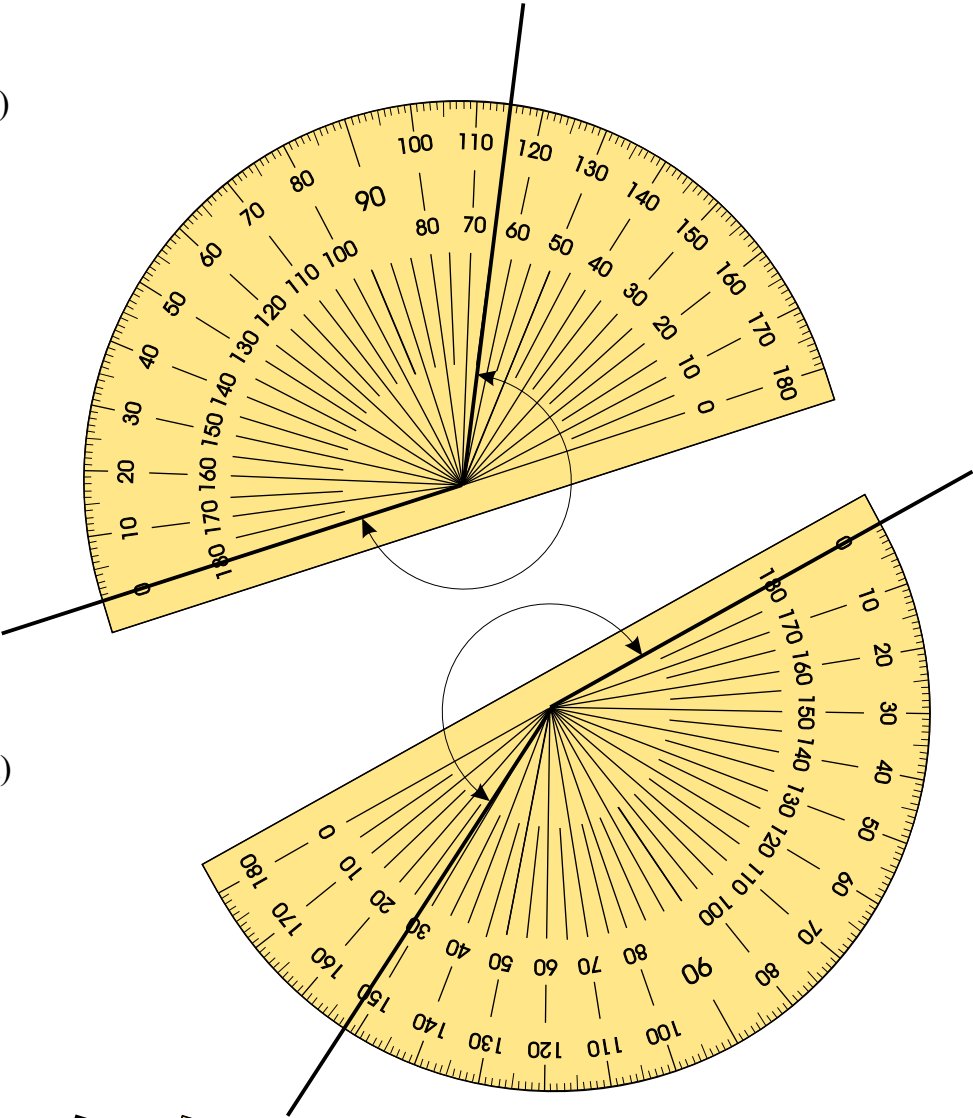
(a)



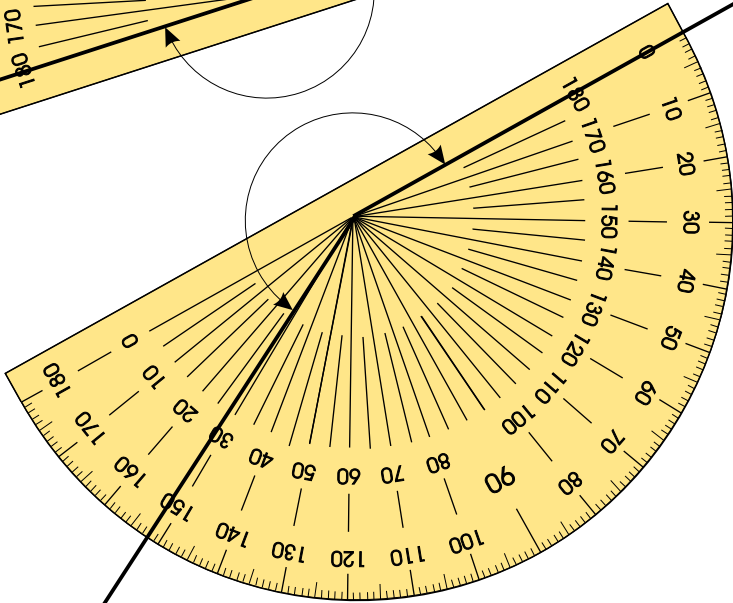
(b)



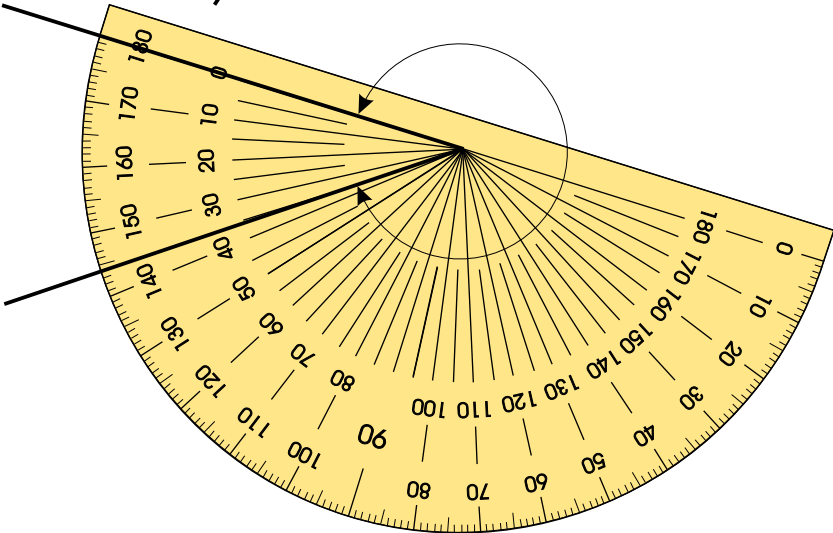
(c)



(d)

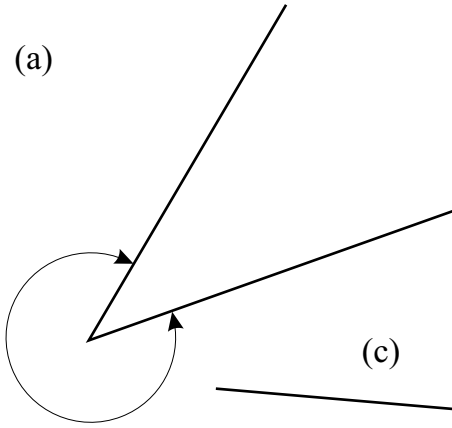


(e)

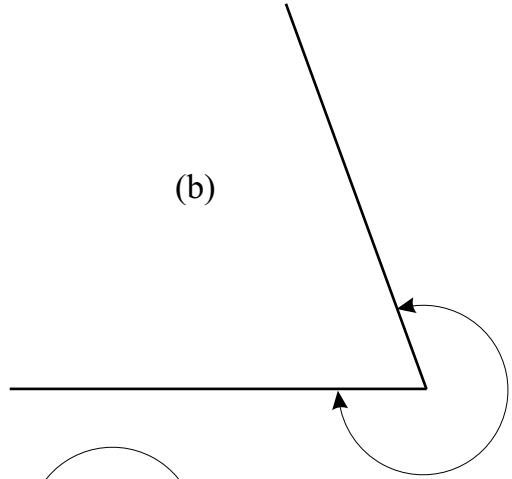


2. Use a protractor to measure the following angles.

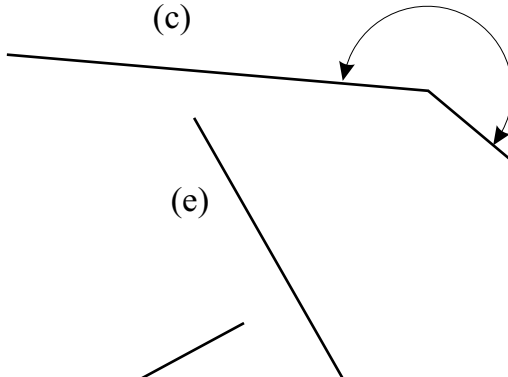
(a)



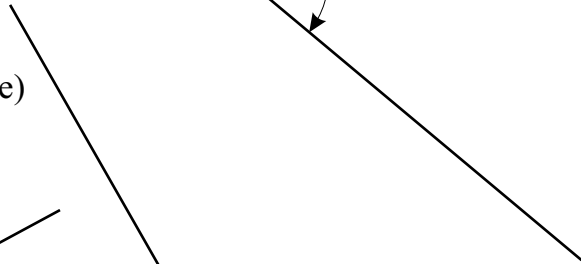
(b)



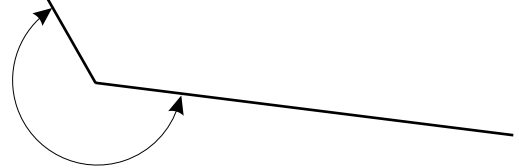
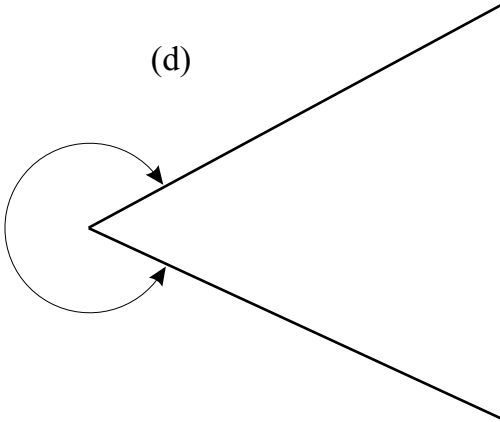
(c)



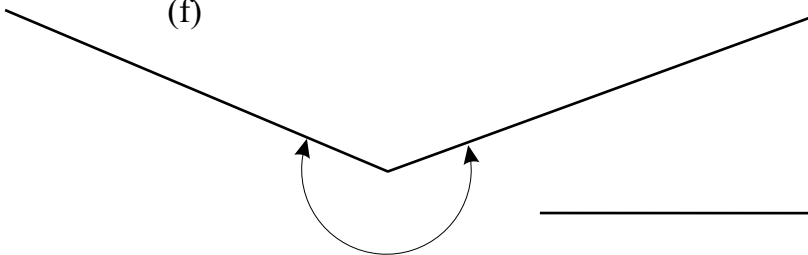
(e)



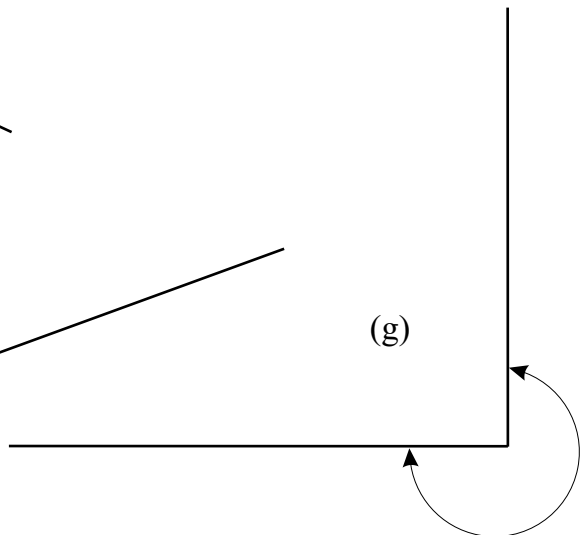
(d)



(f)



(g)



Estimating Angles

EXERCISE 21C

1. (a) Copy the following table into your books.

Angle	Estimate	Measurement	Difference
1.			
2.			
3.			
4.			
5.			
Total			

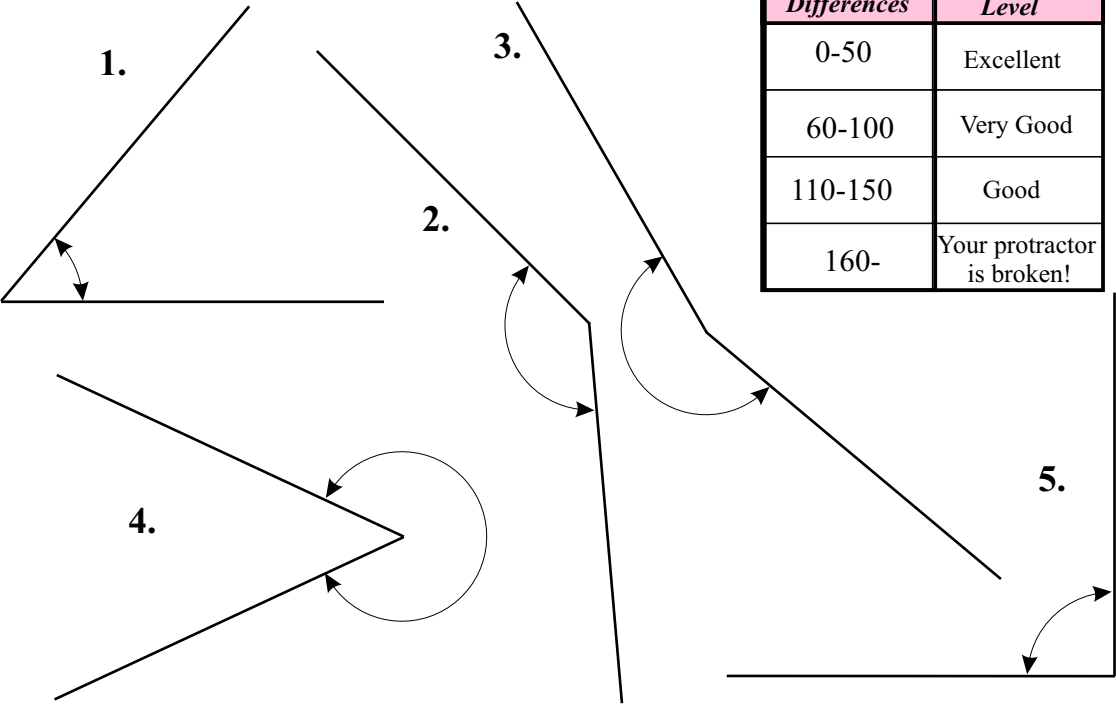
(b) Without using a protractor estimate the size of each of the angles below to the nearest 10°. Record your estimates in the table.

(c) Use a protractor to measure the size of the angles. Record the sizes in the table.

(d) Find the difference between your estimates and the size of the angles. Record the differences in the table.

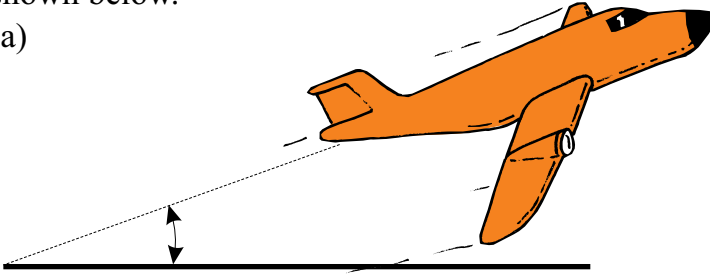
(e) Add all the differences and record the total. Use the table below to see your skill in estimating angles.

Total of Differences	Skill Level
0-50	Excellent
60-100	Very Good
110-150	Good
160-	Your protractor is broken!



2. Without using a protractor choose the best estimate of the angles shown below.

(a)



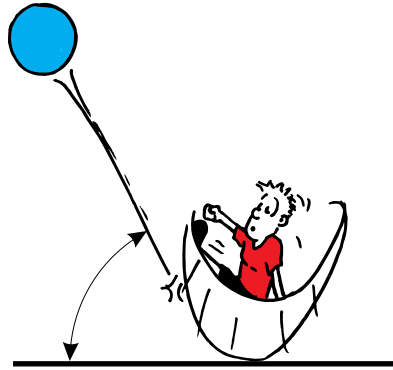
A 5° B 20°

C 40° D 65°

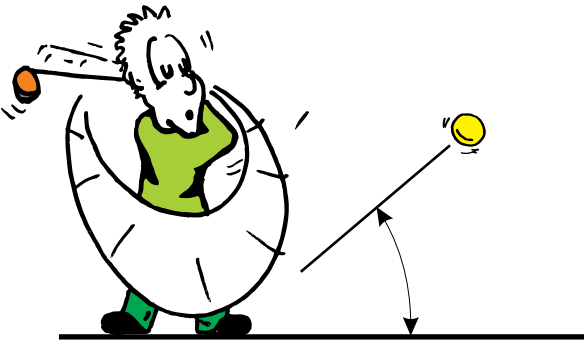
(b)

A 5° B 20°

C 40° D 65°



(c)



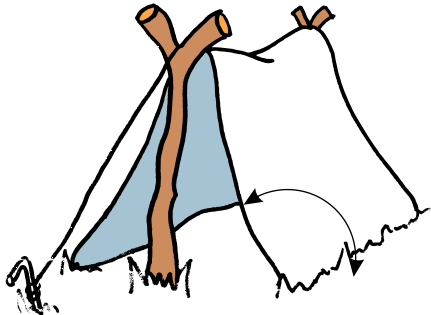
A 5° B 20°

C 40° D 65°

(d)

A 75° B 95°

C 115° D 135°



Types of Angles

Angles are given certain names depending on the size of the angle. The table below shows the names of angles and different size ranges.

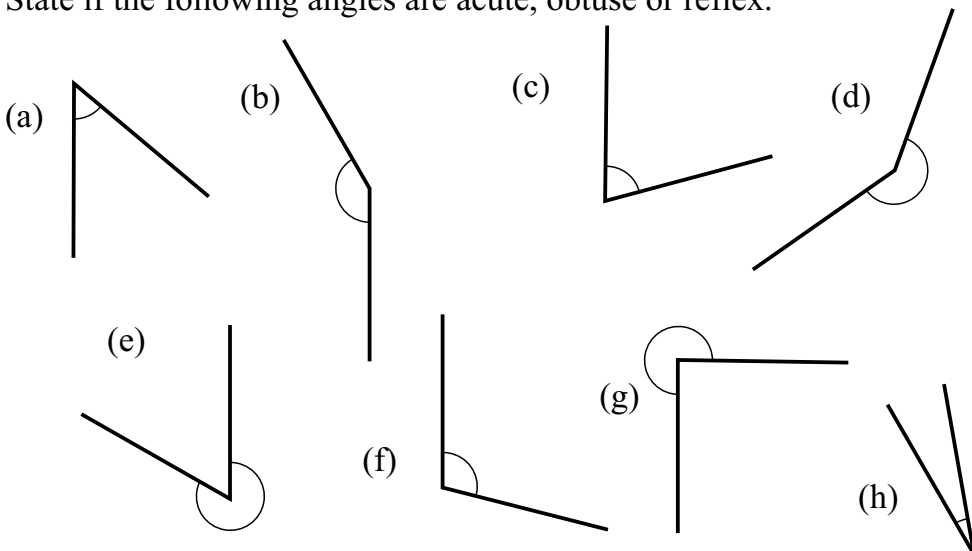
<i>Size of Angle</i>	<i>Name</i>
0 - 90°	Acute
90°	Right Angle
90° - 180°	Obtuse
180°	Straight Angle
180° - 360°	Reflex
360°	Revolution

A **Right Angle** (90°) is given this special symbol



EXERCISE 21D

1. State if the following angles are acute, obtuse or reflex.

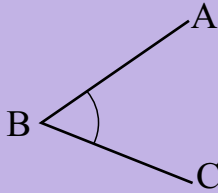


2. (a) How many right angles are in a straight angle?
- (b) How many right angles are in a revolution?

Labelling Angles

EXERCISE 21E

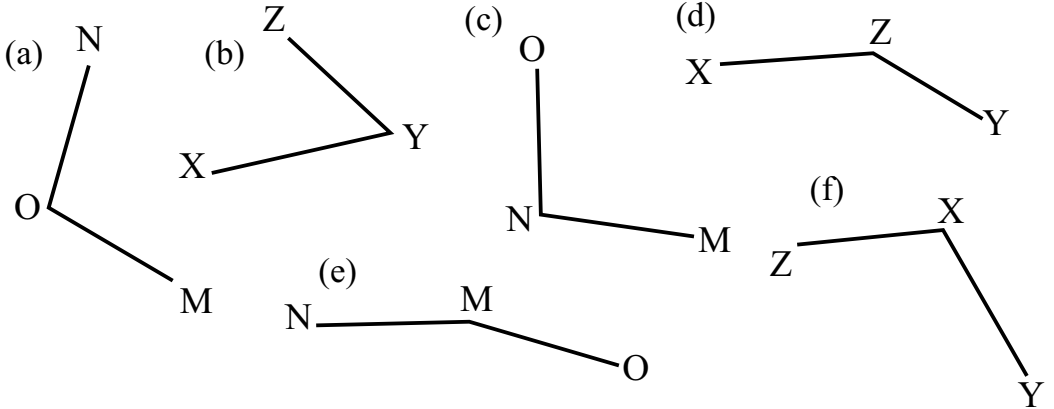
Example



This angle would be labelled as:

$\angle ABC$ or $\angle CBA$

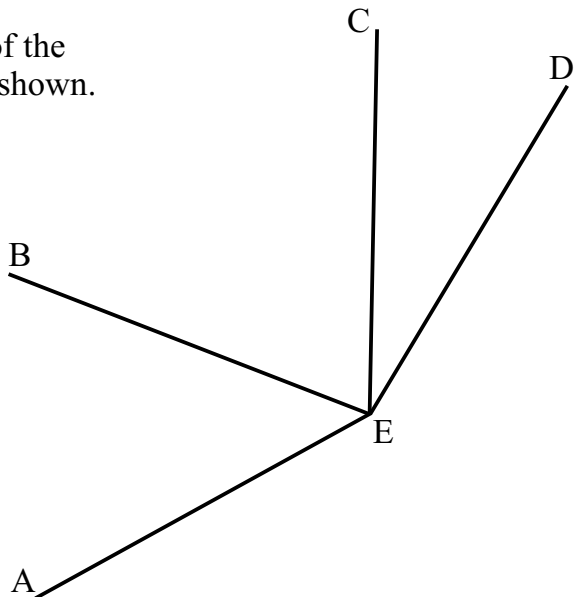
1. From the list below (**A - F**) find the correct label for each of these angles.



A $\angle XYZ$ **B** $\angle YXZ$ **C** $\angle XZY$ **D** $\angle MNO$ **E** $\angle NOM$ **F** $\angle OMN$

2. Use a protractor to find the size of the following angles on the diagram shown.

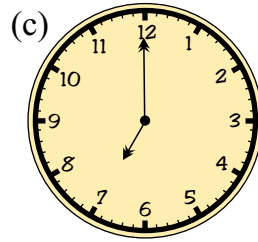
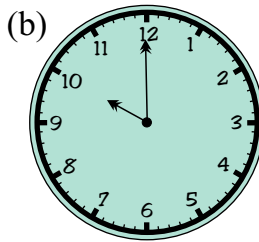
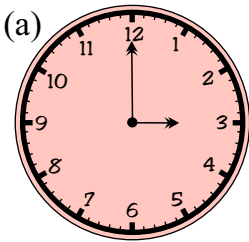
- (a) the acute angle $\angle AEB$
- (b) the acute angle $\angle CEB$
- (c) the obtuse angle $\angle AEC$
- (d) the reflex angle $\angle AED$
- (e) the reflex angle $\angle CEB$
- (f) the obtuse angle $\angle BED$



Finding Angles

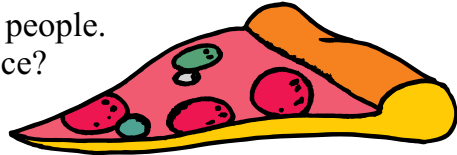
EXERCISE 21F

- How many degrees are in the following angles?
 - A full circle
 - Two circles
 - A third of a circle
 - Half a right angle
 - $\frac{3}{4}$ of a circle
 - $\frac{1}{10}$ of a circle
- Find the smaller angle between the hands of the following clocks.

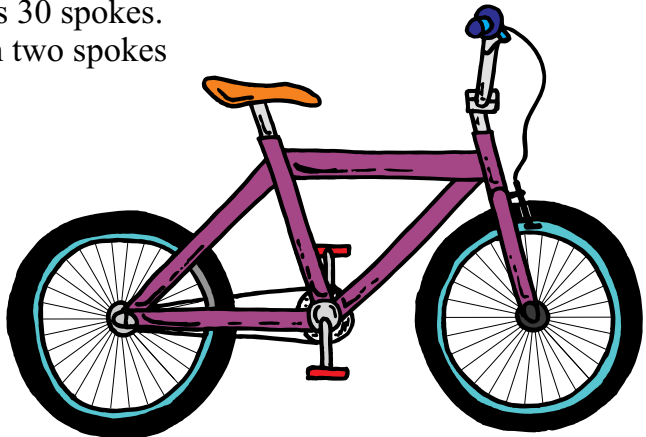


- What angle does the hour hand move through in two hours?
 - What angle does the minute hand move through in half an hour?
 - What angle does the minute hand move through in 40 minutes?
 - What is the reflex angle between the hour and minute hand at 1:30?

- A pizza is divided equally between 6 people.
What should be the angle of each piece?



- The wheel of a bicycle has 30 spokes.
What is the angle between two spokes next to each other?



Calculating Angles

EXERCISE 21G

Angles that add to 90° are called **complementary angles**.
Angles that add to 180° are called **supplementary angles**.

Examples

The following pairs of angles are **complementary**:

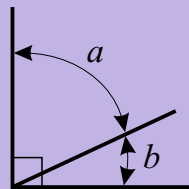
54° and 36° , 72° and 18° , 21° and 69° , 85° and 5°

The following pairs of angles are **supplementary**:

123° and 57° , 149° and 31° , 6° and 174° , 81° and 99°

- From the following list, find the pairs of complementary angles.
 51° 26° 8° 41° 18° 44° 82° 46° 64° 49° 39° 72°
- From the following list, find the pairs of supplementary angles.
 125° 135° 117° 63° 73° 154° 55° 26° 164° 107° 16° 45°
- Find the angle that is complementary to each of the following angles.
(a) 11° (b) 55° (c) 40° (d) 89° (e) 37°
- Find the angle that is supplementary to each of the following angles.
(a) 80° (b) 165° (c) 104° (d) 38° (e) 71°

If a **right angle** is divided into two angles these angles are called **adjacent** because they are next to each other.
They will be complementary because they add to 90° .



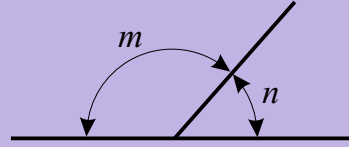
In the diagram shown here angles a and b are adjacent angles.
They are also complementary so:

$$a + b = 90^\circ$$

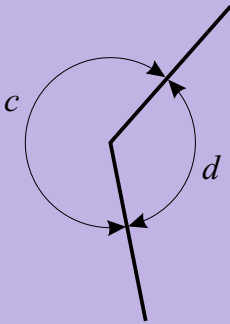
If a straight line is divided into two adjacent angles these will be supplementary.

In the diagram shown here angles m and n are supplementary so:

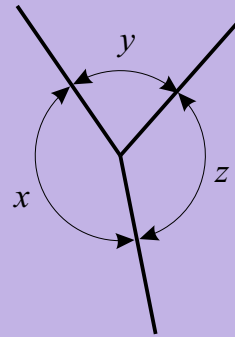
$$m + n = 180^\circ$$



If a circle is divided into adjacent angles these will add to 360° .



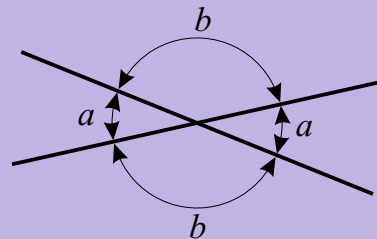
$$c + d = 360^\circ$$



$$x + y + z = 360^\circ$$

When two straight lines intersect, four angles are formed.

The angles opposite each other are called **vertically opposite** and are equal to each other.



In this diagram the two angles marked a are equal to each other and the two angles marked b are equal to each other.

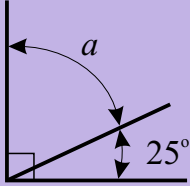
It can also be seen that a and b are supplementary so:

$$a + b = 180^\circ$$

Find the unknown angles in the following diagrams.

Examples

1.



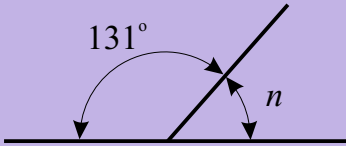
These two angles will add to 90° .

$$a + 25 = 90$$

$$a = 90 - 25$$

$$a = 65^\circ$$

2.



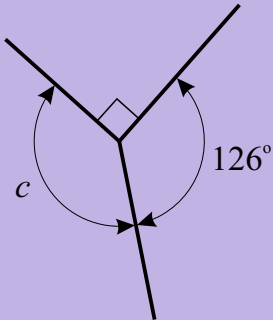
These two angles will add to 180° .

$$n + 131 = 180$$

$$n = 180 - 131$$

$$n = 49^\circ$$

3.



These three angles will add to 360° .

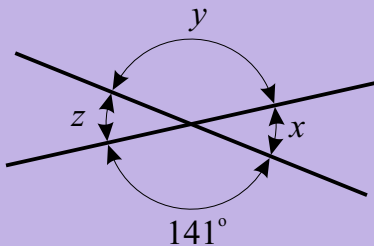
$$c + 90 + 126 = 360$$

$$c + 216 = 360$$

$$c = 360 - 216$$

$$c = 144^\circ$$

4.



$$y = 141^\circ \text{ (vertically opposite)}$$

$$x + y = 180^\circ$$

$$x + 141 = 180$$

$$x = 180 - 141$$

$$x = 37^\circ$$

$$z = x \text{ (vertically opposite)}$$

$$z = 37^\circ$$

EXERCISE 21H

Find the unknown angles in the following diagrams.

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

14.

12.

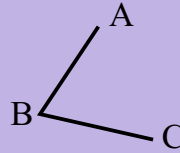
13.

14.

Terms

The **vertex** is the point where two lines meet.

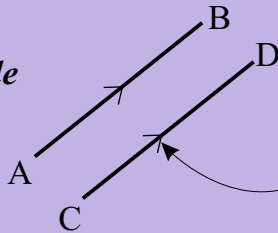
Example



Point B is the **vertex**.

Parallel lines are lines that never meet.

Example

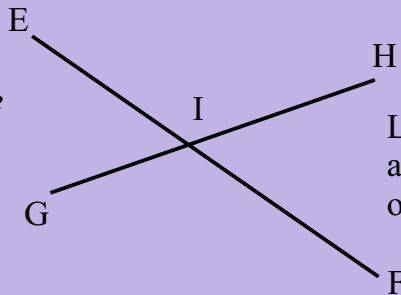


Lines AB and CD are parallel.

These symbols indicate parallel lines.

The **point of intersection** is where lines cross or **intersect**.

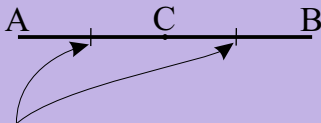
Example



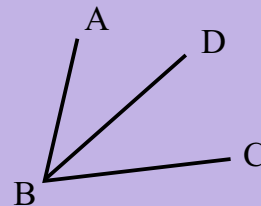
Lines EF and GH intersect at point I. This is the point of intersection.

To **bisect** a line or angle means to divide it into two equal parts.

Examples



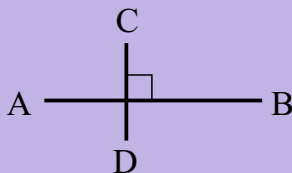
These marks indicate that line AB has been divided into two equal parts. Point C has bisected line AB. So $AC = CB$



If line BD bisects $\angle ABC$ then $\angle ABD = \angle DBC$.

If the angle between two lines is 90° the lines are called **perpendicular**.

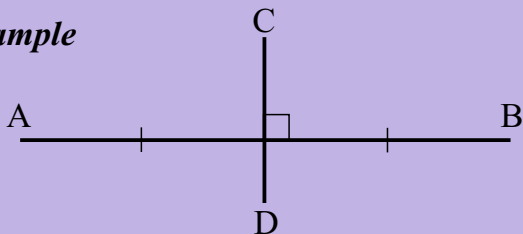
Example



Lines AB and CD are perpendicular.

The **perpendicular bisector** is a line that is perpendicular to another line and divides that line into two equal parts.

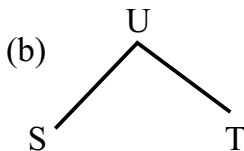
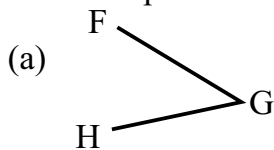
Example



Line CD is a **perpendicular bisector** of line AB.

EXERCISE 21I

1. Which point is the vertex in these angles?



(c) $\angle JKL$

(d) $\angle QSR$

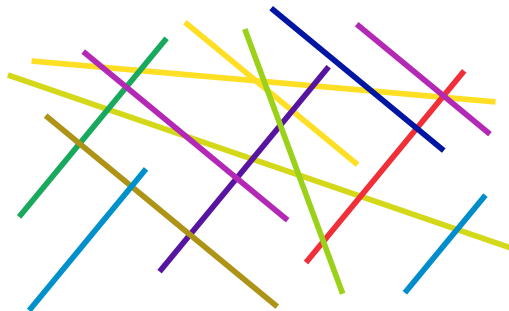
2. Rearrange the letters **VET CRIES** to form the word that is the plural of **VERTEX** (more than one vertex).

3. Rearrange the letters **CAT LIVER** to find a word that describes an object that is perpendicular to a **HORIZONTAL** object.

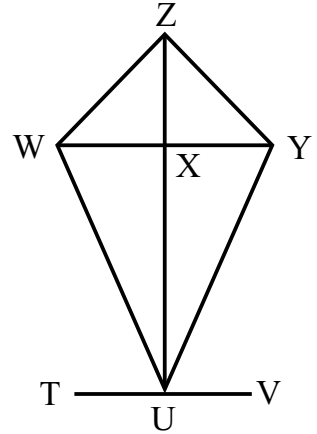
4. (a) How many lines are **parallel** to the red line?

(b) How many lines are **perpendicular** to the red line?

(c) How many lines **intersect** the red line?



5. From this diagram choose the correct answer for each of the following statements.



(a) Which line is parallel to WY?

- A** UZ **B** TV
C WU **D** WZ

(b) Which line is perpendicular to WY?

- A** UZ **B** TV
C WU **D** WZ

(c) Which line bisects WY?

- A** ZW **B** TV **C** WU **D** ZU

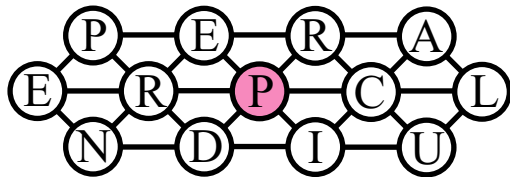
(d) Which line bisects $\angle WUY$?

- A** UZ **B** TV **C** WU **D** WY

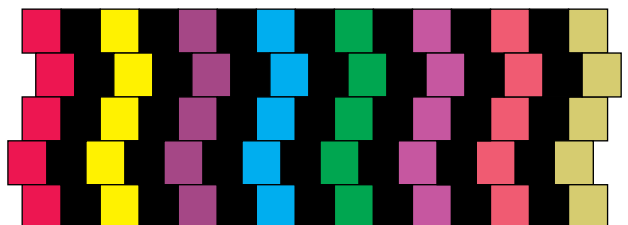
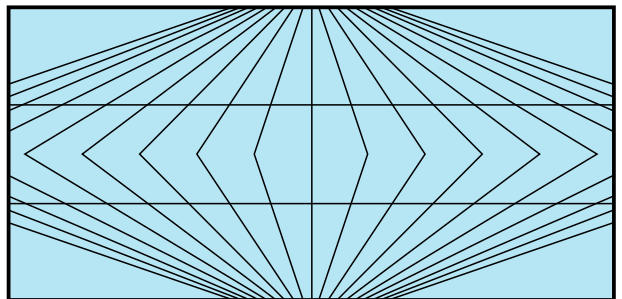
(e) Which is the point of intersection of lines UZ and WY?

- A** W **B** X **C** Y **D** Z

6. Starting with the shaded letter, follow the lines and use all the letters to spell a word from this exercise.



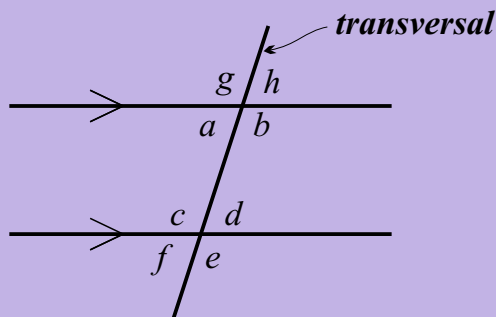
7. Are the horizontal lines in these diagrams parallel?



Parallel Lines

Definitions

If two parallel lines are crossed by a line, this line is called a **transversal**. The angles formed by the transversal are given special names and are related.



a and d are called **alternate angles** and are equal.

b and c are also alternate angles.

b and e are called **corresponding angles** and are equal.

Other pairs of corresponding angles are a and f , c and g , d and h .

a and c are called **allied** (or **co-interior**) **angles** and add to 180° .

b and d are also allied angles.

It can also be seen that there are four pairs of vertically opposite angles.

$$g = b, a = h, c = e, d = f$$

Using this information it can be seen that there are only two different angles formed when a transversal crosses parallel lines and that these angles are supplementary (add to 180°).

$$a = h = d = f$$

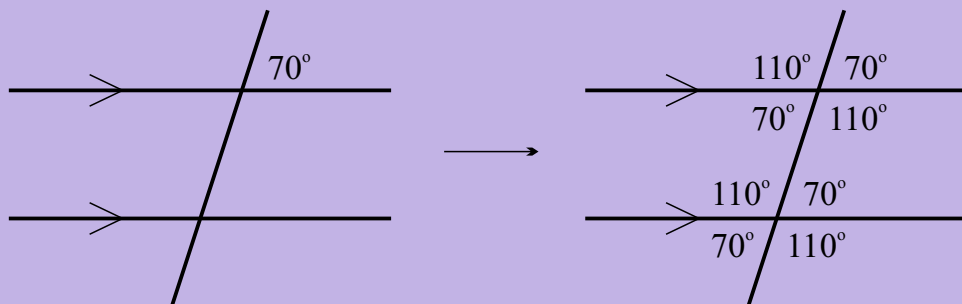
$$b = g = c = e$$

$$a + b = 180^\circ$$

Only one angle is needed to find all the angles formed between a transversal and parallel lines.

Example

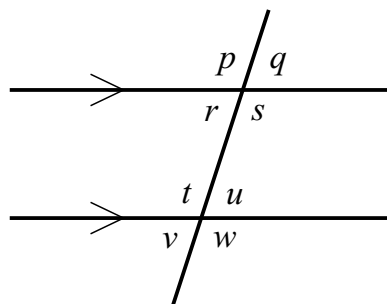
Find all the unknown angles between the transversal and the parallel lines in this diagram.



EXERCISE 21J

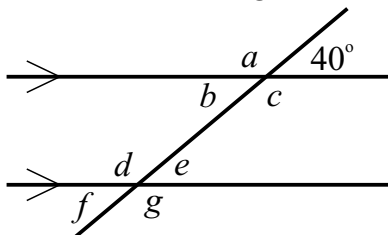
1. From this diagram find the following:

- Two pairs of **alternate angles**.
- Two pairs of **corresponding angles**.
- Two pairs of **allied angles**.
- Two pairs of **vertically opposite angles**.

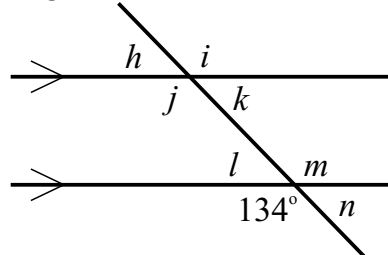


2. Find the unknown angles in the following diagrams.

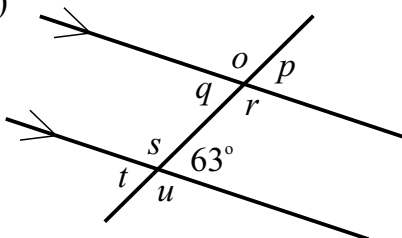
(a)



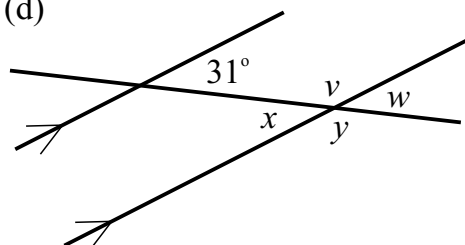
(b)



(c)



(d)

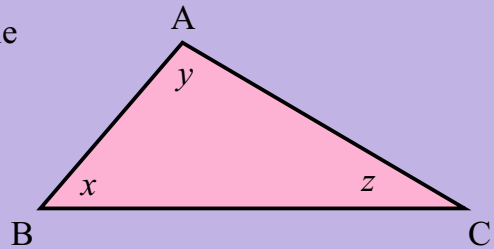


Angles in a Triangle

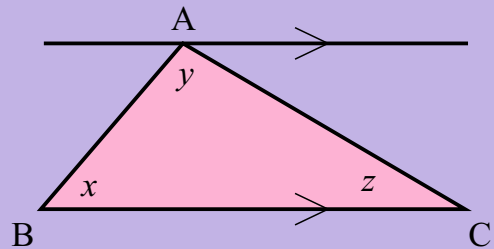
Knowledge of the angles formed by a transversal and two parallel lines can be used to find the sum of the angles in a triangle.

The sum of the angles in the triangle shown here is:

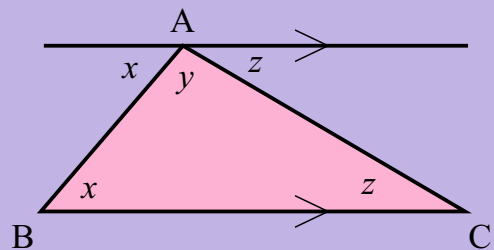
$$x + y + z$$



Step 1 Draw a line parallel to line BC that passes through A.



Step 2 Using lines AB and AC as transversals of these parallel lines note the angles that are alternate angles to x and z .



Step 3 Note that at A:

$x + y + z$ will equal a straight angle (180°)

$$x + y + z = 180^\circ$$

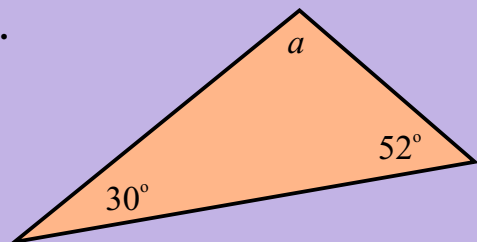
The sum of the angles in a triangle = 180°

Examples

Find the unknown angles in the following triangles.

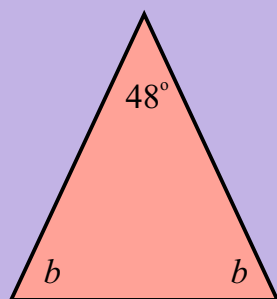
Remember: The sum of the three angles in a triangle is 180° .

1.



$$\begin{aligned} a + 30 + 52 &= 180 \\ a + 82 &= 180 \\ a &= 180 - 82 \\ a &= 98^\circ \end{aligned}$$

2.

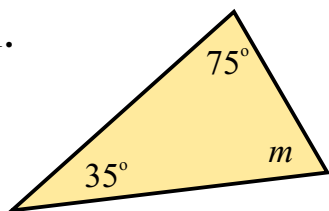


$$\begin{aligned} b + b + 48 &= 180 \\ 2b + 48 &= 180 \\ 2b &= 180 - 48 \\ 2b &= 132 \\ b &= 66^\circ \end{aligned}$$

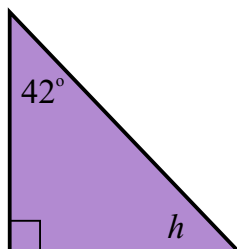
EXERCISE 21K

Find the unknown angles in the following triangles.

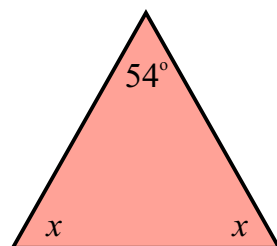
1.



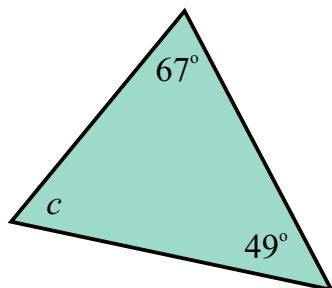
2.



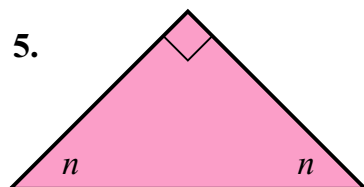
3.



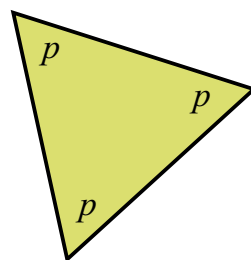
4.



5.

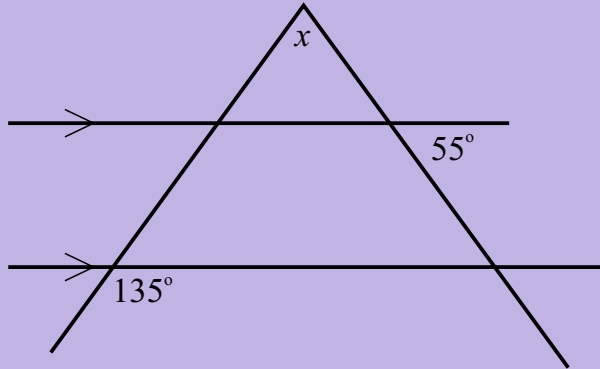


6.



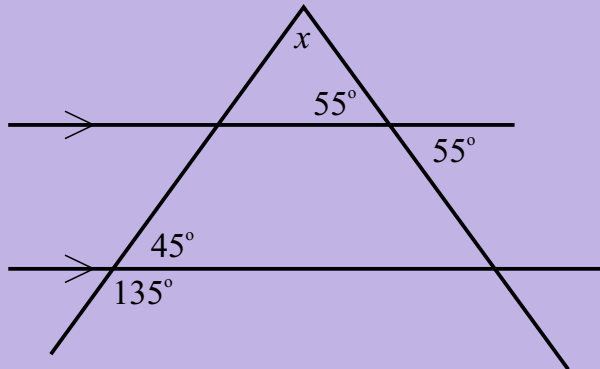
To find an unknown angle it is sometimes necessary to find other angles first.

Example

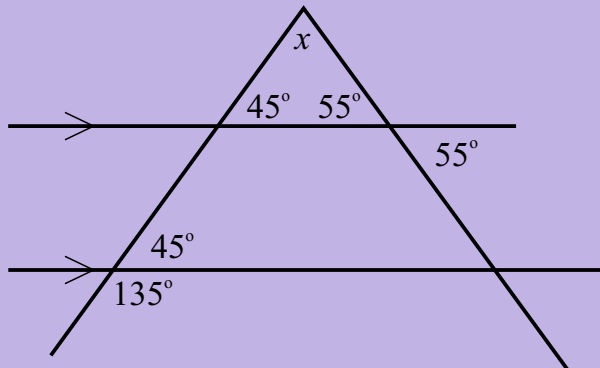


To find x other angles need to be found first.

Step 1



Step 2



Step 3

The sum of the angles in the triangle is 180°

$$x + 45 + 55 = 180$$

$$x + 100 = 180$$

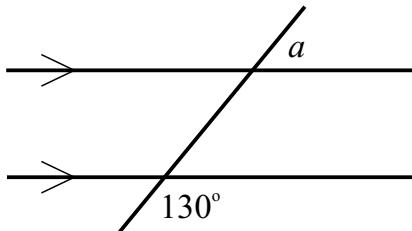
$$x = 180 - 100$$

$$x = 80^\circ$$

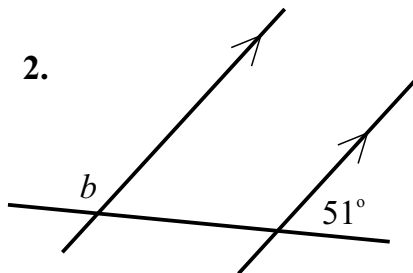
EXERCISE 21L

Find the unknown angles in the following diagrams.

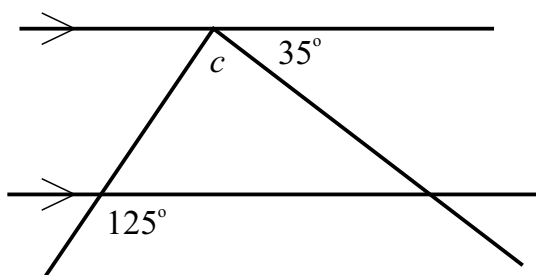
1.



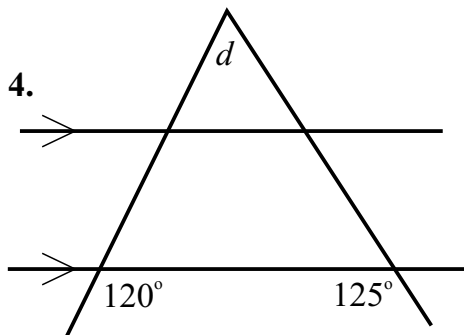
2.



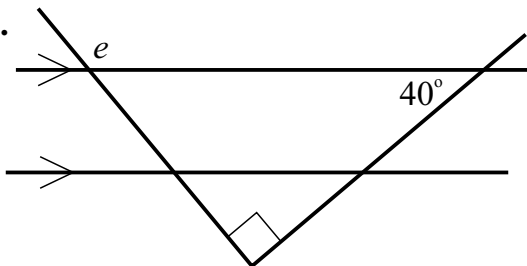
3.



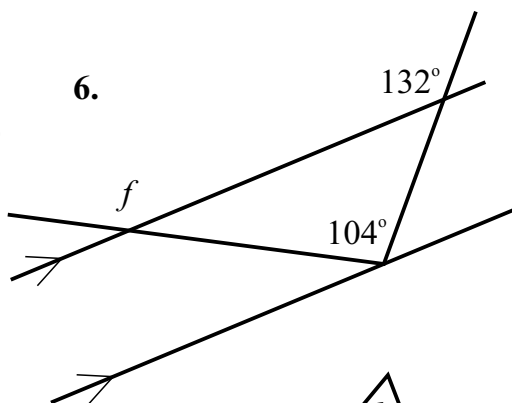
4.



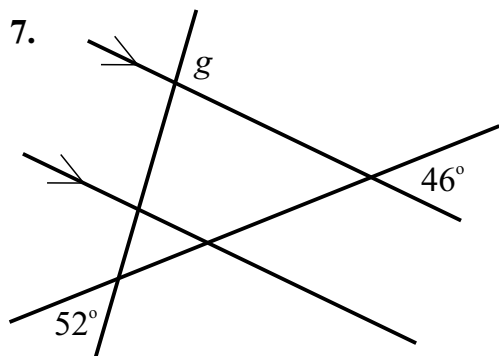
5.



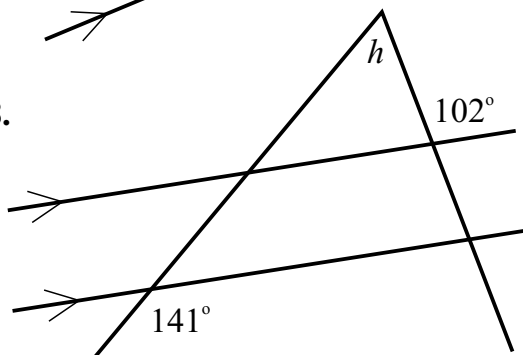
6.



7.



8.



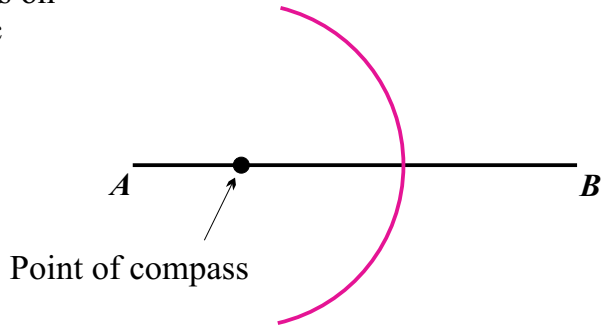
Constructions

1. Constructing a Perpendicular Line

A line perpendicular to a given line can be constructed using a compass.

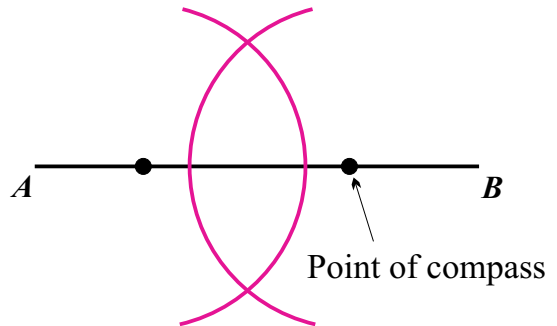
Step 1

Place the point of a compass on the line AB and draw an arc as shown here.



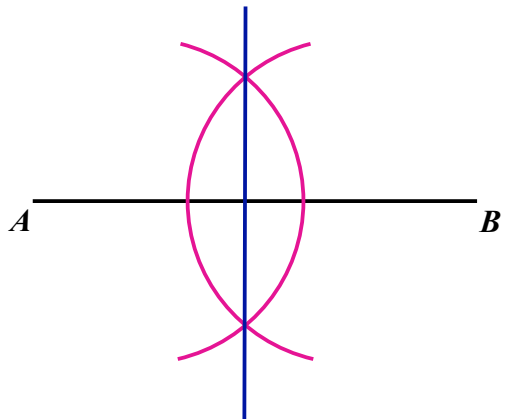
Step 2

Place the point of a compass on another spot and draw an arc as shown here.



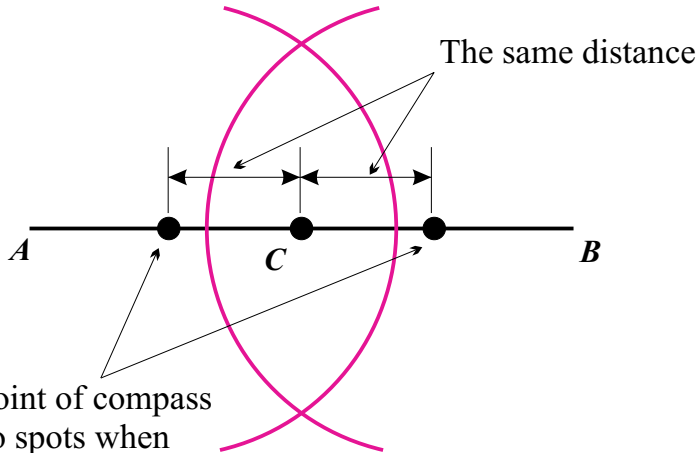
Step 3

Draw a line that connects the points where these two arcs intersect. This line will be perpendicular to line AB .



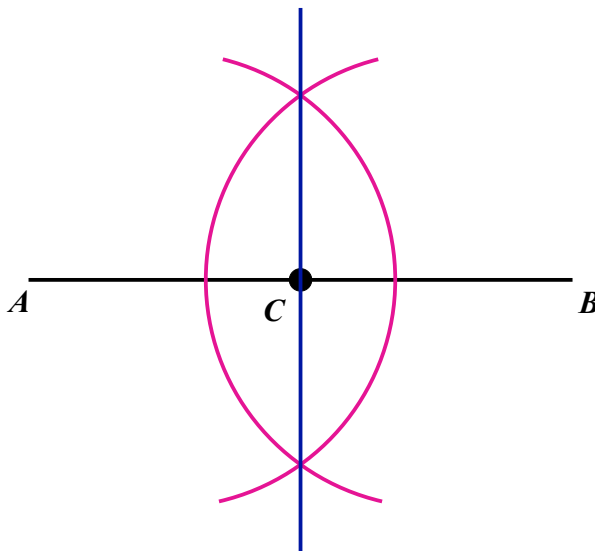
2. Constructing a Perpendicular Line Through a Given Point

Step 1 To draw a line perpendicular to line AB through point C measure an equal distance either side of point C to locate the compass when drawing the arcs.



Place the point of compass at these two spots when drawing the arcs.

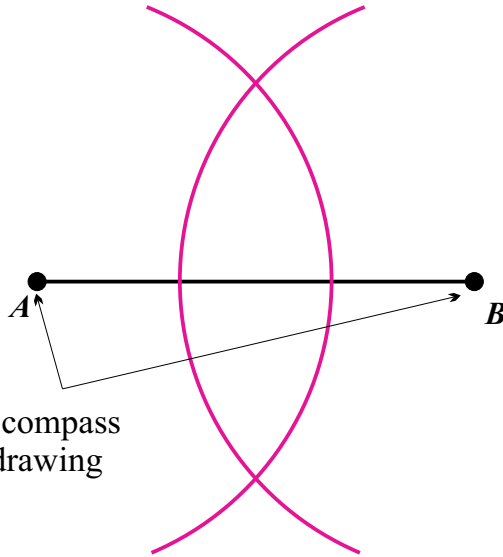
Step 2 The line connecting the points where the two arcs intersect will be perpendicular to line AB and will pass through point C .



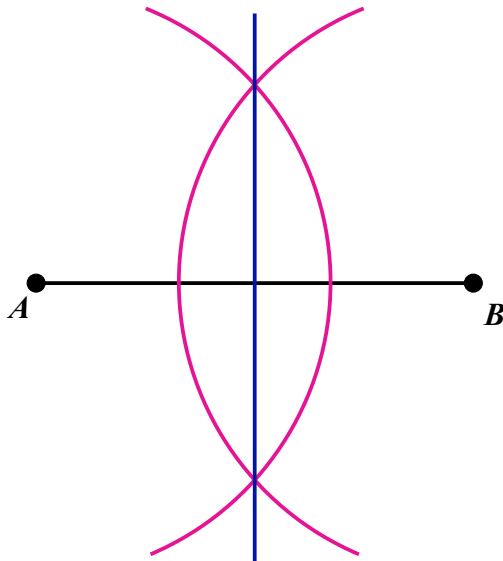
3. Constructing a Perpendicular Bisector of a Line

A perpendicular bisector of a line is a perpendicular line that divides the other line into two equal parts.

Step 1 To draw a perpendicular bisector of line AB locate the compass points at A and B when drawing the arcs.

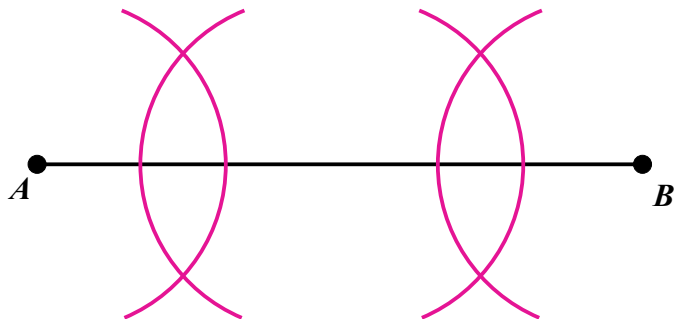


Step 2 The line connecting the points where the two arcs intersect will be a perpendicular bisector of AB .

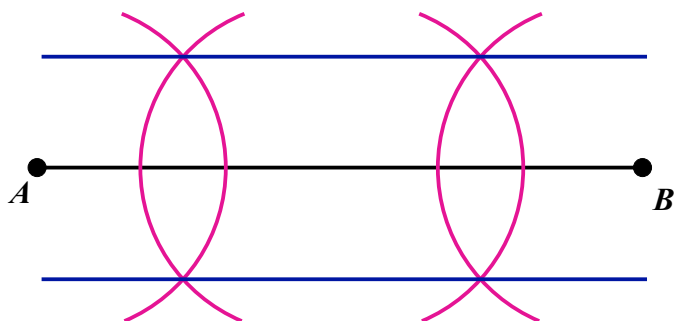


4. Constructing Parallel Lines

Step 1 To draw lines parallel to AB draw two pairs of arcs.



