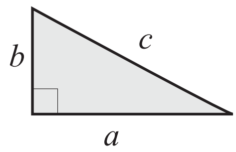
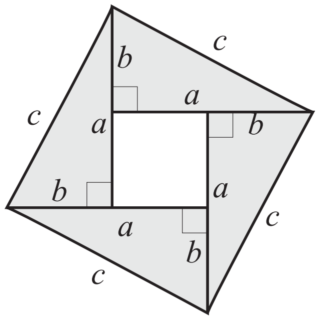
Proofs of Pythagoras' Theorem 1

There are many proof of Pythagoras' Theorem. Here are six of them.

Proof 1



Draw four right-angled triangles that are exactly the same arranged as shown below.
This shape is a square with side length c.
The centre of this large square is a small square with side length $(a - b)$.

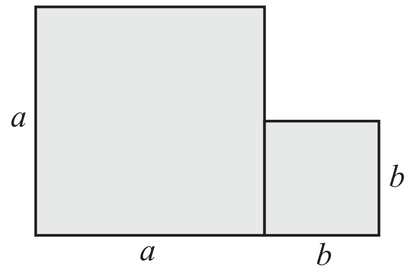


The area of each of the triangles = $\frac{1}{2}ab$
The area of the small square in the centre = $(a - b)^2$
The area of the large square = c^2

The area of the large square is also equal to the sum of the areas of the four triangles and the smaller square.
Therefore:

$$4 \times \frac{1}{2}ab + (a - b)^2 = c^2$$
$$2ab + a^2 - 2ab + b^2 = c^2$$
$$\boldsymbol{a^2 + b^2 = c^2}$$

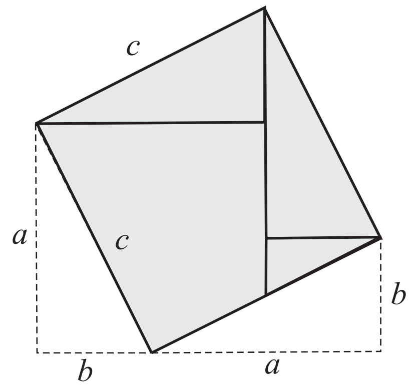
Proof 2



Two squares are drawn next to each other.
One with side length a, the other with side length b.

The total area of these two squares = $a^2 + b^2$

Two right-angled triangles with side lengths a and b are cut from the two squares and placed in the positions shown in the diagram below.



The resultant shape is a square with side length equal to the length of the hypotenuse of the triangle, c.

The area of this square = c^2.

These two areas are equal so:

$$\boldsymbol{a^2 + b^2 = c^2}$$

Proofs of Pythagoras' Theorem 2

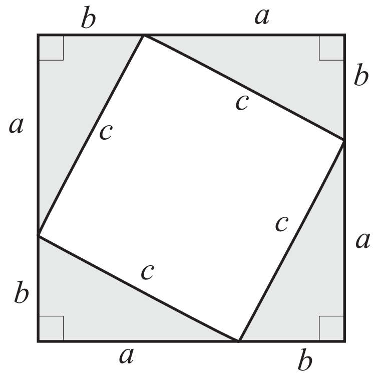
Proof 3

Form a square using four right-angled triangles as shown in this diagram.
The area of each triangle = $\frac{1}{2}ab$
The area of the square in the centre = c^2
The area of the large square = $(a + b)^2$
These two areas area equal.

$$(a + b)^2 = 4 \times \frac{1}{2}ab + c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\boldsymbol{a^2 + b^2 = c^2}$$



Proof 4

Arrange two right-angled triangles as shown and form into a trapezium.
Find the area of this trapezium two different ways:

Using area of trapezium rule = $\frac{1}{2}(a + b) \times (a + b)$
$= \frac{1}{2}(a^2 + 2ab + b^2)$

Using the areas of the three triangles:

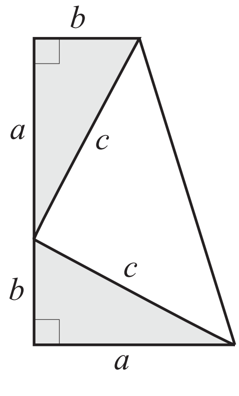
$$2 \times \frac{1}{2}ab + \frac{1}{2}c^2$$
$$= ab + \frac{1}{2}c^2$$

These two areas are equal:

$$\frac{1}{2}(a^2 + 2ab + b^2) = ab + \frac{1}{2}c^2$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

$$\boldsymbol{a^2 + b^2 = c^2}$$



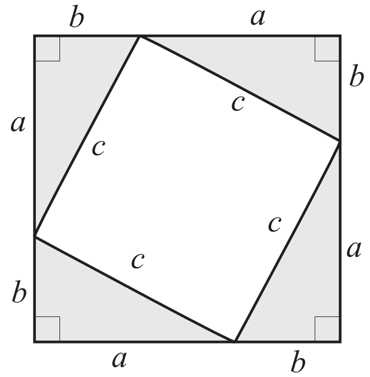
Proofs of Pythagoras' Theorem 3

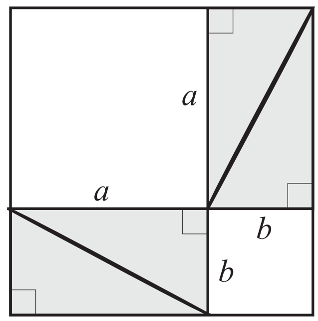
Proof 5

Use four right-angled triangles to form a square as shown.
Move the triangles to the different positions shown in the square.
The area of the centre square in the first diagram = c^2
The area of the two small squares in the second diagram = $a^2 + b^2$

These two areas are equal:

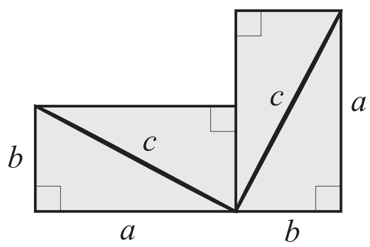
$$\boldsymbol{a^2 + b^2 = c^2}$$



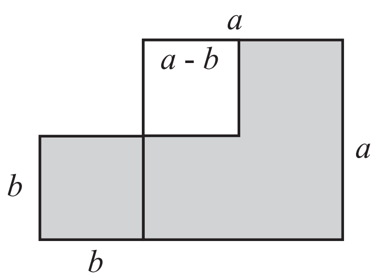


Proof 6

Arrange four right-angled triangles as shown.



Form two squares from these triangles.
One with side length a and the other with side length b.
A smaller square is formed with side length (a - b).



Form a square using the four triangles.
The smaller square in the centre will have side length (a - b).

The area of these two shapes is therefore the same:

$$\boldsymbol{a^2 + b^2 = c^2}$$