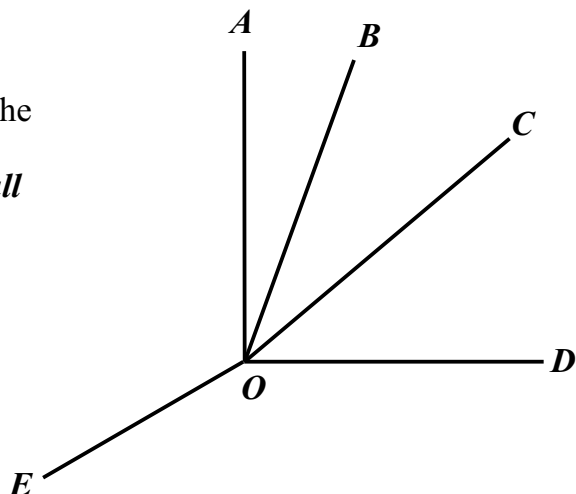
Step 2 The lines connecting the intersection of these arcs will be parallel to AB .



PROBLEM SOLVING

1. Use a protractor to measure the angles in this diagram. Label, and state the size of, *all* the angles in the diagram.

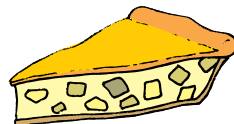
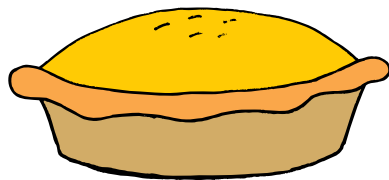
(Hint-there are more than five angles)



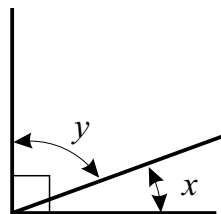
2. Four friends have an apple pie to share. They decide to divide the pie in the following way:

Alex will get twice what Xavier gets.
Alex and Amelia get the same amount.
Meredith gets the same as Xavier.

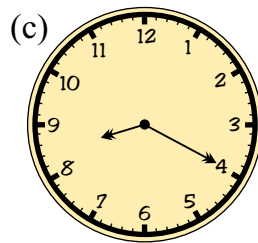
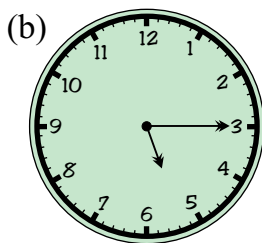
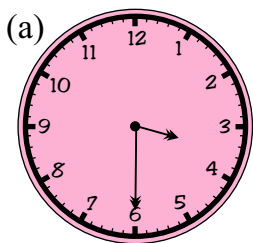
What should be the angle at the centre of the pie for each piece, if the whole pie is to be divided between the four friends?



3. Angle y is *four times* the size of angle x . Find x and y .



4. Find the angle between the hands of each of these clocks.



PUZZLES

1. Using the letters shown, form as many words as you can. The letters can only be used once and all nine letters form a word found in this chapter.

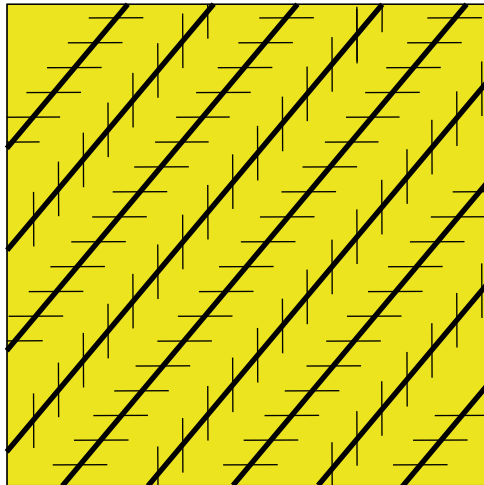
Your targets: 5 words - good
10 words - very good
15 words - excellent

E	N	R
I	E	T
S	T	C

2. The letters shown form five 6-letter words found in this chapter. The beginning, middle and end of the words have been mixed up.
Find the five words.

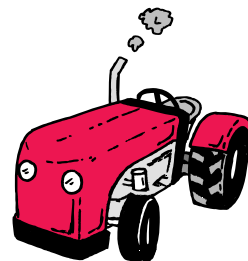
DE	TU	EE
OB	SE	EX
VE	FL	SE
BI	GR	EX
RE	RT	CT

3. Are the diagonal lines in this diagram parallel?



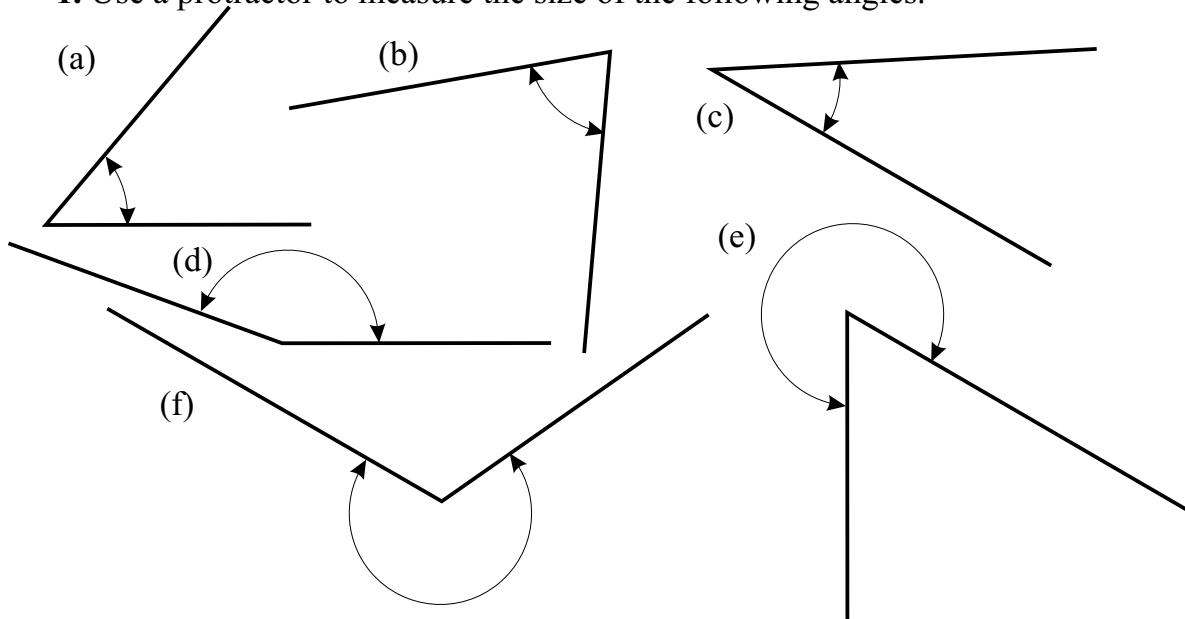
4. Solve this puzzle to find a word from this chapter.

PRO

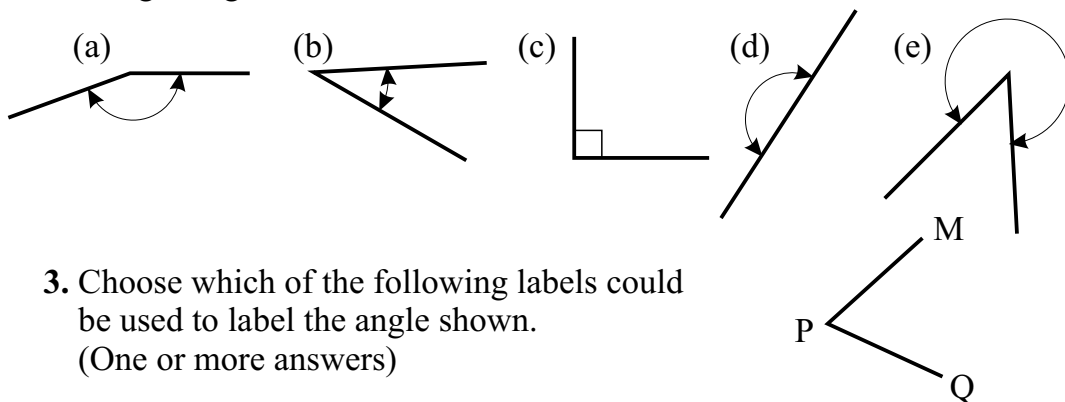


CHAPTER REVIEW

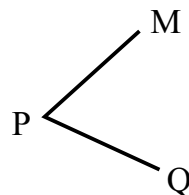
1. Use a protractor to measure the size of the following angles.



2. State if the following angles are acute, obtuse, reflex, right or straight angles.



3. Choose which of the following labels could be used to label the angle shown.
(One or more answers)



A $\angle MPQ$ **B** $\angle PMQ$ **C** $\angle PQM$ **D** $\angle MQP$ **E** $\angle QPM$ **F** $\angle QMP$

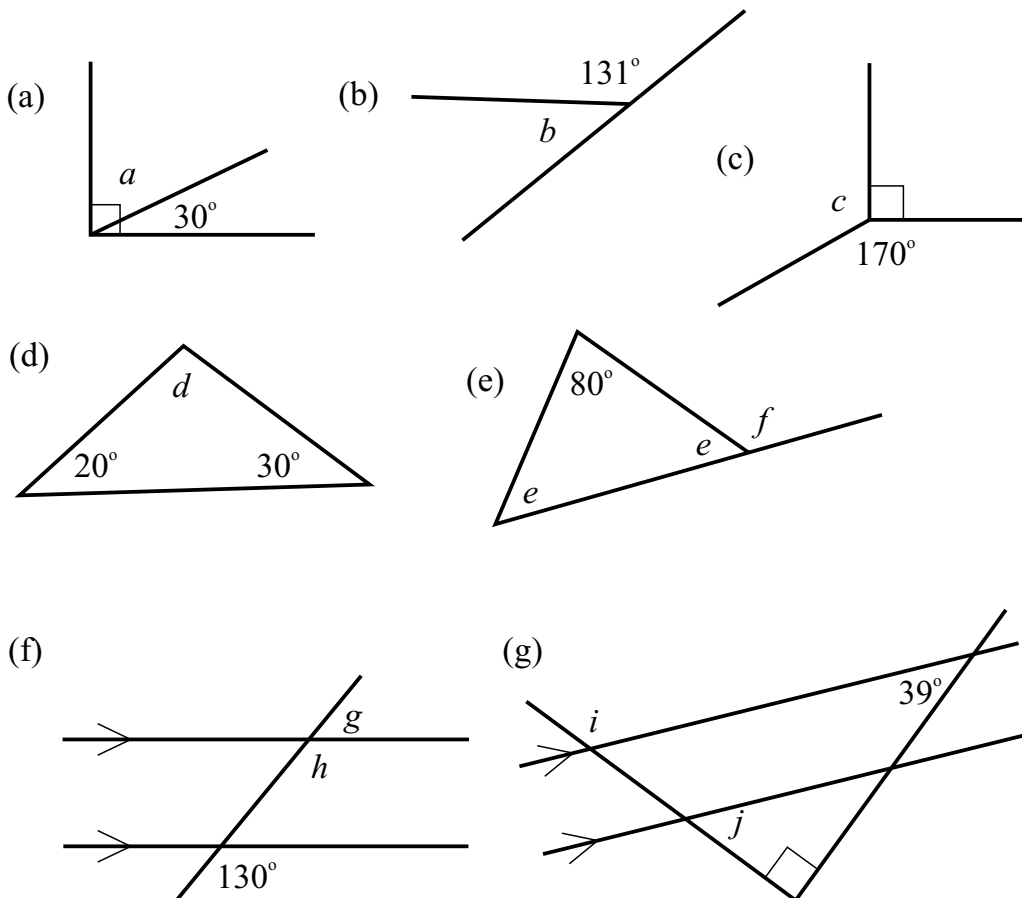
4. How many degrees are in the following angles?

- (a) A right angle (b) Half a circle
(c) One third of a straight angle (d) 3 circles

5. Find the angle between the hands of a clock for the following times.

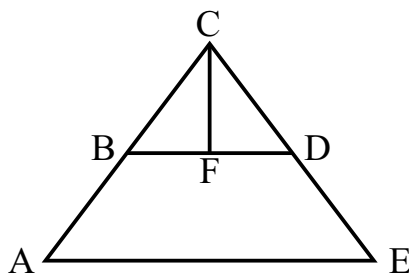
- (a) 8 o'clock (b) 5:30

6. Find the unknown angles.



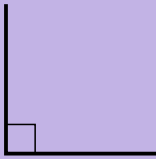
7. A triangle has two angles that are the same size and each of these angles is half the size of the other angle.
Find the sizes of the three angles in this triangle.

8. (a) Which line is parallel to BD?
 (b) Which line is perpendicular to BD?
 (c) Which two lines form $\angle CAE$?
 (d) What angle is formed by the lines BC and CD?
 (e) Which line bisects this angle?
 (f) At which point do lines AC and BD intersect?



Polygons

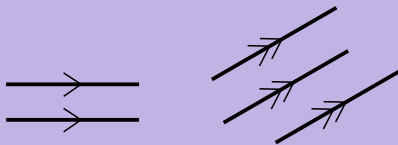
Symbols



A **right angle**.

The angle between these lines is 90° .

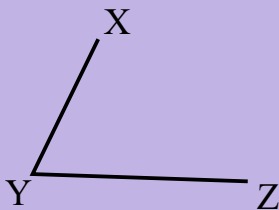
These lines are **perpendicular**.



Sets of **parallel** lines

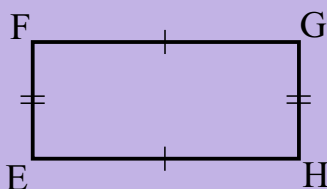


'Line AB' or $\overline{AB}$



Angle XYZ or $\angle XYZ$

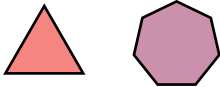
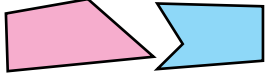
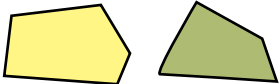
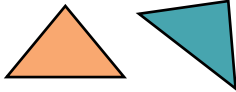
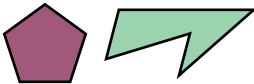
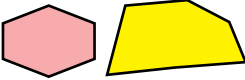
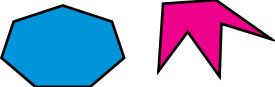
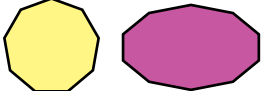
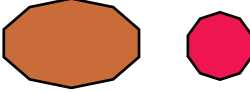
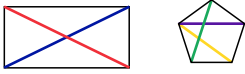
Point Y is the **vertex**.



The length of $\overline{EF}$ = length of $\overline{GH}$

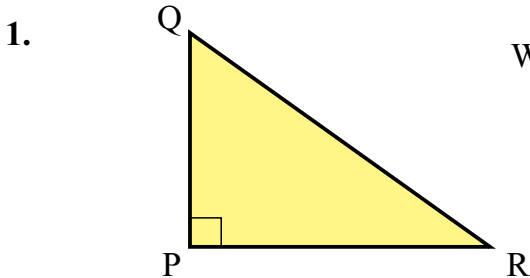
The length of $\overline{FG}$ = length of $\overline{EH}$

Definitions

<i>Name</i>	<i>Description</i>	<i>Examples</i>
<i>Polygon</i>	A <i>closed</i> figure with <i>straight</i> sides	
<i>Regular Polygon</i>	A polygon with <i>all</i> sides and angles the same size	
<i>Irregular Polygon</i>	A polygon with sides and angles that are not all equal	
<i>Convex Polygon</i>	A polygon with all vertex angles less than 180°	
<i>Concave Polygon</i>	A polygon with at least one vertex angle greater than 180°	
<i>Triangle</i>	A three-sided polygon	
<i>Quadrilateral</i>	A four-sided polygon	
<i>Pentagon</i>	A five-sided polygon	
<i>Hexagon</i>	A six-sided polygon	
<i>Heptagon</i>	A seven-sided polygon	
<i>Octagon</i>	An eight-sided polygon	
<i>Nonagon</i>	A nine-sided polygon	
<i>Decagon</i>	A ten-sided polygon	
<i>Diagonal</i>	A line joining vertices across a polygon	

EXERCISE 22A

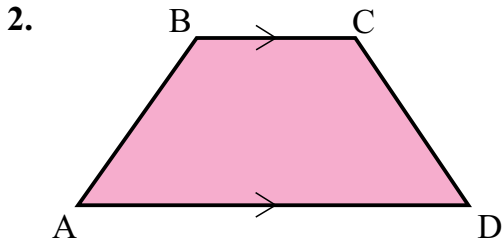
Choose the correct answer for the following questions.



Which angle is a right angle?

A $\angle PQR$ **B** $\angle QRP$

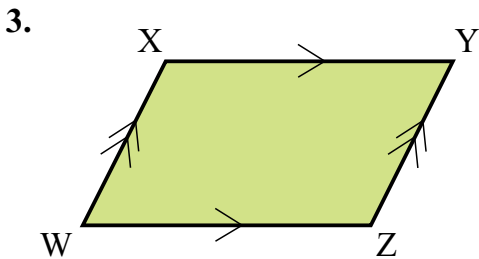
C $\angle RPQ$ **D** $\angle PRQ$



Which pair of lines are parallel?

A $\overline{AB}$ and $\overline{BC}$ **B** $\overline{BC}$ and $\overline{CD}$

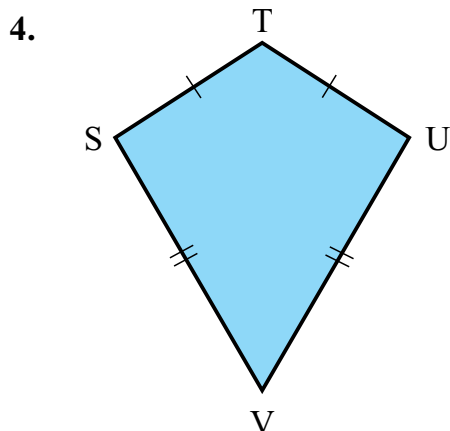
C $\overline{AD}$ and $\overline{DC}$ **D** $\overline{AD}$ and $\overline{BC}$



Which line is parallel to $\overline{WX}$?

A $\overline{WY}$ **B** $\overline{ZY}$

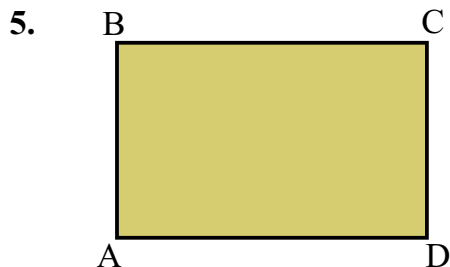
C $\overline{WZ}$ **D** $\overline{XY}$



Which line is the same length as $\overline{ST}$?

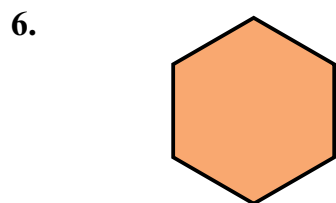
A $\overline{TU}$ **B** $\overline{TV}$

C $\overline{VS}$ **D** $\overline{SU}$



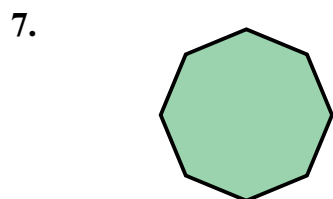
Which of the following lines would both be diagonals of this shape?

- A** $\overline{AB}$ and $\overline{AC}$ **B** $\overline{AC}$ and $\overline{AD}$
C $\overline{CD}$ and $\overline{CB}$ **D** $\overline{CA}$ and $\overline{DB}$



How many diagonals could be drawn in this shape?

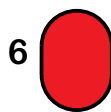
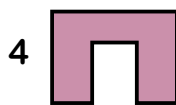
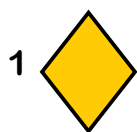
- A** 3 **B** 6 **C** 9 **D** 18



How many *pairs* of parallel lines are in a *regular* octagon?

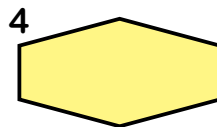
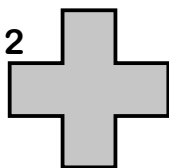
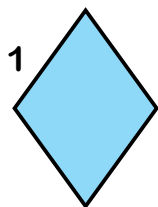
- A** 2 **B** 3 **C** 4 **D** 8

8. Which of these shapes are polygons?



- A** 1,4,5 **B** 1,3,4,5 **C** 1,2,4,5,6 **D** All of the shapes

9. Which of these shapes are concave polygons?



- A** 2,3,5 **B** 1,4 **C** 1,5 **D** All of the shapes

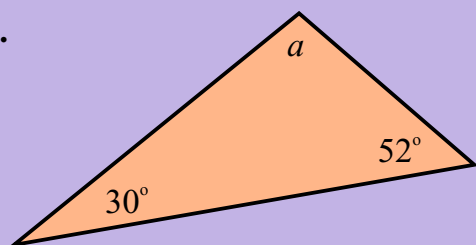
Triangles

Examples

Find the unknown angles in the following triangles.

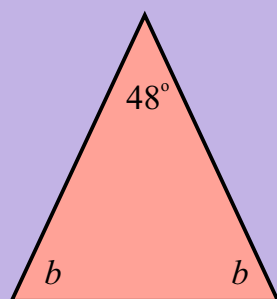
Remember: The sum of the three angles in a triangle is 180° .

1.



$$\begin{aligned} a + 30 + 52 &= 180 \\ a + 82 &= 180 \\ a &= 180 - 82 \\ a &= 98^\circ \end{aligned}$$

2.

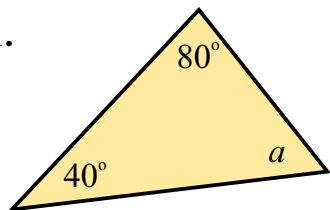


$$\begin{aligned} b + b + 48 &= 180 \\ 2b + 48 &= 180 \\ 2b &= 180 - 48 \\ 2b &= 132 \\ b &= 66^\circ \end{aligned}$$

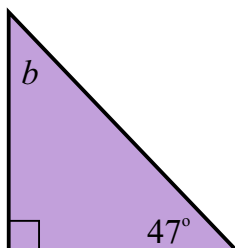
EXERCISE 22B

Find the unknown angles in the following triangles.

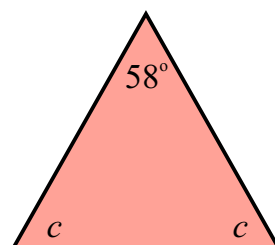
1.



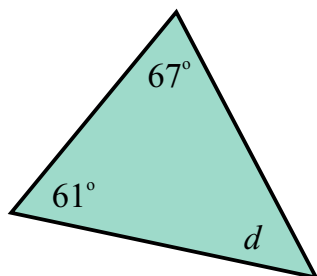
2.



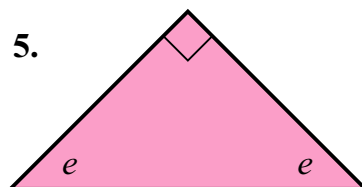
3.



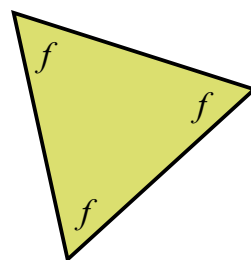
4.



5.

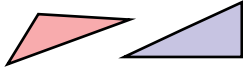
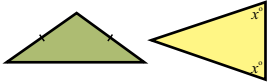
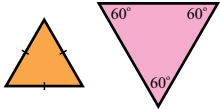


6.



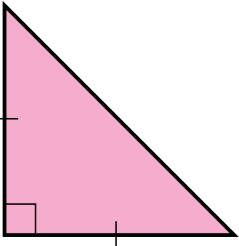
Types of Triangles

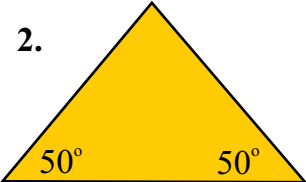
There are three types of triangles - *scalene*, *isosceles* and *equilateral*. Descriptions of these are given in this table.

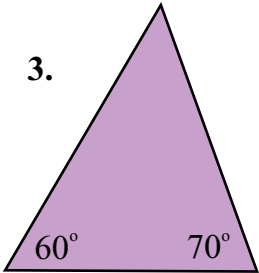
Name	Description	Examples
Scalene Triangle	All sides and angles are different	
Isosceles Triangle	Two sides and two angles are the same	
Equilateral Triangle	All sides and angles are the same	

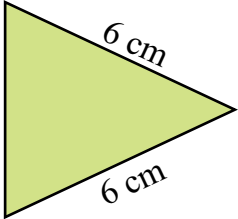
EXERCISE 22C

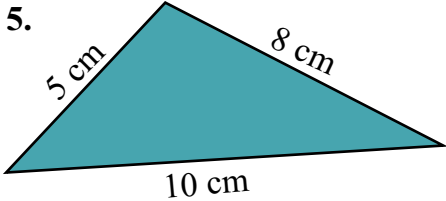
State whether the following triangles are *scalene*, *isosceles* or *equilateral*.

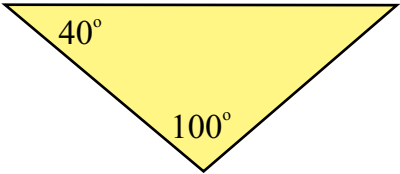
1. 

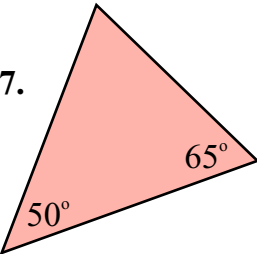
2. 

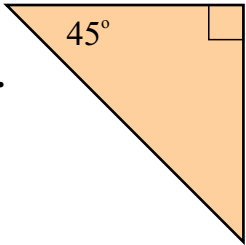
3. 

4. 

5. 

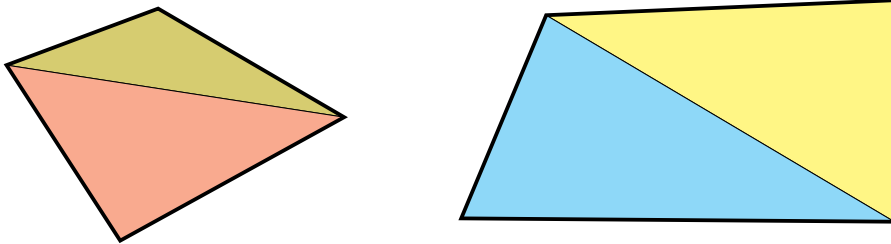
6. 

7. 

8. 

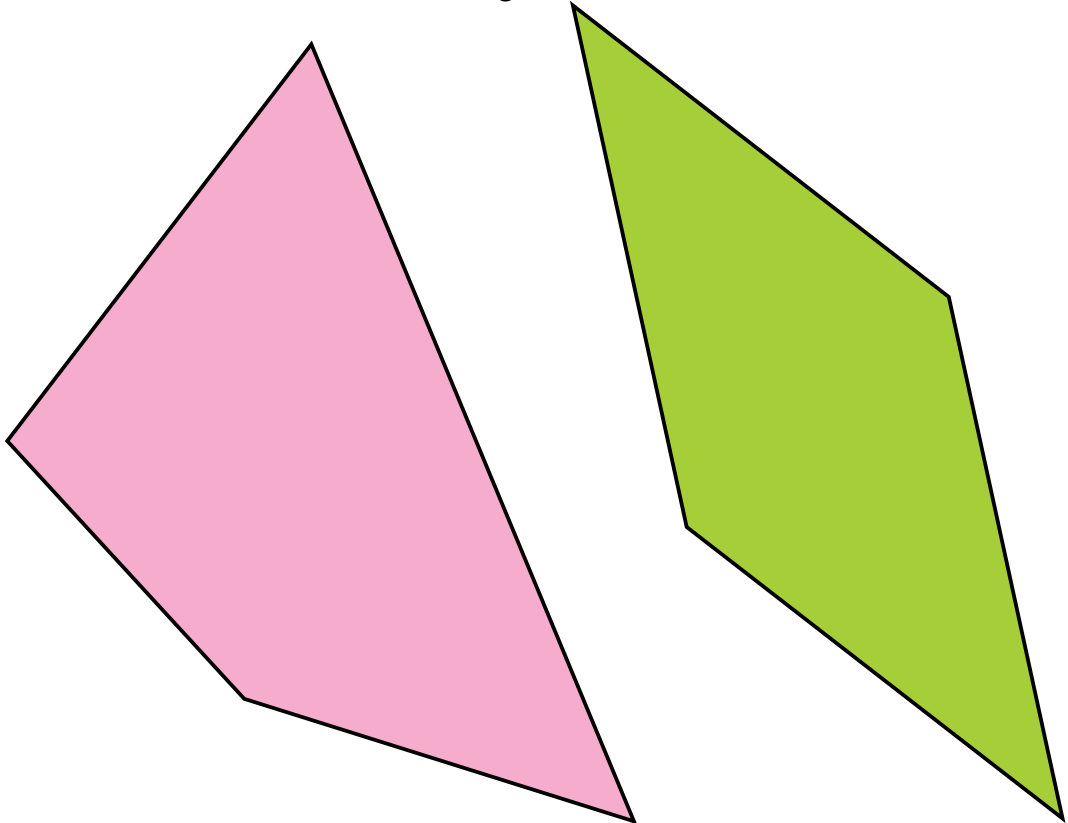
Quadrilaterals

To find the sum of the angles in a quadrilateral it can be seen that any quadrilateral could be divided into two triangles.



We know that the sum of the angles in a triangle is 180° so the sum of the angles in a quadrilateral must be 360° .

To check this use a protractor to measure the angles in the quadrilaterals below. Find the sum of the four angles.

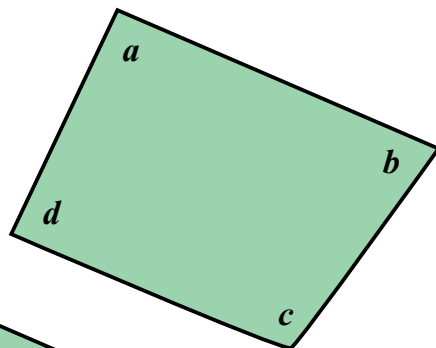


Draw your own quadrilateral, measure the angles and find the sum of these angles.

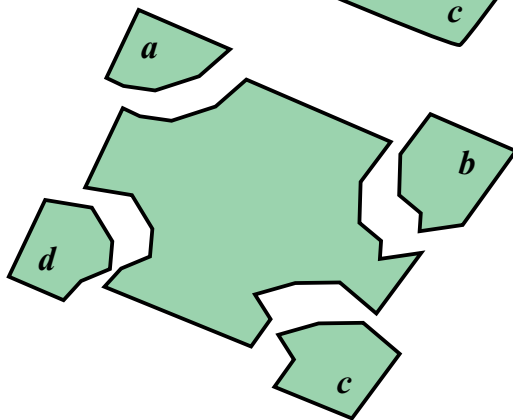
Another way to find the sum of the angles in a quadrilateral is to follow these steps.

Step 1 Cut a quadrilateral out of a piece of paper.

Label all the angles.



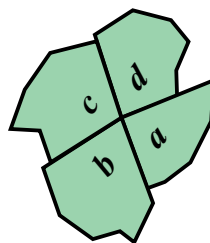
Step 2 Tear off the corners.



Step 3 Join all the corners together.

It can be seen that all the angles form a complete revolution.

$$a + b + c + d = 360^\circ$$



The sum of the four angles in a quadrilateral is 360° .

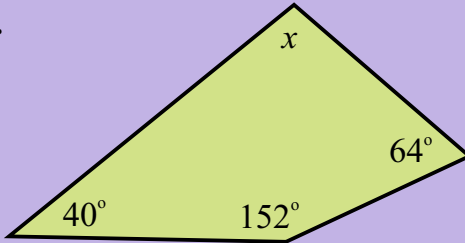
This method can also be used to show that the sum of the angles in a triangle is 180° .

Examples

Find the unknown angles in the following quadrilaterals.

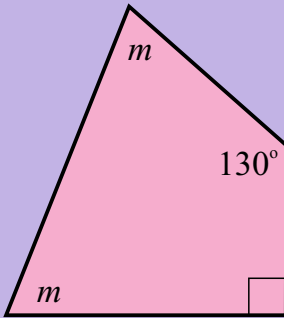
Remember: The sum of the four angles in a quadrilateral is 360° .

1.



$$\begin{aligned}x + 64 + 152 + 40 &= 360 \\x + 256 &= 360 \\x &= 360 - 256 \\x &= 104^\circ\end{aligned}$$

2.

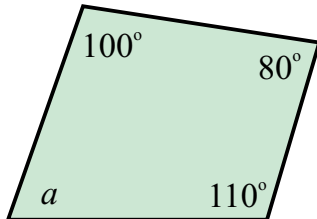


$$\begin{aligned}m + m + 130 + 90 &= 360 \\2m + 220 &= 360 \\2m &= 360 - 220 \\2m &= 140 \\m &= 70^\circ\end{aligned}$$

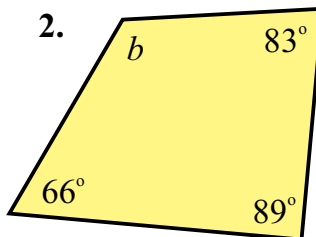
EXERCISE 22D

Find the unknown angles in the following quadrilaterals.

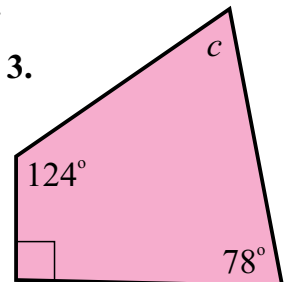
1.



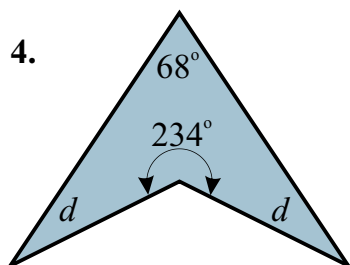
2.



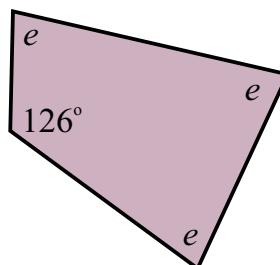
3.



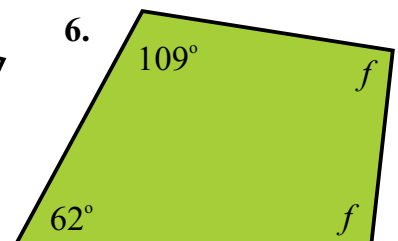
4.



5.

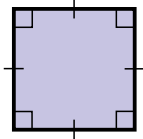
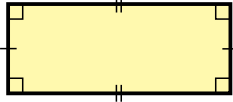
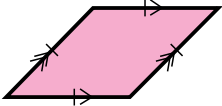
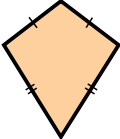
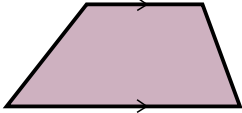
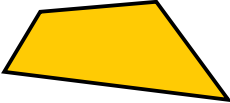


6.



Types of Quadrilaterals

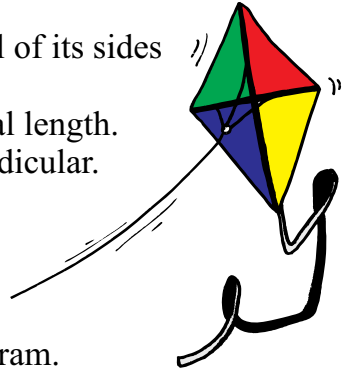
The table below shows and describes the different types of quadrilaterals.

Name	Description	Example
<i>Square</i>	All sides of equal length. All angles equal to 90°.	
<i>Rectangle</i>	Two pairs of sides of equal length. All angles equal to 90°.	
<i>Parallelogram</i>	Two pairs of parallel lines. Angles not equal to 90°.	
<i>Rhombus</i>	Special type of parallelogram with all sides of equal length.	
<i>Kite</i>	To pairs of sides of equal length that are not parallel.	
<i>Trapezium</i>	One pair of parallel lines.	
<i>Irregular Quadrilateral</i>	All sides of different lengths.	

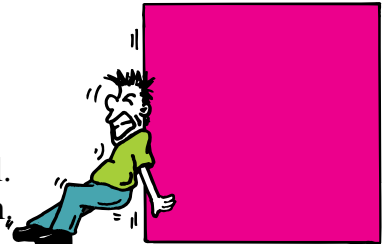
EXERCISE 22E

1. State whether the following statements are **TRUE** or **FALSE**.

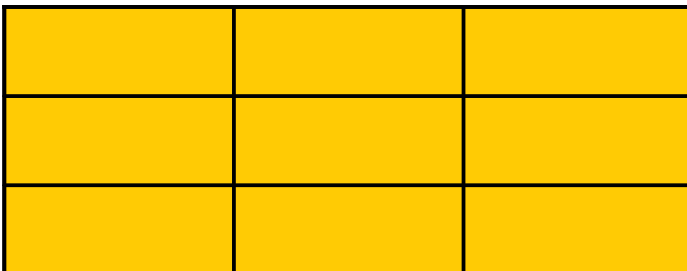
- (a) All the angles in a rhombus are the same.
- (b) All the angles in a rectangle are the same.
- (c) A parallelogram is made with two pairs of parallel lines.
- (d) The diagonals of a square would cross at 90° .
- (e) All the angles in a kite are different.
- (f) A square is the only quadrilateral with all of its sides equal in length.
- (g) The diagonals of a kite would be of equal length.
- (h) The diagonals of a kite would be perpendicular.



- (i) A rhombus is a special type of parallelogram.
- (j) An irregular quadrilateral has four different internal angles.
- (k) It is impossible to form a concave quadrilateral.
- (l) A kite could be formed by joining two different isosceles triangles.
- (m) A parallelogram with internal angles of 90° would be called a square.
- (n) A rhombus is a 'pushed over' square.
- (o) A regular quadrilateral would be called a square.
- (p) All the angles in a parallelogram are equal.
- (q) A square is a special type of parallelogram.



2. How many rectangles are in this drawing?



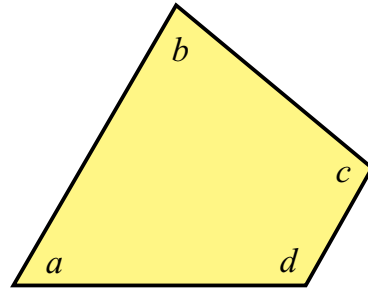
PROBLEM SOLVING

1. Find the size of the angles in the quadrilateral shown if:

$$b = a + 20^\circ$$

$$c = b + 20^\circ$$

$$d = c + 20^\circ$$



2. Find the size of the angles in this quadrilateral if angle y is twice the size of angle x .



PUZZLE

Change **BAGS** to **KITE** in four steps by changing one letter at a time to form a new word.

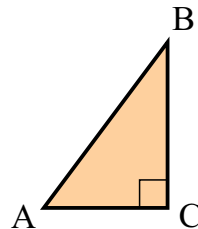
Example **CAT** can be changed to **DOG**
in three steps

CAT
COT
DOT
DOG

CHAPTER REVIEW

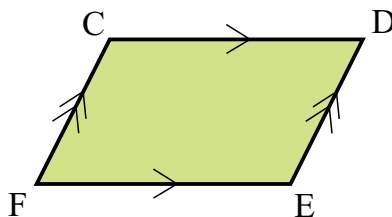
1. Which angle is a right angle in this triangle?

- A** $\angle ABC$ **B** $\angle CAB$ **C** $\angle BCA$ **D** $\angle BAC$



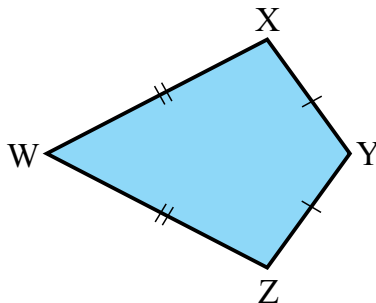
2. Which line is parallel to $\overline{DE}$ in this diagram?

- A** $\overline{EF}$ **B** $\overline{CF}$ **C** $\overline{CD}$ **D** $\overline{DF}$

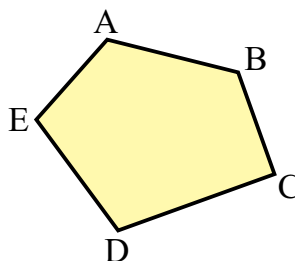


3. Which line is the same length as $\overline{YZ}$ in this diagram?

- A** $\overline{WZ}$ **B** $\overline{WX}$ **C** $\overline{XY}$ **D** $\overline{XZ}$

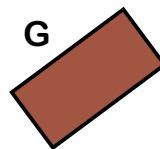
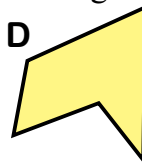
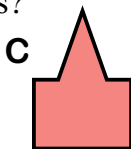
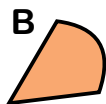
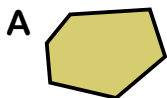


4. List all the lines that would be diagonals in this diagram.

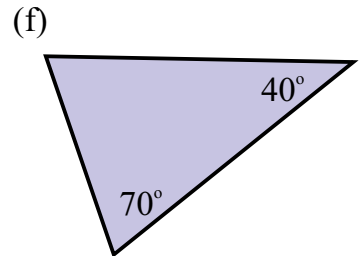
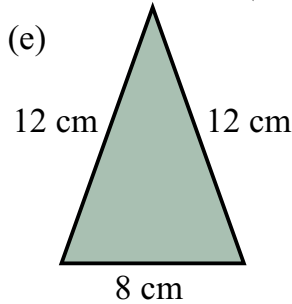
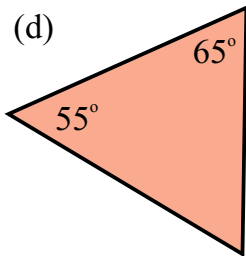
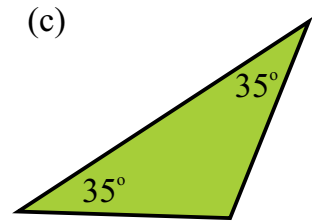
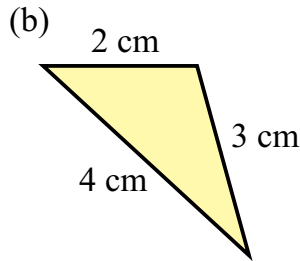
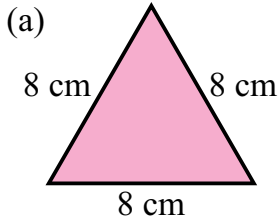


5. From the shapes shown here list the following.

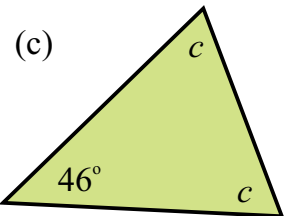
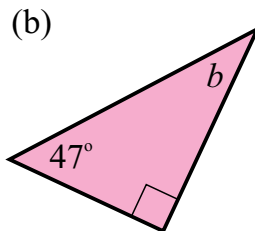
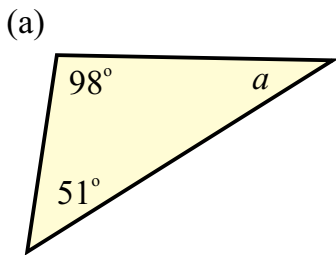
- Which are **not** polygons?
- Which are concave polygons?
- Which are convex polygons?
- Which are hexagons?
- Which are quadrilaterals?



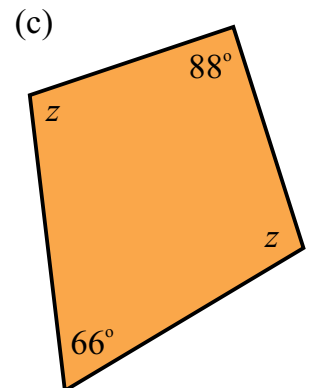
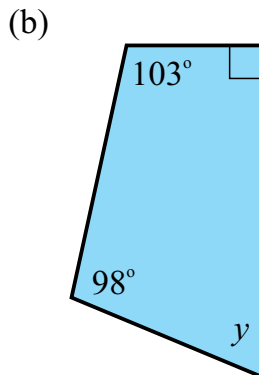
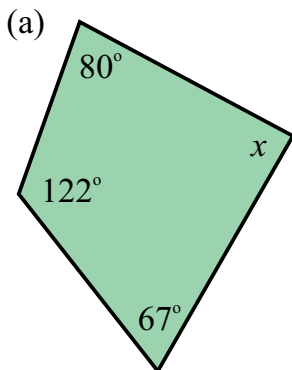
6. State if the following triangles are *scalene*, *isosceles* or *equilateral*.



7. Find the unknown angles in the following triangles.

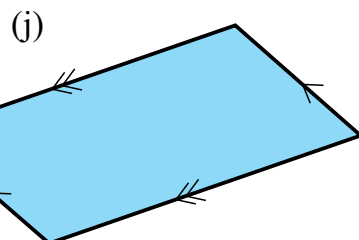
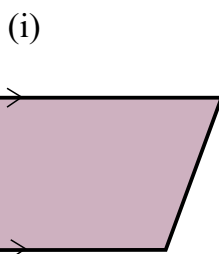
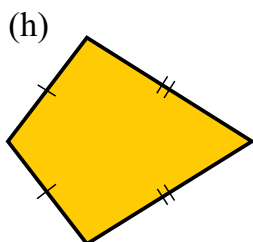
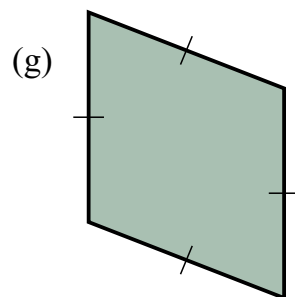
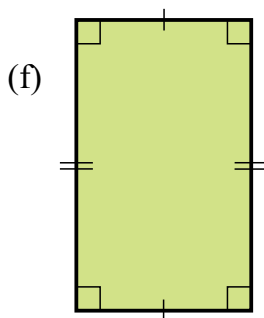
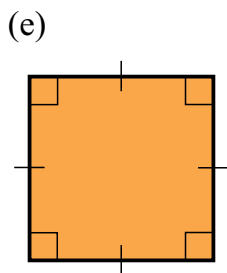
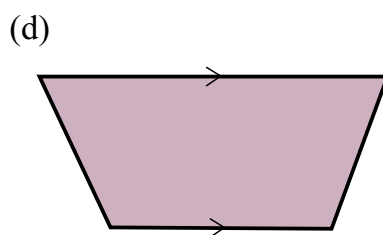
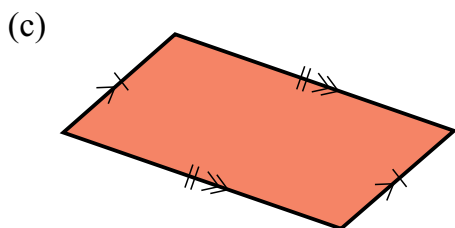
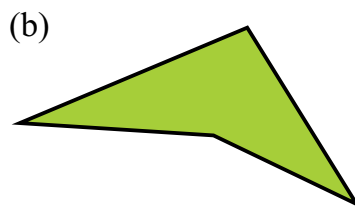
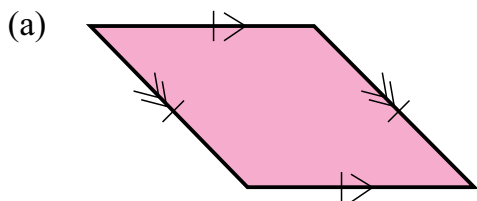


8. Find the unknown angles in the following quadrilaterals.



9. State the name of the following quadrilaterals.
The names for common quadrilaterals are listed here.

Square
Rectangle
Parallelogram
Rhombus
Kite
Trapezium
Irregular Quadrilateral



Probability

Probability

Probability is defined as the chance or likelihood of something occurring. We often talk about the probability of certain events occurring.

Examples

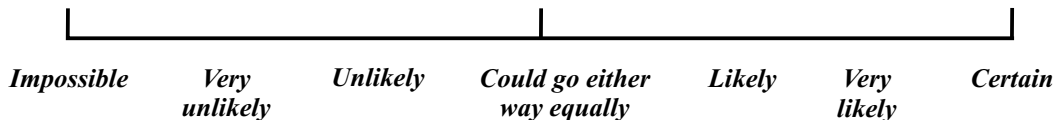
What is the chance of rain tomorrow?

What is the likelihood of meeting someone you know at a party?

What is the chance of a player kicking a goal?

What is the chance of tossing a coin and turning up heads?

Whenever probability is discussed the likelihood of the event occurring ranges on a scale from impossible to certain. The diagram below shows this scale.



Examples

It is very unlikely that it will snow in summer.

It is certain that Wednesday will follow Tuesday.

It is impossible to be born on the 30th of February.

It is likely that someone in a class likes chocolate.

In all situations when probability is discussed, there will be a number of possible **outcomes**.

Examples

When tossing a coin either heads or tails will be the outcome.

The temperature tomorrow will less than 20°, 20° or greater than 20°.

When a baby is born it will be a boy or a girl.

It is equally possible to roll a 1, 2, 3, 4, 5 or 6 on a die.

In the mathematics of probability there are several other terms that are often used.

Equally likely outcomes

When all the possible outcomes of an event have the same probability.

Examples

1. Tossing a coin.

There are two possible outcomes - heads and tails.

There is the same probability of either outcome occurring.

2. Rolling a die.

There are six possible outcomes - 1, 2, 3, 4, 5 or 6.

There is the same probability of any number being rolled.

Outcomes that are not equally likely

When the probability of possible outcomes of an event are not equal.

Examples

1. The winner of a sporting contest.

In many sporting contests the probabilities of the teams/opponents winning are different. There is often a favourite.

2. A class has an uneven number of boys and girls. If a student is chosen randomly the probability of a boy being chosen will be different to that of a girl being chosen.

Sample space

The sample space is the list of all possible outcomes.

Note - { } these brackets are used for a sample space.

Examples

1. Tossing a coin.

Sample space = {heads, tails}

2. Rolling a die.

Sample space = {1, 2, 3, 4, 5, 6}

3. Picking a card from a deck of cards and observing the colour.

Sample space = {red, black}

Chance Events

Chance events are events that occur by accident.

Examples

1. Weather events.
2. Car accidents.

Trials and Experiments

To find the probability that a certain outcome will occur, an experiment can be performed that consists of a number of trials.

Example

A company that produces balloons wants to find the probability of a balloon bursting before it reaches a diameter of 40 cm. It could conduct an experiment involving inflating 1000 balloons (1000 trials). If, for example, 13 balloons burst, the company could then state that, based on this experiment, the probability of a balloon bursting before reaching 40 cm in diameter is 13 in 1000.

Favourable Outcomes

A favourable outcome is that outcome for which the probability is being calculated.

Example

A bag contains 11 balls. Three of the balls are black. What is the probability of randomly choosing a black ball?

In this example the favourable outcome is ***choosing a black ball***.

EXERCISE 23A

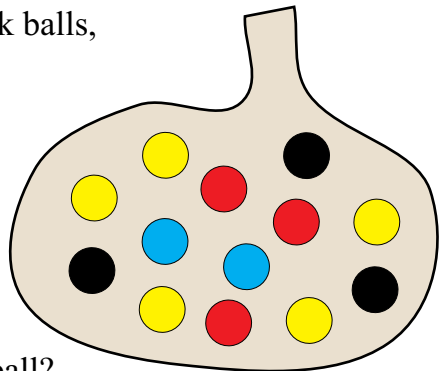
1. For the events below, choose from the alternatives shown here the best description of the probability that they will occur.

A Impossible
B Very unlikely
C Unlikely
D Could go either way equally
E Likely
F Very likely
G Certain

- (a) People will be living on Mars next year.
- (b) The price of a house will be cheaper next year than this year.
- (c) Someone in your class will have the same birthday as you.
- (d) Someone in your class will be born in the same month as you.
- (e) There will be a cyclone in Queensland next year.
- (f) In a football team of 22 players there is at least one left-footer.
- (g) In a basketball team of eight players they are all left-handers.
- (h) Christmas Day will be on the 25th of December.
- (i) There will be some rain in Melbourne during August next year.
- (j) There will be empty seats on a peak hour train in Sydney.
- (k) There will be empty seats on a late night train in Sydney.

2. A bag contains three red balls, three black balls, five yellow balls and two blue balls. The bag is shaken and a ball is randomly taken out.

- (a) What is the most likely colour of this ball?
- (b) What is the least likely colour of this ball?
- (c) Which colours have an equal likelihood of being the colour of the ball?



3. A bag contains seven blue balls, two green balls, three red balls, six yellow balls and one white ball. A ball is randomly taken from the bag. List the colours in order from the *least likely* to the *most likely* to be the colour of the ball.

4. For the following situations state:

(i) the sample space (list of possible outcomes).

Remember - { } these brackets are used for a sample space.

(ii) state if these outcomes are *equally likely* or *not equally likely*.

Examples

1. A coin is tossed.

There are two possible outcomes.

(i) The sample space is {*heads, tails*}.

(ii) These two outcomes are *equally likely*.

2. The result of a football game between the Cats and Tigers.

There are three possible outcomes.

(i) The sample space is {*Cats win, Tigers win, draw*}.

(ii) These outcomes are *not equally likely*.

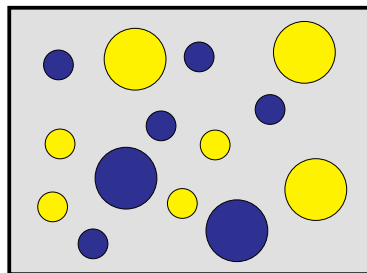
3. A bag contains five red balls and two blue balls. A ball is chosen at random and the *colour* is noted.

There are two possible outcomes.

(i) The sample space is {*red, blue*}

(ii) These two outcomes are *not equally likely*.

(a) The box shown here contains yellow balls and blue balls. Some are large and some small. A ball is chosen at random from the box and the *colour* is noted.



(b) A ball is taken at random from the box and the *size* is noted.

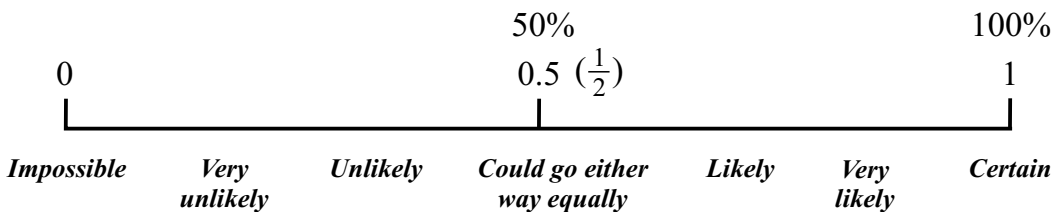
(c) Jack randomly chose a letter from the alphabet and noted if it was a consonant or a vowel.

(d) In a raffle there were 1000 tickets. The prizes were one computer, two phones and three cinema tickets. What are the possible outcomes for someone who buys a ticket?

(e) In a class of 26 students, 13 were girls. A student is chosen at random and their gender noted.

(f) In a tennis club the names of the committee members are Judy, Stephanie, Justine, Adam, Julie, Tim, Mitchell and Eve. The president is elected and their gender is noted.

It has been discussed that probability is often referred to in descriptive terms - impossible, unlikely, likely, certain, etc. In mathematics, probability is given a value that can be a decimal, fraction or percentage. As a decimal or fraction, probability will range from 0 (impossible) to 1 (certain). As a percentage it will range from 0 (impossible) to 100% (certain). This can be shown on the scale from earlier.



The higher the value of the probability (closer to 1) the more likely it is to occur.

The lower the value of the probability (closer to 0) the less likely it is to occur.

Examples

Rate the probability of the following events occurring.
Give answers as decimals, fractions and percentages when not 0 or 1.

1. Rolling a 7 on a standard die.

There is **no chance** of rolling a 7 on a standard die.

The probability = **0**

2. You will get wet if you fall into the sea.

You will **definitely** get wet if you fall into the sea.

The probability = **1**

3. The maximum temperature on New Year's Day in Perth will be above 20°.

It is **very likely** that the maximum temperature will be above 20° in Perth on New Year's Day.

The probability would be about 0.9. (This is an estimate)

$$0.9 = \frac{9}{10} = 90\%$$

Examples continued

4. The maximum temperature on Christmas Day in Sydney will be below 20°.

The maximum temperature is less likely to be below 20° than above 20°.

The probability would be about 0.2. (This is an estimate)

$$0.2 = \frac{2}{10} = \frac{1}{5} \text{ (simplest form)} = 20\%$$

5. When a baby is born it will be a boy.

There is usually a 50-50 chance that a baby will be a boy.

The probability = 0.5.

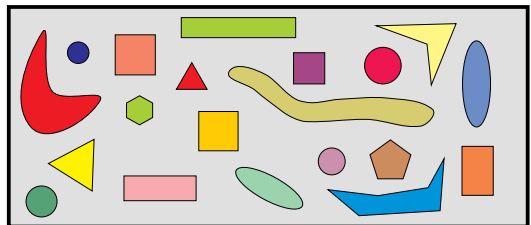
$$0.5 = \frac{5}{10} = \frac{1}{2} \text{ (simplest form)} = 50\%$$

EXERCISE 23B

Rate the probability of the following events occurring.

Give answers as decimals, fractions and percentages when not 0 or 1.

1. It will rain tomorrow.
2. It will be sunny all day tomorrow.
3. Your favourite football team will win the next game it plays.
4. Your favourite football team wins the next premiership.
5. You will pass the next mathematics test.
6. You choose the winner in the next Melbourne Cup.
7. Tomorrow is Wednesday.
8. The maximum temperature next Christmas Day in Adelaide will be over 30°C.
9. There will be snow on Mt. Buller on the 1st of July.
10. You can correctly guess the number of shapes in this box without counting them.
Make a guess then count them to see if you are correct.



Calculating Probabilities

Probabilities can be calculated exactly when the number of favourable outcomes possible and the total number of outcomes are known.

Probabilities can be expressed as fractions, decimals or percentages.

To calculate the probability as a fraction the following rule is used.

$$\text{Probability of a favourable outcome} = \frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$$

Examples

1. What is the probability of tossing a head on a coin?

The favourable outcome is tossing a head.

The number of favourable outcomes possible is

the number of heads on a coin = 1

The total number of outcomes = 2 (head or tail)

$$\text{Probability of a favourable outcome} = \frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$$

$$\begin{aligned}\text{Probability of a tossing a head} &= \frac{1}{2} \quad (\text{fraction}) \\ &= 0.5 \quad (\text{decimal}) \\ &= 50\% \quad (\text{percentage})\end{aligned}$$

2. A bag contains three green balls and one red ball. One ball is chosen at random. What is the probability that the red ball is chosen?

The number of favourable outcomes possible = 1 (one red ball)

The total number of outcomes = 4 (four balls)

$$\text{Probability of a favourable outcome} = \frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$$

$$\begin{aligned}\text{Probability of choosing the red ball} &= \frac{1}{4} \quad (\text{fraction}) \\ &= 0.25 \quad (\text{decimal}) \\ &= 25\% \quad (\text{percentage})\end{aligned}$$

3. A mixed basketball team consists of two boys and three girls. The captain of the team is chosen randomly by putting all their names in a hat and picking one. What is the probability the captain will be a girl?
 The number of favourable outcomes possible = 3 (three girls in the team)
 The total number of outcomes = 5 (five players in the team)

Probability of a favourable outcome = $\frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$

$$\begin{aligned}\text{Probability of the captain being a girl} &= \frac{3}{5} \quad (\text{fraction}) \\ &= 0.6 \quad (\text{decimal}) \\ &= 60\% \quad (\text{percentage})\end{aligned}$$

4. Indie has one pair of red socks, four pairs of yellow socks and three pairs of blue socks in her sock draw. One dark morning she randomly chooses a pair of socks. What is the probability it is blue?
 The number of favourable outcomes = 3 (three pairs of blue socks)
 The total number of outcomes = 8 (eight pairs of socks)

Probability of a favourable outcome = $\frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$

$$\begin{aligned}\text{Probability of choosing a pair of blue socks} &= \frac{3}{8} \quad (\text{fraction}) \\ &= 0.375 \quad (\text{decimal}) \\ &= 37.5\% \quad (\text{percentage})\end{aligned}$$

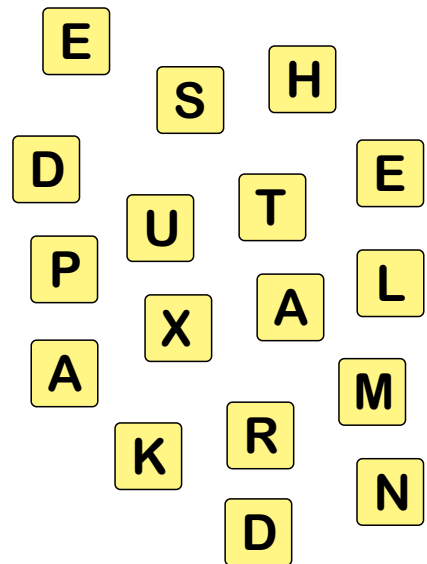
EXERCISE 23C

1. A bag contains one black ball and three white balls.
 - (a) What is the probability of randomly choosing the black ball from the bag?
Give answer as: (i) fraction (ii) decimal (iii) percentage
 - (b) What is the probability of randomly choosing a white ball from the bag?
Give answer as: (i) fraction (ii) decimal (iii) percentage

2. Aaron buys a bag of sweets. There are three caramels, one peppermint and six chocolates.
 - (a) What is the probability of randomly choosing a caramel from the bag?
Give answer as: (i) fraction (ii) decimal (iii) percentage
 - (b) What is the probability of randomly choosing the peppermint from the bag?
Give answer as: (i) fraction (ii) decimal (iii) percentage

3. Kristie has eight pencils in her pencil case. One of them is purple. She randomly chooses a pencil from her pencil case.
 - (a) What is the probability that it is the purple one?
Give answer as: (i) fraction (ii) decimal (iii) percentage
 - (b) What is the probability that it is *not* purple?
Give answer as: (i) fraction (ii) decimal (iii) percentage

4. Jay is playing a word game. The letters shown here are all upturned so he cannot see them. If he randomly chooses one letter find the following.
Give answers as fractions.
 - (a) What is the probability it will be an **X**?
 - (b) What is the probability it will be a **D**?
 - (c) What is the probability it will be a **Z**?
 - (d) What is the probability it will be a vowel?
 - (e) What is the probability it will be a consonant?



Example

Felix has 40 songs saved and 12 are from his favourite band. If he randomly chooses one song, what is the probability it is from his favourite band?

Give answer as a fraction in its simplest form.

To write a fraction in its simplest form the fraction is cancelled down. Refer to chapter on *fractions*.

$$\text{Probability of a favourable outcome} = \frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$$

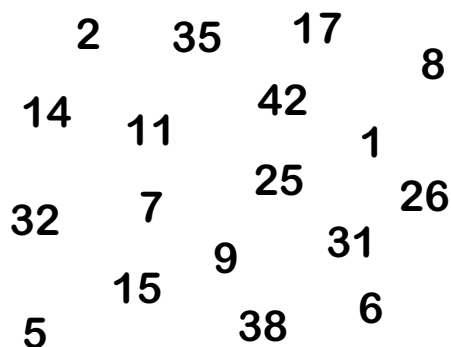
$$\begin{aligned} \text{Probability of choosing song from his favourite band} &= \frac{\text{number of songs from his favourite band}}{\text{total number of songs}} \\ &= \frac{12}{40} \\ &= \frac{3}{10} \quad (\text{fraction in its simplest form}) \end{aligned}$$

EXERCISE 23D

Write the answers to the following questions as fractions in their simplest form.

- A card is chosen at random from a normal deck of playing cards.
 - What is the probability it is a red card?
 - What is the probability it is a diamond?
 - What is the probability it is a king?
 - What is the probability it is the ace of spades?
- In a football team of 22 players, eight are left-footers. A player is chosen at random.
 - What is the probability he is a left-footer?
 - What is the probability he is a right-footer?
- In a class there are 14 boys and 10 girls. A student is chosen at random to do roll duty.
 - What is the probability the student is a boy?
 - What is the probability the student is a girl?

4. A number is chosen randomly from the numbers shown here.



- What is the probability that it is an even number?
- What is the probability that it is an odd number?
- What is the probability that it is a prime number?
- What is the probability that it is larger than 10?

5. A bag contained flavoured sweets. There were six caramel, four strawberry, nine peppermint and five coconut flavoured sweets.

- If a sweet is chosen at random, find the probability that it is:
 - caramel
 - strawberry
 - peppermint
 - coconut
- Five friends take it turns to choose a sweet. Felicity takes a caramel, Simon takes a coconut, Clare and Felix both take a peppermint sweet. Find the probability that the next friend takes a sweet that is flavoured:
 - caramel
 - strawberry
 - peppermint
 - coconut

6. In the game of Scrabble there are small tiles each with a letter of the alphabet. The number of tiles with each of the letters is shown below.

A - 9, B - 2, C - 2, D - 4, E - 12, F - 2, G - 3, H - 2, I - 9, J - 1, K - 1, L - 4, M - 2, N - 6, O - 8, P - 2, Q - 1, R - 6, S - 4, T - 6, U - 4, V - 2, W - 2, X - 1, Y - 2, Z - 1

At the start of the game all of the tiles are upturned so that the letters cannot be seen.

- How many tiles are there?
- What is the probability that the first tile chosen is each of the following letters?
 - E**
 - T**
 - X**
 - A**
- What is the probability that the first tile chosen is a vowel?
- What is the probability that the first tile chosen is a consonant?

Approximate Probabilities

Approximate probabilities can be calculated by performing trials and experiments or by using a set of data.

The size of the experiment or set of data is important for the accuracy of the probability. The more trials that are conducted the more accurate will be the probability.

Example 1

Chaz plays basketball. During a training session she had 40 free throws and scored 28 of them. Based on these figures find the probability that she will score a free throw. Give the probability as a fraction in its simplest form, a decimal and a percentage.

$$\text{Probability of a favourable outcome} = \frac{\text{number of favourable outcomes possible}}{\text{total number of outcomes}}$$

$$\begin{aligned} \text{Probability of scoring a free throw} &= \frac{\text{number of free throws scored}}{\text{total number of attempts}} \\ &= \frac{28}{40} \\ &= \frac{7}{10} \text{ (fraction in its simplest form)} \\ &= 0.7 \text{ (decimal)} \\ &= 70\% \text{ (percentage)} \end{aligned}$$

Example 2

Melanie plays basketball with Chaz and wanted to find the probability that she will score a free throw. She had 10 attempts and scored 8 free throws. Based on these figures what is the probability Melanie will score a free throw and which of the two probabilities (Melanie or Chaz) is more accurate?

Using the same approach as in example 1 the probability that Melanie will score a free throw is:

$$\frac{8}{10} = \frac{4}{5} = 0.8 = 80\%$$

The probability of 0.7 for Chaz will be more accurate than the 0.8 for Melanie because it was based on more trials.

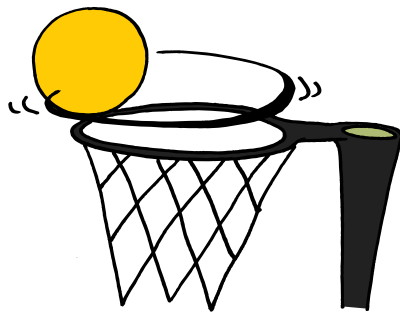
EXERCISE 23E

1. Clara and Gabrielle play in a netball team. To find out who was the better goal shooter they each had a number of shots at goal and the number of goals scored was recorded. These results are shown below:

Clara had 60 shots at goal and scored 45 goals.

Gabrielle had 15 shots at goal and scored 12 goals.

- (a) Based on these figures, find the probability of Clara scoring a goal. Give answer as:
 (i) a fraction in its simplest form (ii) a decimal (iii) a percentage
 (b) Based on these figures, find the probability of Gabrielle scoring a goal. Give answer as:
 (i) a fraction in its simplest form (ii) a decimal (iii) a percentage
 (c) Based on these probabilities, who is the more accurate goal shooter?
 (d) Based on these figures, the probability for which player is more accurate?



2. Bindie made 10 pottery vases and when she put them in the kiln to dry two cracked.

Lachy made 40 pottery vases and when he put them in the kiln to dry six cracked.

- (a) Based on these figures, what is the probability that one of Bindie's vases will crack when put in the kiln? Give answer as:
 (i) a fraction in its simplest form (ii) a decimal (iii) a percentage
 (b) Based on these figures, what is the probability that one of Lachy's vases will crack when put in the kiln? Give answer as:
 (i) a fraction in its simplest form (ii) a decimal (iii) a percentage
 (c) Based on these figures, whose vases are less likely to crack when put in the kiln?
 (d) Based on these figures, the probability of whose vases cracking in more accurate?

3. Sean was a farmer and wanted to plant a crop in September. He wanted to find out the likely rainfall during September so he looked up the records from the Bureau of Meteorology and found the monthly rainfall (in millimetres) for September for the previous 20 years were:

57, 83, 121, 45, 27, 96, 81, 52, 76, 33,
133, 62, 39, 77, 28, 74, 92, 19, 46, 67

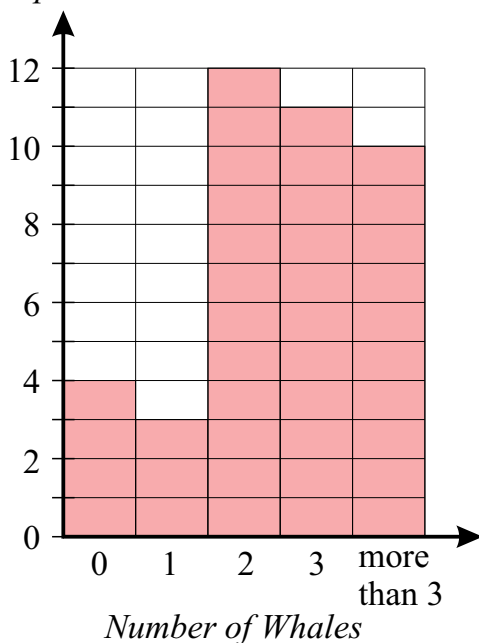


Based on these figures:

- what is the probability, as a **percentage**, that the rainfall in September will be greater than 100 mm?
 - what is the probability, as a **percentage**, that the rainfall in September will be less than 50 mm?
4. Fleur runs whale watching trips and records the number of whales that are seen each trip. The numbers of whales seen for the past 40 trips are shown on the column graph. Based on these figures, what is the probability, as a **percentage**, of seeing the following number of whales on one of Fleur's trips?
- 0
 - more than 3
 - more than 0



Number
of Trips



5. A fruit shop was selling 2 kg bags of apples. The owner conducted a survey to find out if there were any bruised apples in the bags. He asked everyone who bought a bag to tell him how many of the apples in each bag were bruised. The tally sheet shows the results.

- (a) How many people were surveyed?
- (b) Based on the survey, find the probability, as a **percentage**, that:
- there are no bruised apples in a bag.
 - there are less than two bruised apples in a bag.
 - there are two or more bruised apples in a bag.

<i>Number of bruised apples in a bag</i>	<i>Tally</i>
0	
1	
2	
3	
More than 3	

6. Lefty Les is the full forward for the Possums football team. In one season he had 100 kicks at goal and he kicked 60 goals. Based on these figures:
- what is the probability, as a **percentage**, that when Lefty kicked for goal he scored a goal?
 - If Lefty had 10 kicks for goal in a game, how many goals would he be expected to kick?

Zac is another player with the Possums. Zac had 40 kicks for goal and kicked 32 goals.

Based on these figures:

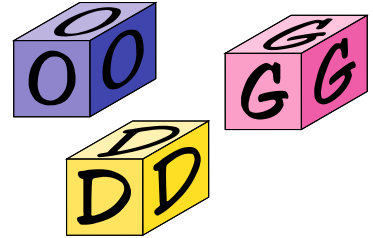
- what is the probability, as a **percentage**, that when Zac kicked for goal he scored?
- If Zac had 10 kicks for goal in a game, how many goals would he be expected to kick?
- Based on these figures who is the more reliable kick for goal?
- The figures for which player are more accurate? Why?



PROBLEM SOLVING

1. A child is playing with the three blocks shown and randomly arranges them in a row.

What is the probability that the three letters spell an actual word?



2. Lucy has a TattsLotto ticket. There are 45 numbers and she has picked six. If her six numbers come up she will win TattsLotto. She watches the draw and to her excitement the first five numbers to come up are five of her numbers. What is the probability that the sixth number to come up will be her sixth number?

PUZZLES

1. Change **ROLL** into **DICE** in six steps by changing one letter at a time to form a new word. Clues for each of the five words between ROLL and DICE are given. The letters do not need to be rearranged to form each word.

R	O	L	L
D	I	C	E

Small model of a person used as a toy

Spicy herb used in pickles

Machine for grinding grains

1.6 kilometres

Plural of mouse

2. Change **HEADS** into **TAILS** in three steps by changing one letter at a time to form a new word. The letters may need to be rearranged to form each word.

CHAPTER REVIEW

1. From the list below match the mathematical probability with the descriptive probability.

0.9 40% $\frac{1}{2}$ 100% 0 $\frac{3}{5}$ 0.2

(a) a certainty (b) could go either way equally (c) unlikely
(d) likely (e) very unlikely (f) no chance (g) very likely

2. Rate the probability, as a *decimal*, of the following events.

(a) Christmas will be in November this year.
(b) If a person is selected at random they will be left-handed.
(c) If you threw a tennis ball at a window it would break.
(d) If you threw a golf ball at a window it would break.
(e) If you rolled a die the number would be 1, 2, 3, 4, 5 or 6.

3. Write the following probabilities as:

(i) fraction in its simplest form (ii) decimal (iii) percentage

(a) A bag contains four red balls and six blue balls.

What is the probability of randomly drawing a blue ball?

(b) In a litter of five puppies one is a male.

What is the probability of randomly choosing the male?

(c) A letter is chosen at random from the word **BASKETBALL**.

What is the probability it is an **A** or a **B**?

4. Sarah had three green pens, four blue pens, five black pens and three red pens in her pen box. She randomly chose a pen. What is the probability that it was a red pen?

Give answer as a fraction in its simplest form.

5. There were two lucky dips at a school fete - one organised by the Year 7 classes and one organised by the Year 8 classes.

Jonah had five tries at the Year 7 lucky dip and won one prize.

Kathy had 20 tries at the Year 8 lucky dip and won five prizes.

(a) Based on these figures, calculate the probability of winning a prize at each lucky dip. Give answers as decimals.

(b) Based on these figures, in which lucky dip are you more likely to win a prize?

(c) Which of the two probabilities are more accurate? Why?

Data 1

Types of Data

The collection of data is very important in many areas of society. Federal, state and local governments need information on which to base their policies in areas such as education, transport, health, environment, etc. Businesses need information to help them decide which products to produce or stock.

Medical researchers need data to see which research is needed and how research is progressing.

Farmers need information on weather trends, which crops to plant or which livestock to farm.

There are companies that collect data using surveys and sell them to organisations, governments and individuals.

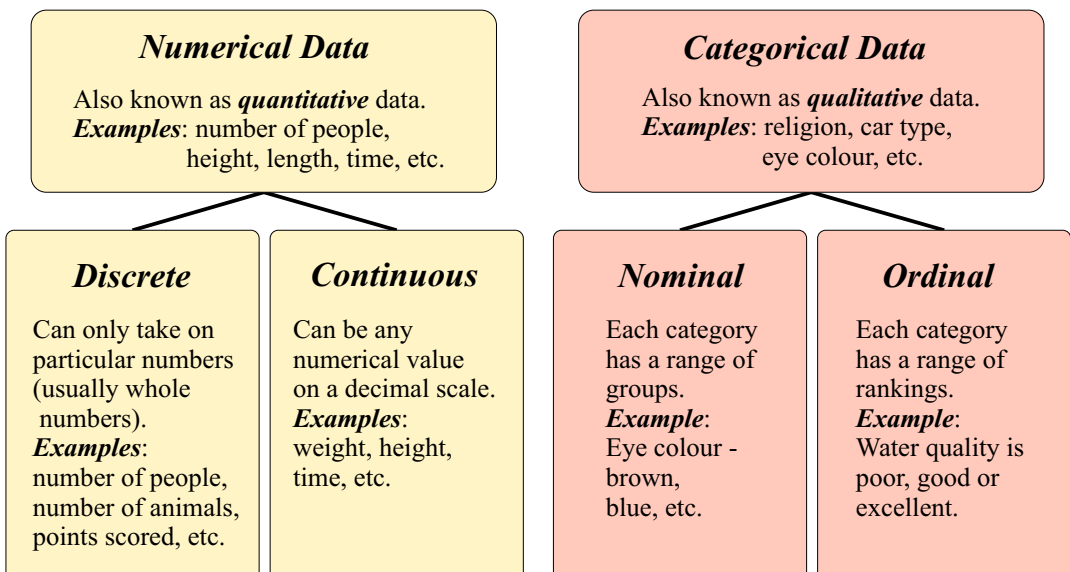
Data can be grouped in several ways.

1. Numerical and Categorical Data

There are two types of numerical data - **discrete** and **continuous**.

There are two types of categorical data - **nominal** and **ordinal**.

The diagram below details these different types of data.



2. Primary and Secondary Data

Primary data

Data that is collected by the people, organisation, etc. who are going to use the data.

Sources

Surveys, questionnaires, observations, experiments, interviews, telephone, focus groups

Examples

- ♦ A medical research group may interview a group of people to investigate the effects of a medical treatment.
- ♦ A local government may conduct a survey to find out the opinions of residents on a proposed development.

Advantages

- ♦ Direct personal investigation can be more accurate.
- ♦ The information can be more reliable.
- ♦ Can target the particular people, areas, etc. that are being studied.

Disadvantages

- ♦ Direct contact with people may lead to bias or reluctance of people to answer truthfully depending on the topic.
- ♦ May be expensive and time consuming.
- ♦ Due to restrictions of money and time may lead to an investigation that is smaller than what is required to obtain accurate results.

Secondary data

Data that has been collected by a group or organisation other than the people who are going to use the information.

Sources

Internet, newspapers, magazines, Australian Bureau of Statistics (ABS)

Examples

- ♦ The government may want information on which form of transport people use to decide whether to build more roads or expand public transport.
- ♦ Medical researchers may need statistics on health areas that need more research.

Advantages

- ♦ In the case of a country wide census the information will be more accurate due to the number of people surveyed.
- ♦ It already exists so there is no cost or time involved with conducting surveys or studies. The only cost may be in the use of the data.
- ♦ Offers access to information that may otherwise be difficult to obtain.
- ♦ May provide information that was obtained using equipment that would otherwise have been difficult to access.
- ♦ Offers access to data obtained over a period of time.
- ♦ Offers access to data obtained in areas that would have been difficult to access - geographically, politically, etc.

Disadvantages

- ♦ It may be difficult to confirm the accuracy, quality or independence of the data.
- ♦ The data may be obsolete.
- ♦ Some data may be expensive to get approval to use.

Samples and Population

It is often impractical and/or unnecessary to survey everyone or everything affected when obtaining information.

When collecting data it is usual to use a **sample** of the **population**. The population is the total number of people, animals, etc. that would be affected by the results of the survey or study.

Examples

1. A football/netball club wants to find out whether to build a carpark. They may survey a **sample** of the total number of people who would use the car park (the **population**).
2. A conservationist wants to find the number of possums in a national park. The park may be too large to count every possum so they might choose an area of the park and use this to predict the total number.

To get an accurate representation of the population the following guidelines need to be followed.

1. Ensure the sample is a true representation of the population. This means a true cross-section of the population needs to be surveyed. This includes age, gender, race, religion, etc.
2. Ensure the sample size is large enough. The larger the sample size the more accurate will be the results. There are several complicated formulae that can be used to determine the minimum sample size. It depends on how accurate the results need to be.
An approximate rule for the minimum sample size is:

$$\text{minimum sample size} = \sqrt{\text{population}} + 1$$

Example A town has a population of 6000 people. The council wants to conduct a survey. What should be the minimum sample size?

$$\begin{aligned} \text{minimum sample size} &= \sqrt{6000} + 1 \\ &= 77 + 1 \\ &= 78 \\ &= \mathbf{78} \end{aligned}$$

Example 1

A local council of a town of 4000 people wants to find out if they should convert a vacant area of land to a picnic area or a skate park. They survey 30 nearby landowners to find out which they prefer.

1. Is the data obtained from this survey **numerical** or **categorical**?
2. Is the data **primary** or **secondary**?
3. Is this survey adequate? If not, how could it be improved?

Answers

1. The data is **categorical**. The results were either those who chose a picnic area or a skate park.
2. The data is **primary** because the people were asked directly to complete the survey.
3. The survey is not adequate for two reasons.
 - (a) Only land owners were surveyed. To get a true idea of what people want, a cross-section of the community should be surveyed. Young and old, men and women.
 - (b) The survey sample size is not large enough. Using the guide:

$$\begin{aligned}
 \text{minimum sample size} &= \sqrt{\text{population}} + 1 \\
 &= \sqrt{4000} + 1 \\
 &= 63 + 1 \\
 &= 64
 \end{aligned}$$



Example 2

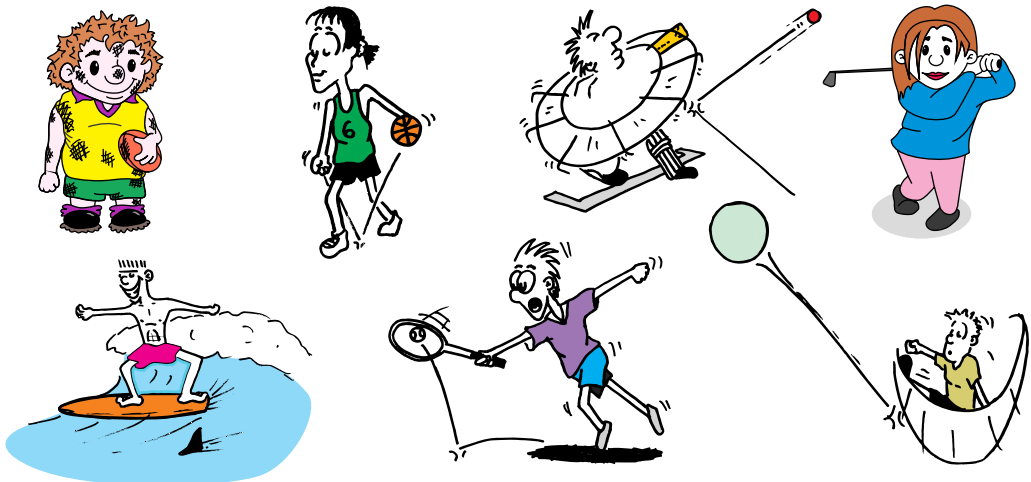
The teachers at a school wanted to find out how long their students spent involved with physical activities each day to see if they needed to include more physical education in school hours.

They looked at a report from the Heart Foundation that stated that less than 17% of young people were spending the recommended time involved in physical activity each day.

1. Is the data obtained from this survey **numerical** or **categorical**?
2. Is the data **primary** or **secondary**?
3. Is the data adequate? How could it be improved?

Answers

1. The data is **numerical**. The **amount** of time spent by young people involved in physical activity was being studied.
2. The data is **secondary** because it was already collected by, and from, people other than those at the school where it will be used.
3. The data can be used as a guide. It could be assumed that data collected and analysed by large organisations has involved an adequate sample size and cross-section of the population. But to get accurate data about students in a particular school it would necessary to survey the students. The survey would need to involve as many students as possible and cover a cross-section of the student population - all year levels and both boys and girls.



EXERCISE 24A

1. State if the following data is *numerical* or *categorical*.
 - (a) A scientist at a seal colony recorded the length of new born seal calves.
 - (b) 400 people were surveyed to find out their preferred breakfast:
A cereal **B** toast **C** eggs **D** other
 - (c) Students recorded the colours of the cars that passed their school.
 - (d) People in a city who used trams to get to work were asked how many minutes they had to wait for their morning tram.
 - (e) Students were asked how many brothers and sisters they had.
 - (f) Students were asked what type of pet they owned.
 - (g) The amount of money earned by students in part-time jobs.
 - (h) The type of part-time jobs in which students work.
2. State if the following numerical data is *discrete* or *continuous*.
 - (a) The length of hair.
 - (b) The number of goals scored by a football team.
 - (c) The number of students studying science at a university.
 - (d) The distance that people jog each day.
 - (e) The number of steps taken by a person running.
 - (f) The amount of energy used by someone exercising.
 - (g) The number of sheep owned by farmers.
 - (h) The weight of wool produced by sheep farmers.
3. State if the following categorical data is *nominal* or *ordinal*.
 - (a) Students were asked to rate the service at the school canteen on a scale from 0 (terrible) to 10 (excellent).
 - (b) Students were asked what foods they would like served at the school canteen.
 - (c) Students were asked if the range of food at the school canteen was poor, adequate or good.
 - (d) A number of people were asked their favourite holiday location.
 - (e) A number of young people were asked to rate the quality of sound at a concert on a scale from 0 (poor) to 5 (excellent).
 - (f) The residents were surveyed by the local council to rate the quality of their tap water.
 - (g) 1000 motorists who used a particular intersection were asked if traffic lights should be installed.
 - (h) 1000 motorists who used a particular road were asked to rate the quality of the road surface from 0 (needs repairing) to 5 (excellent).

4. State if the following statements are *true (T)* or *false (F)*.

- (a) The larger the number of people surveyed, the more accurate will be the results.
- (b) The internet is a source of primary data.
- (c) The number of people at a concert would be classified as categorical data.
- (d) The results of a survey will be more accurate if a cross-section of the population are surveyed.
- (e) The number of students studying the courses at a college is a type of categorical data.
- (f) The types of courses offered by a college is a type of categorical data.
- (g) Two research scientists measured and recorded the length and weight of a number of turtles on the Great Barrier Reef. This would be primary data.
- (h) These research scientists compared their findings with the recorded measurements of turtles taken 100 years ago. These measurements are secondary data.
- (i) The number of turtles on the Great Barrier Reef is discrete numerical data.
- (j) The temperature of the water around the Great Barrier Reef is discrete numerical data.
- (k) A person's religion would be classified as categorical data.
- (l) A person's gender would be classified as numerical data.
- (m) The Australian Bureau of Statistics is a good source of secondary data.



5. Calculate the minimum sample size that should be used in the following surveys or studies.

- (a) A school of 950 students.
- (b) A town of 4500 people.
- (c) An orchard with 800 fruit trees.
- (d) A city with 800 000 people.
- (e) A city with 2 million people.
- (f) A country of 8.9 million people.

6. A football club has 42 000 members. The club president decides to survey 1% of the members to get their thoughts on several issues.
Is this sample size large enough?

7. (a) A school council wants to investigate changing the school uniform. They decide to conduct a survey and consider three different sample groups. These are listed below.
State which of the sample groups would give the most accurate results and give reasons.
Sample Group **A** - 50 students from each year level
Sample Group **B** - 20 students from each year level and 20 teachers
Sample Group **C** - 200 randomly chosen students, 10 teachers and 90 parents
- (b) The state government wants to conduct a survey to find out secondary students' attitudes towards a television advertisement aimed at stopping people drinking and driving.
State which of the sample groups below would give the most accurate results and give reasons.
Sample Group **A** - 50 year 7 students from every secondary school in the state
Sample Group **B** - 50 students randomly chosen from every secondary school in the state
Sample Group **C** - 100 students randomly chosen from every secondary school in the capital city
8. A year 7 class were asked to do a project on the number of houses that had solar electricity panels installed now compared to the number of houses that had them installed 10 years ago. They were asked to include primary and secondary data in their project.
(a) Describe a way that primary data could be found for the project.
(b) Describe a source of secondary data that could be used for the project.



Secondary Data

Secondary data can be obtained from many different sources: the internet, newspapers, magazines, the Australian Bureau of Statistics (ABS), research organisations, non-government organisations (NGOs), tertiary institutions, etc..

There are companies that collect data and sell it to organisations that can use it. Often governments or organisations will employ groups to collect data.

Secondary data is used by many different groups:

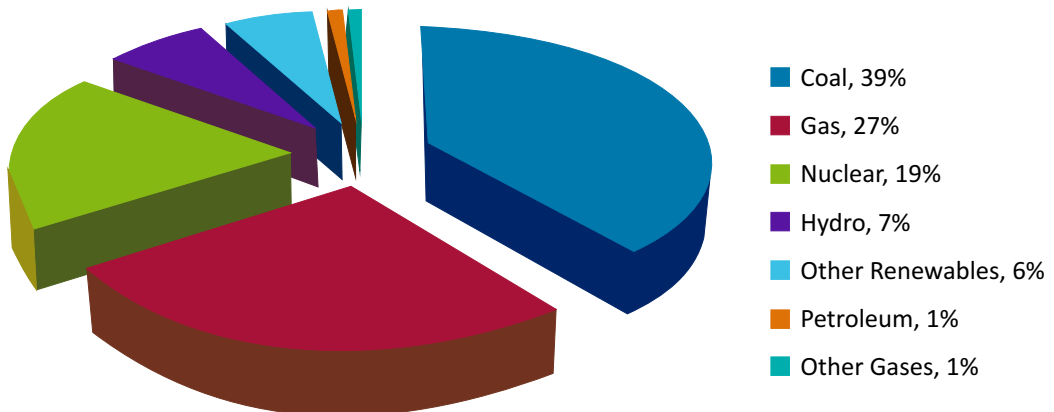
- ♦ governments - to help decide which areas need funding
- ♦ scientific organisations - to assist in deciding which areas of research are most important
- ♦ non-government organisations - to assist in targeting areas of need

Some of the important areas that secondary data is useful in are:

health, environment, energy (consumption and production), education, religion, conservation, agriculture, population, immigration, demographics, politics, technology, etc..

Example

The diagram below shows the resources that were used to produce electricity in the USA in 2013.

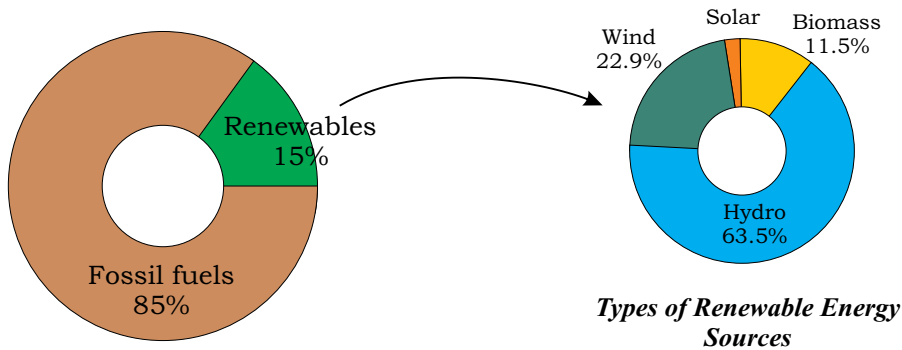


Source: EIA (Energy Information Administration US)

This data could be used by the US government to help with the development of renewable energy targets.

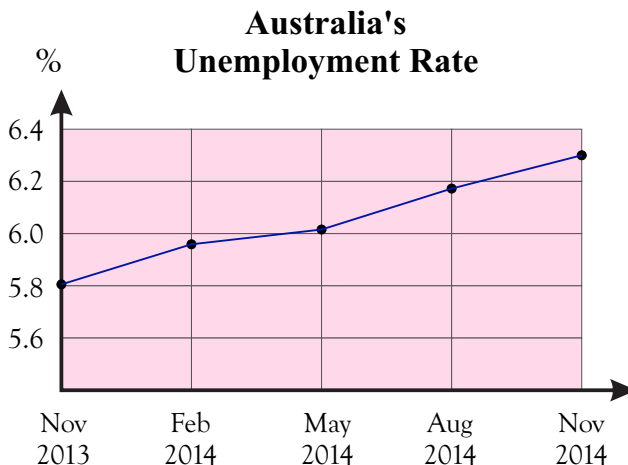
EXERCISE 24B

1. The diagrams below shows the resources that were used to produce electricity in Australia in 2010.



Source: Clean Energy Council Australia 2010

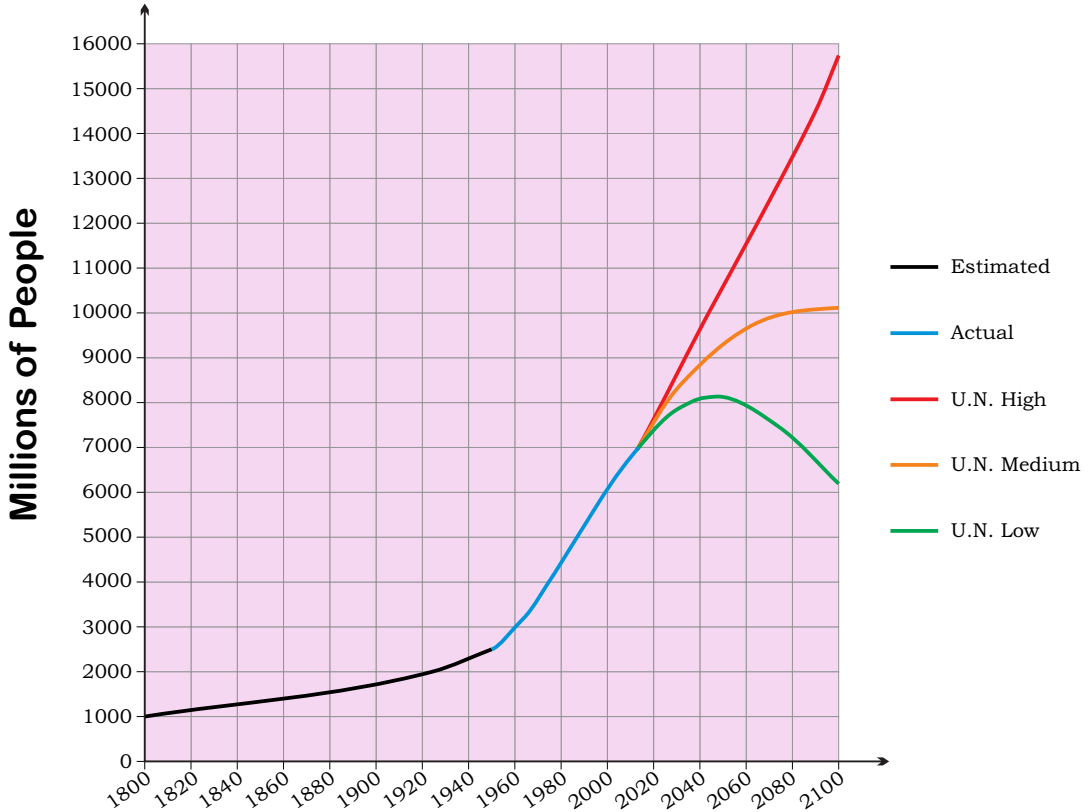
- (a) If the Australian Government changed the amount of energy to be produced from renewable sources to 25%, what would be the percentage that would be produced from fossil fuels?
- (b) Calculate the percentage of renewable energy that was solar energy.
2. The graph below shows Australia's unemployment rate between November 2013 and November 2014.



Source: ABS

- (a) What was Australia's unemployment rate in November 2013?
- (b) What was Australia's unemployment rate in November 2014?
- (c) What was the change in the unemployment rate in the 12 month period?
- (d) In which three month period was the rise in unemployment rate the *least*?

3. The graph below shows the *estimated* world population between 1800 and 1950, the *actual* population between 1950 and 2015 and three *predictions* of the population between 2015 and 2100. These predications were made by the United Nations (U.N.).

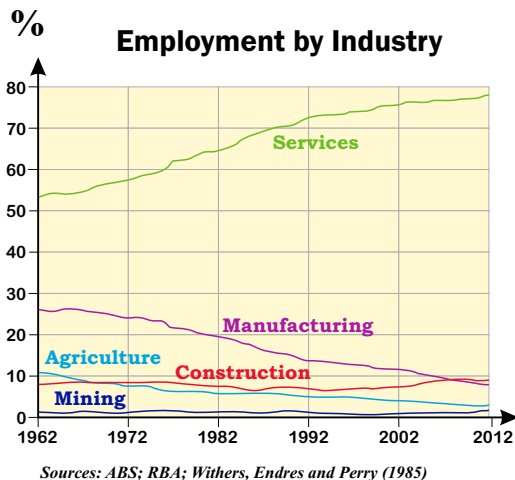


Source: United Nations (U.N.)

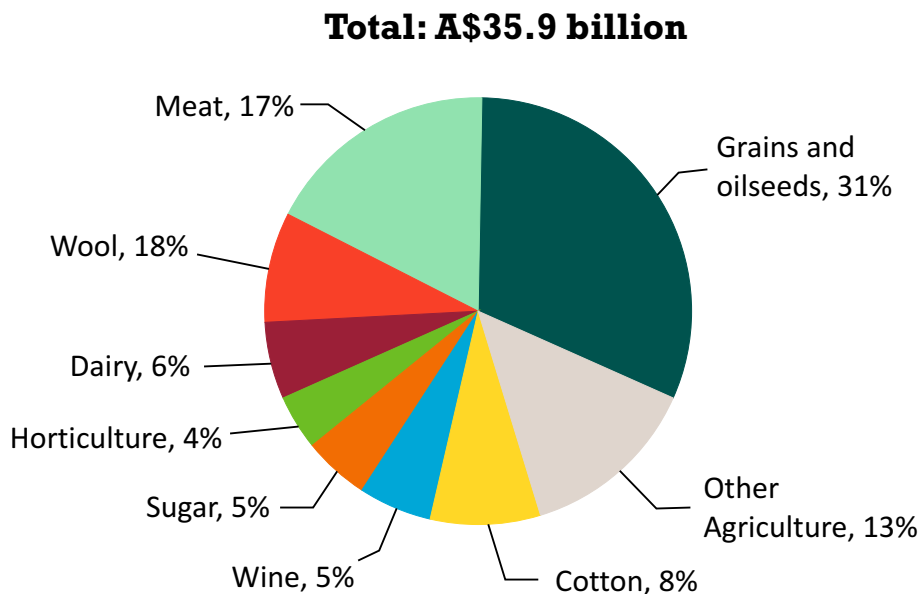
- What was the world's population in 1950?
- What was the world's population in 2015?
- What was the estimated world's population in 1800?
- What was the increase in world's population between 1800 and 1950?
- What was the increase in world's population between 1950 and 2015?
- What is the approximate highest estimate of the world's population in 2100?
- What is the approximate lowest estimate of the world's population in 2100?
- What factors could lead to the low estimate?
- Why are the population figures between 1800 and 1950 estimates?

4. This graph shows the percentage of Australia's workers that were employed in each industry between 1962 and 2012.

- (a) Approximately what percentage of Australia's workers were employed in each industry in:
 (i) 1962? (ii) 2012?
 (Remember they must add to 100%)
- (b) Which industry had the greatest *increase*?
- (c) Which industry had the greatest *decrease*?



5. The graph below shows Australian farm exports for 2011-12.



Source: ABARES 2012

- (a) The source of this graph was ABARES. What do these initials stand for?
- (b) Can you find a mistake with this graph?

PROJECT

1. Find an item of secondary data from the internet, magazine, newspaper, etc..
2. State where you found it.
3. State the source of the data.
4. Describe the information that could be obtained from the data.
5. Describe who or what organisation could use the data.
6. Describe how the information from the data could be used.

PUZZLE

Find the following words in this puzzle.

CATEGORICAL CONTINUOUS DATA DISCRETE NOMINAL
 NUMERICAL ORDINAL POPULATION PRIMARY QUALITATIVE
 QUANTITATIVE SAMPLE SECONDARY STATISTICS SURVEY

E	Z	I	U	H	Q	P	O	K	E	L	A	I	S	V
V	S	O	Y	T	C	J	P	Z	C	E	V	E	E	N
I	W	R	I	J	J	L	O	G	V	F	O	N	C	S
T	P	D	N	U	M	E	R	I	C	A	L	O	O	Z
A	D	I	S	C	R	E	T	E	S	C	N	I	N	R
T	A	N	U	P	N	A	U	C	Z	T	Y	T	D	B
I	Q	A	I	L	T	Y	I	X	I	V	C	A	A	V
T	B	L	J	I	B	T	U	N	B	U	J	L	R	B
N	P	D	L	K	S	N	U	F	S	Z	Y	U	Y	I
A	L	A	C	I	R	O	G	E	T	A	C	P	V	F
U	U	T	T	M	U	M	N	G	C	H	M	O	S	C
Q	P	A	C	S	U	I	L	X	J	L	E	P	Y	U
W	T	M	S	R	W	N	Q	Y	D	H	A	C	L	C
S	M	P	R	I	M	A	R	Y	E	V	R	U	S	E
I	O	R	F	E	C	L	U	R	V	R	D	U	B	D

CHAPTER REVIEW

1. State if the following data is **numerical** or **categorical**.
 - (a) The amount of milk produced by each cow on a dairy farm.
 - (b) The times taken for swimmers to swim 100 metres.
 - (c) The preferred dessert by the customers in a restaurant.
 - (d) The number of cars taken on each crossing of a ferry.
2. State if the following numerical data is **discrete** or **continuous**.
 - (a) The number of passengers on each flight of an aeroplane.
 - (b) The time taken for an aeroplane to complete a flight.
 - (c) The height of trees in a national park.
 - (d) The number of trees in a national park.
3. State if the following categorical data is **nominal** or **ordinal**.
 - (a) The nationality of people visiting an art exhibition.
 - (b) The quality of customer service in a large retail store.
 - (c) The level of printing quality of a number of printers.
 - (d) The type of camera used by a number of photographers.
4. State if the following sets of data are **primary** or **secondary**.
 - (a) A student studies the data on the population of a city over 100 years.
 - (b) A local government worker counts the number of people using a bicycle path.
 - (c) Medical researchers study the affect of a medical trial on people who were trial patients.
 - (d) An education expert investigates how the number of students studying the sciences has changed over the last 20 years.
5. Calculate the minimum sample size that should be used in the following surveys or studies.
 - (a) A city with 300 000 people.
 - (b) A seal colony estimated at 900 seals.
6. When conducting a survey or a study state two important guidelines that need to be followed to ensure the results are as accurate as possible.

Data 2

Frequency Distribution Tables

One easy way of presenting data is a *frequency distribution table*. The *frequency* is the number of times that a type of data occurs. A frequency distribution table consists of three columns:

<i>Data</i>	<i>Tally</i>	<i>Frequency</i>
-------------	--------------	------------------

Frequency distribution tables can be used to display both categorical and numerical data.

Example

50 people were asked what pet they owned and the results are shown in the frequency distribution table below.

<i>Pet</i>	<i>Tally</i>	<i>Frequency</i>
Dog		14
Cat		7
Bird		3
Other		16
None		10
<i>Total</i>		50

In the *Tally* column each tally mark represents one person. Note that the fifth tally mark is drawn through the last four marks to make it easier to find the frequency for each item of data.

EXERCISE 25A

1. Copy and complete the following frequency distribution tables by filling in the frequency of each item and the total.
- (a) A number of people were surveyed and asked how many movies they watched at a cinema last year. The results are shown in this table.

<i>Number of Movies</i>	<i>Tally</i>	<i>Frequency</i>
0		
1		
2		
3		
4		
5		
more than 5		
Total		

- (b) A number of people were surveyed and asked which area of Government they thought was most important. The results are shown in the table below.

<i>Area of Government</i>	<i>Tally</i>	<i>Frequency</i>
Defence		
Education		
Employment		
Environment		
Foreign Affairs		
Health		
Social Services		
Total		

Example

Complete a frequency distribution table for the following data.

50 people were surveyed and asked their favourite fruit. Their responses are shown below.

apple, mango, pineapple, banana, watermelon, strawberry,
 pineapple, apple, mango, orange, mango, grapes, strawberry,
 banana, grapes, mango, banana, watermelon, banana, mango,
 watermelon, mango, watermelon, grapes, apple, watermelon,
 strawberry, strawberry, strawberry, banana, strawberry,
 pineapple, strawberry, orange, strawberry, mango, banana,
 strawberry, grapes, mango, mango, orange, mango, grapes,
 mango, watermelon, mango, orange, watermelon, pineapple

Step 1 Find out how many different fruits have been listed.

There are eight: apple, mango, banana, watermelon, strawberry,
 pineapple, orange, grapes.

Step 2 Draw a frequency distribution table with eight rows.

Step 3 Working through the list make a tally mark for each item of fruit. Using a pencil make a soft line through each item as it is included in the table. This will help to avoid mistakes when the data is transferred to the frequency distribution table.

Step 4 Complete the table by filling in the frequencies and total.

<i>Favourite Fruit</i>	<i>Tally</i>	<i>Frequency</i>
Apple		3
Mango		12
Banana		6
Watermelon		7
Strawberry		9
Pineapple		4
Orange		4
Grapes		5
<i>Total</i>		50

2. Complete a frequency distribution table for each of the following sets of data.

- (a) A number of students were asked what their favourite colour was and the results are listed below.

blue, purple, yellow, red, blue, black, blue, green,
green, blue, purple, red, yellow, orange, red, black,
yellow, blue, orange, black, purple, green, red, orange,
green, blue, blue, green, yellow, orange, green, yellow,
blue, green, red, blue, yellow, orange, purple, blue

- (b) A zoologist was studying dingoes and recorded the number of pups in litters found in a National Park in the Northern Territory. The number of pups in each litter is recorded below.

1, 7, 6, 4, 6, 7, 6, 5, 1, 4, 2, 5, 8, 7, 8, 5, 4, 5, 6, 2, 5, 5, 7, 8, 7, 3, 4,
4, 8, 7, 6, 4, 3, 5, 7, 5, 8, 5, 4, 6, 7, 6, 5, 4, 5, 2, 5, 4, 5, 2, 5, 5, 3, 5,
6, 3, 5, 6, 4, 5, 4, 6, 3, 6, 5, 4, 6, 2, 5, 3, 6, 4, 1, 6, 5, 3, 4, 6, 5, 3, 6,
3, 4, 6, 7, 1, 6, 4, 6, 6, 7, 2, 1, 7, 5, 7, 2, 3, 8, 7, 6, 1, 4, 7, 7, 4, 7, 5
8, 2, 8, 3, 5, 4, 8, 5, 8, 3, 8, 5, 3, 7, 2, 4, 8, 1, 3, 6, 5, 6, 2, 4, 8, 1, 7



Grouped Data

It is sometimes easier to record numerical data in groups.

Example

The runs scored by a cricketer in 30 innings are listed below and recorded in the frequency distribution table.

14, 72, 21, 36, 50, 19, 23, 0, 47, 15, 8, 61, 65, 106, 96
16, 7, 21, 36, 19, 10, 84, 1, 67, 42, 30, 120, 11, 13, 87

<i>Runs</i>	<i>Tally</i>	<i>Frequency</i>
0 - 25		15
- 50		6
- 75		4
- 100		3
- 125		2
<i>Total</i>		30

EXERCISE 25B

1. The ages of 50 people who attended a festival are shown below.
Copy and complete the frequency distribution table for this data.

45, 31, 18, 52, 33, 27, 9, 42, 24, 20, 44, 58, 16, 27, 33, 14, 7,
31, 63, 8, 14, 46, 55, 8, 70, 13, 39, 86, 5, 23, 27, 36, 38, 43,
26, 52, 17, 11, 38, 22, 9, 25, 30, 41, 14, 28, 23, 28, 37, 18

<i>Age</i>	<i>Tally</i>	<i>Frequency</i>
0 - 9		
10 -		
20 -		
30 -		
40 -		
50 -		
60 -		
<i>Total</i>		50

2. A group of scientists at Antarctica were studying penguins. As part of the studies they weighed a number of penguins and those weights (in kg) are listed below. Copy and complete the frequency distribution table.

7.5	5.6	5.2	6.8	3.9	1.2	2.6	4.7	9.5	4.1	5.0	3.3	6.9	6.4	5.2
3.6	2.1	7.0	7.3	6.5	5.2	4.8	8.4	7.7	9.0	6.4	5.6	8.9	1.8	8.1
1.1	9.2	1.9	3.0	6.0	8.3	3.7	4.1	7.1	7.8	6.3	8.7	2.8	1.9	3.8
8.0	1.0	5.7	6.6	9.8	3.5	5.7	7.2	6.0	4.6	7.1	1.9	4.3	7.2	5.9

Weight (kg)	Tally	Frequency
0 - 2		
- 4		
- 6		
- 8		
- 10		
Total		

3. The annual salaries of a number of employees at a company are shown below. Copy and complete the frequency distribution table.

\$54,600	\$76,250	\$38,700	\$33,250	\$24,950	\$38,700	\$44,750
\$65,500	\$44,750	\$68,500	\$24,500	\$56,750	\$44,750	\$44,750
\$65,400	\$48,750	\$44,750	\$48,750	\$24,800	\$60,200	\$48,750
\$44,750	\$48,750	\$24,950	\$78,750	\$44,750	\$48,750	\$72,700
\$85,500	\$57,750	\$44,750	\$59,750	\$82,750	\$44,750	\$29,500

Annual Salary	Tally	Frequency
0 - \$25,000		
- \$50,000		
- \$75,000		
- \$100,000		
Total		

Dot Plots

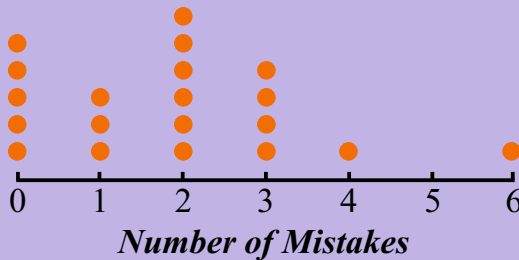
A dot plot is an easy way to represent and compare data.

Example 1

20 students were asked to spell ten words. The number of words spelt incorrectly by each student is shown below:

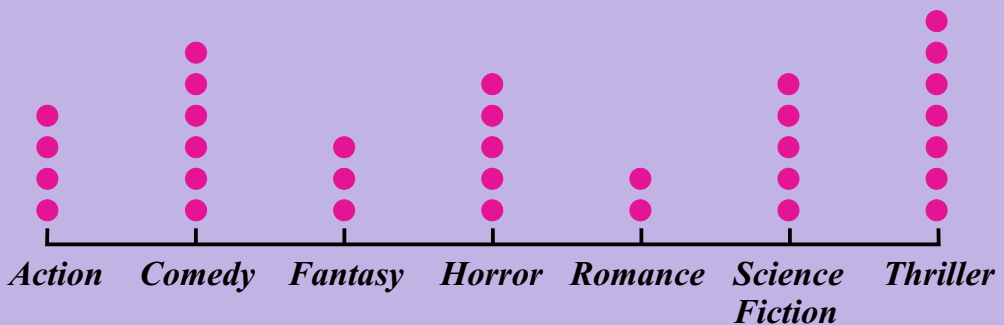
0, 2, 3, 1, 2, 0, 4, 2, 1, 0, 3, 2, 6, 2, 3, 0, 1, 0, 2, 3

A dot plot can be used to display this data.



Example 2

A number of people were asked their favourite type of movie and the results are shown on this dot plot.



1. How many people were surveyed?
2. What was the most popular type of movie?

Answers

1. There were 32 people surveyed (add the number of dots).
2. The most popular type of movie was **thriller** (7 people).

EXERCISE 25C

1. A number of people were surveyed and asked how many overseas holidays they had been on. The results are shown below.

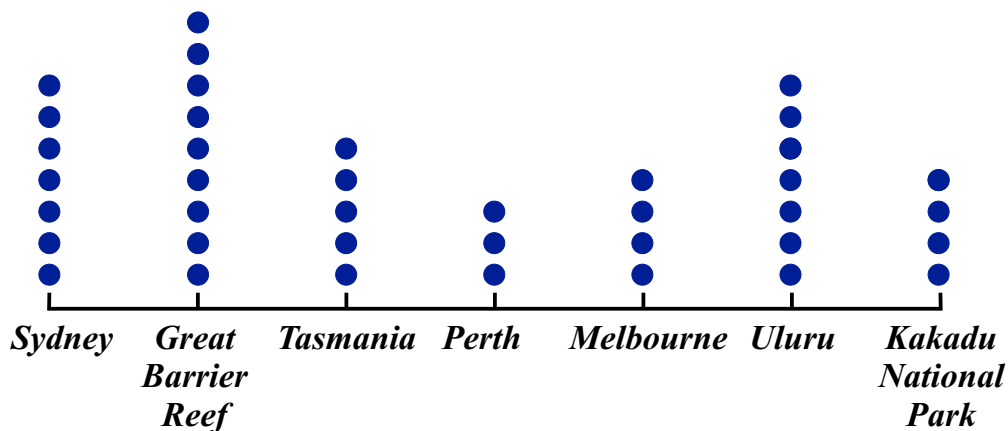
1, 2, 4, 0, 1, 0, 0, 1, 3, 2, 0, 1, 2, 2, 1, 5, 1, 3, 0, 2, 1, 0, 2, 3, 4

- (a) Draw a dot plot to display this data.
 (b) How many people from the survey have not been on an overseas holiday?
2. Twenty-five people were surveyed and asked their favourite animal at the zoo. The results are shown below.

monkey, lion, meerkat, giraffe, monkey,
 penguin, elephant, meerkat, elephant,
 lion, meerkat, monkey, elephant, meerkat,
 monkey, lion, giraffe, seal, penguin,
 monkey, meerkat, seal, elephant,
 monkey, lion



- (a) Draw a dot plot to display this data.
 (b) From the people surveyed which animal was the most popular?
3. A number of people were surveyed and asked their favourite place to visit in Australia. The results are shown on the dot plot below.

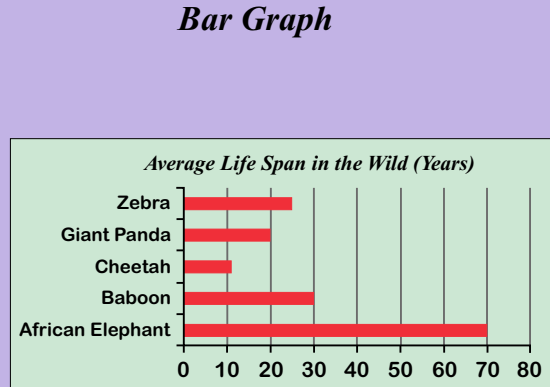
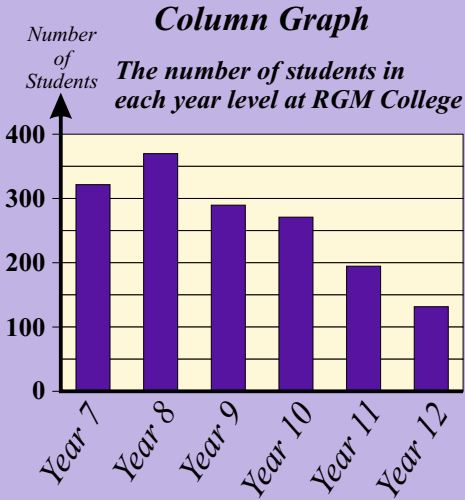


- (a) How many people were surveyed?
 (b) From the people surveyed which of the places listed was the most popular?

Column Graphs and Bar Graphs

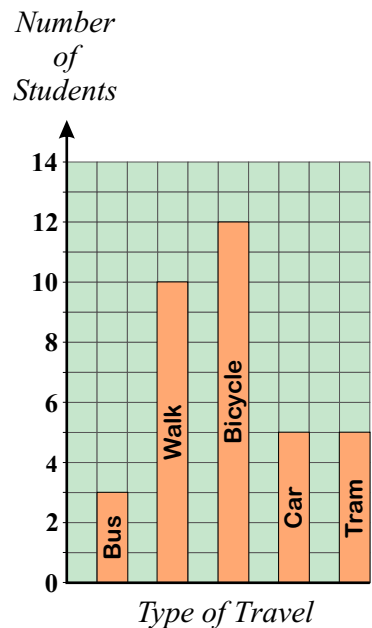
Another way to display and compare data is on column graphs and bar graphs. The main difference between these two types of graphs is that column graphs are vertical and bar graphs are horizontal.

Examples

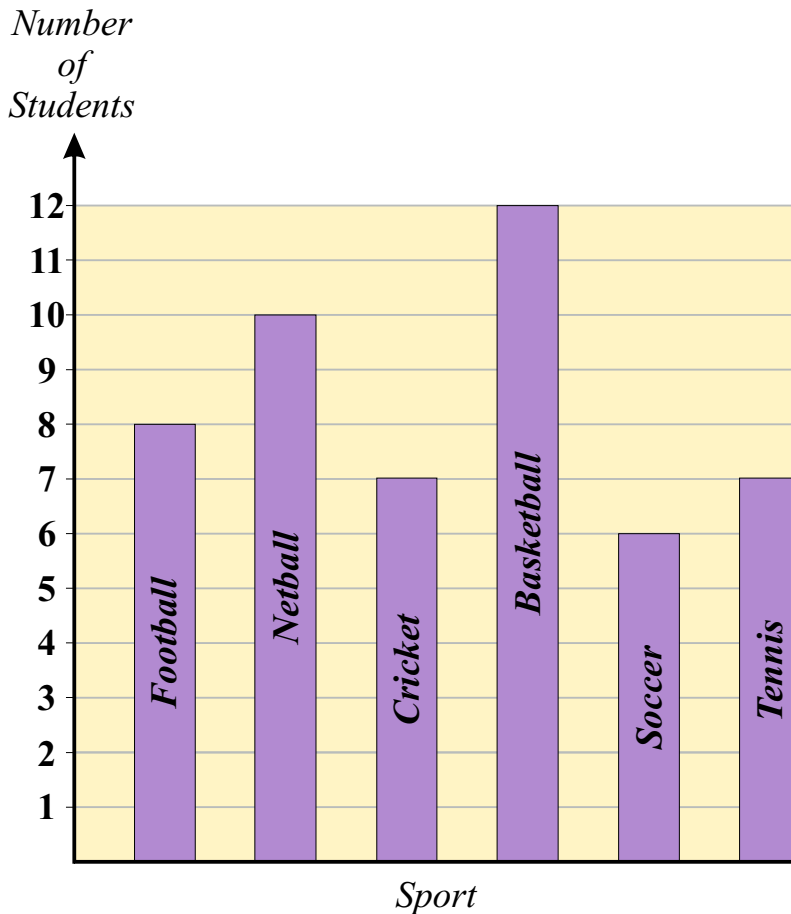


EXERCISE 25D

1. A number of students at T.R.C. College were surveyed and asked how they travelled to school. The results are shown in this column graph.
 - (a) How many students used each of the types of travel?
 - (i) Bus (ii) Walk (iii) Bicycle
 - (iv) Car (v) Tram
 - (b) How many students were surveyed?
 - (c) From the information in the graph is T.R.C. College more likely to be:
 - A** In a large city
 - B** In a small country town

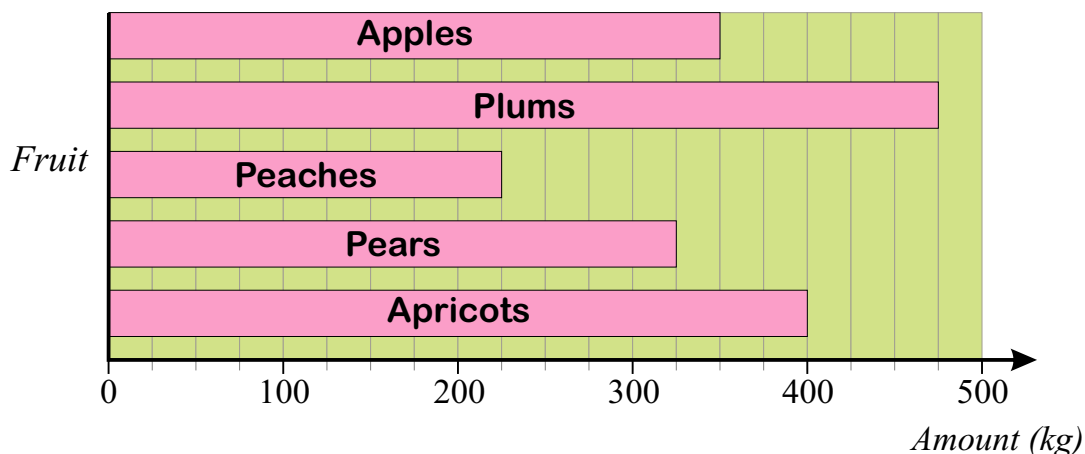


2. A number of students were asked which sport they preferred to play. The results are shown on the column graph below.



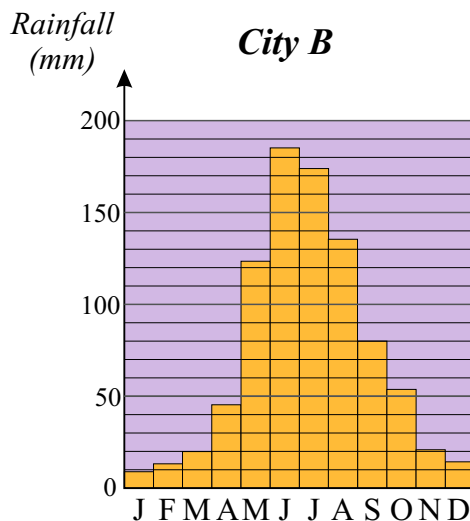
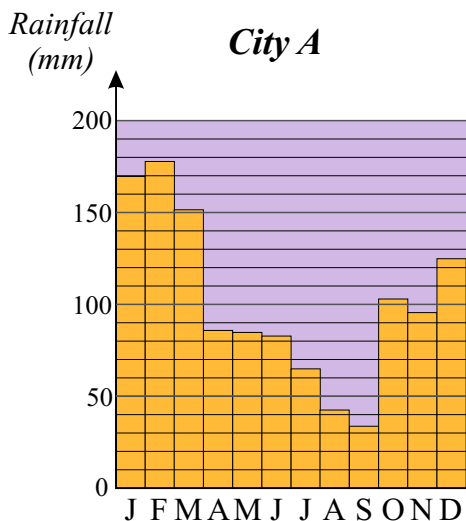
- (a) Which sport was the most popular?
- (b) Which sport was played by the least number of students surveyed?
- (c) Which sports had the same number of students play them?
- (d) If there were four boys who preferred to play tennis, how many girls preferred to play tennis?
- (e) There were the same number of boys as girls that chose basketball. How many girls preferred to play basketball?
- (f) One third of the soccer players were girls. How many girls preferred to play soccer?
- (g) How many students were surveyed?

3. I. M. Picken is a fruit farmer. The bar graph below shows the amount (in kg) of each type of fruit he produced in one year.



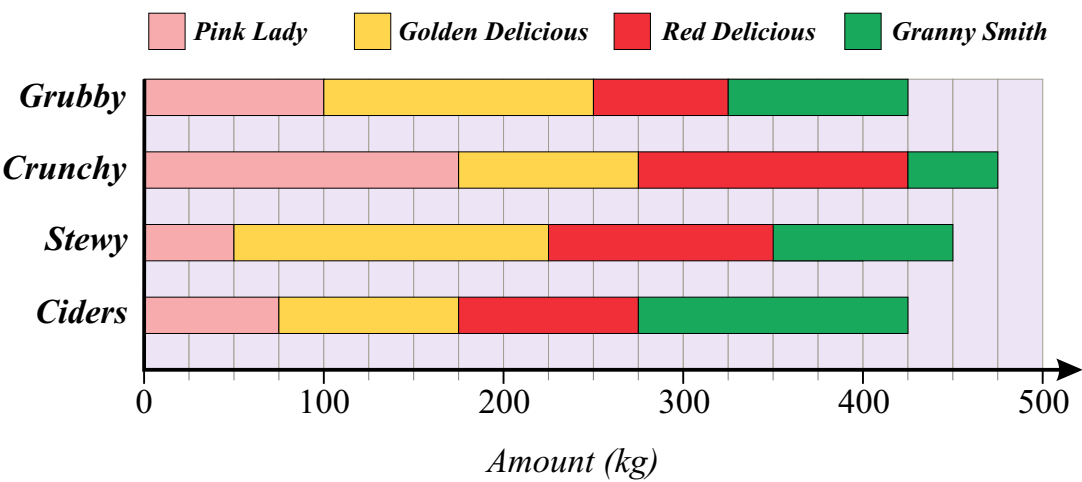
Find the amount (in kg) of each type of fruit he produced.

4. The average monthly rainfall for each month for two Australian cities is shown on the graphs below.



- What is the approximate average December rainfall for city A?
- Which month has the highest average rainfall for city B?
- Find the approximate average *winter* rainfall for city A.
- Find the approximate average *annual* rainfall for city B.
- The information on the two graphs are the actual figures for Perth and Brisbane. Use your knowledge of Australian climate to determine which is City A.

5. Grubby, Crunchy, Stewy and Ciders are apple growers. They each grow four varieties of apples - Pink Lady, Golden Delicious, Red Delicious and Granny Smith. The graph below shows the amount (in kg) of each variety of apple that the growers produced in one year.

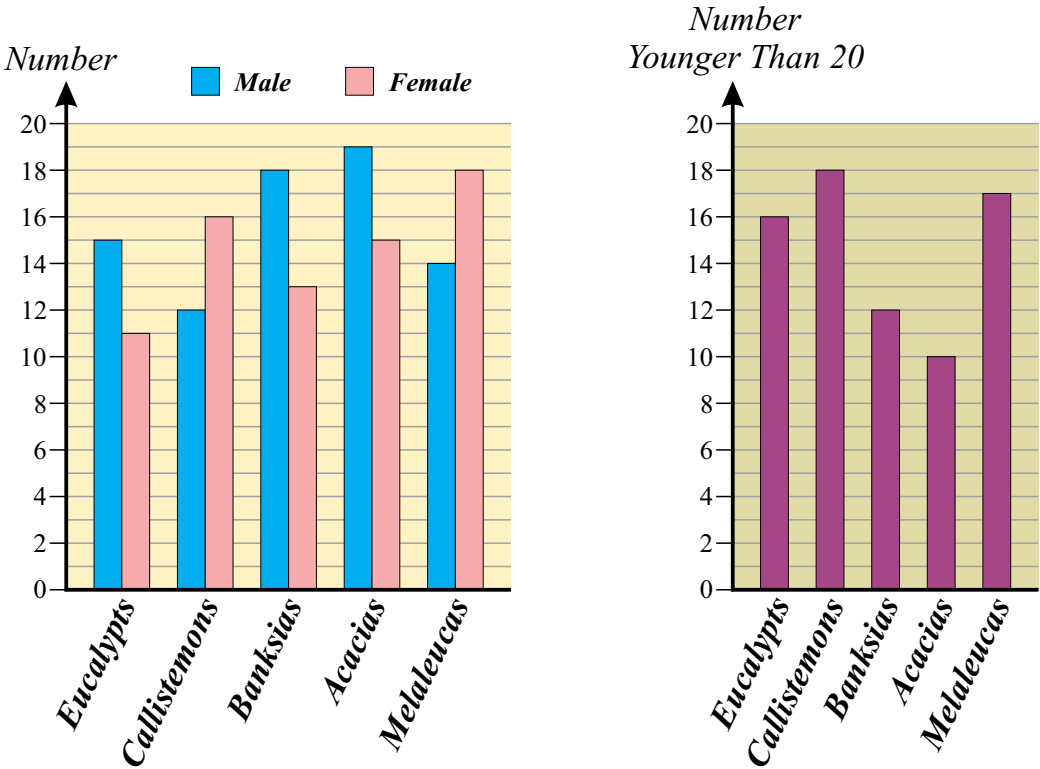


Copy and complete the table below showing the amount of each variety of apple and the total amount of apples that the growers produced.

	Amount (kg)				
Grower	Pink Lady	Golden Delicious	Red Delicious	Granny Smith	Total
Grubby					
Crunchy					
Stewy					
Ciders					



6. All the volunteers at a tree planting day were divided into five groups. Each group was given a different variety of tree to plant - *eucalypts*, *callistemons*, *banksias*, *acacias* and *melaleucas*. The first column graph below shows the number of males and females in each group. The second column graph shows the number of volunteers in each group who were younger than 20.



- (a) Find the total number of volunteers in each group:
(i) eucalypts (ii) callistemons (iii) banksias (iv) acacias (v) melaleucas
- (b) Find the total number of volunteers.
- (c) Find the number of volunteers in each group who were aged 20 or older:
(i) eucalypts (ii) callistemons (iii) banksias (iv) acacias (v) melaleucas
- (d) Rearrange the letters **HOTTER BULBS** to find a common name for a *callistemon*.
- (e) Fill in the letters in the word below to find a common name for an *acacia*.

W _ _ T _ E

- (f) What is a common name for a *eucalypt*?

Stem-and-Leaf Plots

Stem-and-leaf plots are a convenient way of arranging numerical data in order from smallest to largest.

Example

Construct a stem-and-leaf plot for the following numbers.

17, 26, 30, 19, 42, 38, 21, 33, 46, 12, 32, 40, 24, 35

Step 1 Note the number of digits in each of the data items.
In this example all the numbers have *two* digits.

Step 2 Note the range of numbers of the first digit - this is called the *stem*.
In this example the first digit ranges from 1 to 4.

Step 3 Construct a stem-and-leaf plot with these numbers (first digits) as the *stems*.

<i>Stem</i>	<i>Leaf</i>
1	
2	
3	
4	

Step 4 Complete the stem-and-leaf plot by using the second digit of each item of data as the *leaf*. Work through the data items from left to right. This is called a *non-ordered stem-and-leaf plot*.

<i>Stem</i>	<i>Leaf</i>
1	7 9 2
2	6 1 4
3	0 8 3 2 5
4	2 6 0

Step 5 Check that the number of items listed on the stem-and-leaf plot is the same as the original data list to make sure you haven't missed any data. In this example there 14 items of data.

Step 6 Arrange each row of digits in the leaf column in order from the smallest to the largest. This is called an *ordered stem-and-leaf plot*.

Stem	Leaf
1	2 7 9
2	1 4 6
3	0 2 3 5 8
4	0 2 6

Step 7 Include a key to show what numbers the stem-and-leaf plot represent.

$$\begin{array}{c|c} & \\ 1 & 2 \end{array} = 12$$

EXERCISE 25E

1. For the set of numerical data items listed below:

- construct a non-ordered stem-and-leaf plot.
- construct an ordered stem-and-leaf plot.
- state the number of data items.
- state the smallest data item.
- state the largest data item.

45, 37, 51, 23, 35, 50, 41, 38, 30, 42, 53, 29, 38, 38, 57, 40, 31, 51

2. The ages of a number of people who attended a concert are shown below.

33, 20, 41, 56, 18, 62, 38, 45, 60, 40, 42, 38, 31, 18, 22, 32, 28, 23, 28,
41, 20, 29, 50, 42, 37, 28, 26, 29, 31, 27, 19, 25, 39, 34, 27, 26, 25, 33

For this list of ages:

- construct a non-ordered stem-and-leaf plot.
- construct an ordered stem-and-leaf plot.
- state the number of people who were asked their age.
- state the age of the youngest person asked.
- state the age of the oldest person asked.

Example 1

Construct a stem-and-leaf plot for the following data list.

125, 134, 140, 152, 138, 122, 131, 158, 149, 140, 151, 136, 139, 144

For numerical data with three digits use the first two digits as the stems.

Non-ordered stem-and-leaf plot

Stem	Leaf
12	5 2
13	4 8 1 6 9
14	0 9 0 4
15	2 8 1

$$\overline{12} \mid 5 = 125$$

Ordered stem-and-leaf plot

Stem	Leaf
12	2 5
13	1 4 6 8 9
14	0 0 4 9
15	1 2 8

$$\overline{12} \mid 5 = 125$$

Example 2

Construct a stem-and-leaf plot for the following data list.

8.3, 5.6, 4.2, 2.7, 5.3, 7.7, 3.7, 8.4, 7.0, 4.8, 5.1, 7.7, 2.5, 3.7, 5.8, 3.5, 4.2, 7.6, 3.2, 5.9, 4.0, 3.0, 5.9, 4.2, 5.7, 8.0, 3.6, 4.9

Non-ordered stem-and-leaf plot

Stem	Leaf
2	7 5
3	7 7 5 2 0 6
4	2 8 2 0 2 9
5	6 3 1 8 9 9 7
6	
7	7 0 7 6
8	3 4 0

$$\overline{2} \mid 5 = 2.5$$

Ordered stem-and-leaf plot

Stem	Leaf
2	5 7
3	0 2 5 6 7 7
4	0 2 2 2 8 9
5	1 3 6 7 8 9 9
6	
7	0 6 7 7
8	0 3 4

$$\overline{2} \mid 5 = 2.5$$

EXERCISE 25F

1. The heights (in cm) of the students in a year 7 class are shown below.

**163, 170, 159, 163, 151, 172, 164, 169, 158, 162, 160, 158, 178,
179, 182, 168, 171, 173, 164, 173, 159, 160, 170, 172, 163, 172**

- Construct a non-ordered stem-and-leaf plot for this data.
 - Construct an ordered stem-and-leaf plot for this data.
 - What is the height of the shortest student in the class?
 - What is the height of the tallest student in the class?
 - How many students are in the class?
2. The weights (in kg) of a number of dolphins studied near Byron Bay are shown below.

**215, 287, 254, 201, 293, 235, 262,
227, 230, 241, 219, 299, 239, 245,
261, 229, 209, 217, 220, 231, 242,
251, 248, 250, 210, 227, 283, 237,
251, 244, 219, 223, 247, 237, 239,
227, 231, 233, 204, 217, 228, 240**

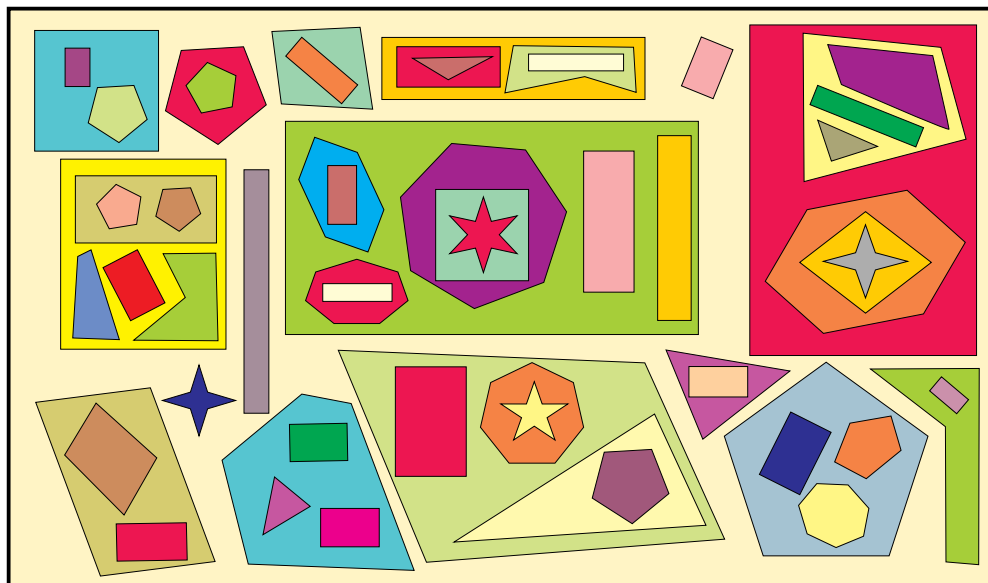


- Construct a non-ordered stem-and-leaf plot for this data.
 - Construct an ordered stem-and-leaf plot for this data.
 - What was the weight of the smallest dolphin in the data list?
 - What was the weight of the largest dolphin in the data list?
 - How many dolphins were studied?
3. In a school athletics meeting twelve students competed in the shot put event. The best distances thrown by each student (in metres) is shown below.

8.7, 7.6, 6.9, 8.3, 7.3, 7.8, 8.0, 9.1, 8.8, 7.9, 9.0, 7.7

- Construct a non-ordered stem-and-leaf plot for this data.
- Construct an ordered stem-and-leaf plot for this data.
- What was the furthest distance thrown?
- What was the shortest distance thrown?

4. Thirty students recorded how long (in seconds) it took each of them to find the *five triangles* in this diagram. The times they took are shown below.



3.5, 4.8, 5.3, 1.9, 2.6, 6.7, 4.6, 3.8, 2.8, 3.8, 4.7, 3.1, 4.2, 4.3, 4.2, 2.9, 3.8, 2.5, 3.7, 8.2, 4.3, 5.3, 2.7, 1.8, 3.1, 3.7, 2.4, 2.1, 3.7, 4.0

- Construct a non-ordered stem-and-leaf plot for this data.
 - Construct an ordered stem-and-leaf plot for this data.
 - What was the fastest time?
 - What was the slowest time?
5. The maximum temperatures (in degrees °) on each day in January in Alice Springs in a particular year are shown below.

42.5, 38.7, 33.4, 29.8, 31.5, 36.7, 42.3, 44.5, 40.7, 39.6, 37.2, 36.5, 33.8, 38.5, 40.3, 39.1, 37.3, 35.4, 32.7, 38.5, 32.6, 33.5, 37.8, 35.9, 30.6, 29.6, 34.6, 37.1, 38.0, 39.0, 34.7

- Construct a non-ordered stem-and-leaf plot for this data.
- Construct an ordered stem-and-leaf plot for this data.
- What was the hottest temperature recorded?
- What was the lowest temperature recorded?

Back-to-Back Stem-and-Leaf Plots

A back-to-back stem-and-leaf plot can be used to compare two sets of numerical data.

Example

The points scored by two netball teams in 12 rounds of a competition are listed below.

Blue Birds: 34, 46, 53, 38, 47, 29, 50, 42, 33, 40, 36, 32

Red Robins: 28, 31, 22, 18, 23, 31, 30, 17, 28, 33, 20, 26

A back-to-back stem-and-leaf plot shows both teams' scores.

The stem is shared by both sets of data.

One team's scores are shown on the right side of the stem being recorded left to right.

The other team's scores are shown on the left side of the stem being recorded right to left.

Non-ordered back-to-back stem-and-leaf plot

Blue Birds		Red Robins	
Leaf	Stem	Leaf	
	1	8 7	
9	2	8 2 3 8 0 6	
2 6 3 8 4	3	1 1 0 3	
0 2 7 6	4		
0 3	5		

$$\begin{array}{r} 2 \overline{) 5} = 25 \end{array}$$

Ordered back-to-back stem-and-leaf plot

Blue Birds		Red Robins	
Leaf	Stem	Leaf	
	1	7 8	
9	2	0 2 3 6 8 8	
8 6 4 3 2	3	0 1 1 3	
7 6 2 0	4		
3 0	5		

$$\begin{array}{r} 2 \overline{) 5} = 25 \end{array}$$

EXERCISE 25G

1. The heart rate (in beats per minute) of a number of people was recorded before and after they exercised. The results are shown in the ordered back-to-back stem-and-leaf plot below.

<i>Heart Rate Before Exercise</i>		<i>Heart Rate After Exercise</i>	
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>	
9 8	5		
9 8 6 6 5 5 3 1	6	9	
8 7 7 4 4 3 2 1 0 0	7	3 5 8 8	
	8	0 1 2 3 6 7 8 8 9 9	
	9	0 1 2 4 6	

$$\begin{array}{c|c} 6 & 9 \end{array} = 69$$

- (a) How many people had their heart rates recorded?
 - (b) What was the slowest heart rate before they exercised?
 - (c) What was the fastest heart rate before they exercised?
 - (d) What was the slowest heart rate after they exercised?
 - (e) What was the fastest heart rate after they exercised?
2. The age of all the females and males who attended an art course are shown in the ordered back-to-back stem-and-leaf plot below.

<i>Age of Females</i>		<i>Age of Males</i>	
<i>Leaf</i>	<i>Stem</i>	<i>Leaf</i>	
9 9 8 7	1		
9 8 6 6 5 3 2 1 0	2	5 6 6 7 9	
8 7 7 4 4 3 2 1 0 0	3	0 1 2 3 5 8 8	
9 4 3	4	0 1 1 2 3 9	

$$\begin{array}{c|c} 1 & 9 \end{array} = 19$$

- (a) How many females attended the course?
- (b) How many males attended the course?
- (c) What was the age of the youngest female who attended the course?
- (d) What was the age of the oldest female who attended the course?
- (e) What was the age of the youngest male who attended the course?
- (f) What was the age of the oldest male who attended the course?

3. A Year 7 mathematics class was set a practice exam, then given two weeks of revision before being set the main exam. The percentage marks for all the students for the practice exam and main exam are shown below.

Percentage marks for practice exam:

45, 56, 27, 48, 38, 58, 40, 50, 63, 52, 48, 39,
41, 52, 37, 49, 36, 43, 47, 39, 28, 45, 40, 47

Percentage marks for main exam:

54, 68, 72, 85, 93, 85, 70, 67, 93, 82, 84, 72,
71, 62, 73, 80, 63, 57, 74, 81, 82, 67, 70, 74

- Construct a non-ordered back-to-back stem-and-leaf plot to display this data.
 - Construct an ordered back-to-back stem-and-leaf plot to display this data.
 - What was the lowest mark on the practice exam?
 - What was the highest mark on the practice exam?
 - What was the lowest mark on the main exam?
 - What was the highest mark on the main exam?
4. Con is a motor mechanic and Cam is his apprentice. The times (in minutes) they each took to service a number of cars are shown below.

Con: 58, 67, 72, 63, 59, 70, 62, 68, 76, 72, 82, 57, 84, 68, 71, 70

Cam: 86, 93, 102, 109, 93, 90, 84, 97, 90, 91, 105, 97, 89, 99, 91

- Construct a non-ordered back-to-back stem-and-leaf plot to display this data.
- Construct an ordered back-to-back stem-and-leaf plot to display this data.
- What was the quickest time that Con took to service a car?
- What was the slowest time that Con took to service a car?
- What was the quickest time that Cam took to service a car?
- What was the slowest time that Cam took to service a car?

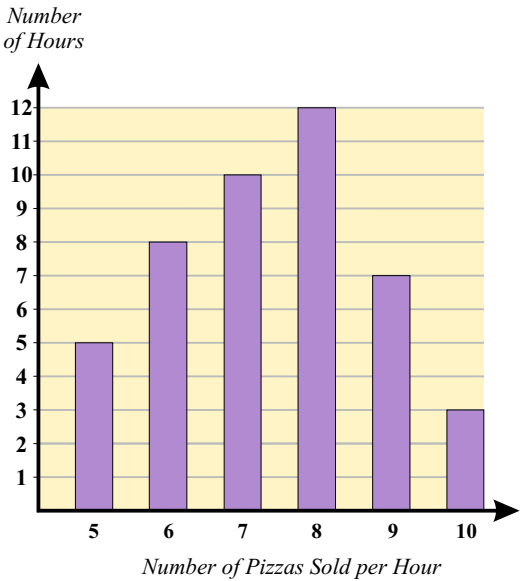
PROBLEM SOLVING

1. A dairy farmer milked 20 cows each day. On one day the amount of milk produced by each cow is shown in this stem-and-leaf plot. On looking at the plot he noticed he had only recorded the amount of milk produced from 19 cows. He noticed that in the milk vat there was a total of exactly 800 litres of milk. How many litres of milk did the cow not recorded on the stem-and-leaf plot produce?

Stem	Leaf
2	8 9
3	0 2 5 6 7 8 9
4	0 1 2 3 3 4 5 7
5	0 1

$$\begin{array}{r} 3 \overline{) 5} = 35 \end{array}$$

2. Rob sells pizzas from his mobile pizza oven for three hours each day for five days each week. He sells his pizzas for \$12 each. He records the number of pizzas he sells every hour and this column graph shows his pizza sales for a number of days.



- (a) How many *weeks* are recorded on the column graph?
- (b) How much did Rob earn from his pizza sales in the period shown?

PUZZLE

Four terms from this chapter are written below but have been partly covered by blobs of paint. Find the four terms.

COLUMN GRAPH

BARGAIN

STEM AND LEAF PLOT

SCATTER DATA

CHAPTER REVIEW

1. A number of people were asked how many brothers and sisters they had. The results are shown below.
Construct a frequency distribution table for this set of data.

0, 1, 3, 0, 1, 1, 2, 4, 3, 2, 1, 1, 1, 0, 1, 2, 2, 1, 0, 1, 1, 0, 2, 3, 1, 1, 2, 2, 1, 1, 1, 0, 2, 3, 4, 5, 1, 2, 2, 1, 1, 1, 0, 2, 3, 1, 2, 0, 1, 1

2. The number of passengers taking a trip on a ferry was recorded every trip for a week. The results are shown below.
Copy and complete the frequency distribution table for this data.

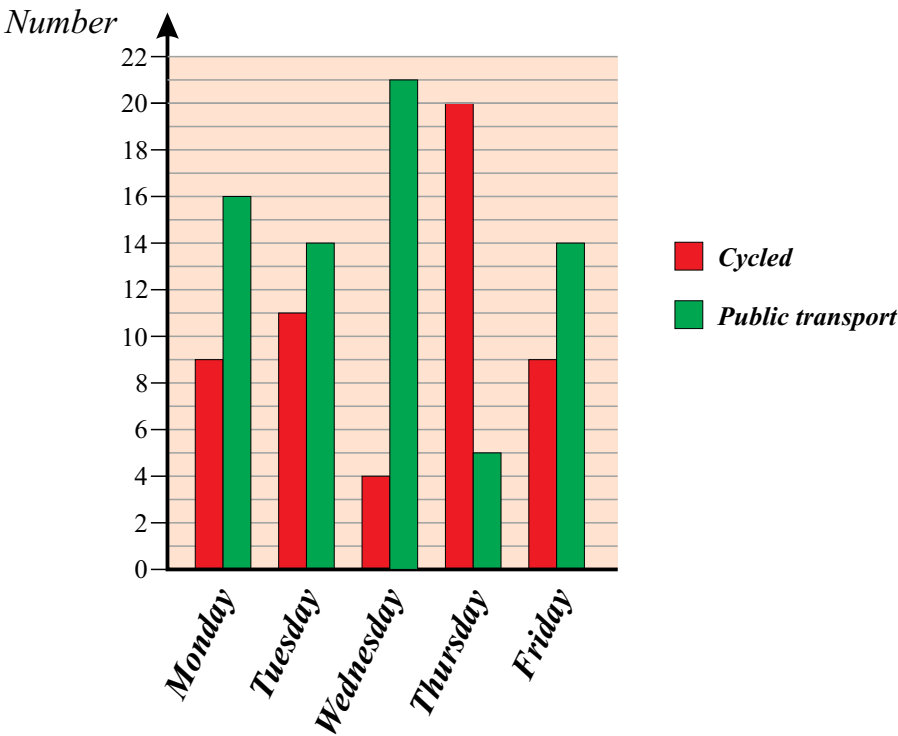
25, 41, 26, 38, 56, 40, 71, 52, 19, 27, 38, 63, 52, 48, 39, 50, 32, 50, 15, 29, 59, 60, 31, 28, 40, 33, 62, 29, 18, 32, 57, 42, 31, 20, 42, 58

<i>Number of Passengers</i>	<i>Tally</i>	<i>Frequency</i>
0 - 20		
- 40		
- 60		
- 80		
<i>Total</i>		

3. A number of Australians at a conference were asked in which state or territory they live. The results are shown below.
- (a) Draw a dot plot to display this data.
 - (b) How many people were surveyed?
 - (c) In which state or territory did the most people surveyed live?

V, NSW, T, NT, ACT, WA, SA, V, Q, T, V, NSW, V, SA, NSW, WA, Q, NSW, T, SA, ACT, V, Q, NT, WA, Q, V, NSW, SA, V, SA, Q, NT

4. The workers at an inner city store either cycled or caught public transport to work. The number who used each way to get to work was recorded for a working week and displayed on the column graph below.



- (a) On which day did two people work at another store?
 - (b) How many people worked at the store?
 - (c) On which day did it rain heavily?
 - (d) Which day was 'ride-to-work day'?
5. A basketball team recorded all the scores they made for a season and all the scores other teams made against them. This data is shown below.
- Scores for:** 57, 68, 48, 50, 47, 69, 72, 48, 53, 42, 59, 44, 68, 71, 66, 58
- Scores against:** 52, 61, 43, 40, 37, 49, 52, 42, 52, 42, 50, 41, 60, 51, 46, 53
- (a) Construct a non-ordered back-to-back stem-and-leaf plot for this data.
 - (b) Construct an ordered back-to-back stem-and-leaf plot for this data.
 - (c) How many games did the team play for the season?
 - (d) What was the highest score made by the team?
 - (e) What was the highest score made against the team?

Data 3

Measures of Centre

There are three measures used to show the centre of a set of numerical data - *mean*, *median* and *mode*.

Mean

The mean is also called the *average*. The mean is calculated by adding all the items of data and dividing by the number of items of data.

Median

The median is the middle number when the data is arranged in numerical order from the smallest to the largest.

Mode

The mode is the number that occurs the most.

Mean

$$\text{Mean} = \frac{\text{the sum of the data items}}{\text{the number of items}}$$

Example 1

Find the mean of the following set of data.

6, 8, 5, 9, 2, 1, 5, 7, 2

$$\begin{aligned}\text{Mean} &= \frac{\text{the sum of the data items}}{\text{the number of items}} \\ &= \frac{6 + 8 + 5 + 9 + 2 + 1 + 5 + 7 + 2}{9} \\ &= \frac{45}{9}\end{aligned}$$

9 ← There are 9 numbers

$$\text{Mean} = 5$$

Example 2

Use a calculator to help find the mean of the following set of data.

34, 44, 50, 66, 29, 37, 81, 63

$$\begin{aligned}
 \text{Mean} &= \frac{\text{the sum of the data items}}{\text{the number of items}} \\
 &= \frac{34 + 44 + 50 + 66 + 29 + 37 + 81 + 63}{8} \\
 &= \frac{404}{8}
 \end{aligned}$$

8 ← There are 8 numbers

Mean = 50.5

Example 3

Find the mean of the following measurements.

7 m, 9 m, 8 m, 4 m, 9 m, 5 m

$$\begin{aligned}
 \text{Mean} &= \frac{\text{the sum of the data items}}{\text{the number of items}} \\
 &= \frac{7 + 9 + 8 + 4 + 9 + 5}{6} \\
 &= \frac{42}{6}
 \end{aligned}$$

6 ← There are 6 numbers

Mean = 7 m

EXERCISE 26A

1. Find the mean of the following sets of data. **Do not** use a calculator.

(a) 4, 7, 2, 1, 3, 8, 6, 1

(b) 8, 10, 3, 12, 2

(c) 2, 4, 5, 5, 7, 9, 10, 12, 9

(d) 15, 20, 45, 50, 60

(e) 6, 0, 7, 3, 1, 0, 4, 2, 0, 7

(f) 0, 0, 1, 2, 3, 4, 4, 5, 6, 7, 13, 15

(g) 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2

(h) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2. Find the mean of the following sets of data. You may use a calculator.
- (a) 0, 8, 12, 31, 26, 18, 27, 26 (b) 45, 16, 9, 37, 24
 (c) 35, 29, 18, 57, 38, 36 (d) 123, 0, 56, 9, 167, 23, 62, 84
 (e) 6.8, 9.7, 13.3, 8.5, 21.7, 15.6 (f) 9.63, 4.59, 17.82, 6.82, 23.81, 31.11
3. Find the mean of the following measurements.
- (a) 8 m, 6 m, 12 m, 15 m, 20 m, 11 m
 (b) 6 kg, 1 kg, 5 kg, 4 kg, 8 kg, 10 kg, 7 kg, 9 kg, 5 kg, 15 kg
 (c) 15 cm, 24 cm, 18 cm, 20 cm, 22 cm, 14 cm, 27 cm
4. A cricketer makes the following runs in each innings for a season.
 Calculate the average (mean) score for each innings for the season.
 27, 11, 53, 162, 8, 0, 0, 28, 72, 49, 99, 31

Median

The median is the middle number when the data is arranged in numerical order from the smallest to the largest.

If there is an even number of data items the median is midway between the two middle numbers.

Example 1

Find the median of the following set of data.

4, 1, 8, 12, 0, 5, 6, 4, 7

Step 1 Arrange the data in numerical order from the smallest to the largest.

0, 1, 4, 4, 5, 6, 7, 8, 12

Step 2 Find the middle number. This is the median.

0, 1, 4, 4, 5, 6, 7, 8, 12

Median = 5

Example 2

Find the median of the following set of data.

2, 6, 9, 10, 8, 11, 6, 3

Step 1 Arrange the data in numerical order from the smallest to the largest.

2, 3, 6, 6, 8, 9, 10, 11

Step 2 Find the middle number.

In this example there is an even number of data items so the median will be midway between the middle two.

2, 3, 6, 6, 8, 9, 10, 11

Median = 7

Example 3

Find the median of the following set of data.

13, 3, 19, 9, 5, 8, 15, 9

Step 1 Arrange the data in numerical order from the smallest to the largest.

3, 5, 8, 9, 9, 13, 15, 19

Step 2 Find the middle number.

3, 5, 8, 9, 9, 13, 15, 19

When the two middle numbers are the same the median is this number.

Median = 9

Example 4

Find the median of the following set of data.

1, 22, 5, 31, 8, 63, 9, 11, 20, 30, 0, 45

Step 1 Arrange the data in numerical order from the smallest to the largest.

0, 1, 5, 8, 9, 11, 20, 22, 30, 31, 45, 63

Step 2 Find the middle number.

0, 1, 5, 8, 9, 11, 20, 22, 30, 31, 45, 63

One way to find the number midway between two numbers is to add the two numbers and divide this by 2.

$$\frac{11 + 20}{2} = \frac{31}{2} = 15.5$$

Median = 15.5

EXERCISE 26B

1. Find the median of the following sets of data.

- | | |
|----------------------------------|--|
| (a) 0, 1, 3, 6, 10, 12, 15 | (b) 6, 7, 9, 9, 10, 11, 16, 17, 17 |
| (c) 5, 6, 7, 7, 8, 8, 9, 9, 9, 9 | (d) 0, 2, 3, 3, 4, 6, 6, 6, 7, 8 |
| (e) 12, 14, 17, 19, 23, 27 | (f) 5, 7, 10, 15, 19, 27, 31, 33 |
| (g) 10, 16, 25, 33, 37, 40 | (h) 7, 8, 10, 12, 13, 13, 16, 18, 19, 20 |
| (i) 2, 3, 3, 4, 4, 5, 6, 7, 7, 8 | (j) 0, 6, 7, 12, 17, 19, 22, 26 |
| (k) 2, 5, 5, 7, 16, 17, 18, 19 | (l) 0, 6, 16, 21, 27, 40, 45, 57, 68, 77 |

2. Find the median of the following sets of data.

- | | |
|-------------------------------------|--|
| (a) 23, 9, 17, 2, 0, 18, 11 | (b) 3, 16, 9, 3, 7, 11, 22, 11, 10 |
| (c) 9, 3, 4, 0, 2, 8, 7, 6, 2, 5, 7 | (d) 11, 17, 7, 9, 19, 0, 15, 2 |
| (e) 23, 9, 19, 45, 27, 6, 33, 2 | (f) 35, 72, 60, 41, 19, 41, 57, 26 |
| (g) 15, 27, 7, 33, 42, 28, 0, 59 | (h) 5, 33, 19, 30, 14, 44, 1, 50, 8, 61 |
| (i) 8, 19, 27, 43, 69, 87, 91, 99 | (j) 9, 42, 56, 15, 29, 48, 17, 69, 6, 70 |

Mode

The mode is the number that occurs the most.

There may be more than one mode or there may be no mode.

Example 1

Find the mode in the following set of data.

0, 1, 3, 3, 4, 4, 4, 5, 5, 6, 7, 9

It can be seen that the number 4 occurs more than any other number.

0, 1, 3, 3, 4, 4, 4, 5, 5, 6, 7, 9

Mode = 4

Example 2

Find the mode in the following set of data.

0, 0, 2, 2, 2, 3, 3, 5, 5, 5, 6, 6, 7, 7

In this set of data there are two modes - 2 and 5.

0, 0, 2, 2, 2, 3, 3, 5, 5, 5, 6, 6, 7, 7

Modes = 2 and 5

Example 3

Find the mode in the following set of data.

6, 8, 2, 9, 6, 8, 7, 8, 2, 9, 8, 3, 9

When the data is random, arrange it in numerical order to find the mode.

2, 2, 3, 6, 6, 7, 8, 8, 8, 8, 9, 9, 9

Mode = 8

EXERCISE 26C

1. Find the mode of the following sets of data.

- (a) 0, 0, 1, 1, 2, 2, 2, 3, 3, 4 (b) 8, 9, 10, 10, 11, 12, 12, 12, 13, 14
 (c) 2, 2, 2, 3, 3, 3, 4, 4, 4, 5 (d) 6, 7, 7, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10

2. Find the mode of the following sets of data.

- (a) 11, 8, 9, 8, 9, 7, 10, 8, 7, 10 (b) 7, 9, 6, 10, 6, 10, 7, 11, 9, 7, 9, 11
 (c) 0, 7, 3, 5, 6, 7, 5, 0, 7, 5, 8 (d) 9, 14, 12, 10, 11, 12, 9, 13, 8, 16

EXERCISE 26D

1. For the following sets of data find:

- (i) the mean (ii) the median (iii) the mode
 (a) 6, 8, 9, 9, 10, 10, 12, 16 (b) 5, 7, 8, 8, 9, 9, 9, 11, 11, 11, 11
 (c) 7, 8, 8, 8, 9, 10, 12, 14, 14 (d) 6, 7, 7, 7, 8, 10, 11, 11, 11, 12

2. For the following sets of data find:

- (i) the mean (ii) the median (iii) the mode
 (a) 6, 8, 3, 2, 4, 6, 7, 0, 9 (b) 0, 1, 2, 3, 4, 4, 5, 6, 6, 6, 7
 (c) 2, 2, 4, 4, 4, 5, 5, 6 (d) 0, 1, 3, 6, 7, 7, 9, 9, 9, 10, 11, 12
 (e) 4, 3, 6, 2, 3, 5, 2, 5, 1, 5, 8 (f) 1, 2, 8, 6, 4, 7, 5, 7, 5, 6, 0, 7, 5, 7
 (g) 2, 3, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 8, 8, 9, 9, 10, 11, 14
 (h) 0, 12, 7, 13, 11, 8, 13, 9, 14, 9, 14, 10, 11, 2, 12, 13, 5, 13, 15, 19

3. A basketball player scored the following amounts in the games played over a season.

11, 24, 14, 20, 15, 18, 16, 17, 17, 18, 9, 18, 12, 18, 15, 22

For this data find:

- (a) the mean (b) the median (c) the mode

4. The ages of people starting an apprentice chef course are shown below.

18, 21, 24, 23, 25, 30, 23, 20, 19, 21, 22, 24, 25, 27, 22, 23, 24

For this data find:

- (a) the mean (b) the median (c) the mode

Measure of Spread

It is often valuable to have a measure of the spread of a set of numerical data. One measure of the spread is the **range**. The range is the difference between the highest and lowest values of the set of data.

$$\text{Range} = \text{highest value} - \text{lowest value}$$

Example 1

Find the range of the following set of data.

3, 4, 6, 7, 8, 9, 12, 33, 45

$$\begin{aligned}\text{Range} &= \text{highest value} - \text{lowest value} \\ &= 45 - 3\end{aligned}$$

$$\text{Range} = 42$$

Example 2

Find the range of the following set of data.

6, 26, 11, 31, 5, 7, 24, 16

Arrange the data in numerical order.

5, 6, 7, 11, 16, 24, 26, 31

$$\begin{aligned}\text{Range} &= \text{highest value} - \text{lowest value} \\ &= 31 - 5\end{aligned}$$

$$\text{Range} = 26$$

EXERCISE 26E

Find the range for the following sets of data.

- 5, 7, 9, 10, 10, 11, 16, 28
- 0, 4, 5, 7, 13, 18, 27
- 9, 11, 27, 16, 2, 17, 27, 32, 31
- 38, 26, 19, 33, 40, 26, 48, 52, 47

EXERCISE 26F

For the following sets of data find:

(a) the mean (b) the median (c) the mode (d) the range

1. 4, 6, 9, 9, 10, 13, 13, 13, 22

2. 0, 2, 3, 3, 5, 5, 6, 6, 6, 7

3. 7, 7, 8, 8, 8, 9, 9, 9, 10, 10

4. 12, 16, 23, 25, 32, 41, 43, 50

5. 11, 12, 14, 12, 13, 12, 11, 9

6. 24, 23, 26, 22, 23, 23, 20, 22, 24

Stem-and-Leaf Plots

As detailed in Chapter 25, stem-and-leaf plots can be used to display numerical data. The mean, median, mode and range can be found for a set of numerical data if it is displayed on a stem-and-leaf plot.

Example

For the data shown on this stem-and-leaf plot find:

1. the mean 2. the median 3. the mode 4. the range

Stem	Leaf
1	8 8 9
2	0 1 1 2 3 3 3 4 6 8 9
3	0 1 2

$$\frac{1}{18} = 18$$

$$1. \text{ Mean} = \frac{18 + 18 + 19 + 20 + 21 + \dots + 30 + 32 + 33}{17}$$

17 ← There are 17 numbers

$$= \frac{408}{17}$$

$$\text{Mean} = 24$$

2. Median

The data is in numerical order so the median can be found by locating the middle number. There are 17 items of data so the median will be the ninth item.

Stem	Leaf
1	8 8 9
2	0 1 1 2 3 3 3 4 6 8 9
3	0 1 2

$$\frac{1}{1} \overline{8} = 18$$

$$\text{Median} = 23$$

3. Mode

Stem	Leaf
1	8 8 9
2	0 1 1 2 3 3 3 4 6 8 9
3	0 1 2

$$\frac{1}{1} \overline{8} = 18$$

$$\text{Mode} = 23$$

4. Range

Stem	Leaf
1	8 8 9
2	0 1 1 2 3 3 3 4 6 8 9
3	0 1 2

$$\frac{1}{1} \overline{8} = 18$$

$$\begin{aligned} \text{Range} &= \text{highest value} - \text{lowest value} \\ &= 32 - 18 \end{aligned}$$

$$\text{Range} = 14$$

EXERCISE 26G

1. For the data displayed in the following stem-and-leaf plots find:

(i) the mean (ii) the median (iii) the mode (iv) the range

(a)

<i>Stem</i>	<i>Leaf</i>
1	9
2	0 2 4 4 6 8
3	0 2

$$\frac{1}{19} = 19$$

(b)

<i>Stem</i>	<i>Leaf</i>
2	7 8 9
3	1 2 3 5 5 7 9
4	0 2

$$\frac{2}{27} = 27$$

(c)

<i>Stem</i>	<i>Leaf</i>
1	5 7 8 8 9
2	0 2 4 4 5 6 6 7 8
3	1 3 5

$$\frac{1}{15} = 15$$

(d)

<i>Stem</i>	<i>Leaf</i>
1	8 9
2	0 2 5 7 8 8 9
3	2 6 6 6 8 9
4	0 1 2 4 6

$$\frac{1}{18} = 18$$

(e)

<i>Stem</i>	<i>Leaf</i>
1	5 7
2	2 4 5 7 8 9
3	0 1 3 6 7 8 9
4	0 1 2 3 5 6 7 9
5	1 2 4 8

$$\frac{1}{15} = 15$$

2. Convert the following non-ordered stem-and-leaf plots to ordered stem-and-leaf plots then find:

(i) the mean (ii) the median (iii) the mode (iv) the range

(a)

<i>Stem</i>	<i>Leaf</i>
1	9 4 3 8
2	4 6 2 1 0 6
3	3 1 2

$$\frac{1}{3} = 13$$

(b)

<i>Stem</i>	<i>Leaf</i>
4	5 4 9 1 7
5	8 6 8 7 3 0 3
6	5 4 7 0 2 1

$$\frac{4}{1} = 41$$

(c)

<i>Stem</i>	<i>Leaf</i>
2	6 5 3 7 4 9 2 5
3	1 3 0 5 2 4 6 3 8 2 9
4	4 3 5 0 1 2 5 4

$$\frac{2}{2} = 22$$

3. For the following lists of data:

- construct a non-ordered stem-and-leaf plot
- construct an ordered stem-and-leaf plot
- find the mean
- find the median
- find the mode
- find the range

(a)

28, 31, 43, 56, 27, 32, 40, 45, 51, 46, 51, 29, 33, 30, 42, 29, 37, 35, 41, 39, 29, 40, 39, 50, 30, 40, 37, 42, 40, 52, 38, 46

(b)

15, 13, 22, 34, 43, 30, 26, 22, 19, 35, 32, 42, 42, 37, 36, 32, 29, 26, 27, 28, 19, 31, 37, 18, 28, 27, 40, 30, 20, 19, 18, 22, 28, 33, 34, 39, 37, 36, 29, 27, 28, 30, 33, 34, 33, 39, 27, 30

Frequency Distribution Tables

Example 1

The number of goals scored by a soccer team in each match of a season are shown below.

0, 1, 2, 0, 1, 2, 2, 3, 1, 2, 0, 0, 2, 1, 3, 2, 1, 1, 2, 2

One way to find the mean, median, mode and range for these scores would be to arrange them in numerical order.

0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3

$$\begin{aligned} \text{Mean} &= \frac{0 + 0 + 0 + \dots + 2 + 3 + 3}{20} \\ &= \frac{28}{20} \end{aligned}$$

$$\text{Mean} = 1.4$$

0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3

$$\text{Median} = 1.5$$

0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3

$$\text{Mode} = 2$$

$$\text{Range} = 3 - 0$$

$$\text{Range} = 3$$

Example 1 continued

0, 1, 2, 0, 1, 2, 2, 3, 1, 2, 0, 0, 2, 1, 3, 2, 1, 1, 2, 2

Another way to represent this data is on a frequency distribution table.

Score	Tally	Frequency
0		4
1		6
2		8
3		2
Total		20

Note: an easy way to calculate the mean from a frequency distribution table is to multiply each data item by its frequency then add these and divide by the total.

$$\begin{aligned}
 \text{Mean} &= \frac{(4 \times 0) + (6 \times 1) + (8 \times 2) + (2 \times 3)}{20} \\
 &= \frac{28}{20}
 \end{aligned}$$

$$\text{Mean} = 1.4$$

The **median** is midway between the 10th and 11th tally mark.

$$\text{Median} = 1.5$$

Score	Tally	Frequency
0		4
1	<u>0</u>	6
2	<u>0</u>	8
3		2
Total		20

The **mode** is the data item with the highest frequency.

$$\text{Mode} = 2$$

$$\text{Range} = 3 - 0$$

$$\text{Range} = 3$$

Example 2

For the data displayed on the frequency distribution table below find:

- (i) the mean
- (ii) the median
- (iii) the mode
- (iv) the range

Number	Tally	Frequency
12		9
13		10
14		16
15		10
Total		45

$$\begin{aligned}
 \text{Mean} &= \frac{(9 \times 12) + (10 \times 13) + (16 \times 14) + (10 \times 15)}{45} \\
 &= \frac{612}{45}
 \end{aligned}$$

$$\text{Mean} = 13.6$$

The **median** is the 23th tally mark.

$$\text{Median} = 14$$

Number	Tally	Frequency
12		9
13		10
14		16
15		10
Total		45

The **mode** is the data item with the highest frequency.

$$\text{Mode} = 14$$

$$\text{Range} = 15 - 12$$

$$\text{Range} = 3$$

Example 3

For the data displayed on the frequency distribution table below find:

- (i) the mean
- (ii) the median
- (iii) the mode
- (iv) the range

<i>Number</i>	<i>Frequency</i>
9	6
10	13
11	15
12	15
13	11
Total	60

$$\begin{aligned} \text{Mean} &= \frac{(6 \times 9) + (13 \times 10) + (15 \times 11) + (15 \times 12) + (11 \times 13)}{60} \\ &= \frac{672}{60} \end{aligned}$$

$$\text{Mean} = 11.2$$

The **median** is midway between the 30th and the 31th numbers. This can be found by looking at the running total of the frequency.

<i>Number</i>	<i>Frequency</i>
9	6
10	13
11	15
12	15
13	11
Total	60

**Running
Total**



6
19
34
49
60



The 30th and 31th numbers will be in this group.

$$\text{Median} = 11$$

There are two modes - both with frequency of 15.

$$\text{Modes} = 11 \text{ and } 12$$

$$\text{Range} = 13 - 9$$

$$\text{Range} = 4$$

EXERCISE 26H

1. For the following sets of data find:

(i) the mean (ii) the median (iii) the mode (iv) the range

(a)

<i>Score</i>	<i>Tally</i>	<i>Frequency</i>
0		5
1		6
2		12
3		7
<i>Total</i>		30

(b)

<i>Score</i>	<i>Tally</i>	<i>Frequency</i>
0		2
1		4
2		13
3		10
4		6
<i>Total</i>		35

(c)

<i>Score</i>	<i>Tally</i>	<i>Frequency</i>
10		8
11		9
12		11
13		13
14		6
<i>Total</i>		47

2. For the following sets of data find:

- (i) the mean (ii) the median (iii) the mode (iv) the range

(a)

<i>Number</i>	<i>Frequency</i>
0	8
1	12
2	16
3	14
4	10
Total	60

(b)

<i>Number</i>	<i>Frequency</i>
2	4
3	8
4	17
5	17
6	19
Total	65

(c)

<i>Number</i>	<i>Frequency</i>
24	4
25	7
26	15
27	9
28	3
Total	38

(d)

<i>Number</i>	<i>Frequency</i>
12	4
13	7
14	9
15	22
16	13
Total	55

3. For the sets of data below:

- (i) construct a frequency distribution table
 (ii) find the mean
 (iii) find the median
 (iv) find the mode
 (v) find the range

(a)

8, 9, 10, 9, 8, 11, 12, 8, 9, 8, 9, 10, 10, 11, 8, 9, 10, 9, 11, 10,
 11, 12, 8, 9, 10, 8, 9, 11, 12, 9, 8, 9, 9, 9, 8, 10, 11, 9, 11, 12

(b)

8, 9, 3, 5, 6, 3, 2, 5, 6, 7, 4, 8, 9, 3, 6, 3, 5, 7, 4, 3, 2, 5, 8, 9, 6,
 2, 6, 7, 8, 4, 6, 9, 7, 5, 6, 4, 6, 4, 7, 6, 5, 3, 5, 6, 7, 5, 4, 7, 5, 4,
 3, 7, 6, 5, 6, 7, 4, 3, 8, 6, 5, 7, 9, 3, 2, 3, 5, 4, 6, 5, 7, 6, 5, 8, 6,
 5, 6, 6, 7, 5, 4, 7, 8, 6, 5, 7, 8, 6, 5, 7, 6, 2, 9, 3, 4, 5, 7, 5, 4, 6

4. The number of people catching a bus from a rural town to the city each day for 40 days is shown below.

19, 23, 15, 17, 24, 22, 16, 18, 19, 25, 24, 20, 21, 23, 19, 15, 25, 25, 22, 18, 17, 20, 23, 24, 16, 18, 19, 20, 17, 23, 20, 19, 18, 25, 15, 20, 22, 23, 19, 20

For this set of data:

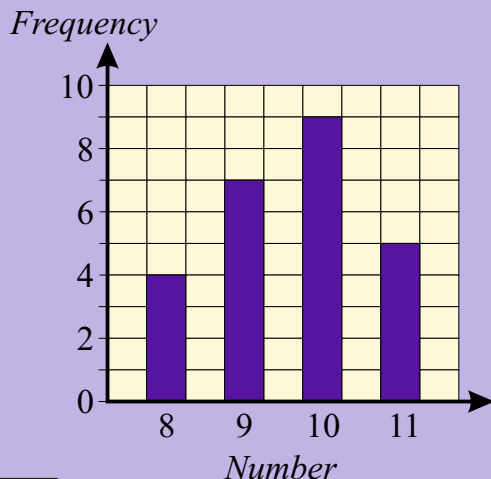
- construct a frequency distribution table
- find the mean
- find the median
- find the mode
- find the range

Graphs

Data from frequency distribution tables can be displayed on graphs. The mean, median, mode and range can be determined from these graphs.

Example

Number	Frequency
8	4
9	7
10	9
11	5
Total	25



Mean = 9.6
Median = 10
Mode = 10
Range = 3

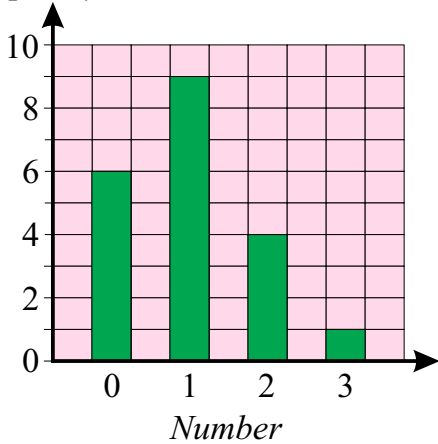
EXERCISE 26I

1. For the data displayed on the following graphs find:

(i) the mean (ii) the median (iii) the mode (iv) the range

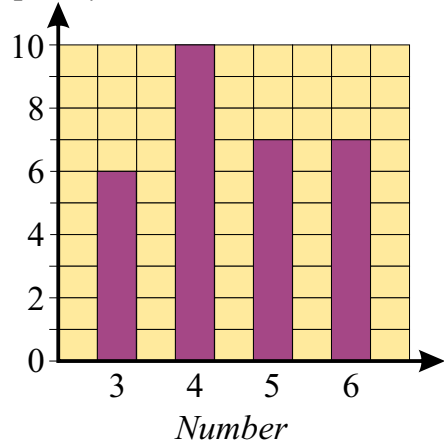
(a)

Frequency

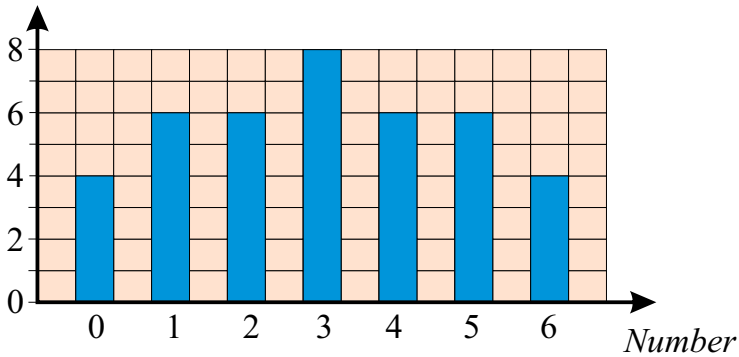


(b)

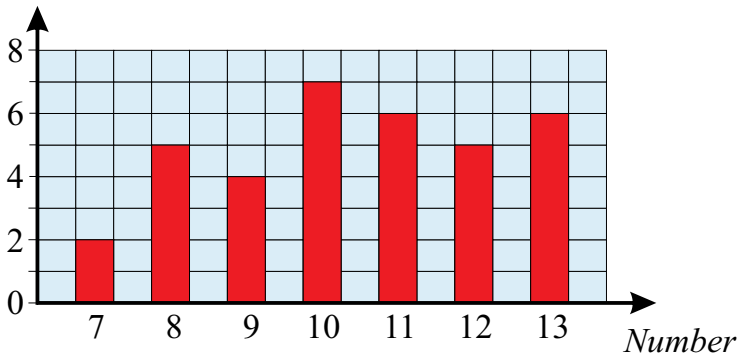
Frequency



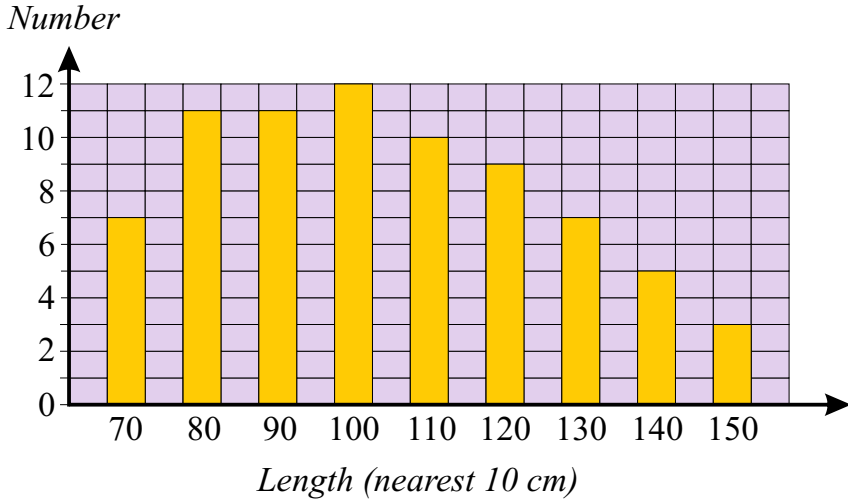
(c) *Frequency*



(d) *Frequency*



2. A herpetologist was studying tiger snakes in southern Australia and recorded the length of a number of them. These lengths were rounded to the nearest 10 cm and are displayed on the graph below.



- (a) How many snakes did the herpetologist measure?
- (b) What was the mean length of the snakes?
- (c) What was the median length of the snakes?
- (d) What was the mode of the length of the snakes?
- (e) What was the range of lengths of the snakes?



Outliers

Sometimes in sets of numerical data there are items that are considerably larger or smaller than the rest of the data. These are called **outliers**. Outliers will affect the mean of the set of data.

Example

A batter in cricket made the following scores in ten innings.

3, 7, 12, 25, 0, 1, 325, 32, 14, 8

Arranged in numerical order:

0, 1, 3, 7, 8, 12, 14, 25, 32, 325

Mean = 42.7

Median = 10

There is a big difference between these and this is because the large score of 325 increases the mean.

If this score was taken out the mean would be 11.3.

When there is an obvious outlier, the median is the better measure of the 'centre' of a set of data.

Another batter had the following scores.

45, 32, 57, 29, 60, 27, 39, 56, 48, 31

Arranged in numerical order:

27, 29, 31, 32, 39, 45, 48, 56, 57, 60

Mean = 42.4

Median = 42

It can be seen when comparing these two batters, the means are similar but the medians are very different and better reflect the overall performance of the batters.

EXERCISE 26J

1. The time taken for a tram to complete its route was recorded each day for 10 days. These times (in minutes) are shown below.

44, 42, 41, 44, 73,
39, 43, 43, 43, 39



- (a) Find the mean of these times.
 (b) Find the median of these times.
 (c) Which time is an outlier?
 (d) What is the mean if the outlier is taken out?
2. Two javelin throwers are competing to see who would represent their school. They each had five throws and the distances, in metres (m), they each threw are shown below.

Spike: 35, 41, 58, 40, 37
Lance: 45, 42, 46, 25, 47



- (a) Find the mean (average) length thrown by each thrower.
 (b) Find the median length thrown by each thrower.
 (c) Which of the javelin throwers should be chosen to represent the school? Give reasons.
3. The weekly wages of each person at two stores are shown below.

Bargains Plus: \$620, \$660, \$680, \$620, \$990, \$630
Best'n Town: \$680, \$660, \$680, \$700, \$680, \$680

- (a) Find the mean wage in each store.
 (b) Find the median wage at each store.
 (c) Which store offers the best wages? Give reasons.

Practical Problems

EXERCISE 26K

1. (a) Dave plays basketball and had the following scores in a season.

19, 23, 24, 15, 22, 28, 12, 30, 22, 16, 17, 12

What was the mean (average) of Dave's scores?

- (b) Belinda also plays basketball and she scored a total of 210 points in a season where she played 15 games. What was Belinda's average score per game?
- (c) Kristie also plays basketball and she made the following scores in the first nine games of a season.

18, 23, 29, 16, 14, 20, 11, 18, 21

How many points should she score in the next game if she wants to have an average of 20 points per game after the ten games?

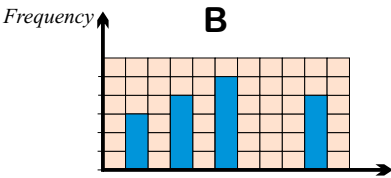
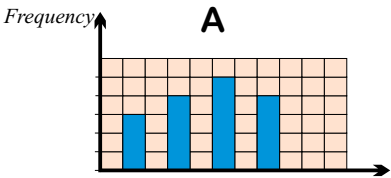
- (d) Busta also plays basketball and after 6 games his average points per game was 12. After his next game his average was 11. How many points did he score in the seventh game?
- (e) Jasmin also plays basketball. In the 9 games she played in a season her average score was 15 points per game. If her highest score was not included she averaged 12.5 for the remaining games. What was her highest score?
2. (a) The range of ages of people on a hike was 28 years. If the youngest person was 19 years old, how old was the oldest person?
- (b) There were three brothers in a family. Their median age was 12 and their average age was 13. The eldest brother was 17. How old was the youngest brother?
- (c) There were three sisters in a family. Their median age was 15 and their average age was 14. The range of their ages was 7 years. What were the ages of the three sisters?
- (d) Jess, Tess and Hess are friends. Their median age and average age are both 14. Hess is not the oldest but is two years older than Jess. How old are the three girls?
- (e) Jack, Jed, Jim and Jock are brothers. The median of their ages is 14. Jed and Jim are twins. Their average age is 15. Jack is the eldest. Jock is two years older than the twins. The range of their ages is 8 years. Find the ages of the four brothers.

3. Sam and Terry are two batters in a cricket team. The average and range of their scores for a season are shown in this table.

	Average	Range
Sam	46	30
Terry	46	190

Describe the types of scores that each player must have made over the season.

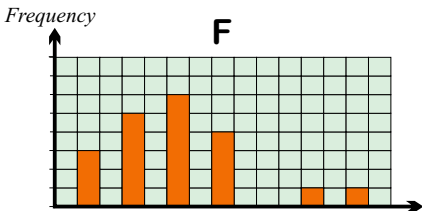
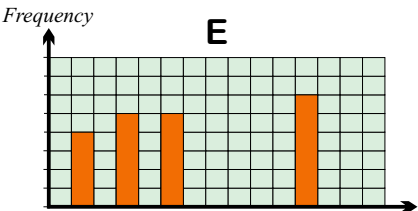
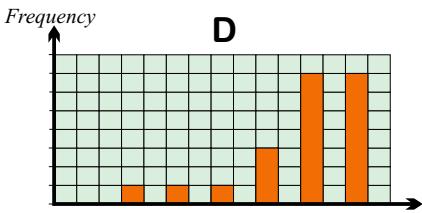
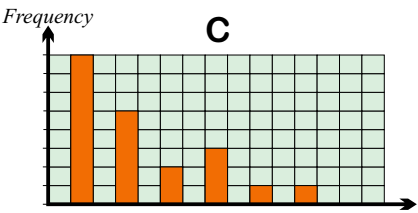
4. Two graphs of data are shown below. The scales are the same on both.



Copy and complete the sentences below by including the words *greater than*, *less than* or *equal to* in the gaps.

- (a) The *mean* of graph **A** is the *mean* of graph **B**.
- (b) The *median* of graph **A** is the *median* of graph **B**.
- (c) The *mode* of graph **A** is the *mode* of graph **B**.
- (d) The *range* of graph **A** is the *range* of graph **B**.

5. Four graphs of data are shown below. The scales are the same on all four.

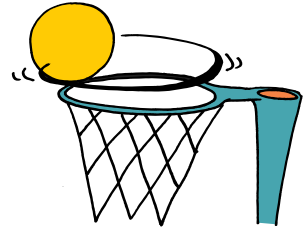


- (a) Which of the four graphs has the *highest mean*?
- (b) Which of the four graphs has the *lowest mean*?
- (c) Which of the four graphs has the *highest median*?
- (d) Which of the four graphs has the *lowest median*?
- (e) Which of the four graphs has the *highest mode*?
- (f) Which of the four graphs has the *highest range*?

PROBLEM SOLVING

1. Eve, Indie and Olive are netball players. At practice they each had 10 shots at goal. From the information below, find out how many goals they each scored.

- ♦ They all scored a different number of goals.
- ♦ The average of their scores was 6 goals.
- ♦ The median of their scores was 7 goals.
- ♦ Indie scored the most number of goals.
- ♦ Eve scored 4 more than Olive.



2. Hilde, Zephyr, Dyllon and Frankie are practising for a spelling competition. They each attempt to spell 20 words. From the information below, find out how many mistakes they each made.

- ♦ The average number of mistakes was 5.
- ♦ The median number of mistakes was 4.
- ♦ Frankie made the most number of mistakes.
- ♦ Hilde made 2 more mistakes than Dyllon.
- ♦ Zephyr made the least number of mistakes - 8 less than Frankie.

PUZZLE

The word **DATA** is hidden four times in the puzzle below.
The words may be found forwards, backwards or diagonally.

D	A	T	D	A	D	T	A	D	D	A	A	T	D	D
A	A	D	A	D	A	T	D	D	T	T	A	T	A	A
T	T	D	A	D	T	A	T	A	A	D	A	T	D	T
T	A	T	A	D	A	A	T	D	A	D	T	A	A	D
A	T	D	A	T	T	A	D	A	T	T	D	A	T	D

CHAPTER REVIEW

1. For the following sets of data find:

(i) the mean (ii) the median (iii) the mode (iv) the range

(a) 5, 5, 6, 7, 7, 8, 8, 8, 9

(b) 0, 1, 3, 5, 7, 7, 8, 9

(c) 6, 7, 5, 6, 4, 4, 5, 6, 7, 2, 3

(d) 15, 5, 6, 20, 16, 8, 10, 0, 13, 18

2. The ages of people on a tram were recorded and are shown below.

20, 33, 71, 37, 25, 44, 52, 28, 18, 34, 56

For this data find:

(a) the mean (b) the median (c) the range

3. For the data displayed in the following stem-and-leaf plots find:

(i) the mean (ii) the median (iii) the mode (iv) the range

(a)

<i>Stem</i>	<i>Leaf</i>
3	5 8 9
4	0 1 2 2 2 8 9
5	1 2 3

$$\overline{3} \mid 8 = 38$$

(b)

<i>Stem</i>	<i>Leaf</i>
1	6 7 7 8 9
2	0 1 2 2 4 5 5 5 8
3	2 2 4 5

$$\overline{1} \mid 6 = 16$$

4. Convert the following non-ordered stem-and-leaf plot to an ordered stem-and-leaf plot then find:

(a) the mean (b) the median (c) the mode (d) the range

<i>Stem</i>	<i>Leaf</i>
2	6 4 8 5
3	2 7 9 4 1 5 0 3 8
4	4 2 6

$$\overline{2} \mid 4 = 24$$

5. For the following list of data:
- construct a non-ordered stem-and leaf-plot
 - construct an ordered stem-and-leaf plot
 - find the mean
 - find the median
 - find the mode
 - find the range

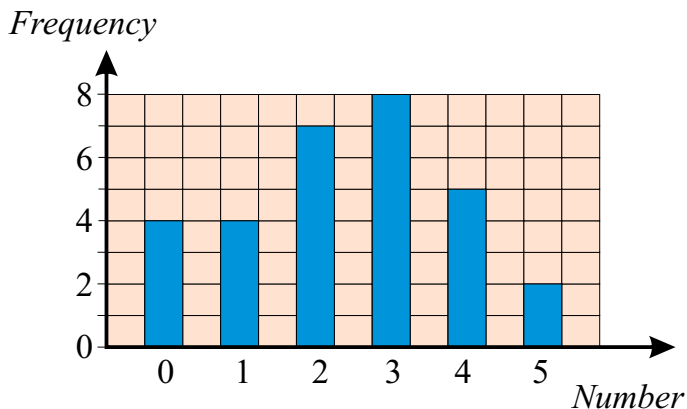
59, 49, 50, 52, 54, 54, 48, 55, 61, 55
56, 72, 60, 62, 57, 49, 65, 68, 55, 75

6. The number of spelling mistakes found on each page of a short story typed by its author are listed below.

0, 1, 2, 1, 0, 0, 1, 2, 3, 3, 2, 2, 2, 1, 3, 4, 5, 2, 3, 1, 0, 0, 1, 2, 3
1, 2, 1, 2, 3, 2, 1, 4, 3, 2, 4, 0, 2, 1, 2, 3, 0, 5, 4, 3, 1, 2, 0, 1, 2

For this set of data:

- construct a frequency distribution table
 - find the mean
 - find the median
 - find the mode
 - find the range
7. For the data displayed on the following graph find:
- the mean
 - the median
 - the mode
 - the range



Answers

Chapter 1

Exercise 1A

- (a) 13 (b) 61 (c) 320 (d) 572
(e) 809 (f) 7811 (g) 14 690
(h) 271 305 (i) 903 040
(j) 380 701 (k) 2 015 958
(l) 401 050 006
- (a) eighteen (b) ninety-five
(c) four hundred and seven
(d) three thousand, two hundred and forty-six
(e) eight thousand and thirty
(f) seventeen thousand, one hundred and seventy-nine
(g) five hundred and three thousand and twenty-seven
(h) six hundred and eighteen thousand, nine hundred and eight
(i) twenty-three million, eight hundred and ten thousand, six hundred and forty-five
(j) nine million and nine
(k) one hundred and eleven thousand, one hundred and eleven
(l) one hundred million ten thousand and one
- (a) twelve (b) four (c) ten
(d) sixty (e) two (f) three
(g) one (h) five (i) eight (j) nine
(k) thousand (l) thirty (m) fifty
(n) seventy (o) million (p) hundred
- (a) 12 (b) 100 (c) 20 (d) 144
(e) 10 (f) 1000 (g) 2
(h) 1 with one hundred zeros
- (a) 321 (b) 9851 (c) 6420
(d) 5321 (e) 97 610 (f) 65 211
(g) 984 320 (h) 998 321
- (a) 123 (b) 1589 (c) 2468
(d) 1235 (e) 11 679 (f) 11 256
(g) 123 489 (h) 123 899

- 32 099, 32 798, 33 720, 35 621,
36 789, 36 821, 43 907, 43 915
- 6093, 6139, 6193, 6259, 6295,
6309, 6390, 6903, 6913, 6925
- (a) $36 > 26$ (b) $123 < 132$
(c) $3078 < 3087$ (d) $1203 > 987$
(e) $578 > 575$ (f) $6237 < 6239$
(g) $6804 > 6799$ (h) $4095 > 467$
(i) $12\ 993 > 12\ 987$ (j) $983 < 990$
(k) $39\ 999 > 4000$
(l) $675\ 238 < 675\ 328$
- (a) 792, 794, 796, 798, 800,
802, 804, 806
(b) 6091, 6093, 6095, 6097,
6099, 6101, 6103, 6105
(c) 16 (d) 29
(e) 831, 381, 813, 183
(f) 3576, 3756, 5376, 5736,
7356, 7536
- (a) 97 614 (b) 12 385

Exercise 1B

- (a) 50°C (b) 15°C (c) 65°C
(d) 32°C (e) 70°C
- (a) 40 km/h (b) 70 km/h (c) 55 km/h
- (a) 9 (b) 35 (c) 70 (d) 160 (e) 22
(f) 300 (g) 275 (h) 1800 (i) 14 (j) 2250

Exercise 1C

- (a) 5336 (b) 3825 (c) 417 (d) 5902
(e) 560 (f) 3009 (g) 25 041
(h) 5 294 108
- (a) 3215 (b) 602 (c) 1020 (d) 9403
(e) 7098 (f) 25 061 (g) 90 909
(h) 54 321
- (a) 83 (b) 196 (c) 762 (d) 1630
(e) 16 (f) 702 (g) 10109 (h) 40 009
- (a) 35 (b) 268 (c) 780 (d) 95
(e) 600 (f) 896 (g) 15 086 (h) 77 991

5. (a) 721 (b) 150 (c) 995 (d) 6173
(e) 104 (f) 13 017 (g) 1099
(h) 730 045
6. (a) 491 (b) 17 (c) 986 (d) 18 951
(e) 900 (f) 21 011 (g) 253 621
(h) 99 900
7. (a) 10 (b) 10 (c) 1000 (d) 100
(e) 10 (f) 100 (g) 100 (h) 10 000
8. (a) 5, 15, 25, 35, **45, 55, 65**
(b) 305, 405, 505, **605, 705, 805**
(c) 1, 10, 100, 1000, **10 000, 100 000**
(d) 30 000, 3000, 300, **30, 3**
9. (a) 60 (b) 110 (c) 140 (d) 10
(e) 1900 (f) 1000 (g) 90 (h) 1110
(i) 13 760 (j) 410 (k) 800 (l) 8070
10. (a) 100 (b) 600 (c) 1700 (d) 900
(e) 3000 (f) 35 900 (g) 700 (h) 1100
(i) 8000 (j) 10 000 (k) 0 (l) 21 100

Chapter Review

1. (a) 17 (b) 57 (c) 2071
(d) 50 609 (e) 84 023 406
2. (a) Fourteen
(b) Five hundred and eighty-six
(c) Eighty thousand, nine hundred and sixty-three
(d) Three million, forty-two thousand, seven hundred and ninety-four
(e) Seven hundred and one million, twenty thousand, eight hundred and seventy-nine
3. (a) (i) 652 (ii) 256
(b) (i) 96 411 (ii) 11 469
(c) (i) 875 332 (ii) 233 578
(d) (i) 999 221 (ii) 122 999
4. Betaton, Zeton, Alphaton, Eton, Gammaton, Deltaton, Epsiton, Theton
5. 709 759 791 809 811 821 826
830 857 890 899 980 988 1000
6. (a) 14 (b) 700
7. (a) 2603 (b) 4005 (c) 65 082
8. (a) 94 (b) 101 (c) 907 (d) 612
(e) 13 (f) 4015 (g) 6404

9. (a) 632 (b) 2202 (c) 89 157
(d) 8985 (e) 109 954
10. (a) 50 (b) 10 (c) 260 (d) 10
(e) 400 (f) 1200 (g) 2000
11. (a) 600 (b) 3300 (c) 100
(d) 7100 (e) 0 (f) 290 000

Chapter 2

Exercise 2A

1. (a) 7 (b) 2 (c) 17 (d) 9 (e) 37
(f) 26 (g) 59 (h) 40 (i) 46 (j) 23
(k) 63 (l) 28 (m) 132 (n) 75
(o) 520 (p) 110 (q) 553 (r) 207
(s) 1263 (t) 285
2. (a) 12 157 (b) 13 945 (c) 1 073 643
(d) 989 669 (e) 57 542 (f) 5864
(g) 857 (h) 4408
3. (a) 840 (b) 2137 (c) 28 (d) 67
(e) 165 cm
(f) Emily - 46 kg, Jay - 52 kg

Exercise 2B

1.

×	2	9	6	4	12	8
1	2	9	6	4	12	8
10	20	90	60	40	120	80
3	6	27	18	12	36	24
11	22	99	66	44	132	88
7	14	63	42	28	84	56
5	10	45	30	20	60	40

×	7	2	12	6	10	3
4	28	8	48	24	40	12
11	77	22	132	66	110	33
1	7	2	12	6	10	3
8	56	16	96	48	80	24
9	63	18	108	54	90	27
5	35	10	60	30	50	15

2. (a) 12 (b) 36 (c) 30 (d) 80 (e) 81
(f) 24 (g) 36 (h) 55 (i) 60 (j) 36
(k) 42 (l) 32 (m) 21 (n) 16 (o) 33
(p) 96 (q) 63 (r) 25 (s) 84 (t) 24

3. (a) 7 (b) 3 (c) 3 (d) 5 (e) 10 (f) 3
(g) 5 (h) 3 (i) 2 (j) 11 (k) 12 (l) 9
(m) 8 (n) 3 (o) 8 (p) 8
4. (a) 56 (b) \$3 (c) 85 (d) 64

Exercise 2C

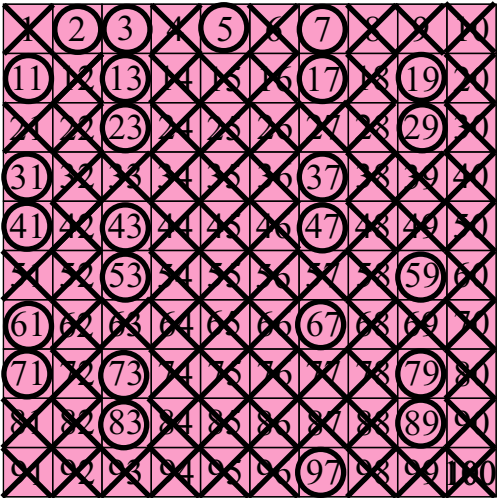
1. (a) 1, 2, 3, 4, 6, 12
(b) 1, 2, 3, 4, 6, 9, 12, 18, 36
(c) 1, 3, 9, 27
(d) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
(e) 1, 2, 4, 8, 14, 28, 56, 112
(f) 1, 2, 4, 7, 8, 14, 28, 56
(g) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60, 75, 100, 150, 300
(h) 1, 23
(i) 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144 (j) 1, 3, 9, 19, 57, 171
2. (a) 1, 2, 4 (b) 1, 3 (c) 1, 2, 4
(d) 1, 2, 3, 4, 6, 12 (e) 1, 2, 4
3. (a) 4 (b) 3 (c) 4 (d) 12 (e) 4
4. (a) 3 m (b) 5 blue, 4 white
5. (a) 8 cm (b) 14
6. (a) \$8 (b) Susan-9, Craig-7
7. (a) 4 cm (b) 45
8. 7 cm
9. (a) 12 m (b) 26
10. 7 days

Exercise 2D

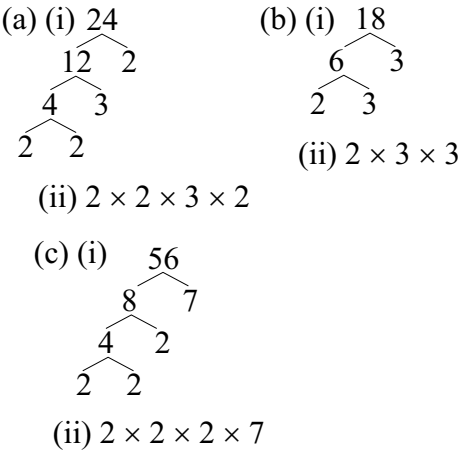
	2	3	4	5	6	8	9	10
1380	✓	✓	✓	✓	✓			✓
7344	✓	✓	✓		✓	✓	✓	
11 496	✓	✓	✓		✓	✓		
46 800	✓	✓	✓	✓	✓	✓	✓	✓
410 400	✓	✓	✓	✓	✓	✓	✓	✓

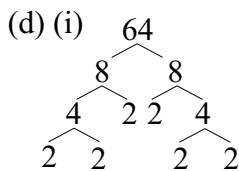
Exercise 2E

1.

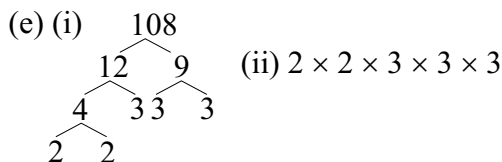


2. 3 & 5, 5 & 7, 11 & 13, 7 & 19,
29 & 31, 41 & 43, 59 & 61, 71 & 73
3. (a) There may be more than one
answer for these.
(i) 31 & 5 (ii) 71 & 3 (iii) 79 & 19
(iv) 97 & 31 (v) 97 & 61
(vi) 97 & 103
(b) 2
4. 1531
5. 161
6. There may be several factor trees
for each number - only one is shown.
The prime factors will be the same.





(ii) $2 \times 2 \times 2 \times 2 \times 2 \times 2$



7. (a) $2^3 \times 3$ (b) 2×3^2 (c) $2^3 \times 7$
 (d) 2^6 (e) $2^2 \times 3^3$
 8. (a) 72 (b) 48 (c) 45 (d) 175
 (e) 54 (f) 75 (g) 6125 (h) 540

Exercise 2F

1. (a) 1 (b) 4 (c) 9 (d) 16 (e) 25
 2. (a) 1, 4, 9, 16, 25, **36, 49, 64, 81, 100, 121, 144**
 (b) (i) 100 (ii) 400 (iii) 1225
 3. (a) 400 (b) 900 (c) 1600 (d) 2500
 (e) 3600 (f) 4900 (g) 6400
 (h) 8100 (i) 10 000 (j) 12 100
 4. (a) B (b) C (c) B (d) D (e) D
 5. (a) C (b) A (c) D (d) B (e) D

Exercise 2G

1. (a) 6 (b) 9 (c) 4 (d) 1 (e) 10
 (f) 7 (g) 8 (h) 12 (i) 2 (j) 20
 2. (a) 7 and 8 (b) 2 and 3 (c) 3 and 4
 (d) 9 and 10 (e) 5 and 6 (f) 6 and 7
 (g) 1 and 2 (h) 4 and 5 (i) 8 and 9
 (j) 10 and 11
 3. (a) 7.8 (b) 2.6 (c) 3.6 (d) 9.7
 (e) 5.5 (f) 6.5 (g) 1.7 (h) 4.4
 (i) 8.9 (j) 10.5
 4. (a) C (b) B (c) E (d) A (e) D (f) F
 5. (a) C (b) D (c) E (d) A (e) F (f) B

Exercise 2H

1. (a) 3, 6, 9, 12, 15, 18, 21, **24**
 8, 16, **24**, 32, 40, 48, 56, 64
 (b) 6, 12, 18, 24, **30**, 36, 42, 48
 10, 20, **30**, 40, 50, 60, 70, 80
 (c) 4, 8, **12**, 16, 20, **24**, 28, 32
 6, **12**, 18, **24**, 30, 36, 42, 48
 (d) 8, 16, **24**, 32, 40, **48**, 56, 64
 12, **24**, 36, 48, **60**, 72, 84, 96
 (e) **6**, **12**, 18, **24**, 30, 36, 42, 48
 3, **6**, 9, **12**, 15, **18**, 21, **24**
 (f) 8, 16, **24**, 32, 40, **48**, 56, 64
 12, **24**, 36, **48**, 60, 72, 84, 96
 (g) 5, 10, 15, 20, 25, 30, **35**, 40
 7, 14, 21, 28, **35**, 42, 49, 56
 (h) 7, 14, **21**, 28, 35, 42, 49, 56
 3, 6, 9, 12, 15, 18, **21**, 24
 2. (a) 56 (b) 60 (c) 24 (d) 60
 3. (a) 20 cm, 25 A dice or 4 B dice
 (b) 30 cm, 25 B dice or 9 C dice
 (c) 60 cm, 225 A dice or 100 B dice
 or 36 C dice
 4. (a) Step 20 (b) Step 40
 5. (a) July 16 (b) September 14

Exercise 2I

1. (a) 30 (b) 185 (c) 492 (d) 582
 (e) 1945 (f) 19 628 (g) 1638
 (h) 20 988 (i) 312 (j) 0 (k) 440
 (l) 54 806
 2. (a) 60 (b) 230 (c) 5100 (d) 20 100
 (e) 200 000 (f) 1 032 000 (g) 300
 (h) 620
 3. 96
 4. (a) 180 (b) \$90 (c) 90
 5. 37 500
 6. (a) 135 (b) 16 goals
 7. (a) 5000 kg (b) 120 oats, 80 wheat

Exercise 2J

1. (a) 3000 (b) 800 (c) 16 000
 (d) 210 000 (e) 2000 (f) 2 000 000
 (g) 60 000 (h) 24 000

2. (a) $20 \times 60 = 1200$ (b) $70 \times 30 = 2100$
 (c) $10 \times 90 = 900$ (d) $50 \times 100 = 5000$
 (e) $100 \times 80 = 8000$
 (f) $30 \times 10 \times 20 = 6000$
 (g) $120 \times 100 = 12\ 000$
 (h) $60 \times 50 \times 100 = 300\ 000$
3. (a) $100 \times 200 = 20\ 000$
 (b) $200 \times 500 = 100\ 000$
 (c) $200 \times 300 = 60\ 000$
 (d) $700 \times 300 = 210\ 000$
 (e) $800 \times 300 = 240\ 000$
 (f) $600 \times 400 = 240\ 000$
 (g) $2400 \times 100 = 240\ 000$
 (h) $1000 \times 3300 = 3\ 300\ 000$

4.

Problem	Answer Using Calculator	Problem with Rounded Numbers	Answer Using Rounded Numbers	Difference Between the Answers
23×57	1311	20×60	1200	111
68×9	612	70×10	700	88
97×31	3007	100×30	3000	7
86×44	3784	90×40	3600	184
79×73	5767	80×70	5600	167

5. C 6. B 7. C 8. B

Exercise 2K

1. (a) 3 (b) 3 (c) 8 (d) 8 (e) 20 (f) 30
 (g) 22 (h) 123 (i) 38 (j) 58 (k) 56
 (l) 270 (m) 234 (n) 87 (o) 131
 (p) 245
2. (a) 456 (b) 458 (c) 14 (d) 154
 (e) 54 (f) 56
3. (a) 20 (b) 200 (c) 400 (d) 20 (e) 12
 (f) 65 (g) 1800 (h) 263
- 4.

Problem	Answer Using Calculator	Problem with Rounded Numbers	Answer Using Rounded Numbers	Difference Between the Answers
$1957 \div 19$	103	$2000 \div 20$	100	3
$611 \div 13$	47	$600 \div 10$	60	13
$899 \div 31$	29	$900 \div 30$	30	1
$2009 \div 49$	41	$2000 \div 50$	40	1
$1368 \div 72$	19	$1400 \div 70$	20	1

5. B 6. B 7. B 8. (a) D (b) B
9. C 10. C**Exercise 2L**

1. (a) 11 (b) 11 (c) 21 (d) 8 (e) 2
 (f) 7 (g) 6 (h) 74 (i) 17 (j) 8
 (k) 10 (l) 20 (m) 11 (n) 18 (o) 0
 (p) 36 (q) 12 (r) 12 (s) 26
 (t) 20 (u) 9 (v) 16 (w) 44 (x) 2
2. There may be other answers to these.
 (a) $6+4 \times 2 = 14$ (b) $10 \div 2 + 4 = 9$
 (c) $5+4-2 = 7$ (d) $3 \times 2 \times 4 = 24$
 (e) $20 \div 2 + 6 = 16$ (f) $4+4 \times 2 = 12$
 (g) $10+4 \times 2-6 = 12$ (h) $6 \div 2 + 3 \times 4 = 15$
3. (a) $2 \times (3+4) = 14$ (b) $2+5 \times (2+3) = 27$
 (c) $6 \times (3+4)-5 = 37$ (d) $10+3 \times (5-2) = 19$
 (e) $(6+3) \times (2+3) = 45$ (f) $8-4 \div (2+2) = 7$
 (g) $(4+7) \times 2+3 = 25$
 (h) $(12-10) \div 2+3 \times 2 = 7$

4. There are many answers to this question. Three examples are:
 $(24+24) \div 6 + 2 \times 3 = 14$
 $24+24 \div (6+2 \times 3) = 26$
 $(24+24) \div (6+2) \times 3 = 18$

Exercise 2M

1. (a) T (b) F (c) T (d) F (e) T
 (f) F (g) T (h) F (i) T (j) F
 (k) T (l) F (m) T (n) T
 (o) F (p) F
2. (a) $3 \times 7 = 7 \times 3$
 (b) $8 + 13 = 13 + 8$
 (c) $21 - 9 \neq 9 - 21$
 (d) $8 \div 5 \neq 5 \div 8$
 (e) $78 + 34 = 34 + 78$
 (f) $128 - 77 \neq 77 - 128$
 (g) $A \times 7 = 7 \times A$
 (h) $B + C = C + B$
 (i) $D \div F \neq F \div D$
 (j) $3a - 7 \neq 7 - 3a$

Exercise 2N

1. (a) T (b) F (c) T (d) F
2. (a)(i) 20 (ii) 20 (b)(i) 4 (ii) 8
 (c)(i) 144 (ii) 144 (d)(i) 1 (ii) 4

Exercise 2O

1. (a) $30 + 15 = 45$, $5 \times 9 = 45$
 (b) $56 + 28 = 84$, $7 \times 12 = 84$
 (c) $54 - 12 = 42$, $6 \times 7 = 42$
 (d) $96 - 60 = 36$, $12 \times 3 = 36$
 (e) $40 + 48 = 88$, $8 \times 11 = 88$
 (f) $72 - 54 = 18$, $9 \times 2 = 18$
2. (a) $7 \times (30 + 2) = 210 + 14 = 224$
 (b) $8 \times (50 + 3) = 400 + 24 = 424$
 (c) $9 \times (90 - 1) = 810 - 9 = 801$
 (d) $6 \times (80 - 2) = 480 - 12 = 468$
 (e) $11 \times (70 + 1) = 770 + 11 = 781$
 (f) $13 \times (100 + 2) = 1300 + 26 = 1326$
 (g) $8 \times (100 - 3) = 800 - 24 = 776$
 (h) $7 \times (70 - 3) = 490 - 21 = 469$
 (i) $15 \times (100 + 3) = 1500 + 45 = 1545$
- (f) $2 \times 33 \times 5 \times 3$
 $= 33 \times 3 \times 2 \times 5$
 $= 99 \times 10 = 990$
 (g) $5 \times 5 \times 2 \times 31$
 $= 5 \times 31 \times 5 \times 2$
 $= 153 \times 10 = 1530$
 (h) $5 \times 7 \times 2 \times 11$
 $= 7 \times 11 \times 5 \times 2$
 $= 77 \times 10 = 770$
 (i) $5 \times 3 \times 2 \times 42$
 $= 3 \times 42 \times 5 \times 2$
 $= 126 \times 10 = 1260$
7. There are many answers - here is one example for each
 (a) $9 \times 4 \times 2 \times 3$
 (b) $8 \times 2 \times 2 \times 32$
 (c) $2 \times 3 \times 2 \times 53$
 (d) $2 \times 30 \times 2 \times 13$
 (e) $7 \times 8 \times 32 \times 3$
 (f) $9 \times 7 \times 2 \times 81$
 (g) $3 \times 23 \times 2 \times 23$
 (h) $2 \times 82 \times 2 \times 173$
 (i) $8 \times 32 \times 2 \times 343$

Exercise 2P

1. (a) A, B, D
2. A, B, C, D, E, F
3. A, B, D
4. A, B, D, E, F
5. (a) E (b) D (c) F (d) A (e) B (f) C
6. (a) $8 \times 2 \times 50 = 8 \times 100 = 800$
 (b) $9 \times 2 \times 25 = 9 \times 50 = 450$
 or $9 \times 2 \times 5 \times 5$
 $= 9 \times 5 \times 2 \times 5$
 $= 45 \times 10 = 450$
 (c) $7 \times 2 \times 5 \times 7$
 $= 7 \times 7 \times 2 \times 5$
 $= 49 \times 10 = 490$
 (d) $9 \times 5 \times 2 \times 12$
 $= 9 \times 12 \times 5 \times 2$
 $= 108 \times 10 = 1080$
 (e) $5 \times 11 \times 2 \times 7$
 $= 11 \times 7 \times 5 \times 2$
 $= 77 \times 10$
 $= 770$
8. (a) $(35 \times 59) + (35 \times 2)$
 (b) $(19 \times 159) + (19 \times 4)$
 (c) $(32 \times 210) + (32 \times 53)$
 (d) $(29 \times 590) + (29 \times 27)$
 (e) $(7 \times 8 \times 59) + (7 \times 8 \times 2)$
 (f) $(7 \times 9 \times 270) - (7 \times 9 \times 1)$
 (g) $(2 \times 23 \times 170) - (2 \times 23 \times 7)$
 (h) $(2 \times 173 \times 590) + (2 \times 173 \times 53)$
 (i) $(2 \times 38 \times 590) + (2 \times 38 \times 11)$

Chapter Review

1. (a) 15 (b) 57 (c) 183 (d) 4 (e) 19
2. (a) 3812 (b) 3827
3. (a) 9 kg (b) Swansey-1600, Duckton-800, Heronville-500
4. (a) 18 (b) 72 (c) 35 (d) 48
 (e) 7 (f) 8 (g) 9 (h) 9

5. 672

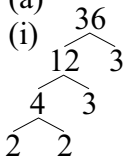
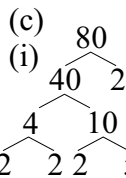
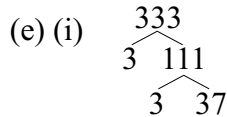
6. (a) 4 (b) 5 (c) 3 (d) 18 (e) 7

7. 6 cm

8. 67, 71, 73, 79, 83

9. 137, 337

10. (a)

(ii) $2^2 \times 3^2$ (ii) $2^4 \times 5$ (e) (i) (ii) $3^2 \times 37$

11. 9 and 10

12. (a) 40 (b) 18 (c) 60

(d) 12 (e) 24

13. (a) 1554 (b) 16 758

(c) 75 215

14. (a) 400 (b) 57 000

(c) 20 000

15. (a) 92 beats per minute

(b) 5 040 000

16. (a) 2800 (b) 6000

(c) 360 000

17. (a) $30 \times 40 = 1200$ (b) $40 \times 140 = 5600$ (c) $90 \times 250 = 22\,500$

18. (a) 21 (b) 321 (c) 514 (d) 100

(e) 6 (f) 2211 (g) 191

19. (a) 32 (b) 12 (c) 4 (d) 10 (e) 17

20. $8 \times (30 + 6) = 240 + 48 = 288$

21. A, B, C, E, F

Chapter 3

Exercise 3A

1. (a) D (b) C (c) B (d) A

2. (a) -4 (b) -3 (c) 0 (d) 2 (e) 5

(f) -8 (g) -4 (h) 4 (i) 10 (j) -11

(k) -8 (l) -2 (m) 3 (n) 9 (o) -18

(p) -12 (q) -6 (r) 8 (s) 22

3. (a) -9, -8, -7, -6, -5, -4, -3, -2, -1, 0

(b) -12, -10, -8, -6, -4, -2, 0, 2, 4, 6

(c) -35, -30, -25, -20, -15, -10, -5,

0, 5, 10

(d) 6, 5, 4, 3, 2, 1, 0, -1, -2, -3

(e) 60, 50, 40, 30, 20, 10, 0, -10,

-20, -30

(f) -37, -32, -27, -22, -17, -12, -7,

-2, 3, 8

(g) 34, 28, 22, 16, 10, 4, -2, -8,

-14, -20

(h) 20, 19, 17, 14, 10, 5, -1, -8,

-16, -25

(i) 100, 95, 85, 70, 50, 25, -5,

-40, -80, -125

(j) -50, -48, -44, -38, -30, -20,

-8, 6, 22, 40

(k) -50, -47, -41, -32, -20, -5,

13, 34, 58, 85

Exercise 3B

1. (a) -8, -5, -4, 3, 4, 6, 9

(b) -23, -14, -11, -5, 0, 9, 17, 21

(c) -12, -10, -8, -4, -3, 0

(d) -6, -4, -2, -1, 0, 1, 3, 5, 7, 8

2. (a) 33 (b) 0

3. (a) -13 (b) -3

4. (a) $-5 < 1$ (b) $-4 > -7$ (c) $9 > 4$ (d) $4 > -9$ (e) $0 < 7$ (f) $7 < 12$ (g) $-8 < -5$ (h) $0 > -7$ (i) $6 > -1$ (j) $-12 < -1$

5. (a) 5 (b) 3 (c) 7 (d) 3 (e) 4

(f) 9 (g) 9 (h) 5 (i) 6 (j) 1

6. (a) 11 (b) 14 (c) 18 (d) 31 (e) 11
(f) 40 (g) 42 (h) 26 (i) 56 (j) 68
(k) 18 (l) 39 (m) 46 (n) 39 (o) 55
7. -51°C
8. 280°C
9. (a)(i) 17°C (ii) 14°C (iii) 8°C
(b) New York (c) New York
(d) Moscow (e) 27°C (f) -5°C
(g) 3°C

10.

City	Min. Temp.	Max. Temp.
Warsaw	-10°C	-1°C
Stockholm	-3°C	12°C
Berlin	-5°C	8°C

Exercise 3C

1. (a) 1 (b) -1 (c) -2 (d) -4 (e) -3
(f) -1 (g) 2 (h) 6 (i) -2 (j) 0 (k) 1
(l) -4 (m) -6 (n) -6 (o) -1 (p) 3
2. (a) 2 (b) -9 (c) -5 (d) -12 (e) -20
(f) -13 (g) 6 (h) -14 (i) -18 (j) -6
(k) -34 (l) -1 (m) 7 (n) -20 (o) 4
(p) -13
3. (a) 2 (b) 6 (c) 5 (d) 5 (e) 3 (f) 7
(g) 12 (h) 9 (i) 8 (j) -3 (k) -3
(l) 3 (m) -6 (n) -6 (o) -5
4. (a) 7 (b) 3 (c) 2 (d) 4 (e) 2 (f) -3
(g) -9 (h) 10 (i) 4 (j) -11 (k) 0
(l) -12 (m) 12 (n) -6 (o) 2 (p) 2
5. (a) 8 (b) -10 (c) 34 (d) -50 (e) -50
(f) -10 (g) 5 (h) -14 (i) 15 (j) 3
(k) -16 (l) -22
6. (a) -2 (b) 4 (c) -2 (d) -3 (e) -6
(f) 6 (g) -4 (h) -7 (i) -6 (j) -6
(k) -6 (l) -5
7. (a) + (b) + (c) - (d) - (e) - (f) +
(g) - (h) - (i) -
8. (a) $2 + -4 - 5 = -7$ (b) $-3 + 5 = -2 - -4$
(c) $6 = 5 + -1 - -2$ (d) $1 - 7 = -9 + 3$
(e) $-3 + 4 - -8 = 9$ (f) $-5 = 6 + -3 - 8$

Chapter Review

1. (a) -11 (b) -7 (c) -3 (d) 4 (e) 8
2. (a) -34, -28, -22, -16, **-10, -4, 2**
(b) 15, 14, 12, 9, **5, 0, -6**
3. -26 -21 -19 -15 -6 5 9 13
4. (a) $-1 < 7$ (b) $-9 > -15$ (c) $0 > -5$
(d) $-6 < -1$
5. (a) 10 (b) 8 (c) 34 (d) 27
6. 23°C
7. (a) -6 (b) 4 (c) -15 (d) 13 (e) -16
(f) 8 (g) -11 (h) 22 (i) -2 (j) 5
(k) 0 (l) 7
8. (a) -2 (b) -3 (c) 10 (d) -4 (e) -4
(f) 1
9. (a) - (b) + (c) + (d) - (e) - (f) -
10. (a) $-6 - 2 = 1 + -9$
(b) $5 = 2 + -3 - -6$

Chapter 4**Exercise 4A**

1. (a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{5}{6}$ (d) $\frac{5}{9}$ (e) $\frac{5}{12}$ (f) $\frac{7}{16}$
2. (a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $\frac{1}{6}$ (d) $\frac{4}{9}$ (e) $\frac{7}{12}$ (f) $\frac{9}{16}$
3. $\frac{1}{5}$
4. $\frac{1}{7}$
5. (a) $\frac{13}{25}$ (b) $\frac{12}{25}$
6. (a) $\frac{8}{31}$ (b) $\frac{23}{31}$
7. (a) $\frac{23}{36}$ (b) $\frac{13}{36}$
8. (a) $\frac{7}{24}$ (b) $\frac{17}{24}$
9. (a) $\frac{1}{60}$ (b) $\frac{1}{24}$ (c) $\frac{5}{26}$
10. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{3}{7}$ (e) $\frac{4}{5}$ (f) $\frac{5}{8}$
(g) $\frac{2}{11}$ (h) $\frac{4}{9}$
11. (a) 2 (b) 3
12. (a) $\frac{1}{7}$ (b) $\frac{3}{5}$
13. (a) $\frac{5}{7}$ (b) $\frac{3}{8}$ (c) $\frac{2}{5}$ (d) $\frac{7}{10}$ (e) $\frac{3}{4}$ (f) $\frac{11}{20}$
14. (a) two-ninths (b) seven-eighths
(c) one-half (d) two-thirds
(e) eleven-thirteenths
(f) twenty-three-thirtieths

Exercise 4B

1. (a) (i) $\frac{2}{3}$ (ii) $\frac{6}{9}$ (iii) $\frac{4}{6}$ (iv) $\frac{8}{12}$
 (b) The same amount of each shape is shaded.
 (c) The fractions are all equal.
2. (a) $\frac{4}{10}$ (b) $\frac{2}{6}$ (c) $\frac{3}{12}$ (d) $\frac{16}{20}$ (e) $\frac{15}{18}$ (f) $\frac{18}{24}$
 (g) $\frac{21}{35}$ (h) $\frac{15}{50}$ (i) $\frac{25}{45}$ (j) $\frac{72}{88}$ (k) $\frac{42}{48}$ (l) $\frac{66}{77}$
3. A
4. D
5. D
6. (a) $\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \dots$
 (b) $\frac{11}{15}, \frac{22}{30}, \frac{33}{45}, \frac{44}{60}, \dots$
 (c) $\frac{7}{13}, \frac{14}{26}, \frac{21}{39}, \frac{28}{52}, \dots$

Exercise 4C

1. (a) $\frac{1}{2}$ (b) $\frac{5}{6}$ (c) $\frac{7}{8}$ (d) $\frac{5}{6}$ (e) $\frac{4}{5}$ (f) $\frac{4}{5}$
 (g) $\frac{2}{7}$ (h) $\frac{6}{7}$ (i) $\frac{1}{2}$ (j) $\frac{2}{3}$ (k) $\frac{1}{1}$ (l) $\frac{1}{2}$
2. $\frac{4}{9}$
3. (a) $\frac{3}{5}$ (b) $\frac{5}{9}$ (c) (i) $\frac{1}{4}$ (ii) $\frac{3}{4}$ (d) $\frac{1}{6}$
 (e) (i) $\frac{4}{5}$ (ii) $\frac{1}{5}$ (f) (i) $\frac{3}{7}$ (ii) $\frac{4}{7}$
 (g) Natasha $\frac{9}{20}$, Melissa $\frac{3}{10}$, Carley $\frac{1}{4}$
 (h) $\frac{1}{2}$ (i) $\frac{2}{7}$ (j) $\frac{2}{11}$ (k) (i) $\frac{2}{3}$ (ii) $\frac{1}{3}$
 (l) Emma $\frac{3}{10}$ (m) (i) $\frac{3}{4}$ (ii) $\frac{2}{3}$

Exercise 4D

1. $\frac{3}{4}$
 2. $\frac{1}{2}$
 3. $\frac{5}{7}$
 4. $\frac{5}{6}$

Exercise 4E

1. $\frac{2}{5}$
 2. $\frac{3}{4}$
 3. $\frac{1}{4}$
 4. $\frac{7}{10}$
 5. $\frac{5}{7}$
 6. $\frac{6}{7}$
 7. $\frac{5}{8}$
 8. $\frac{4}{9}$

Exercise 4F

1. (a) $\frac{3}{5} = \frac{6}{10}, \frac{1}{2} = \frac{5}{10}$ (b) $\frac{1}{3} = \frac{2}{6}, \frac{1}{4} = \frac{3}{12}$
 (c) $\frac{3}{4} = \frac{15}{20}, \frac{1}{5} = \frac{4}{20}$ (d) $\frac{3}{5} = \frac{9}{15}, \frac{2}{3} = \frac{10}{15}$
 (e) $\frac{2}{5} = \frac{4}{10}, \frac{3}{10}$ (f) $\frac{5}{6} = \frac{25}{30}, \frac{2}{5} = \frac{12}{30}$
 (g) $\frac{2}{7} = \frac{16}{56}, \frac{5}{8} = \frac{35}{56}$ (h) $\frac{5}{12} = \frac{10}{24}, \frac{3}{8} = \frac{9}{24}$

2. (a) $\frac{3}{5} > \frac{1}{2}$ (b) $\frac{1}{3} < \frac{1}{2}$ (c) $\frac{3}{4} > \frac{1}{5}$
 (d) $\frac{3}{5} < \frac{2}{3}$ (e) $\frac{2}{5} > \frac{3}{10}$ (f) $\frac{5}{6} > \frac{2}{5}$
 (g) $\frac{2}{7} < \frac{5}{8}$ (h) $\frac{5}{12} > \frac{3}{8}$

3. Justin

4. (a) $\frac{3}{4}$ (b) $\frac{11}{15}$ (c) $\frac{1}{13}$ (d) $\frac{11}{12}$
 5. (a) $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ (c) $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$
 (d) $\frac{13}{24}, \frac{7}{12}, \frac{5}{8}, \frac{3}{4}$

6. Chris

7. (a) Megan (b) Alice

Exercise 4G

1. (a) $\frac{1}{2}, \frac{8}{9}, \frac{3}{4}$
 (b) $\frac{3}{2}, \frac{37}{10}, \frac{13}{7}$
 (c) $9\frac{15}{16}, 3\frac{7}{13}, 1\frac{5}{7}$
2. (a) (i) $2\frac{2}{3}$ (ii) $\frac{8}{3}$
 (b) (i) $4\frac{1}{4}$ (ii) $\frac{17}{4}$
 (c) (i) $5\frac{5}{8}$ (ii) $\frac{45}{8}$
3. (a) 18 (b) (i) 400 mm (ii) 30
 (c) 32 (d) 40
4. (a) 6 (b) 50 (c) 11 (d) 15
5. (a) $\frac{5}{2}$ (b) $\frac{15}{4}$ (c) $\frac{31}{6}$ (d) $\frac{31}{7}$ (e) $\frac{17}{5}$
 (f) $\frac{58}{7}$ (g) $\frac{43}{4}$ (h) $\frac{56}{9}$ (i) $\frac{167}{11}$ (j) $\frac{421}{18}$
6. (a) $2\frac{1}{3}$ (b) $5\frac{1}{2}$ (c) $1\frac{2}{7}$ (d) $3\frac{7}{10}$ (e) $9\frac{1}{8}$
 (f) $5\frac{2}{5}$ (g) $7\frac{3}{4}$ (h) $6\frac{5}{6}$ (i) $13\frac{4}{15}$ (j) $30\frac{11}{20}$
7. (a) $2\frac{2}{3}$ (b) $4\frac{2}{5}$ (c) $3\frac{3}{4}$ (d) $2\frac{3}{7}$
 (e) $1\frac{7}{9}$ (f) $8\frac{1}{2}$
8. (a) $2\frac{1}{2}$ (b) $\frac{7}{3}$ (c) $15\frac{4}{5}$ (d) $\frac{53}{28}$ (e) $8\frac{3}{4}$
 (f) $\frac{2}{1}$ (g) $7\frac{1}{2}$ (h) $3\frac{6}{7}$ (i) $\frac{5}{4}$ (j) $9\frac{2}{3}$

Chapter Review

1. (a) $\frac{3}{10}$ (b) $\frac{5}{9}$
 2. (a) $\frac{14}{21} = \frac{2}{3}$ (b) $\frac{7}{21} = \frac{1}{3}$
 3. (a) $\frac{2}{7}$ (b) $\frac{9}{11}$
 4. $\frac{3}{13}$
5. Seven-elevenths
6. (a) $\frac{9}{15}$ (b) $\frac{8}{28}$ (c) $\frac{27}{90}$ (d) $\frac{16}{36}$
7. (a) $\frac{1}{2}$ (b) $\frac{6}{7}$ (c) $\frac{2}{3}$ (d) $\frac{7}{8}$ (e) $\frac{1}{3}$ (f) $\frac{2}{3}$
8. (a) $\frac{2}{7}$ (b) $\frac{4}{7}$ (c) $\frac{2}{9}$ (d) $\frac{5}{12}$
9. (a) $\frac{3}{6}, \frac{2}{6}$ (b) $\frac{6}{10}, \frac{5}{10}$ (c) $\frac{24}{56}, \frac{35}{56}$ (d) $\frac{18}{60}, \frac{25}{60}$
10. (a) $\frac{7}{2}$ (b) $\frac{11}{4}$ (c) $\frac{47}{8}$ (d) $\frac{107}{10}$ (e) $\frac{68}{7}$
11. (a) $2\frac{2}{3}$ (b) $2\frac{2}{5}$ (c) $9\frac{5}{6}$ (d) $1\frac{7}{11}$ (e) $10\frac{2}{7}$
12. (a) $3\frac{3}{5}$ (b) $2\frac{2}{3}$
13. (a) $3\frac{2}{3}$ (b) $\frac{7}{3}$ (c) $12\frac{2}{3}$ (d) $\frac{13}{5}$
14. $\frac{1}{3}$
15. 24

Chapter 5

Exercise 5A

- (a) $\frac{2}{3}$ (b) $\frac{7}{9}$ (c) $\frac{4}{7}$ (d) $\frac{1}{11}$ (e) $\frac{11}{15}$ (f) $\frac{6}{23}$
(g) $\frac{5}{9}$ (h) $\frac{6}{17}$ (i) $\frac{17}{19}$ (j) $\frac{19}{25}$ (k) $\frac{5}{7}$ (l) $\frac{26}{29}$
- (a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{3}{4}$ (f) $\frac{3}{4}$
(g) $\frac{2}{3}$ (h) $\frac{4}{9}$ (i) $\frac{4}{5}$ (j) $\frac{7}{12}$ (k) $\frac{5}{6}$ (l) $\frac{5}{8}$
- (a) $1\frac{2}{5}$ (b) $1\frac{4}{9}$ (c) $1\frac{7}{8}$ (d) $2\frac{2}{7}$ (e) $3\frac{1}{9}$
(f) $1\frac{6}{7}$ (g) $1\frac{4}{5}$ (h) $2\frac{2}{9}$ (i) $3\frac{6}{11}$
- (a) $2\frac{1}{2}$ (b) $3\frac{2}{3}$ (c) $4\frac{1}{2}$ (d) $3\frac{1}{3}$ (e) $4\frac{1}{6}$
(f) 2 (g) $5\frac{2}{3}$ (h) $2\frac{1}{2}$ (i) 8 (j) 9 (k) $5\frac{1}{3}$
- (a) $11\frac{2}{5}$ (b) $8\frac{1}{3}$ (c) $5\frac{1}{2}$ (d) $8\frac{2}{3}$ (e) $4\frac{1}{2}$
(f) $4\frac{3}{5}$ (g) $10\frac{3}{4}$ (h) $5\frac{1}{2}$ (i) $3\frac{1}{9}$
- (a) $\frac{5}{8}$ (b) $\frac{1}{4}$ (c) $\frac{7}{10}$ (d) $\frac{7}{8}$ (e) $\frac{1}{6}$
(f) $\frac{2}{9}$ (g) $\frac{5}{12}$ (h) $\frac{17}{20}$ (i) $\frac{13}{15}$ (j) $\frac{17}{24}$ (k) $\frac{17}{20}$
(l) $\frac{13}{36}$ (m) $\frac{5}{12}$ (n) $\frac{17}{20}$ (o) $\frac{7}{30}$ (p) 1
- (a) $1\frac{1}{2}$ (b) $1\frac{5}{8}$ (c) $1\frac{7}{15}$ (d) $1\frac{7}{20}$ (e) $1\frac{7}{24}$
(f) $1\frac{27}{40}$ (g) $1\frac{19}{30}$ (h) $1\frac{29}{72}$ (i) $1\frac{1}{2}$ (j) $1\frac{17}{20}$
(k) $1\frac{3}{10}$ (l) $1\frac{1}{4}$
- (a) $6\frac{3}{20}$ (b) $11\frac{1}{6}$ (c) $13\frac{5}{12}$ (d) $5\frac{13}{24}$
(e) $14\frac{1}{10}$ (f) $10\frac{3}{8}$ (g) $5\frac{26}{35}$ (h) $10\frac{1}{2}$
(i) $14\frac{5}{6}$ (j) $6\frac{7}{12}$ (k) $16\frac{1}{8}$
- (a) $4\frac{3}{10}$ (b) $1\frac{7}{12}$ (c) $3\frac{13}{30}$ (d) $2\frac{27}{40}$ (e) $1\frac{8}{15}$
(f) $7\frac{19}{56}$
- (a) $5\frac{17}{20}$ (b) $\frac{5}{6}$ (c) $2\frac{7}{15}$ (d) $4\frac{23}{24}$ (e) $2\frac{8}{15}$
(f) $15\frac{1}{44}$ (g) $1\frac{23}{40}$ (h) $3\frac{9}{20}$ (i) $13\frac{13}{18}$ (j) $8\frac{13}{24}$
(k) $6\frac{5}{12}$ (l) $10\frac{2}{3}$ (m) $5\frac{33}{40}$
- (a) $7\frac{1}{2}$ (b) $2\frac{1}{2}$ (c) 30 (d) \$22.50
- (a) 5 pm (b) 3 hours
- (a) 20 cups (b) 2 cups
(c) 15 cups of soda water
 $9\frac{1}{2}$ cups of orange juice
 $7\frac{1}{2}$ cups of apple juice
 $4\frac{1}{2}$ cups of pineapple juice
 $3\frac{1}{2}$ cups of grapefruit juice
- $\frac{1}{4}$
- $14\frac{5}{6}$ kg
- $3\frac{1}{2}$ kg, $4\frac{1}{2}$ kg
- $7\frac{1}{3}$ m, $6\frac{2}{3}$ m
- (a) $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2
(b) $\frac{1}{3}$, $\frac{5}{6}$, 1, $1\frac{1}{6}$, $1\frac{1}{3}$, $1\frac{1}{2}$, $1\frac{2}{3}$
(c) $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{3}{4}$, $\frac{11}{12}$, $1\frac{1}{12}$, $1\frac{1}{4}$
(d) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{8}$, 0
(e) $\frac{5}{8}$, $\frac{13}{24}$, $\frac{11}{24}$, $\frac{3}{8}$, $\frac{7}{24}$, $\frac{5}{24}$, $\frac{1}{8}$
(f) 1, $\frac{23}{24}$, $\frac{11}{12}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{19}{24}$, $\frac{3}{4}$

Exercise 5B

- (a) $\frac{1}{6}$ (b) $\frac{6}{35}$ (c) $\frac{25}{42}$ (d) $\frac{10}{27}$ (e) $\frac{9}{100}$ (f) $\frac{11}{24}$
(g) $\frac{1}{24}$ (h) $\frac{15}{56}$
- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{5}$ (e) $\frac{9}{28}$ (f) $\frac{4}{7}$
(g) $\frac{8}{25}$ (h) $\frac{4}{15}$
- (a) $3\frac{3}{4}$ (b) $5\frac{1}{3}$ (c) $3\frac{3}{4}$ (d) $2\frac{4}{7}$ (e) $5\frac{1}{3}$
(f) 15 (g) $5\frac{1}{2}$ (h) 14 (i) 10 (j) $7\frac{1}{2}$
(k) $10\frac{1}{2}$ (l) 28
- (a) $12\frac{1}{2}$ (b) $8\frac{4}{5}$ (c) $15\frac{1}{2}$ (d) 26
(e) $14\frac{1}{2}$ (f) $25\frac{2}{3}$ (g) $31\frac{1}{2}$ (h) 46
(i) $26\frac{2}{5}$ (j) 69 (k) $20\frac{1}{4}$ (l) 70
- (a) $4\frac{1}{3}$ (b) 12 (c) $14\frac{2}{5}$ (d) $16\frac{1}{2}$
(e) $32\frac{2}{9}$ (f) $6\frac{4}{5}$ (g) 27 (h) $12\frac{1}{2}$ (i) 35
(j) $19\frac{1}{4}$ (k) $19\frac{1}{6}$ (l) $10\frac{1}{12}$
- (a) 10 (b) 8 (c) 4 (d) 12 (e) 12 (f) 6
(g) 12 (h) 12 (i) 30 (j) 44 (k) 22
(l) 15 (m) $\frac{1}{6}$ (n) $\frac{2}{5}$ (o) $\frac{2}{3}$ (p) $\frac{10}{13}$ (q) $2\frac{1}{3}$
(r) $6\frac{1}{3}$ (s) 5 (t) $6\frac{1}{2}$
- (a) 4 m (b) 15 kg (c) \$50 (d) 200 mm
(e) 18 cm (f) \$75 (g) 150 m
(h) 800 kg (i) 7 g (j) $1\frac{1}{2}$ m (k) $1\frac{1}{2}$ m
(l) 6 kg
- (a)(i) 3 kg (ii) $4\frac{1}{2}$ kg (iii) $6\frac{1}{4}$ kg
(b) 5 litres
(c)(i) 12 m (ii) $22\frac{1}{2}$ m (iii) $37\frac{1}{2}$ m
(d) 6 hectares
(e)(i) 21 kg (ii) 35 kg (iii) $87\frac{1}{2}$ kg
(f)(i) 15 minutes (ii) 40 minutes
(iii) $62\frac{1}{2}$ minutes
- 27 goals
- (a) \$27 (b) \$18
- Devils - 2500, Saints - 1500
- \$600
- 55 minutes
- (a) 400 (b) 1600
- (a) 1 tonne
(b) $\frac{3}{4}$ of a tonne
(c) $\frac{3}{4}$ of a tonne
- 6 eggs
- $\frac{1}{16}$
- (a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{20}$
(d) 150 hectares (e) 450 hectares
- (a) 15 minutes (b) $37\frac{1}{2}$ minutes
(c) $87\frac{1}{2}$ minutes
- (a) $1\frac{1}{3}$ kg
(b)(i) $13\frac{1}{3}$ kg (ii) 20 kg (iii) $33\frac{1}{3}$ kg

Exercise 5C

- (a) $\frac{8}{9}$ (b) $\frac{24}{35}$ (c) $\frac{2}{3}$ (d) $\frac{21}{50}$ (e) $\frac{9}{10}$
 (f) $\frac{1}{2}$ (g) $\frac{8}{11}$ (h) $\frac{7}{9}$ (i) $1\frac{1}{2}$ (j) $1\frac{1}{5}$
 (k) $1\frac{1}{2}$ (l) $1\frac{1}{12}$ (m) $4\frac{2}{3}$ (n) $7\frac{1}{2}$
 (o) $2\frac{4}{9}$ (p) $5\frac{1}{4}$
- (a) $1\frac{3}{7}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) 4 (e) $1\frac{1}{3}$
 (f) 6 (g) $\frac{2}{9}$ (h) $\frac{1}{6}$
- (a) $\frac{3}{5}$ (b) $\frac{17}{22}$ (c) $\frac{8}{9}$ (d) $\frac{1}{6}$ (e) $1\frac{3}{4}$
 (f) $6\frac{3}{4}$ (g) $3\frac{9}{10}$ (h) $6\frac{5}{6}$
- (a) $10\frac{1}{3}$ (b) $6\frac{2}{5}$ (c) $\frac{3}{8}$ (d) $\frac{7}{10}$
 (e) $12\frac{1}{7}$ (f) $\frac{2}{3}$ (g) $8\frac{1}{2}$ (h) $\frac{5}{9}$ (i) $\frac{3}{5}$
 (j) $\frac{1}{3}$
- (a) 3 (b) 10 (c) 16 (d) 30 (e) 7
 (f) 11 (g) 73 (h) 29 (i) 53 (j) 69
- (a) 15 (b) $6\frac{1}{2}$ (c) $3\frac{1}{3}$ (d) $\frac{2}{3}$
 (e) $6\frac{1}{2}$ (f) $5\frac{1}{4}$ (g) $7\frac{2}{5}$ (h) $\frac{8}{9}$
 (i) $3\frac{1}{4}$ (j) $9\frac{1}{3}$
- 9
- $1\frac{1}{3}$ donuts
- (a) $\frac{3}{4}$ m
 (b)(i) 80 steps
 (ii) 440 steps
 (iii) 4000 steps
- 20 blocks
- (a) 20 hectares (b) 125 acres
- 20 buckets
- 12 containers

Exercise 5D

- $\frac{3}{4}$ 2. $\frac{1}{2}$ 3. 3 4. $\frac{5}{6}$ 5. $1\frac{1}{3}$ 6. $\frac{1}{5}$
 7. $1\frac{3}{4}$ 8. $\frac{46}{63}$ 9. 0

Exercise 5E

- $\frac{39}{40}$ 2. $8\frac{9}{35}$ 3. $\frac{299}{315}$ 4. $2\frac{17}{24}$ 5. $9\frac{29}{72}$
 6. $1\frac{113}{360}$ 7. $63\frac{1}{12}$ 8. $\frac{15}{68}$ 9. $2\frac{115}{126}$

Exercise 5F

- (a) 20 kg (b) 8 days
 (c) 6 pellets, 18 seed, 6 corn
- (a) 3 m (b) $4\frac{1}{2}$ m (c) $15\frac{3}{16}$ m
- (a) 1 m (b) $\frac{8}{27}$ m
- (a) 60 km, 40 km, 80 km (b) $\frac{1}{3}$, $\frac{2}{9}$, $\frac{4}{9}$
- (a) $1\frac{1}{2}$ kg (b) $1\frac{3}{4}$ kg (c) $\frac{1}{3}$ kg

Chapter Review

- (a) $\frac{4}{5}$ (b) $\frac{3}{7}$ (c) $5\frac{7}{11}$ (d) $3\frac{3}{7}$ (e) $\frac{5}{8}$ (f) $\frac{3}{4}$
 (g) $\frac{5}{8}$ (h) $1\frac{4}{15}$ (i) $\frac{1}{12}$ (j) $\frac{45}{56}$ (k) $\frac{19}{30}$ (l) $\frac{13}{14}$
- (a) $3\frac{17}{20}$ (b) $1\frac{11}{12}$ (c) $1\frac{5}{7}$ (d) $10\frac{1}{28}$
- (a) 5 cups (b) 10 days
 (c) $4\frac{2}{3}$ cups of wheat flakes
 3 cups of oats
 $1\frac{1}{3}$ cups of sultanas
 $\frac{1}{2}$ a cup of sunflower seeds
 $\frac{1}{2}$ a cup of crushed nuts
- (a) $\frac{1}{12}$ (b) $\frac{3}{10}$ (c) $\frac{2}{9}$ (d) $\frac{7}{15}$
- (a) 4 (b) 10 (c) $12\frac{2}{3}$ (d) $13\frac{1}{2}$
- (a) \$20 (b) 10 kg (c) \$180 (d) 84 m
- (a) 320 students (b) $\frac{3}{5}$
- $\frac{1}{27}$
- (a) $\frac{3}{4}$ (b) $\frac{4}{7}$ (c) $\frac{5}{21}$ (d) $\frac{12}{25}$
- (a) 4 (b) $\frac{1}{4}$ (c) $1\frac{1}{2}$ (d) $1\frac{13}{35}$
- (a) 20 (b) $1\frac{2}{3}$ (c) 19 (d) 12
- $\frac{3}{5}$
- (a) $\frac{27}{40}$ (b) $\frac{7}{8}$ (c) 1
- (a) $\frac{3}{4}$ kg (b) 21 kg of clay,
 $11\frac{1}{2}$ kg of sand, $3\frac{1}{2}$ kg of cement

Chapter 6**Exercise 6A**

1.

10000	1000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten Thousandths

- (a)(i) eight thousandths (ii) $\frac{8}{1000}$
 (b)(i) five tenths (ii) $\frac{5}{10}$
 (c)(i) six hundredths (ii) $\frac{6}{100}$
 (d)(i) nine hundred-thousandths (ii) $\frac{9}{10000}$
 (e)(i) one ten-thousandth (ii) $\frac{1}{10000}$
- (a) 53.36 (b) 382.5 (c) 4.17 (d) 5.902
 (e) 0.56 (f) 3.009 (g) 25.041
 (h) 52.94108 (i) 4.0541 (j) 3.065204

4. (a)(i) three tenths (ii) $\frac{3}{10}$
 (b)(i) three hundred (ii) 300
 (c)(i) three hundredths (ii) $\frac{3}{100}$
 (d)(i) three units (ii) 3
 (e)(i) three ten-thousandths (ii) $\frac{3}{10000}$
 (f)(i) three thousand (ii) 3000
 (g)(i) three tens (ii) 30
 (h)(i) three ten-thousands (ii) 30000
 (i)(i) three millionths (ii) $\frac{3}{1000000}$
 (j)(i) three thousandths (ii) $\frac{3}{1000}$
5. (a) 700, 70, 7, 0.7, **0.07, 0.007, 0.0007**
 (b) 0.0002, 0.002, 0.02, 0.2, **2, 20, 200**
6. (a) 0.07 (b) 0.002 (c) 0.9
 (d) 0.0004 (e) 0.005
 (f) 0.00008 (g) 0.06
7. (a) 0.18 (b) 0.087 (c) 0.206
 (d) 0.0039 (e) 0.4 (f) 2.7
 (g) 15.18 (h) 6.279 (i) 123.082
 (j) 7.2 (k) 8.7 (l) 1.63
 (m) 20.5 (n) 7.099 (o) 8050.1
8. (a) 5.67 (b) 4.129 (c) 0.874
 (d) 6.02 (e) 9.402 (f) 30.0701
 (g) 0.874 (h) 6.903 (i) 809.15
 (j) 102.63 (k) 5050.50505
9. (a) $\frac{310}{1000}$, $\frac{4}{10}$, 0.5, $\frac{63}{100}$, 0.9
 (b) 0.022, 0.202, 0.22, 2.02, 2.2
 (c) 0.01, $\frac{8}{100}$, 0.089, $\frac{9}{10}$, 0.91
 (d) 6.5163, 6.5361, 6.5621, 6.6351, 6.6531
10. (a) $0.3 < 0.4$ (b) $1.8 < 2.1$
 (c) $0.305 < 0.31$ (d) $0.721 > 0.719$
 (e) $3.605 > 3.596$ (f) $7.136 < 7.141$
11. (a) 0.7 (b) 1.9 (c) 0.37 (d) 4.1
 (e) 307.1 (f) 6.132 (g) 0.322
 (h) 0.101 (i) 1 (j) 3.99 (k) 16.02
 (l) 21 (m) 210 (n) 60 (o) 1000
12. (a) 3.6225 (b) 0.2058 (c) 2.541
 (d) 18.001 (e) 3.071 (f) 6.033
 (g) 0.29 (h) 0.00101 (i) 1.0009
 (j) 20
13. (a) 0.1, 0.2, 0.3, 0.4, **0.5, 0.6, 0.7, 0.8**
 (b) 0.70, 0.71, 0.72, 0.73, **0.74, 0.75**
0.76, 0.77
 (c) 3.0, 3.2, 3.4, 3.6, **3.8, 4.0, 4.2, 4.4**
 (d) 0.25, 0.26, 0.27, 0.28, **0.29, 0.30**
0.31, 0.32
 (e) 5.95, 5.96, 5.97, 5.98, **5.99, 6.00**
6.01, 6.02
14. (a) Many answers eg. 6, 7, 8.5, 9.1
 (b) Many answers eg. 4.1, 4.2, 4.75
 (c) Many answers eg. 0.21, 0.22, 0.248
 (d) Many answers eg. 5.671, 5.672
15. (a) 15 (b) 8.5 (c) 2.15 (d) 7.855
16. (a) 10 (b) 200 (c) 30 (d) 500
17. 37.4°C
18. 1. Charley 11.9 seconds
 2. Sharon 12.2 seconds
 3. Yvonne 12.6 seconds
19. Damian Dill 1 minute 31.292 seconds
 Jack Badman 1 minute 31.315 seconds
20. Bradon by 0.04 metres
21. Rocky Reservoir, Peaceful Pond,
 Long Lake, Deep Dam, Wide Waters

Exercise 6B

1. (a) 7 (b) 2.5 (c) 0.4 (d) 0.35 (e) 14
 (f) 0.8 (g) 1.8 (h) 0.04 (i) 1.72
 (j) 21.64
2. (a) 2.5°C (b) 0.3°C (c) 0.35°C
 (d) 1.25°C (e) 0.12°C
3. (a) 0.4 (b) 1.4 (c) 0.035 (d) 0.05
 (e) 0.06 (f) 0.016

Exercise 6C

1. (a) \$8.50 (b) \$10.80 (c) \$15.35
 (d) \$68.15 (e) \$22.20 (f) \$22.00
 (g) \$6.05 (h) \$1.00 (i) \$0.65 (j) \$0.35
2. (a) \$8.50 (b) \$10.80 (c) \$15.30
 (d) \$68.20 (e) \$22.20 (f) \$22.00
 (g) \$6.10 (h) \$1.00 (i) \$0.70
 (j) \$0.30 (k) \$8.40 (l) \$60.00
 (m) \$9.00 (n) \$100.00 (o) \$76.80

3. (a) \$9.00 (b) \$11.00 (c) \$15.00
(d) \$68.00 (e) \$22.00 (f) \$22.00
(g) \$6.00 (h) \$1.00 (i) \$1.00 (j) \$0
(k) \$8.00 (l) \$60.00 (m) \$9.00
(n) \$100.00 (o) \$77.00
4. (a) 1 (b) 3 (c) 4 (d) 3 (e) 5 (f) 4
(g) 5 (h) 3 (i) 0 (j) 6
5. (a) 56.71 (b) 8.2 (c) 8.4507
(d) 123.6 (e) 0.57 (f) 45.322
(g) 11.50 (h) 782.0 (i) 47.46
(j) 0.01 (k) 14.09010 (l) 245.700
6. (a) 45.8 (b) 610.8 (c) 93.1 (d) 0.3
(e) 9.0 (f) 13.5 (g) 101.0 (h) 0.1
7. (a) two (b) thousandth (c) four
8. (a) 567.03 (b) 9.46 (c) 17.89
(d) 23.60 (e) 6.10 (f) 102.30
(g) 0.57 (h) 7.65
9. (a) 567.027 (b) 9.457 (c) 17.893
(d) 23.597 (e) 6.095 (f) 102.303
(g) 0.568 (h) 7.654
10. (a) 68.7 m (b) 9.7 m (c) 35.1 m
(d) 0.9 m (e) 120.6 m (f) 35.0 m
(g) 9.0 m (h) 60.0 m
11. (a) 5.674 sec (b) 0.048 sec
(c) 19.209 sec (d) 59.139 sec
(e) 107.969 sec (f) 53.900 sec
12. (a) 67.85 kg (b) 7.97 kg
(c) 41.78 kg (d) 108.99 kg
(e) 0.67 kg (f) 0.10 kg
13. (a) 7 kg (b) 12 kg (c) 20 kg
(d) 1 kg (e) 10 kg (f) 7 kg
(g) 29 kg (h) 4 kg
14. (a) 70 kg (b) \$140 (c) 69.2 kg
(d) 69 kg (e) \$138
15. (a) Glenn - \$21.00
Linda - \$22.00
(b) \$20.97
(c) \$20.95, Change - \$29.05

Exercise 6D

1. (a) 0.1 (b) 0.03 (c) 0.27
(d) 0.009 (e) 0.081 (f) 0.451
2. (a) 0.5 (b) 0.75 (c) 0.2
(d) 0.125 (e) 0.45 (f) 2.75

3. (a) 0.4 (b) 0.6 (c) 0.8
4. (a) 3.5 (b) 8.25 (c) 10.2
(d) 6.75 (e) 25.6 (f) 17.8

Exercise 6E

1. (a) $0.\dot{7}$ (b) $0.\overline{25}$ (c) $0.6\dot{1}$ (d) $0.97\overline{1245}$
(e) $6.\overline{47}$ (f) $23.\dot{8}$ (g) $17.96\overline{21}$ (h) $3.04\overline{59}$
2. (a) $0.\dot{3}$ (b) $0.8\dot{3}$ (c) $0.\dot{7}$ (d) 0.416
(e) $0.4285\overline{71}$ (f) $5.91\dot{6}$
3. (a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) $\frac{8}{9}$ (d) $\frac{1}{6}$ (e) $\frac{1}{7}$

Chapter Review

1. (a) 353.6 (b) 3.504 (c) 1.427
2. (a) (i) five hundredths (ii) 0.05
(b) (i) six tenths (ii) 0.6
(c) (i) seven and twenty-eight thousandths (ii) 7.028
(d) (i) twenty-three and seventeen ten-thousandths (ii) 23.0017
(e) (i) eight and two-hundred and four thousandths (ii) 8.204
3. (a) 3.79 (b) 60.32 (c) 57.207
4. (a) $\frac{7}{100}$, 0.09, 0.4, $\frac{63}{100}$, $\frac{714}{1000}$, $\frac{8}{10}$
(b) 0.6123, 0.6125, 0.6152, 0.6215, 0.6251, 0.6312, 0.6321
5. (a) 20 (b) 300
6. (a) 3.488 (b) 0.225 (c) 6.31
(d) 8.2073 (e) 0.0101
7. (a) 0.8 (b) 0.03 (c) 0.4 (d) 8.16
8. (a) (i) \$0.90 (ii) \$1.00
(b) (i) \$3.40 (ii) \$3.00
(c) (i) \$15.80 (ii) \$16.00
(d) (i) \$10.50 (ii) \$10.00
(e) (i) \$101.60 (ii) \$102.00
9. (a) $0.7\dot{5}$ (b) 0.8 (c) 8.5 (d) $13.\dot{3}$
(e) $65.\dot{2}$
10. Andrew 58.96 sec, Luke 59.04 sec, Aidan 59.06 sec

Chapter 7

Exercise 7A

- (a) 9.9 (b) 19.5 (c) 29.88 (d) 14.7
(e) 37.9 (f) 0.899 (g) 394.2 (h) 13.1
(i) 24.6 (j) 16.36 (k) 12.68
(l) 99.698 (m) 13.353 (n) 5.076
(o) 46.818
- \$14.90
- (a) 2.15, 2.45, 2.75, 3.05
(b) 0.90, 0.93, 0.96, 0.99
(c) 3.4, 4.4, 5.6, 7.0
(d) 25.5, 33, 42, 52.5
- 17.055 MB
- (a) (i) 4.5 m (ii) 8.5 m
(b) 1.2 m (c) 15 years
(d) thirtieth year

Exercise 7B

- (a) 1.3 (b) 13.3 (c) 25.43 (d) 1.9
(e) 18.6 (f) 0.667 (g) 259 (h) 3.5
(i) 3.37 (j) 8.76 (k) 23.52
(l) 474.482 (m) 7.907 (n) 4.904
(o) 7605.8555
- (a) 1.38 m (b) 0.16 m (c) 0.34 m

Exercise 7C

- Salad Roll \$1.40, Donut \$0.70,
Orange Juice \$0.90
- 3.5 kg
- 1.5, 2.5
- 8.2, 2.8
- \$3.55
- 2.87 litres
- (a) 7.2 mL, 6.0 mL, 4.8 mL, 3.6 mL,
2.4 mL, 1.2 mL
(b) 25.2 mL (c) 4.8 mL
- \$8.05
- (a) Jan, Feb, Mar, Sep, Oct, Nov, Dec
(b) 21.8°C (c) 14.3°C (d) 7.5°C
(e) 1.6°C decrease (f) 0.8°C increase
(g) Feb (h) Jul

Exercise 7D

- (a) 1 (b) 2 (c) 1 (d) 3 (e) 3 (f) 3
(g) 0 (h) 4
- (a) 588.0 (b) 54.45 (c) 80.58
(d) 12.481 (e) 43.2 (f) 145.88
(g) 1132.080 (h) 715.815 (i) 2770.740
- (a) 0.75 kg (b) 3 kg (c) 5.4 kg
- \$70
- (a) 59.15 kWh (b) 3084.25 kWh
- Length 3.6 m, height 2.72 m

Exercise 7E

- (a) 28.35 (b) 60.45 (c) 83.08
(d) 2.584 (e) 0.432 (f) 0.416
(g) 1.132 (h) 1.5015 (i) 0.4003
(j) 0.01005 (k) 0.14122 (l) 24.582
(m) 0.00002 (n) 0.01 (o) 0.000064
- (a) \$1.88 (b) \$13.16 (c) \$56.87

Exercise 7F

- (a) 1.3 (b) 3.2 (c) 12.3 (d) 48.9
(e) 12.9 (f) 24.75 (g) 48.36
(h) 25.46 (i) 5.247 (j) 1.254
(k) 0.026 (l) 0.0048
- (a) 1.5 (b) 4.5 (c) 3.4 (d) 0.15
(e) 15.5 (f) 3.75 (g) 0.0125
(h) 0.688 (i) 3.8875 (j) 9.062
(k) 5.9325 (l) 25.0025
- \$4.55
- \$34.25
- 23.6 kL
- 0.54 m

Exercise 7G

- (a) 4 (b) 9 (c) 0.3 (d) 90 (e) 410
(f) 0.42 (g) 2 (h) 10.1 (i) 40
(j) 40 (k) 12.3 (l) 80 (m) 3450
(n) 10 (o) 500
- 15 oranges
- 12 pieces of wood
- 500 sheets
- 40 blood cells

Exercise 7H

- \$31.25
- \$0.85
- 25.5 mm
- 0.568 million dollars
- 50 bottles
- 0.055 kg
- 0.35 kg
- 50 bolts

Exercise 7I

- (a) 78.3 (b) 6208 (c) 21.4
(d) 31 000 (e) 0.08 (f) 5500
(g) 27.8 (h) 8 270 000 (i) 5000
(j) 0.56 (k) 651.75 (l) 2967
- (a) 3.52 (b) 0.6573 (c) 0.06
(d) 50 (e) 0.07935 (f) 89.13
(g) 2.2 (h) 0.000038 (i) 0.0059
(j) 0.075145 (k) 4.5278 (l) 0.054001
- (a) 98.3 (b) 1.628 (c) 80.4
(d) 0.457 (e) 5 (f) 8.439 (g) 44.8
(h) 8.2702 (i) 0.0005 (j) 16
(k) 950.75 (l) 0.2967

Exercise 7J

- (a) 3000 (b) 5640 (c) 2 (d) 25.4
(e) 2 000 000 (f) 56 000 (g) 21 500
(h) 785 (i) 5 (j) 78.6
(k) 400 000 000 (l) 0.256
(m) 0.00437 (n) 0.00089
- (a) 7500 (b) 2.5 (c) 0.35 (d) 3650
(e) 0.035 (f) 678 500 (g) 0.25
(h) 0.008 (i) 0.00625
(j) 230 000 000 (k) 19 000
(l) 0.0000454
- 895 000 000 watts
- 0.0075 amps
- (a) 90 mL (b) 0.09 L
- 3 000 000 litres
- 0.000058 grams
- (a) 0.00035 metres
(b) 350 micrometres

Exercise 7K

- 1.74 m
- 25.6 kg
- 12.4
- 56.575 sec
- (a) 49 (b) 59

Exercise 7L

- (a) 39.2 (b) 8.7 (c) 437.76 (d) 6.6
(e) 68.45 (f) 35.3
- 2.35 kg
- 378 kg
- 10.08 m
- 53.8 sec
- \$23.15

Exercise 7M

- (a)(i) $7 \times 9 = 63$ (ii) 62.56
(b)(i) $48 \div 6 = 8$ (ii) 7.8
(c)(i) $20 \times 6 = 120$ (ii) 120.17
(d)(i) $72 \div 8 = 9$ (ii) 9.2
(e)(i) $(6 + 2) \times 7 = 8 \times 7 = 56$ (ii) 59.86
(f)(i) $80 \div (16 - 8) = 10$ (ii) 10.1
(g)(i) $(16 - 7) \times (24 - 17) = 9 \times 7 = 63$
(ii) 56.96
(h)(i) $(7 + 8 + 9) \times (6 + 4) = 24 \times 10 = 240$
(ii) 237.16
- (a) $9 \times 3 = \$27$ (b) \$27.72
- (a) $(7 + 8 + 8 + 7 + 8) \times 3 = 38 \times 3 = \$114$
(b) \$109.155 (c) \$109.20
- (a) $12 \times 5 = 60 \text{ m}^2$ (b) 59.29 m^2

Chapter Review

- (a) 6.8 (b) 19.6 (c) 15.2 (d) 104
(e) 8.86 (f) 28.094
- (a) 2.7 (b) 12.32 (c) 4.77 (d) 8.95
(e) 226.97 (f) 18.72
- (a) 18.6 (b) 129.5 (c) 976.8
(d) 25.42 (e) 10.44 (f) 13.1285
- (a) 2.7 (b) 6.7 (c) 0.27 (d) 3.25
(e) 1.44 (f) 6.3875
- (a) 9 (b) 10.4 (c) 321.5

6. (a) 865 (b) 3.7 (c) 6.83 (d) 560
(e) 0.0894 (f) 0.008
7. (a) 7000 m (b) 0.065 g (c) 8.3 m
(d) 6.832 m (e) 5600 L (f) 57.8 g
8. 20
9. 22.82 km
10. 1.75 m
11. \$45

Chapter 8

Exercise 8A

1. (a)(i) 1% (ii) 99%
(b)(i) 10% (ii) 90%
(c)(i) 20% (ii) 80%
(d)(i) 36% (ii) 64%
(e)(i) 20% (ii) 80%
(f)(i) 60% (ii) 40%
2. 37%
3. 85%
4. 40%
5. Milo 30%, Helena 20%, Cherie 50%
6. 82%
7. (a) 48% (b) Russell
8. 14 %
9. B
10. (a) B (b) A
11. (a) 50% (b) 50%

Exercise 8B

1. (a) 10% (b) 30% (c) 90% (d) 50%
(e) 20% (f) 75% (g) 35% (h) 65%
(i) 48% (j) 42%
2. (a) 25% (b) 75%
3. (a)(i) 40% (ii) 60%
(b)(i) 80% (ii) 20%
(c)(i) 50% (ii) 50%
4. (a) Fractions - 85%, Decimals - 80%,
Numbers - 90%, Graphs - 88%
(b) Numbers
5. (a) 80% (b) 70%
6. (a) 40% (b) 60%
7. (a) 900 (b) Blue - 25%, Green - 20%,
Orange - 55%
8. (a) 16 (b) 75% (c) 25%

9. (a) 50 (b) American - 18%,
German - 14%, French - 8%,
English - 12%, Japanese - 16%,
Chinese - 6%, Australian - 26%
(c) 34% (d) 22%
10. Garden - 30%, Bathroom - 25%,
Laundry - 15%, Toilet - 20%,
Kitchen - 10%
11. Surfing 50%, Body Boarding 30%
Swimming 10%, Diving 10%

Exercise 8C

1. (a) $\frac{1}{100}$ (b) $\frac{11}{100}$ (c) $\frac{19}{100}$ (d) $\frac{73}{100}$ (e) $\frac{99}{100}$
2. (a) $\frac{1}{20}$ (b) $\frac{3}{25}$ (c) $\frac{9}{50}$ (d) $\frac{1}{4}$ (e) $\frac{7}{20}$
(f) $\frac{1}{2}$ (g) $\frac{3}{5}$ (h) $\frac{19}{25}$ (i) $\frac{19}{20}$ (j) 1
3. (a) $\frac{39}{50}$ (b) $\frac{21}{100}$ (c) $\frac{1}{100}$
4. (a) $\frac{17}{20}$ (b) $\frac{11}{50}$
5. ACT - $\frac{1}{50}$, NSW - $\frac{8}{25}$, NT - $\frac{1}{100}$,
Qld - $\frac{1}{5}$, SA - $\frac{2}{25}$, Tas - $\frac{1}{50}$, Vic - $\frac{1}{4}$,
WA - $\frac{1}{10}$

Exercise 8D

1. 33% 2. 19% 3. 87% 4. 6%
5. 40% 6. 29.5% 7. 5.6% 8. 0.8%
9. 200% 10. 10.57%

Exercise 8E

1. 0.59 2. 0.24 3. 0.76 4. 0.3
5. 0.08 6. 0.1 7. 1 8. 0.654
9. 0.058 10. 0.0025

Exercise 8F

1.	Percentage	Fraction	Decimal
	50%	$\frac{1}{2}$	0.5
	10%	$\frac{1}{10}$	0.1
	25%	$\frac{1}{4}$	0.25
	20%	$\frac{1}{5}$	0.2
	90%	$\frac{9}{10}$	0.9
	66%	$\frac{33}{50}$	0.66
	18%	$\frac{9}{50}$	0.18
	32%	$\frac{8}{25}$	0.32
	85%	$\frac{17}{20}$	0.85
	9%	$\frac{9}{100}$	0.09
	3%	$\frac{3}{100}$	0.03
	4%	$\frac{1}{25}$	0.04

2. (a) 0.9 (b) 80% (c) 0.01 (d) 10%
 (e) 0.09 (f) 0.08 (g) $\frac{3}{5}$ (h) 6%
 (i) 100% (j) 0.1%
 3. (a) 80% (b) $\frac{1}{5}$ (c) 60%

Exercise 8G

Percentage	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%
Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$	$\frac{1}{5}$
Decimal	0.25	0.5	0.75	0. $\dot{3}$	0. $\dot{6}$	0.1	0.2

Exercise 8H

1. (a) 4 m (b) \$10 (c) \$5 (d) 3 kg
 (e) 6 kg (f) 9 m (g) \$60 (h) \$100
 (i) 10 km (j) 25 kg (k) \$4000
 (l) 100 kg (m) \$2.50 (n) \$0.30
 (o) \$2.60
 2. (a) 120 kg (b) 90 m (c) \$96 (d) \$210
 (e) 1350 m (f) 44 m (g) \$51 (h) \$21
 (i) 28 km (j) 40 kg (k) \$31.50
 (l) 134.4 kg (m) \$0.10 (n) \$2.28
 (o) \$42.96
 3. (a) 2 (b) 80
 4. 4
 5. 18
 6. (a) 3300 (b) 3630
 7. (a) \$2250 (b) \$47 250
 8. \$1600
 9. 5.2 kg
 10. (a) 40% (b) canola - 6000 hectares,
 wheat - 18 000 hectares,
 oats - 16 000 hectares

Exercise 8I

1. \$64
 2. \$27
 3. \$450
 4. \$23 750
 5. \$245
 6. \$132

Exercise 8J

1. (a) 279 kg (b) 161.5 m (c) \$593.40
 (d) \$1128 (e) 3013.50 m (f) 0.385 m
 (g) \$48.75 (h) \$3570 (i) \$10799.25

2. \$112.50
 3. 371
 4. (a) 96 (b) 6
 5. 1.75 kg
 6. (a) 5.6% (b) 628 125 (c) 75 375

Chapter Review

1. (a) 68% (b) 80%
 2. (a) 5% (b) $\frac{1}{20}$
 3. (a) 70% (b) red - 2 litres,
 yellow - 4 litres, white - 14 litres
 4.

Percentage	10%	25%	50%	20%	1%	$33\frac{1}{3}\%$	70%
Fraction	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{100}$	$\frac{1}{3}$	$\frac{7}{10}$
Decimal	0.1	0.25	0.5	0.2	0.01	0. $\dot{3}$	0.7

5. (a) \$8 (b) 50 m (c) 30 kg (d) \$90
 (e) 2 m (f) 12 kg
 6. (a) \$60 (b) 406 m (c) 255 kg
 (d) \$0.56 (e) 69 m (f) \$41.25
 7. 4500
 8. (a) \$120 (b) \$72

Chapter 9

Exercise 9A

1. (a) \$10 (b) 6 kg (c) 12 m (d) \$4
 (e) \$100 (f) 8 litres
 2. (a)(i) 3 (ii) \$12
 (iii) Iris \$12, Lily \$24
 (b) (i) Iris \$24, Lily \$12
 (ii) Iris \$9, Lily \$27
 (iii) Iris \$27, Lily \$9
 (iv) Iris \$6, Lily \$30
 (v) Iris \$32, Lily \$4
 (vi) Iris \$28, Lily \$8
 (vii) Iris \$20, Lily \$16

3.

Amount to be Divided	Ratio	Total Number of Parts	Size of each Part	Size of First Amount	Size of Second Amount
\$24	3:1	4	\$6	\$18	\$6
\$30	1:4	5	\$6	\$6	\$24
28 m	4:3	7	4 m	16 m	12 m
35 kg	2:3	5	7 kg	14 kg	21 kg
100 m	7:3	10	10 m	70 m	30 m
80 km	3:5	8	10 km	30 km	50 km

4. Sachin \$40, Amir \$50
 5. 10 boxes of oats, 25 boxes of lucerne
 6. Ernie 6 cubic metres, Carla 9 cubic metres
 7. Sebastian 25 hectares, Christian 15 hectares
 8. Dry food 840 grams, fresh meat 560 grams
 9. Sharks 4000, Dolphins 6000
 10. 16 cups of plaster powder, 4 cups of water
 11. (a) 5 litres (b) 6 litres
 12. \$6000
 13. 1250 mm and 500 mm
 14. (a) 2:1 (b) 16 (c) 8
 15. (a) Increasing (b) 49

Exercise 9B

1. (a) 2:1 (b) 1:4 (c) 5:4 (d) 2:3 (e) 9:7 (f) 3:4
 2. (a) 4:1 (b) 1:6 (c) 3:2 (d) 7:9 (e) 2:7 (f) 9:7 (g) 2:3 (h) 4:3 (i) 8:9
 3. 17:5
 4. 9:2

Exercise 9C

1. (a) 2:3 (b) 15 kg (c) 20 kg of chicken meat, 30 kg of vegetables
 2. (a) 1:24 (b) 600 mL (c) 50 mL (d) 80 mL of oil, 1920 mL of petrol
 3. (a) 1:4 (b) 16 (c) 3 (d) 5 of cement, 20 of sand

4. (a) 1:20 (b) 1500 mL (c) 100 mL (d) 500 mL of fertilizer, 10 000 mL of water
 5. (a) 8:5 (b) 125 cm (c) 240 cm
 6. (a) 7:5 (b) 7000 roses, 5000 carnations (c) 9800 roses
 7. (a) 2:5 (b) 2250 mL (c) 2400 mL (d) 2600 mL of ethanol, 6500 mL of petrol

Exercise 9D

1. 1:2
 2. (a) $\frac{3}{4}$ (b) 3:1
 3. $\frac{1}{6}$
 4. (a) $\frac{2}{5}$ (b) 2:3
 5. (a) horse $\frac{1}{8}$, elephant $\frac{1}{6}$, pig $\frac{1}{3}$, sloth $\frac{5}{12}$, dog $\frac{11}{24}$, cat $\frac{1}{2}$, platypus $\frac{7}{12}$, tiger $\frac{2}{3}$, koala $\frac{3}{4}$, brown bat $\frac{5}{6}$
 (b) horse 1:7, elephant 1:5, pig 1:2, sloth 5:7, dog 11:13, cat 1:1, platypus 7:5, tiger 2:1, koala 3:1, brown bat 5:1
 (c)(i) 1:2 (ii) 1:6 (iii) 3:2

Exercise 9E

1. 80%
 2. 70%
 3. 1:9
 4. 3:17

Exercise 9F

1. (a) 10 % cement, 30% sand, 60 % stones
 2. (a) 3:2 (b) Felix \$3600, Trent \$2400
 3. 200m long, 100 m wide
 4. 300 mL
 5. (a) 30 mL (b) 10 mL
 6. 10, 15, 32, 44

Chapter Review

1. (a) \$32, \$16 (b) 9 kg, 63 kg (c) 40 m, 50 m (d) 100 L, 60 L
 2. Gary 36 kg, Melanie 54 kg
 3. (a) 2:3 (b) 4:1 (c) 8:3

4. (a) 5:3 (b) 9 kg (c) 100 kg of sunflower seeds, 60 kg of grain
5. (a) $\frac{3}{5}$ (b) 3:2
6. 3:1
7. 3:7
8. (a) 10% (b) 270 mL
9. 40 mL

Chapter 10

Exercise 10A

1. (a) 200 g (b) 1400 g (c) 2000 g
2. (a) \$15 (b) \$105
3. (a) \$4 (b) \$80
4. (a) $\frac{1}{2}$ minute (b) 20 minutes
5. 36 kg
6. \$90
7. (a) 20 litres (b) 800 km
8. (a) 24 minutes (b) 3000 litres
9. 70 minutes
10. (a) \$400 (b) \$1000 (c) \$4000
11. 400 hours
12. 15 years

Exercise 10B

1. **A (\$1.80)**, B (\$1.90), C (\$1.85)
2. A (90c), **B (82c)**, C (85c)
3. A (\$9.00), B (\$9.20), **C (\$8.90)**
4. A (\$1.35), B (\$1.40), **C (\$1.32)**
5. A (\$3.35), B (\$3.30), **C (\$3.28)**
6. A (\$2.24), B (\$2.25), **C (\$2.22)**

Exercise 10C

1. A (\$1.70), B (\$1.72), **C (\$1.68)**
2. A (\$2.30), B (\$2.25), **C (\$2.20)**
3. **A (15c)**, B (16c), C (17c)
4. A (33c), **B (32c)**, C (34c)
5. A (\$1.56), B (\$1.52), **C (\$1.42)**, D (\$1.48)
6. **A (\$1.10)**, B (\$1.15), C (\$1.12)

Exercise 10D

1. 5 m/sec
2. 20 km/h
3. 900 km/h
4. 14 km/h
5. (a) 18 days (b) 18 km/h (c) 90 000 m (d) 300 minutes (e) 300 m/min (f) 5 m/sec
6. Peregrine falcon (105 m/sec)
Black marlin (36 m/sec)
Cheetah (30 m/sec)
Ostrich (26 m/sec)
Southern giant dragonfly (24 m/sec)
Greyhound (21 m/sec)
Kangaroo (18 m/sec)
Horse (16 m/sec)

Exercise 10E

1. (a) 200 m (b) 480 m (c) 125 sec (2 minutes 5 seconds) (d) 500 sec (8 minutes 20 seconds)
2. (a) 75 km (b) 1800 km (c) 108 000 km
3. (a) 480 m (b) 7200 m (c) 25 sec (d) 500 sec (8 min 20 sec)
4. (a) 1000 m (1 km) (b) 6000 m (6 km) (c) 144 000 m (144 km) (d) 40 sec (e) 2500 sec (41 min 40 sec)
5. (a) 5 m/sec (b) 1000 sec (c) 16 min 40 sec (d) 36 000 m (36 km)
6. (a) 30 km/h (b) 900 km (c) 114 hours (d) 19 days
7. (a) 75 m/min (b) No (7 hours 32 minutes)
8. 0.6 sec
9. 500 sec (8 min and 20 seconds)

Chapter Review

1. (a) 5 minutes (b) 50 minutes
2. (a) \$2 (b) \$50
3. 125 minutes
4. **A (\$2.45)**, B (\$2.55), C (\$2.52)
5. A (\$1.46), B (\$1.42), **C (\$1.40)**
6. A (\$2.24), B (\$2.22), **C (\$2.21)**
7. (a) 6 km/h (b) 48 km (c) 50 days
8. (a) 40 km/h (b) 4 hours (c) 960 km

Chapter 11

Exercise 11A

- 2, 4, 6, 8, **10, 12, 14.**
 - 20, 17, 14, 11, **8, 5, 2.**
 - 1, 3, 9, 27, **81, 243, 729.**
 - 0, 4, 8, 12, **16, 20, 24.**
 - 100, 90, 80, 70, **60, 50, 40.**
 - 64, 32, 16, 8, **4, 2, 1.**
 - 3, 6, 12, 24, **48, 96, 192.**
 - 100 000, 10 000, 1000, **100, 10, 1.**
- 0.2, 2, 20, 200, **2000, 20 000, 200 000.**
 - 5, 10, 15, 20, **25, 30, 35.**
- +2 (b) -3 (c) $\times 3$ (d) +4 (e) -10 (f) $\div 2$ (g) $\times 2$ (h) $\div 10$ (i) $\times 10$ (j) +5
- 7, 10, 13, 16, 19.
 - 50, 45, 40, 35, 30.
 - 1, 10, 100, 1000, 10 000.
 - 10, 20, 30, 40, 50.
 - 2187, 729, 243, 81, 27.
- 28 (b) 15 (c) 10 000 000 (d) 80 (e) 1
- B - Multiplying the step number by 5.
C - Multiplying the step number by 6.
D - Multiplying the step number by 10.
E - Adding 2 to the step number.
 - B 40 matches
C 48 matches
D 80 matches
E 10 matches

Exercise 11B

- $m = 5 \times s$
C $m = 6 \times s$
D $m = 10 \times s$
E $m = s + 2$
- (i) 6 (ii) 12 (iii) 18 (iv) 24 (v) 36

2. (b)

s	m
1	6
2	12
3	18
4	24
5	30
6	36

(c) 120 (d)(i) 10 (ii) 12

(e) $m = 6 \times s$

3. (a)

s	m
1	8
2	9
3	10
4	11
5	12
6	13

(b) $m = s + 7$

4. (a)(i)

(b)(i)

s	m
1	4
2	8
3	12
4	16
5	20
6	24

s	m
1	14
2	13
3	12
4	11
5	10
6	9

(ii) $m = 4 \times s$ (ii) $m = 15 - s$

(c)(i)

(d)(i)

s	m
1	8
2	16
3	24
4	32
5	40
6	48

s	m
1	2
2	4
3	6
4	8
5	10
6	12

(ii) $m = 8 \times s$ (ii) $m = 2 \times s$

4. (e)(i)

s	m
1	4
2	8
3	12
4	16
5	20
6	24

(f)(i)

s	m
1	8
2	9
3	10
4	11
5	12
6	13

(ii) $m = 4 \times s$ (ii) $m = s + 7$ 5. (a) $m = 5 \times s$ (b) $B = A + 4$ (c) $q = p - 1$ (d) $t = 10 \times c$ (e) $y = x + 20$ (f) $T = F \div 2$ (g) $T = W - 6$ (h) $k = h \div 5$ (i) $\beta = \alpha + 8$

6. (a)

m	n
10	15
20	25
30	35
40	45
50	55
60	65

(b)

m	n
10	110
20	220
30	330
40	440
50	550
60	660

(c)

m	n
10	25
20	35
30	45
40	55
50	65
60	75

(d)

m	n
10	1
20	2
30	3
40	4
50	5
60	6

6. (e)

m	n
10	4
20	14
30	24
40	34
50	44
60	54

(f)

m	n
10	5
20	10
30	15
40	20
50	25
60	30

(g)

m	n
10	20
20	40
30	60
40	80
50	100
60	120

(h)

m	n
10	20
20	30
30	40
40	50
50	60
60	70

(i)

m	n
10	9
20	19
30	29
40	39
50	49
60	59

7. (a)(i) 12 (ii) 16 (iii) 28 (iv) 44

(b)(i) 1 (ii) 4 (iii) 20 (iv) 37

8. (a)

x	y
3	8
6	11
25	30
12	17
21	26
55	60

(b)

x	y
3	9
6	18
10	30
12	36
21	63
20	60

8. (c)

x	y
3	18
6	21
15	30
12	27
21	36
45	60

(e)

x	y
3	1
6	4
32	30
12	10
21	19
62	60

(g)

x	y
3	30
6	60
3	30
12	120
21	210
6	60

(i)

x	y
3	2
6	5
31	30
12	11
21	20
61	60

(d)

x	y
3	1
6	2
90	30
12	4
21	7
180	60

(f)

x	y
3	6
6	12
15	30
12	24
21	42
30	60

(h)

x	y
3	13
6	16
20	30
12	22
21	31
50	60

10. (a)(i) 16 (ii) 24 (iii) 45

(b) B (c)(i) 20 (ii) 32 (iii) 47 (d) C

11. (a)(i) 12 km (ii) 28 km (b) A

(c)(i) 4 (ii) 6 (d) D

Exercise 11C

1. (a)

i	1	4	9	15
r	7	10	15	21

(b)

i	2	3	9	11
r	10	15	45	55

(c)

i	8	13	19	26
r	1	6	12	19

(d)

i	3	9	21	36
r	1	3	7	12

2. (a)

i	2	6	10	16
r	5	9	13	19

(b)

i	3	4	7	10
r	6	8	14	20

(c)

i	8	11	15	19
r	2	5	9	13

(d)

i	8	20	32	44
r	2	5	8	11

9. (a)(i) 20 (ii) 50 (iii) 100 (iv) 200

(b) A (c)(i) 3 (ii) 7 (iii) 50 (d) D

3. (a)

<i>i</i>	3	5	9	13
<i>r</i>	7	9	13	17

(b)

<i>i</i>	2	4	6	8
<i>r</i>	6	12	18	24

(c)

<i>i</i>	8	11	15	22
<i>r</i>	3	6	10	17

(d)

<i>i</i>	6	8	14	22
<i>r</i>	3	4	7	11

(e)

<i>i</i>	8	13	17	26
<i>r</i>	0	5	9	18

4. (a)

<i>i</i>	1	3	7	9
<i>r</i>	5	9	17	21

(b)

<i>i</i>	3	5	6	10
<i>r</i>	8	16	20	36

(c)

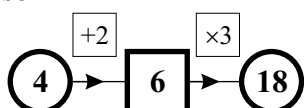
<i>i</i>	6	9	15	21
<i>r</i>	9	10	12	14

(d)

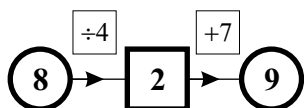
<i>i</i>	8	10	13	17
<i>r</i>	6	10	16	24

Exercise 11D

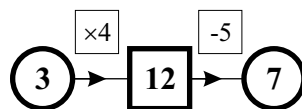
1. (a)



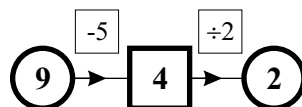
(b)



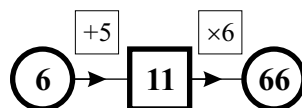
1. (c)



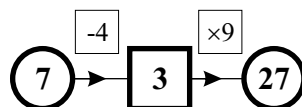
(d)



(e)



(f)



2. 20

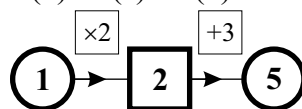
3. (a) $b = 4$ (b) $b = 5$ (c) $b = 7$ (d) $b = 15$

4.

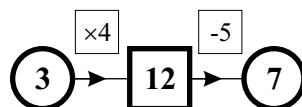
<i>a</i>	<i>b</i>
1	16
2	20
3	24
4	28

5. (a) C (b) A (c) D (d) B

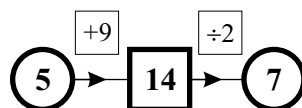
6. (a)



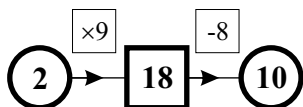
(b)



(c)

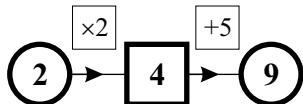


2. (d)

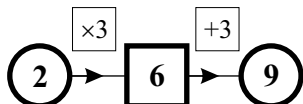
**Exercise 11E**

1. (a) $b = 3a + 4$ (b) $y = 6x - 2$
 (c) $W = 2T - 5$ (d) $F = 8G + 3$

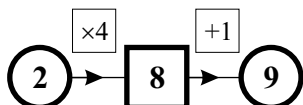
2. (a)



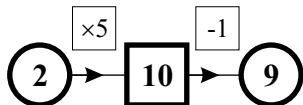
(b)



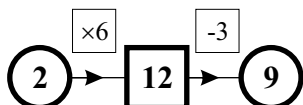
(c)



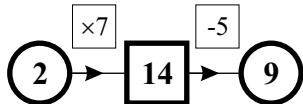
(d)



(e)

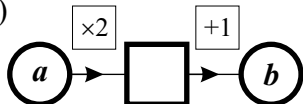


(f)

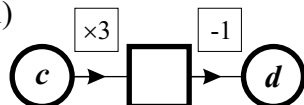


3. (a) $B = 2A + 5$ (b) $B = 3A + 3$
 (c) $B = 4A + 1$ (d) $B = 5A - 1$
 (e) $B = 6A - 3$ (f) $B = 7A - 5$

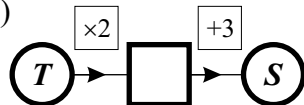
4. (a)(i)

(ii) $b = 2a + 1$

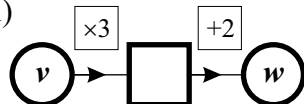
4. (b)(i)

(ii) $d = 3c - 1$

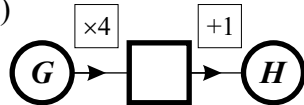
(c)(i)

(ii) $S = 2T + 3$

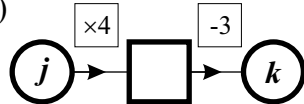
(d)(i)

(ii) $w = 3v + 2$

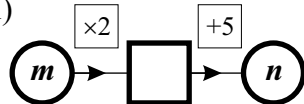
(e)(i)

(ii) $H = 4G + 1$

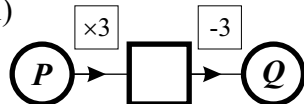
(f)(i)

(ii) $k = 4j - 3$

(g)(i)

(ii) $n = 2m + 5$

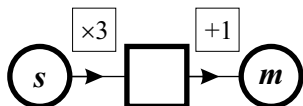
(h)(i)

(ii) $Q = 3P - 3$

5. (a)

s	m
1	4
2	7
3	10
4	13

(b)

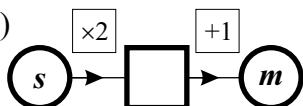
(c) $m = 3s + 1$

(d) (i) 19 (ii) 31

6. (a)(i)

s	m
1	3
2	5
3	7
4	9

(ii)

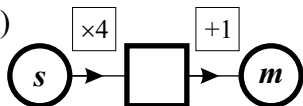
(iii) $m = 2s + 1$

(iv) 21

(b)(i)

s	m
1	5
2	9
3	13
4	17

(ii)

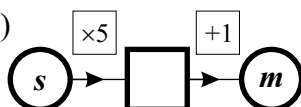
(iii) $m = 4s + 1$

(iv) 41

6. (c)(i)

s	m
1	6
2	11
3	16
4	21

(ii)

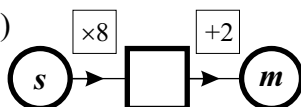
(iii) $m = 5s + 1$

(iv) 51

(d)(i)

s	m
1	10
2	18
3	26
4	34

(ii)

(iii) $m = 8s + 2$

(iv) 82

7. (a)

t	c
1	6
2	10
3	14
4	18

(b) $c = 4t + 2$

(c) 26

(d) the table

(e) There are two combinations that require 9 tables:

$$6 \times 1 + 1 \times 3$$

$$5 \times 1 + 2 \times 2$$

Number of Tables in each Group				
	1	2	3	4
Number of Groups	6		1	
		5		
	5	2		
	3		1	1
	2	2		1
	2	1	2	
	1	3	1	
			1	2

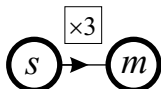
Chapter Review

1. (a) 1, 4, 7, 10, 13, 16, 19.
 (b) 10, 20, 40, 80, 160, 320, 640.

2. (a)

s	m
1	3
2	6
3	9
4	12
5	15
6	18

(b)

(c) $m = 3 \times s$

(d) 24

(e) 10

3. (a) $m = s + 4$ (b) $B = 5 \times A$ (c) $q = p - 10$

4. (a)(i) 10 (ii) 40 (iii) 70

(b)(i) 2 (ii) 5 (iii) 10

5.

A	B
1	7
3	9
7	13
9	15
19	25

6. (a)

p	o
1	3
2	6
3	9
4	12

(b) $o = 3 \times p$

7. (a)

i	1	5	7	11
r	3	15	21	33

(b)

i	2	6	11	17
r	6	10	15	21

(c)

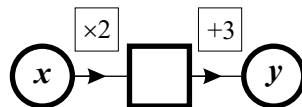
i	2	8	18	24
r	1	4	9	12

(d)

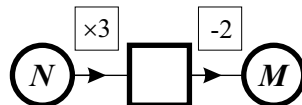
i	1	5	7	10	11	12
r	4	16	22	31	34	37

8. (a) A (b) B

9. (a)(i)

(ii) $y = 2x + 3$

(b)(i)

(ii) $M = 3N - 2$

Chapter 12

Exercise 12A

1. (a) $c = d + 8$ (b) $L = M - 3$ (c) $a = 7b$ (d) $f = \frac{g}{5}$ (e) $W = XY$ (f) $q = p + r$ (g) $z = \frac{p}{t}$ (h) $W = Imt$ (i) $G = \frac{H}{5}$ (j) $A = B + 5$ (k) $C = D - 6$ 2. (a) $T = B + G$ (b) $P = 4L$ (c) $L = \frac{P}{4}$ (d) $A = lw$ (e) $P = \frac{W}{t}$ (f) $S = \frac{d}{t}$ (g) $P = I - E$ (h) $C = \pi D$ (i) $p = mv$ (j) $I = PRt$ (k) $A = l^2$ (l) $V = lwh$ (m) $P = a + b + c$ (n) $E = mc^2$

Exercise 12B

1. (a) 14 (b) 11 (c) 36 (d) 6 (e) 3 (f) 8

(g) 144 (h) 27 (i) 38 (j) 9 (k) 39 (l) 52

2. (a) 31 (b) 14 (c) 27 (d) 14 (e) 16 (f) 12

3. (a) 10 (b) 28 (c) 4 (d) 11 (e) 15 (f) 9

(g) 16 (h) 22 (i) 4 (j) 9 (k) 35 (l) 22

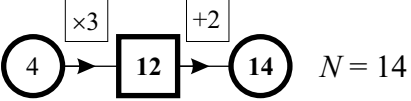
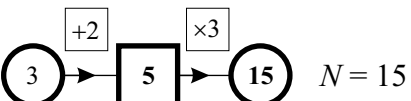
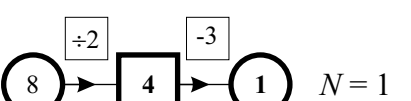
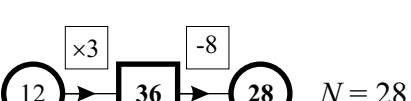
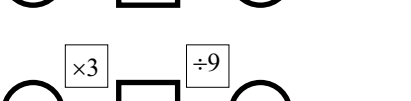
(m) 2 (n) 60

4. (a) $R = 14$ (b) $Q = 9$ (c) $n = 31$ (d) $X = 0$ (e) $D = 16$ (f) $j = 4$ (g) $V = 104$ (h) $H = 37$ (i) $N = 41$ (j) $y = 120$ (k) $Z = 42$ (l) $d = 60$

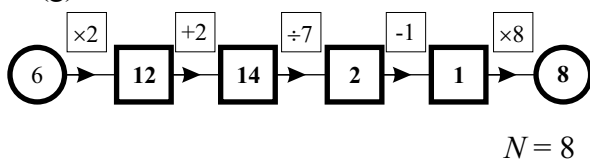
Exercise 12C

1. (a) I (b) C (c) I (d) C (e) C
(f) C (g) I (h) C (i) C
2. (a) 4 (b) 4 (c) 6 (d) 11 (e) 27
(f) 5
3. (a) $8 \times 2 = 16$ (b) $6 \times 10 = 60$
(c) $25 \div 5 = 5$
4. (a) 2 (b) 14 (c) 7 (d) 23 (e) 4
(f) 7 (g) 16 (h) 63 (i) 2 (j) 6
(k) 2 (l) 6 (m) 16 (n) 15 (o) 5
(p) 10 (q) 7 (r) 3
5. (a) 3 (b) 3 (c) 10 (d) 9 (e) 7
(f) 6 (g) 3 (h) 5 (i) 7 (j) 10
6. (a) 17 years old (b) 5 and 6
(c) 3 and 8 (d) 4×11 kg and 1×6 kg
(e) paint brush - \$3, pencil - \$2
(f) 4 left-handers

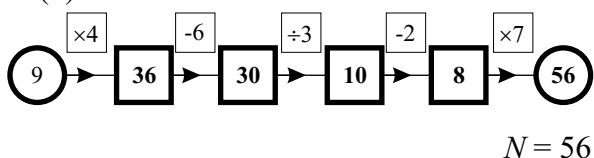
Exercise 12D

1. (a)  $N = 14$
- (b)  $N = 15$
- (c)  $N = 1$
- (d)  $N = 28$
- (e)  $N = 20$
- (f)  $N = 2$

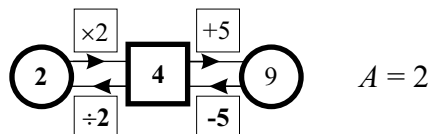
(g)



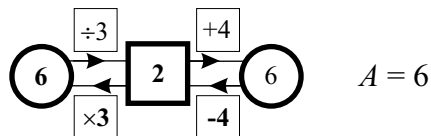
(h)



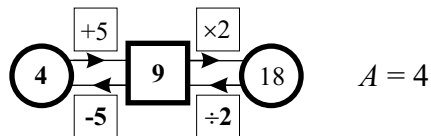
2. (a)



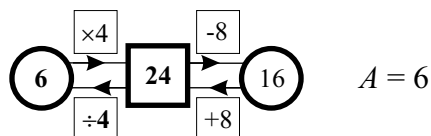
(b)



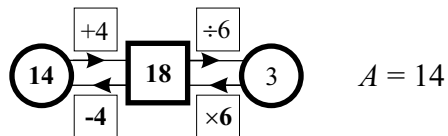
(c)



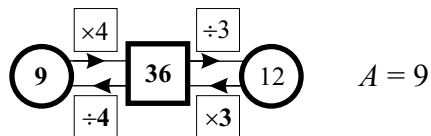
(d)



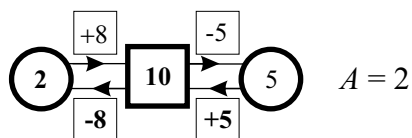
(e)



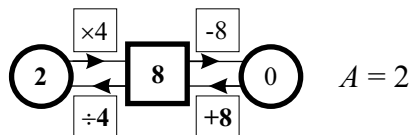
(f)



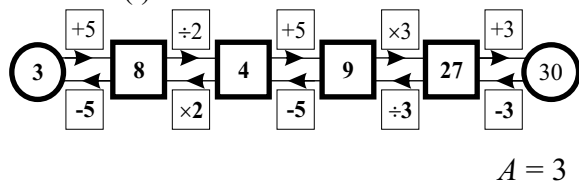
2. (g)



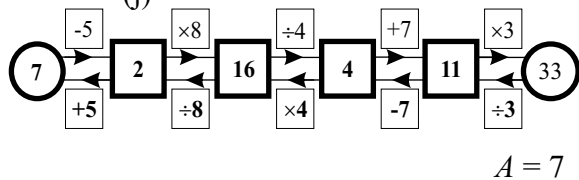
(h)



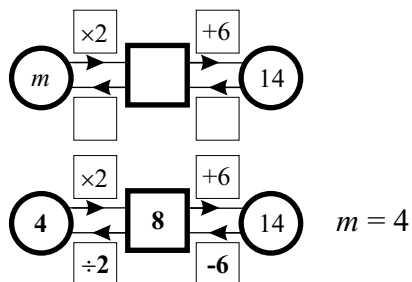
(i)



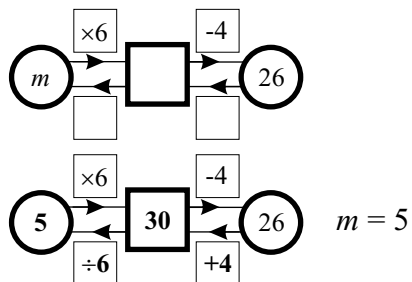
(j)



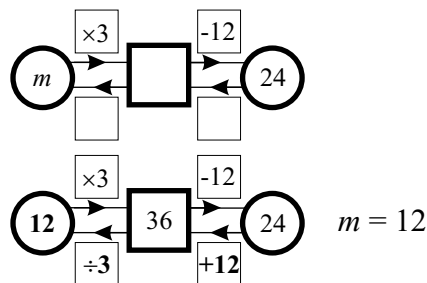
3. (a)



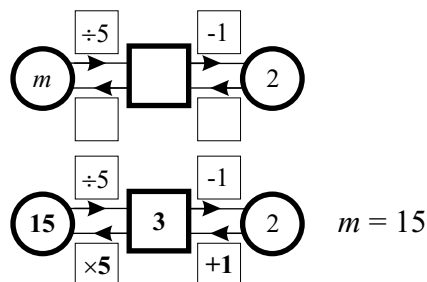
(b)



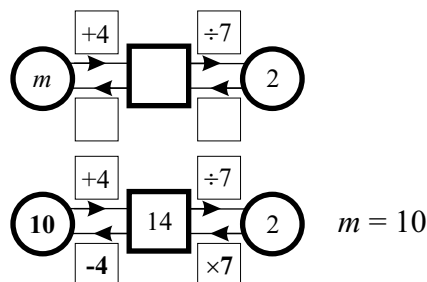
(c)



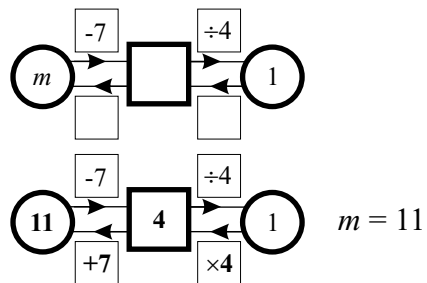
(d)



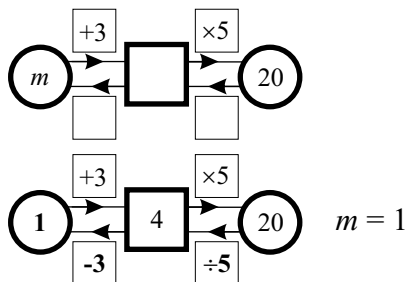
(e)



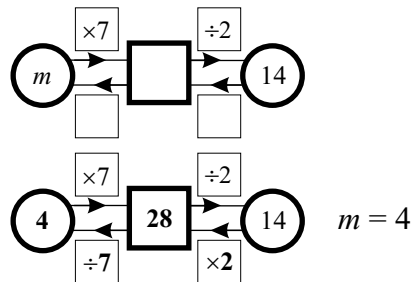
(f)



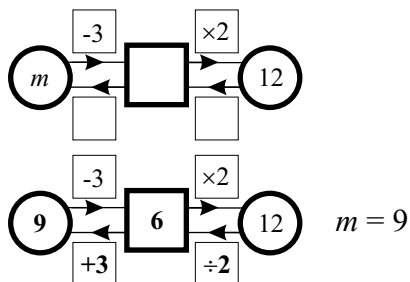
3. (g)



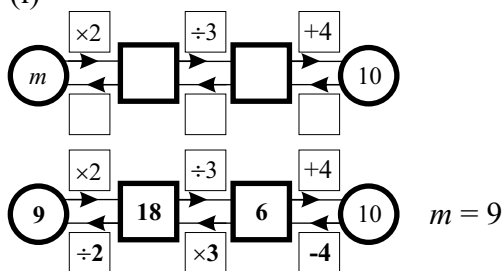
(k)



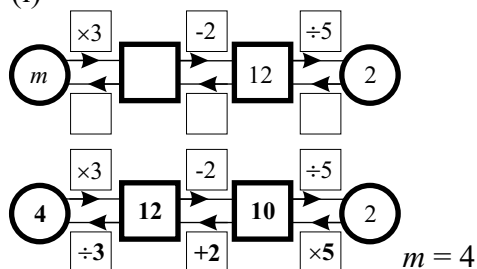
(h)



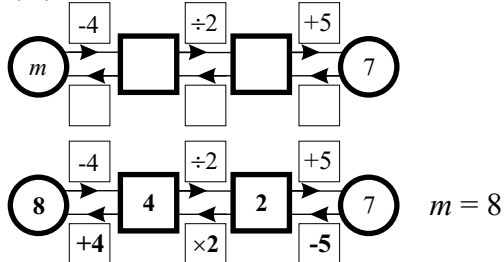
(l)



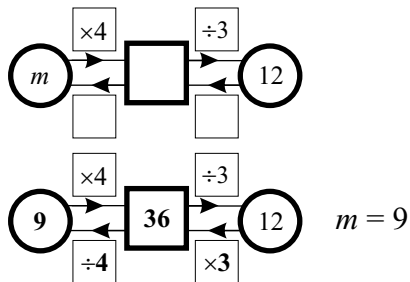
(i)



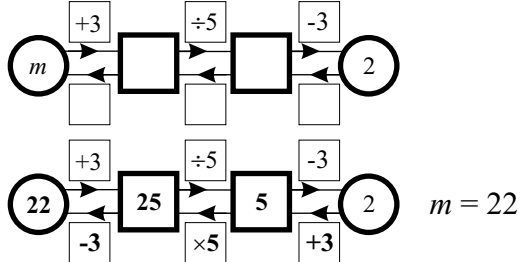
(m)



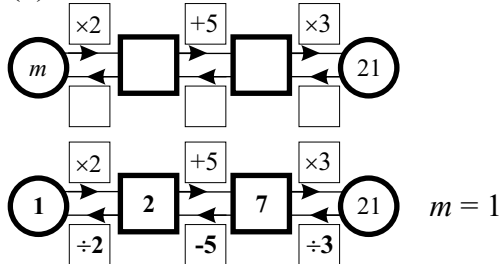
(j)



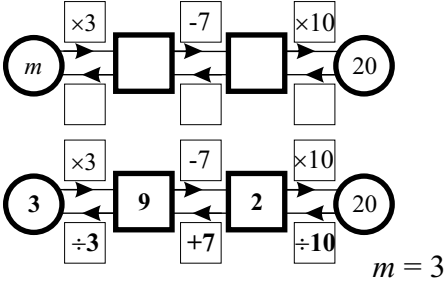
(n)



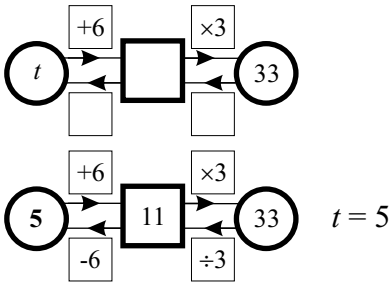
(o)



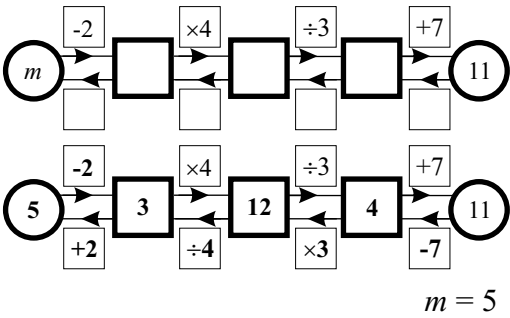
3. (p)



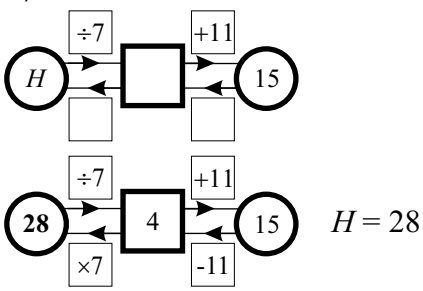
(b) $3(t + 6) = 33$



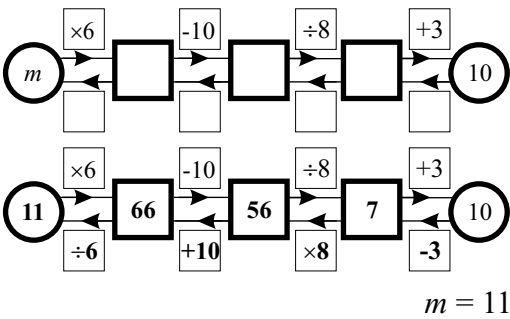
(q)



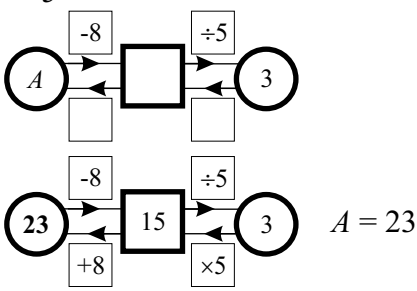
(c) $\frac{H}{7} + 11 = 15$



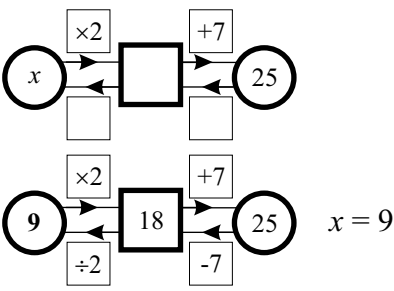
(r)



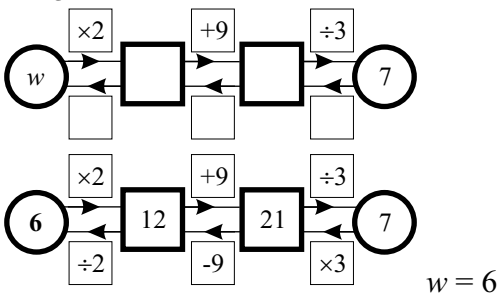
(d) $\frac{A - 8}{5} = 3$



4. (a) $2x + 7 = 25$



(e) $\frac{2w + 9}{3} = 7$

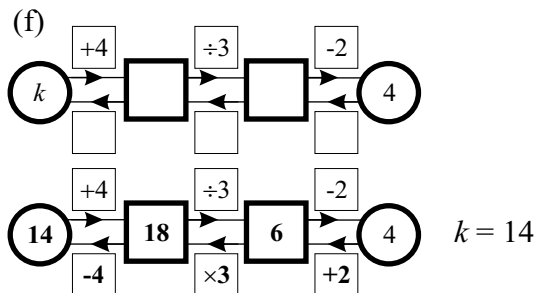
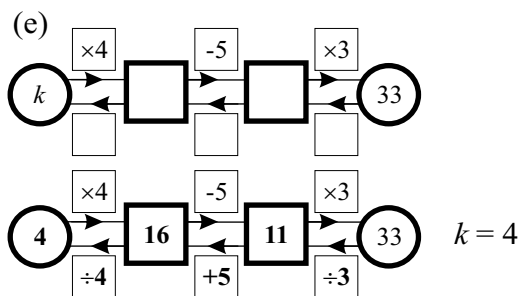
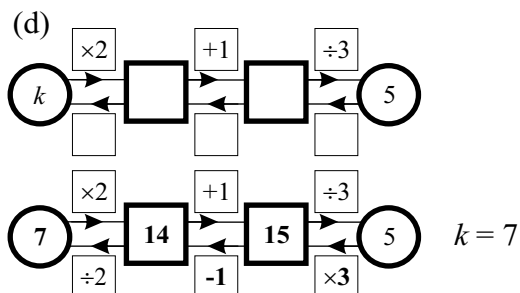
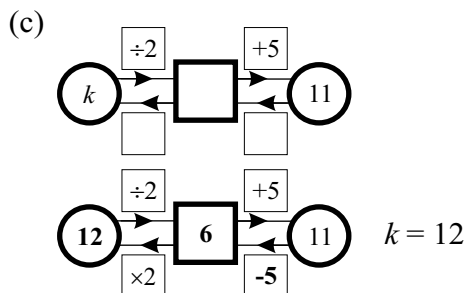
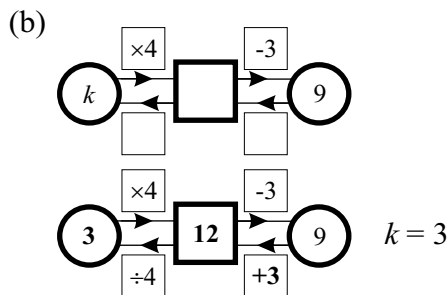
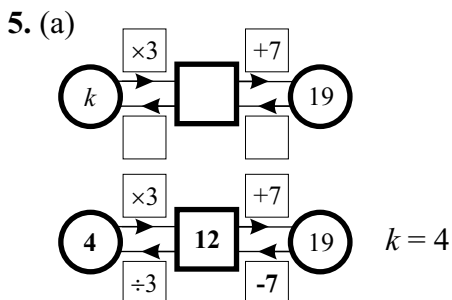
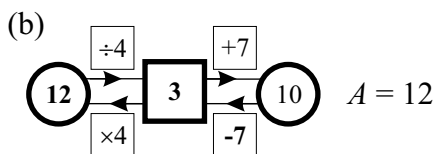
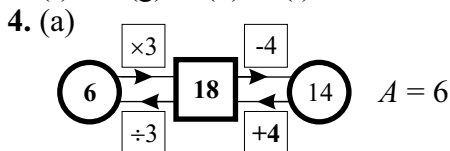


Exercise 12E

1. (a) 3 (b) 16 (c) 7 (d) 20 (e) 3
 (f) 2 (g) 5 (h) 3 (i) 7 (j) 8 (k) 25
 (l) 12 (m) 6 (n) 11 (o) 19 (p) 8
 (q) 10 (r) 6 (s) 4 (t) 6 (u) 4
2. (a)(i) $3m + 5 = 26$ (ii) $m = 7$
 (b)(i) $\frac{a}{4} + 7 = 12$ (ii) $a = 20$
 (c)(i) $\frac{x+6}{5} = 3$ (ii) $x = 9$
 (d)(i) $\frac{4n}{3} = 12$ (ii) $n = 9$
3. (a) $6b = 24$ (b) $b = 4$ kg
 4. (a) $3t = 24$ (b) $t = 8$ minutes
 5. (a) $6b + 2 = 50$ (b) $b = 8$ litres

Chapter Review

1. (a) $A = MQ$ (b) $B = T + W$
 (c) $C = MU + 12$ (d) $D = E + 5$
2. (a) 4 (b) 9 (c) 20 (d) 25 (e) 3
 (f) 4
3. (a) 3 (b) 5 (c) 12 (d) 5 (e) 7
 (f) 12 (g) 5 (h) 3 (i) 4



6. (a) $3k + 7 = 19$

$$3k + 7 - 7 = 19 - 7$$

$$3k = 12$$

$$\frac{3k}{3} = \frac{12}{3}$$

$$k = 4$$

(b) $4k - 3 = 9$

$$4k - 3 + 3 = 9 + 3$$

$$4k = 12$$

$$\frac{4k}{4} = \frac{12}{4}$$

$$k = 3$$

(c) $\frac{k}{2} + 5 = 11$

$$\frac{k}{2} + 5 - 5 = 11 - 5$$

$$\frac{k}{2} = 6$$

$$k = 12$$

(d) $\frac{2k+1}{3} = 5$

$$\frac{2k+1}{3} \times 3 = 5 \times 3$$

$$2k + 1 = 15$$

$$2k + 1 - 1 = 15 - 1$$

$$2k = 14$$

$$k = 7$$

(e) $3(4k - 5) = 33$

$$\frac{3(4k - 5)}{3} = \frac{33}{3}$$

$$4k - 5 = 11$$

$$4k - 5 + 5 = 11 + 5$$

$$4k = 16$$

$$\frac{4k}{4} = \frac{16}{4}$$

$$k = 4$$

(f) $\frac{k+4}{3} - 2 = 4$

$$\frac{k+4}{3} - 2 + 2 = 4 + 2$$

$$\frac{k+4}{3} = 6$$

$$\frac{k+4}{3} \times 3 = 6 \times 3$$

$$k + 4 = 18$$

$$k + 4 - 4 = 18 - 4$$

$$k = 14$$

7. (a) Boofer - 6 kg, Woofer - 4 kg

(b) Noah - 6, Lisa - 1

Chapter 13

Exercise 13A

1. $9b$

2. $15w$

3. $12c$

4. $25s$

5. $9n$

6. $16p$

7. (a) $12b$ (b) $7b$

8. (a) $15v$ (b) $4v$

9. $21c$

Exercise 13B

1. (a) $7n$ (b) $6y$ (c) $8t$ (d) $2k$

2. (a) $3c$ (b) $8u$ (c) $10g$ (d) $10e$ (e) $15h$
(f) $16d$ (g) $21u$ (h) $15s$ (i) $20A$

3. (a) $3y$ (b) $5m$ (c) v (d) 0

4. (a) $5n$ (b) $8p$ (c) $12y$ (d) $8a$

5. (a) $16m$ (b) 0 (c) $15K$ (d) $7P$ (e) $10q$
(f) d (g) $4x$ (h) $2y$

6. (a) $9a + 5b$ (b) $7x + 9y$ (c) $3p + 7q$
(d) $4m + 2n$ (e) $6k + 7h$ (f) $11T + 8U$

7. (a) $7a + 8b$ (b) $12g + 9h$ (c) $3m + 11n$
(d) $15x + 5y$ (e) $5N + 6M$ (f) $5A + 21B$
(g) $9e + 5f$ (h) $6c + 12d$ (i) $13p + q$
(j) $3w + 15v$

8. (a) $3a + 5b + 9c$ (b) $8x + 9y + 7z$
(c) $11A + 14B + 13C$ (d) $8m + 14n + 6p$
(e) $3f + 16g + 20h$

9. (a) $3x + 7y$ (b) $9m + 3n$
 (c) $10a + 5b$ (d) $5C + 12D$
 (e) $3p + 10q$ (f) $11M + 9N$
 (g) $8r + 9t$ (h) $12J + 14K$
10. (a) $2x + 3y$ (b) $5m + 5n$
 (c) $3p + 2q$ (d) $4g + 6h$
 (e) $y + 2z$ (f) $10M + N$ (g) A
 (h) 0
11. (a) $5a + 6b + 5c$ (b) $12m + 3n + 3q$
 (c) $9A + 12B + 8C$ (d) $5x + 6y + 6z$
 (e) $16g + 14h + 15k$
 (f) $31P + 19Q + 12R$
 (g) $7L + 6W + 15G$
12. (a) $4m + 2n + 2p$ (b) $3x + 4y + 4z$
 (c) $3c + 10d + 3e$ (d) $7a + b + 8c$
 (e) $3p + 15q$ (f) $11X + 3Y + 13Z$
 (g) $a + 2b + c$ (h) $m + n + 10p$
 (i) $x + y + z$ (j) A

Exercise 13C

1. (a) $15a$ (b) $14b$ (c) $42y$ (d) $24f$
 (e) $40m$ (f) $45g$ (g) $21n$ (h) $20c$
2. (a) $2xy$ (b) $7fg$ (c) $12kl$ (d) $16cd$
 (e) $30mn$ (f) $12ab$ (g) $28gh$
 (h) $27pq$ (i) $16jk$ (j) $28yz$
 (k) $24rt$ (l) $40MP$
3. (a) $3abc$ (b) $7ghi$ (c) $6pqr$
 (d) $16ABC$ (e) $24XYZ$ (f) $60stu$
 (g) $50hjk$ (h) $8def$ (i) $48NPQ$
 (j) $42def$ (k) $84uvw$ (l) $144HKM$

Exercise 13D

1. (a) a^5 (b) p^3 (c) m^6 (d) $4b^4$
 (e) $12w^2$ (f) $16n^4$ (g) $8v^2$ (h) $30t^2$
 (i) $6m^4$ (j) $60k^3$ (k) $48r^3$ (l) $32x^5$
2. (a) a^2b^5 (b) m^5n^3 (c) h^5g^3 (d) x^5y^5
 (e) $12c^3d^2$ (f) $20j^5k^3$ (g) $36v^3w^4$
 (h) $64p^4q^4$ (i) $48a^6b$ (j) $80x^8y$

Exercise 13E

1. (a) $2n$ (b) $4a$ (c) $8b$ (d) $3c$ (e) $4u$
 (f) $\frac{4h}{3}$ (g) $\frac{4x}{3}$ (h) $\frac{4}{5m}$ (i) $\frac{9v}{8}$

(j) $\frac{5a}{8}$ (k) $\frac{i}{3}$ (l) $\frac{3}{5a}$ (m) $\frac{1}{7n}$ (n) $\frac{c}{5}$

(o) $\frac{1}{2p}$

2. (a) 6 (b) 4 (c) 3 (d) $\frac{1}{2}$ (e) $\frac{1}{5}$ (f) $\frac{b}{6}$

(g) $\frac{1}{6y}$ (h) $\frac{2n}{3}$ (i) $\frac{5}{4w}$ (j) $\frac{1}{4l}$ (k) $7f$

(l) $\frac{1}{5b}$ (m) $\frac{m}{7}$ (n) $\frac{9}{20d}$ (o) $\frac{5u}{8}$

3. (a) $\frac{1}{3z}$ (b) $2b$ (c) $\frac{8}{9q}$ (d) $3c$ (e) $\frac{1}{4v}$

(f) $\frac{b}{7c}$ (g) $\frac{2u}{3y}$ (h) $\frac{7r}{9x}$ (i) $\frac{1}{3}$ (j) $\frac{8k}{9w}$

(k) $\frac{8c}{9g}$ (l) $\frac{7u}{6y}$ (m) $\frac{rt}{4qv}$ (n) $\frac{2ae}{df}$

(o) $\frac{4aht}{bgs}$

4. (a) $2x^2$ (b) $\frac{1}{5a^4}$ (c) $\frac{5n^3}{3}$ (d) $\frac{3}{4c^4}$

(e) $2v^2$ (f) $\frac{1}{4m^4}$ (g) $\frac{3}{4x}$ (h) $\frac{3}{4v^2}$

(i) $\frac{4r^7}{3}$ (j) $\frac{5}{6s^4}$

5. (a) $\frac{5x^2}{6z}$ (b) $\frac{3b}{2a^5}$ (c) $\frac{11r}{9q}$ (d) $\frac{7c}{9}$

(e) $\frac{15w^6}{16v}$ (f) $\frac{a^3b}{2c}$ (g) $\frac{4uz^5}{y}$ (h) $\frac{m^4r}{4x}$

(i) $\frac{10m^3}{3}$ (j) $\frac{4k}{3v^3w}$

6. (a) $\frac{2a}{3}$ (b) $\frac{4}{m}$ (c) $\frac{x}{3}$ (d) $\frac{1}{2}$

(e) $\frac{3}{2}$ (f) $\frac{1}{6}$ (g) $\frac{2b}{3}$ (h) x

(i) $\frac{1}{4n}$ (j) $\frac{5b}{6}$ (k) $\frac{9mn}{14}$ (l) $\frac{1}{4y}$

(m) $\frac{2x}{5}$ (n) $\frac{7m^2}{6}$ (o) $\frac{7a^2b}{4}$

Chapter Review

- $6p$
- (a) $6a$ (b) $8d$ (c) $5u$ (d) $8x$
- (a) $7n + 5p$ (b) $7a + 7b$
(c) $6m + 4n$ (d) $y + 4z$
- (a) $6a + 8b + 3c$ (b) $5x + 4y + 3z$
- (a) $20a$ (b) $14n$ (c) $3tu$ (d) $15xy$
- (a) $24abc$ (b) $70fgh$
- (a) c^5 (b) $12m^3n^4$
- (a) $3n$ (b) $\frac{5}{3a}$ (c) $\frac{7}{3}$ (d) $\frac{4d}{9}$
(e) $\frac{2}{3v}$ (f) $\frac{v}{4}$ (g) $\frac{5d}{4b}$ (h) $\frac{16a^4}{9}$
(i) $\frac{1}{a^3}$ (j) $\frac{4n^2}{9m^4}$
- (a) $\frac{3}{5n}$ (b) $\frac{3b}{2}$ (c) $\frac{7m}{5}$ (d) $\frac{3h^3}{10g^3}$

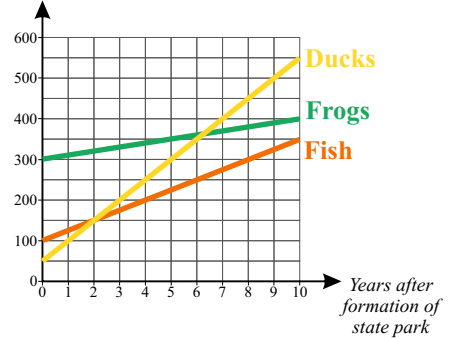
Chapter 14

Exercise 14A

- (a) 6°C (b) 30°C (c) 18°C (d) 24°C
- (a)(i) 12°C (ii) 17°C (iii) 24°C
(iv) 14°C
(b) 4 am, 8 am, midnight
(c) 25°C (d) 8°C (e) 4 pm
(f) 6 am (g) 17°C (h) 2°C
- (a) 8 am, 8 pm (b) 2 pm, 2 am
(c) 12 hours (d) 2.6 metres
(e) 1.4 metres (f) 1.2 metres
(g) 4 pm
- (a) Blue gum - 0.5 metres
Apple tree - 2.0 metres
(b) Blue gum - 5.5 metres
Apple tree - 4.5 metres
(c) 6 years (d) 3.5 metres
(e) Blue gum - 2.5 metres
Apple tree - 3.0 metres
(f) 7 years (g) 8 years (h) 2 years
(i) 10 years
(j) Blue gum - 0.5 metres
Apple tree - 0.25 metres
(k) The blue gum

- (i) Blue gum - 6.5 metres
Apple tree - 5.0 metres
(ii) Blue gum - 10.5 metres
Apple tree - 7.0 metres

5. (a) *Number of animals*

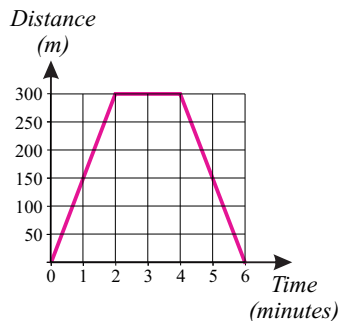


- (d) Ducks (e) Frogs
(f) Frogs - 450
Fish - 475
Ducks - 800

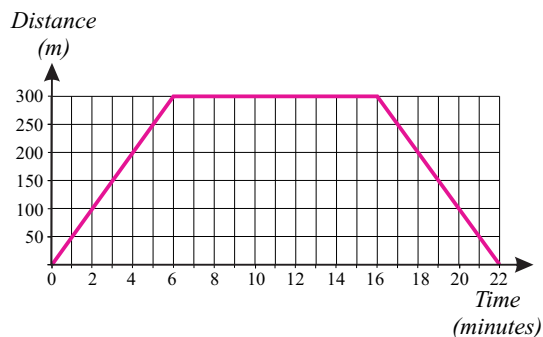
- (a) decreasing (b) decreasing (c) dam 1
(d) dam 2 (e) dam 1
- (a)(i) Alton - 250, Granton - 500,
Baybury - 200, Kleetor - 550
(ii) Alton - 350, Granton - 450,
Baybury - 250, Kleetor - 400
(iii) Alton - 450, Granton - 400,
Baybury - 300, Kleetor - 250
(b) Alton, Baybury (c) Granton, Kleetor
(d) Alton (e) Kleetor

Exercise 14B

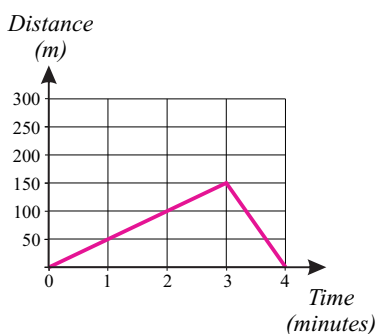
- (a) 300 m (b) 6 minutes (c) 50 m/min
- (a) 2 minutes (b) 150 m/min
- 8 minutes
- 7 minutes
- (a)



5. (b)

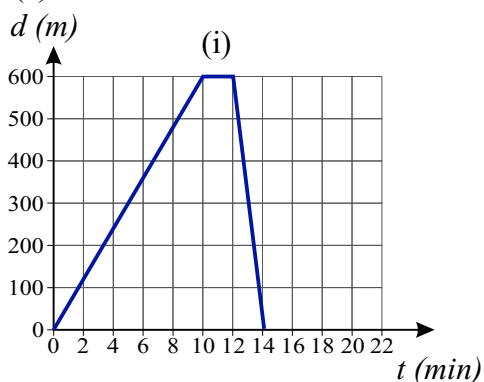


(c)

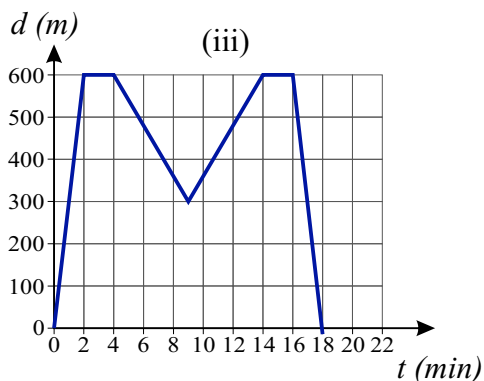
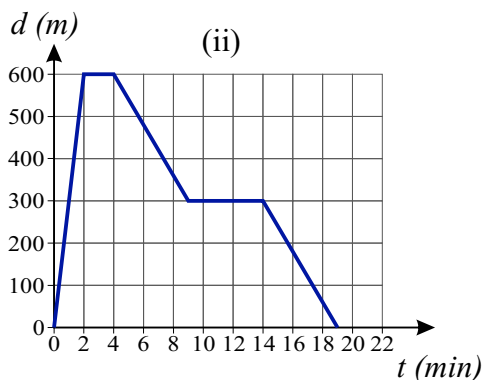


6. (a)(i) 600 m (ii) 10 minutes
 (iii) 2 minutes (iv) 50 m
 (b)(i) 2 minutes (ii) 300 m
 (c) Alanna **ran** to the shop, bought some groceries and **walked** home.
 (d)(i) C (ii) F (iii) B (iv) E (v) A
 (vi) D

(e)



(e)



7. (a) The person moves in one direction 20 metres in 15 seconds then moves back again. They then stop for 30 seconds and repeat the movement four times.
 An example of this motion could be a model displaying clothes on a catwalk.
 (b) The person moves in one direction 5 metres in 1 second then back again. They then move 10 metres in 2 seconds then back again. They then move 15 metres in 3 seconds then back again. They then move 20 metres in 4 seconds then back again.
 An example of this motion could be a training drill for sport where the athlete is running forwards then back varying amounts.

8. (a) 9 am (b) 11 am
 (c) 11 am, 2 pm (d) half an hour
 (e) 2 pm (f) 360 km (g) 120 km
 (h) Ruth - 4 pm, Fagan - 3 pm
 (i) Ruth - 7 hours, Fagan - 4 hours
 (j)(i) 120 km/hr (ii) 80 km/hr
9. (a)(i) A, D, H (ii) C, F (iii) B, E, I
 (b) G (c) 10 seconds
 (d) 200 km/hr (e) 5 seconds
 (f) ACCELERATING
 (g) DECELERATING

Chapter Review

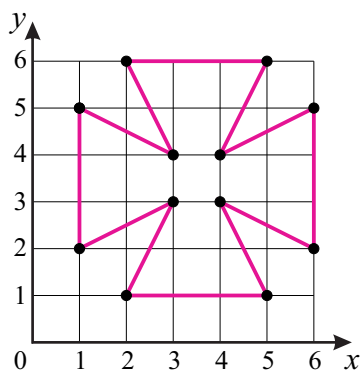
1. (a) 4 m (b) 1 m (c) April, October
 (d) January, February, March, November, December
 (e) June (f) Increase of 1 metre
2. (a) Ken - 24 minutes
 Ben - 8 minutes
 (b) Ben - 4 minutes before Ken
 (c) 6000 metres
 (d) Ken - 250 metres/minute
 Ben - 750 metres/minute
 (e) 4500 metres
 (f) 16 minutes

Chapter 15

Exercise 15A

1. $A = (0,7)$ $B = (2,6)$ $C = (5,8)$
 $D = (7,7)$ $E = (3,4)$ $F = (6,5)$
 $G = (0,3)$ $H = (1,3)$ $I = (8,3)$
 $J = (0,0)$ $K = (2,0)$ $L = (5,0)$
 $M = (7,1)$

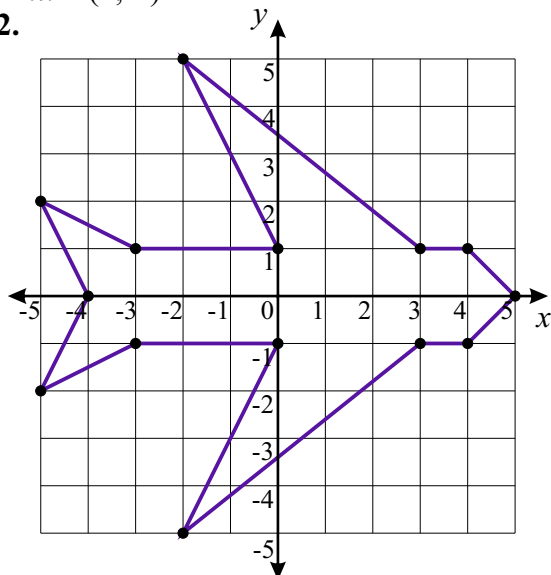
2.



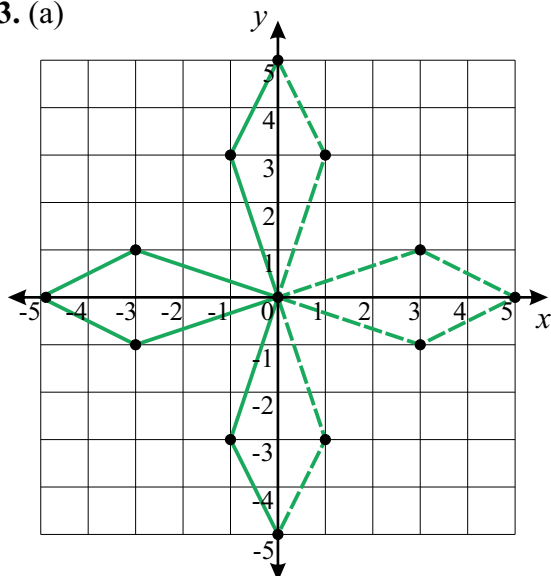
Exercise 15B

1. $A = (-3,4)$ $B = (-2,2)$ $C = (0,3)$
 $D = (3,2)$ $E = (4,3)$ $F = (-3,0)$
 $G = (1,0)$ $H = (0,-1)$ $I = (3,-1)$
 $J = (-4,-2)$ $K = (-3,-3)$ $L = (1,-3)$
 $M = (4,-4)$

2.



3. (a)



- (d) $(0,-5)$ $(1,-3)$ $(0,0)$ $(3,-1)$ $(5,0)$ $(3,1)$
 $(0,0)$ $(1,3)$ $(0,5)$

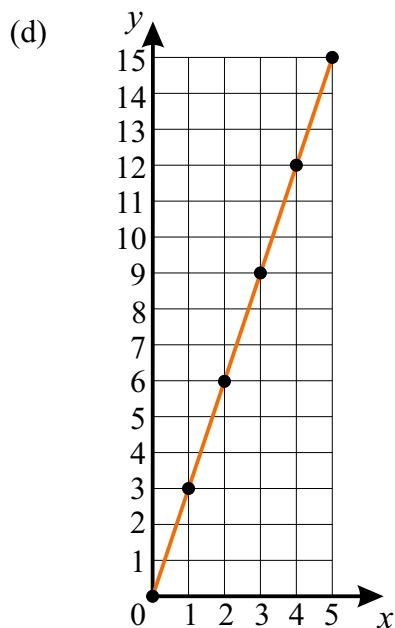
Exercise 15C

1. (a) (0,0) (1,3) (2,6) (3,9) (4,12) (5,15)

(b) $y = 3x$

(c)

x	0	1	2	3	4	5
y	0	3	6	9	12	15

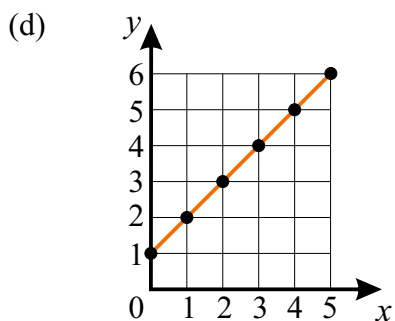


2. (a) (0,1) (1,2) (2,3) (3,4) (4,5) (5,6)

(b) $y = x + 1$

(c)

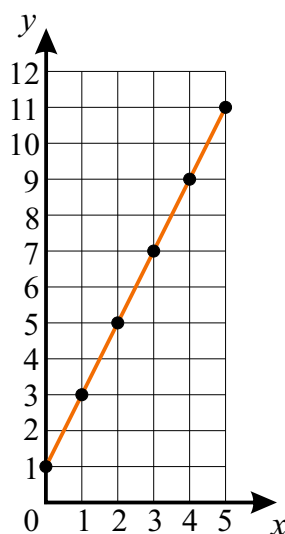
x	0	1	2	3	4	5
y	1	2	3	4	5	6



3. (a)

x	0	1	2	3	4	5
y	1	3	5	7	9	11

(b)



4. (a)

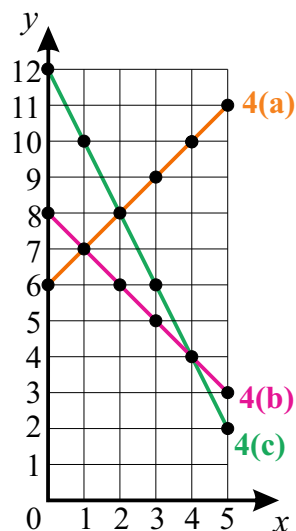
x	0	1	2	3	4	5
y	6	7	8	9	10	11

(b)

x	0	1	2	3	4	5
y	8	7	6	5	4	3

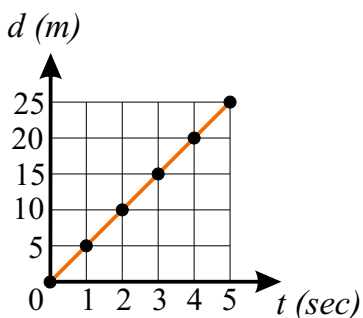
(c)

x	0	1	2	3	4	5
y	12	10	8	6	4	2



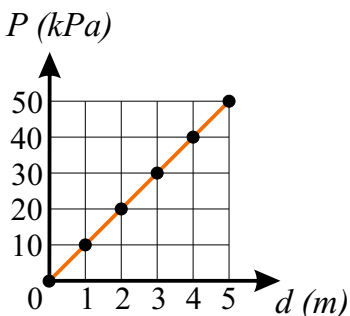
Exercise 15D

1. (a)

(b) $d = 5t$

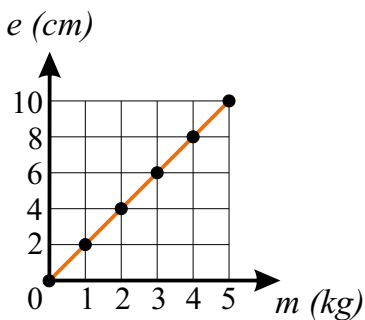
(c) (i) 100 m (ii) 600 m

2. (a)

(b) $P = 10d$

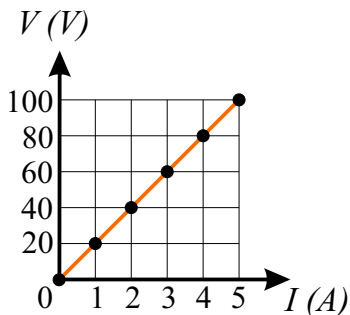
(c) 80 kPa

3. (a)

(b) $e = 2m$

(c) 18 cm

4. (a)

(b) $V = 20I$

(c) (i) 120 V (ii) 300 V (iii) 500 V

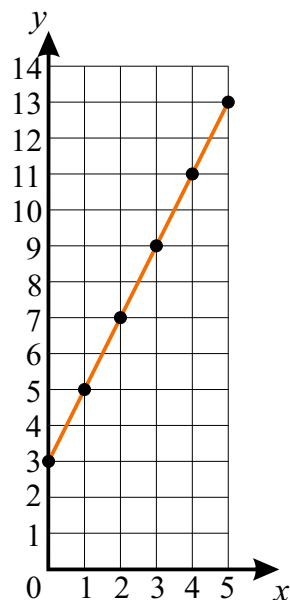
Chapter Review

1. **A** = (-2,3) **B** = (-2,2) **C** = (1,2)**D** = (0,1) **E** = (3,1) **F** = (-1,0)**G** = (0,0) **H** = (2,0) **I** = (-2,-1)**J** = (1,-1) **K** = (-1,-3) **L** = (0,-3)**M** = (2,-3)

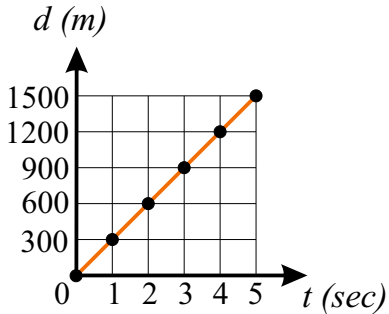
2. (a)

<i>x</i>	0	1	2	3	4	5
<i>y</i>	3	5	7	9	11	13

(b)



3. (a)



(b) $d = 300t$

(c)(i) 3000 m (ii) 18 000 m

Chapter 16

Exercise 16A

1. C

2. (a) B (b) A (c) B (d) A (e) C (f) C
(g) C (h) D (i) A (j) B (k) A (l) D3. (a) C (b) B (c) D (d) A (e) C (f) A
(g) D (h) B (i) B (j) D

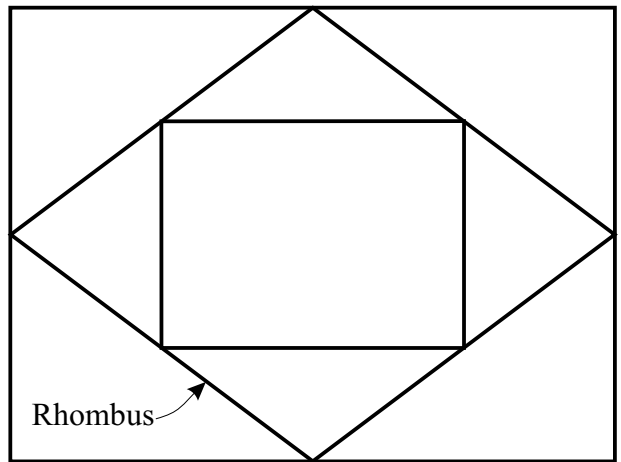
4. These are approximate:

(a) 22 cm (b) 12 cm (c) 1500 km
(d) 15 mm (e) 28 mm (f) 4 m5. 10 km, 20 km/hr, 60 mm, 100 m,
half a mm, 17 m, 18 m, 175 cm, 1 m,
20 cm, half a kg.

Exercise 16B

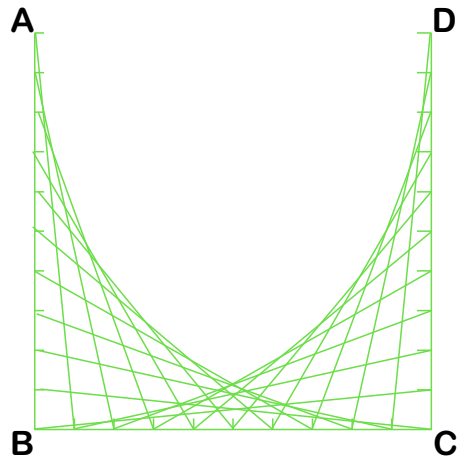
1. (a) 6 (b) 14 (c) 28 (d) 15 (e) 45
(f) 15 (g) 45 (h) 90 (i) 60 (j) 220
(k) 380 (l) 800 (m) 2300 (n) 3600
(o) 300 (p) 1200 (q) 1650 (r) 75
(s) 150 (t) 3252. (a) 1 cm (b) 4 cm (c) 2.5 cm
(d) 1.5 cm (e) 3 cm (f) 4.5 cm
(g) 0.5 cm (h) 10 cm (i) 8 cm
(j) 2 cm (k) 5 cm (l) 9 cm
(m) 11 cm (n) 9.5 cm (o) 6 cm3. (a) 15 mm (b) 42 mm (c) 46 mm
(d) 49 mm (e) 52 mm (f) 61 mm
(g) 18 mm (h) 75 mm (i) 32 mm
(j) 88 mm (k) 108 mm (l) 117 mm
(m) 24 mm (n) 8 mm (o) 15 mm
(p) 97 mm4. A 8 cm, B 5 cm, C 10 cm, D 6 cm,
E 3 cm, F 2 cm, G 4 cm, H 12 cm
I 1 cm, J 3 cm5. Bedroom 1 - 4 m $\times$ 3 m
Dining Room - 4 m $\times$ 3 m
Kitchen - 5 m $\times$ 3 m
Bedroom 2 - 3 m $\times$ 3 m
Laundry - 3 m $\times$ 2 m
Lounge Room - 5 m $\times$ 3 m
House - 14 m $\times$ 6 m

6.



The quadrilateral in the middle is a rectangle with side lengths of 4 cm and 3 cm - half the size of the side lengths of the original rectangle.

7.



Exercise 16C

- (a) 60 mm (b) 800 cm (c) 20 000 m
(d) 23 000 mm (e) 400 mm
(f) 59 000 m (g) 89 mm
(h) 375 cm (i) 2300 m (j) 100 mm
(k) 70 m (l) 267 mm (m) 0.3 cm
(n) 23.57 mm (o) 578 m
(p) 30 000 cm (q) 430 000 mm
(r) 1.7 m
- (a) 5 cm (b) 2 m (c) 7 km
(d) 9 m (e) 80 cm (f) 930 km
(g) 0.6 cm (h) 0.33 m
(i) 0.058 km (j) 0.003 m
(k) 5.8 cm (l) 3.8 m
(m) 0.3502 m (n) 7.851 m
(o) 0.0037 m (p) 0.005 km
(q) 0.03 km (r) 872.1 m
- (a) 1 cm (b) 320 cm (c) 820 mm
(d) 0.35 m (e) 90 mm
(f) 47.5 m (g) 0.04 cm
(h) 0.0235 m (i) 0.123456 km
(j) 22 mm (k) 1.25 cm (l) 0.8 cm
(m) 0.00003 m (n) 0.0005 km
(o) 0.00235 m
- (a) 2.11 m (b) 3.2 cm (c) 6.06 m
(d) 20.2 m (e) 5.02 km (f) 18.05 m
(g) 5.08 m (h) 6.238 m (i) 4.007 m
(j) 20.4 cm
- (a) 0.063 m, 7.5 cm, 83 mm
(b) 620 mm, 74 cm, 0.812 m
(c) 953 mm, 1.32 m, 178 cm
(d) 2.8 m, 345 cm, 3921 mm
(e) 0.0076 m, 8 mm, 0.82 cm
(f) 796 mm, 4.77 m, 598 cm
- Keiran
- (a) 62 cm (b) 359 cm
- (a) 2.5 km (b) 15 minutes
- (a) 7 (b) 20.3 m
- 400
- (a) 60 metres (b) 4 metres

Exercise 16D

- (a) 3 m (b) 66 m (c) 14 m (d) 30 m
(e) 16 m (f) 20 m
- (a) 3 m (3.15 m) (b) 3 m (2.8 m)
(c) 16 m (15.56 m) (d) 4 m (4.04 m)
(e) 8 m (8.09 m) (f) 10 m (10.089 m)
(g) 11 m (11.256 m) (h) 6 m (5.78 m)
(i) 48 m (48.458 m)
- (a) 3 cm (b) 3 cm (c) 16 cm
(d) 1 cm (e) 14 cm (f) 26 cm
(g) 1 cm (h) 7 cm (i) 20 cm

Exercise 16E

- (a) 61 cm (b) 41 km (c) 122 m
(d) 54.3 m
- (a) 104 mm (b) 142 cm (c) 96 m
(d) 130 cm (e) 124 m (f) 336 m
- (a) 414 mm (b) 5674 m
- 101.4 cm
- (a) 16 cm (b) 14 cm, 7 cm
- 108 m
- 190 cm

Chapter Review

- (a) B (b) B (c) D
- (a) 8 (b) 13 (c) 29 (d) 20 (e) 160
(f) 440 (g) 50 (h) 175 (i) 325
- (a) 70 mm (b) 1000 m (c) 2000 cm
(d) 60 m (e) 85 cm (f) 0.6 cm (g) 9.7 m
(h) 3 cm (i) 0.05 cm (j) 0.04 m
(k) 710.5 m (l) 0.21 m
- (a) 3.67 m (b) 8.9 cm (c) 10.07 m
(d) 5.3 km (e) 1.05 km (f) 7.06 m
(g) 1.1 cm (h) 8.342 m
- (a) 6 m (b) 38 m (c) 100 m (d) 1 m
(e) 15 m (f) 5 m
- (a) 6 cm (b) 11 cm (c) 9 cm (d) 2 cm
(e) 3 cm (f) 36 cm
- 300 cm
- 200
- (a) 44 cm (b) 60 cm (c) 36 m
(d) 61.5 cm
- 10 cm
- 300 metres

Chapter 17

Exercise 17A

- (a) 25 (b) 20 (c) 24 (d) 15
(e) 24 (f) 28
- (a) 9 (b) 10 (c) 18
- 100
- (a) C (b) C (c) B (d) C (e) D
(f) A
- (a) B (b) F (c) D (d) A (e) C
(f) E
- C, F, D, B, A, E
- (a) A square of side length 10 cm
(b) Circle (c) Square **A** (d) **D**

Exercise 17B

- 6 cm^2
- 12 cm^2
- 8 cm^2
- 11 cm^2
- 9 cm^2
- 16 cm^2
- 17 cm^2
- 17 cm^2
- 26 cm^2

Exercise 17C

- $12\text{--}13 \text{ cm}^2$
- $24\text{--}26 \text{ cm}^2$

Exercise 17D

- (a) 16 cm^2 (b) 10 cm^2 (c) 21 cm^2
(d) 20 cm^2 (e) 5 cm^2
- (i) (a) length = width = 4 cm
(b) length = 5 cm, width = 2 cm
(c) length = 7 cm, width = 3 cm
(d) length = 5 cm, width = 4 cm
(e) length = 5 cm, width = 1 cm
(ii)(iii) Area = length $\times$ width
- 27 cm^2
- (a) 36 cm^2 (b) 40 m^2 (c) 27 mm^2
(d) 80 m^2 (e) 180 mm^2 (f) 1008 cm^2
(g) 106.25 mm^2 (h) $70\,000 \text{ m}^2$
- (a) 64 m^2 (b) 99 cm^2

- 27 m^2
- 5400 m^2
- (a) $7 \text{ m} \times 1 \text{ m}$, $6 \text{ m} \times 2 \text{ m}$, $5 \text{ m} \times 3 \text{ m}$,
 $4 \text{ m} \times 4 \text{ m}$
(b) 7 m^2 , 12 m^2 , 15 m^2 , 16 m^2
(c) $4 \text{ m} \times 4 \text{ m}$ (d) Square (e) 100 m^2
- (a) 40 m^2 (b) 176 m^2

Exercise 17E

- 12 m^2
- 72 m^2
- 189 m^2
- 139 m^2
- 16 m^2
- 52 m^2
- 48 m^2
- 490 m^2

Exercise 17F

- (a)(i) 16 cm^2 (ii) 8 cm^2
(b)(i) 12 cm^2 (ii) 6 cm^2
(c)(i) 18 cm^2 (ii) 9 cm^2
(d)(i) 24 cm^2 (ii) 12 cm^2
(e)(i) 16 cm^2 (ii) 8 cm^2
(f)(i) 24 cm^2 (ii) 12 cm^2
- Yes. The area of the triangle is half the area of the rectangle.
- (a) 24 cm^2 (b) 9 cm^2 (c) 100 mm^2
(d) 17.5 mm^2 (e) 112 m^2
(f) 88 cm^2 (g) 9000 m^2 (h) 7.5 m^2

Exercise 17G

- 33 m^2
- 35 m^2
- 16 m^2
- 100 cm^2
- 260 m^2
- 164 cm^2
- 1400 cm^2

Exercise 17H

- (a) \$30 (b) \$24 (c) \$28
- (a)(i) 4 ha (ii) 8 ha
(iii) 30 ha (iv) 250 ha
(b)(i) 2 ha (ii) 5 ha
(iii) 12 ha (iv) 100 ha
(c) 500 m (d) 400 m
- (a) 3 litres (b) 2 litres
- Germany 356 733 km²
India 3 165 596 km²
Australia 7 682 292 km²
Brazil 8 511 965 km²
China 9 571 300 km²
Canada 9 970 610 km²

Chapter Review

- D
- 28-29 cm²
- (a) 64 m² (b) 45 cm²
(c) 1500 mm² (d) 240 mm²
(e) 160 cm² (f) 111 m²
- 63 m²
- 30 hectares

Chapter 18**Exercise 18A**

- (a) 19 (b) 17 (c) 18 (d) 14 (e) 19
(f) 12 (g) 11 (h) 16 (i) 25
- (a) 19 cm³ (b) 17 cm³ (c) 18 cm³
(d) 14 cm³ (e) 19 cm³ (f) 12 cm³
(g) 11 cm³ (h) 16 cm³ (i) 25 cm³
-

Object	Length	Width	Height	Volume
A	3	3	2	18
B	3	2	2	12
C	3	3	3	27
D	4	3	2	24
E	4	1	2	8
F	4	2	2	16
G	3	3	4	36
H	5	2	2	20
I	5	1	3	15

- (a) The volume of a rectangular prism can be found by multiplying the length, width and height.
(b) Volume = length $\times$ width $\times$ height
- (a) 40 cm³ (b) 300 cm³ (c) 64 cm³
(d) 60 cm³ (e) 480 cm³
- (a) 30 cm³ (b) 400 cm³ (c) 120 cm³
(d) 6000 cm³ (e) 432 cm³
- 1000 cm³
- (a) 24 cm $\times$ 1 cm $\times$ 1 cm
8 cm $\times$ 3 cm $\times$ 1 cm
6 cm $\times$ 4 cm $\times$ 1 cm
12 cm $\times$ 2 cm $\times$ 1 cm
6 cm $\times$ 2 cm $\times$ 2 cm
4 cm $\times$ 3 cm $\times$ 2 cm
(b) 6 cm (c) 4 cm $\times$ 3 cm $\times$ 2 cm
- 4 cm (5 cm $\times$ 5 cm $\times$ 4 cm)
- (a) 220 cm³ (b) 880 cm³

Exercise 18B

- (a) 8 litres (b) 12 litres (c) 200 litres
(d) 0.5 litres (e) 9.6 litres (f) 35.7 litres
(g) 0.41 litres (h) 0.02 litres
(i) 0.085 litres (j) 3.9765 litres
(k) 0.0134 litres (l) 0.2587 litres
(m) 0.001 litres (n) 0.0085 litres
(o) 0.0007 litres
- (a) 40 litres (b) 0.3 litres
- (a) 2 litres (b) 10 km
- (a) 2000 cm³ (b) 7000 cm³
(c) 13 000 cm³ (d) 80 000 cm³
(e) 4100 cm³ (f) 7150 cm³
(g) 12 685 cm³ (h) 10 080 cm³
(i) 5555.5 cm³ (j) 900 cm³
(k) 675.2 cm³ (l) 5000.8 cm³
(m) 5.2 cm³ (n) 10 cm³ (o) 0.07 cm³
- 10 glasses
- 20 cm
- (a) 1 litre
(b) A- 400 cm³, B- 800 cm³, C- 200 cm³
- 6 buckets

9. (a) 4000 litres (b) 12 000 litres
 (c) 50 000 litres (d) 500 litres
 (e) 1200 litres (f) 10 litres
 (g) 5080 litres (h) 6987.5 litres
10. (a) 6000 litres (b) 8000 litres
 (c) 60 000 litres (d) 10 000 litres
 (e) 3200 litres (f) 10 800 litres
 (g) 480 litres (h) 540 litres
 (i) 1000 litres (j) 560 litres
 (k) 540 litres (l) 1500 litres
11. (a)(i) 45 m^3 (ii) 45 000 litres
 (b) 3 m
12. (a) 1000 litres (b) 10 minutes
 (c) 5 days

Chapter Review

1. (a) 125 cm^3 (b) 48 cm^3
 (c) 21 cm^3 (d) 72 cm^3
2. (a) 4000 litres (b) 18 000 litres
 (c) 10 000 litres (d) 5000 litres
 (e) 10 000 litres (f) 1800 litres
 (g) 24 litres (h) 16 litres
 (i) 1 litre (j) 48 litres
 (k) 540 litres (l) 1500 litres
3. 2 cm
 4. 32 cm
 5. 18 litres

Chapter 19

Exercise 19A

1. (a) Cube (b) Rectangular Prism
 (c) Cylinder (d) Triangular Prism
 (e) Hexagonal Prism (f) Tetrahedron
 (g) Pyramid (h) Trapezoidal Prism
 (i) Sphere (j) Cone
2. A and L, B and G, C and I,
 D and J, E and H, F and K
3. A and L, B and E, C and F,
 D and K, G and J, H and I
4. C, E and G

5.

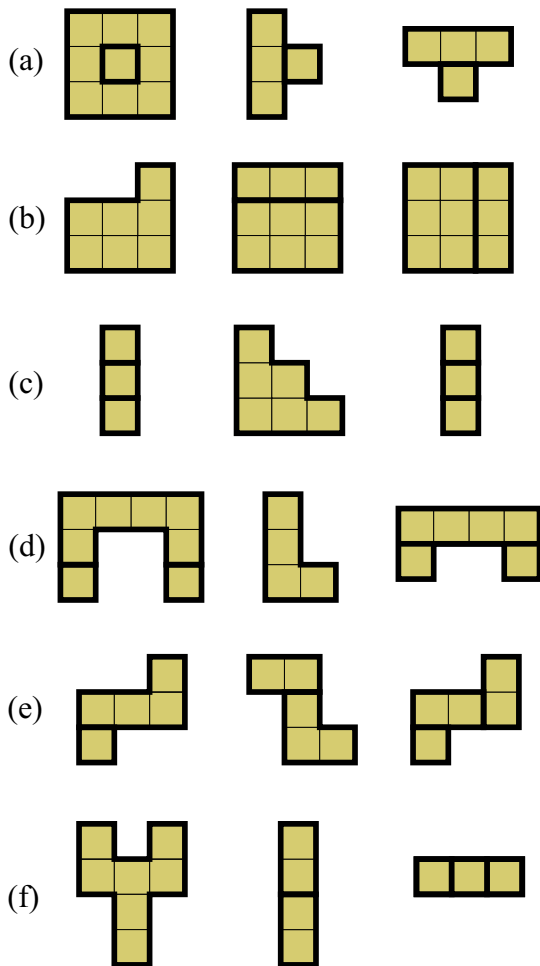


Exercise 19C

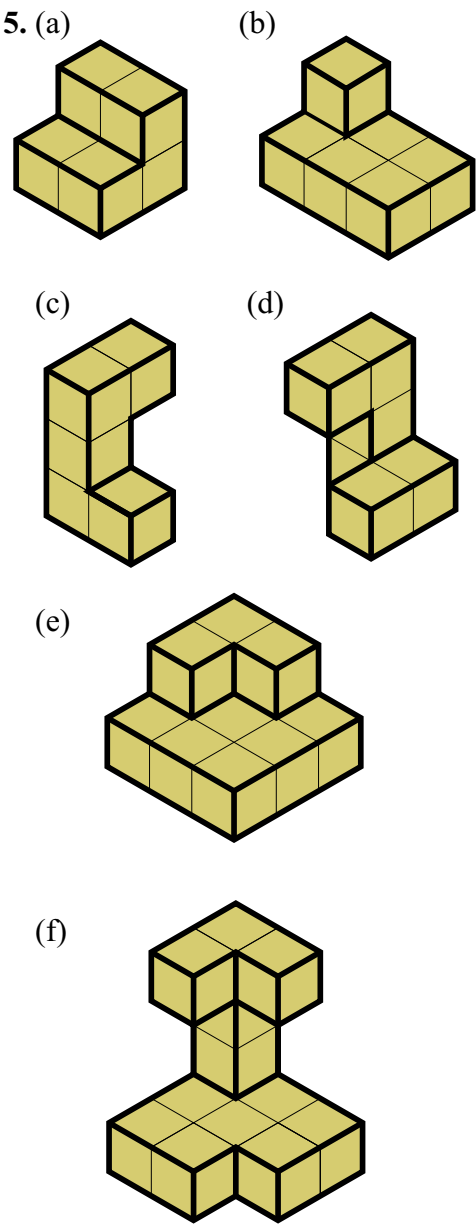
1. (a)(i) C (ii) D (iii) A
 (b)(i) D (ii) A (iii) C

2. (a) D (b) E (c) F (d) A

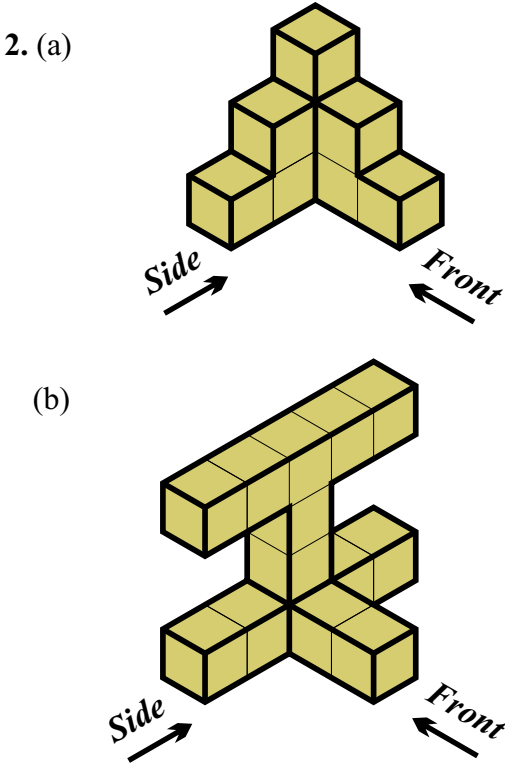
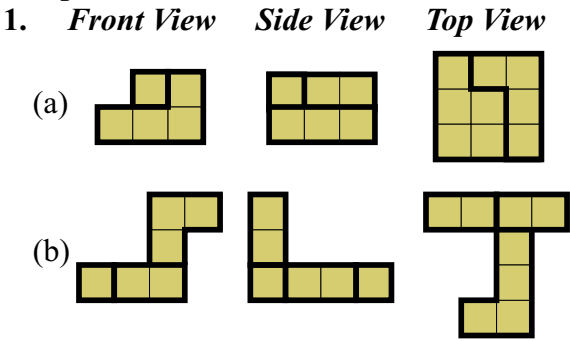
3. *Front View* *Side View* *Top View*



4. (a) Cube (b) Pyramid (c) Cone
 (d) Sphere (e) Cylinder
 (f) Triangular Prism (g) Rectangular Prism
 (h) Octagonal Prism



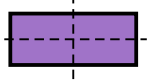
Chapter Review



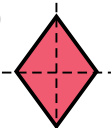
Chapter 20

Exercise 20A

1. (a)



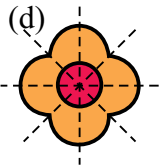
(b)



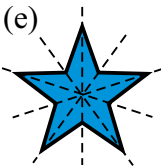
(c)



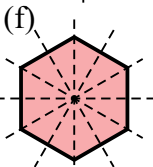
(d)



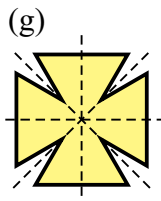
(e)



(f)



(g)



(h)



(i)



(h)



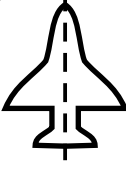
(i)



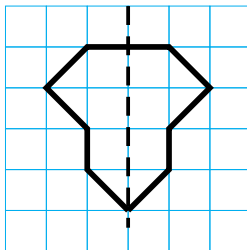
(j)



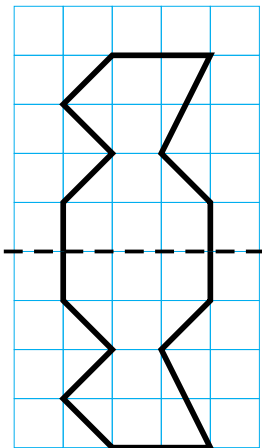
(k)



2. (a)



(b)



2. (a) A, H, I, M, O, T, U, V, W, X

(b) B, D, E, H, I, K, O, X

(c) H, I, O, X

3. (a)

W W M V
A I O O
X T U M
Y H T

(b) Any word using letters that have a vertical line of symmetry.

4. (a) BOX CHECK DICE

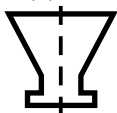
(b) Any word using letters that have a horizontal line of symmetry.

(c) Examples: MUM, TOOT

5. (a) A, E (b) D, F (c) B

Exercise 20B

1. (a)



(b)



(c)



(d)



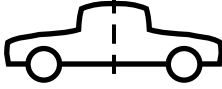
(e)



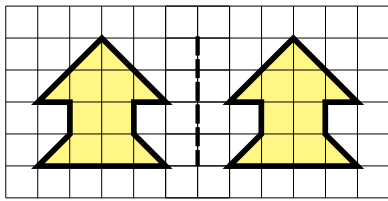
(f)

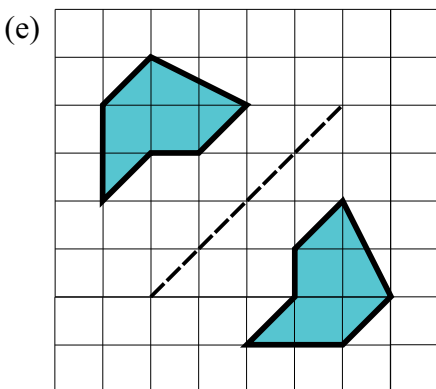
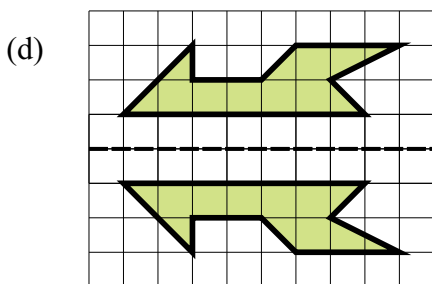
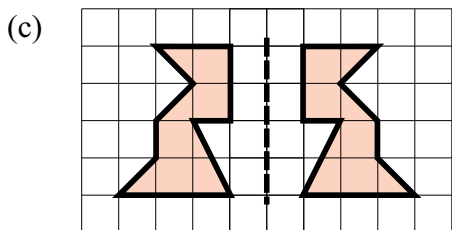
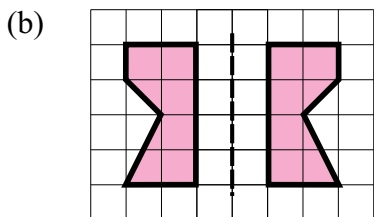


(g)



4. (a)





5. (a) An ambulance.

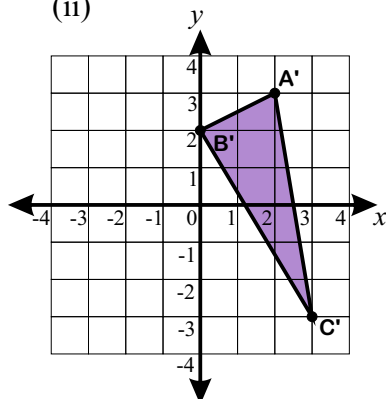
(b) When drivers driving in front of an ambulance look in their rear vision mirror they will see the reflection which will look like the word **AMBULANCE**.

6. (a) 202 (b) 2 hours and 54 minutes

Exercise 20C

1. (a)(i) $A = (-2,3)$, $B = (0,2)$, $C = (-3,-3)$

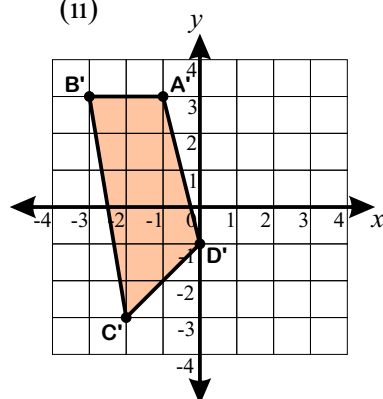
(ii)



$A' = (2,3)$
 $B' = (0,2)$
 $C' = (3,-3)$

(b)(i) $A = (1,3)$, $B = (3,3)$, $C = (2,-3)$,
 $D = (0,-1)$

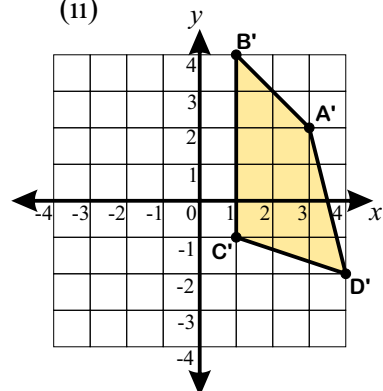
(ii)



$A' = (-1,3)$
 $B' = (-3,3)$
 $C' = (-2,-3)$
 $D' = (0,-1)$

(c)(i) $A = (-3,2)$, $B = (-1,4)$, $C = (-1,-1)$,
 $D = (-4,-2)$

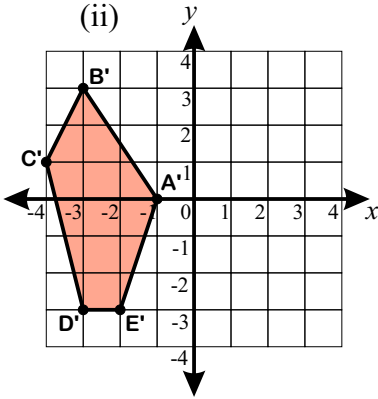
(ii)



$A' = (3,2)$
 $B' = (1,4)$
 $C' = (1,-1)$
 $D' = (4,-2)$

- (d)(i) $A = (1,0)$, $B = (3,3)$, $C = (4,1)$
 $D = (3,-3)$, $E = (2,-3)$

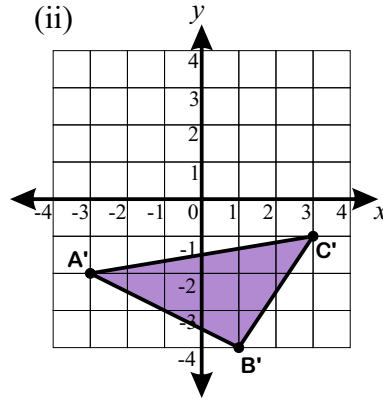
(ii)



$$\begin{aligned} A' &= (-1,0) \\ B' &= (-3,3) \\ C' &= (-4,1) \\ D' &= (-3,-3) \\ E' &= (-2,-3) \end{aligned}$$

2. (a)(i) $A = (-3,2)$, $B = (1,4)$, $C = (3,1)$

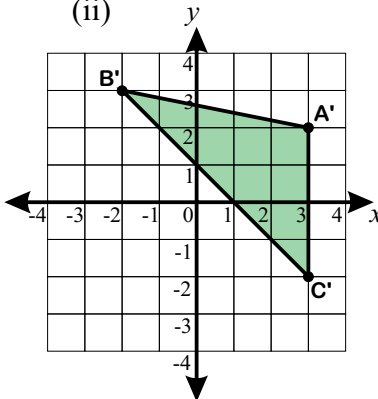
(ii)



$$\begin{aligned} A' &= (-3,-2) \\ B' &= (1,-4) \\ C' &= (3,-1) \end{aligned}$$

- (e)(i) $A = (-3,2)$, $B = (2,3)$, $C = (-3,-2)$

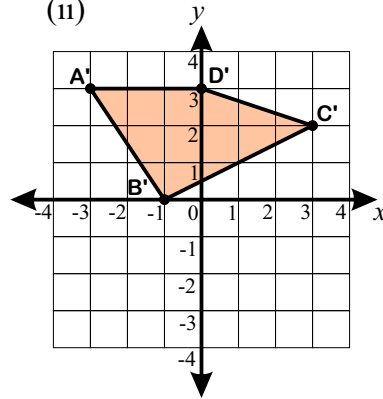
(ii)



$$\begin{aligned} A' &= (3,2) \\ B' &= (-2,3) \\ C' &= (3,-2) \end{aligned}$$

- (b)(i) $A = (-3,-3)$, $B = (-1,0)$, $C = (3,-2)$
 $D = (0,-3)$

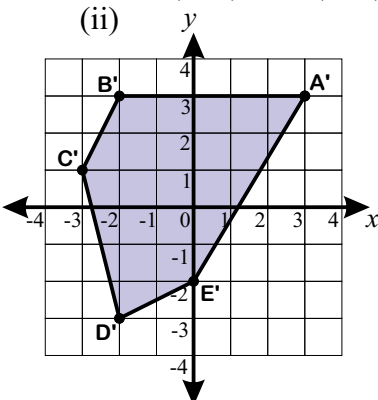
(ii)



$$\begin{aligned} A' &= (-3,3) \\ B' &= (-1,0) \\ C' &= (3,2) \\ D' &= (0,3) \end{aligned}$$

- (f)(i) $A = (-3,3)$, $B = (2,3)$, $C = (3,1)$
 $D = (2,-3)$, $E = (0,-2)$

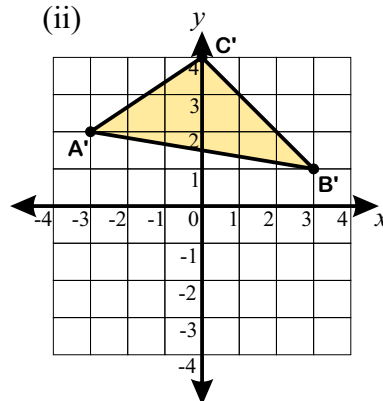
(ii)



$$\begin{aligned} A' &= (3,3) \\ B' &= (-2,3) \\ C' &= (-3,1) \\ D' &= (-2,-3) \\ E' &= (0,-2) \end{aligned}$$

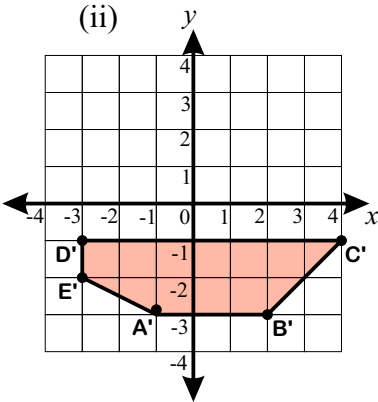
- (c)(i) $A = (-3,-2)$, $B = (3,-1)$, $C = (0,-4)$

(ii)

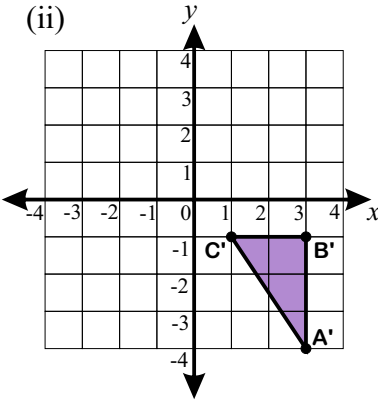


$$\begin{aligned} A' &= (-3,2) \\ B' &= (3,1) \\ C' &= (0,4) \end{aligned}$$

2. (d)(i) $A = (-1,3)$, $B = (2,3)$, $C = (4,1)$ 3. (a)(i) $A = (-3,4)$, $B = (-3,1)$, $C = (-1,1)$
 $D = (-3,1)$, $E = (-3,2)$ (ii)

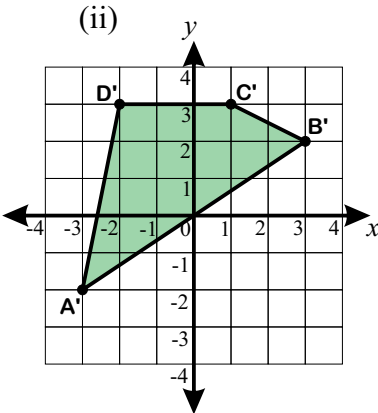


$A' = (-1,-3)$
 $B' = (2,-3)$
 $C' = (4,-1)$
 $D' = (-3,-1)$
 $E' = (-3,-2)$



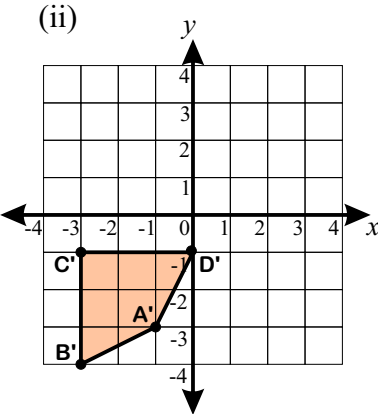
$A' = (3,-4)$
 $B' = (3,-1)$
 $C' = (1,-1)$

- (e)(i) $A = (-3,2)$, $B = (3,2)$, $C = (1,-3)$
 $D = (-2,-3)$



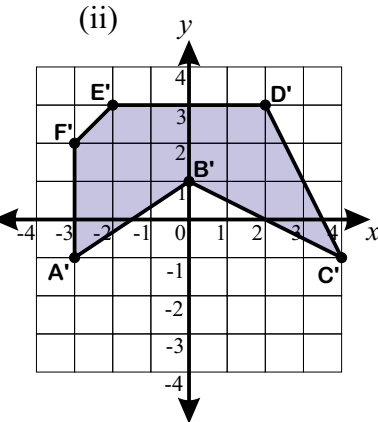
$A' = (-3,-2)$
 $B' = (3,2)$
 $C' = (1,3)$
 $D' = (-2,3)$

- (b)(i) $A = (1,3)$, $B = (3,4)$, $C = (3,1)$
 $D = (0,1)$



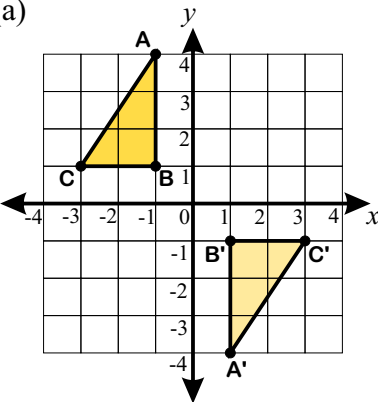
$A' = (-1,-3)$
 $B' = (-3,-4)$
 $C' = (-3,-1)$
 $D' = (0,-1)$

- (f)(i) $A = (-3,1)$, $B = (0,-1)$, $C = (4,1)$
 $D = (2,-3)$, $E = (-2,-3)$, $F = (-3,-2)$



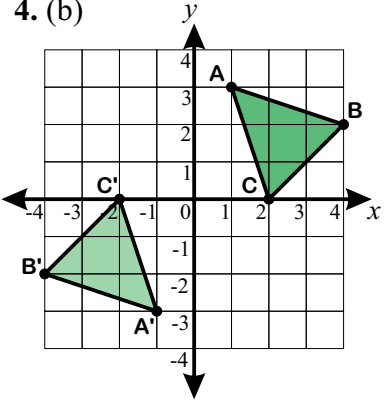
$A' = (-3,-1)$
 $B' = (0,1)$
 $C' = (4,1)$
 $D' = (2,3)$
 $E' = (-2,3)$
 $F' = (-3,2)$

4. (a)



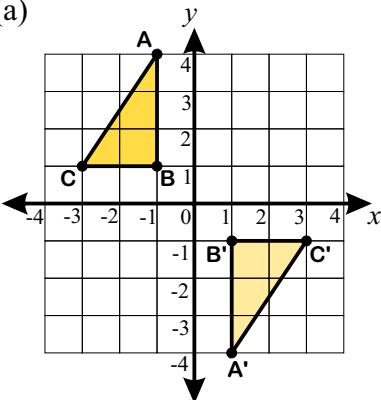
$A' = (1,-4)$
 $B' = (1,-1)$
 $C' = (3,-1)$

4. (b)



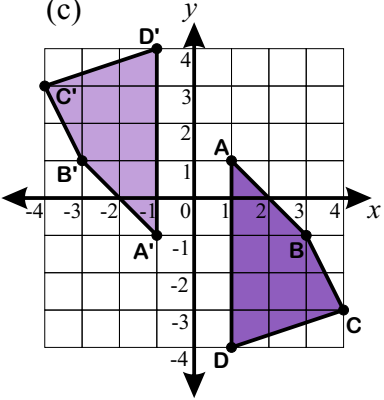
$A' = (-1, -3)$
 $B' = (-4, -2)$
 $C' = (-2, 0)$

5. (a)

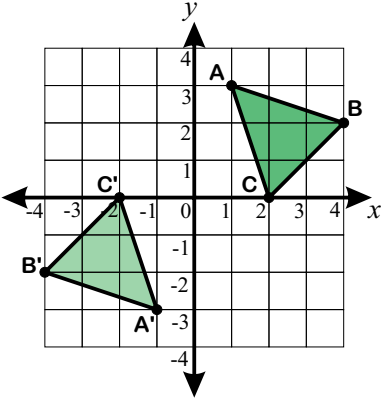


$A' = (1, -4)$
 $B' = (1, -1)$
 $C' = (3, -1)$

(c)

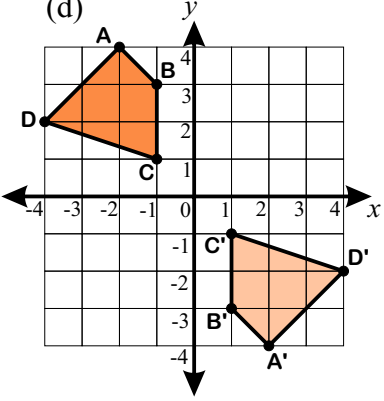


$A' = (-1, -1)$
 $B' = (-3, 1)$
 $C' = (-4, 3)$
 $D' = (-1, 4)$



$A' = (-1, -3)$
 $B' = (-4, -2)$
 $C' = (-2, 0)$

(d)



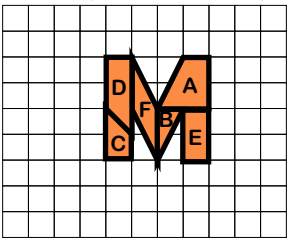
$A' = (2, -4)$
 $B' = (1, -3)$
 $C' = (1, -1)$
 $D' = (3, -2)$

- (b) Yes.
6. The sign of the coordinates changes.
Negative coordinates become positive.
Positive coordinates become negative.

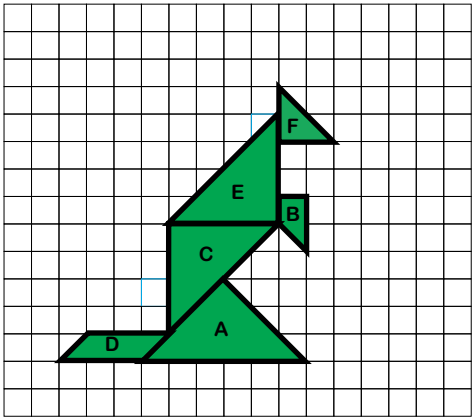
- (a) $A' = (2, -4)$, $B' = (0, -1)$, $C' = (4, -2)$
(b) $A' = (1, 0)$, $B' = (1, 3)$, $C' = (4, 2)$,
 $D' = (3, 1)$

- Exercise 20D**
1. (a) **E**
(b) **A** - L8, U2 **B** - L4, U2 **C** - R5, U2
 D - R7, D4 **F** - R2, D4 **G** - R5, D2

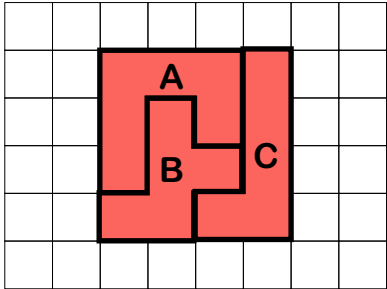
2.



3. Kangaroo



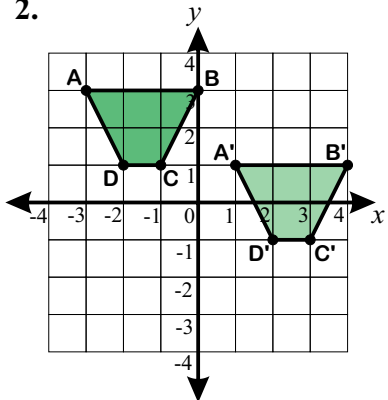
4. **A** - R2, D1 **B** - R1, U1 **C** - L2, U1



Exercise 20E

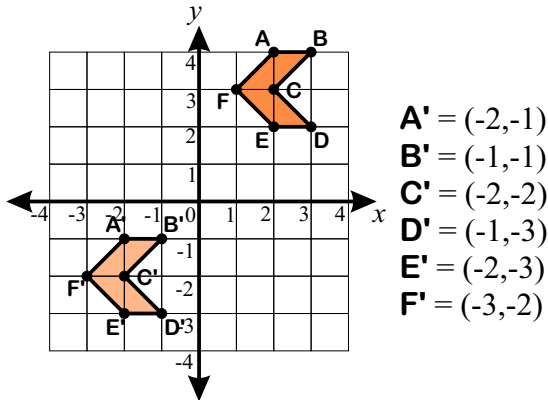
1. (a) $A' = (-2,1)$, $B' = (-1,0)$,
 $C' = (-2,-2)$, $D' = (-3,0)$
- (b) $A' = (2,4)$, $B' = (3,1)$,
 $C' = (1,0)$, $D' = (0,3)$

2.



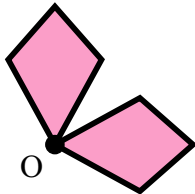
- $$\begin{aligned} A' &= (1,1) \\ B' &= (4,1) \\ C' &= (3,-1) \\ D' &= (2,-1) \end{aligned}$$

3.

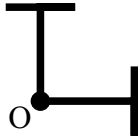


Exercise 20F

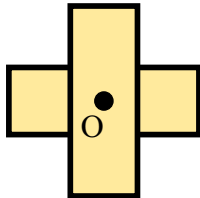
1. (a)



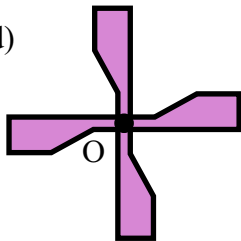
(b)



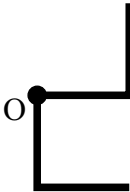
(c)



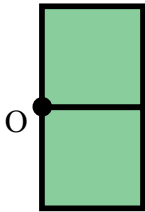
(d)



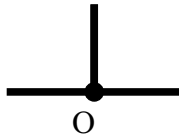
(e)



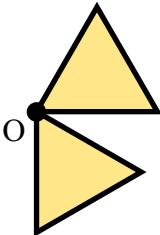
(f)



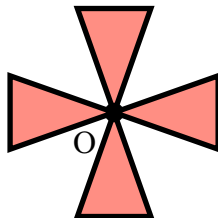
(g)

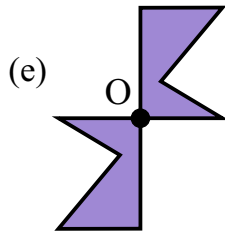
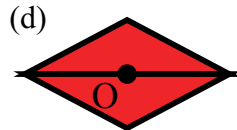
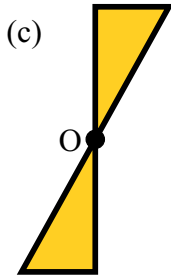
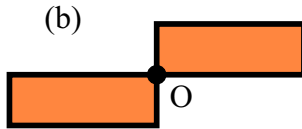
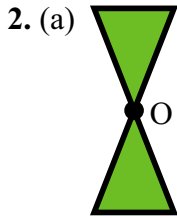
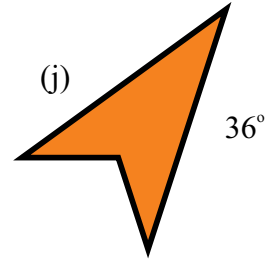
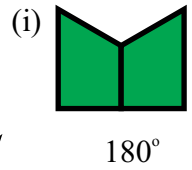
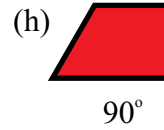
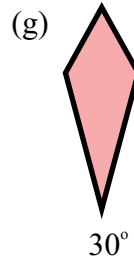
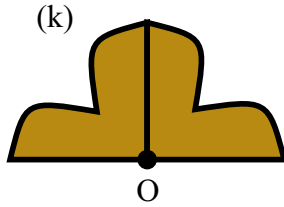
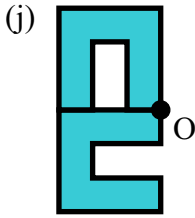


(h)



(i)





Exercise 20G

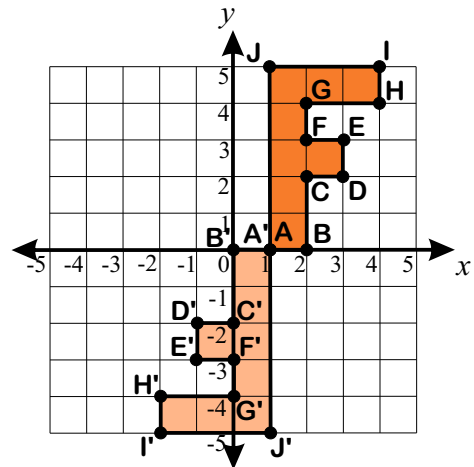
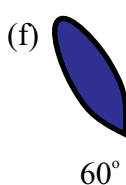
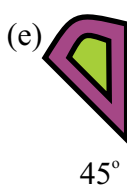
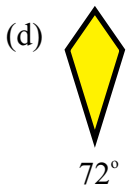
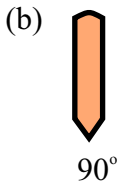
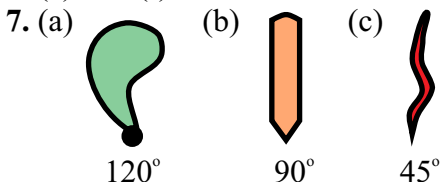
- (a) $A' = (-2, 1)$, $B' = (0, 1)$, $C' = (-2, -3)$
 (b) $A' = (-1, 3)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, 3)$
- (a) $A' = (-1, 0)$, $B' = (-1, 4)$, $C' = (1, 4)$
 (b) $A' = (-1, -1)$, $B' = (-1, 3)$, $C' = (1, 3)$, $D' = (1, -1)$
- (a) $A' = (0, 0)$, $B' = (1, 3)$, $C' = (-4, 3)$
 (b) $A' = (0, 1)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, 1)$
-

3. Square, Regular Octagon

4. Square, Rectangle, Rhombus, Parallelogram, Regular Hexagon, Regular Octagon

5. (a) B (b) C (c) D

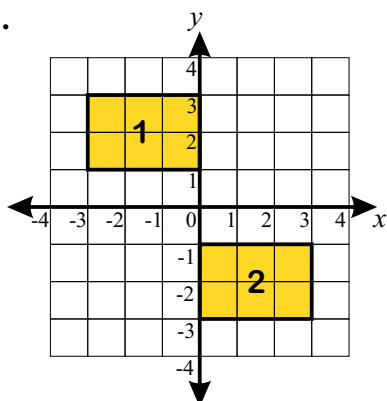
6. (a) 180° (b) 120° (c) 60° (d) 120°
 (e) 72° (f) 45°



- (d) $A' = (1, 0)$, $B' = (0, 0)$, $C' = (0, -2)$, $D' = (-1, -2)$, $E' = (-1, -3)$, $F' = (0, -3)$, $G' = (0, -4)$, $H' = (-2, -4)$, $I' = (-2, -5)$, $J' = (1, -5)$

Exercise 20H

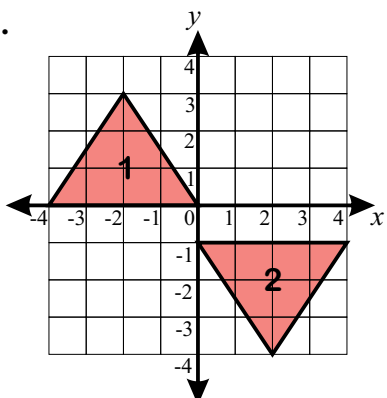
1.



(c) One other way:

Translate it to the right three units and down four units (R3, D4).

2.

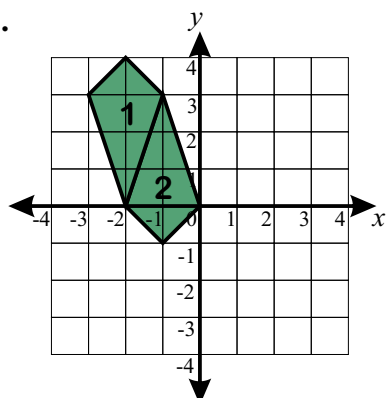


(c) One other way:

Step 1 Reflect it about the x -axis.

Step 2 Translate it to the right four units and down one unit (R4, D1).

3.

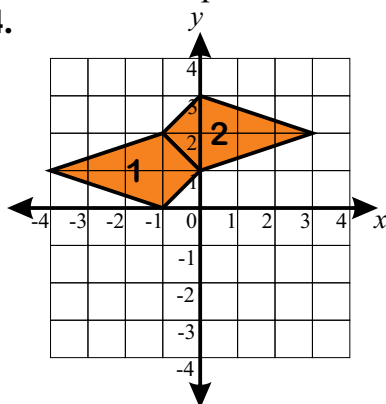


3. (c) One other way:

Step 1 Rotate it 180° about $(-2, 0)$.

Step 2 Translate it to the right one unit and up three units (R1, U3).

4.

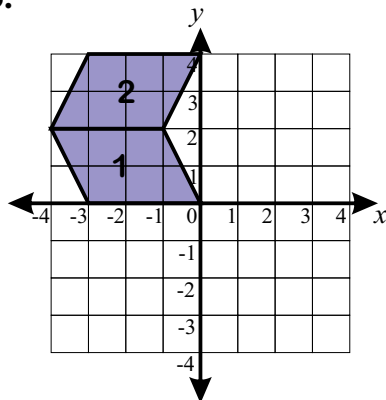


(c) One other way:

Step 1 Reflect it about the y -axis.

Step 2 Translate it to the left one unit and up one unit (L1, U1).

5.

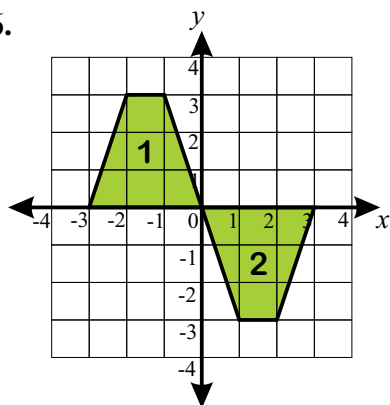


(c) One other way:

Step 1 Reflect it about the y -axis.

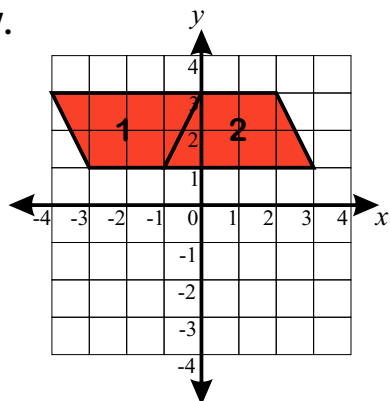
Step 2 Translate it to the left four units and up two units (L4, U2).

6.



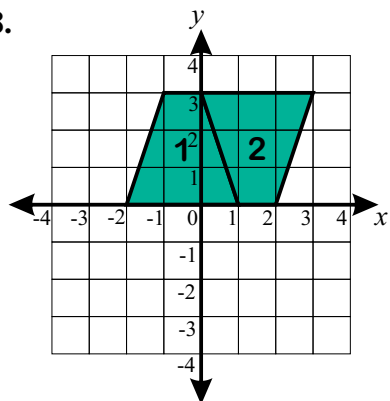
(c) One other way:
Rotate it 180° about $(0,0)$.

7.



(c) One other way:
Step 1 Rotate it 180° about $(-1,1)$.
Step 2 Translate it to the right one unit and up two units (R1, U2).

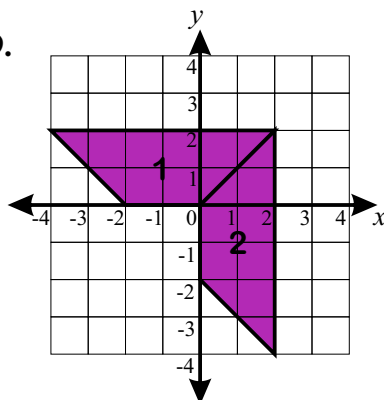
8.



8. (c) One other way:

Step 1 Reflect it about the x -axis.**Step 2** Translate it to the right two units and up three units (R2, U3).

9.



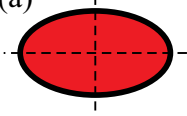
(c) One other way:

Step 1 Rotate it 90° clockwise about $(-2,0)$.**Step 2** Translate it to the right two units (R2).**Exercise 20I**

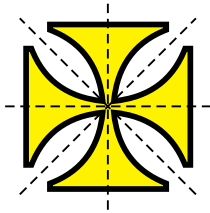
1. (a) Rectangles
- (b) Equilateral triangles
- (c) Octagons and squares
- (d) Equilateral triangles and squares
- (e) Hexagons and equilateral triangles
- (f) Parallelograms
- (g) Hexagons
- (h) Hexagons, equilateral triangles and squares
- (i) Kites
- (j) Dodecagons and equilateral triangles
- (k) Dodecagons, squares and hexagons
- (l) Trapeziums
- (m) Parallelograms and triangles
- (n) Hexagons and equilateral triangles
- (o) Squares and equilateral triangles
- (p) Trapeziums, rhombuses, hexagons and equilateral triangles

Chapter Review

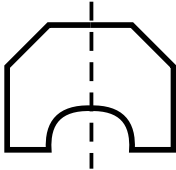
1. (a)



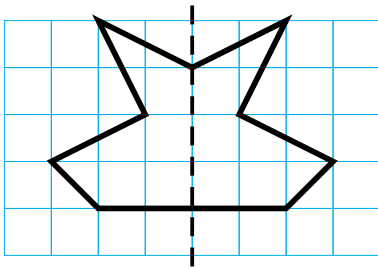
(b)



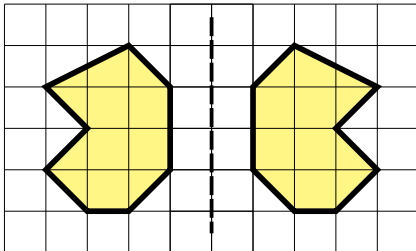
2. (a)



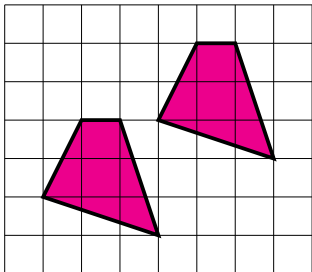
(b)



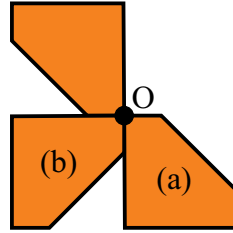
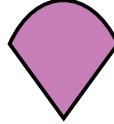
3.



4.



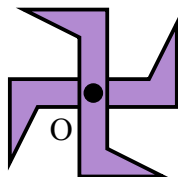
6.

7. (a) 90° (b) 60° 8. (a) 72° (b) 36° 9. (a) $A = (1,2)$, $B = (3,3)$, $C = (4,1)$,
 $D = (1,1)$ (b) $A' = (-1,2)$, $B' = (-3,3)$, $C' = (-4,1)$,
 $D' = (-1,1)$ (c) $A' = (1,-2)$, $B' = (3,-3)$, $C' = (4,-1)$,
 $D' = (1,-1)$ (d) $A' = (-2,-2)$, $B' = (0,-1)$, $C' = (1,-3)$,
 $D' = (-2,-3)$ (e) $A' = (2,1)$, $B' = (3,-1)$, $C' = (1,-2)$,
 $D' = (1,1)$ (f) $A' = (0,-1)$, $B' = (-2,0)$, $C' = (-3,-2)$,
 $D' = (0,-2)$ (g) $A' = (2,-3)$, $B' = (1,-1)$, $C' = (3,0)$,
 $D' = (3,-3)$

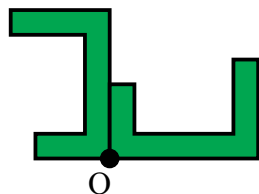
10. Two methods are:

Method 1**Step 1** Reflect it about the y -axis.**Step 2** Reflect it about the x -axis.**Method 2****Step 1** Rotate it 180° about $(-1,-1)$.**Step 2** Translate it to the right two units and up two units (R2, U2).

5. (a)



(b)



Chapter 21

Exercise 21A

- (a) 40° (b) 20° (c) 70° (d) 45°
(e) 15° (f) 53° (g) 37° (h) 130°
(i) 160° (j) 165° (k) 103°
(l) 90° (m) 172° (n) 30° (o) 68°
(p) 125° (q) 116°
- (a) 40° (b) 35° (c) 120° (d) 105°
(e) 42° (f) 90° (g) 83° (h) 152°
(i) 75°

Exercise 21B

- (a) 340° (b) 250° (c) 245°
(d) 208° (e) 324°
- (a) 320° (b) 290° (c) 215°
(d) 307° (e) 233° (f) 223°
(g) 270°

Exercise 21C

- 1 - 50° , 2 - 140° , 3 - 200° , 4 - 310° ,
5 - 90°
- (a) B (b) D (c) C (d) C

Exercise 21D

- (a) acute (b) obtuse (c) acute
(d) reflex (e) reflex (f) obtuse
(g) reflex (h) acute
- (a) 2 (b) 4

Exercise 21E

- (a) E (b) A (c) D (d) C (e) F
(f) B
- (a) 50° (b) 70° (c) 120° (d) 210°
(e) 290° (f) 100°

Exercise 21F

- (a) 360° (b) 720° (c) 120° (d) 45°
(e) 270° (f) 36°
- (a) 90° (b) 60° (c) 150°
- (a) 60° (b) 180° (c) 240° (d) 225°
- 60°
- 12°

Exercise 21G

- 51° and 39° , 26° and 64° , 8° and 82° ,
 41° and 49° , 18° and 72° , 44° and 46°
- 125° and 55° , 135° and 45° , 117° and 63° ,
 73° and 107° , 154° and 26° , 164° and 16°
- (a) 79° (b) 35° (c) 50° (d) 1° (e) 53°
- (a) 100° (b) 15° (c) 76° (d) 142° (e) 109°

Exercise 21H

- $a = 60^\circ$ 2. $b = 110^\circ$ 3. $c = 67^\circ$
- $d = 48^\circ$ 5. $e = 80^\circ$
- $f = 35^\circ$, $g = 145^\circ$, $h = 145^\circ$
- $i = 140^\circ$ 8. $j = 34^\circ$ 9. $k = 76^\circ$
- $l = 24^\circ$ 11. $m = 60^\circ$ 12. $n = 48^\circ$
- $o = 38^\circ$, $p = 52^\circ$, $q = 38^\circ$ 14. $r = 40^\circ$

Exercise 21I

- (a) G (b) U (c) K (d) S
- VERTICES
- VERTICAL
- (a) 4 (b) 5 (c) 5
- (a) B (b) A (c) D (d) A (e) B
- PERPENDICULAR
- Yes

Exercise 21J

- (a) r and u , s and t
(b) r and v , p and t , q and u , s and w
(c) r and t , s and u
(d) p and s , q and r , t and w , u and v
- (a) $a = 140^\circ$, $b = 40^\circ$, $c = 140^\circ$, $d = 140^\circ$,
 $e = 40^\circ$, $f = 40^\circ$, $g = 140^\circ$
(b) $h = 46^\circ$, $i = 134^\circ$, $j = 134^\circ$, $k = 46^\circ$,
 $l = 46^\circ$, $m = 134^\circ$, $n = 46^\circ$
(c) $o = 117^\circ$, $p = 63^\circ$, $q = 63^\circ$, $r = 117^\circ$,
 $s = 117^\circ$, $t = 63^\circ$, $u = 117^\circ$
(d) $v = 149^\circ$, $w = 31^\circ$, $x = 31^\circ$, $y = 149^\circ$

Exercise 21K

- $m = 70^\circ$ 2. $h = 48^\circ$ 3. $x = 63^\circ$ 4. $c = 64^\circ$
- $n = 45^\circ$ 6. $p = 60^\circ$

Exercise 21L

1. $a = 50^\circ$ 2. $b = 129^\circ$ 3. $c = 90^\circ$
 4. $d = 65^\circ$ 5. $e = 130^\circ$ 6. $f = 152^\circ$
 7. $g = 98^\circ$ 8. $h = 63^\circ$

Chapter Review

1. (a) 50° (b) 75° (c) 33° (d) 160°
 (e) 300° (f) 245°
 2. (a) Obtuse (b) Acute (c) Right
 (d) Straight (e) Reflex
 3. A and E
 4. (a) 90° (b) 180° (c) 60° (d) 1080°
 5. (a) 120° (b) 15°
 6. (a) $a = 60^\circ$ (b) $b = 49^\circ$ (c) $c = 100^\circ$
 (d) $d = 130^\circ$ (e) $e = 50^\circ$, $f = 130^\circ$
 (f) $g = 50^\circ$, $h = 130^\circ$
 (g) $i = 129^\circ$, $j = 51^\circ$
 7. 90° , 45° , 45°
 8. (a) AE (b) CF (c) CA and AE
 (d) $\angle BCD$ (e) CF (f) B

Chapter 22**Exercise 22A**

1. C 2. D 3. B 4. A 5. D
 6. C 7. C 8. A 9. A

Exercise 22B

1. $a = 60^\circ$ 2. $b = 43^\circ$ 3. $c = 61^\circ$
 4. $d = 52^\circ$ 5. $e = 45^\circ$ 6. $f = 60^\circ$

Exercise 22C

1. Isosceles 2. Isosceles
 3. Scalene 4. Equilateral
 5. Scalene 6. Isosceles
 7. Isosceles 8. Isosceles

Exercise 22D

1. $a = 70^\circ$ 2. $b = 122^\circ$ 3. $c = 68^\circ$
 4. $d = 29^\circ$ 5. $e = 78^\circ$ 6. $f = 94.5^\circ$

Exercise 22E

1. (a) false (b) true (c) true (d) true
 (e) false (f) false (g) false (h) true
 (i) true (j) true (k) false (l) true
 (m) false (n) true (o) true (p) false
 (q) false
 2. 36

Chapter Review

1. C 2. B 3. C
 4. $\overline{AC}$, $\overline{AD}$, $\overline{BD}$, $\overline{BE}$, $\overline{CE}$
 5. (a) B, H (b) C, E, D, I, J (c) A, F, G
 (d) A, J (e) G, I
 6. (a) equilateral (b) scalene (c) isosceles
 (d) scalene (e) isosceles (f) isosceles
 7. (a) $a = 31^\circ$ (b) $b = 43^\circ$ (c) $c = 67^\circ$
 8. (a) $x = 91^\circ$ (b) $y = 69^\circ$ (c) $z = 103^\circ$
 9. (a) rhombus (b) irregular quadrilateral
 (c) parallelogram (d) trapezium
 (e) square (f) rectangle (g) rhombus
 (h) kite (i) trapezium (j) parallelogram

Chapter 23**Exercise 23A**

1. (a) A (b) C (c) B (d) E (e) F (f) F (g) B
 (h) G (i) F (j) B (k) F
 2. (a) yellow (b) blue (c) red and black
 3. white, green, red, yellow, blue
 4. (a)(i) {blue, yellow} (ii) equally likely
 (b)(i) {large, small} (ii) not equally likely
 (c)(i) {consonant, vowel}
 (ii) not equally likely
 (d)(i) {win computer, win phone,
 win cinema ticket, not win a prize}
 (ii) not equally likely
 (e)(i) {male, female} (ii) equally likely
 (f)(i) {male, female} (ii) not equally likely

Exercise 23B

- 1 - 7 and 10 will vary.
 8. ~ 0.7
 9. ~ 0.9

Exercise 23C

- (a)(i) $\frac{1}{4}$ (ii) 0.25 (iii) 25%
(b)(i) $\frac{3}{4}$ (ii) 0.75 (iii) 75%
- (a)(i) $\frac{3}{10}$ (ii) 0.3 (iii) 30%
(b)(i) $\frac{1}{10}$ (ii) 0.1 (iii) 10%
- (a)(i) $\frac{1}{8}$ (ii) 0.125 (iii) 12.5%
(b)(i) $\frac{7}{8}$ (ii) 0.875 (iii) 87.5%
- (a) $\frac{1}{17}$ (b) $\frac{2}{17}$ (c) 0 (d) $\frac{5}{17}$ (e) $\frac{12}{17}$

Exercise 23D

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$
- (a) $\frac{4}{11}$ (b) $\frac{7}{11}$
- (a) $\frac{7}{12}$ (b) $\frac{5}{12}$
- (a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{5}{18}$ (d) $\frac{11}{18}$
- (a)(i) $\frac{1}{4}$ (ii) $\frac{1}{6}$ (iii) $\frac{3}{8}$ (iv) $\frac{5}{24}$
(b)(i) $\frac{1}{4}$ (ii) $\frac{1}{5}$ (iii) $\frac{7}{20}$ (iv) $\frac{1}{5}$
- (a) 98
(b)(i) $\frac{6}{49}$ (ii) $\frac{3}{49}$ (iii) $\frac{1}{98}$ (iv) $\frac{9}{98}$
(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

Exercise 23E

- (a)(i) $\frac{3}{4}$ (ii) 0.75 (iii) 75%
(b)(i) $\frac{4}{5}$ (ii) 0.8 (iii) 80%
(c) Gabrielle (d) Clara
- (a)(i) $\frac{1}{5}$ (ii) 0.2 (iii) 20%
(b)(i) $\frac{3}{20}$ (ii) 0.15 (iii) 15%
(c) Lachy (d) Lachy
- (a) 10% (b) 35%
- (a) 10% (b) 25% (c) 90%
- (a) 50 (b)(i) 50% (ii) 80% (iii) 20%
- (a) 60% (b) 6 (c) 80% (d) 8
(e) Zac (f) Lefty Les - more samples

Chapter Review

- (a) 100% (b) $\frac{1}{2}$ (c) 40% (d) $\frac{3}{5}$
(e) 0.2 (f) 0 (g) 0.9
- (a) 0 (b) ~ 0.3 (c) ~ 0.1 (d) ~ 0.9
(e) 1
- (a)(i) $\frac{3}{5}$ (ii) 0.6 (iii) 60%
(b)(i) $\frac{1}{5}$ (ii) 0.2 (iii) 20%
(c)(i) $\frac{2}{5}$ (ii) 0.4 (iii) 40%
- $\frac{1}{5}$
- (a) Year 7 - 0.2, Year 8 - 0.25
(b) Year 8 (c) Year 8 - more samples

Chapter 24**Exercise 24A**

- (a) numerical (b) categorical (c) categorical
(d) numerical (e) numerical (f) categorical
(g) numerical (h) categorical
- (a) continuous (b) discrete (c) discrete
(d) continuous (e) discrete (f) continuous
(g) discrete (h) continuous
- (a) ordinal (b) nominal (c) ordinal
(d) nominal (e) ordinal (f) ordinal
(g) nominal (h) ordinal
- (a) T (b) F (c) F (d) T (e) F (f) T (g) T
(h) T (i) T (j) F (k) T (l) F (m) T
- (a) 32 (b) 68 (c) 29 (d) 895 (e) 1415
(f) 2984
- Yes
- (a) **C** - a cross-section of the school community is surveyed
(b) **B** - more of a cross-section of students is surveyed
- (a) Students could survey people and ask if they have solar panels now or had them 10 years ago. They could walk or ride around the streets and observe if houses have solar panels.
(b) Websites such as the Clean Energy Council

Exercise 24B

- (a) 75% (b) 2.1%
- (a) 5.8% (b) 6.3% (c) rise of 0.5%
(d) Feb 2014 - May 2014
- (a) 2500 million (b) 7000 million
(c) 1000 million (d) 1500 million
(e) 4500 million (f) 15 700 million
(g) 6200 million (h) Maybe natural disasters or a pandemic (i) Accurate population statistics were not taken in all countries of the world
- (a)(i) services 53%, manufacturing 26%, agriculture 11%, construction 8%, mining 2%
(ii) services 78%, manufacturing 8%, agriculture 3%, construction 9%, mining 2%

4. (b) services (c) manufacturing
5. (a) Australian Bureau of Agriculture and Resource Economics and Sciences
(b) The total of the percentages is not 100%

Chapter Review

1. (a) numerical (b) numerical (c) categorical (d) numerical
2. (a) discrete (b) continuous (c) continuous (d) discrete
3. (a) nominal (b) ordinal (c) ordinal (d) nominal
4. (a) secondary (b) primary (c) primary (d) secondary
5. (a) 549 (b) 31
6. ♦ Include as many people as possible.
♦ Survey a cross-section of people.

Chapter 25

Exercise 25A

1. (a)

Number of Movies	Tally	Frequency
0		6
1		9
2		15
3		17
4		11
5		8
more than 5		4
Total		70

(b)

Area of Government	Tally	Frequency
Defence		14
Education		18
Employment		25
Environment		22
Foreign Affairs		8
Health		31
Social Services		12
Total		130

2. (a)

Favourite Colour	Tally	Frequency
blue		10
purple		4
yellow		6
red		5
black		3
green		7
orange		5
Total		40

(b)

Number of Pups	Tally	Frequency
1		8
2		10
3		14
4		21
5		28
6		23
7		18
8		13
Total		135

Exercise 25B

1.

Age	Tally	Frequency
0 - 9		6
10 -		9
20 -		12
30 -		10
40 -		6
50 -		4
60 -		3
Total		50

2.

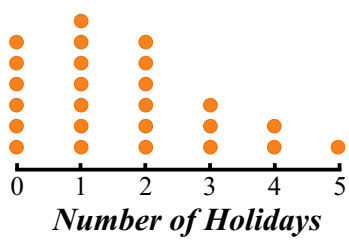
Weight (kg)	Tally	Frequency
0 - 2		7
- 4		10
- 6		17
- 8		17
- 10		9
Total		60

3.

Annual Salary	Tally	Frequency
0 - \$25,000		4
- \$50,000		18
- \$75,000		9
- \$100,000		4
Total		35

Exercise 25C

1. (a)



(b) 6
2. (a)



(b) monkey
3. (a) 39 (b) Great Barrier Reef

Exercise 25D

1. (a)(i) 3 (ii) 10 (iii) 12 (iv) 5 (v) 5
(b) 35 (c) A
2. (a) basketball (b) soccer
(c) cricket and tennis (d) 3 (e) 6
(f) 2 (g) 50
3. apples - 350 kg, plums - 475 kg,
peaches - 225 kg, pears - 325 kg,
apricots - 400 kg
4. (a) 125 mm (b) June (c) 190 mm
(d) 870 mm (e) Brisbane

5.

Grower	Amount (kg)				Total
	Pink Lady	Golden Delicious	Red Delicious	Granny Smith	
Grubby	100	150	75	100	425
Crunchy	175	100	150	50	475
Stewy	50	175	125	100	450
Ciders	75	100	100	150	425

6. (a)(i) 26 (ii) 28 (iii) 31 (iv) 34 (v) 32
(b) 151
(c)(i) 10 (ii) 10 (iii) 19 (iv) 24 (v) 15
(d) BOTTLEBRUSH
(e) WATTLE (f) GUM

Exercise 25E

1. (a)

Stem	Leaf
2	3 9
3	7 5 8 0 8 8 1
4	5 1 2 0
5	1 0 3 7 1

$2 \overline{) 3} = 23$

(b)

Stem	Leaf
2	3 9
3	0 1 5 7 8 8 8
4	0 1 2 5
5	0 1 1 3 7

$2 \overline{) 3} = 23$

- (c) 18 (d) 23 (e) 57

2. (a)

Stem	Leaf
1	8 8 9
2	0 2 8 3 8 0 9 8 6 9 7 5 7 6 5
3	3 8 8 1 2 7 1 9 4 3
4	1 5 0 2 1 2
5	6 0
6	2 0

(b)

Stem	Leaf
1	8 8 9
2	0 0 2 3 5 5 6 6 7 7 8 8 8 9 9
3	1 1 2 3 3 4 7 8 8 9
4	0 1 1 2 2 5
5	0 6
6	0 2

$1 \overline{) 8} = 18$

2. (c) 38 (d) 18 (e) 62

Exercise 25F

1. (a)

Stem	Leaf
15	9 1 8 8 9
16	3 3 4 9 2 0 8 4 0 3
17	0 2 8 9 1 3 3 0 2 2
18	2

$$\overline{15} \overline{9} = 159$$

(b)

Stem	Leaf
15	1 8 8 9 9
16	0 0 2 3 3 3 4 4 8 9
17	0 0 1 2 2 2 3 3 8 9
18	2

$$\overline{15} \overline{9} = 159$$

(c) 151 cm (d) 182 cm (e) 26

2. (a)

Stem	Leaf
20	1 9 4
21	5 9 7 0 9 7
22	7 9 0 7 3 7 8
23	5 0 9 1 7 7 9 1 3
24	1 5 2 8 4 7 0
25	4 1 0 1
26	2 1
27	
28	7 3
29	3 9

$$\overline{20} \overline{1} = 201$$

(b)

Stem	Leaf
20	1 4 9
21	0 5 7 7 9 9
22	0 3 7 7 7 8 9
23	0 1 1 3 5 7 7 9 9
24	0 1 2 4 5 7 8
25	0 1 1 4
26	1 2
27	
28	3 7
29	3 9

$$\overline{20} \overline{1} = 201$$

(c) 201 kg (d) 299 kg (e) 42

3. (a)

Stem	Leaf
6	9
7	6 3 8 9 7
8	7 3 0 8
9	1 0

$$\overline{6} \overline{9} = 6.9$$

(b)

Stem	Leaf
6	9
7	3 6 7 8 9
8	0 3 7 8
9	0 1

$$\overline{6} \overline{9} = 6.9$$

(c) 9.1 m (d) 6.9 m

4. (a)

Stem	Leaf
1	9 8
2	6 8 9 5 7 4 1
3	5 8 8 1 8 7 1 7 7
4	8 6 7 2 3 2 3 0
5	3 3
6	7
7	
8	2

$$\overline{1} \overline{9} = 1.9$$

(b)

Stem	Leaf
1	8 9
2	1 4 5 6 7 8 9
3	1 1 5 7 7 7 8 8 8
4	0 2 2 3 3 6 7 8
5	3 3
6	7
7	
8	2

$$\overline{1} \overline{9} = 1.9$$

(c) 1.8 sec (d) 8.2 sec

5. (a)

Stem	Leaf
29	8 6
30	6
31	5
32	7 6
33	4 8 5
34	6 7
35	4 9
36	7 5
37	2 3 8 1
38	7 5 5 0
39	6 1 0
40	7 3
41	
42	5 3
43	
44	5

$$\begin{array}{r} 29 \overline{)8} = 29.8 \end{array}$$

(b)

Stem	Leaf
29	6 8
30	6
31	5
32	6 7
33	4 5 8
34	6 7
35	4 9
36	5 7
37	1 2 3 8
38	0 5 5 7
39	0 1 6
40	3 7
41	
42	3 5
43	
44	5

$$\begin{array}{r} 29 \overline{)8} = 29.8 \end{array}$$

(c) 44.5° (d) 29.6°

Exercise 25G

1. (a) 20 (b) 58 (c) 78 (d) 69 (e) 96

2. (a) 26 (b) 18 (c) 17 (d) 49 (e) 25

(f) 49

3. (a)

Practice Exam		Main Exam	
Leaf	Stem	Leaf	Stem
	8 7	2	
	9 6 7 9 8	3	
7 0 5 7 3 9	1 8 0 8 5	4	
	2 2 0 8 6	5	4 7
	3	6	8 7 2 3 7
		7	2 0 2 1 3 4 0 4
		8	5 5 2 4 0 1 2
		9	3 3

$$\begin{array}{r} 2 \overline{)7} = 27 \end{array}$$

(b)

Practice Exam		Main Exam	
Leaf	Stem	Leaf	Stem
	8 7	2	
	9 9 8 7 6	3	
9 8 8 7 7 5	5 3 1 0 0	4	
	8 6 2 2 0	5	4 7
	3	6	2 3 7 7 8
		7	0 0 1 2 2 3 4 4
		8	0 1 2 2 4 5 5
		9	3 3

$$\begin{array}{r} 2 \overline{)7} = 27 \end{array}$$

(c) 27% (d) 63% (e) 54% (f) 93%

4. (a)

Con		Cam	
Leaf	Stem	Leaf	Stem
	7 9 8	5	
	8 8 2 3 7	6	
0 1 2 6 0 2	7		
	4 2	8	6 4 9
		9	3 3 0 7 0 1 7 9 1
		10	2 9 5

$$\begin{array}{r} 5 \overline{)8} = 58 \end{array}$$

(b)

Con		Cam	
Leaf	Stem	Leaf	Stem
	9 8 7	5	
	8 8 7 3 2	6	
6 2 2 1 0 0	7		
	4 2	8	4 6 9
		9	0 0 1 1 3 3 7 7 9
		10	2 5 9

$$\begin{array}{r} 5 \overline{)8} = 58 \end{array}$$

(c) 57 min (d) 84 min (e) 84 min

(f) 109 min

Chapter Review

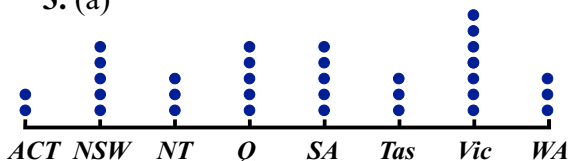
1.

Number of B & S	Tally	Frequency
0		8
1		22
2		12
3		5
4		2
5		1
Total		50

2.

Number of Passengers	Tally	Frequency
0 - 20		4
- 40		16
- 60		13
- 80		3
Total		36

3. (a)



(b) 33 (c) Vic

4. (a) Friday (b) 25 (c) Wednesday
(d) Thursday

5. (a)

Scores for		Scores against
Leaf	Stem	Leaf
4 2 8 7 8	3	7
8 9 3 0 7	4	3 0 9 2 2 1 6
6 8 9 8	5	2 2 2 0 1 3
1 2	6	1 0
	7	

$$\begin{array}{c|c} 3 & 7 \end{array} = 37$$

(b)

Scores for		Scores against
Leaf	Stem	Leaf
8 8 7 4 2	3	7
9 8 7 3 0	4	0 1 2 2 3 6 9
9 8 8 6	5	0 1 2 2 2 3
2 1	6	0 1
	7	

$$\begin{array}{c|c} 3 & 7 \end{array} = 37$$

(c) 16 (d) 72 (e) 61

Chapter 26

Exercise 26A

- (a) 4 (b) 7 (c) 7 (d) 38 (e) 3 (f) 5
(g) 2 (h) 5
- (a) 18.5 (b) 26.2 (c) 35.5 (d) 65.5
(e) 12.6 (f) 15.63
- (a) 12 m (b) 7 kg (c) 20 cm
- 45

Exercise 26B

- (a) 6 (b) 10 (c) 8 (d) 5 (e) 18 (f) 17
(g) 29 (h) 13 (i) 4.5 (j) 14.5 (k) 11.5
(l) 33.5
- (a) 11 (b) 10 (c) 5 (d) 10 (e) 21
(f) 41 (g) 27.5 (h) 24.5 (i) 56 (j) 35.5

Exercise 26C

- (a) 2 (b) 12 (c) 2, 3 and 4 (d) 7 and 9
- (a) 8 (b) 7 and 9 (c) 5 and 7
(d) 9 and 12

Exercise 26D

- (a)(i) 10 (ii) 10 (iii) 10
(b)(i) 9 (ii) 9 (iii) 11
(c)(i) 10 (ii) 9 (iii) 8
(d)(i) 9 (ii) 9 (iii) 7 and 11
- (a)(i) 5 (ii) 6 (iii) 6
(b)(i) 4 (ii) 4 (iii) 6
(c)(i) 4 (ii) 4 (iii) 4
(d)(i) 7 (ii) 8 (iii) 9
(e)(i) 4 (ii) 4 (iii) 5
(f)(i) 5 (ii) 5.5 (iii) 7
(g)(i) 7 (ii) 7 (iii) 8
(h)(i) 10.5 (ii) 11.5 (iii) 13
- (a) 16.5 (ii) 17 (iii) 18
- (a) 23 (b) 23 (c) 23 and 24

Exercise 26E

- 23
- 27
- 30
- 33

Exercise 26F

1. (a) 11 (b) 10 (c) 13 (d) 18
 2. (a) 4.3 (b) 5 (c) 6 (d) 7
 3. (a) 8.5 (b) 8.5 (c) 8 and 9 (d) 3
 4. (a) 30.25 (b) 28.5 (c) no mode (d) 38
 5. (a) 11.75 (b) 12 (c) 12 (d) 5
 6. (a) 23 (b) 23 (c) 23 (d) 6

Exercise 26G

1. (a)(i) 25 (ii) 24 (iii) 24 (iv) 13
 (b)(i) 34 (ii) 34 (iii) 35 (iv) 15
 (c)(i) 24 (ii) 24 (iii) 18, 24 and 26 (iv) 20
 (d)(i) 32.3 (ii) 34 (iii) 36 (iv) 28
 (e)(i) 37 (ii) 38 (iii) no mode (iv) 43

2. (a)

Stem	Leaf
1	3 4 8 9
2	0 1 2 4 6 6
3	1 2 3

$$1 \overline{) 3} = 13$$

(i) 23 (ii) 22 (iii) 26 (iv) 20

(b)

Stem	Leaf
4	1 4 5 7 9
5	0 3 3 6 7 8 8
6	0 1 2 4 5 7

$$4 \overline{) 1} = 41$$

(i) 55 (ii) 56.5 (iii) 53 and 58 (iv) 26

(c)

Stem	Leaf
2	2 3 4 5 5 6 7 9
3	0 1 2 2 3 3 4 5 6 8 9
4	0 1 2 3 4 4 5 5

$$2 \overline{) 2} = 22$$

(i) 34 (ii) 33
 (iii) 25, 32, 33, 44 and 45
 (iv) 23

3. (a)(i)

Stem	Leaf
2	8 7 9 9 9
3	1 2 3 0 7 5 9 9 0 7 8
4	3 0 5 6 2 1 0 0 2 0 6
5	6 1 1 0 2

$$2 \overline{) 8} = 28$$

(ii)

Stem	Leaf
2	7 8 9 9 9
3	0 0 1 2 3 5 7 7 8 9 9
4	0 0 0 0 1 2 2 3 5 6 6
5	0 1 1 2 6

$$2 \overline{) 8} = 28$$

(iii) 39 (iv) 39.5 (v) 40 (vi) 29

(b)(i)

Stem	Leaf
1	5 3 9 9 8 9 8
2	2 6 2 9 6 7 8 8 7 0 2 8 9 7 8 7
3	4 0 5 2 7 6 2 1 7 0 3 4 9 7 6 0 3 4 3 9 0
4	3 2 2 0

$$1 \overline{) 5} = 15$$

(ii)

Stem	Leaf
1	3 5 8 8 9 9 9
2	0 2 2 2 6 6 7 7 7 7 8 8 8 8 9 9
3	0 0 0 0 1 2 2 3 3 3 4 4 4 5 6 6 7 7 7 9 9
4	0 2 2 3

$$1 \overline{) 5} = 15$$

(iii) 29.5 (iv) 30 (v) 27, 28 and 30
 (vi) 30

Exercise 26H

1. (a)(i) 1.7 (ii) 2 (iii) 2 (iv) 3
 (b)(i) 2.4 (ii) 2 (iii) 2 (iv) 4
 (c)(i) 12 (ii) 12 (iii) 13 (iv) 4
 2. (a)(i) 2.1 (ii) 2 (iii) 2 (iv) 4
 (b)(i) 4.6 (ii) 5 (iii) 6 (iv) 4
 (c)(i) 26 (ii) 26 (iii) 26 (iv) 4
 (d)(i) 14.6 (ii) 15 (iii) 15 (iv) 4

3. (a)(i)

Number	Tally	Frequency
8		9
9		13
10		7
11		7
12		4
Total		40

(ii) 9.6 (iii) 9 (iv) 9 (v) 4

(b)(i)

Number	Tally	Frequency
2		5
3		11
4		12
5		20
6		22
7		16
8		8
9		6
Total		100

(ii) 5.53 (iii) 6 (iv) 6 (v) 7

4. (a)

Number of People	Tally	Frequency
15		3
16		2
17		3
18		4
19		6
20		6
21		1
22		3
23		5
24		3
25		4
Total		40

(b) 20.2 (c) 20 (d) 19 and 20 (e) 10

Exercise 26I

1. (a)(i) 1 (ii) 1 (iii) 1 (iv) 3
 (b)(i) 4.5 (ii) 4 (iii) 4 (iv) 3
 (c)(i) 3 (ii) 3 (iii) 3 (iv) 6
 (d)(i) 10.4 (ii) 10 (iii) 10 (iv) 6
 2. (a) 75 (b) 104 cm (c) 100 cm
 (d) 100 cm (e) 80 cm

Exercise 26J

1. (a) 45.1 minutes (b) 43 minutes
 (c) 73 minutes (d) 42 minutes
 2. (a) Spike: 42.2 m, Lance: 41 m
 (b) Spike: 40 m, Lance: 45 m
 (c) Lance because without the two outliers (Spike: 58 m, Lance: 25 m) Lance threw further distances.
 3. (a) Bargains Plus: \$700
 Best'n Town: \$680
 (b) Bargains Plus: \$645
 Best'n Town: \$680
 (c) Best'n Town because all of its wages are \$660 or more.

Exercise 26K

1. (a) 20 (b) 14 (c) 30 (d) 5 (e) 35
 2. (a) 47 (b) 10 (c) 10, 15 and 17
 (d) Jess - 12, Hess - 14, Tess - 16
 (e) Jed, Jim - 12, Jock - 16, Jack - 20
 3. Sam had a more consistent season with a range of 30. Terry must have made a few large scores and many low scores.
 4. (a) The mean of graph **A** is *less than* the mean of graph **B**.
 (b) The median of graph **A** is *equal to* the median of graph **B**.
 (c) The mode of graph **A** is *equal to* the mode of graph **B**.
 (d) The range of graph **A** is *less than* the range of graph **B**.
 5. (a) **D** (b) **C** (c) **D** (d) **C** (e) **C** (f) **F**

Chapter Review

1. (a)(i) 7 (ii) 7 (iii) 8 (iv) 4
 (b)(i) 5 (ii) 6 (iii) 7 (iv) 9
 (c)(i) 5 (ii) 5 (iii) 6 (iv) 5
 (d)(i) 11.1 (ii) 11.5 (iii) no mode
 (iv) 20

2. (a) 38 (b) 34 (c) 53

3. (a)(i) 44 (ii) 42 (iii) 42 (iv) 18
 (b)(i) 24 (ii) 23 (iii) 25 (iv) 19

4.

Stem	Leaf
2	4 5 6 8
3	0 1 2 3 4 5 7 8 9
4	2 4 6

$$\begin{array}{r|l} & 4 \\ 2 & \end{array} = 24$$

- (a) 34 (b) 33.5 (c) no mode (d) 22

5. (a)

Stem	Leaf
4	9 8 9
5	9 0 2 4 4 5 5 6 7 5
6	1 0 2 5 8
7	2 5

$$\begin{array}{r|l} & 8 \\ 4 & \end{array} = 48$$

(b)

Stem	Leaf
4	8 9 9
5	0 2 4 4 5 5 5 6 7 9
6	0 1 2 5 8
7	2 5

$$\begin{array}{r|l} & 8 \\ 4 & \end{array} = 48$$

- (c) 57.8 (d) 55.5 (e) 55 (f) 27

6. (a)

Number of Mistakes	Tally	Frequency
0		8
1		12
2		15
3		9
4		4
5		2
Total		50

- (b) 1.9 (c) 2 (d) 2 (e) 5

7. (a) 2.4 (b) 2.5 (c) 3 (d) 5